

# Big brain matrix eigenvalue lightspeed fourier transform for the great good solar light

## A 4-bit waltz in frequency space

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**Abstract.** Identification of sounds has immense applications in the embedded systems space, ranging from simple detection of sounds to complete voice transcription. Being able to do this on low power devices is an area of active interest. We present an approach to this problem involving bypassing a complete Fourier Transform and approximating its results using a cross-correlation based approach pruning a tree of (preset) frequencies. Our method returns present frequencies with reasonable accuracy whilst maintaining the speed expected of such an embedded system.

## 1 Project Details

### 1.1 Motivation

**Why try to make better Fourier Transforms?** Fourier transforms are the most important tools used in data processing. The relative amplitudes of the signal at different frequencies give us a lot of characterising data of the signal, and changing the signal in frequency space has a lot of applications, like auto-tuning, removing noise etc.

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt \quad (1)$$

**Fourier is Overkill.** Looking at the structure of the Fourier transform, it is easy to see that it is closely resembles the idea of a correlation, essentially extracting from the signals its *similarity* to a given frequency. These coefficients, gathered using all frequencies, can produce a one to one mapping to a function space in the frequency realm. However, the entire function is far too much information. For most tasks involving categorization and identification of sounds, fingerprinting is more than good enough. That is, considering the delta response of the system, its restriction to a tiny subset of points in frequency space.

As such, simply extracting the info for these frequencies alone is sufficient, and can be done whilst incorporating several statistical approximations. This is easily done with a correlation of the signal with a pure mode.

However, when frequencies are aggregated, the phase difference is absorbed into the coefficients of multiple frequencies, but in the correlation scenario, no such hope exists. This is where cross-correlation comes in.

$$(f * g)(t) \triangleq \int_{-\infty}^{+\infty} \overline{f(\tau)} \cdot g(t + \tau) d\tau \quad (2)$$

By considering the similarities of shifted signals, we can infer more precise information about their similarity in space, by completely disregarding the temporal dimension. This is immensely useful in matching an externally sourced wave of unknown phase with a synthetic mode of known or unknown phase to identify the wave.

In particular, the global maximum of the cross-correlation may be taken as the phase independent correlation of two signals.

$$\text{corr}(f, g) \triangleq \max((f * g)(t))_t \quad (3)$$

The main idea of this project is to extract some characteristic audio data from the signal without having to resort to memory and computationally expensive fourier transforms. The 2KiB SRAM of the Arduino is the biggest bottleneck here for doing any processing. In order to do this processing, we came up with multiple optimizations.

## 1.2 Optimizations

**Cross Correlations** Instead of doing a FFT on the data, we will extract a few characteristic frequencies components by correlating the signal with a set of frequencies. The expression for crosscorrelation of 2 discrete signals  $x, y$  is given by

$$R_{xy}[k] = \sum_i x[i]y[i - k] \quad (4)$$

Along with some normalization. The harmonics form a linearly independent set and return zero-correlation when the product is integrated over several time periods, i.e.

$$\frac{1}{\pi} \int_0^{2\pi} \sin(n_1 x) \sin(n_2 x) = \delta_{nn'} \quad (5)$$

Where  $\delta$  is the usual Kronecker delta. We can normalize the correlation with the auto-correlation at 0 to get a number between -1 and 1 that characterized the coefficients.

$$c_{xy} = \frac{R_{xy}[0]}{\sqrt{R_{xx}[0]R_{yy}[0]}} \quad (6)$$

However, the above expression does not account for the phase shift  $\phi$  between the harmonics which reduces the correlation by a factor of  $\cos(\phi)$ . Therefore we modify check the signal with phase shifted test harmonics by modifying the expression to:

$$c_{xy} = \frac{\max\{R_{xy}[k]\}}{\sqrt{R_{xx}[0]R_{yy}[0]}} \quad (7)$$

This ensures that the loss due to potential phase shift is avoided by getting an estimate of the phase. Further, these correlations avoid the complex multiplications required in FFT calculations while giving more characteristic data.

**Memory Management** We store the signal using an array of 4 bit numbers, i.e. numbers from -7 to 7. This has multiple benefits, such as being able to outright store more numbers, and more subtly, perform calculations in a smaller time period as the Arduino processor is based around an 8-bit bus. Instead of requiring multiple clock cycles to process 16 / 32 bit datatypes like `float` adn `int`, we can process the signals much faster.

**Frequency Space Pruning** Beginning with a Dirac comb in frequency space, we prune a lattice tree of the frequency power set via depth first binary search. This bypasses the computationally expensive cross correlation calculations for a lot of (wrong) frequencies.

### 1.3 Major blocks

The flow of control begins at the bottom left of [Figure 1](#), with a mic providing audio input, which is captured by MATLAB (above) and converted to an array of integers. In ideal conditions and availability of modules, this would be done on the arduino, with little to no performance detriment. The time between serial inputs (based on the baud rate) is utilised to process the incoming data into the format used through the program, which is as in the Doobit storage system.

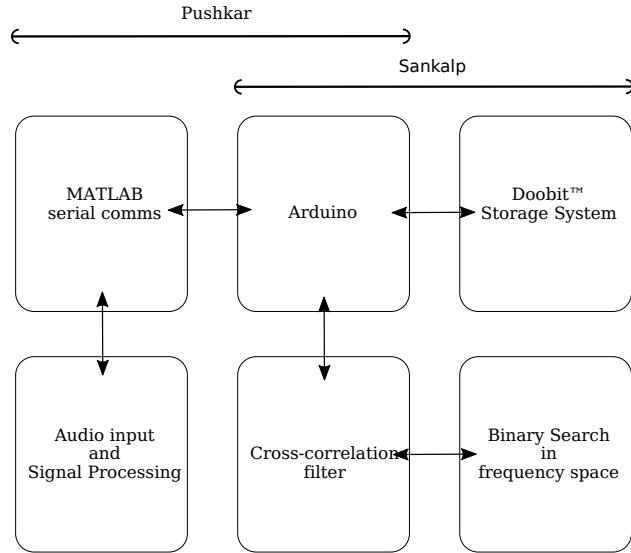


Fig. 1: Block diagram and work distribution.

After the signal has been completely read, a Dirac comb (in frequency space) is generated from a preset range of frequencies. Ideally, a Fourier transform would be utilized for this, but that method carries far too much resolution for just identifying certain frequencies. A limitation that still preserves a huge space of applications. Identification of a combination of frequencies (a trivial extension of the current algorithm, a simple check at the end, or even better if specific, a smaller starting set) could allow for identifying more complicated speech patterns and more processing on subsets of the input (would be stretching the limits of the arduino) could help in preliminary speech recognition, possibly as a low power first gate in an always-on speech recognition system. For example, a lower level circuit that listening for "Alexa" and activates a more power hungry and performant circuit for speech recognition.

To avoid processing all the frequencies, we process the entire comb at once, and if it passes a certain threshold, we infer that one or more of the frequencies in the comb is present

in the sample. Proceeding, the comb is split halfway in frequency space, processed depth first, to identify the frequencies present, rejecting the entire set at any point the global maxima of its cross correlation drops below a given threshold.

A possible optimization, abandoned in the interest of time, is to compute the cross correlation at only a few points, utilizing optimization techniques instead of brute forcing to find the maximum. The first thing to do is check whether the correlation function gives us an appropriate response with this sampling rate. In order to do this, we plot the correlation coefficient of a signal and a pure harmonic vs a change in frequency about the pure harmonic for  $N$  datapoints.

The code for plotting these graphs in MATLAB is given here:

- Code for correlation of 2 signals

```

1 function outp = corel(A,B)
2     outp = 0;
3     normA = 0;
4     normB = 0;
5     for i = 1:length(A)
6         outp = outp+(A(i)*B(i));
7         normA = normA+(A(i)^2);
8         normB = normB+ (B(i)^2);
9     end
10    outp = outp/sqrt(normA*normB);
11    end

```

- Code for getting graphs

```

1 function grp = coreldraw(f1,f2,f3,f4,phi4)
2 t = -0.5:0.005:0.5; %Change the step size to get different
%resolutions
3 signal = round(7/3*(sin(f1*t) + sin(f2*t) + sin(f3*t))) ; %This
%ensures we have a number between \(\pm 7\)
4 df = -200:0.1:200;
5 grp = zeros(length(df));

6
7 for i = 1:length(df)
8 signal2 = round(7*sin(((f4+df(i))*t) + phi4));
9 grp(i) = corel(signal,signal2);
10 end
11 plot(df,grp)
12 xlabel('df')
13 ylabel('Correlation Coefficient')
14 s = strcat('Graph of correlation of' ,int2str(f1), 'Hz' ,int2str(f2)
,' Hz' ,int2str(f3), ' Hz with ' ,int2str(f4), ' Hz vs change in'
,'int2str(f4));
15 title(s)
16 end

```

If we first try correlating with 100 data points, we get Clearly, this does not provide a good enough degree of correlation as there are several significant analogous peaks. We would expect only 2 significant peaks corresponding to 200 and 300 Hz. Next we try

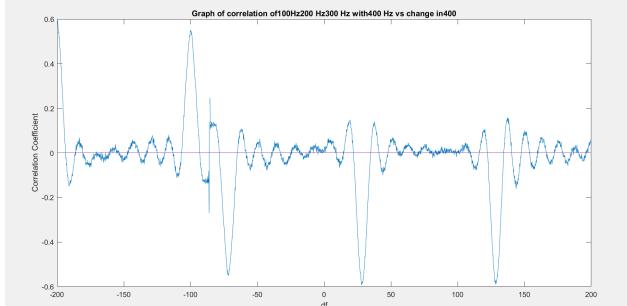


Fig. 2: Correlation with 100 point sampling

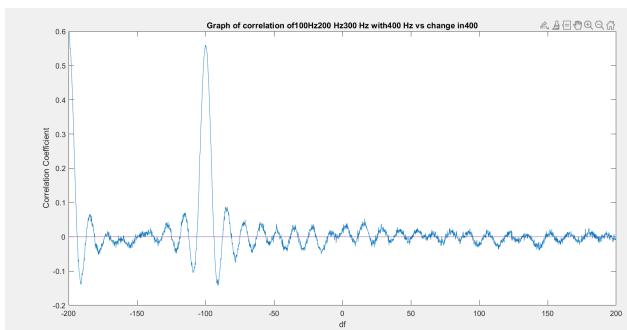


Fig. 3: Correlation with 200 point sampling

with 200 points. This gives us very clear peaks at the right frequencies. It appears that 200 points sampling is good enough. In order to check the resolution limit, we also try a signal where the frequencies are close: The 3 peaks expected are missed. This implies that we cannot separate frequencies very close to each other, and this is the price we have to pay for avoiding a FFT. From these graphs it is clear that a threshold greater than 0.1 will suffice for testing our audio signals.

#### 1.4 Doobit™

Working on the Arduino with signals very quickly turned into a constant battle of resolution and storage, limited by its relatively tiny memory. After most optimizations we went through, managing memory manually very closely and ensuring sequentialized execution contexts, we were terribly bottlenecked by the storage. To work around this physical limit, we manage each byte of the stored signal manually and instead of one, store two signal data points in every byte. This reduces our data resolution by way of being limited to 4 bits, but grants us unmatched robustness to noise by comparison, by doubling possible data processing capabilities. The system has been affectionately named Doobit.

```

1 // bit mask storage
2 struct doobit{
3     uint_fast8_t data;
4
5     doobit(int8_t x = 0, int8_t y = 0){ // handles all our casts too

```

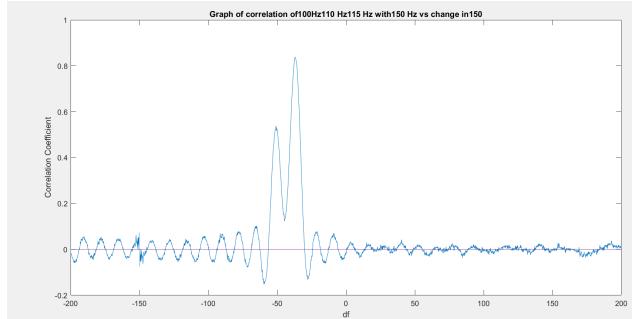


Fig. 4: Correlation with 200 point sampling for 5Hz difference

```

6     this->storelow(x);
7     this->storehigh(y);
8 }
9
10    void storelow(int8_t);
11    void storehigh(int8_t);
12
13    int8_t getlow();
14    int8_t gethigh();
15
16    int16_t operator*(doobit& b);
17 };

```

A struct provides us fast access with very little memory and performance overhead, something that only becomes more and more negligible as we increase our (now doubled!) data numbers.

Unpacking the code block, `data` is the actual storage, an unsigned 8-bit type, chosen this way to avoid accidental signed interpretation and any unwanted processing by the compiler. Reliance on unsigneds in a case such as this is common even within the compiler itself, where it would convert signeds to unsigneds before evaluation to avoid ambiguity. In particular, a two's complement could scramble the data beyond recognition quite quickly.

The `fast` part of `uint_fast8_t` indicates to the compiler that we are looking for a type that is atleast 8 bits, but is the fastest among those. On an Arduino Uno, of course, this is just the 8 bit unsigned integer, but on more exotic embedded systems this could end up being a 9 or 10 bit type, if not more. This notation allows for some compatibility between systems, though as is with embedded systems, one would try to create more efficient structures that exploit the architecture of those systems.

The storage of the signal is handled by the `storelow()` and `storehigh()` functions, which store data into the 4 least significant and 4 most significantbits of the storage `data` respectively. We look at one of the functions:

```

1 void doobit::storelow(int8_t x){
2     this->data &= 0b11110000; // clear for storage
3
4     x += 7; // remove signed component
5     assert((!(x & 0b11110000)) && "doobit range violation");
6

```

```
7     this->data |= x;
8 }
```

This function takes in a value `x`, to be stored in the lower 4 bits of `data`, and preparing for it, clears the lower 4 bits via an AND operation with a bitmask `11110000`. Following this, a manual conversion is made to ensure `x` is unsigned. The operation moves `x`'s previous range, -7 to +7, to now 0 to 14, with 15 remaining generally unused, rushing to somehow occupy this position leads to little benefit and after much trial to squeeze extra storage out of the system, was abandoned.

The `assert` exists merely for testing purposes to ensure our data can indeed fit in 4 bits. Finally, having ensured `x` has only its lower bits populated, an OR operation with the storage inserts it in.

The `storehigh()` function works in a similar manner, albeit with a bit shift and an inverted bit mask to work on the 4 higher bits.

Now remains the issue of retrieving data from this storage, and is done as very much the inverse of how it is stored:

```
1 int8_t doobit::getlow(){
2     int8_t x = (data & 0b00001111); // bitmask
3     return (x-7); // reinsert sign
4 }
```

The *unrequired* data is removed via a bit mask, and would be moved rightwards via a bit shift in the case of `gethigh()`, and its signed nature is restored by shifting it back to the original range.

The conversion to unsigned is quite important to have to not manage the carry bits arising from a two's complement operation. The number -7 in C++ could ambiguously be coming from an `int` (internally `int32_t`) in which case the signed bit is the 32nd, while it could also be coming from an 8 or 16 bit type, making the signed bit unclear and its extraction slow and painful. Asking for a change of range was the fastest of the operations we tested, included several bitwise only operations.

The storage could be optimized for several arithmetic operations, but for our case in particular, for the correelation setups, we need only multiplication. As of now, this is done simply by retrieving the numbers individually and multiplying them pairwise. This is done as opposed to attempting bitwise operations as (1), multiplication operations are quite optimized on a circuit level in modern processors anyway, and more importantly (2), 4 bit being such a limited storage type, would cause an overflow for most possible multiplication operands.

```
1 int16_t operator*(doobit& b){
2     auto highprod = this->gethigh() * b.gethigh();
3     auto lowprod = this->getlow() * b.getlow();
4     return (highprod + lowprod);
5 }
```

This definition is obviously not compatible with all arithmetic operations, but quite specific for our limited operations, kept this way to prevent overcomplication of the rather heavily utilized routine.

The full code for all the functions may as always be found in the [Appendix](#).

## 1.5 Serial Communication and the Great Horse Race

The input of the system is handled using MATLAB. This is appealing for several reasons, (1), we can immediately visualise and process the sound array that we get, making it easy to debug, and (2), we can use the inbuilt system microphone. MATLAB by default offers several different ways to get this audio data, the lowest resolution being a 1000Hz Sampling frequency and 8-bit sampling, using the `audiorecorder` object. Once we record the relevant sample, we extract an array with this data and after doing some basic filtering, to remove any high frequency noise that may affect the signal, we send the data to the Arduino via the Serial port.

The important part here is understanding how the serial communication with the Arduino works. The serial port can only send data character by character to the Arduino and waits for it to be processed. Therefore the processing of the integers is done as follows:

1. Collect multiple audio samples from the microphone. We take 2000 datapoints, assuming a periodic sample. The points are all `int8`.
2. Apply a filter to remove any peaks above  $\pm 7$  to ensure that the data doesn't overflow in the arduino. Then we apply a Moving average filter by taking 3 200-datapoint samples from the 2000 datapoints to remove any anomalous noise in the data.
3. In order to communicate this data, we convert the integer Array in MATLAB to strings using the `int2str` method so that it can be sent via the Serial port. The terminator that we use is the newline character '`\n`'.
4. We open the serial monitor and send the data character by character using the `fprintf` function in Matlab.
5. We check for available data in the Arduino using the `Serial.available()` function.
6. Then we read the data into an array in Arduino using the `Serial.parseInt()` function. We wait for 2 bytes to arrive and store them in a single byte on the arduino to work with our processing scheme.
7. When all the expected data (200 points) are read, stop recording and do the processing to find the matching frequencies.
8. Send the frequencies back to MATLAB to print using the Serial port.

This protocol can be seen in this block diagram

However, we ran into some issues here, where several values were not detected by the Arduino. We tried multiple approaches to solve this

1. We increased the Baud rate to 115200 to ensure that the transmission wasn't causing any loss of data.
2. We added a small delay of 30ms between subsequent transmissions to ensure that the Arduino had time to process the data.
3. The Serial protocol has a buffer that stores the incoming values before they are processed by the Arduino. In the Arduino Uno, this `SERIAL_RX_BUFFER` memory that stores the incoming data from the Serial Port is only 64 bytes. We changed the header files to make the buffer 128 bytes in `HardwareSerial.h` so that more characters could be stored without losing them in transmission.

In order to ensure that this method is working correctly we have several checks

1. We ran a simple program that turns the pin13 LED On and Off based on whether the character it received was 0 or -7.

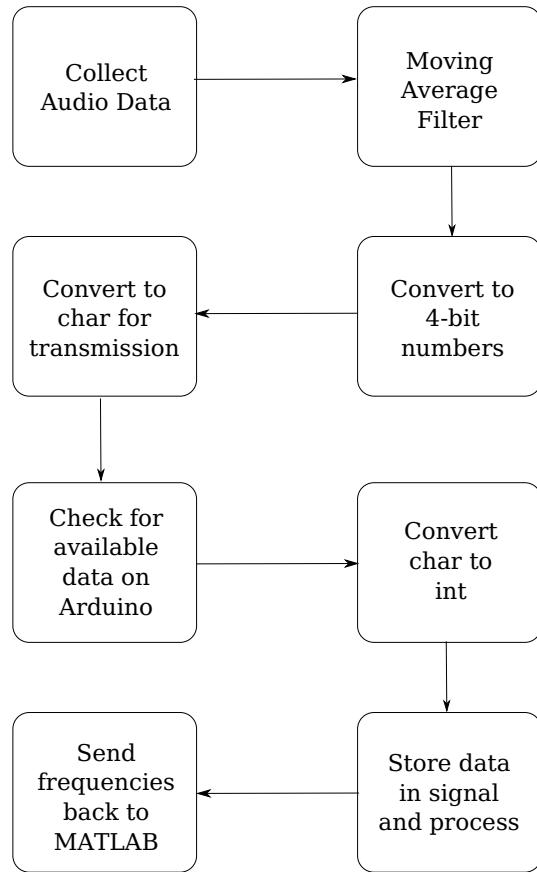


Fig. 5: Block diagram for communication protocol.

2. In our main code, pin 13 is HIGH when waiting for data and recording. When all the expected data has been found, it stops glowing.
3. We also plot the raw data that we get from the Mic to ensure that a reasonable waveform has been detected.

The code for testing the glowing of the LEDs is found here:

```

1 const int led = 13;
2 String text;
3 int k;
4 void setup() {
5     //pinMode(led, OUTPUT);
6     Serial.begin(9600);
7     k = 0;
8 }
9 void loop() {

```

```

10  if  (Serial.available()){
11      text = Serial.readStringUntil('$');
12      k = text.toInt();
13      if(k == -7){
14          digitalWrite(led, 1);
15      }
16      if(k == 0){
17          digitalWrite(led, 0);
18      }
19  }
20 }
```

The MATLAB code for controlling this unit is found here:

```

1 Arduino = serial("COM4","BaudRate",9600);
2 fopen(Arduino) %This opens a monitor
3 fprintf(Arduino ,'%s$',int2str(0));
4 fprintf(Arduino ,'%s$',int2str(-7));
5 fclose(Arduino);
```

## 2 Main components and Invectory

The project mainly revolves around processing onboard the Arduino, so not much hardware is required:

- Arduino
- USB Cable for serial communication
- Mic – replaced by laptop with MATLAB here
- LEDs for displaying frequencies (optional extra)

## 3 Results

### 3.1 Synthetic Data

First, tests were run on data generated in situ from a list of frequencies, coefficients, and phase differences

$$\begin{aligned} c &= \{c_1, c_2, \dots, c_n\} \\ \omega &= \{\omega_1, \omega_2, \dots, \omega_n\} \\ \phi &= \{\phi_1, \phi_2, \dots, \phi_n\} \end{aligned}$$

which generate a 'generating' function for the discrete signal to follow

$$f(x) = \sum_{i=1}^n c_i \cdot \sin(w_i x + \phi_i)$$

implemented as a lambda-function in code

```

1 auto f_gen = [w, c, p](int x){
2     float sum = 0;
3     for(int i = 0; i < wlist.size(); i++){
4         sum += c[i] * sin(w[i]*x + p[i]);
5     }
6     return 7.0*sum/float(std::accumulate(c.begin(), c.end
7     (), 0));
8 };
9 auto f = new signal[SIZE];
10
11 for(int x = 0; x < SIZE; x++){
12     f[i] = signal(f_gen(2*x), f_gen(2*x + 1));
13 }

```

The normalising factor may be succinctly written as

$$a_N = \frac{7.0}{\sum_{i=1}^n c_i}.$$

It simply ensures that the cross correlation of the signals do not blow up and that a single signal agnostic threshold value may be chosen as a metric for matching. The factor of 7 scales the float value to the range (-7, 7) so as to maintain reasonable resolution after conversion to integer type to save memory.

The signal is generated as even/odd pairs to facilitate pairwise storage implemented by **Doobit™**. The typename **signal** is an alias for **doobit**, kept as such for modularity amongst storage backends, and to possibly expand across architectures, saving major edits.

Playing with the described parameters, experiments were performed, and the results have been tabulated in [Figure 6](#). Since we look for a global maxima of cross correlation, the phase is irrelevant to the results, but for the sake of completeness, an indication has been provided where phase shifts were tested. Several arbitrary values were tested, with the result being unaffected as expected.

All synthetic tests were performed with a resolution of 1ms, and 400 data points.

### 3.2 Real Inputs

Not having a microphone module for the Arduino, we resorted to passing a recorded array to the device via MATLAB, opening its own can of worms by way of buffer overflows and race conditions, as described in detail in [??](#).

For currently unknown reasons, we had memory overflows with the same size of real data as we ran synthetic tests for. Curiously, in this case, the Arduino continuously prints 'w' on the Serial line. After much testing we have linked this behaviour to memory overflow, but are yet to find resources confirming it.

Reducing the size, however, allows us to perform some tests, limited by the data transfer speed as well, with the serial line taking up to 30 seconds to transfer all the data successfully, yet with significant packet loss. The best transfer ratio we were able to obtain after fine tuning the transfer rates and timing was 400 packets received for 420 sent, but at this delicate parameter island, the Arduino would, with roughly half probability, run out of memory. It may be dependent on buffering of incoming packets. Buffering just a few more packets may have been pushing it over the edge.

NCC Threshold (t)	Input frequencies and coefficients (kHz)	Output (kHz)
0.1	0.3, 0.5, 0.8	0.3, 0.5, 0.8
0.1	0.3, 0.5, 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.1	0.3, 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.2	0.3, 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.2	0.3, 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.2	0.3, 4× 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.4	0.3, 4× 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5, 0.8
0.5	0.3, 4× 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.3, 0.5
0.6	0.3, 4× 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	0.5
0.8	0.3, 4× 0.5 <sup>†</sup> , 0.8 <sup>†</sup>	∅
0.1	0.3, 0.5, 0.35 <sup>†</sup>	0.3, 0.4, 0.5
0.2	0.3, 0.5, 0.35 <sup>†</sup>	0.3, 0.5

Fig. 6: Experiments with varying synthetic inputs and thresholds.

†. Phase shifted. Weight unit unless otherwise specified

For generating required frequencies, the Android app [Frequency Sound Generator](#) was used.

All real tests were performed with a resolution of 1ms, and 200 data points.

### 3.3 Proofs

**Code and compilation** screenshots of code compiled and running on arduino

**Arduino and Humans** photo video uploads

NCC Threshold (t)	Input signals	Output (kHz)
0.1	0.4 kHz (generated using a phone speaker)	INSERT
0.1	0.5 kHz (generated using a phone speaker)	INSERT
0.1	0.6 kHz (generated using a phone speaker)	INSERT
0.2	0.6 kHz (generated using a phone speaker)	INSERT
0.3	0.6 kHz (generated using a phone speaker)	INSERT
0.5	0.6 kHz (generated using a phone speaker)	INSERT
0.1	Speech	INSERT
0.1	SOME SHIT CHORD	INSERT
0.1	SOME SHIT CHORD	INSERT
0.1	SOME SHIT CHORD	INSERT

Fig. 7: Experiments with real inputs and thresholds.

## Appendix A Arduino Code

Here is the full code sent to the Arduino, verbatim.

```

1 // main.ino
2
3 #include "ArduinoSTL.h"
4
5 #define assert(x) (void*)0
6 #define SIZE 200
7
8 // bit mask storage
9 struct doobit{
10     uint_fast8_t data;
11
12     doobit(int8_t x = 0, int8_t y = 0){ // handles all our casts too
13         this->storelow(x);
14         this->storehigh(y);
15     }
16
17     void storelow(int8_t);
18     void storehigh(int8_t);
19
20     int8_t getlow();
21     int8_t gethigh();
22
23     int16_t operator*(doobit& b){
24         auto highprod = this->gethigh() * b.gethigh();
25         auto lowprod = this->getlow() * b.getlow();
26         return (highprod + lowprod);
27     }
28 };

```

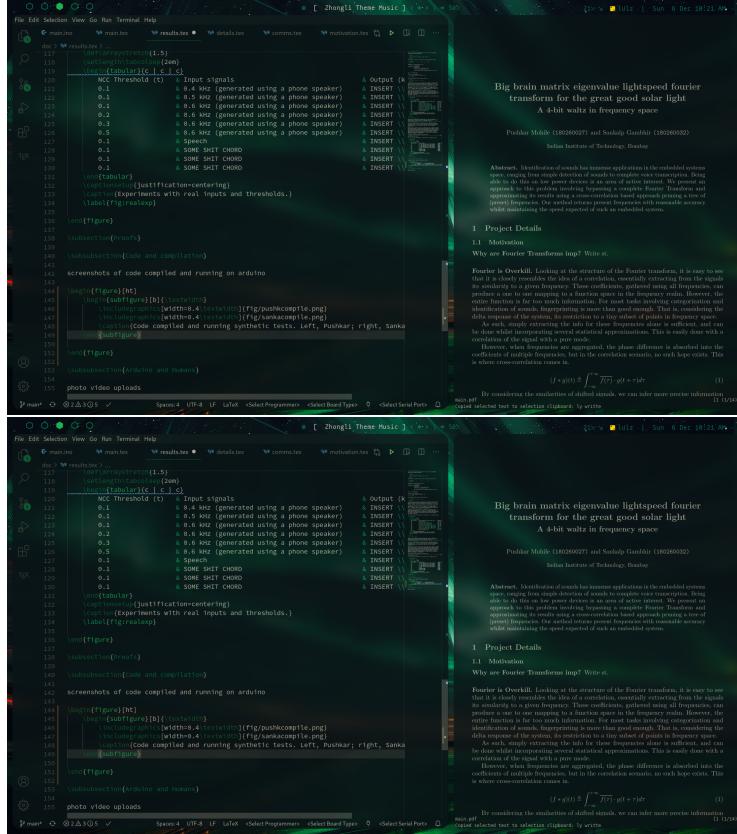


Fig. 8: Code compiled and running synthetic tests. Left, Pushkar; right, Sankalp.

```

29 typedef doabit signal;
30
31 // functions and constants
32 int correlation(signal*, signal*);
33 int crosscorrelation(signal*, signal*, int = 0);
34 std::vector<float> checkcorr(signal*, std::vector<float>);
35
36 std::vector<float> freq{};
37
38 const float corr_threshold = 0.1;
39
40 // data input
41 bool recording = false;
42 uint16_t recorded = 0;
43 String text;
44 uint8_t has_num = 0;
45 uint8_t k[] = {0, 0};
46
47 signal f[SIZE];
48

```



Fig. 9: Code running. Left, Pushkar; right, Sankalp.

```

49 void setup(){
50   Serial.begin(115200);
51
52   // clear data just in case
53   for(int i = 0; i < SIZE; i++){
54     f[i] = 0;
55   }
56
57   for(float i = 0.1; i < 1.0; i += 0.1){
58     freq.push_back(i);
59   }
60
61   // recording indication
62   pinMode(13, 1);
63   digitalWrite(13,1);
64   recording = true;
65 }
66
67 void loop(){
68

```

```

69     if(recording){
70         // recording data
71         if (Serial.available()){
72             k[has_num] = Serial.parseInt();
73             has_num++;
74         }
75         if ((has_num >= 2) && recorded < SIZE) {
76             f[recorded++] = signal(k[0], k[1]);
77             has_num = 0;
78         }
79         if(recorded >= SIZE){
80             Serial.write("G"); // we Good
81             digitalWrite(13,0);
82             recording = false;
83         }
84     }
85     else{
86         // calculate with the data
87
88         // f coming from data
89         /*
90         auto f_gen = [](int x){
91             return (7.0*(sin(0.3 * x) + 4*sin(0.5 * x) + sin
92             (0.6*x + 0.6))/6.0);
93         };
94
95         for(int i = 0; i < SIZE; i++){
96             f[i] = signal(f_gen(2*i), f_gen(2*i+1));
97         }
98         */
99
100        auto wpresent = checkcorr(f, freq);
101
102        for(auto w : wpresent){
103            Serial.println(w);
104        }
105        Serial.println("---LOOPEND---");
106    }
107
108    return;
109}
110
111 // definitions
112
113
114 void doobit::storelow(int8_t x){
115     this->data &= 0b11110000; // clear for storage
116
117     x += 7; // remove signed component
118     assert((!(x & 0b11110000)) && "doobit range violation");
119

```

```

120     this->data |= x;
121 }
122
123 void doobit::storehigh(int8_t x){
124     this->data &= 0b00001111; // clear for storage
125
126     x += 7; // remove signed component
127     assert((!(x & 0b11110000)) && "doobit range violation");
128
129     this->data |= (x << 4);
130 }
131
132 int8_t doobit::getlow(){
133     int8_t x = (data & 0b00001111); // bitmask
134     return (x-7); // reinsert sign
135 }
136
137 int8_t doobit::gethigh(){
138     int8_t x = ((data & 0b11110000) >> 4); // bitmask and shift
139     return (x-7); // reinsert sign
140 }
141
142
143 int correlation(signal* f, signal* g){
144     int sum = 0;
145
146     for(int i = 0; i < SIZE; i++){
147         sum += f[i] * g[i];
148     }
149     return sum;
150 }
151
152 int crosscorrelation(signal* f, signal* g, int m){
153     int sum = 0;
154
155     if(m >= 0){
156         for(int i = 0; i < SIZE - m; i++){
157             sum += f[i] * g[i+m];
158         }
159         for(int i = 0; i < m; i++){
160             sum += f[i+SIZE-m] * g[i];
161         }
162     }
163     else{
164         m = -m;
165         for(int i = 0; i < m; i++){
166             sum += f[i] * g[i+SIZE-m];
167         }
168         for(int i = m; i < SIZE; i++){
169             sum += f[i] * g[i-m];
170         }
171     }

```

```

172     return sum;
173 }
174
175 std::vector<float> checkcorr(signal* f, std::vector<float> wlist){
176
177     if(wlist.size() == 0) return wlist;
178
179     float *maxcorr = new float(-1);
180
181     auto g_gen = [wlist](int x){
182         float sum = 0;
183         for(auto w : wlist){
184             sum += sin(w*x);
185         }
186         return 7.0*sum/float(wlist.size());
187     };
188
189     auto g = new signal[SIZE];
190
191     for(int i = 0; i < SIZE; i++){
192         g[i] = signal(g_gen(2*i), g_gen(2*i + 1));
193     }
194
195     float* norm_coeff = new float(0);
196     *norm_coeff = sqrt(((double)correlation(f, f) * (double)correlation(
197         g, g)));
198     *norm_coeff = 1/ (*norm_coeff);
199
200     for(int i = -(SIZE/5) + 1; i < SIZE/5; i++){
201         auto corr = crosscorrelation(f, g, i);
202         *maxcorr = *maxcorr > corr ? *maxcorr : corr;
203         if (*maxcorr < corr){
204             *maxcorr = corr;
205             if ((*maxcorr)*(*norm_coeff) > corr_threshold) break;
206         }
207     }
208
209     // clean memory just in case it isn't deallocated
210     // before recursion else we run over quota
211     delete[] g;
212
213     if ((*maxcorr)*(*norm_coeff) < corr_threshold){
214         delete norm_coeff;
215         delete maxcorr;
216         return std::vector<float>{};
217     }
218     delete norm_coeff;
219     delete maxcorr;
220
221     if(wlist.size() == 1) return wlist;

```

```
222     auto wl = checkcorr(f, std::vector<float>(wlist.begin(), wlist.begin()
223         () + (wlist.size()/2)));
224     auto wr = checkcorr(f, std::vector<float>(wlist.begin() + (wlist.
225         size()/2), wlist.end()));
226
227     wl.insert(wl.end(), wr.begin(), wr.end());
228
229     return wl;
230 }
```

## Appendix B MATLAB

The code used inside MATLAB to record and send audio to the Arduino.

```
1 function carr = getAudioAndSend2()
2 a = audiorecorder(1000,8,1);
3 recordblocking(a,2); %200 data points recorded
4 d = getaudiodata(a,'int8');
5 %d = round(7.0*sin(0.6*(1:1:500)));
6 indices = find(d > 7);
7 d(indices) = 7;
8 indices = find(d < -7);
9 d(indices) = -7;
10 plot(d);
11
12 Ard = serial("COM3","BaudRate",115200);
13 fopen(Ard);
14 for i = 1:430
15     fprintf(Ard,'%s\n',int2str(d(i)));
16     pause(3/100);
17 end
18 pause(1);
19 for i=1:15
20     y = fscanf(Ard,'%s');
21     fprintf('%s\n', y);
22 end
23 fclose(Ard);
24 end
```