35 kW of power is to be transmitted at 450 rpm (pinion speed) to a shaft with a speed ratio of 3.5 through $20^0$ stub spur gear drive. The load is steady and continuous. The center distance is 400 mm approximately. Design the gear drive for strength. Check the design for dynamic load and wear. Allowable static stress of forged steel pinion is 172 MPa and cast steel gear is 137 MPa. Pinion is hardened to BHN 350 and gear is hardened to BHN 250

Given $P = 35$ kW $\quad N_p = 450$ rpm $\quad \frac{N_p}{N_g} = \frac{T_g}{T_p} = \frac{D_g}{D_p} = 3.5$

$\sigma_p = 172$ MPa $\quad \sigma_g = 137$ MPa

Centre distance between the shaft $= \frac{D_p}{2} + \frac{D_g}{2}$

$$\frac{D_p}{2} + \frac{D_g}{2} = 400$$

$$D_p + 3.5 D_p = 800 \Rightarrow D_p = 177.7 \text{ mm}$$

let's take $D_p = 180$ mm

$$D_g = 180 \times 3.5 = 630 \text{ mm}$$

Actual centre distance $= 180/2 + 630/2 = 405$ mm

Torque $T = \frac{60P}{2\pi N} = \frac{60 \times 35 \times 1000}{2\pi \times 450} = 742.7$ Nm

Transmitted load (Tangential force)

$$F_t = \frac{T}{D_p/2} = \frac{742.7}{180/2 \times 10^{-3}} = 8252 \text{ N}$$

Pitch line velocity

$$V = \frac{\pi D_p N_p}{60} = \frac{\pi \times 180 \times 10^{-3} \times 450}{60}$$

$$= 4.24 \text{ m/s}$$

Velocity factor $C_v = \frac{3}{3+V} = 0.41$

To determine Lewis form factor, we will assume at least 15 number of teeth for the pinion. In order to satisfy the required velocity ratio,

$$T_p = 16$$

$$T_g = 16 \times 3.5 = 56$$

Find $y_p$ and $y_g$ from table

$20^\circ$ stub involute, $T_p = 16$, $y_p = 0.115$

$T_g = 56$, $y_g = 0.151 + \frac{(0.154 - 0.151)}{10} \times 6 = 0.1528$

Determination of weaker gear

$$\sigma_p y_p = 172 \times 0.115 = 19.8$$

$$\sigma_g y_g = 137 \times 0.1528 = 20.9$$

Since $\sigma_p y_p < \sigma_g y_g$, pinion is weaker. Hence the design is based on pinion.

Find the module from Lewis equation

Choose $b = 4\pi m$

$$F_t = C_v F_b = C_v \sigma_p y_p \pi \, 4\pi m \, m$$

$$m^2 = \frac{F_t}{C_v \sigma_p y_p 4\pi^2} = \frac{8252}{0.41 \times 19.8 \times 10^6 \times 4\pi^2}$$

$$m^2 = 2.57 \times 10^{-5}$$

$$m = 5.07 \times 10^{-3} \, m = 5.07 \, mm$$

Choose the standard module, $m = 6$ mm

$$b = 4\pi m = 75.4 \, mm$$

$$T_p = \frac{D_p}{m} = \frac{180}{6} = 30$$

$$T_g = 3.5 \times 30 = 105$$

Again find $y_p$ and $y_g$

20° stub, $T_p = 30$, $y_p = 0.139$

20° stub $T_g = 105$, $y_g = 0.1614$

Determine weaker gear of pinion and gear again

$\sigma_p y_p = 172 \times 0.139 = 23.9$ MPa

$\sigma_g y_g = 137 \times 0.1614 = 22.1$ MPa

Since $\sigma_g y_g < \sigma_p y_p$, gear is weaker

$C_v F_b = C_v \sigma_g y_g \pi b m$

$= 0.41 \times 22.1 \times 10^6 \times \pi \times 75.4 \times 10^{-3} \times 6 \times 10^{-3}$.

$= 12878$ N

As $C_v F_b > F_t$ (8252 N), the design is safe from the standpoint of strength.

Adjust the width of gear

$$b = \frac{F_t}{C_v \sigma_g y_g \pi m} = \frac{8252}{0.41 \times 22.1 \times 10^6 \times \pi \times 6 \times 10^{-3}}$$

$= 0.048$ m $= 48$ mm

Optimum range $b \rightarrow 3\pi m$ to $4\pi m$

$b = 3\pi \times 6 = 56.55$ mm

We choose $b = 57$ mm

Dynamic tooth load - Buckingham's equation

$$F_d = \frac{21V\,(bc + F_t)}{21V + \sqrt{bc + F_t}} + F_t$$

For $V = 4.24\ m/s$ permissible error $\simeq 0.076\ mm$

Select a first class commercial gear for which error $\simeq 0.06$ for $m = 6\ mm$ which is less than permissible error.

for $20^\circ$ stub with steel pinion and steel gear

$$C = 600.4 + \frac{(900.6 - 600.4)}{(0.075 - 0.050)} \times (0.060 - 0.050)$$

$$= 720\ kN/m$$

$$F_d = \frac{21 \times 4.24 \times [57 \times 10^{-3} \times 720 \times 10^{3} + 8252]}{21 \times 4.24 + \sqrt{57 \times 10^{-3} \times 720 \times 10^{3} + 8252}} + 8252$$

$$= 22367\ N$$

$$F_b = \sigma_g y_g \pi b m = 22.1 \times 10^{6} \times \pi \times 57 \times 10^{-3} \times 6 \times 10^{-3}$$

$$= 23744\ N$$

Hence the design is safe from the standpoint of dynamic load.

Wear strength - Buckingham's equation

$$F_w = D_p\, b\, Q\, K$$

$D_p = 180\ mm$ $\qquad b = 57\ mm$

$$Q = \frac{2T_g}{T_p+T_g} = \frac{2\times105}{30+105} = 1.55$$

From the table for pinion BHN 350 and gear BHN 250

$$K = 1.3518\ MPa$$

$$F_w = 180\times10^{-3}\times57\times10^{-3}\times1.55\times1.3518\times10^{6}$$

$$= 21498\ N$$

Since $F_w < F_d$, design is not safe from the standpoint of wear

$$F_d = 22367\ N$$

Hence we select a carefully cut gear

expected error $\approx 0.026$ for $m = 6\ mm$

$$C = 312\ kN/m$$

$$F_d = \frac{21v(bC+F_t)}{21v+\sqrt{bC+F_t}} + F_t$$

$$= \frac{21\times4.24(57\times312+8252)}{21\times4.24+\sqrt{57\times312+8252}} + 8252$$

$$= 17510\ N$$

Hence $F_b = 23744\ N$ $\quad F_d = 17510\ N$ $\quad F_w = 21498\ N$

Since $C_v F_b > F_t$ - strength

$F_b > F_d$ - dynamic load

$F_w > F_d$ - wear

Design is safe

# Dimensions of the gear

i) Number of teeth on the pinion $(T_p)$ = 30
ii) Number of teeth on the gear $(T_g)$ = 105
iii) Module (m) = 6 mm
(iv) Face width (b) = 57 mm
(v) Pitch circle diameter of the pinion $(D_p)$ = 180 mm
(vi) Pitch " gear $(D_g)$ = 630 mm
(vii) Addendum (0·8m) = 4·8 mm
(viii) Dedendum (1m) = 6 mm
(ix) Clearence (0·2m) = 1·2 mm
x) Tooth thickness (1·5708m) = 9·425 mm
xi) Fillet radius (0·4m) = 2·4m

We select a carefully cut gear for our design