Sep 21, 2022 Wed

$LL(D|w,\sigma^2)$

prior w. $P(w) \sim \mathcal{N}(w|0, \tau^2 I)$

multivariate Gaussian

$$\tau^2 I = \begin{bmatrix} \tau^2 & & 0 \\ & \ddots & \\ 0 & & \tau^2 \end{bmatrix}$$

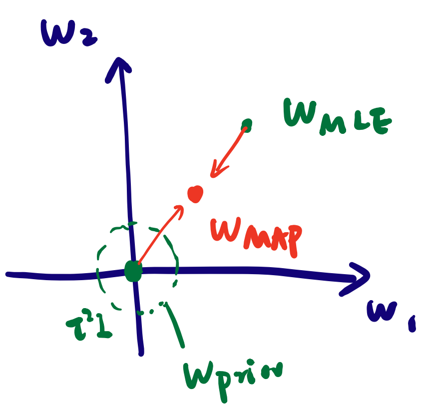
$p(w|D) \propto P(D|w,\sigma^2)\cdot P(w)$ Bayes Rule

$$= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(y_i - w^T x_i)^2\right)\cdot$$

$$\frac{1}{(2\pi)^{D/2}\tau}\exp\left(-\frac{1}{2\tau^2}w^Tw\right)$$

$$\hat{w}_{MAP} = \arg\max_{w} \sum_{i=1}^{N} -\frac{1}{2\sigma^2}(y_i - w^Tx_i)^2 - \frac{1}{2\tau^2}w^Tw$$

$$= \arg\min_{w} \sum_{i=1}^{N} (y_i - w^Tx_i)^2 + \boxed{\frac{\sigma^2}{\tau^2}w^Tw}$$



Ridge Regression L_2 regularization

$$\lambda = \frac{\sigma^2}{\tau^2}$$

$$\hat{w}_{MAP} = (X^TX + \lambda I)^{-1}X^Ty$$

$x = [15, 5, 8]$

$Y = [98, 62, 85]$

$$\underline{X} = \begin{bmatrix} 1 & 15 \\ 1 & 5 \\ 1 & 8 \end{bmatrix} \qquad Y = \begin{bmatrix} 98 \\ 62 \\ 85 \end{bmatrix} \qquad w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$\phi(x) = [1, x, x^2]$

$$\underline{X} = \begin{bmatrix} 1 & 15 & 225 \\ 1 & 5 & 25 \\ 1 & 8 & 64 \end{bmatrix} \qquad Y = \begin{bmatrix} 98 \\ 62 \\ 85 \end{bmatrix} \qquad w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$$

$w_1 = [0.30, 0.50]$

$$J(w) = \sum_{i=1}^{N} (y_i - w^T x_i)^2$$

$$= (y - Xw)^T (y - Xw)$$

$\phi = [1, x, x^2, x^3, x^4]$

$$w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

$$J(w) = (y - Xw)^T (y - Xw) + \lambda w^T w$$

$$= (y - Xw)^T (y - Xw) + \frac{\sigma^2}{\tau^2} w^T w$$

$$\lambda = \frac{\sigma^2}{\tau^2} = \frac{0.01}{0.01}$$