Your Name: _____   Your Student ID: _____

Problems	1	2	3	Total
Max. Score	24	24	32	80
Your Score				

Problem 1   For each of the following 4problems, state (i) whether the problem is known to be in NP, and (ii) whether the problem is known to be in Co-NP. For problems (1b) and (1d), if your answer for (i) (or (ii)) is yes, you need to give the certificate and the certifier that establishes that the problem is in NP (or Co-NP).

(1a) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether G contains an independent set of size s.

(1b) Given two circuits C_1 and C_2, each with m input variables $z_1, z_2, \cdots, z_m$, decide if the two circuits compute the same function. That is, whether C_1 and C_2 give the same output for every boolean assignment of z-variables.

(1c) Given a graph $G = (V, E)$, decide if G is 3-colorable.

(1d) Given a graph $G = (V, E)$, decide if G is 2-colorable.

Problem 2   Let NPC be the set of NP-Complete problems. Prove the following statements:

- If $P \neq NP$, then $P \cap NPC = \emptyset$.
- If $P = NP$ then $P = Co\text{-}NP$.

Problem 3   In the class, you learned that we can solve shortest path problem on graphs with negative weights, with one caveat: If the graph contains a negative cycle, we can not force the algorithm to find the *shortest simple path* from s to t (a simple path is a path that does not visit a vertex twice).

In this problem, you need to prove that HP $\leq_P$ Shortest-Simple-Path, where Shortest-Simple-Path is the shortest simple path problem, and HP is the Hamiltonian path problem. (Though we did not prove this in class, it is known that HP is NP-complete. So this suggests Shortest-Simple-Path is also NP-complete.)

You can assume the Shortest-Simple-Path problem is a decision problem: we are given a graph $G = (V, E)$ with (possibly negative) edge weights $w : E \to \mathbb{R}$, two different vertices $s, t \in V$ and a threshold $L \in \mathbb{R}$. We need to decide if the shortest simple path from s to t in G has weight at most L or not.

Also, recall that in the HP problem, we are given a graph $G = (V, E)$ and two different vertices s and t, and we need to output if there is a Hamiltonian path between s and t in G or not.