CSE 431/531: Algorithm Analysis and Design Spring 2022

# Homework 6

*Instructor: Shi Li* **Deadline: No**

Your Name: _____ Your Student ID: _____

**Problem 1** For each of the following 5 problems, state (i) whether the problem is known to be in NP, and (ii) whether the problem is known to be in Co-NP. For problems (1c), (1d) and (1e), if your answer for (i) (or (ii)) is yes, you need to give the certificate and the certifier that establishes that the problem is in NP (or Co-NP).

(1a) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether $G$ contains an independent set of size $s$.

(1b) Given two circuits $C_1$ and $C_2$, each with $m$ input variables $z_1, z_2, \cdots, z_m$, decide if the two circuits compute the same function. That is, whether $C_1$ and $C_2$ give the same output for every boolean assignment of $z$-variables.

(1c) Given a graph $G = (V, E)$, decide if $G$ is 3-colorable.

(1d) Given a graph $G = (V, E)$, decide if $G$ is 2-colorable.

(1e) An undirected graph $G = (V, E)$ is called a 1-expander if for every $U \subseteq V$, the number of edges between $U$ and $V \setminus U$ in $G$ is at least $\min\{|U|, |V \setminus U|\}$. Given a graph $G$, decide if $G$ is a 1-expander.

**Problem 2** Let NPC be the set of NP-Complete problems. Prove the following statements:

- If P $\neq$ NP, then P $\cap$ NPC $= \emptyset$.
- If P = NP, then P = NPC.
- If P = NP then P = Co-NP.