

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
- A. 0.3875
B. 0.2676
C. 0.5
D. 0.6987

Answer -: B

The work begin after 10 min, so the average time increase from 45min to 55min.

for normal distribution -:

$$\begin{aligned} z &= (X - \mu) / \sigma \\ &= (60 - 55) / 8 \\ &= 0.625 \end{aligned}$$

Find probability

$$\begin{aligned} &= \text{round}(1 - \text{stats.norm.cdf}(\text{abs}(0.625)), 5) \\ &= 0.26599 \end{aligned}$$

The probability that the service manager cannot meet his commitment is 0.2659

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2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

- A. More employees at the processing center are older than 44 than between 38 and 44.

Answer -:

False. Because the probability for employees at the processing center are more between 38 and 44 than older than 44.

Employees older than 44 yrs of age

$$= 1 - \text{stats.norm.cdf}(44, \text{loc}=38, \text{scale}=6)$$

$$= 0.15865525393145707$$

Employees between 38 to 44 yrs of age

$$= \text{stats.norm.cdf}(44, 38, 6) - \text{stats.norm.cdf}(38, 38, 6)$$

$$= 0.3413447460685429$$

- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer -: True

Employees under 30 yrs of age

$$= \text{stats.norm.cdf}(30, 38, 6)$$

$$= 0.09121121972586788$$

No. of employees attending training program from 400 nos. is

$$N * P(X < 30)$$

$$= 400 * \text{stats.norm.cdf}(30, 38, 6)$$

$$= 36.484487890347154$$

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer -:

The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then ($X_1 +$

X_2) and $(2X_1)$ tends to have Normal distribution only If X_1 and X_2 are i.i.d and n is Large.

The Difference between $2X_1$ and $(X_1 + X_2)$ is the magnitude they hold of two different sample subsets (X_1 and X_2) from the same source(population). X_1 and X_2 can be a different subset of a sample from a similar source (population) but If $X_1 \sim N(\mu, s^2)$ then, $2X_1 \sim N(2\mu, 4s^2)$ If $X_1 \sim N(\mu, s^2)$ and $X_2 \sim N(\mu, s^2)$ are iid normal random variables then $(X_1 + X_2) \sim N(\mu + \mu, s^2 + s^2) = N(2\mu, 2s^2)$ Hence, $2X_1 - (X_1 + X_2) \sim N(2\mu - 2\mu, 4s^2 + 2s^2) = N(0, 6s^2)$ The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

Answer :- D

`= stats.norm.interval(0.99,100,20)`

`= (48.48341392902199, 151.516586070978)`

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer -:

Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

B. Specify the 5th percentile of profit (in Rupees) for the company

Answer -:

The 5TH Percentile of profit for the company is 17 Crore Rupees

C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer -:

The Division #2 (Profit2 ~ N(7, 42)) has a larger probability of making a loss in a given year
