PH423 Assignment 1

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Question 1.

[Sankalp: I got this one.]

We consider the action of $\hat{\mathbf{P}}$ on an arbitrary state ϕ,

$$\hat{\mathbf{P}}\phi(x) \tag{1}$$

to derive a weak equivalence with the desired result.

Starting with ??, we first write it in bra-ket notation as

$$\hat{\mathbf{P}}\phi(x) = \langle x| \hat{\mathbf{P}} |\phi\rangle . \tag{2}$$

On the right hand side, we write $\hat{\mathbf{P}} = \hat{\mathbf{P}} \cdot \hat{\mathbf{1}}$ and use the completeness relation for the position space, $\int \mathrm{d}x' \, |x'\rangle\langle x'| = \hat{\mathbf{1}}$, to obtain

$$\hat{\mathbf{P}}\phi(x) = \int \mathrm{d}x' \, \langle x| \hat{\mathbf{P}} |x'\rangle \langle x'|\phi\rangle \tag{3}$$

Question 2.

[Parth: Question 2 is mine] Starting from the abstract ket form of the Schrodinger eqn. derive the Schrodinger eqn. in the co-ordinate basis

The Schrodinger eqn. in the abstract ket form is given as follows -

$$\hat{\mathbf{H}} \; |\psi\rangle = E \; |\psi\rangle \, (5)$$

In the position basis, $\hat{\mathbf{H}}$ can be expressed in the form of a matrix, while $|\psi\rangle$ can be expressed in terms of components in the basis. In the position basis, the components of $|\psi\rangle$ form the wavefunction.

$$\langle x|\psi\rangle = \psi(x).(7)$$

The Hamiltonian becomes a matrix in the position basis, with components given by -

$$H_{xy} = \langle x| \hat{\mathbf{H}} |y\rangle \,.(9)$$

Applying this to equation ??. We multiply by $\langle x|$ on the left side and insert an identity on the right hand side. The identity is given by -

$$1 = \int dy\ \ |y\rangle \langle y| \,(11)$$

So now we obtain -

$$\int dy\ \ \langle x| \hat{\mathbf{H}} |y\rangle \langle y|\psi\rangle = E \langle x|\psi\rangle \,.(13)$$

simplifying to components, we get -

$$\int dy\, H_{xy}\psi(y) = E\,\psi(x).(15)$$

now our components are -

$$H_{xy} = -\frac{\hbar^2}{2m}\delta''(x-y) + V\,\delta(x-y)(17)$$

we plug this in to equation ??. We get -

$$\int dy\ -\frac{\hbar^2}{2m}\delta''(x-y)\psi(y) + \int dy\, V\,\psi(y)\delta(x-y) = E\,\psi(x)(19)$$

$$\frac{\hbar^2}{2m}\int dy\,\delta''(x-y)\psi(y)+V\psi(x)=E\,\psi(x)(21)$$

Using Integration by Parts on the integral twice, we obtain -

$$\frac{\hbar^2}{2m}\int dy\,\delta(x-y)\psi''(y)+V\psi(x)=E\,\psi(x)(23)$$

$$\frac{\hbar^2}{2m}\psi''(x)+V\,\psi(x)=E\,\psi(x)(25)$$

Generalising this a bit -

$$-\frac{\hbar^2}{2m}\nabla^2\psi+V\psi=E\psi(27)$$

This is quite obviously, our Schrodinger's Equation in co-ordinate form.

Question 3.

Goodbye World