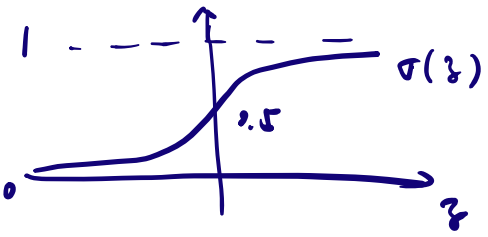
Sep 28, 2022. Wed

$p(y|x) \sim Ber(\theta)$

$\theta = \frac{1}{1+\exp(-w^T x)}$ sigmoid

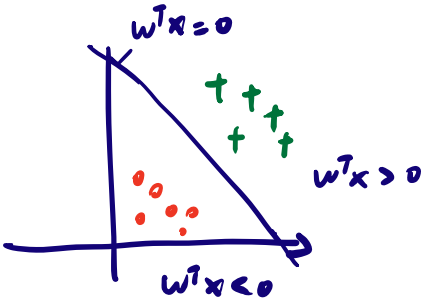


$\theta_i > 0.5 \Rightarrow y_i = 1$

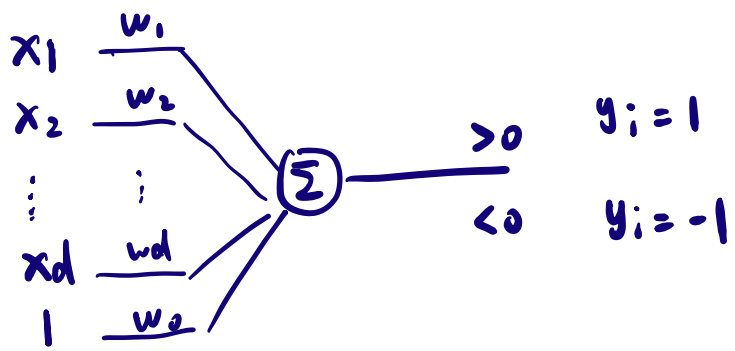
$z_i > 0$

$w^T x_i > 0 \Rightarrow y_i = 1$

$w^T x_i < 0 \Rightarrow y_i = -1$



Perceptron



Logistic regression training

$$w_k = w_{k-1} - \eta \frac{df(w)}{dw}$$

$$= w_{k-1} - \eta \frac{dNLL}{dw}$$

$$= w_{k-1} - \eta (\theta_i - y_i) x_i$$

$$\approx w_{k-1} - \eta (\hat{y}_i - y_i) x_i$$

Logistic regression online learning / perceptron learning

$y_i = \{-1, +1\}$

$D = \{(x_i, y_i)\}$ $i = 1, \cdots, N$

y_i : label

$\hat{y}_i$: output from perceptron

if $\hat{y}_i = y_i$, then $w_k = w_{k-1}$

if $\hat{y}_i \neq y_i$, then $w_k = w_{k-1} + 2\eta y_i x_i$ ($w_{k-1} + \eta y_i x_i$)

if $\hat{y}_i = 1$ $y_i = -1$

then $w_k = w_{k-1} - 2\eta x_i$

if $\hat{y}_i = -1$ $y_i = 1$

then $w_k = w_{k-1} + 2\eta x_i$

Psendo code for perceptron learning Rule :

Given $\{x_i, y_i\}$ $x_i \in \mathbb{R}^D$, $y_i \in \{-1, +1\}$

```
initialize w_0
repeat for all x_i, y_i
    ŷ_i = w^T x_i
    if ŷ_i y_i < 0
        w_k = w_{k-1} + η y_i x_i
    end if
until converged
```