High accuracy determination of rheological properties of drilling fluids using the Marsh funnel

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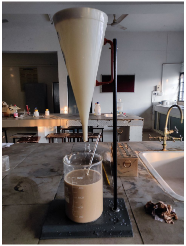
Introduction



- At present, the determination of the rheological properties—apparent viscosity, plastic viscosity and yield stress—of drilling fluids is very complex.
- For efficient drilling operations, these properties are continuously determined and monitored in the oil field using a ZNN-6 viscometer.
- Although highly accurate, this viscometer has a complex structure, involves tedious calculations, is costly and cannot function without a power source.
- In contrast, the Marsh funnel—a device with a simple structure, convenient operation, economic price and requiring no power source—determines the Marsh Funnel Viscosity (MFV) of drilling fluids in very little time.
- But, MFV is considered to be purely empirical and of no fundamental significance, since it is only a rough indicator of viscosity and hence cannot characterise actual rheology.

Background & motivation

- Till now, a series of mathematical models have been proposed to characterise actual fluid rheology using the Marsh funnel, but each of these models have led to some shortcomings.
- With growing computational power and complexity of processes, the CFD modelling approach of fluid flows is becoming an integral part of the chemical and petroleum industry.
- Very few accurate and efficient CFD models have been developed to determine the velocity and pressure distributions of the fluid flow inside the Marsh funnel.
- The preexisting models only deal with two-phase flows.

Based on these, we set and achieved the following objectives...

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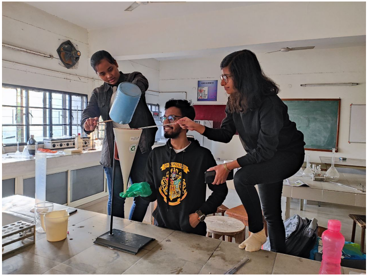
Objectives achieved & future work



What we did?

1. Relevant literature survey.
2. Developed a simple, efficient and economic mathematical model to calculate the rheological properties with high accuracy (average systematic errors less than 5%).
3. Verified the present model using experiments.
4. Validated the present model with ZNN-6 viscometer measurements, Li et al. (2020) model, Sedaghat (2017) model and Guria et al. (2013) model.

What we still wish to do?

1. Improve the existing CFD methods to determine the velocity and pressure field of the fluid volume fraction inside the Marsh funnel (average systematic errors less than 2%) for two-phase non-Newtonian drilling fluids.
2. Simulate a dynamic 3D model of the flowing fluid inside the Marsh funnel.
3. Extend the above method to non-Newtonian fluid flows with multiphases.

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Mapping of COs and POs

Our project maps the program objectives (POs) and course objectives (COs) as follows:

POs & COs	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	S	S	W	S	S	M	S	S	S	S	M	M	M	S
CO2	S	S	S	S	S	S	S	M	S	S	M	M	M	S
CO3	S	M	M	S	S	M	S	S	S	S	M	M	M	S

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Classification of fluids

- τ_w: Average wall shear stress
- $\dot{\gamma}_w$: Average wall shear rate

For drilling fluids, the Herschel-Bulkley model is recommended:

$$\text{Herschel-Bulkley model: } \tau_w = \tau_0 + \eta \dot{\gamma}_w^n \tag{1}$$

where τ_0 is the yield point, η is the consistency index, and n is the flow index.

Type	**Fluid name**	**Yield point**	**Flow index**
1	Shear thinning (pseudoplastic)	$\tau_0 = 0$	$n < 1$
2	Newtonian	$\tau_0 = 0$	$n = 1$
3	Shear thickening (dilatants)	$\tau_0 = 0$	$n > 1$
4	Shear thinning with yield stress	$\tau_0 > 0$	$n < 1$
5	Bingham plastic	$\tau_0 > 0$	$n = 1$
6	Shear thickening with yield stress	$\tau_0 > 0$	$n > 1$

Traditional determination of rheological parameters

Rheological parameters:

- Apparent viscosity: μ_a
- Plastic viscosity: μ_p
- Yield point: τ_0
- Determination using R1-B1-F1 combination of the ZNN-6 viscometer:

$$\mu_a(\text{mPa s}) = \frac{\theta_{600}}{2} \tag{2}$$

$$\mu_p(\text{mPa s}) = \theta_{600} - \theta_{300} \tag{3}$$

$$\tau_0(\text{Pa}) = \theta_{300} - \mu_p = 2\theta_{300} - \theta_{600} \tag{4}$$

where θ_ω is the dial reading at the Fann rotational speed of ω rpm.

- Determination using Marsh funnel:

$$\mu_a(\text{mPa s}) = \frac{\tau_{1020}}{1020} \times 1000 = 0.980392(\tau_{1020}) \tag{5}$$

$$\mu_p(\text{mPa s}) = \frac{\tau_{1020} - \tau_{510}}{1020 - 510} \times 1000 = 1.9607843(\tau_{1020} - \tau_{510}) \tag{6}$$

$$\tau_0(\text{Pa}) = \frac{3}{4}(\mu_a - \mu_p) = 1.4705882(\tau_{510}) - 0.7352942(\tau_{1020}) \tag{7}$$

where τ_γ (in Pa) is the average wall shear stress at the average wall shear rate $\dot{\gamma} = \gamma$ (in s^{-1}).

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Standard Marsh funnel dimensions

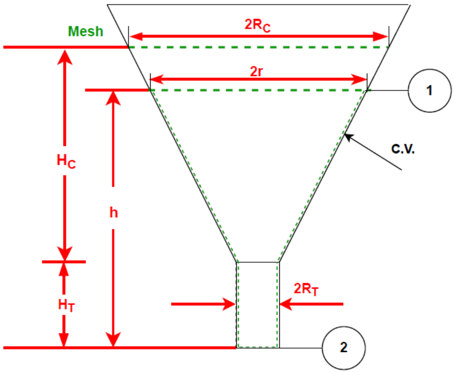

Figure: The schematic geometry of the Marsh funnel

The dimensions of a standard Marsh funnel are as follows:

Dimension	In inches	In metres
Radius of cone at mesh (R_C)	2.75	0.06985
Height of cone at mesh (H_C)	11	0.2794
Radius of copper tube (R_T)	$\frac{3}{32}$	0.00238125
Height of copper tube (H_T)	2	0.0508

Fluid volume in the Marsh funnel

- Relation between the *liquid surface radius*, r, and the *fluid/temporal height*, h:

$$r = \begin{cases} b_1 h - b_2 & , \ h \geq H_T \\ R_T & , \ h \leq H_T \end{cases} \tag{8}$$

where

$$b_1 = \frac{R_C - R_T}{H_C} \text{ and } b_2 = H_T \left(\frac{R_C - R_T}{H_C} \right) - R_T .$$

- Relation between the *fluid volume*, V_{CV}, and the fluid height:

$$V_{CV} = \begin{cases} a_3 h^3 + a_2 h^2 + a_1 h + a_0 & , \ h \geq H_T \\ \pi R_T^2 h & , \ h \leq H_T \end{cases} \tag{9}$$

where

$$a_3 = \frac{\pi b_1^2}{3}, \ a_2 = \frac{\pi}{3}(R_T b_1 - 2 b_1 b_2 - H_T b_1^2), \ a_1 = \frac{\pi}{3}(b_2^2 + R_T^2 + 2 H_T b_1 b_2 - R_T H_T b_1 - R_T b_2),$$

$$\text{and } a_0 = \left[\frac{\pi}{3}(R_T H_T b_2 - H_T b_2^2 - R_T^2 H_T) + \pi R_T^2 H_T \right] .$$

- Total volume of the Marsh funnel:

$$V = a_3 (H_C + H_T)^3 + a_2 (H_C + H_T)^2 + a_1 (H_C + H_T) + a_0 = 1.576 \text{ litres.}$$

Discharge rate, discharge time and flow factor

- Continuity equation for the deforming control volume:

$$\begin{cases} (3a_3 h^2 + 2a_2 h + a_1)\frac{dh}{dt} = -Q_o & , \; h \geq H_T \\ A_T \frac{dh}{dt} = -Q_o & , \; h \leq H_T \end{cases} \tag{10}$$

  where $Q_o = A_T f \sqrt{2gh}$ is the outlet discharge rate and f is the *flow factor*.

- Relation between the time, t, required to change the fluid height from $H_C + H_T$ to h:

$$t = \begin{cases} -\frac{1}{A_T f \sqrt{2g}} \int_{H_C+H_T}^{h} (3a_3 h^{3/2} + 2a_2 h^{1/2} + a_1 h^{-1/2}) dh & , \; h \geq H_T \\ -\frac{1}{A_T f \sqrt{2g}} \int_{H_C+H_T}^{H_T} (3a_3 h^{3/2} + 2a_2 h^{1/2} + a_1 h^{-1/2}) dh - \frac{1}{f\sqrt{2g}} \int_{H_T}^{h} h^{-1/2} dh & , \; h \leq H_T \end{cases} \tag{11}$$

- Relation between the *discharge time*, t_d, and the fluid height, h:

$$t_d \text{ (in s)} = t(0) - t(h) = \begin{cases} (c_1 + c_2 h^{1/2} + c_3 h^{3/2} + c_4 h^{5/2})/f & , \; h \geq H_T \\ (d_1 + d_2 h^{1/2})/f & , \; h \leq H_T \end{cases} \tag{12}$$

- For any fluid, the *final discharge time*, t_F is given by:

$$\begin{aligned} t_F &= t_d(H_T) - t_d(H) \\ &= \frac{1}{f}\left[c_2\left(H_T^{1/2} - H^{1/2}\right) + c_3\left(H_T^{3/2} - H^{3/2}\right) + c_4\left(H_T^{5/2} - H^{5/2}\right)\right] \\ &= \alpha/f \end{aligned} \tag{13}$$

- So, f can be directly calculated by experimentally determining the final discharge time t_F using

$$f = \frac{\alpha}{t_F}$$

Rheological parameters

- The average wall shear stress on the Marsh funnel:

$$\tau_w = \frac{1}{2}\rho g h(1 - f^2)\frac{R_T}{H_T} \tag{15}$$

- The average wall shear rate on the Marsh funnel:

$$\text{Newtonian:} \quad -\dot{\gamma}_w = \frac{7\sqrt{2g}}{4R_T\sqrt{H_T}} fh = 14435.41372fh \text{ (in s}^{-1}\text{)} \tag{16}$$

$$\text{Nonweighted non-Newtonian:} \quad -\dot{\gamma}_w = \frac{7\sqrt{2g}}{4R_T H_T} fh^{\frac{3}{2}} = 64046.79164fh^{3/2} \text{ (in s}^{-1}\text{)} \tag{17}$$

$$\text{Weighted non-Newtonian:} \quad -\dot{\gamma}_w = \frac{7\sqrt{g}}{8R_T H_T} fh^{\frac{3}{2}} = 22643.96034fh^{3/2} \text{ (in s}^{-1}\text{)} \tag{18}$$

- For Newtonian fluids:

$$\mu = 1.59114 \times 10^{-2}\rho\left(\frac{1 - f^2}{f}\right) \text{ (in mPa/s)} \tag{19}$$

- For nonweighted non-Newtonian fluids:

$$\mu_a = 0.0142541\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa/s)} \tag{20}$$

$$\mu_p = 0.0105493\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa/s)} \tag{21}$$

$$\tau_0 = 3.7048 \times 10^{-3}\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa)} \tag{22}$$

Rheological parameters contd...

- For weighted non-Newtonian fluids:

$$\mu_a = 0.0285082\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa/s)} \tag{23}$$

$$\mu_p = 0.0211\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa/s)} \tag{24}$$

$$\tau_0 = 7.4082 \times 10^{-3}\rho(1 - f^2)f^{-\frac{2}{3}} \text{ (in mPa)} \tag{25}$$

- Using only f and ρ of the fluid, the rheological properties of all fluids can be determined:
  - μ of any Newtonian fluid $\rightarrow$ equation (19).
  - μ_a, μ_p and τ_0 of any nonweighted non-Newtonian fluid $\rightarrow$ equations (20), (21) and (22) respectively.
  - μ_a, μ_p and τ_0 of any weighted non-Newtonian fluid $\rightarrow$ equations (23), (24) and (25) respectively.

Contents

Analysis for Newtonian fluids

- **M1**: ZNN-6 viscometer measurements
- **M2**: Present model
- **M3**: Li et al. (2020) model
- **M4**: Guria et al. (2013) model
- **M5**: Sedaghat (2017) model

Fluid	**M1 & M2**	**M1 & M3**	**M1 & M4**	**M1 & M5**
Mineral oil	0.54%	1.52%	23.69%	1.52%
Synthetic oil	0.16%	1.64%	29.18%	1.64%
Fuel oil	0.82%	1.03%	34.64%	1.03%
Engine oil	0.05%	1.08%	11.05%	1.08%
Glycerin	0.38%	1.46%	9.25%	1.46%
Average	**0.06%**	**1.35%**	**21.56%**	**1.35%**

Table: Comparison of systematic errors in dynamic viscosity obtained using different models

Conclusions for Newtonian fluids:

- The systematic errors between the present model and ZNN-6 viscometer measurements are small.
- They are lesser than the systematic errors between the Li et al. (2020) model and the ZNN-6 viscometer measurements.
- The average value is 0.06% and all systematic errors are less than 1% with the lowest error being 0.05 %.
- Therefore, our model closely matches with M1, M3 and M5 for Newtonian fluids.
- By contrast, the average systematic error between the Guria et al. (2013) model and the ZNN-6 viscometer measurements is relatively large.

Non-Newtonian test fluid data

Fluid	Composition	Density (kg/cm^3)	Flow factor f	t_F (s)	Weighted/Non-weighted
Fluid 1	4.0% bentonite + 0.2% XC	1040.0	0.418	90.25	Non-weighted
Fluid 2	4.0% bentonite + 0.2% XC + 10.0% barite	1125.0	0.502	75.10	Weighted
Fluid 3	4.0% bentonite + 0.2% XC + 10.0% calcium carbonate	1125.0	0.509	74.08	Weighted
Fluid 4	10.0% PEG + 20.0% sodium chloride	1035.0	0.708	53.29	Non-weighted
Fluid 5	10.0% PEG + 20.0% sodium chloride + 10.0% barite	1119.0	0.766	49.28	Weighted
Fluid 6	10.0% PEG + 20.0% sodium chloride + 10.0% calcium carbonate	1119.0	0.789	47.81	Weighted
Fluid 7	6.0% bentonite + 0.5% PAC LV + 0.1% XC	1050.0	0.558	66.99	Non-weighted
Fluid 8	6.0% bentonite + 0.5% PAC LV + 0.1% XC + 10.0% barite	1135.0	0.689	56.20	Weighted
Fluid 9	6.0% bentonite + 0.5% PAC LV + 0.1%XC + 10.0% calcium carbonate	1135.0	0.715	54.73	Weighted
Fluid 10	0.15% sodium hydroxide + 0.5% PAC LV + 2.0% sodium silicate + 0.2% XC	1015.0	0.369	102.36	Non-weighted
Fluid 11	0.15% sodium hydroxide + 0.5% PAC LV + 2.0% sodium silicate + 0.2% XC + 10.0% barite	1065.0	0.503	74.95	Weighted
Fluid 12	0.15% sodium hydroxide + 0.5% PAC LV + 2.0% sodium silicate + 0.2% XC + 10.0% calcium carbonate	1065.0	0.472	79.86	Weighted

Table: 12 non-Newtonian (drilling) fluids and their composition

Results for non-Newtonian fluids (μ_a & μ_p)

Fluid	μ_a (mPa s)					μ_p (mPa s)				
	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
Fluid 1	20.00	21.50	21.88	32.76	21.88	14.00	15.92	15.35	38.92	15.35
Fluid 2	37.50	37.45	38.08	46.14	19.04	27.00	27.72	28.18	51.27	7.27
Fluid 3	38.25	36.87	36.09	39.28	19.05	27.50	27.29	26.71	43.80	12.66
Fluid 4	9.50	8.99	8.53	14.29	8.53	6.00	6.66	5.99	12.63	5.99
Fluid 5	16.50	15.21	14.55	26.11	7.28	12.00	11.26	10.77	33.74	5.10
Fluid 6	14.50	13.55	13.20	22.35	6.60	10.50	10.03	9.77	29.09	4.63
Fluid 7	15.00	15.64	16.08	30.34	16.08	10.00	11.58	11.28	26.82	11.28
Fluid 8	22.50	21.40	20.62	28.21	10.31	15.00	15.84	15.26	35.94	7.23
Fluid 9	21.00	19.24	18.71	27.38	9.36	14.50	14.24	13.85	38.10	6.56
Fluid 10	27.00	24.29	24.16	39.26	24.16	18.00	17.98	16.95	47.93	16.95
Fluid 11	38.00	35.36	35.29	49.70	17.65	26.50	26.17	26.12	52.75	12.38
Fluid 12	40.00	38.41	37.42	47.35	18.71	29.00	28.43	27.69	58.60	13.12

Table: Apparent viscosity and plastic viscosity of the 12 drilling fluids obtained using different models

Results for non-Newtonian fluids (τ_0)

Fluid no.	τ_0 **(mPa)**				
	M1	**M2**	**M3**	**M4**	**M5**
Fluid 1	6.86	5.68	6.53	-	4.90
Fluid 2	11.88	9.86	9.90	-	8.83
Fluid 3	11.33	9.67	9.38	-	4.79
Fluid 4	3.61	2.40	2.53	1.25	1.91
Fluid 5	4.60	4.09	3.78	-	1.64
Fluid 6	4.63	3.66	3.43	-	1.48
Fluid 7	5.83	3.95	4.80	2.64	3.60
Fluid 8	7.61	5.65	5.36	-	2.31
Fluid 9	6.67	5.13	4.86	-	2.10
Fluid 10	10.31	6.32	7.21	-	5.41
Fluid 11	12.02	9.30	9.17	-	3.95
Fluid 12	11.68	10.10	9.73	-	4.19

Table: Yield point of the 12 drilling fluids obtained using different models

Comparison of systematic errors in non-Newtonian fluids

Fluid	M1 & M2			M1 & M3			M1 & M5		
	μ_a	μ_p	τ_0	μ_a	μ_p	τ_0	μ_a	μ_p	τ_0
Fluid 1	9.35%	15.57%	17.20%	9.40%	9.64%	4.81%	9.40%	9.64%	28.57%
Fluid 2	1.15%	3.96%	17.00%	1.54%	4.37%	16.67%	49.23%	73.07%	25.67%
Fluid 3	2.67%	0.22%	14.65%	5.64%	2.87%	17.21%	50.20%	53.96%	57.72%
Fluid 4	2.53%	14.17%	33.52%	10.21%	0.17%	29.92%	10.21%	0.17%	47.09%
Fluid 5	4.48%	2.83%	11.09%	11.82%	10.25%	17.83%	55.88%	57.50%	64.35%
Fluid 6	2.90%	0.76%	20.95%	8.97%	6.95%	25.92%	54.48%	55.90%	68.03%
Fluid 7	1.27%	12.40%	32.25%	7.20%	12.80%	17.67%	7.20%	12.80%	38.25%
Fluid 8	3.38%	7.27%	25.75%	8.36%	1.73%	29.57%	54.18%	51.80%	69.65%
Fluid 9	5.90%	0.83%	23.09%	10.90%	4.48%	27.13%	55.43%	54.76%	68.52%
Fluid 10	9.96%	0.05%	38.70%	10.52%	5.83%	30.07%	10.52%	5.83%	47.52%
Fluid 11	5.76%	0.00%	22.63%	7.13%	1.43%	23.71%	53.55%	53.28%	67.14%
Fluid 12	2.82%	0.79%	8.84%	6.45%	4.52%	16.70%	53.23%	54.76%	64.13%
Average	**2.39%**	**4.16%**	**22.14%**	**8.18%**	**5.42%**	**21.43%**	**38.62%**	**40.29%**	**53.89%**

Table: Comparison of systematic errors in μ_a, μ_p and τ_0 of the 12 drilling fluids obtained using different models

Analysis for non-Newtonian fluids and overall analysis

Conclusions for non-Newtonian fluids:

- The average systematic error of μ_a between M1 and the present model is 2.39%; the maximum value is 9.96% (Fluid 10) and the minimum value is 1.15% (Fluid 2).
- The systematic error in μ_p using the present model ranges between 0.00% and 15.57%, with an average of 4.16%.
- μ_a and μ_p determined using the present model match the M1 measurements, and the average systematic error is less than 5%.
- The average systematic error of μ_a and μ_p obtained using the present model is lesser than that of Li et al. (2020) model, 8.18% and 5.42% respectively.
- However, the systematic error in τ_0 is 22.14%, more than that obtained using M3 (21.43%),
- The present model is somewhat inaccurate in terms of calculating τ_0 as compared to that of M3.
- Although, this model's results do not remarkably differ from the ZNN-6 viscometer measurements.

Overall conclusions:

- The obtained results show consistency between the present model and M1 and M3 for the Newtonian and non-Newtonian fluids studied.
- The proposed mathematical models for determining μ, μ_a, μ_p and τ_0 are concise and accurate, and we need to measure only ρ and t_F.
- Hence, in addition to M3, the present model is available for industrial applications.

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Thank you