PH563 Assignment 1

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Question 1.

(a) B. $\left| \vec{r}_1 - \vec{r}_2 \right|$
In a translationally invariant context, relative positioning can be the only independent parameter.

(b) C. $-p - 2$
The identity corresponding to the given group operation is -1, so using $p * p^{-1} = p + p^{-1} + 1 = \hat{\mathbf{1}} = -1$, we get the result.

(c) B. $R_{\frac{\pi}{4}}$
A rotation of $\frac{\pi}{4}$ over $\frac{7\pi}{4}$ generates the identity, i.e. 2π.

(d) C. $(2, 0, 1, 1, 0, 0, 0, 0, 0)$
Counting columns from the right, 2 of length 1, 1 of length 3, and 1 of length 4.

Question 2.

(a) False
The symmetric potential gives rise to a Hamiltonian with inversion symmetry about origin. The solutions 'inherit' this symmetry, being either even or odd, with position expectation 0.

(b) True
Due to the construction of the coset (i.e. $G \cdot a$), every element of the group is included in atleast one coset, by action on the identity. Thus, the union of all cosets of a group forms the entire group.

(c) True
The exponential map $\exp(x)$ maps $\mathbb{R}$ to $\mathbb{R}^+$ homomorphically, and inversely, the natural logarithm maps $\mathbb{R}^+$ to $\mathbb{R}$. The composition gives the identity morphisms in either direction.

(d) True
A 2-cycle and a 4-cycle can be written as a composition of 1 and 3 transpositions respectively, for a total of 4, which is even. Hence, the elements do belong to $\mathfrak{U}(6)$.

Question 3.

(a) **4.**

The classes are:

- $C_1 = \{E\}$
- $C_2 = \{Q, R, QR\}$
- $C_3 = \{P, PQ, PR, PQR\}$
- $C_4 = \left\{P^2, P^2Q, P^2R, P^2QR\right\}$

(b) **105.**

The order of the conjugacy class is the number of permutations in a (2, 2, 1, 1, 1) partition of 7 elements,

$$|C| = \frac{\binom{7}{2} \cdot \binom{5}{2}}{2!} \cdot \frac{\binom{3}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}}{3!}.$$

We get $|C| = 105$.

The same can be gotten using the cycle decomposition formula.