PH563 Assignment 2

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Question 1.

(a) **ACD.** The group is defined by the direct product in A. Since D_n is not Abelian for $n \geq 3$, D_3 isn't and neither is its direct product, hence C. Finally, since $D_3 \subset D_6$ their direct products with σ_h will follow their hierarchy, so we get D.

(b) **B.** The identity element is not conjugate to anything, obviously. The next conjugacy class is given by rotational symmteries while preserving the order of atoms. Finally, we get the last set, with rotational symmetries of the molecule with an altered atomic order.

(c) **AB.** The symmetric irreducible representation invariably has dimension 1, and the number of irreducible representation gives us the number of conjugacy classes as well.

(d) **D.** Assume the number of elements in each conjugacy class are x_n for $n \in \{1, 2, 3, 4, 5\}$. The character vectors must be orthogonal for the different irreducible representations. Disregarding the unknown representation, we get $\binom{4}{2}$ equations. Solving them using a computer, we get x_5 as an independent parameter, but it must be a multiple of 6 for integer solutions for the rest. So, assuming x_5 to be 6, we get $(x_n) = [1, 8, 3, 6, 6]$ and we can solve to get $[p, q, r, s, t] = [-3, 0, 1, -1, 1]$ as a reduced solution.

Question 2.

(a) We can write down the conjugacy classes and the number of elements in each using combinatorial split of 6 elements into the number of required cycles as

Cycle structure	Number of elements in class
(6, 0, 0, 0, 0, 0)	1
(4, 1, 0, 0, 0, 0)	15
(2, 2, 0, 0, 0, 0)	45
(0, 3, 0, 0, 0, 0)	15
(3, 0, 1, 0, 0, 0)	40
(1, 1, 1, 0, 0, 0)	120
(0, 0, 2, 0, 0, 0)	40
(2, 0, 0, 1, 0, 0)	90
(0, 1, 0, 1, 0, 0)	90
(1, 0, 0, 0, 1, 0)	144
(0, 0, 0, 0, 0, 1)	120

Start		Start	
	1 2 3		1 2 3
P_1		P_2	
	2 1 3		1 3 2
P_2		P_1	
	2 3 1		3 1 2
P_1		P_2	
	3 2 1		3 2 1

Figure 1: Similarity swapping with a common element.

Left action (126)(45)(3) | Right action (13)(24)(56). We can calculate it simply by multiplying the individual cycles to get a final cycle structure.

(b)

- **No.**
  Take $b = a + 1$. There is a clear commutation violation.
- $i \in \{-1, 0, +1\}$.
  Since P_a is its own inverse, the operation $P_a \cdot \cdot P_a$ is simply the inner operation in a "morphed space" where P_a had acted, and then transorming back, the same intuition carried for similarity actions of matrices. The equivalence is trivially violated if P_a and P_{a+i} share no common elements, and trivially holds if $i = 0$ as they are the same elements. If $i \notin \{-1, +1\}$, i.e. in the case where there is one element is common, it is easily demostrated this holds considering three elements as in Figure 1. It is simply a 3 element reversal.

(c) The stereographic projection is as in Figure 2.

The conjugacy classes will be:

- $\{e\}$
- $\{C_4, C_4^3\}$
- $\{C_2\}$
- $\{U_2, U_2C_4, U_2C_2, U_2C_4^3\}$
- $\{\sigma_d, \sigma_d C_4, \sigma_d C_2, \sigma_d C_4^3\}$
- $\{U_2\sigma_d, U_2\sigma_d C_2\}$
- $\{U_2\sigma_d C_4, U_2\sigma_d C_4^3\}$

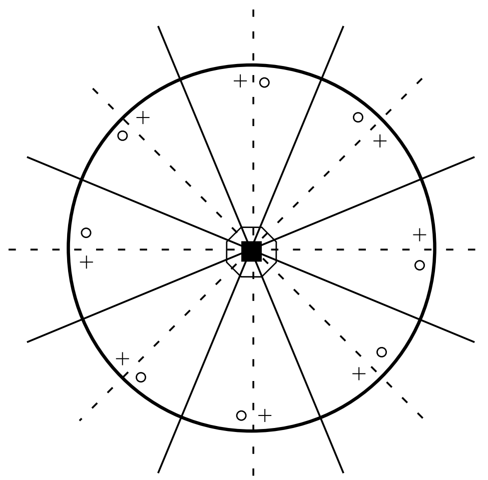

Figure 2: Stereographic Projection / Schoenflies Notation.