Calculus →

1) why we need calculus?

optimization → To find the best set of weight vectors.

Q→ Can it be done with Linear Algebra?

— Yes, but it would be slow as the search-space is pretty large.

— Linear Algebra uses brute force (try every possible combination of weight vector) to find the optimal set of weights.

→ Calculus can do this optimization faster.

— using gradient descent.

Calculus Basics →

1) Maxima / minima → The maximum & minimum value of a fn.

Q→ How to use calculus with ML →

Let there be a fn we want to optimize.

$\hat{y} = f(x)$ Data → $\{x, y\}$

Goal → Modify the optimization fn such that $\hat{y} \approx y$.

in → optimize loss fn to get the best fit.

$$\text{gain}(\{x_i, y_i\}, \vec{w}, w_0) = \sum_{i=1}^{n} \frac{w^T x_i + w_0}{\|\vec{w}\|} \cdot y_i$$

why sum, why not product?

— with a zero in the middle the π would become zero.

2) Domain / Range of a fn → The space for inputs & outputs.

A fn relates an input in Domain to an output in Range.

→ Value that a fn attains for very small change in x.

3) Limit & Continuity →

↳ Let there be a point x.
let $\Delta x \to 0$ be a very minimal quantity.

$x + \Delta x$ → right hand limit.
$x - \Delta x$ → left hand limit.

$\lim\limits_{x \to a} fn(x)$ → Limit exists at $fn(x)$ iff

1) left-hand limit == right-hand limit.

2) $fn(x)$ exists at point x.

why use limits →

1) Define the value of a fn at a point where it may not be defined.

2) Define instant rate of change.

Continuity → $fn(x)$ → Continuous / discontinuous

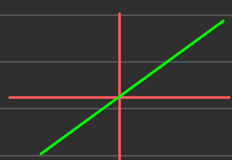
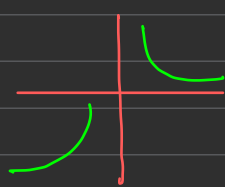
→ Smooth curve

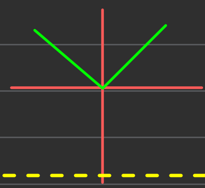
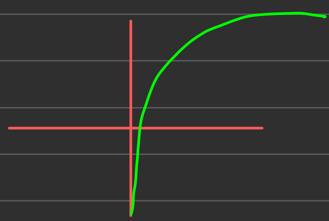
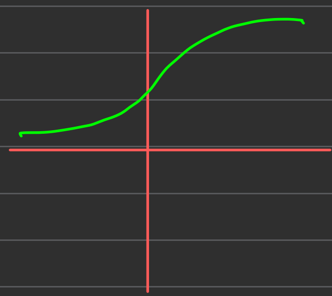
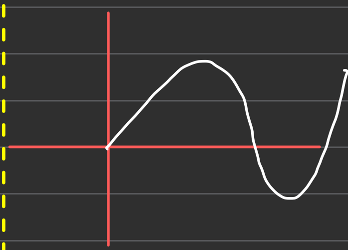
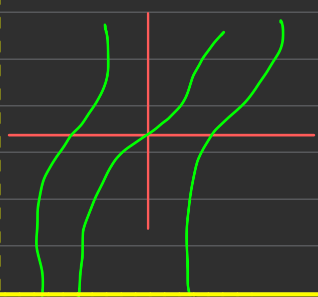
↳ steps are holes in fn.

A fn is continuous at a point x iff:

1) fn is defined at the point x.
2) Limit of $f(x)$ exists at x.
3) The value of $fn(x) = \lim\limits_{x \to a} fn(x)$.

Important fns →

Function	Domain	Range	Continuous	Graph
1) $y = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	Yes	
2) $y = 1/x$	$(-\infty, \infty)$	$(-\infty, \infty)$	NO	

3) $y = e^{x}$	$(-\infty, \infty)$	$(0, \infty)$	Yes	
4) $y = \lvert x \rvert$	$(-\infty, \infty)$	$(0, \infty)$	Yes	
5) $y = \log(x)$	$(0, \infty)$	$(-\infty, \infty)$	Yes	
6) $y = 1/(1+e^{-x})$	$(-\infty, \infty)$	$(0, 1)$	Yes	
7) $y = \sin(x)$	$(-\infty, \infty)$	$[-1, 1]$	Yes	
8) $y = \tan\theta$	(∞, ∞)	$(-\infty, \infty)$	no	

Derivatives :-

$$y = f(x), \quad \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

ex :-

$$f(x) = x^2$$

$$f'(x) = \frac{d f(x)}{dx} = \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + \Delta x^2 + 2x\Delta x - x^2}{\Delta x} = \frac{\Delta x(\Delta x + 2x)}{\Delta x} = 2x$$

1) $\frac{d\, x^n}{dx} = n x^{n-1}$

2) $\frac{d \sin x}{dx} = \cos x$

3) $\frac{d \cos x}{dx} = -\sin x$

4) $\frac{d \log x}{dx} = 1/x$

---

1) Sum rule :- $\frac{d(fx + gx)}{dx} = f'x + g'x$

2) Product rule :- $\frac{d(fx \cdot gx)}{dx} = f'x\, gx + g'x\, fx$

3) Quotient rule :- $\frac{d(fx/gx)}{dx} = \frac{f'x\, gx - g'x\, fx}{(gx)^2}$

4) Chain rule :- $\frac{d(f(g(x)))}{dx} = f'(gx) \cdot g'x$

Gradient Descent →

$f(x) = x^2 - x + 2$

$f'(x) = 2x - 1 \longrightarrow$ minima/maxima co-ordinate.

$f''(x) = 2 \longrightarrow$ $>0 \rightarrow$ minima
$<0 \rightarrow$ maxima

$2x - 1 = 0 \Rightarrow x = 1/2 \longrightarrow$ minima at $x = 1/2$.

$f'(x)$ — slope of tangent at x on the fn.

+ve → slope +ve
−ve → slope −ve