

Notes on Algebra

Donu Arapura

December 5, 2017

Contents

1	The idea of a group	3
1.5	Exercises	8
2	The group of permutations	11
2.11	Exercises	13
3	Rotations and reflections in the plane	15
3.6	Exercises	17
4	Cyclic groups and dihedral groups	19
4.8	Exercises	22
5	Finite sets, counting and group theory	24
5.17	Exercises	28
6	More counting problems with groups	29
6.12	Exercises	34
7	Kernels and quotients	36
7.12	Exercises	39
8	Rings and modular arithmetic	40
8.17	Exercises	43
9	\mathbb{Z}_p^* is cyclic	45
9.9	Exercises	47
10	Matrices over \mathbb{Z}_p	49
10.7	Exercises	50
11	The sign of a permutation	52
11.9	Exercises	54
12	Determinants	56
12.8	Exercises	59

13 The 3 dimensional rotation group	60
13.9 Exercises	63
14 Finite subgroups of the rotation group	64
14.8 Exercises	67
15 Quaternions	69
15.5 Exercises	71
16 The Spin group	73
16.8 Exercises	75