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To solve linear programming using R studio, we need to install lpsolve package Install.packages("lpsolve")

PRACTICAL 1

GRAPHICAL METHOD USING R PROGRAMMING

```
# R Program
#Find a geometrical interpretation and solution as well for the following LP problem
#Max z = 3x1 + 5x2
#subject to constraints:
#x1+2x2<=2000
#x1+x2<=1500
#x2<=600
#x1,x2>=0
# Load lpSolve
require(lpSolve)
## Set the coefficients of the decision variables -> C of objective function
C <- c(3,5)
# Create constraint martix B
A \leftarrow matrix(c(1, 2,
        1, 1,
       0, 1
), nrow=3, byrow=TRUE)
# Right hand side for the constraints
B <- c(2000,1500,600)
```

```
# Direction of the constraints
constranints_direction <- c("<=", "<=", "<=")
# Create empty example plot
plot.new()
plot.window(xlim=c(0,2000), ylim=c(0,2000))
axis(1)
axis(2)
title(main="LPP using Graphical method")
title(xlab="X axis")
title(ylab="Y axis")
box()
# Draw one line
segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, x1 
# Find the optimal solution
optimum <- lp(direction="max",
                       objective.in = C,
                       const.mat = A,
                       const.dir = constranints_direction,
                       const.rhs = B,
                       all.int = T)
# Print status: 0 = success, 2 = no feasible solution
print(optimum$status)
# Display the optimum values for x1,x2
best sol <- optimum$solution
names(best sol) <- c("x1", "x2")
print(best_sol)
```

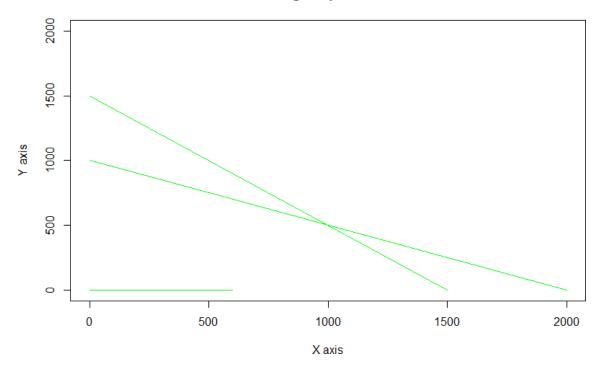
Check the value of objective function at optimal point print(paste("Total cost: ", optimum\$objval, sep=""))

OUTPUT:

```
[Workspace loaded from ~/.RData]
> # Right hand side for the constraints
> B < -c(2000, 1500, 600)
> # R Program
> # Load lpSolve
> require(lpSolve)
Loading required package: lpSolve
> ## Set the coefficients of the decision variables -> C
> C < -c(3,5)
> # Create constraint martix B
> A <- matrix(c(1, 2,
                1, 1,
                 0, 1
+ ), nrow=3, byrow=TRUE)
> # Right hand side for the constraints
> B < -c(2000, 1500, 600)
> # Direction of the constraints
> constranints_direction <- c("<=", "<=", "<=")
> # Create empty example plot
> #plot(2000, 2000, col = "white", xlab = "", ylab = "")
> plot.new()
> plot.window(xlim=c(0,2000), ylim=c(0,2000))
> axis(1)
> axis(2)
> title(main="LPP using Graphical method")
> title(xlab="X axis")
> title(ylab="Y axis")
> box()
> # Draw one line
> segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
> segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
> segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, col = "green")
> # Find the optimal solution
> optimum <- lp(direction="max",</pre>
                objective.in = C_{\bullet}
                const.mat = A_{r}
                const.dir = constranints direction,
                const.rhs = B_{r}
                all.int = T)
```

```
> # Print status: 0 = success, 2 = no feasible solution
> print(optimum$status)
[1] 0
> # Display the optimum values for x1,x2
> best_sol <- optimum$solution
> names(best_sol) <- c("x1", "x2")
> print(best_sol)
    x1     x2
1000    500
> # Check the value of objective function at optimal point
> print(paste("Total cost: ", optimum$objval, sep=""))
[1] "Total cost: 5500"
```

LPP using Graphical method



Simplex Method with 2 variables using Python

from scipy.optimize import linprog							
#Max z=3x1+2x2							
#subject to							
#x1 + x2 <=4							
#x1 - x2 <=2							
#x1,x2>=0							
obj = [-3, -2]							
Ihs_ineq = [[1, 1], # Red constraint left side [1, -1]] # Blue constraint left side							
rhs_ineq = [4, # Red constraint right side 2] # Blue constraint right side							
bnd = [(0, float("inf")), # Bounds of x (0, float("inf"))] # Bounds of y							
>>> opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,							
<pre> bounds=bnd,method="revised simplex") >>> opt</pre>							
opt.fun							
opt.success							
opt.x							

Simplex Method with 3 variables using Python

from scipy.optimize import linprog #Min z = x1-3x2+2x3#subject to #3x1-x2+3x3<=7 #-2x1+4x2<=12 #-4x1+3x2+8x3<=10 #x1,x2,x3>=0obj = [1, -3, 2]Ihs ineq = [[3, -1, 3], # Red constraint left side [-2, 4, 0], # Blue constraint left side [-4, 3, 8]] # Yellow constraint left side rhs_ineq = [7, # Red constraint right side 12, # Blue constraint right side 10] # Yellow constraint right side bnd = [(0, float("inf")), # Bounds of x (0, float("inf")), (0, float("inf"))] # Bounds of y >>> opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq, bounds=bnd, method="revised simplex") >>> opt

Trusted Python 3

```
File Edit View Insert Cell Kernel Widgets Help
     + :|< |',') | | + +
                                          Run 🔳 C
                                                                    Code
   In 127]: M from !5cipy.optimiz import linprog
   In {28]: H #Min 2= x1-3x2+2x3
                      tsubp::ct to
#3xl-x.2+3x3<-7
f-2x1+4x2<=12
f-4xl+Jx2+8x3<=10
#xl,x.2,x3,>=0
                      obJ = 11, -3, 2]
   In 129]: \mathbf{M} lh.9_ineq = \mathbf{I}[ 3, -1, 3], : Red constraint left side (-2, 4, 0], | I Bl11e constraint left side [-4, 3, 8]] i Yellov constraint left side
   In JJO]: M rh!5_J.n q = 17, i Red constraint right side 12, i Blue constraint right side 10] f YslloJo'constraint right side
   In {33}: \boldsymbol{H} bnd = I {0, float {"inf")}, i Bot1nds of x (0, float("inf")), (0, float ("l.nf"))] # Bounds of y
   In {34}: M >>>opt= linprog(c=obJ, A_ub=ln.5_J.n q, b_ub=rh.5_J.n q, bound.9=bnd, method="revised simplex")
                      >>> opt
        0Ut[34]:
                              array([], dtyp-e::float64) fun: -11.0
                        me5.5age: 'Optimization terminated .5ucce.5.5fully.' nit: 2
slack: array{[ 0., 0., 11.])
                          .5tatu.5: 0
                                     array([4,, 5., 0,])
```

Simplex Method with Equality Constraints Using Python

```
from scipy.optimize import linprog
\#Max z=x+2y
#subject to
#2x+y<=20
#-4x+5y<=10
#-x+2y>=-2
\#-x+5y=15
\#x,y>=0
obj = [-1, -2]
lhs_ineq = [[ 2, 1], # Red constraint left side
        [-4, 5], # Blue constraint left side
         [1, -2]] # Yellow constraint left side
rhs_ineq = [20, # Red constraint right side
         10, # Blue constraint right side
         2] # Yellow constraint right side
lhs_eq = [[-1, 5]] # Green constraint left side
                 # Green constraint right side
rhs_eq = [15]
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
          A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
          method="revised simplex")
```

method ="revised simplex" solves linear programming problem using two phase simplex method.

```
con: array([0.])
      fun: -16.8181818181817
 message: 'Optimization terminated successfully.'
   slack: array([ 0. , 18.18181818, 3.36363636])
  status: 0
 success: True
         x: array([7.72727273, 4.54545455])
 File Edit View Insert Cell Kernel Widgets Help
                                                                                        Trusted Python 3 O
 ~
    In [1]: M from scipy.optimize import linprog
    In [2]: ► #Max z=x+2y
            #Max 2-x+2y
#subject to
#2x+y<=20
#-4x+5y<=10
            \#-4x+5y<=10

\#-x+2y>=-2

\#-x+5y=15

\#x,y>=0

obj = [-1, -2]
    In [5]: | lhs_eq = [[-1, 5]] # Green constraint left side
    In [6]: M rhs_eq = [15] # Green constraint right side
    In [8]: M opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
                  A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
method="revised simplex")
    In [9]: 🔰 opt
      Out[9]:
              con: array([0.])
fun: -16.818181818181817
             message: 'Optimization terminated successfully.'
    nit: 3
             slack: array([ 0. , 18.18181818, 3.36363636])
status: 0
success: True
                 x: array([7.72727273, 4.54545455])
```

BigM Simplex Method using Python

Solve Following linear programming problem using Big M Simplex method.

Min z = 4x1 + x2

subjected to:

$$3x1 + 4x2 >= 20$$

$$x1 + 5x2 >= 15$$

$$x1, x2 >= 0$$

from scipy.optimize import linprog

$$obj = [4, 1]$$

lhs_ineq = [[-3, -4], # left side of first constraint

... [-1, -5]] # right side of first constraint

rhs_ineq = [-20, # right side of first constraint

... -15] # right side of Second constraint

bnd = [(0, float("inf")), #Bounds of x1]

... (0, float("inf"))] # Bounds of x2

>>> opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,

... bounds=bnd,method="interior-point")

>>> opt

method =" interior-point" solves linear programming problem using default simplex method.

Solve Following linear programming problem using Big M Simplex method.

```
Min z= 4x1 + x2
subjected to:

3x1 + 4x2 >= 20
X1 + 5x2 >= 15
X1, x2 >= 0

In (41]: H from .9cipy.optimize import linprog
In (42]: H obj (4, 1]

In (43]: H lhs_ineq = [[ -3, -4], f left sids of first constraint
(-1, -5]] # right sid of first constraint
In (44]: H rh!!_ineq = {-20, i right: sid of first constraint
-15] right side of Second constraint

In (45]: H bnd = {(0, float("inf")), I Bounds of x2

In (48]: M >>> opt = linprog(e=obj, A_ub=!1!!!_i.neq, b_ub=rn!!_i.neq, bub=rn!!_i.neq, b
```

RESOURCE ALLOCATION PROBLEM BY SIMPLEX METHOD

Use SciPy to solve the resource allocation problem stated as follows:

Max z=20x1 + 12x2 + 40x3 + 25x4(profit)

subjected to:

from scipy.optimize import linprog

obj = [-20, -12, -40, -25] #profit objective function

lhs_ineq = [[1, 1, 1, 1], # Manpower

... [3, 2, 1, 0], # Material A

... [0, 1, 2, 3]] # Material B

rhs_ineq = [50, # Manpower

... 100, # Material A

... 90] # Material B

opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,

... method="revised simplex")

Opt

Use SciPy to solve the resource allocation problem stated as follows:

```
Max z= 20x1 + 12x2 +40x3 + 25x4......
                                              ...(profit)
           subjocled to:
              x1 + x2 + x3 + x4
                                       50----- (manpower)
              3x1 + 2x2 + K3
                                      100 ----- (m.atei::ial A)
                       x.2 + 2x3
                                       90 ----- (m.atei::ial 8)
                   x.1. x.2, x3, x.4 >= 0
In [12]: M rrom :3cipy.optimize import linpi::og
In [13]: M obj = [-20, -12, -40, -25]
                                                    #prQfit objective function
In [14]: M lh:3 i.neq = [[1, 1, 1, 11, # l'.,gnpover [3, 2, 1, 0], # i'f.:teric.l - [0, 1, 2, 311 # Materia.1 B
In [15]: M :i::h:3_ineq = 50, # ivr.,gnpover
                                  100, #- i'lateria.1 A
90] # i'la teria.1 B
```

```
\label{eq:ling_problem}  \begin{array}{ll} \text{In } [161; & II \text{ opt } & \text{li.nprog}(c = obj, \ A\_ub = lh: 3\_1.neq, \ b\_ub = :t: h: 3 \ ineq, \\ & method = ".revi: 3ed \ : 3implex") \end{array}
```

The i::e:3ult tell:3 yo,u that the m.axim.alpi::ofit i:31900 and coi::re:3pc-nd:3 to x1 = 5 and ib = 45. I-c':3 not p?:Dfitable to p:::oduce the :3econd and fo,urth p::::Dduct:3 unde::: the given condition:3 You can draw :3evei::al interi::e:3ting conclu:3ion:3 he!:e:

The thii::d preduct b:::ing:3 the lai::ge:3t pi::ofi t pe::: unit, :30the factor}; will produce it the mo:3"t.

The fir:3t :3lack i:3 O, whim mea.n:3 that the value:3 of the left and right :3ide:3 of the manpower (:fir:3t) on:3traint the :3ame. The factoi::ypi::oduce:3 50 unit:3pe::: day, a.nd tha-c':3 it:3 full capacity.

The :3econd :3lack i:3 40 becau:3e the factm:y con:3ume:3 60 unit:3 of raw materizarial A (15 unit:3 for the :fir:3t principal to plue:3 45 for the third) out of a potential 100 unit:3.

The thiii:d:3lack i:3 0, whim mea.n:3-cha-c the :factoi::y con:3ume:3 all 90 unit:3 o,f the i::aw matei::ial B. Thi:3 entii::e amount i:3 con:3umed :fm: the thii::dpr:oduct. That':3 why the :factoi::y can't pr:oduce the :3econd or: fourthpr:oduct at all a.nd ca.n't p:::oduce mo:::e than 45 unit:3 of the thi:::dproduct. It lack:3 the i::aw material B.

opt .:3tatu:3 i:3 0 and opt

i:3 True, indicating that the op-cimization problem wa:3 :3ucce:3:3:fully :301-..red with the

INFEASIBILITY IN SIMPLEX METHOD

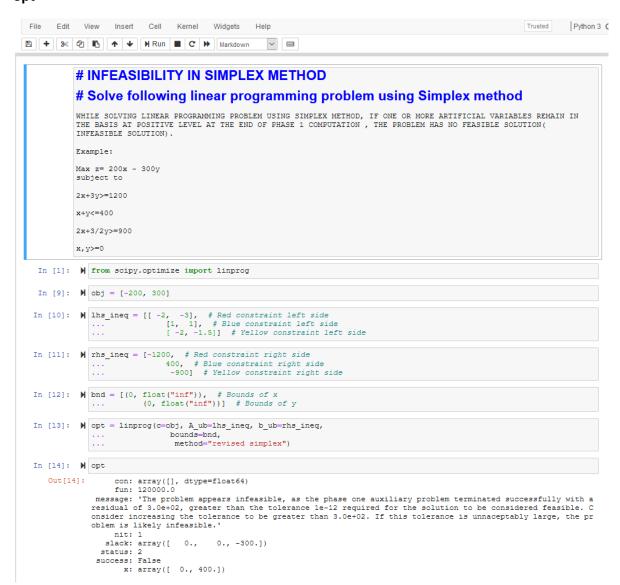
Solve following linear programming problem using Simplex method

WHILE SOLVING LINEAR PROGRAMMING PROBLEM USING SIMPLEX METHOD, IF ONE OR MORE ARTIFICIAL VARIABLES REMAIN IN THE BASIS AT POSITIVE LEVEL AT THE END OF PHASE 1 COMPUTATION, THE PROBLEM HAS NO FEASIBLE SOLUTION(INFEASIBLE SOLUTION).

```
Example:
Max z = 200x - 300y
subject to
2x+3y>=1200
x+y<=400
2x+3/2y>=900
x,y>=0
from scipy.optimize import linprog
obj = [-200, 300]
lhs ineq = [[ -2, -3], # Red constraint left side
... [1, 1], # Blue constraint left side
        [-2, -1.5]] # Yellow constraint left side
rhs_ineq = [-1200, # Red constraint right side
         400, # Blue constraint right side
         -900] # Yellow constraint right side
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
         bounds=bnd,
```

... method="revised simplex")

opt



DUAL SIMPLEX METHOD

##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R PROGRAMMING

Max z=40x1+50x2

#subject to

#2x1 + 3x2 <= 3

#8x1 + 4x2 <= 5

x1, x2>=0

Import IpSolve package

library(lpSolve)

Set coefficients of the objective function

f.obj <- c(40, 50)

Set matrix corresponding to coefficients of constraints by rows

Do not consider the non-negative constraint; it is automatically assumed

f.con <- matrix(c(2, 3,

Set unequality signs

```
"<=")
```

Set right hand side coefficients

f.rhs <- c(3,

5)

Final value (z)

lp("max", f.obj, f.con, f.dir, f.rhs)

Variables final values

lp("max", f.obj, f.con, f.dir, f.rhs)\$solution

Sensitivities

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.from lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.to

Dual Values (first dual of the constraints and then dual of the variables)

Duals of the constraints and variables are mixed

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals

Duals lower and upper limits

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.from lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.to

OUTPUT:

```
##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R PROGRAMM
> # Max z=40x1+50x2
> #subject to
> #2x1 + 3x2 <= 3
> #8x1 + 4x2 <= 5
> \# x1, x2>=0
> # Import lpSolve package
> library(lpSolve)
> # Set coefficients of the objective function
> f.obj <- c(40, 50)
> # Set matrix corresponding to coefficients of constraints by rows
> # Do not consider the non-negative constraint; it is automatically assumed
> f.con <- matrix(c(2, 3,
                    8, 4), nrow = 2, byrow = TRUE)
> # Set unequality signs
> f.dir <- c("<=",
             "<=")
> # Set right hand side coefficients
> f.rhs <- c(3,
             5)
> # Final value (z)
> lp("max", f.obj, f.con, f.dir, f.rhs)
Success: the objective function is 51.25
> # Variables final values
> lp("max", f.obj, f.con, f.dir, f.rhs)$solution
[1] 0.1875 0.8750
> # Sensitivities
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.from
[1] 33.33333 20.00000
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.to
[1] 100 60
> # Dual Values (first dual of the constraints and then dual of the variables)
> # Duals of the constraints and variables are mixed
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals
[1] 15.00 1.25 0.00 0.00
> # Duals lower and upper limits
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.from
[1] 1.25e+00 4.00e+00 -1.00e+30 -1.00e+30
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.to
[1] 3.75e+00 1.20e+01 1.00e+30 1.00e+30
```

TRANSPORTATION PROBLEM

##sOLVE FOLLOWING TRANSPORTATION PROBLEM IN WHICH CELL ENTRIES REPRESENT UNIT COSTS USING R PROGRAMMING.

"Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY

#sUPPLIER 1	10	2	20	11	15
#sUPPLIER 1	12	7	9	20	25
#sUPPLIER 1	4	14	16	18	10
#DEMAND	5	15	15	15	

Import IpSolve package

library(lpSolve)

Set transportation costs matrix

4, 14, 16, 18), nrow = 3, byrow = TRUE)

Set customers and suppliers' names

Set unequality/equality signs for suppliers

row.signs <- rep("<=", 3)

```
# Set right hand side coefficients for suppliers

row.rhs <- c(15, 25, 10)

# Set unequality/equality signs for customers

col.signs <- rep(">=", 4)

# Set right hand side coefficients for customers

col.rhs <- c(5, 15, 15, 15)

# Final value (z)

TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)

# Variables final values

lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution

print(TotalCost)
```

OUTPUT:

```
> ##solve following transportation problem in which cell entries represent unit costs u
              "Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY
> #sUPPLIER 1 10
                                             20
                                                           11
                                                                        15
                12
> #sUPPLIER 1
                             7
                                             9
                                                           20
                                                                        25
> #sUPPLIER 1
                4
                            14
                                             16
                                                           18
                                                                        10
> #DEMAND
                 5
                             15
                                             15
                                                           15
> # Import lpSolve package
> library(lpSolve)
> # Set transportation costs matrix
> costs <- matrix(c(10, 2, 20, 11,
                   12, 7, 9, 20,
                   4, 14, 16, 18), nrow = 3, byrow = TRUE)
> # Set customers and suppliers' names
> colnames(costs) <- c("Customer 1", "Customer 2", "Customer 3", "Customer 4")</pre>
> rownames(costs) <- c("Supplier 1", "Supplier 2", "Supplier 3")</pre>
```

```
> # Set unequality/equality signs for suppliers
> row.signs <- rep("<=", 3)</pre>
> # Set right hand side coefficients for suppliers
> row.rhs <- c(15, 25, 10)
> # Set unequality/equality signs for customers
> col.signs <- rep(">=", 4)
> # Set right hand side coefficients for customers
> col.rhs <- c(5, 15, 15, 15)
> # Final value (z)
> TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
> # Variables final values
> lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution
    [,1] [,2] [,3] [,4]
[1,]
      0 5 0 10
[2,]
      0 10 15 0
                0
                     5
[3,]
      5 0
> print(TotalCost)
Success: the objective function is 435
```

>

ASSIGNMENT PROBLEM

#SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMING

```
# Assignment Problem
```

```
# JOB1 JOB2 JOB3

#W1 15 10 9

#W2 9 15 10

#W3 10 12 8
```

Import IpSolve package

library(lpSolve)

Set assignment costs matrix

```
costs <- matrix(c(15, 10, 9,
9, 15, 10,
10, 12,8), nrow = 3, byrow = TRUE)
```

Print assignment costs matrix

costs

Final value (z)

lp.assign(costs)

Variables final values

lp.assign(costs)\$solution

OUTPUT:

- > #SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMI
- > # Assignment Problem
- > # JOB1 JOB2 JOB3

```
> #W1 15 10 9
> #W2 9 15 10
> #W3 10 12 8
                         10
> # Import lpSolve package
> library(lpSolve)
> # Set assignment costs matrix
> costs <- matrix(c(15, 10, 9,
                    9, 15, 10,
                    10, 12 ,8), nrow = 3, byrow = TRUE)
> # Print assignment costs matrix
> costs
    [,1] [,2] [,3]
[1,] 15 10 9
[2,]
      9 15 10
[3,] 10 12
                8
> # Final value (z)
> lp.assign(costs)
Success: the objective function is 27
> # Variables final values
> lp.assign(costs)$solution
    [,1] [,2] [,3]
[1,] 0 1 0
[2,] 1 0 0
[3,] 0 0 1
```

>