Macroeconomic Theory I
Problem Set 4

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NOTE: Code to generate the tables and figures in this document are available in this GitHub repository - in the interest of brevity and document hygiene, I do not display code directly in this document. However, the data build is fully replicable from the code in the GitHub repository.

The README contains the link to the folder with the raw data.

Collect annual time-series data on real, fixed price (ideally) or chained GDP, Y_t, current price GDP, $NOMY_t$, current price gross investment, $NOMI_t$, hours worked, H_t, and the working age population, POP_t for

- *United States*
- *United Kingdom*

for the same sample period. Ideally, the sample period should be at the latest from 1970 through 2019. It can be a little shorter if one of the countries has a shorter sample period than this available.

Do not attempt to collect data for 2020 or 2021, as they are affected by the pandemic. Check BEA (US), Eurostat, FRED, IMF, OECD, World Bank, and UK-specific sources. Construct the working age population of each country using U.N. Population Estimates data by summing over the population aged 16 through 65 or 20 through 69 or similar.

Make the following calculations for each of the countries.

a. **Construct a real investment series by calculating**

$$I_t = \frac{NOMI_t}{P_t}$$

where $NOMI_t$ is the current price investment and P_t is the GDP fixed price deflator. The latter is the ratio of current price to constant price GDP,

$$P_t = \frac{NOMY_t}{Y_t}$$

Answer: I obtain US macroeconomic data from FRED. Specifically, the following variables are used:

- Y_t: Real GDP, Billions of Chained 2017 Dollars
- $NOMY_t$: Nominal GDP, Billions of Dollars
- $NOMI_t$: Gross Private Domestic Investment, Billions of Dollars
- P_t: GDP implicit price deflator, $Index_{2017} = 100$

For the UK, I obtained data from the Office for National Statistics, the official statistical agency for the United Kingdom. Specifically, the following variables are used:

- Y_t: Gross domestic product at market prices: Chained volume measure
- $NOMY_t$: Gross domestic product at current prices
- $NOMI_t$: Gross capital formation, Millions of Pounds
- I_t: Gross capital formation, Millions of Chained 2019 Pounds
- P_t: GDP implicit price deflator, $Index_{2017} = 100$

For UK, I do not construct I_t by dividing $NOMI_t$ by P_t since UK has a separate deflator for gross capital formation. I instead use chained value of gross capital formation[1].

I now plot the real investment to real GDP ratio for both countries from 1970 to 2019 in Figure 1.

The US investment to GDP ratio has fluctuated between $\sim$ 13.3% and 20.5% between 1970 and 2019 while the UK investment to GDP ratio has fluctuated between 15.8% to 21.1% in the same period. However, US investment to GDP ratio has shown more volatility (as measured by long-run standard deviation) compared to UK.

[1] I additionally validated the accuracy of the gross capital formation deflator - $\frac{NOMI_t}{I_t}$ was equal to the deflator value with an upto 10^{-2} digits

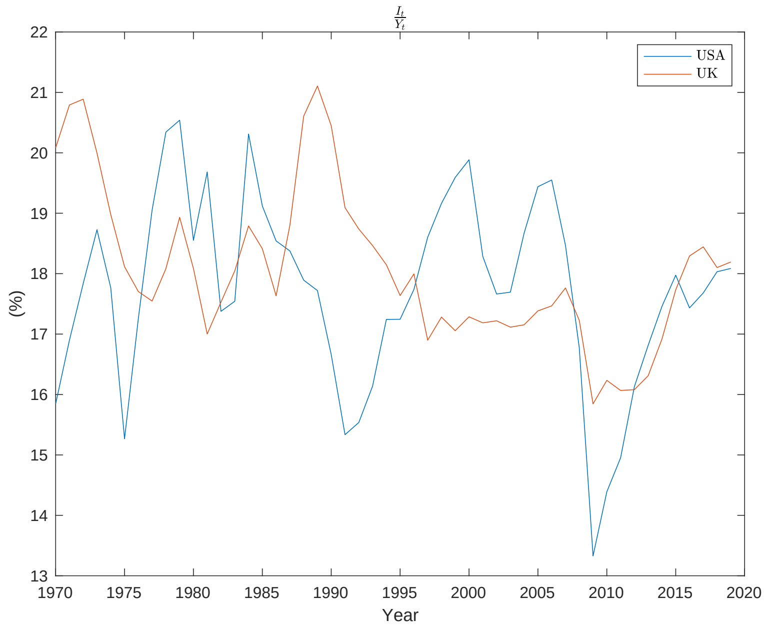

Figure 1: Investment-GDP ratio comparison

2. **Use the real investment series, the capital accumulation equation**

$$K_{t+1} = (1-\delta)K_t + I_t$$

and follow the numerical method we discussed in class to jointly determine the values of δ, K_0, and $\{K_t\}_{t=1}^{T}$. Specifically, you will choose the initial capital stock to satisfy

$$\frac{K_0}{Y_0} = \left(\frac{1}{10}\right)\left(\sum_{t=1}^{10}(K_t/Y_t)\right)$$

given data on initial real GDP, and GDP for the following 10 periods. You will choose the depreciation rate such that the sample average depreciation expenditure as a portion of GDP calculated in the data using each country's National Income and Product Accounts (NIPA) data equals that implied by the sample average product of your constructed (real) capital stock and depreciation rate as a ratio of (real) GDP. And use the capital accumulation equation to calculate the time series of capital stocks over your sample period. You should not use independent sources of information on the depreciation rate, or initial capital stock in making these calculations.

Answer: I use numerical optimization on MATLAB (*fsolve*) to jointly determine the values of δ, K_0, and $\{K_t\}_{t=1}^{T}$ between 1970 and 2019.

The UK has a higher rate of depreciation than the US. However, the capital stock in the US is higher than in the UK by a magnitude of 10, indicating significant differences in the level of capital stock formation.

Table 1: K_0 value

Country	Arithmetic mean (n=10)	δ
United States	9.7794	0.0682
United Kingdom	1.482	0.1056

3. **Repeat part 2 using a geometric ten-year average of capital-output ratios in the initial capital stock formula**, rather than a simple arithmetic average, and compare the two capital stock series you have constructed for each country by plotting them and calculating

- their simple correlation,
- their relative standard deviations,
- their relative growth rates.

Discuss how and why they differ.

By the AM-GM inequality, we know that the geometric mean would be lesser than the arithmetic mean.

Table 2: K_0 value

Country	Geometric mean (n=10)	Arithmetic mean (n=10)	δ
United States	9.5187	9.7794	0.0682
United Kingdom	0.1475	0.1482	0.1056

Additionally, we are required to calculate the correlation between the 10-year arithmetic and geometric mean as well as the relative standard deviation and growth rates.

I now plot the capital series for each country.

Note that the differences in the capital stock series constructed using the capital accumulation equation have very minimal differences which suggests that the estimates are robust to the method employed.

Table 3: Comparison of Capital series for US

Moment	Arithmetic mean (n=10)	Geometric mean (n=10)
Correlation	1	0.99999
Relative SD	1	1.0065
Average growth rate	1	1.019

Table 4: Comparison of Capital series for the UK

Moment	Arithmetic mean (n=10)	Geometric mean (n=10)
Correlation	1	1
Relative SD	1	1.003
Average growth rate	1	1.0048

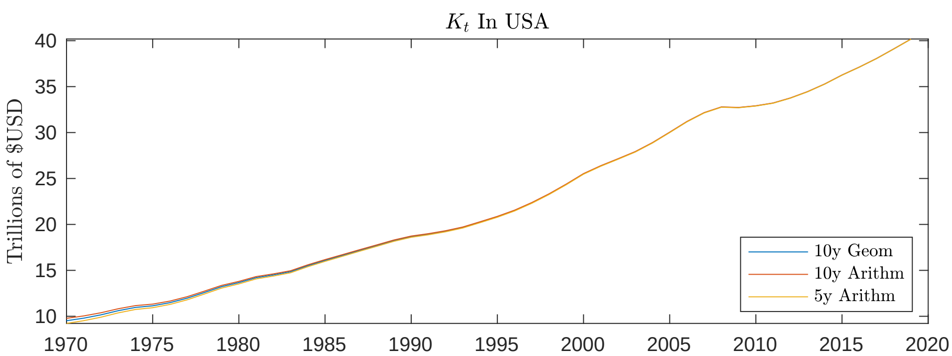

Figure 2: Capital series for US

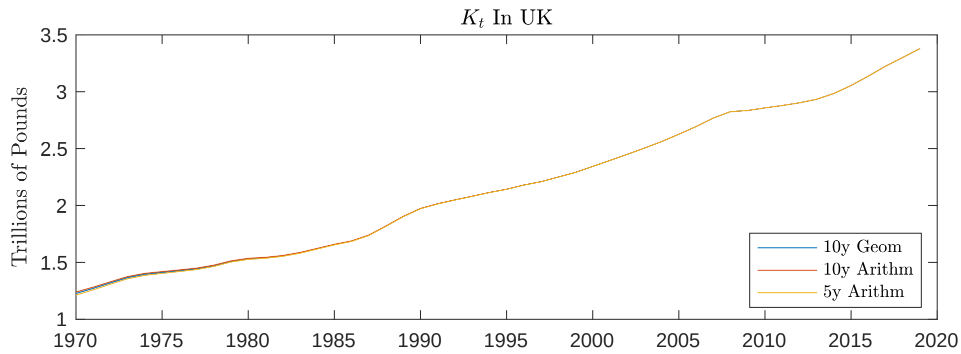

Figure 3: Capital series for the UK

4. **Repeat part 2 using the five-year arithmetic mean of capital-output ratios after the initial period rather than the 10-year arithmetic mean to calculate the initial capital stock**. Does this matter, quantitatively, for the capital stock series you compute for each country? Again, plot the two series and compare their statistical properties (first and second moments).

Answer: I repeat the same exercise for both countries using 5-year and 10-year averages. The plots above also show the 5-year series, which is nearly identical to the 10-year arithmetic mean and 10-year geometric mean capital stock series.

I show the statistical properties of the 5-year arithmetic average capital stock series.

Table 5: Comparison of Capital series for the US with 5-year arithmetic average

Moment	Arithmetic mean (n=10)	Geometric mean (n=10)	Arithmetic mean (n=5)
Correlation	1	0.99999	0.99997
Relative SD	1	1.0065	1.0139
Average growth rate	1	1.019	1.041

Similar to part 3, we see that the correlation between the 10-year and 5-year arithmetic averages is near-perfect, indicating the robustness of the capital stock estimates.

5. **If there are capital stock series available online for your three countries, for example, the PPP series in the Penn World Tables or the**

Table 6: Comparison of Capital series for the UK with 5-year arithmetic average

Moment	Arithmetic mean (n=10)	Geometric mean (n=10)	Arithmetic mean (n=5)
Correlation	1	1	0.99986
Relative SD	1	1.003	0.99656
Average growth rate	1	1.0048	0.94215

nominal series in the IMF's capital and investment dataset, compare your constructed series for each country to the published series and try to uncover any reasons for the differences. Again, plot the series against one another and compare their first and second moments.

Answer: I use data from the Penn World Tables (v10.01), running from 1950 to 2019. I now plot a comparison of the capital stock series for each country against their corresponding PWT capital stock series.

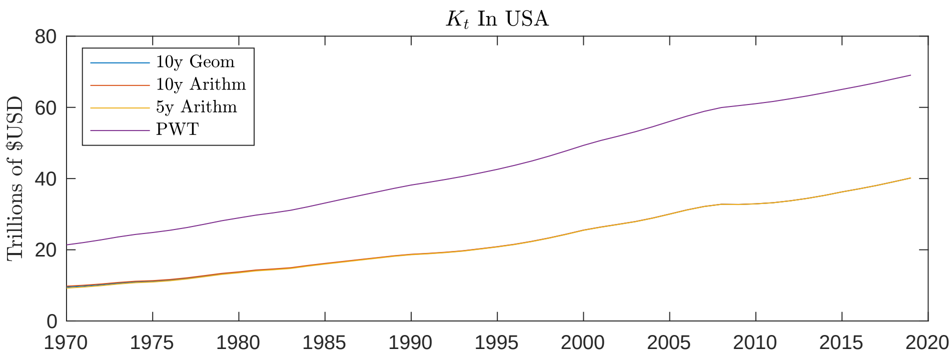

Figure 4: Capital series for US

A plausible reason for the differences between the Penn series and the series I construct can be explained by the fact that I do not use the PPP rate for the UK and US, which may have biased the estimation, hence the improper shape.

6. **Assume a Cobb-Douglas form for the aggregate production function in each of the countries, with TFP. Calibrate the value of capital's share of income for the two countries using NIPA data as we discussed in class**. This is always possible in the United States. It may be more challenging for the United Kingdom, and you should document thoroughly the efforts you made to attain the appropriate data, the data series on income sources that are available, and the data series not available. If all else fails, set the capital income

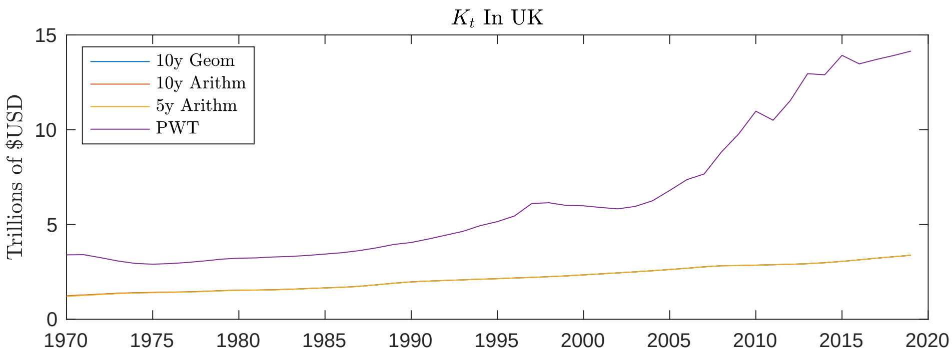

Figure 5: Capital series for UK

share for the United Kingdom equal to that for the United States but first show all the data you collected and explain carefully why that data yields a poor estimate of capital's income share.

Answer: I use the available data to compute $\alpha_{USA} = 0.244324$, as the average of $\frac{CE_t}{Y_t - NOS_t - (T_t - S_t)}$ between 1970 and 2019, which is reasonable.

For the UK, I could not find reliable estimates for net operating surplus so I could not estimate α for the UK. I set α_{UK} the same as that of the USA.

However, I use $\alpha = \frac{1}{3}$ for the remainder of the exercise since it is the value found in the literature.

7. **Conduct a growth accounting exercise for each country individually, by decomposing movements in real GDP per working-age person at each date into components attributable to growth in a technology factor, growth in a capital factor, and growth in a labor factor using (ideally) aggregate hours worked to measure labor input, using the (growth rate version of the) following equation**

$$\left(\frac{Y_t}{POP_t}\right) = A_t^{1/(1-\alpha)} \left(\frac{K_t}{Y_t}\right)^{\alpha/(1-\alpha)} \frac{N_t}{POP_t}$$

To do this, you must impute the growth rate of the TFP factor using the production function.

a) **According to equation (1), what is the balanced growth path growth rate of output per working-age person?** Use the notes from class to re-express this growth rate as a common global long-run growth rate. Based on US data, what is the value of this long-run growth rate over the sample period?

Answer: Solving for A_t gives $A_t = \left(\frac{\frac{Y_t}{N_t}}{\left(\frac{K_t}{Y_t}\right)^{1-\alpha} \frac{L_t}{N_t}} \right)^{1-\alpha}$. I then take natural logarithms and compute their difference (denoted by hats) to get

$$\widehat{\frac{Y_t}{N_t}} = \frac{1}{1-\alpha}\widehat{A_t} + \frac{\alpha}{1-\alpha}\widehat{\frac{K_t}{Y_t}} + \widehat{\frac{L_t}{N_t}}$$

The balanced growth path growth rate for output-per-working age person is 2.1%. I show the complete decomposition for long-run growth rates in 7(b).

b) **Calculate sample average annual growth rates for each growth accounting variable in equation (1) for each country**. Discuss the implied sources of GDP per working-age person (people ages 16-65 or 20-69 or whatever you study define as the working-age population) growth over the sample period for all three countries.

I decompose the long-run growth rate following the formula provided, which produces the average results presented below:

Table 7: Growth Accounting, 1980-2019 (%)

Country	$\widehat{\frac{Y}{N}}$	$\frac{1}{1-\alpha}\widehat{A}$	$\frac{\alpha}{1-\alpha}\widehat{\frac{K}{Y}}$	$\widehat{\frac{L}{N}}$
United Kingdom	1.7138	1.3253	-0.058219	0.44676
United States	1.4881	0.19961	0.055302	1.2332

Where the values are in percentage units (%). For both countries, we get the usual result from the literature: A major TFP component, followed by a considerable capital-output ratio effect, and finally a modest contribution of the population increment.

To check whether there is balanced growth, it suffices to plot the decomposition for both countries, which is shown below.

Under balanced growth, each component should be a straight horizontal line. Looking at the variability of the growth rates in both countries, the hypothesis of balanced growth would be more likely to hold in the USA than in the UK.

The following potential sources can explain the growth in $\widehat{\frac{Y}{N}}$ in the US and UK over the last 50 years:

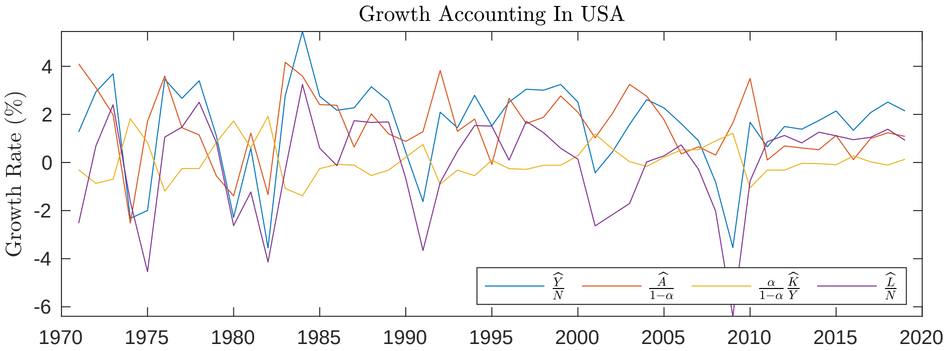

Figure 6: Growth accounting for US

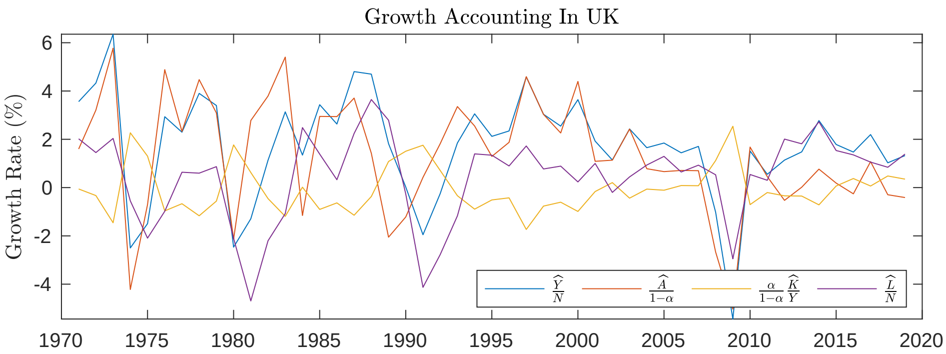

Figure 7: Growth accounting for the UK

- **Capital deepening**: Increased capital investment per worker, through more equipment, machinery, technology, etc. This makes workers more productive.

- **Technological progress**: New innovations, discoveries, and technologies that allow workers to produce more output for the same amount of input. Examples include IT and computers, automation, and new production techniques.

- **Education/skills**: An increase in the education and skill level of the workforce over time.

- **Managerial/organizational innovations**: Better management practices, corporate restructurings, just-in-time inventory systems, supply chain management, etc. can improve the efficiency of workers, making them more productive.

c) **If you divide your sample period into two or three equally long sub-periods and calculate average growth rates over these sub-periods, do the same results hold?**

I split the sample into 2 periods: 1970-1994 and 1995-2019. The growth decomposition is provided separately for each period in the subsequent tables.

Table 8: Growth Accounting by period, 1970-2019 (%)

Period	Country	$\widehat{\frac{Y}{N}}$	$\frac{1}{1-\alpha}\widehat{A}$	$\frac{\alpha}{1-\alpha}\widehat{\frac{K}{Y}}$	$\widehat{\frac{L}{N}}$
1970-1994	United Kingdom	1.8647	1.8755	-0.0041	-0.0066
1995-2019		1.5459	0.7764	-0.09341	0.8629
1970-1994	United States	1.469	-0.0199	-0.01564	1.5046
1995-2019		1.5056	0.47759	0.12465	0.90337

The results are similar to the overall levels of each growth component. There was a significant increase in $\widehat{A}$ in the US between 1995 and 2019 while it declined in the UK. Additionally, the growth rate of capital-output ratio increased significantly in the US. Furthermore, the growth rate of labor-per-capita declined in the US as expected.

d) **Plot GDP per working-age person against each of the growth factors on the right-hand side of equation (1) in a single figure for each country, normalizing all variables to equal 100 in the initial sample year.**

Based on these results, would you say that any of the three countries was likely on, or near, a balanced growth path during the sample period? Explain your answer.

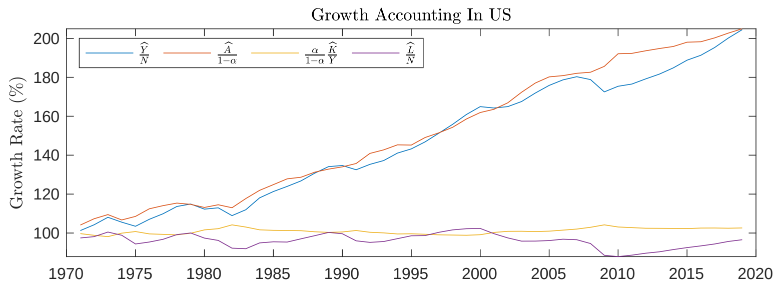

Figure 8: Normalized growth decomposition for US

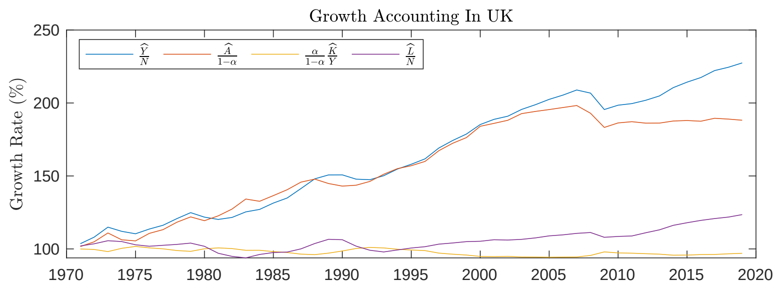

Figure 9: Normalized growth decomposition for UK

Based on these results, it is difficult to conclude that either the US or the UK was on a balanced growth path since that would require both countries to have a horizontal slope for each growth component. However, in a relative sense, the BGP hypothesis is more likely to hold in the US than in the UK.