# Hydrodynamic Interactions of Microswimmers Near Boundaries

by

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# Chapter 1

# Hydrodynamics of self-propulsion near a boundary

We consider the hydrodynamic interactions of a microswimmer with a nearby smooth, planar wall, $S_w$, the $x$-$y$ plane or the $z = 0$ plane, see Fig. 1.1. The active agent is suspended in an incompressible, low-Reynolds number flow with fluid viscosity $\mu$. The quasi-steady fluid velocity $\boldsymbol{u}(\boldsymbol{r})$ and pressure fields $p(\boldsymbol{r})$ are described by the continuity and Stokes equations,

$$\text{Continuity equation: } \nabla \cdot \boldsymbol{u} = 0, \tag{1.1}$$

$$\text{Stokes equation: } \mu \nabla^2 \boldsymbol{u} = \nabla p. \tag{1.2}$$

The fluid velocity on the surface of these microswimmers can be decomposed into a translational velocity $\mathbf{U}$, flow induced by the body rotation at angular velocity, $\boldsymbol{\Omega}$, and a disturbance velocity field $\boldsymbol{u}_S$, which can vary locally over the particle surface, $S_p$. In the laboratory frame the boundary conditions of a microswimmer near a plane wall are

$$\boldsymbol{u}(\boldsymbol{r}) = \boldsymbol{u}_S(\boldsymbol{r}) + \mathbf{U}(\boldsymbol{r}) + (\boldsymbol{r} \times \boldsymbol{\Omega})(\boldsymbol{r}), \ \boldsymbol{r} \in S_p, \tag{1.3}$$

$$\boldsymbol{u}(\boldsymbol{r}) = \mathbf{0}, \ \boldsymbol{r} \in S_w \cup S_\infty, \tag{1.4}$$

where $S_\infty$ denotes the bounding surface at infinity.

We model the microswimmer to be spherical with radius $a$ and located at the position $\boldsymbol{r}_0 = [x_s, y_s, h]$ (position of the centre of the sphere or the microswimmer's centroid). Since here the flow field in close proximity of the swimmer is not of interest but the far-field behaviour is, we will use the mathematical framework of the *far-field description* of the swimmer. Physically, it is equivalent to zooming out and observing the swimmer

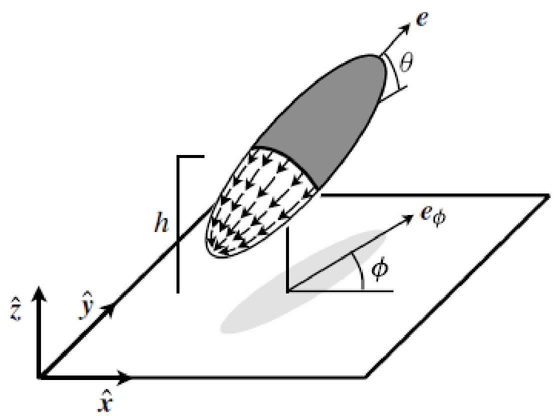


**Figure 1.1:** A model swimmer (prolate ellipsoid) near a wall. The body swims in a direction $\hat{\boldsymbol{e}}$, which may be decomposed into a horizontal part along $\hat{\boldsymbol{e}}_\phi$ and a vertical part along $\hat{\boldsymbol{e}}_z$. The pitch angle $\theta = \cos^{-1}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}}_\phi)$ is indicated, as is the distance from the wall $h$ of the body centroid. The polar angle, $\phi$, denotes the angle of the horizontal swimming component relative to the $x$-axis, and for the present study any variations in $\phi$ are decoupled from the pitching and height dynamics of $\theta$ and $h$. Ignoring the effect of the rotlet dipole, we may set $\phi$ and $\hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_x$ (2D dynamics) without loss of generality.

over length scales much larger than its intrinsic length. We note that under this far-field approximation, the geometrical details of the swimmer are unimportant and hence can be used for generic low-Reynolds-number swimming.

To formulate such a perspective, it is useful first to introduce the Green's function for the Stokes equations, see Eqns. (1.1) and (1.2). This is obtained by placing a point force, $f\hat{\boldsymbol{e}}\boldsymbol{\delta}(\boldsymbol{r}, \boldsymbol{r}_0)$, in an otherwise quiescent infinite fluid, where $\boldsymbol{\delta}(\boldsymbol{r}, \boldsymbol{r}_0)$ is the Dirac delta function centered at $\boldsymbol{r}_0$, $\hat{\boldsymbol{e}}$ represents the direction of the point force, and $f$ the magnitude of the force. The forced Stokes equations are given by

$$\text{Continuity equation: } \nabla \cdot \boldsymbol{u} = 0, \tag{1.5}$$

$$\text{Forced Stokes equation: } \mu\nabla^2\boldsymbol{u} + f\hat{\boldsymbol{e}}\boldsymbol{\delta} = \nabla p. \tag{1.6}$$

The solution $\boldsymbol{u}(\boldsymbol{r})$ can be obtained by a variety of methods, such as Fourier transformation and superposition of vector harmonic functions, and is given by

$$\boldsymbol{u}(\boldsymbol{r}) = \alpha\mathbf{G}(\boldsymbol{r}, \boldsymbol{r}_0; \hat{\boldsymbol{e}}), \tag{1.7}$$

where the Green's function,

$$\mathbf{G}(\hat{\boldsymbol{e}}) \equiv \mathbf{G}(\boldsymbol{r}, \boldsymbol{r}_0; \hat{\boldsymbol{e}}) = \frac{\hat{\boldsymbol{e}} + (\hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{r}})\hat{\boldsymbol{r}}}{\hat{r}}, \tag{1.8}$$

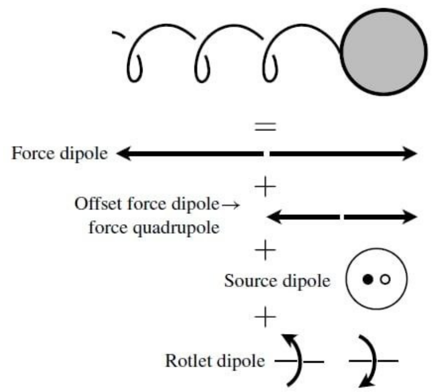


**Figure 1.2:** The fluid velocity far from a swimming *E. coli* is modelled at leading order as that of a Stokeslet dipole. At the next order, the flow in the far field varies due to the length asymmetry between the backward-pushing propeller and the forward-pushing cell body (producing a Stokeslet quadrupole), the finite size of the cell body (producing a source dipole), and the rotation of the flagellum and counter-rotation of the cell body (producing a rotlet dipole).

with $\hat{r} = |\boldsymbol{r} - \boldsymbol{r}_0|$ and $\hat{\boldsymbol{r}} = (\boldsymbol{r} - \boldsymbol{r}_0)/\hat{r}$. This fundamental singular solution $\boldsymbol{u}(\boldsymbol{r})$ is known as the *Stokeslet singularity* and $\alpha$ is known as the *Stokeslet singularity strength*, and is related to the point force magnitude $f$ and the dynamic viscosity $\mu$ via $\alpha = f/(8\pi\mu)$. Note that the Stokeslet decays as $1/\hat{r}$ and is therefore long-ranged. The flow field of a Stokeslet is the one given by a translating sphere in the far field as in both cases a net force is exerted on the fluid. Physically, as we increasingly zoom-out from a translating sphere, it becomes sufficiently small that it can be regarded as a point acted on by a force, and thus a Stokeslet may be physically understood as the far-field approximation of a translating sphere. Since a swimmer does not exert a net force on the surrounding fluid, only force dipoles and other higher-order singularities can be used to approximate the same and the flow field generated by it, as we now detail.

The above flow field generated in an unbounded domain, $\boldsymbol{u}(\boldsymbol{r})$, can be decomposed in terms of a multipole expansion (see Fig. 1.2),

$$\boldsymbol{u}(\boldsymbol{r}) = \boldsymbol{u}_{\mathrm{FD}}(\boldsymbol{r}) + \boldsymbol{u}_{\mathrm{SD}}(\boldsymbol{r}) + \boldsymbol{u}_{\mathrm{FQ}}(\boldsymbol{r}) + \boldsymbol{u}_{\mathrm{RD}}(\boldsymbol{r}) + \mathcal{O}(|\boldsymbol{r} - \boldsymbol{r}_0|^{-4}), \tag{1.9}$$

where the terms correspond to contributions of a force (Stokeslet) dipole (FD), $\boldsymbol{u}_{\mathrm{FD}} = \alpha_{FD}\mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$, source dipole (SD), $\boldsymbol{u}_{\mathrm{SD}} = \alpha_{SD}\mathbf{G}_{\mathrm{SD}}(\hat{\boldsymbol{e}})$, force (Stokeslet) quadrupole (FQ), $\boldsymbol{u}_{\mathrm{FQ}} = \alpha_{FQ}\mathbf{G}_{\mathrm{FQ}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$, and rotlet dipole (RD), $\boldsymbol{u}_{\mathrm{RD}} = \alpha_{RD}\mathbf{G}_{\mathrm{RD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$. In the preceding expressions, $\mathbf{G}_{\mathrm{FD}}$ and $\mathbf{G}_{\mathrm{FQ}}$ are higher-order singularity solutions of the forced Stokes equations (since the forced Stokes equations are linear, a directional derivative of any order of a Stokeslet is also a solution), $\mathbf{G}_{\mathrm{SD}}$ is a higher-order singularity solution of Laplace's equations (substituting $p = 0$ in Eqn. (1.2)), and $\mathbf{G}_{\mathrm{RD}}$ is a directional derivative of a rotlet (a combination of two force dipoles), and $\alpha_{\mathrm{FD}}$, $\alpha_{\mathrm{SD}}$, $\alpha_{\mathrm{FQ}}$ and $\alpha_{\mathrm{RD}}$ are the corresponding singularity strengths. We will discuss more on these later.

In an unbounded fluid, the fluid velocity in the far field generated by a microswimmer (or more generally, an active body) behaves as described in Eqn. (1.9). When a boundary such as a plane wall is present, however, the velocity everywhere is altered due to the additional boundary condition, Eqn. (1.4). Using the *method of images* adapted from electrostatics, the boundary condition on the surface can be satisfied by the placement of additional singularities at the image point (*image singularity*), $\boldsymbol{r}_0^* = \boldsymbol{r}_0 - 2(\boldsymbol{r}_0 \cdot \hat{\boldsymbol{e}}_z)\hat{\boldsymbol{e}}_z = \boldsymbol{r}_0 - 2h\hat{\boldsymbol{e}}_z$, inside the wall.

To cancel the effect of each Stokeslet singularity placed parallel or perpendicular to a no-slip wall, we require an image system consisting of a Stokeslet, a Stokes doublet, and a source dipole as image singularities. The image system for a 'tilted' Stokeslet (a Stokeslet directed at an angle relative to the wall) is simply a linear combination of the wall-parallel and wall-perpendicular image systems. The image systems for higher-order singularities, however, are not simply linear combinations of wall-parallel and wall-perpendicular image systems. The images for each of the axisymmetric singularities in Eqn. (1.9) will be denoted by an asterisk. For instance, the effect of a Stokeslet dipole, placed at some arbitrary point $\boldsymbol{r}_1$, along with its image system, placed at the image point $\boldsymbol{r}_1^*$, evaluated on the wall surface $z = 0$, returns

$$\mathbf{G}_{\mathrm{FD}}(z = 0, \boldsymbol{r}_1; \hat{\boldsymbol{e}}, \hat{\boldsymbol{e}}) + \mathbf{G}^*_{\mathrm{FD}}(z = 0, \boldsymbol{r}_1^*; \hat{\boldsymbol{e}}, \hat{\boldsymbol{e}}) = \mathbf{0}.$$

Likewise, we denote by $\boldsymbol{u}^*(\boldsymbol{r})$ the fluid velocity generated by the entire collection of image singularities needed to cancel, on the no-slip wall, the swimmer-generated velocity description in Eqn. (1.9). This leads us to the following expression of $\boldsymbol{u}^*(\boldsymbol{r})$, corresponding to Eqn. (1.9):

$$\boldsymbol{u}^*(\boldsymbol{r}) = \boldsymbol{u}^*_{\mathrm{FD}}(\boldsymbol{r}) + \boldsymbol{u}^*_{\mathrm{SD}}(\boldsymbol{r}) + \boldsymbol{u}^*_{\mathrm{FQ}}(\boldsymbol{r}) + \boldsymbol{u}^*_{\mathrm{RD}}(\boldsymbol{r}) + \mathcal{O}(|\boldsymbol{r} - \boldsymbol{r}_0^*|^{-4}), \tag{1.10}$$

where the terms correspond to contributions of the image system of corresponding higher-order singularities; $\boldsymbol{u}^*_{\mathrm{FD}} = \alpha_{FD}\mathbf{G}^*_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$, $\boldsymbol{u}^*_{\mathrm{SD}} = \alpha_{SD}\mathbf{G}^*_{\mathrm{SD}}(\hat{\boldsymbol{e}})$, $\boldsymbol{u}^*_{\mathrm{FQ}} = \alpha_{FQ}\mathbf{G}^*_{\mathrm{FQ}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$, and $\boldsymbol{u}^*_{\mathrm{RD}} = \alpha_{RD}\mathbf{G}^*_{\mathrm{RD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$.

Thus, the resultant velocity field induced by the swimmer, $\boldsymbol{u}^{(0)}$, can be decomposed into the velocity field generated in an unbounded domain, $\boldsymbol{u}$, and the disturbance velocity field due to the nearby wall (or equivalently, due to the image singularity system placed at the image point), $\boldsymbol{u}^*$, as follows:

$$\boldsymbol{u}^{(0)}(\boldsymbol{r}) = \boldsymbol{u}(\boldsymbol{r}) + \boldsymbol{u}^*(\boldsymbol{r}). \tag{1.11}$$

Before moving further, let us discuss a bit more on the higher order singularities, particularly, force dipole, force quadrupole, source dipole and rotlet dipole, and the corresponding singularity strengths.

The flow field of a force dipole, oriented along $\hat{\boldsymbol{e}}$, with two point forces, $f\hat{\boldsymbol{e}}$ and $-f\hat{\boldsymbol{e}}$, each at a distance of $l/2$ from $\boldsymbol{r}_0$ along some arbitrary direction $\hat{\boldsymbol{a}}$ can be obtained as

$$\begin{aligned}\boldsymbol{u}_{\mathrm{FD}} &= \alpha\left[\mathbf{G}\left(\boldsymbol{r},\boldsymbol{r}_0+\frac{l}{2}\hat{\boldsymbol{a}};\hat{\boldsymbol{e}}\right)+\mathbf{G}\left(\boldsymbol{r},\boldsymbol{r}_0-\frac{l}{2}\hat{\boldsymbol{a}};-\hat{\boldsymbol{e}}\right)\right] \\ &= \alpha\left[\mathbf{G}\left(\boldsymbol{r},\boldsymbol{r}_0+\frac{l}{2}\hat{\boldsymbol{a}};\hat{\boldsymbol{e}}\right)-\mathbf{G}\left(\boldsymbol{r},\boldsymbol{r}_0-\frac{l}{2}\hat{\boldsymbol{a}};\hat{\boldsymbol{e}}\right)\right] \\ &\simeq \alpha_{\mathrm{FD}}\hat{\boldsymbol{a}}\cdot\nabla_0\mathbf{G}(\hat{\boldsymbol{e}}),\end{aligned} \tag{1.12}$$

where the last relation hold true only for small $l$ and $\alpha_{\mathrm{FD}}=\alpha l$ is defined as the *force dipole strength*. The gradient $\nabla_0$ acts on the singularity position $\boldsymbol{r}_0=[x_s,y_s,h]$, that is,

$$\nabla_0=\left[\frac{\partial}{\partial x_s},\frac{\partial}{\partial y_s},\frac{\partial}{\partial h}\right].$$

This allows us to introduce the *force dipole singularity solution*, which is defined as the directional derivative of $\mathbf{G}(\hat{\boldsymbol{e}})$ along the predefined direction $\hat{\boldsymbol{a}}$

$$\mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})\equiv\mathbf{G}_{\mathrm{FD}}(\boldsymbol{r},\boldsymbol{r}_0;\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})=\hat{\boldsymbol{a}}\cdot\nabla_0\mathbf{G}(\hat{\boldsymbol{e}})=\left(\hat{\boldsymbol{a}}\cdot\nabla_0\right)\mathbf{G}(\hat{\boldsymbol{e}}). \tag{1.13}$$

Note that, $\nabla_0\mathbf{G}(\hat{\boldsymbol{e}})$ is a tensor, $\hat{\boldsymbol{a}}\cdot\nabla_0\mathbf{G}(\hat{\boldsymbol{e}})$ is a vector, and $\hat{\boldsymbol{a}}\cdot\nabla_0$ is a scalar. A Stokes dipole decays as $1/\hat{r}^2$, one order of magnitude faster than a Stokeslet. From the above description, one can infer that, physically, a Stokes dipole can be understood as the limit when the distance between two Stokeslets of equal magnitudes but opposite directions becomes vanishingly small and the strength is adjusted to give a finite flow field. Further, the following is referred to as the *stresslet singularity solution*:

$$\mathbf{G}_{\mathrm{S}}(\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})=\mathbf{G}_{\mathrm{S}}(\boldsymbol{r},\boldsymbol{r}_0;\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})=\frac{1}{8\pi\mu}\left[-\frac{(\hat{\boldsymbol{e}}\cdot\hat{\boldsymbol{a}})\boldsymbol{r}}{\hat{r}^3}+3\frac{(\hat{\boldsymbol{e}}\cdot\boldsymbol{r})(\hat{\boldsymbol{a}}\cdot\boldsymbol{r})\boldsymbol{r}}{\hat{r}^5}\right], \tag{1.14}$$

physically representing straining motion of the fluid, and the following is referred to as the *rotlet singularity*:

$$\mathbf{G}_{\mathrm{R}}(\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})=\mathbf{G}_{\mathrm{R}}(\boldsymbol{r},\boldsymbol{r}_0;\hat{\boldsymbol{e}},\hat{\boldsymbol{a}})\frac{(\hat{\boldsymbol{a}}\times\hat{\boldsymbol{e}})\times\boldsymbol{r}}{8\pi\mu\hat{r}^3}, \tag{1.15}$$

physically representing the flow due to a point torque, and we have:

$$\mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}) = \mathbf{G}_{\mathrm{S}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}) + \mathbf{G}_{\mathrm{R}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}). \tag{1.16}$$

Note that $\mathbf{G}_{\mathrm{S}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}})$ represents the flow due to the symmetric part of the Stokes dipole tensor and $\mathbf{G}_{\mathrm{R}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}})$ represents the flow due to the antisymmetric part.

Similar relations hold for the other oriented higher-order singularities. In particular, the *force quadrupole singularity solution* can be obtained by taking the directional derivative of $\mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}})$ along some predefined direction $\hat{\boldsymbol{b}}$ as follows:

$$\mathbf{G}_{\mathrm{FQ}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) \equiv \mathbf{G}_{\mathrm{FQ}}(\boldsymbol{r}, \boldsymbol{r}_0; \hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) = \hat{\boldsymbol{b}} \cdot \nabla_0 \mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}) = \left(\hat{\boldsymbol{b}} \cdot \nabla_0\right) \mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{a}}). \tag{1.17}$$

The source dipole singularity solution can be expressed in terms of the Stokeslet solution via

$$\mathbf{G}_{\mathrm{SD}}(\hat{\boldsymbol{e}}) \equiv \mathbf{G}_{\mathrm{SD}}(\boldsymbol{r}, \boldsymbol{r}_0; \hat{\boldsymbol{e}}) = -\frac{1}{2} \nabla_0^2 \mathbf{G}(\hat{\boldsymbol{e}}). \tag{1.18}$$

Alternatively, it is the directional derivative of a *source singularity*, the fundamental solution of the Laplace's equations, given by

$$\mathbf{G}_{\mathrm{S}}(\boldsymbol{r}, \boldsymbol{r}_0) = \frac{\hat{\boldsymbol{r}}}{\hat{r}^2}, \tag{1.19}$$

and hence, the source dipole is given by taking the direction derivative of $\mathbf{G}_{\mathrm{S}}(\boldsymbol{r}, \boldsymbol{r}_0)$ along $\hat{\boldsymbol{e}}$:

$$\mathbf{G}_{\mathrm{SD}}(\hat{\boldsymbol{e}}) = \hat{\boldsymbol{e}} \cdot \nabla_0 \mathbf{G}_{\mathrm{S}}(\boldsymbol{r}, \boldsymbol{r}_0) = (\hat{\boldsymbol{e}} \cdot \nabla_0)\, \mathbf{G}_{\mathrm{S}}(\boldsymbol{r}, \boldsymbol{r}_0). \tag{1.20}$$

Finally, the *rotlet dipole singularity solution* is obtained by taking the directional derivative of a rotlet along $\hat{\boldsymbol{c}}$ :

$$\mathbf{G}_{\mathrm{RD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{c}}) \equiv \mathbf{G}_{\mathrm{RD}}(\boldsymbol{r}, \boldsymbol{r}_0; \hat{\boldsymbol{e}}, \hat{\boldsymbol{c}}) = \hat{\boldsymbol{c}} \cdot \nabla_0 \mathbf{G}_{\mathrm{R}}(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) = (\hat{\boldsymbol{c}} \cdot \nabla_0)\, \mathbf{G}_{\mathrm{R}}(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}), \tag{1.21}$$

with unit vectors $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ satisfying $\hat{\boldsymbol{a}} \times \hat{\boldsymbol{b}} = \hat{\boldsymbol{e}}$.

The associated singularity strengths $\alpha_{\mathrm{FD}}$, $\alpha_{\mathrm{SD}}$, $\alpha_{\mathrm{FQ}}$, $\alpha_{\mathrm{RD}}$, depend on the details of the swimming mechanisms. The force dipole strength is used for distinguishing between particles that produce contractile flow fields, known as *pullers* ($\alpha_{\mathrm{FD}} < 0$), and extensile flow fields, known as *pushers* ($\alpha_{\mathrm{FD}} > 0$).

The source dipole describes the far-field hydrodynamics induced by the finite size of the swimming object. The sign of source dipole strength indicates the attraction or repulsion from a wall for a swimmer with orientation towards or away from the wall. Typically, for ciliated microswimmers, the source dipole strength is positive, $\alpha_{\rm SD} > 0$, while for flagellated organisms it is negative, $\alpha_{\rm SD} < 0$.

The flow fields generated by a swimming body with fore-aft symmetry can be modeled by a force quadrupole; specifically, $\alpha_{\rm FQ} > 0$ corresponds to swimmer with long flagella compared to the body size and vice versa for $\alpha_{\rm FQ} < 0$.

Finally, the rotlet dipole allows to predict the flow field produced by the rotation of the flagellum and the cell body, which can induce clockwise ($\alpha_{\rm RD} > 0$) or counter-clockwise ($\alpha_{\rm RD} < 0$) swimming motion along surfaces.

The flow generated by the image singularities indicates the alteration to the fluid motion everywhere due to the presence of the wall. The effects of this induced fluid motion on the swimming trajectory are provided by Faxén's law, which can be written exactly for a prolate spherical body geometry. For a sphere with radius $a$, aligned in the direction $\hat{\boldsymbol{e}}$, the translational velocity $\mathbf{U}^*(\boldsymbol{r}_0)$ and angular velocity $\boldsymbol{\Omega}^*(\boldsymbol{r}_0)$ induced on the microswimmer, located at (an arbitrary) position $\boldsymbol{r}_0$, due to its experience of the flow $\boldsymbol{u}^*(\boldsymbol{r})$ may be written as

$$\mathbf{U}^*(\boldsymbol{r}_0) = \boldsymbol{u}^*(\boldsymbol{r}_0) + \mathcal{O}(a^2\nabla^2\boldsymbol{u}^*|_{\boldsymbol{r}_0}), \tag{1.22}$$

$$\boldsymbol{\Omega}^*(\boldsymbol{r}_0) = \frac{1}{2}\nabla \times \boldsymbol{u}^*(\boldsymbol{r}_0) + \mathcal{O}\left(a^2\nabla^2(\nabla \times \boldsymbol{u}^*)|_{\boldsymbol{r}_0}\right). \tag{1.23}$$

At the order of our consideration in this study (via Eqn. 1.9), the strength of the singularities representing the body motion are not changed by the presence of the wall. If the singularities required to represent the motion differed in rate of decay by more than one degree of separation (we currently include only terms decaying at order $\hat{r}^{-2}$ and $\hat{r}^{-3}$), then Faxén's law above would indicate a problematic interaction of the wall effect with the measurement of the singularity strengths. The approach above, then, must be handled with more care in the event that a Stokeslet singularity is required, or if higher-order terms than those considered here are to be included in Eqn. (1.9). However, as long as the distance of the body to the wall is sufficiently large relative to the body size, the higher-order terms in Eqns. (1.22) and (1.23) may be neglected.

Then the translational and rotational velocities of the microswimmer near a plane wall evaluate to

$$\mathbf{U}^{(0)}(\boldsymbol{r}_0) = \mathbf{U}_{\rm free} + \mathbf{U}^*(\boldsymbol{r}_0), \tag{1.24}$$

$$\boldsymbol{\Omega}^{(0)}(\boldsymbol{r}_0) = \boldsymbol{\Omega}_{\rm free} + \boldsymbol{\Omega}^*(\boldsymbol{r}_0), \tag{1.25}$$

where we have defined $\mathbf{U}_{\rm free} = U\hat{\boldsymbol{e}}$ to be the translational velocity and $\boldsymbol{\Omega}_{\rm free} = \Omega\hat{\boldsymbol{e}}_\phi^\perp$ to be the rotational velocity of the microswimmer in an unbounded domain, and $\mathbf{U}^*$, $\boldsymbol{\Omega}^*$ include contributions from the swimmer–wall interactions, as discussed above. Here,

$\hat{\boldsymbol{e}} = \cos\theta\hat{\boldsymbol{e}}_\phi + \sin\theta\hat{\boldsymbol{e}}_z$ denotes the swimmer's direction of swimming and $\hat{\boldsymbol{e}}_\phi^\perp = \hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{e}}_\phi$, with $\hat{\boldsymbol{e}}_\phi = \cos\phi\hat{\boldsymbol{e}}_x + \sin\phi\hat{\boldsymbol{e}}_y$ being the unit vector measuring the swimming direction $\hat{\boldsymbol{e}}$ in a plane parallel to the plane wall $S_w$ (see Fig. 1.1). The *polar angle*, $\phi$, characterizes the swimmer's orientation in the $x$-$y$ plane and the *pitch angle*, $\theta$, is measured from a horizontal line parallel to the plane wall, $S_w$, at the height $h$ through the swimmer center (see Fig. 1.1).

We note that $\Omega = 0$ for non-chiral swimmers and $\Omega \neq 0$ for chiral swimmers. Further, for microswimmers modelled as prolate ellipsoids with major and minor axis lengths $2a$ and $2b$, we define a dimensionless parameter $\Gamma = (1-e^2)/(1+e^2)$, with $e = b/a$ being the body aspect ratio. In this study, we are interested in spherical non-chiral microswimmers, so $\Omega = 0$ and $\Gamma = 0$.

Since in Eqn. (1.10), the wall-induced flow field, $\boldsymbol{u}^*(\boldsymbol{r})$, is a linear combination of the flow fields due to the image systems of the higher-order singularities in Eqn. (1.9), the wall-induced velocities of the microswimmer can also be written as a linear combination of the contributions due to each higher-order singularity as follows:

$$\mathbf{U}^*(\boldsymbol{r}_0) = \mathbf{U}^*_{\mathrm{FD}}(\boldsymbol{r}_0) + \mathbf{U}^*_{\mathrm{SD}}(\boldsymbol{r}_0) + \mathbf{U}^*_{\mathrm{FQ}}(\boldsymbol{r}_0) + \mathbf{U}^*_{\mathrm{RD}}(\boldsymbol{r}_0), \tag{1.26}$$

$$\boldsymbol{\Omega}^*(\boldsymbol{r}_0) = \boldsymbol{\Omega}^*_{\mathrm{FD}}(\boldsymbol{r}_0) + \boldsymbol{\Omega}^*_{\mathrm{SD}}(\boldsymbol{r}_0) + \boldsymbol{\Omega}^*_{\mathrm{FQ}}(\boldsymbol{r}_0) + \boldsymbol{\Omega}^*_{\mathrm{RD}}(\boldsymbol{r}_0), , \tag{1.27}$$

where

$$\mathbf{U}^*_{\mathrm{i}}(\boldsymbol{r}_0) = \boldsymbol{u}^*_{\mathrm{i}}(\boldsymbol{r}_0) = \alpha_{\mathrm{i}}\mathbf{G}^*_{\mathrm{i}}(\boldsymbol{r}_0, \boldsymbol{r}^*_0; \hat{\boldsymbol{e}}), \tag{1.28}$$

$$\boldsymbol{\Omega}^*_{\mathrm{i}}(\boldsymbol{r}_0) = \frac{1}{2}\nabla \times \boldsymbol{u}^*_{\mathrm{i}}(\boldsymbol{r}_0), \tag{1.29}$$

where $\mathrm{i} \in \{\mathrm{FD}, \mathrm{SD}, \mathrm{FQ}, \mathrm{RD}\}$.

## 1.1 Tilted image systems of the fundamental singularities

### 1.1.1 Tilted Stokeslet (force) dipole image system

Unlike the 'tilted' Stokeslet image system, the image system for a tilted symmetric Stokeslet dipole near a wall is not simply a linear combination of wall-parallel and wall-perpendicular Stokeslet dipole image systems. Instead, we may determine the image system for a tilted Stokeslet dipole by taking derivatives of the Stokeslet image system where derivatives along

$\hat{\boldsymbol{e}}_z$ incorporate the $h$-dependence of the singularity coefficients. The resulting image system is denoted by $\mathbf{G}_{\mathrm{FD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$ and its full expression can be found in Appendix B.2 of Spagnolie and Lauga [2012].

Using Faxén's law (Eqns. (1.28) and (1.29)), the wall-induced translational and rotational velocities of the microswimmer due to a force dipole alone is given by $\mathbf{U}^*_{\mathrm{FD}} = U^*_{\phi,\mathrm{FD}}\hat{\boldsymbol{e}}_\phi + U^*_{z,\mathrm{FD}}\hat{\boldsymbol{e}}_z$ and $\boldsymbol{\Omega}^*_{\mathrm{FD}} = \Omega^{\perp *}_{\phi,\mathrm{FD}}\hat{\boldsymbol{e}}^{\perp}_\phi = \dot{\theta}_{\mathrm{FD}}\hat{\boldsymbol{e}}^{\perp}_\phi$ respectively, where

$$U^*_{\phi,\mathrm{FD}} = \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{8h^2}, \tag{1.30}$$

$$U^*_{z,\mathrm{FD}} = \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1), \tag{1.31}$$

$$\begin{aligned} \Omega^{\perp *}_{\phi,\mathrm{FD}} &= \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3}\left(1 + \frac{\Gamma}{2}(1+\sin^2\theta)\right) \\ &= \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3}. \end{aligned} \tag{1.32}$$

### 1.1.2 Tilted Stokeslet (force) quadrupole image system

Similarly, directional derivatives of the tilted Stokeslet dipole image system yield the image system of the axisymmetric Stokeslet quadrupole, denoted by $\mathbf{G}_{\mathrm{FQ}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$. Its full expression can be found in Appendix B.3 of Spagnolie and Lauga [2012].

We then use Faxén's law to obtain the wall-induced translational and rotational velocities associated with the tilted Stokeslet quadrupole:

$$U^*_{\phi,\mathrm{FQ}} = \frac{\alpha_{\mathrm{FQ}}\cos\theta}{16h^3}(7 - 27\sin^2\theta), \tag{1.33}$$

$$U^*_{z,\mathrm{FQ}} = \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7 - 9\sin^2\theta), \tag{1.34}$$

$$\begin{aligned} \Omega^{\perp *}_{\phi,\mathrm{FQ}} &= \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}\left(1 - 3\sin^2\theta + \frac{\Gamma}{4}(11 - 3\sin^4\theta)\right) \\ &= \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1 - 3\sin^2\theta). \end{aligned} \tag{1.35}$$

### 1.1.3 Tilted source dipole image system

As in the case of the tilted Stokeslet singularity, the tilted dipole image system, $\mathbf{G}_{\text{SD}}(\hat{\boldsymbol{e}})$, is simply a linear combination of wall-parallel and wall-perpendicular image systems (see Appendix B.4 of Spagnolie and Lauga [2012]).

Using Faxén's law, the associated wall-induced translational and rotational velocities are

$$U^*_{\phi,\text{SD}} = -\frac{\alpha_{\text{SD}}\cos\theta}{4h^3}, \tag{1.36}$$

$$U^*_{z,\text{SD}} = -\frac{\alpha_{\text{SD}}\sin\theta}{h^3}, \tag{1.37}$$

$$\begin{aligned} \Omega^{\perp*}_{\phi,\text{SD}} &= -\frac{3\alpha_{\text{SD}}\cos\theta}{8h^4}\left(1+\frac{3\Gamma}{2}(1+\sin^2\theta)\right) \\ &= -\frac{3\alpha_{\text{SD}}\cos\theta}{8h^4}. \end{aligned} \tag{1.38}$$

### 1.1.4 Tilted rotlet dipole image system

The image system for a tilted (axisymmetric) rotlet dipole is denoted by $\mathbf{G}_{\text{RD}}(\hat{\boldsymbol{e}}, \hat{\boldsymbol{e}})$ and its full expression is given in Appendix B.6 of Spagnolie and Lauga [2012].

The wall-induced velocities are

$$U^*_{\phi,\text{RD}} = 0, \tag{1.39}$$

$$U^*_{z,\text{RD}} = 0, \tag{1.40}$$

$$\begin{aligned} \Omega^{\perp*}_{\phi,\text{RD}} &= \frac{3\alpha_{\text{RD}}\sin(2\theta)}{64h^4}\left(6-\Gamma(1+3\sin^2\theta)\right)\hat{\boldsymbol{e}}_\phi \\ &\quad - \frac{3\alpha_{\text{RD}}\sin\theta}{32h^4}\left(1-3\sin^2\theta-\Gamma\cos^2\theta(1+3\sin^2\theta)\right)\hat{\boldsymbol{e}}_z \\ &= \frac{9\alpha_{\text{RD}}\sin(2\theta)}{32h^4}\hat{\boldsymbol{e}}_\phi - \frac{3\alpha_{\text{RD}}\sin\theta}{32h^4}(1-3\sin^2\theta)\hat{\boldsymbol{e}}_z. \end{aligned} \tag{1.41}$$

## 1.2 Wall-induced velocities in Cartesian coordinates (2D and 3D hydrodynamics)

### 1.2.1 Induced velocities due to the force dipole

Using Eqns. (1.30), (1.31) and (1.32), the wall-induced translational velocity of the microswimmer due to a force dipole alone can be represented as follows:

$$
\begin{aligned}
\mathbf{U}^*_{\mathrm{FD}} &= U^*_{\phi,\mathrm{FD}}\hat{\boldsymbol{e}}_\phi + U^*_{z,\mathrm{FD}}\hat{\boldsymbol{e}}_z \\
&= \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{8h^2}\hat{\boldsymbol{e}}_\phi + \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1)\hat{\boldsymbol{e}}_z \\
&= \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)\cos\phi}{8h^2}\hat{\boldsymbol{e}}_x + \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)\sin\phi}{8h^2}\hat{\boldsymbol{e}}_y + \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1)\hat{\boldsymbol{e}}_z,
\end{aligned}
\tag{1.42}
$$

and the corresponding induced rotational velocity can be represented as follows:

$$
\begin{aligned}
\boldsymbol{\Omega}^*_{\mathrm{FD}} &= \Omega^{\perp *}_{\phi,\mathrm{FD}}\hat{\boldsymbol{e}}^\perp_\phi \\
&= \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3}(-\sin\phi\hat{\boldsymbol{e}}_x + \cos\phi\hat{\boldsymbol{e}}_y) \\
&= -\frac{3\alpha_{\mathrm{FD}}\sin(2\theta)\sin\phi}{16h^3}\hat{\boldsymbol{e}}_x + \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)\cos\phi}{16h^3}\hat{\boldsymbol{e}}_y.
\end{aligned}
\tag{1.43}
$$

Without loss of generality, if the swimmer were to move in the $x$-$z$ plane, we would have $\phi = 0$. Accordingly, the induced translational and rotational velocities would be given by:

$$
\mathbf{U}^*_{\mathrm{FD}} = \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{8h^2}\hat{\boldsymbol{e}}_x + \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1)\hat{\boldsymbol{e}}_z, \tag{1.44}
$$

$$
\boldsymbol{\Omega}^*_{\mathrm{FD}} = \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3}\hat{\boldsymbol{e}}_y, \tag{1.45}
$$

respectively.

### 1.2.2 Induced velocities due to the force quadrupole

Using Eqns. (1.32), (1.33) and (1.34), the wall-induced translational velocity of the microswimmer due to a force quadrupole alone can be represented as follows:

$$
\begin{aligned}
\mathbf{U}_{\mathrm{FQ}}^{*} &= U_{\phi,\mathrm{FQ}}^{*}\hat{\boldsymbol{e}}_{\phi} + U_{z,\mathrm{FQ}}^{*}\hat{\boldsymbol{e}}_{z} \\
&= \frac{\alpha_{\mathrm{FQ}}\cos\theta}{16h^3}(7-27\sin^2\theta)\hat{\boldsymbol{e}}_{\phi} + \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta)\hat{\boldsymbol{e}}_{z} \\
&= \frac{\alpha_{\mathrm{FQ}}\cos\theta\cos\phi}{16h^3}(7-27\sin^2\theta)\hat{\boldsymbol{e}}_{x} \\
&\quad + \frac{\alpha_{\mathrm{FQ}}\cos\theta\sin\phi}{16h^3}(7-27\sin^2\theta)\hat{\boldsymbol{e}}_{y} + \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta)\hat{\boldsymbol{e}}_{z},
\end{aligned}
\tag{1.46}
$$

and the corresponding induced rotational velocity can be represented as follows:

$$
\begin{aligned}
\boldsymbol{\Omega}_{\mathrm{FQ}}^{*} &= \Omega_{\phi,\mathrm{FQ}}^{\perp *}\hat{\boldsymbol{e}}_{\phi}^{\perp} \\
&= \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1-3\sin^2\theta)(-\sin\phi\hat{\boldsymbol{e}}_{x} + \cos\phi\hat{\boldsymbol{e}}_{y}) \\
&= \frac{3\alpha_{\mathrm{FQ}}\cos\theta\sin\phi}{8h^4}(3\sin^2\theta-1)\hat{\boldsymbol{e}}_{x} + \frac{3\alpha_{\mathrm{FQ}}\cos\theta\cos\phi}{8h^4}(1-3\sin^2\theta)\hat{\boldsymbol{e}}_{y}.
\end{aligned}
\tag{1.47}
$$

Again, as for the force dipole case, if the swimmer were to undergo a 2D motion in the $x$-$z$ plane ($\phi = 0$), the induced translational and rotational would be given by:

$$
\mathbf{U}_{\mathrm{FQ}}^{*} = \frac{\alpha_{\mathrm{FQ}}\cos\theta}{16h^3}(7-27\sin^2\theta)\hat{\boldsymbol{e}}_{x} + \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta)\hat{\boldsymbol{e}}_{z}, \tag{1.48}
$$

$$
\boldsymbol{\Omega}_{\mathrm{FQ}}^{*} = \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1-3\sin^2\theta)\hat{\boldsymbol{e}}_{y}, \tag{1.49}
$$

respectively.

### 1.2.3 Induced velocities due to the source dipole

Using Eqns. (1.36), (1.37) and (1.38), the wall-induced translational and rotational velocities of the microswimmer due to a source dipole alone are given by

$$
\begin{aligned}
\mathbf{U}_{\mathrm{SD}}^{*} &= U_{\phi,\mathrm{SD}}^{*}\hat{\boldsymbol{e}}_{\phi} + U_{z,\mathrm{SD}}^{*}\hat{\boldsymbol{e}}_{z} \\
&= -\frac{\alpha_{\mathrm{SD}}\cos\theta}{4h^3}\hat{\boldsymbol{e}}_{\phi} - \frac{\alpha_{\mathrm{SD}}\sin\theta}{h^3}\hat{\boldsymbol{e}}_{z} \\
&= -\frac{\alpha_{\mathrm{SD}}\cos\theta\cos\phi}{4h^3}\hat{\boldsymbol{e}}_{x} - \frac{\alpha_{\mathrm{SD}}\cos\theta\sin\phi}{4h^3}\hat{\boldsymbol{e}}_{y} - \frac{\alpha_{\mathrm{SD}}\sin\theta}{h^3}\hat{\boldsymbol{e}}_{z},
\end{aligned}
\tag{1.50}
$$

$$
\begin{aligned}
\boldsymbol{\Omega}^*_{\mathrm{SD}} &= \Omega^{\perp *}_{\phi,\mathrm{SD}} \hat{\boldsymbol{e}}^{\perp}_{\phi} \\
&= -\frac{3\alpha_{\mathrm{SD}} \cos\theta}{8h^4}(-\sin\phi \hat{\boldsymbol{e}}_x + \cos\phi \hat{\boldsymbol{e}}_y) \\
&= \frac{3\alpha_{\mathrm{SD}} \cos\theta \sin\phi}{8h^4} \hat{\boldsymbol{e}}_x - \frac{3\alpha_{\mathrm{SD}} \cos\theta \cos\phi}{8h^4} \hat{\boldsymbol{e}}_y,
\end{aligned}
\tag{1.51}
$$

respectively.

For the 2D motion of the swimmer in the $x$-$z$ plane ($\phi = 0$), the induced translational and rotational velocities are given by

$$
\mathbf{U}^*_{\mathrm{SD}} = -\frac{\alpha_{\mathrm{SD}} \cos\theta}{4h^3} \hat{\boldsymbol{e}}_x - \frac{\alpha_{\mathrm{SD}} \sin\theta}{h^3} \hat{\boldsymbol{e}}_z, \tag{1.52}
$$

$$
\boldsymbol{\Omega}^*_{\mathrm{SD}} = -\frac{3\alpha_{\mathrm{SD}} \cos\theta}{8h^4} \hat{\boldsymbol{e}}_y, \tag{1.53}
$$

respectively.

### 1.2.4 Induced velocities due to the rotlet dipole

Using Eqns. (1.39), (1.40) and (1.41), the wall-induced translational and rotational velocities of the microswimmer due to a rotlet dipole alone are given by

$$
\mathbf{U}^*_{\mathrm{RD}} = U^*_{\phi,\mathrm{RD}} \hat{\boldsymbol{e}}_{\phi} + U^*_{z,\mathrm{RD}} \hat{\boldsymbol{e}}_z = \mathbf{0}, \tag{1.54}
$$

$$
\begin{aligned}
\boldsymbol{\Omega}^*_{\mathrm{RD}} &= \Omega^{\perp *}_{\phi,\mathrm{RD}} \hat{\boldsymbol{e}}^{\perp}_{\phi} \\
&= -\frac{5\alpha_{\mathrm{RD}} \cos\theta}{16h^3} \hat{\boldsymbol{e}}_{\phi} - \frac{\alpha_{\mathrm{RD}} \sin\theta}{8h^3} \hat{\boldsymbol{e}}_z \\
&= -\frac{5\alpha_{\mathrm{RD}} \cos\theta \cos\phi}{16h^3} \hat{\boldsymbol{e}}_x - \frac{5\alpha_{\mathrm{RD}} \cos\theta \sin\phi}{16h^3} \hat{\boldsymbol{e}}_y - \frac{\alpha_{\mathrm{RD}} \sin\theta}{8h^3} \hat{\boldsymbol{e}}_z,
\end{aligned}
\tag{1.55}
$$

respectively.

For the 2D motion of the swimmer, the induced translational and rotational velocities are given by

$$
\mathbf{U}^*_{\mathrm{RD}} = \mathbf{0}, \tag{1.56}
$$

$$
\boldsymbol{\Omega}^*_{\mathrm{RD}} = -\frac{5\alpha_{\mathrm{RD}} \cos\theta}{16h^3} \hat{\boldsymbol{e}}_x - \frac{\alpha_{\mathrm{RD}} \sin\theta}{8h^3} \hat{\boldsymbol{e}}_z, \tag{1.57}
$$

respectively.

## 1.3 Swimmer velocities near a plane wall

Combining Eqns. (1.24), (1.26), (1.42), (1.46), (1.50) and (1.54), the translational velocity of the swimmer is given by

$$\begin{aligned}\mathbf{U}^{(0)} &= \mathbf{U}_{\text{free}} + \mathbf{U}^{*} \\ &= \mathbf{U}_{\text{free}} + \mathbf{U}^{*}_{\text{FD}} + \mathbf{U}^{*}_{\text{SD}} + \mathbf{U}^{*}_{\text{FQ}} + \mathbf{U}^{*}_{\text{RD}} \\ &= \left(U\cos\theta\cos\phi + \frac{3\alpha_{\text{FD}}\sin(2\theta)\cos\phi}{8h^2} + \frac{\alpha_{\text{FQ}}\cos\theta\cos\phi}{16h^3}(7-27\sin^2\theta)\right. \\ &\qquad \left. - \frac{\alpha_{\text{SD}}\cos\theta\cos\phi}{4h^3}\right)\hat{\boldsymbol{e}}_x + \left(U\cos\theta\sin\phi + \frac{3\alpha_{\text{FD}}\sin(2\theta)\sin\phi}{8h^2}\right. \\ &\qquad \left. + \frac{\alpha_{\text{FQ}}\cos\theta\sin\phi}{16h^3}(7-27\sin^2\theta) - \frac{\alpha_{\text{SD}}\cos\theta\sin\phi}{4h^3}\right)\hat{\boldsymbol{e}}_y \\ &\quad + \left(U\sin\theta + \frac{3\alpha_{\text{FD}}}{8h^2}(3\sin^2\theta - 1) + \frac{\alpha_{\text{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta) - \frac{\alpha_{\text{SD}}\sin\theta}{h^3}\right)\hat{\boldsymbol{e}}_z.\end{aligned} \tag{1.58}$$

Combining Eqns. (1.25), (1.27), (1.43), (1.47), (1.51) and (1.55), the rotational velocity of the swimmer is given by

$$\begin{aligned}\boldsymbol{\Omega}^{(0)} &= \boldsymbol{\Omega}_{\text{free}} + \boldsymbol{\Omega}^{*} \\ &= \boldsymbol{\Omega}^{*}_{\text{FD}} + \boldsymbol{\Omega}^{*}_{\text{SD}} + \boldsymbol{\Omega}^{*}_{\text{FQ}} + \boldsymbol{\Omega}^{*}_{\text{RD}} \\ &= \left(-\frac{3\alpha_{\text{FD}}\sin(2\theta)\sin\phi}{16h^3} + \frac{3\alpha_{\text{FQ}}\cos\theta\sin\phi}{8h^4}(3\sin^2\theta - 1) + \frac{3\alpha_{\text{SD}}\cos\theta\sin\phi}{8h^4}\right. \\ &\qquad \left. - \frac{5\alpha_{\text{RD}}\cos\theta\cos\phi}{16h^3}\right)\hat{\boldsymbol{e}}_x + \left(\frac{3\alpha_{\text{FD}}\sin(2\theta)\cos\phi}{16h^3} + \frac{3\alpha_{\text{FQ}}\cos\theta\cos\phi}{8h^4}(1-3\sin^2\theta)\right. \\ &\qquad \left. - \frac{3\alpha_{\text{SD}}\cos\theta\cos\phi}{8h^4} - \frac{5\alpha_{\text{RD}}\cos\theta\sin\phi}{16h^3}\right)\hat{\boldsymbol{e}}_y - \frac{\alpha_{\text{RD}}\sin\theta}{8h^3}\hat{\boldsymbol{e}}_z.\end{aligned} \tag{1.59}$$

Substituting $\phi = 0$ in Eqns. (1.58) and (1.59), we would obtain the 2D translational and rotational velocities of the swimmer:

$$\begin{aligned}
\mathbf{U}^{(0)} &= \dot{x}_s\hat{\boldsymbol{e}}_x + \dot{h}\hat{\boldsymbol{e}}_z \\
&= \left(U\cos\theta + \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{8h^2} + \frac{\alpha_{\mathrm{FQ}}\cos\theta}{16h^3}(7-27\sin^2\theta) - \frac{\alpha_{\mathrm{SD}}\cos\theta}{4h^3}\right)\hat{\boldsymbol{e}}_x \\
&\quad + \left(U\sin\theta + \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1) + \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta) - \frac{\alpha_{\mathrm{SD}}\sin\theta}{h^3}\right)\hat{\boldsymbol{e}}_z,
\end{aligned} \tag{1.60}$$

$$\begin{aligned}
\boldsymbol{\Omega}^{(0)} &= \dot{\theta}_x\hat{\boldsymbol{e}}_x + \dot{\theta}_y\hat{\boldsymbol{e}}_y + \dot{\theta}_z\hat{\boldsymbol{e}}_z \\
&= -\frac{5\alpha_{\mathrm{RD}}\cos\theta}{16h^3}\hat{\boldsymbol{e}}_x \\
&\quad + \left(\frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3} + \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1-3\sin^2\theta) - \frac{3\alpha_{\mathrm{SD}}\cos\theta}{8h^4}\right)\hat{\boldsymbol{e}}_y - \frac{\alpha_{\mathrm{RD}}\sin\theta}{8h^3}\hat{\boldsymbol{e}}_z.
\end{aligned} \tag{1.61}$$

Ignoring the effect of the rotlet dipole, the translational velocity of the swimmer remains the same as in Eqn. (1.60), however, the expression for rotational velocity changes and is given by

$$\begin{aligned}
\boldsymbol{\Omega}^{(0)} &= \dot{\theta}_y\hat{\boldsymbol{e}}_y \\
&= \left(\frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3} + \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1-3\sin^2\theta) - \frac{3\alpha_{\mathrm{SD}}\cos\theta}{8h^4}\right)\hat{\boldsymbol{e}}_y.
\end{aligned} \tag{1.62}$$

Note that, in this case, $\boldsymbol{\Omega}^{(0)} = \boldsymbol{\Omega}^* = \Omega_\phi^{\perp *}\hat{\boldsymbol{e}}_\phi^\perp = \dot{\theta}\hat{\boldsymbol{e}}_\phi^\perp = \dot{\theta}\hat{\boldsymbol{e}}_y = \dot{\theta}_y\hat{\boldsymbol{e}}_y$. This means $\dot{\theta}_y = \dot{\theta}$.

Therefore, in this case, we have the following (non-dimensionalised) equations of motion:

$$\frac{dx}{dt} = U\cos\theta + \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{8h^2} + \frac{\alpha_{\mathrm{FQ}}\cos\theta}{16h^3}(7-27\sin^2\theta) - \frac{\alpha_{\mathrm{SD}}\cos\theta}{4h^3}, \tag{1.63}$$

$$\frac{dh}{dt} = U\sin\theta + \frac{3\alpha_{\mathrm{FD}}}{8h^2}(3\sin^2\theta - 1) + \frac{\alpha_{\mathrm{FQ}}\sin\theta}{4h^3}(7-9\sin^2\theta) - \frac{\alpha_{\mathrm{SD}}\sin\theta}{h^3}, \tag{1.64}$$

$$\frac{d\theta}{dt} = \frac{3\alpha_{\mathrm{FD}}\sin(2\theta)}{16h^3} + \frac{3\alpha_{\mathrm{FQ}}\cos\theta}{8h^4}(1-3\sin^2\theta) - \frac{3\alpha_{\mathrm{SD}}\cos\theta}{8h^4}. \tag{1.65}$$

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