

09/12/2022 Mon

$$P(y=1) = 0.004$$

$$P(x=1|y=1) = 0.8$$

$$P(x=1|y=0) = 0.1$$

$$P(y=1|x=1) = \frac{P(x=1|y=1)P(y=1)}{P(x=1|y=1)P(y=1) + P(x=1|y=0)P(y=0)}$$

Bayes rule

$$x = [x_1 \dots x_j \dots x_d]$$

$$x_j \in [0, 1]$$

$$y \in [0, 1]$$

$$P(y=1|x^*)$$

$$d=1$$

$$P(x|y=1) \sim \text{Ber}(\theta)$$

$$P(x_1=0|y=1) = \theta$$

$$P(x_1=1|y=1) = 1-\theta$$

$$2-1$$

$$d=2$$

$$x = [x_1, x_2]$$

$$P(x|y=1)$$

$$P(x_1=0, x_2=0|y=1)$$

$$P(x_1=0, x_2=1|y=1)$$

$$P(x_1=1, x_2=0|y=1)$$

$$P(x_1=1, x_2=1|y=1)$$

$$2^2-1$$

$$d=3$$

$$x = [x_1, x_2, x_3]$$

$$P(x|y=1)$$

$$P(0,0,0|y=1)$$

$$\vdots$$

$$P(1,1,1|y=1)$$

$$2^3-1$$

$$2(2^D-1) \Rightarrow 2D$$

$$P(x|y=1)$$

$$P(x|y=0)$$

$$P(x_1, x_2, x_3 | y=1) \\ = \underbrace{P(x_1 | y=1)}_{\theta_1} \cdot \underbrace{P(x_2 | y=1)}_{\theta_2} \cdot \underbrace{P(x_3 | y=1)}_{\theta_3}$$

$$P(y=1) = \frac{N_1}{N} = \frac{5}{10} \quad P(y) \sim \text{Ber}(\theta)$$

$$P(y=0) = 1 - \frac{5}{10} = \frac{5}{10}$$

$$P(\text{cir} | y=1) = \frac{N_{\text{cir},1}}{N_1} = \frac{3}{5} \quad P(\text{ovl} | y=1) = 1 - \frac{3}{5} = \frac{2}{5}$$

$$P(\text{small} | y=1) = \frac{1}{5}$$

$$P(\text{light} | y=1) = \frac{2}{5}$$

$$P(\text{cir} | y=0) = \frac{2}{5}$$

$$P(\text{small} | y=0) = \frac{3}{5}$$

$$P(\text{light} | y=0) = \frac{3}{5}$$

$$P(y=1 | \text{cir}, \text{small}, \text{light})$$

$$= \frac{P(\text{cir}, \text{small}, \text{light} | y=1) P(y=1)}{\sum_y P(\text{cir}, \text{small}, \text{light} | y) P(y)}$$

$$\propto P(\text{cir}, \text{small}, \text{light} | y=1) P(y=1)$$

$$= P(\text{cir} | y=1) P(\text{small} | y=1) P(\text{light} | y=1) P(y=1)$$

$$= \frac{3}{5} \cdot \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{5}{10} = \frac{3}{125}$$

$$P(y=0 | \text{cir, small, light}) = \frac{9}{125}$$

$$P(y=1 | \text{cir, small, light}) = \frac{\frac{3}{125}}{\frac{3}{125} + \frac{9}{125}} = 0.25$$

$$P(y=0 | x^*) = 0.75$$