

Production Function: Concepts & Theories

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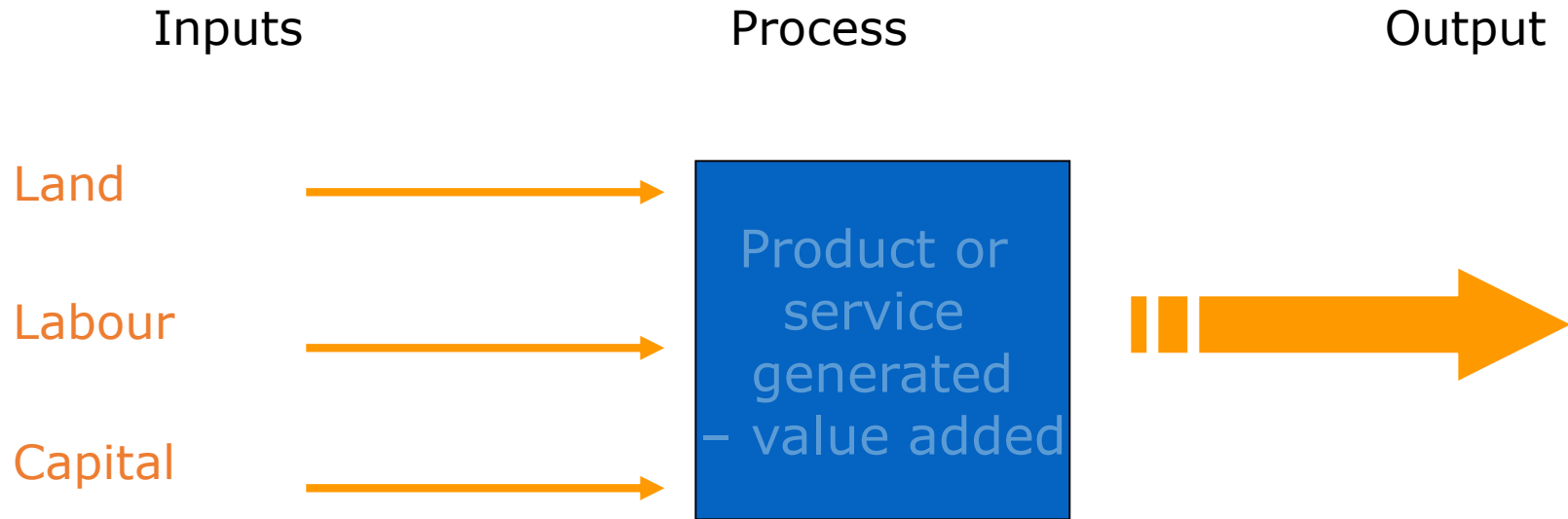
Definition of Production function

- *The production function is a mathematical equation determining the relationship between the factors and quantity of input for production and the number of goods it produces most efficiently.*
- In simple words, it describes the method that will enable the maximum production of goods by technically combining the major factors of production- land, enterprise, labour and at a certain timeframe using a specific technology most efficiently. It changes with development in technology.

Production function

- Production function refers to the functional relationship between the quantity of a good produced (output) and factors of production (inputs).
- The production function is a technical or engineering relation between input and output.

Production Function



Production function

The production function can be mathematically written as

$$Q = F(L, Lb, K, T, t, \dots)$$

L=land.

Lb=labor.

K=capital.

T=Technology.

t=time.

- States the relationship between inputs and outputs
- **Inputs** – the factors of production classified as:
 - **Land** – all natural resources of the earth .
Price paid to acquire land = **Rent**
 - **Labour** – all physical and mental human effort involved in production
 - Price paid to labour = **Wages**
 - **Capital** – buildings, machinery and equipment not used for its own sake but for the contribution it makes to production
 - Price paid for capital = **Interest**

- Let us consider a famous garments company that produces the latest designer wear for customers.
- It requires three types of inputs for producing the designer garments: cloth, industrial sewing machine, and tailor as an employee.
- The variables- cloth, tailor, and industrial sewing machine is the variable that combines to constitute the function. The Production function will then determine the quantity of output of garments as per the number of inputs used. The industrial sewing machine can sew ten pieces of garments every hour. The tailor can use these sewing machines to produce upto five pieces of garment every 15 minutes. The length of clothing that the tailor will use per piece of garment will be 2 meters.

- After including the data into the above formula, which is
- Quantity of output, $Q = \min(\text{input-1}, \text{input-2}, \text{input-3})$ where input1= cloth, input 2= industrial sewing machine and input 3 = tailor
- Production function Q , in one hour = $\min(\text{input 1}, \text{input 2}, \text{input 3}) = \min(\text{cloth} + \text{tailor} + \text{industrial sewing machine}) = \min(2\text{mtrs per piece}, 20 \text{ pieces by tailor}, 20 \text{ pieces by machine}) = \min(40 \text{ meters}, 20 \text{ pieces}, 20 \text{ pieces})$
- From the above, it is clear that if there are:
- Only 100 mtrs cloth are there then only 50 pieces of the garment can be made in 1 hour
- With only one machine, 20 pieces of production will take place in 1 hour.
- Only one tailor can help in the production of 20 pieces.
- Therefore, the best product combination of the above three inputs – cloth, tailor, and industrial sewing machine- is required

Production function –Short run & Long run

- The production function is differently defined in the short run and in the long run. The distinction is based on the nature of factor inputs.
- Those inputs that vary directly with the output are called *variable factors*. These are the factors that can be changed. Variable factors exist in both, the short run and the long run. Examples of variable factors include daily-wage labour, raw materials, etc.
- On the other hand, those factors that cannot be varied or changed as the output changes are called *fixed factors*. These factors are normally characteristic of the short run or short period of time only. Fixed factors do not exist in the long run.
- Production function in the short run is called : law of variable proportions OR Law of diminishing marginal returns
- Production function in the long run is called the Returns to Scale.

- Consequently, we can define two production functions:
- short-run and long-run.
- The *short-run production function*, namely the *law of diminishing marginal returns of the law of variable proportions* defines the relationship between one variable factor (keeping all other factors fixed) and the output.
- The long run production function is called the *returns to scale*. Here, all factors are varied in the same proportion. The law that is used to explain this is called the *law of returns to scale*. It measures by how much proportion the output changes when inputs are changed proportionately.

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- Consequently, we can define two production functions: short-run and long-run. The short-run production function defines the relationship between one variable factor (keeping all other factors fixed) and the output which is called the **law of variable proportions or law of diminishing marginal returns**.

- The law of diminishing returns operates in the short run when we can't change all the factors of production.
- Technically, the law states that as we increase the quantity of one input which is combined with other fixed inputs, the marginal physical productivity of the variable input must eventually decline.
- the total productivity, for a given state of technology, is bound to increase with an increase in the quantity of a variable input. However, as the quantity of the inputs keeps on increasing, the marginal product rises to a maximum, then starts to decline and eventually becomes negative.
- This is because the crowding of inputs eventually leads to a negative impact on the output.

- **Total Product:** Total product is the total output obtained from the combined efforts of all the factors of production. Further, if we wish to find the effect of one factor of production, say labour, on the total product, we need to keep all the other factors constant. In this case, the total product would vary with the factor kept variable.
- **Marginal Product:** The change in the total product when one more unit is added to the variable factor is known as the marginal product.
- **Average Product:** Average product is the total product per unit of the variable factor. In other words, it is the ratio of total product to the quantity of variable factor.

Analysis of the short run

- In the short run **at least one factor fixed** in supply but all other factors capable of being changed
- Reflects ways in which firms respond to changes in output (demand)
- Increase in total capacity only possible in the long run

Law of variable proportion

Also called as Law of Diminishing marginal returns

LAWS OF DIMINISHING RETURNS

- The Law of Variable Proportions

- It refers to the behaviour of output as the quantity of one input is increased while the other inputs are held constant.
- It states that as successive units of a variable resource say labor are added to a fixed resource say land, so beyond some point the extra or marginal product will decline.

LAWS OF DIMINISHING RETURNS

It proclaims that while increasing one input and maintaining other inputs constant initially helps in increasing the output, a further increase in the input will have a restricted effect and will ultimately have nil consequence or a pessimistic effect on the output.

The Law of Diminishing Marginal Productivity helps in understanding why increased manufacturing isn't always the best way to increase profitability.

ASSUMPTIONS OF THE LAW OF VARIABLE PROPORTION

- Short Run
- Constant Technology
- Homogeneous Factors

Basic Concepts of the Law

- Total Product (TP)
- Marginal Product (MP)
- Average Product (AP)

- The law of diminishing marginal returns comes into play whenever a firm tries to increase output by applying additional variable inputs to a fixed factor.
- Production requires the combination of both fixed and variable factors to create an output.
- Economic theory predicts that if firms increase the number of variable factors they use, such as labour, while keeping one factor fixed, such as machinery, the extra output or returns from each additional, marginal unit of the variable factor must eventually diminish.

- **Total Product** means the total amount of product produced by a producer in a given period.
- **Average product** is always computed per unit of an input. That is, average product per labour means number of output produced by a producer per the number of labour employed .
- **Marginal product** is computed by differentiating the total product function. That is, it refers to change in output as a result of one unit change in input . Or, marginal product of labour means that what is the net addition to total output as a result of addition of one more labour as an input.

LAW OF VARIABLE PROPORTION

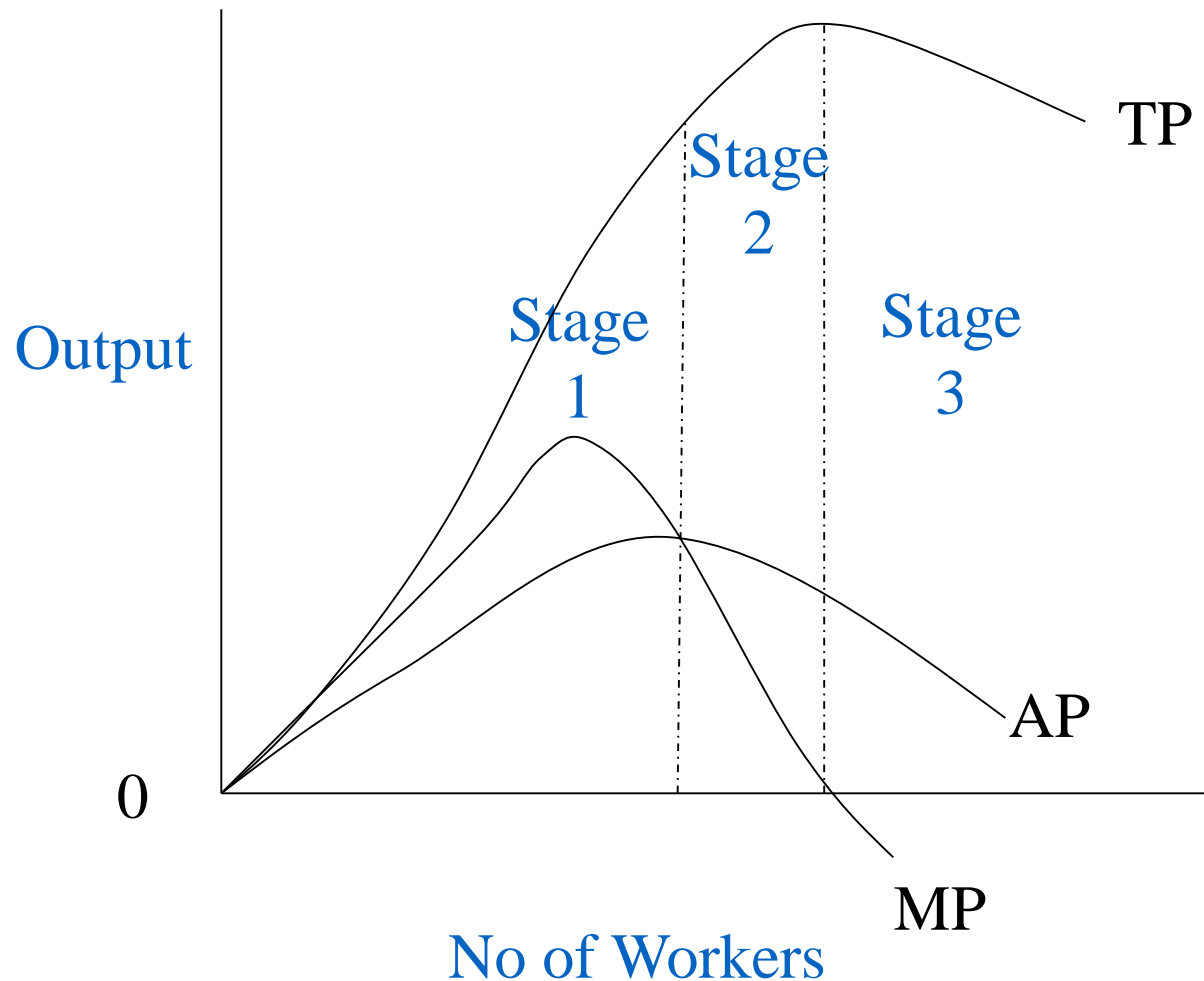
No. of Workers	Total Product	Marginal Product	Average Product
0	0	-	-
1	10	10	10
2	25	15	12.50
3	45	20	15
4	60	15	15
5	70	10	14
6	75	5	12.5
7	75	0	10.71
8	70	-5	8.75

Increasing
Marginal
Return

Diminishing
Marginal
Return

Negative
Marginal
Return

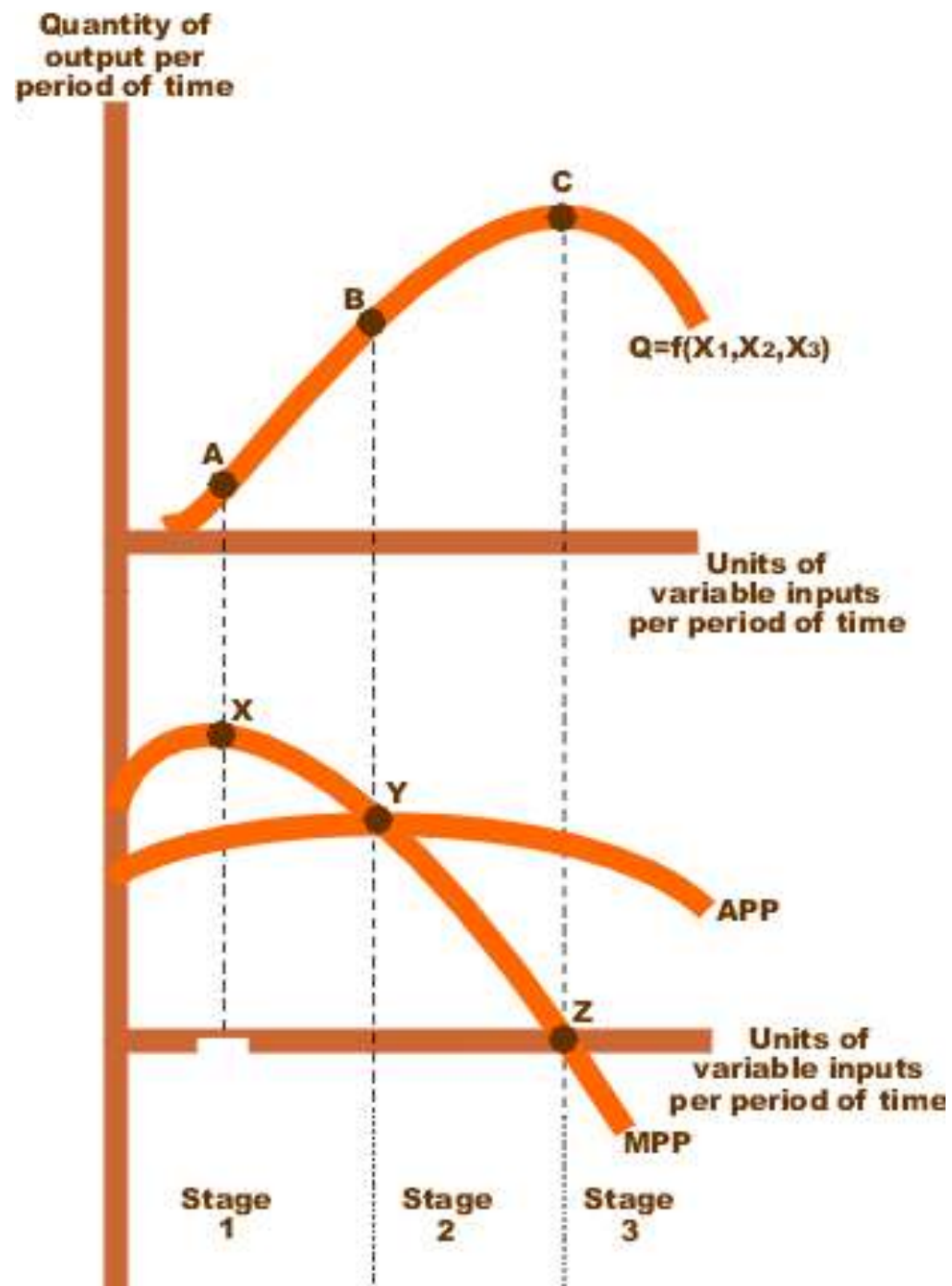
LAW OF VARIABLE PROPORTION



LAW OF VARIABLE PROPORTION

- **Stage 1:** Increasing Returns, TP increases at increasing rate, MP increases at decreasing rate, decreases and is greater than AP, AP goes to the maximum point.
- **Stage 2:** Diminishing Returns, TP increases at decreasing rate and reaches maximum point, MP goes on diminishing, reaches to zero and is less than AP, AP starts decreasing.
- **Stage 3:** Negative returns, TP starts decreasing, MP goes to negative and AP goes on decreasing but greater than MP.

- A rational producer will never operate in stages I and III.
- In stage III, the contribution of additional labour to the total product (marginal product) is negative so it will be unwise to employ an input that reduces the total product.
- In stage I where the average product is rising, so an increase in input increases output in greater proportion.
- Rational producer should produce in stage II



The Relationship between Average Product and Marginal Product

- When there is a rise in the average product due to an increase in the quantity of the variable input, the marginal product is more than the average product.
- The maximum average product is equal to the marginal product. Simply put, the maximum point of the average product curve is also a point on the marginal product curve, a point where both of these curves intersect.
- When the average product falls, the marginal product is less than the average product.

- **Key Points**

1. Diminishing Marginal Returns occur when an extra additional production unit produces a reduced level of output.
2. Some of the causes of diminishing marginal returns include: fixed costs, limited demand, negative employee impact, and worse productivity.
3. The law of diminishing marginal returns is closely associated with disceconomies of scale – where the business starts to become less efficient due to its size.

- Technically, the law states that as we increase the quantity of one input which is combined with other fixed inputs, the marginal physical productivity of the variable input must eventually decline.
- In simpler words, the total productivity, for a given state of technology, is bound to increase with an increase in the quantity of a variable input. However, as the quantity of the inputs keeps on increasing, the marginal product rises to a maximum, then starts to decline and eventually becomes negative.
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Analysis of the long run function

- The long run is defined as the period of time taken to vary all factors of production
 - By doing this, the firm is able to increase its **total capacity** – not just short term capacity
 - Associated with a change in the **scale of production**
 - The period of time varies according to the firm and the industry

Returns to Scale

RETURNS TO SCALE

- The laws of Returns to Scale study the behaviour of production when all the productive factors or inputs are increased or decreased simultaneously in the same ratio.
- We analyze here the effect of doubling, trebling and so on of all the inputs of productive resources on the output of the product.
- It has also three distinct stages
 - Increasing Returns to Scale
 - Constant Returns to Scale
 - Diminishing Returns to Scale

RETURNS TO SCALE

S. No.	Scale of Production	Total Returns	Marginal Returns	
1	1 Worker + 3 Acres of land	2	2	
2	2 workers + 6 Acres of land	5	3	Increasing Returns
3	3 +9	9	4	
4	4 +12	14	5	
5	5 +15	19	5	Constant Returns
6	6 +18	24	5	
7	7 +21	28	4	
8	8 +24	31	3	Diminishing Returns
9	9 +27	33	2	

Alternatives of the long run production

- **Constant returns to the scale**

output increases in the same proportion as the increase in the input.

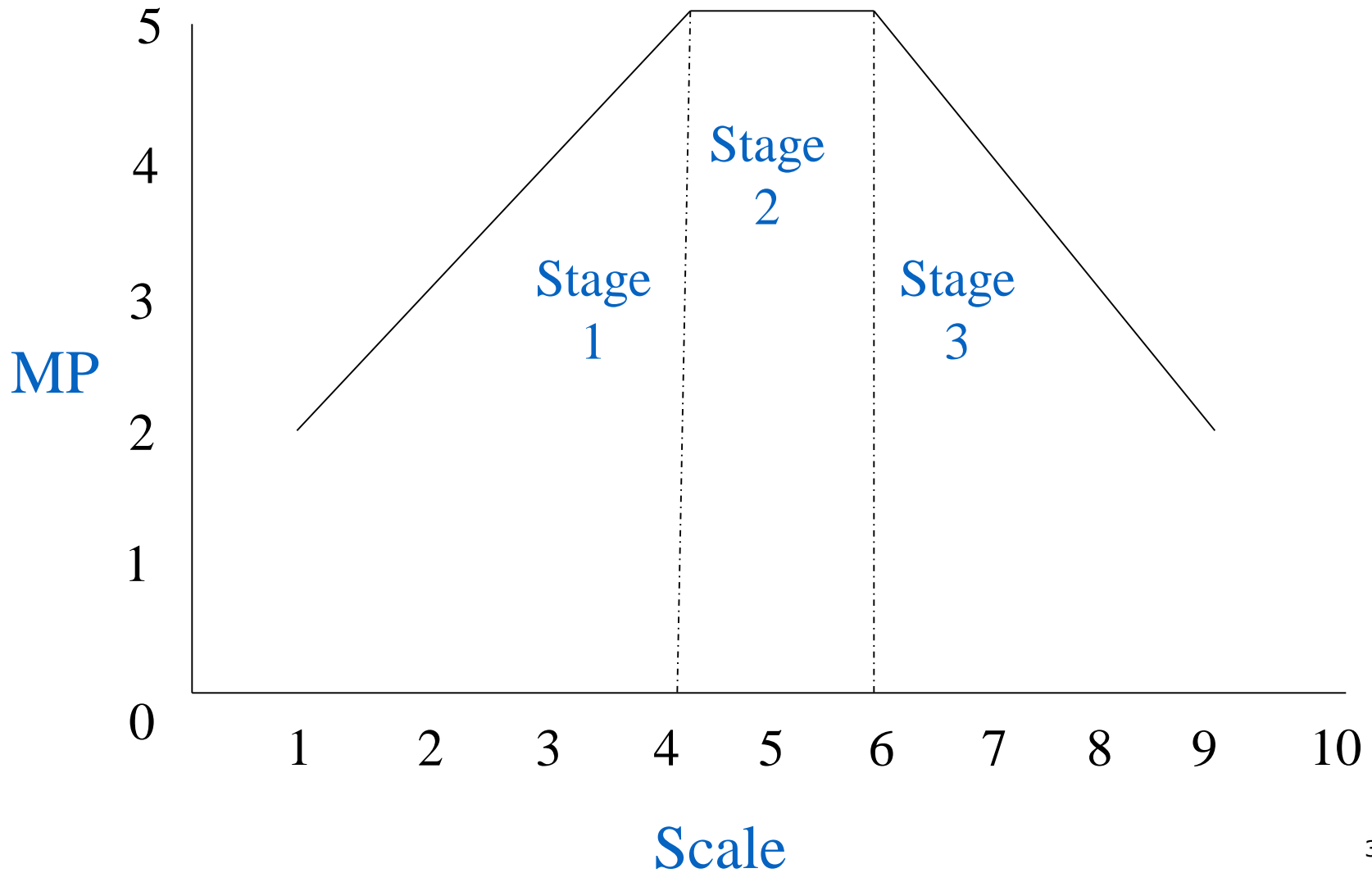
- **Increasing return to scale**

output increases by a greater proportion than the increase in inputs.

- **Decreasing returns to the scale.**

output increases in the lesser proportion than the increase in the inputs.

RETURNS TO SCALE



- **Increasing returns to scale** or diminishing cost refers to a situation when all factors of production are increased, output increases at a higher rate. It means if all inputs are doubled, output will also increase at the faster rate than double. Hence, it is said to be increasing returns to scale.
- **Diminishing returns** or increasing costs refer to that production situation, where if all the factors of production are increased in a given proportion, output increases in a smaller proportion. It means, if inputs are doubled, output will be less than doubled. If 20 percent increase in labour and capital is followed by 10 percent increase in output, then it is an instance of diminishing returns to scale.
- The main cause of the operation of diminishing returns to scale is that internal and external economies. Constant returns to scale or constant cost refers to the production situation in which output increases exactly in the same proportion in which factors of production are increased. In simple terms, if factors of production are doubled output will also be doubled.