Modular Arithmetic :-

Basic Properties :-

1) $(a+b) \% m = (a \% m + b \% m) \% m$
2) $(a \times b) \% m = (a \% m \times b \% m) \% m$
3) $(a-b) \% m = (a \% m - b \% m + m) \% m$
4) $a^b \% m = (a \% m)^b \% m$
5) $(a+m) \% m = (a \% m + m \% m) \% m = (a \% m) \% m$
$= a \% m$

Fast Power with $\% m$

```
int pow (a, n, m)
    if (n == 0) return 1;
    half = pow(a, n/2, m);
    if (n % 2 == 0)
        return (half * half) % m;
    else
        return ((a * half) % m * half) % m;
```

Important to divide at what step % m should be done.

half → [0 - m-1]

Ex :-

pow(4, 7, 5)

half = pow(4, 3, 5) = 4

half = pow(4, 1, 5)

half = pow(4, 0, 5) = 1

return = 4

n → odd

return

((4 × 4) % 5 × 4) % 5 = 4 % 5 = 4

GCD → Greatest common divisor.

gcd(15, 25) = 5

gcd(12, 30) = 6

15 = 1 × 3 × 5 × 15

25 = 1 × 5 × 25

12 = 1 × 2 × 3 × 4 × 6 × 12

30 = 1 × 2 × 3 × 5 × 6 × 10 × 15 × 30

gcd(0, a) = a

if a % y = 0 → y is a factor of x.

0 % any # = 0 → every # is a factor of zero.

min gcd = 1 → 1 is a factor to all the #s.

Properties →

1) gcd(a, b) = gcd(b, a)

2) gcd(0, a) = a

3) gcd(a, b, c) = gcd(a, gcd(b, c)) = gcd(gcd(a, b), c)

4) gcd(a, b) = gcd(a − b, b)

5) gcd(a, b) = gcd(a % b, b)

Proof gcd(a, b) = gcd(a − b, b)

let gcd(a, b) = d

→ a % d = 0 & b % d = 0

(a − b) % d = 0

let gcd(a − b, b) = t

→ (a − b) % t = 0 & b % t = 0

If $a \% b = 0$

d → factor of $a-b$ & b & a.
t → factor of a & b & $a-b$.

$\Rightarrow d = t$.

$\Rightarrow gcd(a, b) = gcd(a-b, b)$

```
int gcd(a, b)                    TC < O(log(max(a,b)))
    if(a == 0) return b;
    return gcd(b%a, a)   // swap & mod at the same step.
```

Q → Find gcd of all the elements in the array

```
ans = gcd(0, a[0])
for(i = 1 to n-1) ans = gcd(ans, a[i]);
return ans;
```

TC → $O(n \times \log(\max\#))$
SC → $O(1)$

Permutation & Combination →

→ Addition & Multiplication rule →

Q → How many couples can be formed with 10 guys & 7 girls.

Ans = $10 \times 7 = 70$

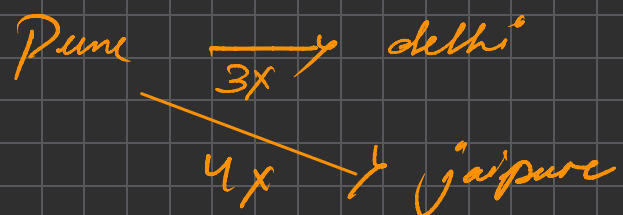
Pune —3→ Delhi —2→ Agra

Q → how many ways can you travel from pune to agra via delhi?

Ans: $2 \times 3 = 6$

And → multiply

or → addition



Q → How many ways can you travel from Pune to delhi via jaipure?

Ans = 7.

Permutation → Arrangement of objects.

$$ {}^{n}P_{r} = \frac{\lfloor n}{\lfloor n-r} = \frac{n \times n-1 \times \ldots (n-r) \times \ldots \rfloor}{(n \times r)} $$

$n \times n-1 \times \ldots (n-r+1)$

→ # of choosable elements.