PH423 Assignment 1

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August 25, 2020

Question 1.

[Sankalp: I got this one.]

We consider the action of $\hat{\mathbf{P}}$ on an arbitrary state ϕ,

$$\hat{\mathbf{P}}\phi(x) \tag{1}$$

to derive a weak equivalence with the desired result.

Starting with ??, we first write it in bra-ket notation as

$$\hat{\mathbf{P}}\phi(x) = \langle x| \hat{\mathbf{P}} |\phi\rangle . \tag{2}$$

On the right hand side, we write $\hat{\mathbf{P}} = \hat{\mathbf{P}} \cdot \hat{\mathbf{1}}$ and use the completeness relation for the position space, $\int \mathrm{d}x' \, |x'\rangle\langle x'| = \hat{\mathbf{1}}$, to obtain

$$\hat{\mathbf{P}}\phi(x) = \int \mathrm{d}x' \, \langle x| \hat{\mathbf{P}} |x'\rangle \langle x'|\phi\rangle \tag{3}$$

Question 2.

[Parth: Question 2 is mine] Starting from the abstract ket form of the Schrodinger eqn. derive the Schrodinger eqn. in the co-ordinate basis

The Schrodinger eqn. in the abstract ket form is given as follows -

$$\hat{\mathbf{H}} \; |\psi\rangle = E \; |\psi\rangle \, (5)$$

In the position basis, $\hat{\mathbf{H}}$ can be expressed in the form of a matrix, while $|\psi\rangle$ can be expressed in terms of components in the basis. In the position basis, the components of $|\psi\rangle$ form the wavefunction.

$$\langle x|\psi\rangle = \psi(x).(7)$$

The Hamiltonian becomes a matrix in the position basis, with components given by -

$$H_{xy} = \langle x | \hat{\mathbf{H}} | y \rangle . (9)$$

Applying this to equation ??. We multiply by $\langle x|$ on the left side and insert an identity on the right hand side. The identity is given by -

$$1 = \int dy \ |y\rangle \langle y| \, (11)$$

So now we obtain -

$$\int dy \ \langle x | \hat{\mathbf{H}} | y \rangle \langle y | \psi \rangle = E \langle x | \psi \rangle . (13)$$

simplifying to components, we get -

$$\int dy \, H_{xy} \psi(y) = E \, \psi(x) . (15)$$

now our components are -

$$H_{xy} = -\frac{\hbar^2}{2m} \delta''(x-y) + V \delta(x-y) (17)$$

we plug this in to equation ??. We get -

$$\int dy \ -\frac{\hbar^2}{2m} \delta''(x-y) \psi(y) + \int dy \, V \psi(y) \delta(x-y) = E \, \psi(x) (19)$$

$$\frac{\hbar^2}{2m}\int dy\, \delta''(x-y)\psi(y) + V\psi(x) = E\,\psi(x) \quad (21)$$

Using Integration by Parts on the integral twice, we obtain -

$$\frac{\hbar^2}{2m}\int dy\, \delta(x-y)\psi''(y) + V\psi(x) = E\,\psi(x) \quad (23)$$

$$\frac{\hbar^2}{2m}\psi''(x) + V\,\psi(x) = E\,\psi(x) \quad (25)$$

Generalising this a bit -

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \quad (27)$$

This is quite obviously, our Schrodinger's Equation in co-ordinate form.

Question 3.

Goodbye World