1. $m(a+bx_i) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

$= \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i \Rightarrow \cancel{\frac{Na}{N}} + \frac{b}{N}\sum_{i=1}^{N} x_i$

$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right) = a + b\,m(x)$ ✓

$\boxed{\therefore m(a+bx_i) = a+b\,m(x)}$

use rule from part 1

2. $cov(x, a+bY) = \frac{1}{N}\sum_{i=1}^{N}\left[(x_i - m(x))\left((a+by_i) - m(a+bY)\right)\right]$

$= \frac{1}{N}\sum_{i=1}^{N}\left[(x_i - m(x))\left((\cancel{a}+by_i) - (\cancel{a}+b\,m(Y))\right)\right]$

$= \frac{1}{N}\sum_{i=1}^{n}\left[(x_i - m(x))\left(b(y_i - m(Y))\right)\right]$

pull constant out

$= b \cdot \frac{1}{N}\sum_{i=1}^{N}\sum\left[(x_i - m(x))(y_i - m(Y))\right]$

$= b \cdot cov(X,Y) \Rightarrow \boxed{\therefore cov(x, a+by) = b \cdot cov(X,Y)}$

3. $cov(a+bx, a+bx) = \frac{1}{N}\sum_{i=1}^{N}\left[\left((a+bx_i) - m(a+bx)\right)^2\right]$

we know from question 2 that this term equals $b(x_i - m(x))$

$= \frac{1}{N}\sum_{i=1}^{N}\left[b(x_i - m(x))\right]^2 = \frac{1}{N}\sum_{i=1}^{N}\left[b^2 (x_i - m(x))^2\right]$

$= b^2 \frac{1}{N}\sum_{i=1}^{N}\left[(x_i - m(x))^2\right]$

standard dev $s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2}$

so this is equal to s^2

$cov(a+bx, a+bx) = b^2 s^2$

$cov(x,x) = \frac{1}{N}\sum_{i=1}^{N}\left[(x_i - m(x))(x_i - m(x))\right] = \frac{1}{N}\sum_{i=1}^{N}\left[(x - m(x))^2\right] \Rightarrow \therefore \boxed{cov(x,x) = s^2}$

$cov(a+bx, a+bx) = b^2 s^2 = b^2 cov(x,x)$

$\boxed{\therefore cov(a+bx, a+bx) = b^2 cov(x,x)}$