Week 1:FDM Applied to Solve Laplace Equation for Systems with Finite Boundary Conditions

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Abstract

Finite Difference Method (FDM), used to numerically solve differential equations by approximating derivatives as differences. Here, it is used to solve an 2D electrostatic problem where one edge of a rectangle is given a nonzero voltage and the other three edges are grounded. The simulation has been performed in MATLAB, where two approaches have been used to calculate the iterations. Analysis of the simulation have also been done.

1 Introduction

FDM approximates the second derivative at a point as the average of the four surrounding points. At the boundary, the V value is not updated. The given area is split into a rectangular grid of points, and the derivative can be computed at these points using the above approximation. These above conditions can be put into an algorithm.

2 Implimentation

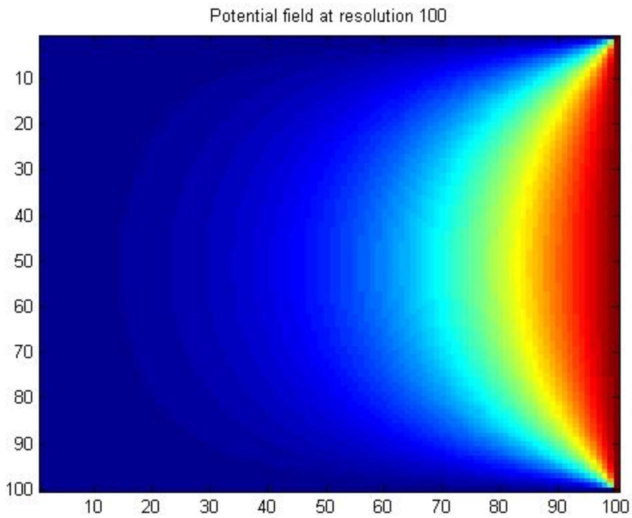
One possible algorithm which converges would be to initialize all voltages to zero in the beginnning, and then starting from the first point, using the above average , compute the updated value. then move to the adjascent point and compute the average again and so on. Carry our this process after sufficient number of iterations. This way of implementation is done in the program with for loop, where the for loops are used to iterate through the grid points.

Another way is to caculate average of each point with the surrounding points' in the grid, and then update the values all at once. Then continue with the iteration. This has been implimented using the matrix operations available in MATLAB, and the implementation turns out to be elegant.

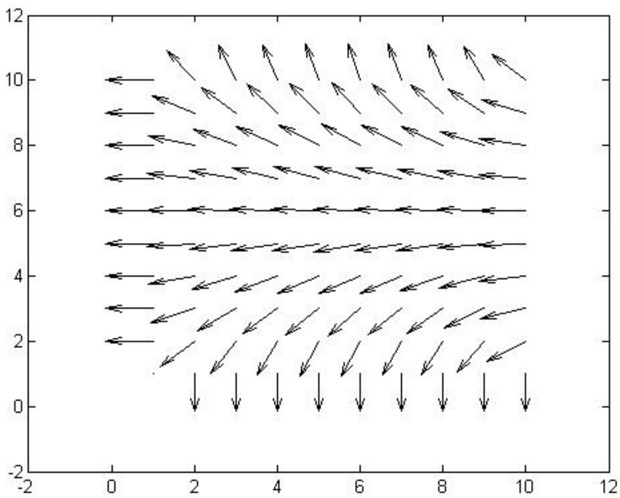
One would expect the first method would use fewer number of iterations than the second as we're updating the values at a slower rate. But since 'for' loops in MATLAB are costly when in comes to computation time, the second method might take lesser time than the first.

3 Observation

Both the algorithms converge and yield reasonable solutions. The iterations have been carried out for the number of points ranging from 20X20 to 100X100, at steps of 5. The numerical solution obtained is as follows (for a 100X100 grid):



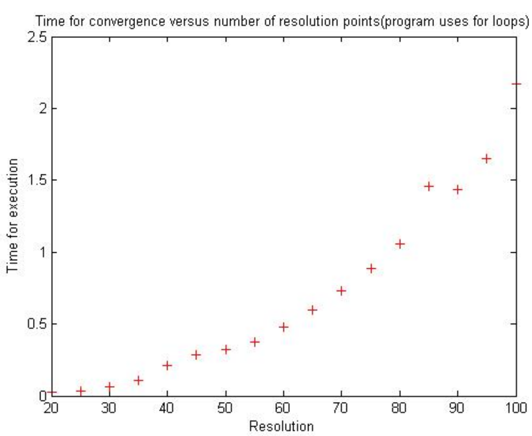
The normallized Electric Field with its direcrtion is as shown(taken from solution for 30X30 grid):

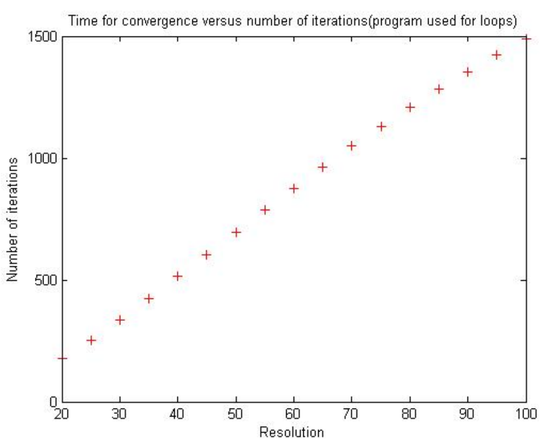


As one can notice, the boundary conditions are satisfied as the electric field is perpendicular to the edges.

4 Analysis of algorithms

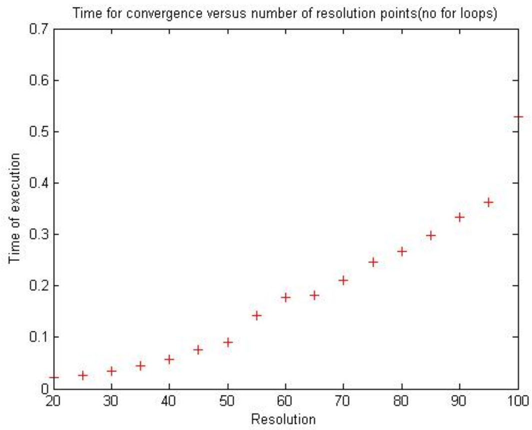
When using the for loop algorithm, the following time and number of iterations characteristics are obtained with respect to grid size(on one axis):

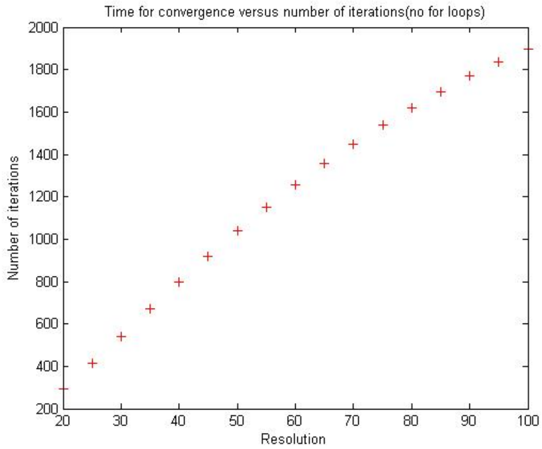




As one can see, it takes around 2 seconds to run through around 1500 iterations for a 100X100 grid.

For the second algorithm, the corresponding time and number of iterations characteristics versus the grid size is as follows:





Even the 100X100 grid takes 0.35 millisecconds on the second algorithm, which ran 1850 iterations.

5 Result and Discussion

From the graphs, using the for loop algorithm, the time taken varies almost parabolically with the grid size, that is, it grows linerly with the number of grid points. This seems reasonable as each time it iterates through each point inside the for loop. Also, since the for loop is inefficient in MATLAB, it's effect can be observed as it takes a full 2 seconds to compute solution for the 100X100 grid.

As with the other case, as expecte, it has many number of iterations than the first one, but it has much lesser execution time, around 0.3 seconds even for the 100X100 case, since it was done sing the matrix operations offered my MATLAB. Also, it an be seen, the execution time is closer to a linear graph in this case, unlike the squared case in the one above.