

CS738 Assignment 1

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1. Prove that the LTL formulae $\mathbf{F} \phi$ and $\neg \mathbf{G} (\neg \phi)$ are equivalent.

Let $M = (S, R, S_0, L)$ be an arbitrary model, and $\pi \in \text{Paths}(M, s)$ an arbitrary path with $s \in S_0$. We prove that $\mathbf{F} \phi$ holds iff $\neg \mathbf{G} (\neg \phi)$ does.

(\Rightarrow direction). If $M, \pi \models \mathbf{F} \phi$, there exists an $i \in \mathbb{N}$ such that $M, \pi^i \models \phi$ per the definition of the operator \mathbf{F} . Consider now the formula $\mathbf{G} (\neg \phi)$ — the semantics of the operator \mathbf{G} imply that for this formula to hold, $\forall j \in \mathbb{N} : M, \pi^j \models \neg \phi$, but we know that this does not hold for $j = i$, as $M, \pi^i \models \neg \phi$ iff $M, \pi^i \not\models \phi$ for any k , as per the definition of \neg . Thus, $M, \pi \not\models \mathbf{G} (\neg \phi)$ and by extension $M, \pi \models \neg \mathbf{G} (\neg \phi)$.

(\Leftarrow direction). If $M, \pi \models \neg \mathbf{G} (\neg \phi)$, then, reducing using the definition of the operator \neg , $M, \pi \not\models \mathbf{G} (\neg \phi)$. Negating the definition of the operator \mathbf{G} , we see that there must exist an $i \in \mathbb{N}$ such that the subformula does not hold, that is, $M, \pi^i \not\models \neg \phi$. Reducing this using the double negation axiom, we get that $M, \pi^i \models \phi$. This is precisely the condition for $\mathbf{F} \phi$ to hold. So we get that $M, \pi \models \mathbf{F} \phi$.

Thus, the two formulae imply each other and are thus equivalent for the pair $\langle M, \pi \rangle$. But, since both were chosen arbitrarily from the respective domains, the equivalence holds true for all $\langle M, \pi \rangle$. Thus, the two formulae are equivalent.