

Assignment

Name : Desireddy Sai Sankheerthana
Student ID : 50465523

Problem 1:

1(a):

(i) $f(n) = \log_2(n^3)$ $g(n) = 10 \log_2 \sqrt{n}$

$$3 \log_2 n$$

$$10 \log_2 n^{1/2}$$

$$3 \log_2 n$$

$$\frac{10}{2} \log_2 n$$

$$3 \log_2 n \leq 5 \log_2 n \Rightarrow f(n) \leq g(n) \Rightarrow O \Rightarrow f(n) = O(g(n))$$

ignore constants $\log_2 n = \log_2 n \Rightarrow f(n) = g(n) = \Theta \Rightarrow f(n) = \Theta(g(n))$

O	Ω	Θ
Yes	Yes	Yes

(ii) $f(n) = 5n^2 + n$ $g(n) = n \log n$

$$n^2 \geq n$$

$$f(n) \geq g(n) \Rightarrow f(n) = \Omega(g(n))$$

O	Ω	Θ
No	Yes	No

(iii) $f(n) = 10n^2 + n + 10$ $g(n) = n^3$

taking highest coefficient in $f(n)$

$$n^2 \leq n^3$$

$$f(n) \leq g(n)$$

$$f(n) = O(g(n))$$

O	Ω	Θ
Yes	No	No

(iv) $f(n) = e^n$ $g(n) = 2^{2n}$

$$n = 1$$

$$e^1 \leq 2^2$$

$\therefore e^1$ value is 2.71

$$2.71 \leq 4$$

$$f(n) \leq g(n)$$

$$f(n) = O(g(n))$$

O	Ω	Θ
Yes	No	No

$$1(b): \quad [10n\sqrt{n}] = O(n\sqrt{n})$$

$$f(n) = [10n\sqrt{n}]$$

if we want to prove using the definition of O -notation

~~it~~ ~~should~~ to satisfy $O(n\sqrt{n})$ it should ~~$f(n) \leq c \cdot g(n)$~~

Satisfy $f(n) \leq c \cdot g(n)$ where $g(n) = n$

So for every $c \geq 1$ and $n \geq 1$ the value of $f(n)$

will always be less than $c \cdot g(n)$

hence we can say $f(n) \leq c \cdot g(n) \Rightarrow O(n)$

problem 2 :

undirected-cycle-graph(v , visited, parent)

mark vertex(v) as "visited" [visited[v] = True]

add v to visited-vertices-list [visited-vertices-list.append(v)]

for all neighbours x of v do

if x is equal to parent then

continue

if x is "not visited" then

undirected-cycle-graph(x , visited, v)

~~else~~

else if x is not equal to parent then

return true

return false.

problem 3:

$\text{visited}[v] \leftarrow \text{true}$, $\text{top} \leftarrow \text{top} + 1$, $\text{stack}[\text{top}] \leftarrow v$, $\text{pos}[v] \leftarrow \text{top}$

for every outgoing edges (v, u) of v do

if $\text{pos}[u] \neq 0$ then

exit the whole algorithm, by returning the cycle $\text{stack}[\text{pos}[u] \dots \text{top}]$

elseif $\text{visited}[u] = \text{false}$ then

DFS(u)

$\text{pos}[v] \leftarrow 0$ $\text{top} \leftarrow \text{top} - 1$

the DFS for a vertex v have running time $O(1 + d^{\text{out}}(v))$, where $d^{\text{out}}(v)$ is the out-degree of v . Every v will be handled in atmost one iteration of DFS so the running time will be

$O(n+m)$, $n = O(m)$