CSE 431/531: Algorithm Analysis and Design Spring 2022

Homework 5

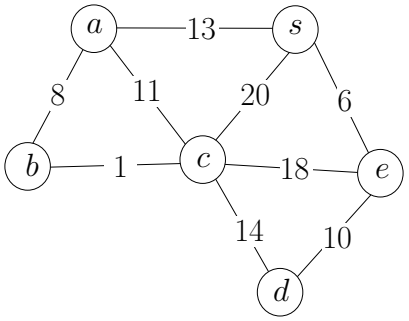
Instructor: Shi Li **Deadline: 5/4/2022**

Your Name: _____ Your Student ID: _____

Problems	1	2	3	4	Total
Max. Score	25	25	25	25	80
Your Score					

The total score for the 4 problems is 100, but your score will be truncated at 80.

Problem 1 Consider the following graph G with non-negative edge weights.



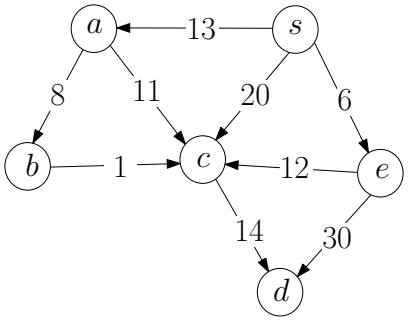
Use Prim's algorithm to compute the minimum spanning tree of G. You need to give the minimum spanning tree and its weight.

You need to use the following table to describe the execution of the algorithm. If $d[v] = \infty$, then $\pi[v] =$"$\perp$". Also, when a vertex v has been added to S, you can leave its d and π values empty, to make the table clean. But it is not required to do so.

iteration	vertex added to S	a		b		c		d		e	
		d	π	d	π	d	π	d	π	d	π
1	s	13	s	∞	$\perp$	20	s	∞	$\perp$	6	s
2											
3											
4											
5											
6											

Table 1: Prim's Algorithm for Minimum Spanning Tree

Problem 2 Consider the following directed graph G with non-negative edge weights. Use Dijkstra's algorithm to compute the shortest paths from s to all other vertices in G.



You need to fill the following table. When $d[v] = \infty$, we set $\pi[v] =$"$\perp$". Also, when a vertex v has been added to S, you can leave its d and π values empty, to make the table clean. But it is not required to do so.

iteration i	vertex added to S in iteration i	a		b		c		d		e	
		d	π	d	π	d	π	d	π	d	π
1	s	13	s	∞	$\perp$	20	s	∞	$\perp$	6	s
2											
3											
4											
5											
6											

Table 2: Dijkstra's algorithm for Shortest Paths

Problem 3 We are given an undirected graph $G = (V, E)$ with non-negative edge weights $(w_e)_{e \in E}$. Assume all the weights are different.

(3a) Let T be the unique minimum spanning tree of G. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then T is still the unique minimum spanning tree of G. Justify your answer.

(3b) Let s and t be two distinct vertices in V. Let P be the unique shortest path from s to t. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then P is still the unique shortest path from s to t. Justify your answer.

By justifying your answer, we mean the following: If the answer is yes, you need to give a proof. If your answer is no, you need to give a counter-example.

Problem 4 Suppose there is a negative cycle over a directed graph $G = (V, E)$ with edge weights $w : \mathbb{E} \to \mathbb{R}$. Then show that for any array $d : V \to \mathbb{R}$ over vertices, there exists some edge $(u, v) \in E$ such that $d[u] + w(u, v) < d[v]$.

(So if we do not put an upper bound on the number of iterations the Bellman-Ford algorithm runs, it will run forever when there is a negative cycle.)