Pendulum with Friction

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Abstract

The problem in hand is to simulate the movement of a pendulum with friction. For this a mass-less pendulum with a bob at it's end is chosen. The Ipython notebook will generate an animation showing the motion of the pendulum using matplotlib and the python script will also generate a plot which will contain $\dot{\theta}$ and θ plots with respect to time. The python code generated these plots by taking the mass, length and damping constant of the pendulum and $\dot{\theta}$ and θ as arguments for the function and these can be changed which will automatically reflect in the pdf report once the make file in run.

- Public git repository with open source code can be found at `https://github.com/sankasuraj/sdesproject1`
- Ipython2 notebook version 1.2.1 or higher is required to run the Ipython code
- Python 2.7 version is required to run the Python code
- Numpy with version 1.8.2 or higer is required
- Matplotlib with version 1.3.1 or higher is required
- A media player is required to play the mp4 animation created using the ipython notebook [1]

Equation of Motion

Equation of motion for a mass-less pendulum with a bob at the end is

$$\ddot{\theta} + \frac{b}{m}\dot{\theta} + \frac{g}{L} sin\theta = 0[2] \tag{1}$$

where,
b = damping constant in $\frac{kg}{s}$
m = mass of the bob in kg
g = acceleration due to gravity which is taken to be $9.8\frac{m}{s^2}$
L = length of the pendulum in m
θ = Angular displacement of the pendulum from the vertical in rad
$\dot{\theta}$ = Angular velocity of the pendulum in $\frac{rad}{s}$
$\ddot{\theta}$ = Angular acceleration of the pendulum in $\frac{rad}{s^2}$
In equation 1 using small angle approximation the equation can be re-written as

$$\ddot{\theta} + \frac{b}{m}\dot{\theta} + \frac{g}{L}\theta = 0 \tag{2}$$

Solution

The equation can be solved assuming the general solution to be $\theta = e^{\lambda}$ Substituting this in equation 2 we get

$$\lambda^2 + \frac{b}{m}\lambda + \frac{g}{L} = 0 \tag{3}$$

$$\lambda_1 = \frac{-\frac{b}{m} + \sqrt{(\frac{b}{m})^2 - \frac{4g}{L}}}{2} \tag{4}$$

$$\lambda_2 = \frac{-\frac{b}{m} - \sqrt{(\frac{b}{m})^2 - \frac{4g}{L}}}{2} \tag{5}$$

$$\tag{6}$$

Solution can be written as

$$\theta = c_1 e^{\lambda_1} + c_2 e^{\lambda_2} \tag{7}$$

Using initial conditions for θ and $\dot{\theta}$, constants c_1 and c_2 in equation 7 can be written as

$$c_1 = \frac{\dot{\theta}(0) - \lambda_2 \theta(0)}{\lambda_1 - \lambda_2} \tag{8}$$

$$c_2 = \frac{\lambda_1 \theta(0) - \dot{\theta}(0)}{\lambda_1 - \lambda_2} \tag{9}$$

This solution will work for both underdamped and overdamped cases. It won't work for critical damped case because $\lambda_1 = \lambda_2$ in critical damped case. However for practical values of damping constant 'b' the pendulum will always be in underdamped condition so the critical damped case is ignored where the above solution won't work.

Results

Plot of θ and $\dot{\theta}$ with respect to time for the pendulum with friction is:

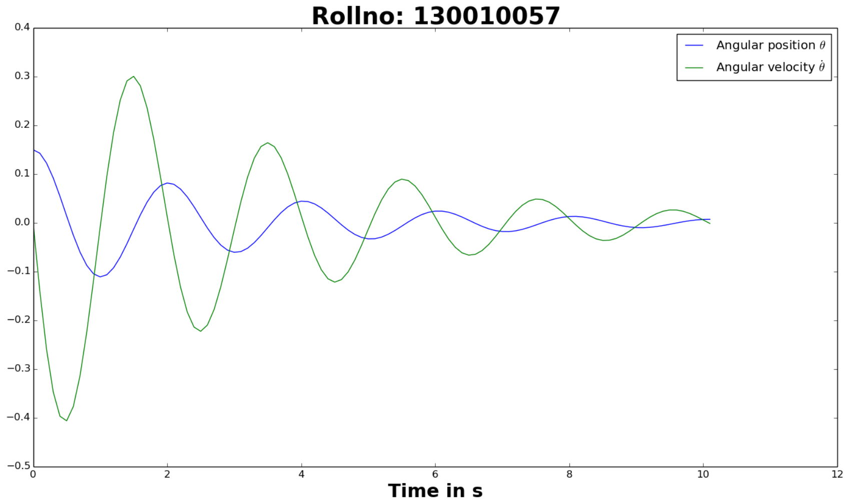

Figure 1: $\dot{\theta}$ and θ w.r.t time

Bibliography

[1] Matplotlib Animation. http://matplotlib.org/examples/animation/moviewriter.html.

[2] Pendulum Dynamics. http://nrich.maths.org/content/id/6478/paul-not%20so%20simple%20pendulum%202.pdf.