# PH563 Assignment 1

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## Question 1.

**(a)** **B.** $\left| \vec{r_1} - \vec{r_2} \right|$
In a translationally invariant context, relative positioning can be the only independent parameter.

**(b)** **C.** $-p - 2$
The identity corresponding to the given group operation is $-1$, so using $p * p^{-1} = p + p^{-1} + 1 = \hat{\mathbf{1}} = -1$, we get the result.

**(c)** **B.** $R_{\frac{\pi}{4}}$
A rotation of $\frac{\pi}{4}$ over $\frac{7\pi}{4}$ generates the identity, i.e. $2\pi$.

**(d)** **C.** $(2, 0, 1, 1, 0, 0, 0, 0, 0, 0)$
Counting columns from the right, 2 of length 1, 1 of length 3, and 1 of length 4.

## Question 2.

**(a)** **False**
The symmetric potential gives rise to a Hamiltonian with inversion symmetry about origin. The solutions 'inherit' this symmetry, being either even or odd, with position expectation 0.

**(b)** **True**
Due to the construction of the coset (i.e. $G \cdot a$), every element of the group is included in atleast one coset, by action on the identity. Thus, the union of all cosets of a group forms the entire group.

**(c)** **True**
The exponential map $\exp(x)$ maps $\mathbb{R}$ to $\mathbb{R}^+$ homomorphically, and inversely, the natural logarithm maps $\mathbb{R}^+$ to $\mathbb{R}$. The composition gives the identity morphisms in either direction.

**(d)** **True**
A 2-cycle and a 4-cycle can be written as a composition of 1 and 3 transpositions respectively, for a total of 4, which is even. Hence, the elements do belong to $\mathfrak{U}(6)$.

**Question 3.**

**(a) 4.**
The classes are:

- $C_1 = \{E\}$
- $C_2 = \{Q, R, QR\}$
- $C_3 = \{P, PQ, PR, PQR\}$
- $C_4 = \{P^2, P^2Q, P^2R, P^2QR\}$

**(b) 105.**
The order of the conjugacy class is the number of permutations in a (2, 2, 1, 1, 1) partition of 7 elements,

$$|C| = \frac{\binom{7}{2} \cdot \binom{5}{2}}{2!} \cdot \frac{\binom{3}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}}{3!}.$$

We get $|C| = 105$.

The same can be gotten using the cycle decomposition formula.