Faculty of Science & Technology

Third Semester B.Tech. (Computer Engineering) (IT/CT/CE) (C.B.C.S.) Examination

MATHEMATICS–III

Time : Three Hours] [Maximum Marks : 70

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve Question No. 1 **OR** Question No. 2.

(3) Solve Question No. 3 **OR** Question No. 4.

(4) Solve Question No. 5 **OR** Question No. 6.

(5) Solve Question No. 7 **OR** Question No. 8.

(6) Solve Question No. 9 **OR** Question No. 10.

(7) Use of non-programmable calculator is permitted.

1. (a) Find $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$ and hence evaluate $\int_0^{\infty}\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}dt$. 5

(b) Using convolution theorem find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$. 5

(c) Find Fourier transform of $f(x) = \begin{cases} 1 & , |x|<1 \\ 0 & , |x|>1 \end{cases}$. 4

OR

2. (a) Solve by method of Laplace transform $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4e^{2t}$, $y(0) = -3$ and $y'(0) = 5$. 7

(b) Solve the integral equation $\int_0^{\infty} f(x)\cos\alpha x\,dx = \begin{cases} 1-\alpha & , 0\le\alpha\le 1 \\ 0 & , \alpha>1 \end{cases}$ and hence evaluate $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$. 7

3. (a) Prove that $Z[n^p] = -Z\frac{d}{dZ}[n^{p-1}]$, where p is any positive integer and hence deduce that $Z[n] = \frac{Z}{(Z-1)^2}$ and $Z[n^2] = \frac{Z(Z+1)}{(Z-1)^3}$. 7

(b) Find inverse z-transform of $F(z)=\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$. 7

OR

4. (a) Using z-transform solve the difference equation :

$y_{n+2} + 5y_{n+1} + 6y_n = 6^n$ given $y_0 = 0$, $y_1 = 1$. 7

(b) Show that $Z^{-1}\left[e^{2/z}\right] = \dfrac{2^n}{n!}$ or $\dfrac{1}{n!} * \dfrac{1}{n!} = \dfrac{2^n}{n!}$. 7

5. (a) Reduce the given matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to a diagonal form. 7

(b) Use Sylvester's theorem to show $\sin^2 A + \cos^2 A = I$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$. 7

OR

6. (a) Find largest eigen value and corresponding eigen vector for the matrix $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$. 7

(b) Find the singular values of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the singular value decomposition of A. 7

7. (a) A random variable X is defined by $f(x) = \begin{cases} -2 & , \text{ prob. } \frac{1}{3} \\ 3 & , \text{ prob. } \frac{1}{2} \\ 1 & , \text{ prob. } \frac{1}{6} \end{cases}$.

Find E(X), E(2X + 3), $E(X^2)$, $E(X^2 + 5X)$, Var(X). 7

(b) If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs :

(i) More than 5

(ii) Between 1 and 3

(iii) At the most 2

(iv) At least 2 bulbs will be defective. 7

OR

8. (a) Find the moment generating function and first four moments about origin for random variable X given by :

$$X = \begin{cases} \frac{1}{2} & \text{prob. } \frac{1}{2} \\ -\frac{1}{2} & \text{prob. } \frac{1}{2} \end{cases}$$ 7

(b) The marks obtained in a certain exam follow normal distribution with mean 45 and SD 10. If 1,300 students appear at the examination, calculate the number of students scoring :

(i) less than 35 marks and

(ii) more than 65 marks. 7

9. (a) In a distribution of three variables it is observed that $r_{12} = 0.70$, $r_{13} = 0.61$, $r_{23} = 0.40$. Calculate the value of $r_{12.3}$, $r_{13.2}$. 4

(b) For the following frequency distribution calculate median, quartiles, 4^{th} decile and 27^{th} percentile :

X	0	1	2	3	4	5	6	7	8
f	1	9	26	59	72	52	29	7	1

7

(c) Let x_1, x_2, x_3 are three variates measured from their mean with $n = 10$, $\Sigma x_1^2 = 90$, $\Sigma x_2^2 = 160$, $\Sigma x_3^2 = 40$, $\Sigma x_1 . x_2 = 60$, $\Sigma x_1 . x_3 = 40$, $\Sigma x_2 . x_3 = 60$. Calculate the multiple Correlation Coeff. $R_{1.23}$. 3

OR

10. (a) Calculate coefficient of skewness of the following distribution :

X	0	1	2	3	4	5	6	7	8
f	7	12	32	56	70	56	28	8	1

7

(b) Find the measure of skewness on the basis of moments :

Class Interval	0 – 2	3 – 5	6 – 8	9 – 11
Freq.	3	7	4	1

7

B.Tech. (Computer Technology / Computer Engineering / Information Technology)
Third Semester (C.B.C.S.)

Applied Mathematics-III

P. Pages : 2

PSM/KW/23/2575/2581/2587

Time : Three Hours

* 2 7 6 7 *

Max. Marks : 70

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Use of non programmable calculator is permitted.

1. a) If $L\{tf(t)\} = -\frac{d}{ds}\bar{f}(s)$, then find $L\{t\cos 2t\}$. **4**

b) Find $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$ by convolution theorem. **5**

c) Find the Fourier sine transform of $e^{-|x|}$ and hence show that **5**

$\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}, m>0$

OR

2. a) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y = e^{-t}\sin t, y(0)=0$ and $y'(0)=1$, using Laplace transform. **7**

b) Express $f(x)=\begin{cases}1, |x|<1\\ 0, |x|>1\end{cases}$, as Fourier integral and hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda}\,d\lambda$. **7**

3. a) Find z - transform of $\frac{1}{n+1}$. **7**

b) Find inverse z - transform of $\frac{z^2+z}{(z-1)\left(z^2+1\right)}$ by partial fraction method. **7**

OR

4. a) Solve $y_{n+2}+3y_{n+1}+2y_n = u_n$, given that $y_0 = 1, y_n = 0$ for $n<0$. **7**

b) Find inverse z – transform of $\frac{z^3}{(z-2)^3}, |z|>2$. **7**

5. a) Investigate the linear dependence of the vectors **7**

$X_1 = (1,2,4),\ \ X_2 = (2,-1,3)$

$X_3 = (0,1,2),\ \ X_4 = (-3,7,2)$

and if possible find the relation between them.

b) Use Sylvester's theorem to show that $\sin^2 A + \cos^2 A = I$, where $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. **7**

OR

6. a) Find the singular values decomposition of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. **7**

b) Find largest eigen value and corresponding eigen vector for the matrix $A = \begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}$. **7**

7. a) Let X be the random variable having density function **7**

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find (i) E (X) (ii) Var (X) (iii) σ_X

b) An insurance salesman sells policies to 5 men all of identical age and in good health. The probability that a man of this particular age will be alive 30 yrs. is 2/3. Find the probability that in 30 yrs, (i) all 5 man, (ii) at least 3 man (iii) only 2 man (iv) at least 1 man (v) at most 1 man will be alive. **7**

OR

8. a) A random variable X has density function given by, **7**

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the moment generating function and also first four moment about the origin.

b) If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determines the probability that out of 2,000 individuals **7**
a) exactly 3 b) more than 2 c) at least 4 will suffer a bad reaction.

9. a) Find multiple linear regression equation of X_1, X_2 and X_3 from the data relating to three variables given below: **7**

X_1	4	6	7	9	13	15
X_2	15	12	8	6	4	3
X_3	30	24	20	14	10	4

b) Calculate mode for the following frequency distribution. **7**

Size	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
Frequency	9	12	15	16	17	15	10	13

OR

10. a) The following are the marks of 150 students in an examination. **7**

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students	10	40	20	0	10	40	16	14

Calculate Karl Person's coefficient of Skewness.

b) Find the measure of Skewness on the basis of moments. **7**

x	2	3	4	5	6
f	1	3	7	3	1

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B.Tech. Third Semester (Computer Engineering / Computer Technology / Information Technology)
(C.B.C.S.) Summer 2023

Mathematics-III / Applied Mathematics-III

P. Pages : 3 **MSP/KS/23/2535/2541/2547**

Time : Three Hours Max. Marks : 70

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Use of non programmable calculator is permitted.

1. a) Find $L\left\{\frac{\sin 2t}{t}\right\}$, Hence evaluate $\int_0^\infty e^{-t}\frac{\sin 2t}{t}dt$ **4**

b) Find $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ using convolution theorem. **6**

c) Find the Fourier sine and cosine transform of e^{-ax}. **4**

OR

2. a) Solve the integral equation $\int_0^\infty f(x)\cos\alpha x\,dx = \begin{cases}1-\alpha, 0\le\alpha\le 1\\ 0, \alpha>1\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt$ **7**

b) Solve by method of Laplace transform $y''' + 2y'' - y' - 2y = 0$, given that $y(0) = y'(0) = 0, y''(0) = 6$ **7**

3. a) Find Z-transform of $\frac{a^n}{n!}$ and also find $\frac{a^n}{(n+1)!}$ **7**

b) Find invers Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$ **7**

OR

4. a) Solve the difference equation $6y_{k+2} - y_{k+1} + y_k = 0, y_0 = 0, y_1 = 1$ by using Z-transform. **7**

b) Show that $Z^{-1}\left[e^{2/z}\right] = \frac{2^n}{n!}$ Or $\frac{1}{n!} * \frac{1}{n!} = \frac{2^n}{n!}$ **7**

5. a) Show that the vectors $[0,1,-2],[1,-1,1][1,2,1]$ form a linearly independent set. **7**

b) Find the eigen values and eigen vectors and model matrix for the matrix **7**

$$A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$$

OR

6. a) Find largest eigen value and corresponding eigen vector by iteration method, where **7**

$$A=\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

b) Find the SDV of the matrix $A=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}$ **7**

7. a) A density function of a random variable X is **7**

$$f(x)=\begin{cases}2e^{-2x}, & x\ge 0\\ 0, & \text{otherwise}\end{cases}$$

Find:

i) E(X)

ii) V(X) and σ_x

iii) $E\left[(X-1)^2\right]$

b) Out of 800 families with 5 children each, how many would you expect to have **7**
i) 3 boys ii) 5 girls and iii) Either 2 or 3 boys?
Assume equal probabilities for boys and girls.

OR

8. a) A random variable X can assume the values+1 and -1with probability $\frac{1}{2}$ each. Find **7**

i) Moment generating function.

ii) First four moment about origin.

b) In a normal distribution 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **7**

9. a) If the simple correlation coefficient have the values $R_{12}=0.6$, $R_{13}=0.65$ and $R_{23}=0.8$ then find the multiple correlation coefficient of $R_{1.23}$ **3**

b) A proof reads through 73 pages manuscript. The number of mistakes found on each of the pages are summarized in the table below determine the mean number of mistakes found per page. **4**

No. of Mistake	1	2	3	4	5	6	7
No. of Pages	5	9	12	17	14	10	6

c) The following table shows age distribution of persons in a particular region: **7**

Age(years)	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80
No. of Persons (in thousand)	2	5	9	12	14	15	15.5	15.6

Find the median age.

OR

10. a) A student's marks in 5 subjects are 75, 68, 80, 92, 56. Find the average of his marks. Using short cut method. **3**

b) The following are the marks of 150 students in an examination. **7**

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students	10	40	20	0	10	40	16	14

Calculate Karl Pearson's coefficient of skewness.

c) The first four raw moments of a distribution are 2, 136, 320 and 40,000. Find out coefficients of skewness. **4**

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B.Tech. Third Semester (Computer Technology / Computer Engineering / Information Technology) (C.B.C.S.) Winter 2022

Mathematics-III / Applied Mathematics-III

P. Pages : 3 **SPM/KW/22/2535/2541/2547**

Time : Three Hours Max. Marks : 70

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Use of non programmable calculator is permitted.

1. a) Find L.T. of $\frac{e^{-at}-e^{-bt}}{t}$, hence evaluate $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}dt$ **5**

b) Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ by using Convolution theorem. **4**

c) Find Fourier transform of $f(x)=\begin{cases}1, & \text{for} |x|<1 \\ 0, & \text{for} |x|>1\end{cases}$, hence find $\int_0^\infty \frac{\sin x}{x}dx$ **5**

OR

2. a) Find $L\left\{\int_0^t \sin u du\right\}$ **4**

b) Solve $\frac{d^2y}{dt^2}+9y=\cos 2t$, given $y(0)=1,\ y(\pi/2)=-1$ **6**

c) Solve the integral equation **4**

$$\int_0^\infty f(x)\cos\lambda x dx = e^{-\lambda},\ \lambda>0$$

3. a) Find Z-transform $\sin n\theta \cos n\theta$. **7**

b) By using convolution theorem, **7**

$$z^{-1}\left\{\frac{z^2}{(z-1)(z-3)}\right\}$$

OR

4.	a)	Using Power series method, find $z^{-1}\left\{\dfrac{1}{z^2-3z+2}\right\}$, for the region $\lvert z\rvert < 1$.	**7**
	b)	Solve the difference equation by Z-transform. $y_{n+2}+5y_{n+1}+6y_n=6^n$, given $y(0)=0, y(1)=1$.	**7**
5.	a)	Reduce the given matrix in diagonal form, $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & 1\\ 2 & -1 & 3\end{bmatrix}$	**7**
	b)	Using Sylvester's Theorem, solve if $A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$, show that $\log_e e^A = A$.	**7**

OR

6.	a)	Find the singular values of the matrix $A=\begin{bmatrix}0 & 1 & 1\\ \sqrt{2} & 2 & 0\\ 0 & 1 & 1\end{bmatrix}$ and find the singular value decomposition of A.	**7**
	b)	Find the largest eigen value and the corresponding eigen vector for the matrix. $\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$	**7**
7.	a)	A density function of random variable X is. $f(x)=\begin{cases}2e^{-2x}, & x\ge 0\\ 0, & \text{otherwise}\end{cases}$ Find i) $E(X)$ ii) $Var(X)$ iii) σ_X iv) $E\left[(X-1)^2\right]$	**7**
	b)	If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs- i) Exactly 2 ii) More than 5 iii) Between 1 and 3 iv) At the most 2, v) At least 2 bulbs will be defective	**7**

OR

8.	a)	Find the moment generating function of the random variable $X=\begin{cases}1/2, & \text{Prob.}1/2\\ -1/2 & \text{Prob.}1/2\end{cases}$ Also find first four moments about the origin.	**7**

b) A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts produced by this machine. **7**

i) Between 30 and 50

ii) at the most 30,

iii) 55 or more of the bolts will be defective.

9. a) Find the mode from the following data. **7**

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

b) Calculate the mean and standard deviation for the following data. **7**

Size of item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

OR

10. a) Let x_1, x_2, x_3 are three variates measured from their mean with **7**

$n = 10, \Sigma x_1^2 = 90, \Sigma x_2^2 = 160, \Sigma x_3^2 = 40, \Sigma x_1 x_2 = 60, \Sigma x_1 x_3 = 40$ and $\Sigma x_2 x_3 = 60$

calculate the multiple correlation coefficient $R_{1.23}$.

b) Calculate coefficient of skewness of the following distribution. **7**

x	0	1	2	3	4	5	6	7	8
f	7	12	32	56	70	56	28	8	1

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