PH423 Assignment 2

Parth Sastry
180260026

Sahas Kamat
180260030

Sankalp Gambhir
180260032

September 22, 2020

1. Your question here.

[Sankalp: I got this one.]

We start with the expansion of the operators $\hat{\mathbf{J}}_x$ and $\hat{\mathbf{J}}_y$ in terms of the ladder operators

$$\hat{\mathbf{J}}_x = \frac{1}{2} \cdot (\hat{\mathbf{J}}_+ + \hat{\mathbf{J}}_-) \tag{1}$$

and

$$\hat{\mathbf{J}}_y = \frac{1}{2\iota} \cdot (\hat{\mathbf{J}}_+ - \hat{\mathbf{J}}_-) \ . \tag{2}$$

The application of the ladder operators on a state $|j, m\rangle$ changes it to a state of the form $c \cdot |j, m \pm 1\rangle$ for some $c \in \mathbb{C}$. So, given the orthogonality of the $|j, m\rangle$ states, we get that

$$\langle j, m|\hat{\mathbf{J}}_x|j, m\rangle = \langle j, m|\hat{\mathbf{J}}_y|j, m\rangle = 0 \qquad \forall\, |j, m\rangle \ . \tag{3}$$

Squaring Equation 1 and 2, we get the operators $\hat{\mathbf{J}}_x^2$ and $\hat{\mathbf{J}}_y^2$ in terms of the ladder operators. With the same argument as before, we see that only terms with equal powers of the two ladder operators will contribute, and using

$$\hat{\mathbf{J}}_\pm |j, m\rangle = \hbar\sqrt{(j \mp m)(j \pm m + 1)} \ \ |j, m \pm 1\rangle \ , \tag{4}$$

we get

$$\begin{aligned}
\langle j, m|\hat{\mathbf{J}}_y^2|j, m\rangle &= \langle j, m|\hat{\mathbf{J}}_x^2|j, m\rangle && (5)\\
&= \langle j, m|\frac{1}{4} \cdot (\hat{\mathbf{J}}_+^2 + \hat{\mathbf{J}}_+\hat{\mathbf{J}}_- + \hat{\mathbf{J}}_-\hat{\mathbf{J}}_+ + \hat{\mathbf{J}}_-^2)|j, m\rangle && (6)\\
&= \langle j, m|\frac{1}{4} \cdot (\hat{\mathbf{J}}_+\hat{\mathbf{J}}_- + \hat{\mathbf{J}}_-\hat{\mathbf{J}}_+)|j, m\rangle && (7)\\
&= \langle j, m| \ \frac{1}{2} \cdot \Big(\sqrt{(j + m + 1)(j - m)}\sqrt{(j - m)(j + m + 1)} \\
&\quad + \sqrt{(j - m)(j + m + 1)}\sqrt{(j + m + 1)(j - m)} \ \Big) \cdot |j, m\rangle && (8)
\end{aligned}$$

$$= (j + m + 1)(j - m) \tag{9}$$

The values for x and y are not separately calculated as a trivial calculation shows they're equal. The same is easily argued using symmetry in the x-y plane.

2. Your question here.

[Parth: I got this one.]

3. Your question here.

[Sahas: I got this one.]

4. Your question here.

[Sankalp: I got this one.]

5. Your question here.

[Parth: I got this one.]

6. Your question here.

[Sahas: I got this one.]