Information Theoretic Bounds on Variational Quantum Algorithms

B.Tech Project II

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Outline

- Noisy Intermediate-Scale Quantum Systems
- NISQy Problems and NISQy Solutions
- Variational Quantum Algorithms
- Information Theoretic Bounds *(Our results)*

Noisy Intermediate-Scale Quantum Devices

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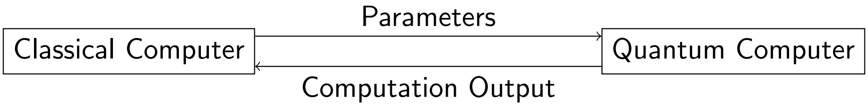
Any actual examples to speak of?

- Quantum Approximate Optimization Algorithms (QAOAs)
- Quantum Variational Eigensolvers (QVEs)
- Ground State Estimation
- Polynomial Unconstrained Binary Optimization (PUBO)[1]

[1] Lennart Bittel and Martin Kliesch. "Training Variational Quantum Algorithms Is NP-Hard". In: *Phys. Rev. Lett.* 127 (12 Sept. 2021), p. 120502. DOI: 10.1103/PhysRevLett.127.120502. URL: https://link.aps.org/doi/10.1103/PhysRevLett.127.120502.

Fixing the leaks

Use as a subroutine!



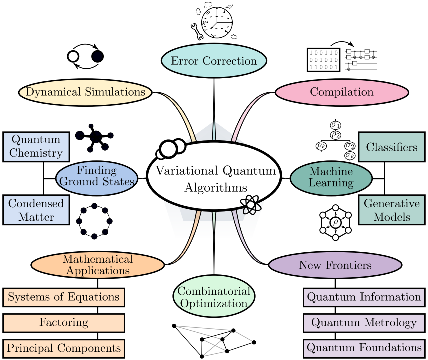
Quantum Bandaid

Tides us over while people look for exotic materials and configurations to attain reasonable coherence!

- ✓ Computational efficiency of a quantum computer
- ✓ Control efficiency of a classical system
- ✓ Allows alternate explorations for quantum supremacy

A Solution

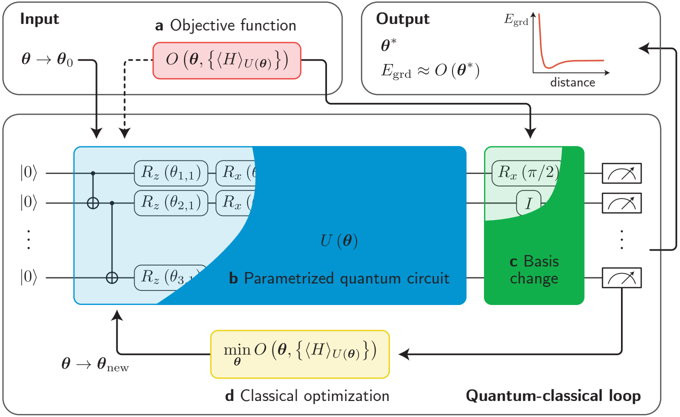
Variational Quantum Algorithms[2]



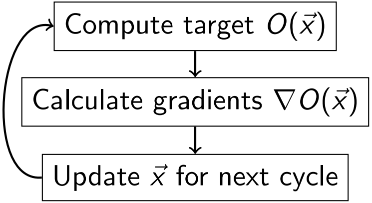
[2] M. Cerezo et al. "Variational quantum algorithms". In: *Nature Reviews Physics* 3.9 (Sept. 2021), pp. 625–644. ISSN: 2522-5820. DOI: 10.1038/s42254-021-00348-9. URL: https://doi.org/10.1038/s42254-021-00348-9.

Variational Quantum Algorithms

Variational Quantum Algorithms (VQA) form the idea of using a proposed architecture for generalised learning problems on NISQ systems [3].



Transitioning



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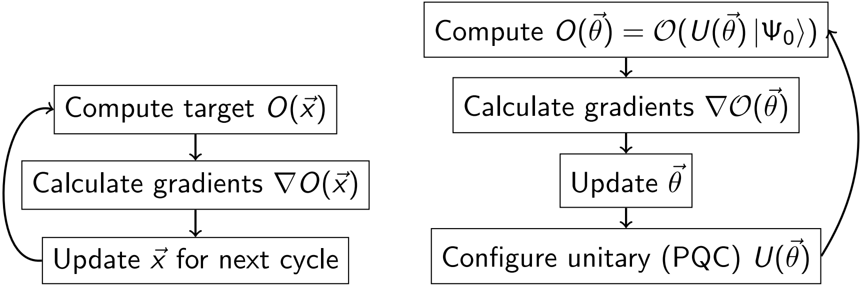

$$\vec{\theta} \in \mathbb{R}^M, \vec{x} \in \mathbb{R}^N, M \sim \lceil \log_2 N \rceil$$

Parametrized Quantum Circuits (PQCs)

$$U(\vec{\theta}) = \prod_{l=1}^{L} U_l(\vec{\theta}_l), \quad U_l(\vec{\theta}) = \prod_{k=1}^{K} e^{-\iota\theta_{lk} H_k} \ . \tag{1}$$

for a set of generators $\{H_k\}$ with $\vec{\theta} = (\vec{\theta}_1, \vec{\theta}_2, \ldots, \vec{\theta}_k)$.

[3] Martin Larocca et al. *Diagnosing barren plateaus with tools from quantum optimal control*. 2021. arXiv: 2105.14377 [quant-ph].

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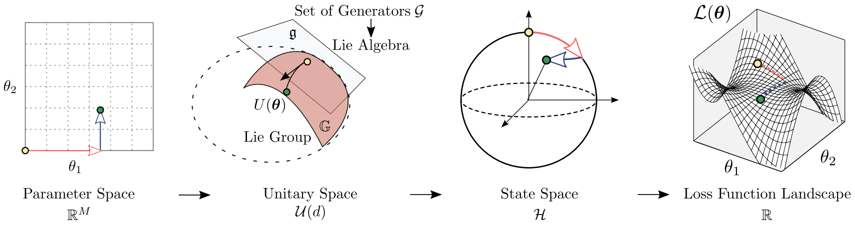
for a set of generators $\{H_k\}$ with $\vec{\theta} = (\vec{\theta}_1, \vec{\theta}_2, \ldots, \vec{\theta}_k)$.
The choice of generators can strongly affect the optimization process[3]. See the report for a more detailed study.

[3] Martin Larocca et al. *Diagnosing barren plateaus with tools from quantum optimal control*. 2021. arXiv: 2105.14377 [quant-ph].

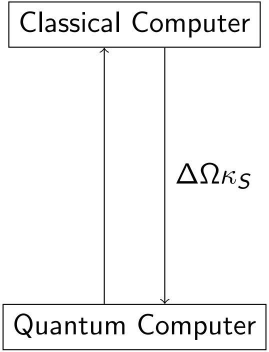
Quantum Landscape Theory

Spaces and maps involved in VQA calculation[4]:



[4] Martin Larocca et al. "Theory of overparametrization in quantum neural networks". In: *arXiv preprint arXiv:2109.11676* (2021).

Information Theoretic Limits



The classical system transferring information about the parameters to the PQC has a limit on how much information it can transfer in a given time T given by

$$b = T\Delta\Omega\kappa_s$$

where $\Delta\Omega$ is the channel bandwidth and κ_s is the bit depth of the control signal (amount of information *per* trasnfer).

Example — Bounds on Quantum Optimal Control

For the specific example of a control pulse for a quantum system [6]

$$\dot{\rho} = \mathcal{L}(\rho, \gamma(t)) \ ,$$

where ρ is the state density matrix, $\mathcal{L}$ is the Liouvillian operator, and $\gamma(t)$ is the control pulse. We have the Hamiltonian

$$\hat{H} = \hat{H}_D + \gamma(t) \cdot \hat{H}_C \ .$$

Example — Bounds on Quantum Optimal Control

We have

$$\kappa_s = \log\left(1 + \frac{\Delta\gamma}{\delta\gamma}\right) ,$$

where $\Delta\gamma$ and $\delta\gamma$ are the maximum and minimum variation in the control field. We have the error bound[5] $\|\rho - \rho_*\| > \epsilon$, with

$$\epsilon \geq 2^{-\frac{T\Delta\Omega\kappa_s}{\mathcal{D}_{\mathcal{W}^+}}} ,$$

where $\mathcal{D}_{\mathcal{W}^+}$ is the dimension of the relevant (polynomially reachable) space of density matrices.

[5] S Lloyd and Simone Montangero. "Information theoretical analysis of quantum optimal control". In: *Physical review letters* 113.1 (2014), p. 010502.

Parameter Space Dimension - Fisher Information

$$\left[F_\mu(\vec{\theta})\right] = 4\mathrm{Re}\left[\left\langle \partial_i\psi_\mu(\vec{\theta}) \middle| \partial_j\psi_\mu(\vec{\theta})\right\rangle - \left\langle \partial_i\psi_\mu(\vec{\theta}) \middle| \psi_\mu(\vec{\theta})\right\rangle \left\langle \psi_\mu(\vec{\theta}) \middle| \partial_j\psi_\mu(\vec{\theta})\right\rangle\right] \tag{2}$$

where ψ_μ varies over the input state set.
F_μ is $M \times M$ where M is the number of parameters in the control.

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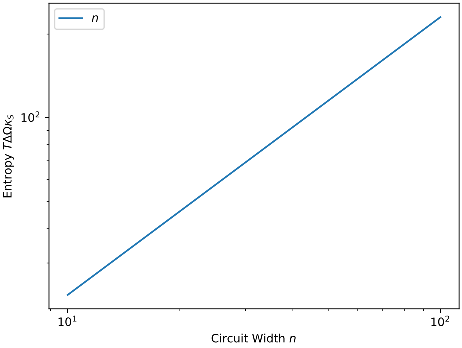
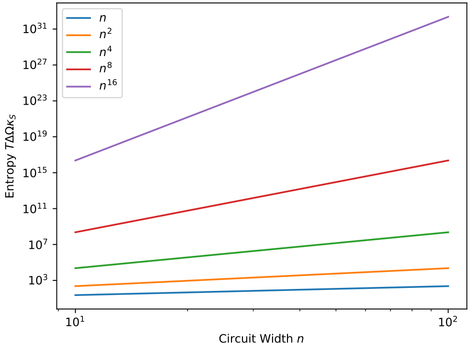
Increase M till rank plateaus.

Parameter Scaling Bounds



- Linear?

Parameter Scaling Bounds



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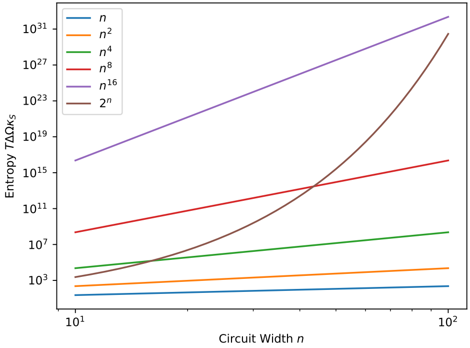

- Linear?
- Polynomial?
- Exponential, in general.

Trainability Tradeoff

What problems does this bring?

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$$\epsilon \geq e^{-\frac{T\Delta\Omega\kappa_S}{\mathcal{D}_{\mathcal{W}^+}}}, \tag{3}$$

$$T\Delta\Omega\kappa_S \geq \mathcal{D}_{\mathcal{W}^+} \ln\left(\frac{1}{\epsilon}\right). \tag{4}$$

Consequences?

- Not a complexity bound![6]

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Consequences?

- Not a complexity bound![6]
- Specification constraints!

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