Equivalence Between Systems Stronger than Resolution

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Resolution...?

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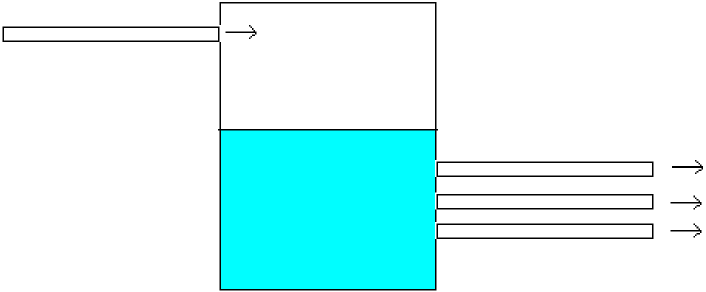
What’s stronger?

I hope not more than 5 minutes have passed

Circular Proofs



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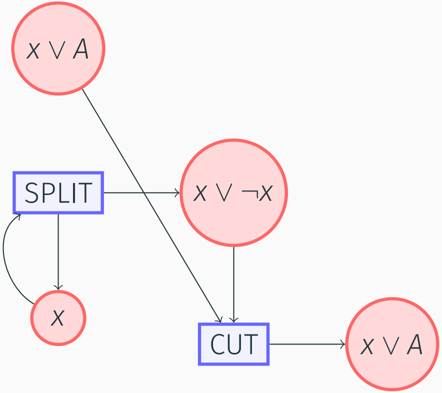

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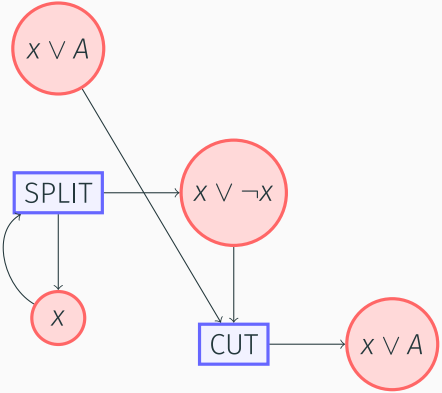

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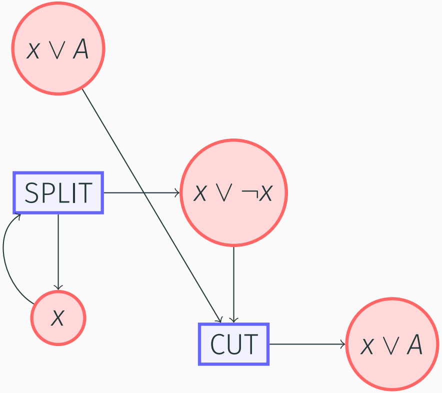

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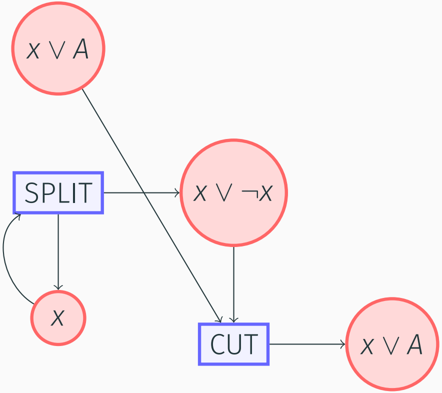

SPLIT and CUT have *flows*. These flows allow us to calculate the balance on each formula node. For each formula in the system, we must have a non-negative balance, and for our goal in particular, we would like the tank to be filled and not just passing water, per the analogy, so we want a positive balance. [AL19]

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Where does n lie? What could $n = -2$ mean?

Equivalence and Dual Rail Proofs

As much as I'd like to think I have 5 more minutes...

Equivalence

So take the balance we discussed for circular proofs, and start with those as weights for your weighted proof. Traversing the nodes, each of the inference rules, having a *flow* associated to it, can be seen as combining and modifying the weights. [BL20]

Equivalence

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For the other direction, start with nodes corresponding to the hypothesis, and for each step of the weighted proof, introduce nodes for the inference rules, with appropriate flow values inferred from the weights used. The main issue is proving, again, that the final weights satisfy our conditions.

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Why do I care? Graphs!

Infact, the actual proof of equivalence relies on introducing terms of the form $(A, n), (A, -n)$. This inherently circular ideology allows us to simplify our proofs by introducing clauses before they're available, making promises for their arrival. All's fine, *as long as we keep our promises.*

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References

- Albert Atserias and Massimo Lauria, *Circular (yet sound) proofs*, International Conference on Theory and Applications of Satisfiability Testing, Springer, 2019, pp. 1–18.
- Maria Luisa Bonet and Jordi Levy, *Equivalence between systems stronger than resolution*, International Conference on Theory and Applications of Satisfiability Testing, Springer, 2020, pp. 166–181.