

Boosting

Myra O'Regan

November 22, 2017

Boosting - Adaboost

- For weak classifiers slightly better than random
- Two very different ways of looking at this
- N observations in the training sets
- Sample with replacement N observations
- Look to see which obs are misclassified
- Weight the cases which are misclassified more
- Sample again
- Repeat above steps
- Weighted Combination at the end

Boosting - Adaboost Original form

- y is coded as -1 and +1
- Initial compute initial weights $= w_i^{(0)} = 1/N$
- Let M = no. of trees
- For $m=1$ to M {
- Fit a classifier $T_m(x)$ to training data with $w_i^{(m)}$
- Compute $err_m = \frac{\sum_{i=1}^N w_i^{(m)} I(y_i \neq T_m(x))}{\sum w_i^{(m)}}$
- Compute $\alpha_m = \ln((1 - err_m)/err_m)$
- Update weight $w_i^{m+1} = w_i^m \exp[\alpha_m \cdot I(y_i \neq T_m(x))]$
- In some versions weights are normalised to sum to 1
- Output $\text{sign}(\sum_{m=1}^M \alpha_m \cdot T_m(x))$

Some points about Boosting

- Compute $\alpha_m = \ln((1 - \text{err}_m)/\text{err}_m)$
- A classifier with 50% error is given zero weight
- A classifier with error rate $< 50\%$ given a negative weight

Illustration of boosting

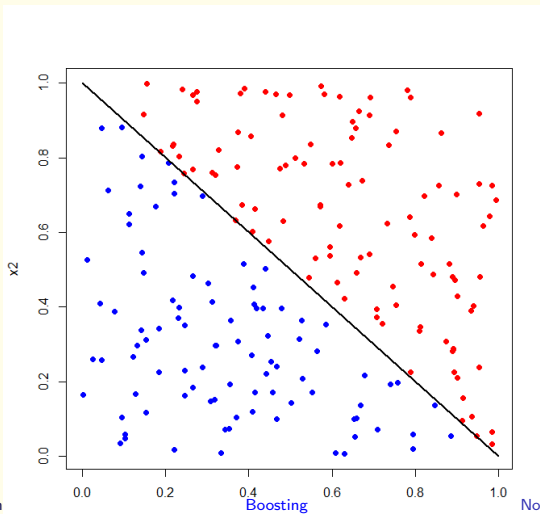
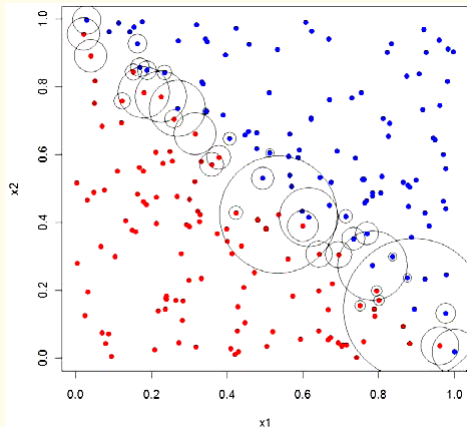


Illustration of boosting



Weights after a few iterations

Generic algorithm for Ensembles

Step 1- Choose $\{\mathbf{p}_m\}$

$$F_0(\mathbf{x}) = \mathbf{0}$$

For $m=1$ to M {

$$\mathbf{p}_m = \operatorname{argmin}_{\mathbf{p}} \sum_{\mathbf{i} \in \mathbf{S}_{n(\eta)}} \mathbf{L}(\mathbf{y}_i, \mathbf{F}_{m-1}(\mathbf{x}_i) + \mathbf{T}(\mathbf{x}_i; \mathbf{p}))$$

$$T_m(\mathbf{x}) = \mathbf{T}(\mathbf{x}; \mathbf{p})$$

$$F_m(\mathbf{x}) = \mathbf{F}_{m-1}(\mathbf{x}) + \nu \mathbf{T}_m(\mathbf{x})$$

}

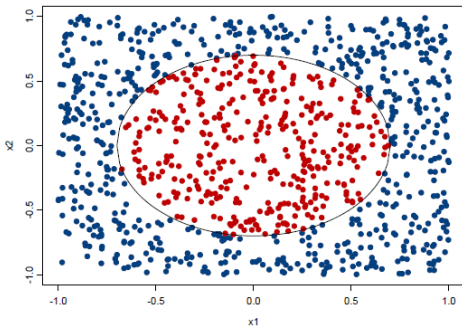
write $\{T_m(x)\}_1^M$

Boosting - Adaboost

- Second way
- y is coded as $+1$ and -1
- $L(y, \hat{y}) = \exp(y \cdot \hat{y})^2$
- $v = 1$
- $\eta = N$
- $T_m(x)$: any weak learner
- $c_0 = 0, \{c_m\}_1^M$; sequential partial regression coefficients
- Use **gbm** package in R

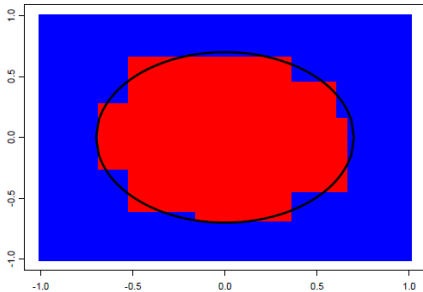
Illustration of boosting, bagging and single trees

Visualizing Bagging and AdaBoost (2-dimensional, 2-class example)



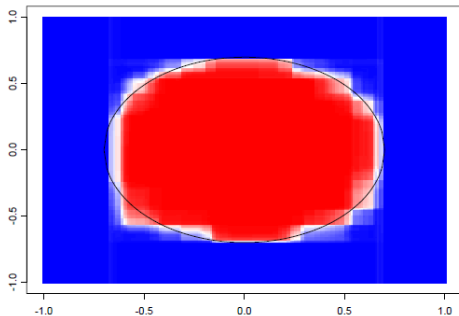
Single tree

Decision boundary of a single tree



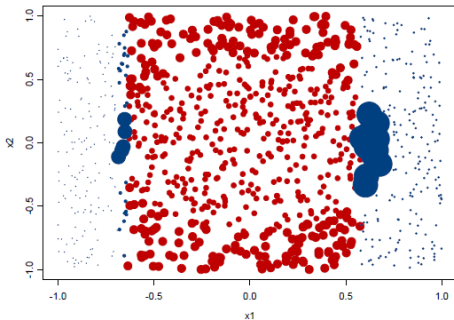
100 bagged trees

100 bagged trees leads to smoother boundary



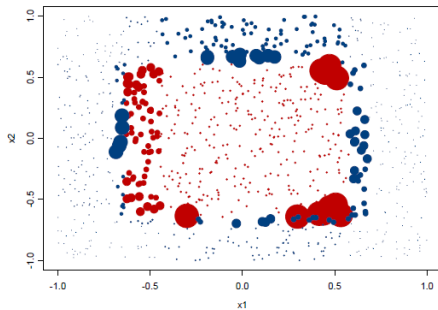
Boosting after 1 iteration

AdaBoost, after one iteration (CART splits, larger points have great weight)



Boosting after 3 iterations

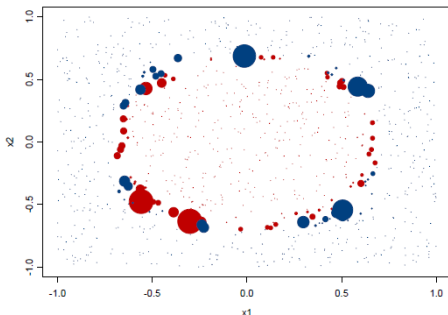
After 3 iterations of AdaBoost



Boosting after 20 iterations

Appendix 1

After 20 iterations of AdaBoost



Boosting after 100 iterations

Decision boundary after 100 iterations of AdaBoost

