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Readme file for frames.py

Attitude Determination and Control Subsystem

latlon (x)

Author: Anant Joshi
Date:
To obtain the latitude and longitude in degrees given the position in earth centered earth fixed (ECEF) frame.
Input: position in ECEF frame (ndarray)
Output: latitude and longitude in degrees (tuple)
In these calculation it is assumed that the shape of earth is perfectly spherical.
Latitude is a geographic coordinate that specifies the north–south position of a point on the Earth's surface. These form circles parallel to equator on earth's surface. Equator is 0^0 latitude. It range from 0^0 to 90^0 from equator to north pole and 0^0 to -90^0 from equator to south pole.

$$latitude = \begin{cases} \cos^{-1}(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}) & z \geq 0 \\ -\cos^{-1}(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}) & z < 0 \end{cases}$$

Longitude is given as an angular measurement ranging from 0^0 at the Prime Meridian to $+180^0$ eastward and -180^0 westward.
For $y = 0$ prime meridian ($x > 0$) is zero longitude and anti-prime-meridian is 180^0 longitude.Thus

$$longitude = \begin{cases} 0 & y = 0, x \geq 0 \\ 180 & y = 0, x < 0 \\ \cos^{-1}\frac{x}{\sqrt{x^2+y^2}} & y > 0 \\ -\cos^{-1}\frac{x}{\sqrt{x^2+y^2}} & y < 0 \end{cases}$$

sgn (x)

Author: Anant Joshi
Date:
This is signum function.

$$sgn(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

ecif2ecef(v_x_i,t)

Author: Anant Joshi
Date:
Input: vector with components in eci frame and time since epoch in seconds
Output: vector with components in ecef frame

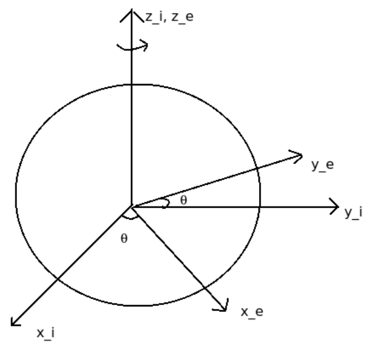

Figure 1: ECIF and ECEF

From this figure the co-ordinate transformation can be written as

$$v_e = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} v_I$$

ecif2ecefR(t)

Author: Anant Joshi
Date:
Input: time since epoch in seconds
Output: returns co-ordinate transformation matrix
This function is same as ecif2ecef. Only difference is that the time used is sidereal time. To obtain sidereal time multiply the time by STEPRUT (1.002738).

ecef2ecif(v_x_e,t)

Author: Anant Joshi
Date:
Input : vector components in ecef, time since epoch in seconds Output : vector components in eci frame

From figure 1, the co-ordinate transformation is

$$v_I = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} v_e$$

ecif2orbit(v_pos_i,v_vel_i,v_x_i)

Author: Sanket Chirame
Date:
Input : vector components in ecif, position and velocity in eci frame
Output : vector components in orbit frame

$$\vec{z} = -\vec{r_I}/|\vec{r_I}|$$

$$\vec{y} = \frac{\vec{v_I} \times \vec{r_I}}{|\vec{v_I} \times \vec{r_I}|}$$

$$\vec{x} = \frac{y \times z}{|y \times z|}$$

$$\vec{v}_o = [\,\vec{x}\;\vec{y}\;\vec{z}\,]^T\;\vec{v}_i$$

qBI2qBO(v_q_BI,v_pos_i,v_vel_i)

Author: Sanket Chirame
Date:
Input : unit quaternion transforming ECI frame compnents of vector to body frame components, position and velocity in ecif Output : unit quaternion transforming orbit frame compnents of vector to body frame components From the definition of orbit frame

$$\vec{z} = -\vec{r_I}/|\vec{r_I}|$$

$$\vec{y} = \frac{\vec{v_I} \times \vec{r_I}}{|\vec{v_I} \times \vec{r_I}|}$$

$$\vec{x} = \frac{y \times z}{|y \times z|}$$

Where $\vec{x},\ \vec{y},\ \vec{z}$ are column vector in contrast to numpy arrays in code of dimension 1×3.

$$DCM_{IO} = [\,\vec{x}\;\vec{y}\;\vec{z}\,]$$

q_{IO} is calculated using this DCM using function $rotmat2quat$ function in this module.
The composition of quaternion is calculated using formula given in reference [1], (Equation 3.97),

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \beta_0'' & -\beta_1'' & -\beta_2'' & -\beta_3'' \\ \beta_1'' & \beta_0'' & \beta_3'' & -\beta_2'' \\ \beta_2'' & -\beta_3'' & \beta_0'' & \beta_1'' \\ \beta_3'' & \beta_2'' & -\beta_1'' & \beta_0'' \end{bmatrix} \begin{bmatrix} \beta_0' \\ \beta_1' \\ \beta_2' \\ \beta_3' \end{bmatrix} \tag{1}$$

where $\vec{\beta} = q_{BO},\ \vec{\beta}'' = q_{BI},\ \vec{\beta}' = q_{IO}$. The sign of quaternion is flipped so that scalar part has positive sign. This is to ensure that quaternion corresponds to smallest rotation. Then it is normalized again to return unit quaternion.

qBO2qBI(v_q_BO,v_pos_i,v_vel_i)

Author: Sanket Chirame
Date:
Input: unit quaternion transforming orbit frame compnents of vector to body frame components, position and velocity in ecif
Output: unit quaternion transforming ECI frame compnents of vector to body frame components
From the definition of orbit frame

$$\vec{z} = -\vec{r_I}/|\vec{r_I}|$$

$$\vec{y} = \frac{\vec{v_I} \times \vec{r_I}}{|\vec{v_I} \times \vec{r_I}|}$$

$$\vec{x} = \frac{y \times z}{|y \times z|}$$

Where $\vec{x},\ \vec{y},\ \vec{z}$ are column vector in contrast to numpy arrays in code of dimension 1×3.

$$DCM_{OI} = [\ \vec{x}\ \vec{y}\ \vec{z}\ ]^T$$

Quaternion corresponding to this DCM is obtained using $rotmat2quat$ function. q_{BI} is calculated using Equation (1) with $\vec{\beta} = q_{BI},\ \vec{\beta}'' = q_{BO},\ \vec{\beta}' = q_{OI}$

wBIb2wBOb(v_w_BI_b,v_q_BO,v_w_IO_o)

Author: Sanket Chirame
Date:
Input : angular velocity of body w.r.t. ecif in body frame, unit quaternion transforming orbit frame compnents of vector to body frame components, angular velocity of ecif wrt orbit frame in orbit frame
Output : angular velocity of body frame w.r.t. orbit frame in body frame

$$\vec{w}_{BO} = \vec{w}_{BI} + \vec{w}_{IO}$$

Since we want components of $\vec{w}_{BO}$ in body frame, all angular velocities on RHS should be expressed in body frame for component-wise addition.

wBOb2wBIb(v_w_BO_b,v_q_BO,v_w_IO_o)

Author: Sanket Chirame
Date:
Input : angular velocity of body w.r.t. orbit frame in body frame, unit quaternion transforming orbit frame compnents of vector to body frame components, angular velocity of ecif wrt orbit frame in orbit frame
Output : angular velocity of body frame w.r.t. eci frame in body frame

$$\vec{w}_{BI} = \vec{w}_{BO} - \vec{w}_{IO}$$

Since we want components of $\vec{w}_{BI}$ in body frame, all angular velocities on RHS should be expressed in body frame for component-wise addition.

ned2ecef(v_ned,lat,lon)

Author: Sanket Chirame
Date: 27/7/2018
Input:

v: 1×3 array with vector compnents in North-East-Down (NED) frame

lat: Latitude in degrees. Positive values correspond to northen hemisphere and negative values correspond to southern hemisphere.

lon: Longitude in degrees. -180 to 180, -180 excluded.

Output:

1×3 array with vector compnents in earth centered earth fixed (ECEF) frame

North east down (NED), also known as local tangent plane (LTP), is a geographical coordinate system for representing state vectors that is commonly used in aviation. It consists of three numbers: one represents the position along the northern axis, one along the eastern axis, and one represents vertical position.[2] This frame is not defined at poles.

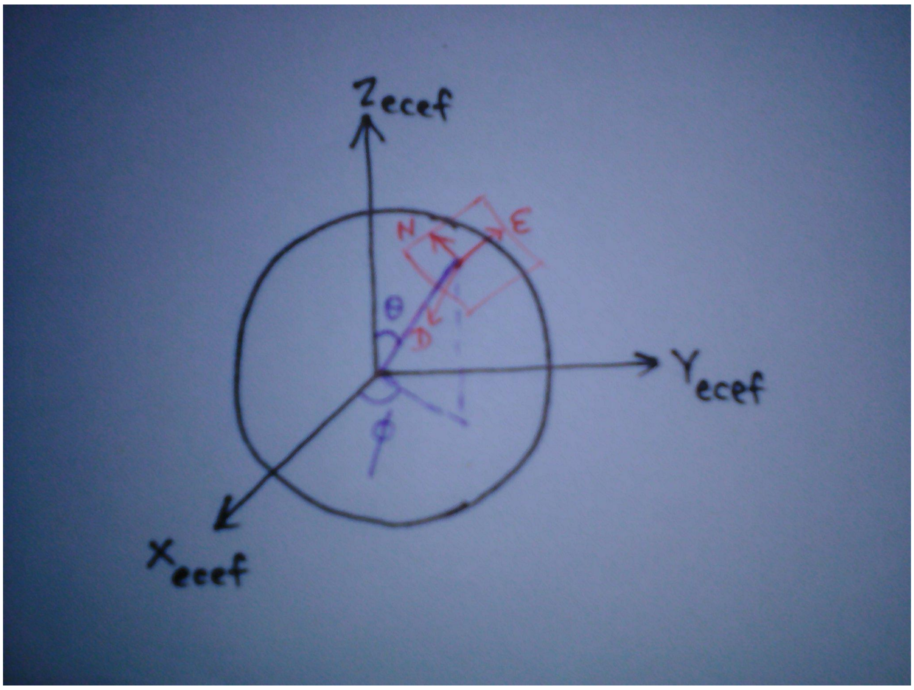

Figure 2: North-East-Down frame

First convert latitude and longitude to spherical polar co-ordinates and change unit to radians.

1. θ = 90 - latitude
2. Convert (-180,180] range of longitude to [0,360) i.e $[0, 2\pi)$

From figure, the direction cosine matrix for required transformation can be deduced.

- Notice that unit vector in down direction is opposite of radial unit vector. Thus from figure, third column (transformed components of original third basis vector) will be

$$[-\sin\theta\cos\phi\ ,\ -\sin\theta\sin\phi\ ,\ -\cos\theta]^T$$

- North basis vector makes angle θ with ECEF z axis and it's projection in $x-y$ plane is negative of projection of position vector in $x-y$ plane. Thus First column of DCM (transformed components of original third basis vector) will be

$$-\cos\theta\cos\phi\ ,\ -\cos\theta\sin\phi\ ,\ \sin\theta]^T$$

- Second column can be found out by demading the frame to be right handed and from orthogonality of DCM.

$$v_{ECEF} = \begin{bmatrix} -\cos\theta\cos\phi & -\sin\phi & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \cos\phi & -\sin\theta\sin\phi \\ \sin\theta & 0 & -\cos\theta \end{bmatrix} v_{NED} \tag{2}$$

References

[1] Hanspeter Schaub and John L Junkins. *Analytical mechanics of space systems*. Aiaa, 2003.

[2] Wikipedia contributors. *North east down — Wikipedia, The Free Encyclopedia*. [Online; accessed 27-July-2018]. 2017. URL: https://en.wikipedia.org/w/index.php?title=North_east_down&oldid=795989002.