PRS/KS/24/2622/2628/2634/2640

Faculty of Science & Technology

Fourth Semester B.Tech. (Computer Science and Engineering) CE/IT/CT (C.B.C.S.) Examination

DISCRETE MATHEMATICS AND GRAPH THEORY

Time : Three Hours] [Maximum Marks : 70

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.
(2) Solve Question **1 OR** Question No. **2**.
(3) Solve Question **3 OR** Question No. **4**.
(4) Solve Question **5 OR** Question No. **6**.
(5) Solve Question **7 OR** Question No. **8**.
(6) Solve Question **9 OR** Question No. **10**.
(7) Illustrate your answers wherever necessry with the help of neat sketches.
(8) Use of non programmable calculator is permitted.

1. (a) By the principle of mathematical induction, show that

$$1^2 + 2^2 + 3^2 + - - - + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$ 7

(b) Let F be the set of all one-one and onto mappings from X to X, where X = {1, 2, 3}. Find all elements of F and also find inverse of each element. 7

OR

2. (a) If the relation matrices of two relations R and S are given by

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

then find R,S and show that $M_{ROS} \neq M_{SOR}$ 7

(b) Define characteristic function. Using property of characteristic function prove that :

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $(A')' = A$ 7

3. (a) If u = {a, b, c, d}, A and B are two fuzzy sets defined on U as given below :

	a	b	c	d
A	0.5	0.8	0.0	0.3
B	0.2	1.0	0.1	0.7

Find :

(i) $A \cup B$

(ii) $A \cap B$ and show that $(A \cup B)' = A' \cap B'$ 7

(b) Define the following terms :

(i) α–cut

(ii) Normal fuzzy set

(iii) Height of fuzzy set.

Also find 0.4 – cut and height of fuzzy set A, where

A = {(1, 0.2), (2, 0.6), (3, 0.5), (4, 0.0), (5, 0.3), (6, 0.7), (7, 0.0), (8, 0.0), (9, 0.1), (10, 0.4)} 7

OR

4. (a) If $A = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3}$ and

$B = \frac{0.1}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3}$ then find

(i) A–B and

(ii) $A \oplus B$. 7

(b) If R and S are fuzzy relations given by

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}, \quad S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.8 \\ 0.7 & 0.9 \end{bmatrix} \end{array}$$

then find $R \cup S$, $R \cap S$, $\overline{R}, \overline{S}$. 7

5. (a) Show that the set A = {1, 2, 3} under multiplication modulo 4 is not a group, but set B = {1, 2, 3, 4} is a group under multiplication modulo 5. 7

(b) Prove that every field is an integral domain. 7

OR

6. (a) If R is a ring then prove that for all a, b∈ R

(i) a. 0 = 0. a = 0

(ii) a(–b) = –(ab) = (–a) b

(iii) a. (b–c) = ab–ac

(iv) (–a) (–b) = (ab) 7

(b) Prove that the set Q^+ of all positive rational numbers form an abelian group for operation * defined as $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. 7

7. (a) Define :

(i) Null graph

(ii) Node base

(iii) Path

(iv) Isomorphic graphs

(v) Trail

(vi) Reachable Node

(vii) Root of the tree. 6

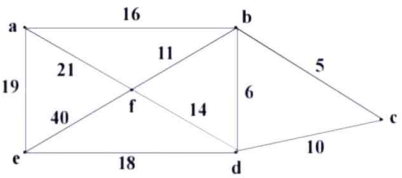
(b) Draw the diagraphs corresponding to adjacency matrices $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ and

$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ also show that these graphs are isomorphic to each other. 6

(c) Construct a tree diagram for the following algebraic expression. Also find corresponding binary tree $[3(1-x) \div \{4+\{7-(y+2)\}\}]. \{7+(x \div y)\}$ 6

OR

8. (a) Apply Prims algorithm to construct a minimal spanning tree for the weighted graph given below : 6



(b) Draw the diagraphs corresponding to matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Find AA^T, $A^T A$, A^2 and interprete the result. 6

(c) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
T = {(2,3), (2,1), (4,5), (4,6), (5,8), (6,7), (4,2), (7,9), (7,10)}
Identify the root and show that T is a rooted tree. Also find corresponding binary tree. 6

9. (a) Solve the recurrence relation
$a_n - 8\, a_{n-1} + 21\, a_{n-2} - 18\, a_{n-3} = 0$,
$a_0 = 1$, $a_1 = 1$, $a_2 = 2$. 5

(b) Find the minimum number of student in a class to be sure that four out of them are born in the same month. 5

OR

10. (a) Find the closed form of generating function for the following :

(i) 3, −3, 3, −3, 3, −3, ...

(ii) 1, 0, −1, 0, 1, 0, −1, 0, ... 5

(b) 5 men and 4 women are required to seat in a row such that the women occupy the even places. How many arrangements are possible ? 5

B.Tech. Fourth Semester (Computer Science & Engineering / Computer Technology / Computer Engineering / Information Technology) (C.B.C.S.) Summer 2023

Discrete Mathematics & Graph Theory

P. Pages : 4 **MSP/KS/23/2582/2588/2594/2600**

Time : Three Hours Max. Marks : 70

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Assume suitable data whenever necessary.
8. Use of non programmable calculator is permitted.

1. a) Prove that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. **7**

b) Let f be the set of all one-one and onto mappings from X to X, where $X = \{1, 2, 3\}$. Find all elements of f and find inverse of each element. **7**

OR

2. a) If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$, then find $R, S, R_o S, S_o R, \tilde{R}, \tilde{S}$ **7**

b) Using the properties of characteristics function, show that **7**

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $(A^1)^1 = A$

3. a) Define the following **7**

i) Equality of two fuzzy sets

ii) Union of two fuzzy sets

iii) Difference of two fuzzy sets

iv) Algebraic sum of two fuzzy sets

v) Algebraic product of two fuzzy sets

b) If $A = \left\{\frac{0.3}{30}, \frac{0.7}{60}, \frac{1}{100}, \frac{0.2}{120}\right\}$ and $B = \left\{\frac{0.2}{20}, \frac{0.4}{40}, \frac{0.6}{60}, \frac{0.8}{80}\right\}$ **7**

Are fuzzy sets, then find the relation $R = A \times B$. Also find R^2.

OR

4. a) Consider the fuzzy sets A, B, C defined on the interval X = [0, 10] of integers by the membership grade function $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1+10(x-2)^2}$. **7**

Determine the mathematical formula for

i) $\bar{A}$

ii) $A \cup B \cup C$

b) If R and S are fuzzy relations given by **7**

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}, \quad S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.8 \\ 0.7 & 0.9 \end{bmatrix} \end{array}$$

Then find $R \cup S$, $R \cap S$, $\bar{R}$, $\bar{S}$

5. a) Prove that the set {1, 2, 3, 4, 5, 6} of order 6 is a finite abelian group under multiplication modulo 7 as composition. **7**

b) Show that the set of numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers, is a field. **7**

OR

6. a) Define : (i) Subgroup (ii) Normal subgroup. Show that the intersection of any two normal subgroups of a group G, is a normal subgroup of G. **7**

b) If R is a ring such that $a^2 = a$, $\forall a \in R$, then show that **7**

i) $a + a = 0$, $\forall a \in R$

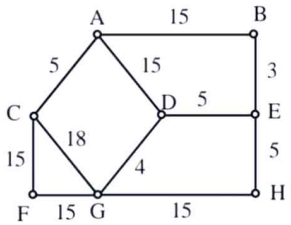
ii) $a + b = 0 \Rightarrow a = b$, $\forall a, b \in R$

iii) R is a commutative ring

7. a) Draw a digraph corresponding to the following adjacency matrix and interpreter the result AA^T, A^TA, A^2, A^3, where **6**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

b) Determine a railway network of minimal cost for the following cities using Prim's algorithm. **6**



c) Represent the following algebraic expressions using binary tree. Also draw the Venn diagrams. **6**

i) $((a+b)/c)+(x+y)$

ii) $(x+(y+z))*(a\times(b+c))$

iii) $(3-2(-(11-(9-4))))\div(2+(3+(4+7)))$

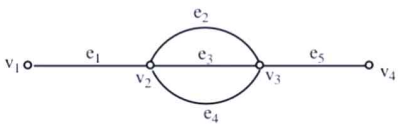
OR

8. a) Draw the digraphs corresponding to the adjacency matrices A, B, A^T, B^T, where **6**

$$A=\begin{bmatrix}0&0&1&1\\1&0&1&0\\0&1&0&0\\0&1&1&0\end{bmatrix},\quad B=\begin{bmatrix}0&1&1&0\\1&0&0&0\\0&1&0&1\\1&1&0&0\end{bmatrix}$$

show that the digraphs corresponding to A^T and B^T are isomorphic.

b) Consider the following graph. **6**



i) How many simple paths are there from v_1 to v_4

ii) How many trials are there from v_1 to v_4

iii) How many paths are there from v_1 to v_4

c) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, **6**
T = {(2, 3), (2, 1), (4, 5), (4, 6), (5, 8), (6, 7), (4, 2), (7, 9), (7, 10)}
Identify the root and show that T is a rooted tree. Also give the corresponding binary tree.

9. a) A box contains 5 red and 6 white marbles. In how many ways can 6 marbles be selected so that there are atleast two balls of each colour. **5**

b) Find the closed form of generating function for **5**

i) $a_n = 2 + 3n$

ii) $a_n = 3^n$

OR

10. a) Find the minimum number of students in a class to be sure that four out of them are born in the same month? **5**

b) Solve the following recurrence relation using generating function. **5**
$a_n - 9a_{n-1} + 20a_{n-2} = 0,\ a_0 = -3,\ a_1 = -10$

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B.Tech. (Computer Science & Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.C.S.) Winter 2022

Discrete Mathematics & Graph Theory

P. Pages : 4 **SPM/KW/22/2582/2588/2594/2600**

Time : Three Hours * 0 4 6 1 * Max. Marks : 70

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Due credit will be given to neatness and adequate dimensions.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) Prove that $1^2+2^2+3^2+-----+n^2=\frac{n(n+1)(2n+1)}{6}, n\geq 1$ by principle of mathematical induction. **4**

b) Write negation and contrapositive of the statement "If I have time and I am not fired, then I will go to the market". **4**

c) Define characteristics function and prove that **6**

i) $f_{A'}(x)=1-f_A(x)$

ii) $f_{A\cup B}(x)=f_A(x)+f_B(x)-f_A(x)\cdot f_B(x)$ for all x

OR

2. a) Let $A=\{1,2,3,4\}$ and $R=\{(1,2),(2,3),(3,4)\}$ be a relation on A. Find transitive closure of R. **4**

b) If $f: x\to y$ and $g: y\to z$ and both f and g are one - one onto then show that **5**

i) $g\circ f$ is also one – one , onto

ii) $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$

c) Let $X=\{2,3,6,12,24,36\}$ Define relation R on X, xRy if x divides y. Draw Hasse diagram of (X, R) **5**

3. a) Define **4**

i) Fuzzy set

ii) Normalized fuzzy set

b) Find $A \cap B$, $A \cup B$ and $A \times B$ where **6**

$A = \frac{0.9}{1} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.3}{6}$, $B = \frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{0.8}{5}$

Are defined on the universe $U = \{1,2,3,4,5,6\}$

c) Find α - cuts of the Fuzzy set **4**

$A = \frac{0.4}{v} + \frac{0.2}{w} + \frac{0.5}{x} + \frac{0.4}{y} + \frac{1}{z}$ for $\alpha = 0.2, 0.5, 0.9$ and 1

OR

4. a) Given a Fuzzy set $A = \frac{0}{x_1} + \frac{0.6}{x_2} + \frac{0.5}{x_3}$ **5**

Find the measure of Fuzziness of a set A.

b) If R and S are fuzzy relations given by $R = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix}$ and $S = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.8 \\ 0.7 & 0.9 \end{bmatrix}$, **6**

Find $R \cup S$ and $R \cap S$.

c) Find the bounded sum for the Fuzzy sets. **3**

$A = \frac{0.5}{4} + \frac{1}{5} + \frac{0.6}{7}$ and $B = \frac{0.3}{4} + \frac{0.6}{5}$

5. a) Prove that any two right cosets of a subgroup H are either disjoint or identical. **5**

b) Show that the set of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $a, b \in R$ is a field with respect to matrix addition and multiplication. **5**

c) In a group G, show that $(ab)^{-1} = b^{-1}a^{-1}$ $\forall a, b \in G$ **4**

OR

6. a) Prove that every field is an integral domain. **4**

b) Prove that the set $S = \{0,1,2,3,4\}$ is a ring with respect to the operation of addition and multiplication modulo 5. **6**

c) Prove that the fourth root of unity forms an abelian group under multiplication. **4**

7. a) Define. **6**

i) Pseudo graph

ii) Pendent node

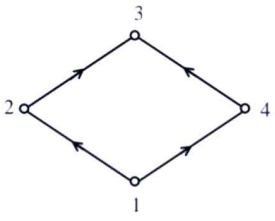
iii) Strongly connected graph.

iv) Forest

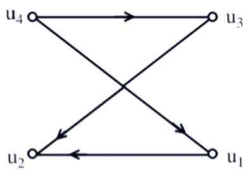
b) Draw binary tree for the following expression: **6**

$(x+(y-(x+y)))\times(3\div(2\times7)\times4)$

c) Show that the two graphs G_1 and G_2 given below are isomorphic. **6**



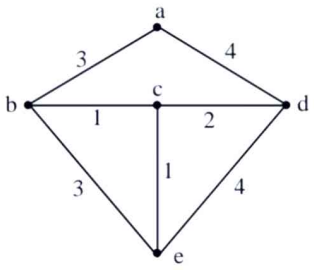
G_1



G_2

OR

8. a) Apply Kruskal's algorithm to construct a minimal spanning tree for the weighted graph given below. **6**

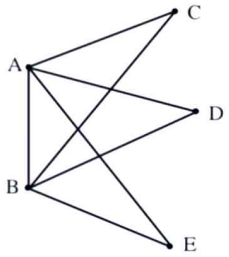


b) Draw the digraph corresponding to matrix : **6**

$$A=\begin{bmatrix}0&1&0&0\\0&0&1&1\\1&1&0&1\\1&0&0&0\end{bmatrix}$$

Interpret the results AA^T, A^TA, and A^2.

c) Define Eulerian path and Eulerian circuit. Show that the graph given below is an Eulerian graph and circuit. **6**



9. a) Prove that $c(n+1,r) = c(n,r) + c(n,r-1)$ **5**

b) Solve the following recurrence relation by using generating function **5**
$a_{n+2} - 2a_{n+1} + a_n = 2^n,\ a_0 = 2,\ a_1 = 1$

OR

10. a) Show that if seven numbers from 1 to 12 are chosen, then two of them will add upto 13. **5**

b) Find the generating function of the sequence $\{a_k\}$ if $a_k = 2 + 3k$. **5**

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PRS/KS/24/2199/2204/2209/2214

Faculty of Science and Technology

B.E. (Computer Technology/CSE/IT/CE) Fourth Semester (C.B.S.) Examination

DISCRETE MATHEMATICS AND GRAPH THEORY

Paper–I

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.
(2) Solve Question **1 OR** Question No. **2**.
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(4) Solve Question **5 OR** Question No. **6**.
(5) Solve Question **7 OR** Question No. **8**.
(6) Solve Question **9 OR** Question No. **10**.
(7) Solve Question **11 OR** Question No. **12**.
(8) Due credit will be given to neatness and adequate dimensions.
(9) Assume suitable data whenever necessary.
(10) Illustrate your answers wherever necessary with the help of neat sketches.
(11) Use of non programmable calculator is permitted.

1. (a) Is the following argument is valid ? 5

"If I study, then I will not fail in Mathematics. If I do not play basketball, then I will study. But if failed in mathematics. Therefore I played basketball".

(b) In a group of students, 70 have a personal computer, 120 have a personal stereo and 41 have both. How many own at last one of these devices ? Draw an appropriate Venn diagram also. 5

OR

2. (a) Show by Mathematical Induction that 5

$$1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(n+2)}{6},\ n \geq 1$$

(b) Prove that : 5

(i) $A-B = A \cap B'$,

(ii) $|A \cap B|' = A' \cup B'$

(iii) $|A \cup B|' = A' \cap B'$

3. (a) If R be a relation in the set of integers Z defined by : 7

$R = \{(x,y) : x \in Z, y \in Z, x-y \text{ is divisible by } 6\}$ then

prove that R is an equivalence relation.

(b) If $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$, then prove that : 6

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $(A - B) \times C = (A \times C) - (B \times C)$

(c) If $f : X \to Y$ and $g : Y \to Z$ and both f and g are one-one onto, then show that $g \circ f$ is also one-one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 5

OR

4. (a) The relation R defined on a set $A = \{0, 1, 2, 3\}$ is $R = \{(0,1), (1,2), (2,3)\}$. 6
Find the transitive closure of R.

(b) Let $f : A \to B$ defined by $f(x) = 2x^3 - 1$.
Prove that f is one-one and onto. 6

(c) Let R and S be the relation on $\{1, 2, 3, 4\}$ defined by 6
$R = \{(1,1), (1,2), (3,4), (4,2)\}$ and
$S = \{(1,1), (2,1), (3,1), (4,4), (2,2)\}$
Find $R \circ S$, $M_{R \circ S}$. Draw diagraph of $R \circ S$.

5. (a) Show that the set of matrices : 6

$$\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}, \quad a \in R$$ forms a monoid.

(b) Show that the set $H = \{a + ib / a^2 + b^2 = 1\}$ is a subgroup of $(G, *)$, where $*$ is multiplication of complex numbers. 6

OR

6. (a) Define normal subgroup and proved that every subgroup of an abelian group is a normal subgroup. 6

(b) State and Prove Lagrange's theorem on subgroups. 6

7. (a) Let $(R, +, .)$ be a ring in which o is the additive identity and $-a$ denotes the additive inverse of $a \in R$. Prove that 6

(i) $a * o = o, \ \forall a \in R$

(ii) $a(b - c) = ab - ac$

(b) Show that the intersection of two sub-rings of a ring R is a sub-ring. 6

OR

8. (a) Show that $(z6, +6, \times 6)$ is a ring but not an integral domain. 6

(b) Construct a switching circuit for Boolean polynomial $(A \cdot B) + [A' \cdot (A + B + B')]$. Simplify and draw equivalence switching circuit. 6

9. (a) Draw a directed tree with 4 node at level 1, 6 node at level 2. Obtain the corresponding binary tree. 6

(b) Draw diagraph corresponding to : 6

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ and interpret the results AA^T, $A^T A$, A^2, A^3, A^4.

(c) Construct the tree for the following expression : 6

$(((5(1-x)5*a)+(3-(6*a)))+(a-(3*b))$

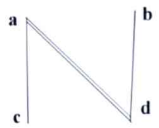
OR

10. (a) Draw diagraph corresponding to : 6

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$ find its complement and write the matrix of that complemented graph.

(b) Draw tree representation for the tree given by :
A = {1, 2, 3, 4, 5, 6, 7}; R = {(1,2), (1,3), (1,4), (2,5), (4,6), (4,7)}and draw corresponding binary tree. 6

(c) Find the minimal spanning tree of the weighted graph of given fig. using Prim's Algorithm. 6



11. (a) Three persons enter into the car, where there are 5 seats. In how many ways can they take up their seats ? 5

(b) Find a general solution for : 5

$a_r + a_{r-1} = (3r)2^r$

OR

12. (a) Prove that C(n,r) = C(n–1, r–1) + C(n–1, r). 5

(b) Find the generating function for the sequence 1, a, a^2, ------, where a is fixed constant. 5