$y = mx + c$

$ax + by + c = 0$

$by = -c - ax$

$\rightarrow y = -\frac{a}{b} x - \frac{c}{b}$

y - intercept.

$m = -a/b \, , \quad c = -\frac{c}{b}$

$\hookrightarrow$ slope.

let line-1 $\rightarrow$ $y = m_1 x + c_1$

$y = m_2 x + c_2$

if $m_1 \times m_2 = -1$ $\rightarrow$ Intersecting lines.

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

let they intersect at (x_1, y_1)

$a_1 x + b_1 y + c_1 = a_2 x + b_2 y + c_2 = 0$

$y = m_1 x + c_1$

$y = m_2 x + c_2$

$(b_1 - b_2) y = (a_2 - a_1) x + c_2 - c_1$

$\rightarrow y = \left(\frac{a_2 - a_1}{b_1 - b_2} \right) x + \frac{c_2 - c_1}{(b_1 - b_2)}$

D-2

1) Matrix multiplication.
2) Dot Product
3) Cosine Similarity
4) unit vector
5) Vector Projection.

Goal →

1) Get the best line.

2) Classify the new point to +ve/-ve.

1) Dot Product →

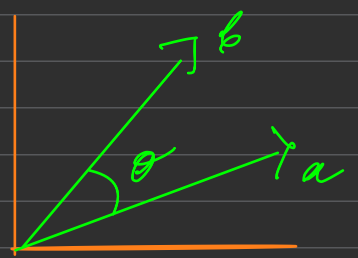
$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \qquad w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}_{3\times 1}$$

Column vectors.

$$w^T x = w_1 x_1 + w_2 x_2 + w_3 x_3$$
$$= \sum_{i=1}^{d} w_i x_i$$

Physics definition → $a^T \cdot b = |a| \cdot |b| \cdot \cos\theta$

↳ angle b/w a & b.



Ex →

	x_1	x_2	x_3	x_4	x_5
a	10	5	0	1.5	5
b	1	0.5	0	1.5	0.5
c	0	10	5	0	0

$a \cdot b = 10 \times 1 + 5 \times 0.5 + 1.5 \times 1.5 + 5 \times 0.5 = 37.5$

$a \cdot c = 5 \times 10 = 50$

$a \cdot c > a \cdot b$ (In reality b is more similar to a than c).

Limitation

- but a.c has a higher magnitude than a.b.
- They are highly similar according to dot product but differ significantly in reality.

2) Cosine Similarity →

$x \cdot y = \|x\| \cdot \|y\| \cos\theta$

$\Rightarrow \cos\theta = \dfrac{x^T y}{|x| \cdot |y|}$

- Checks similarity w.r.t direction.

$$\cos(a,b) = \frac{a \cdot b}{|a| \cdot |b|} = \frac{37.5}{19.36 \times 1.936} \simeq 1$$

$|a| = 19.36$
$|b| = 1.936$
$|c| = 12.25$

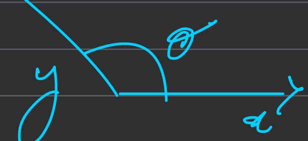
$$\cos(a,c) = \frac{a \cdot c}{|a| \cdot |c|} = \frac{50}{19.36 \times 12.25} = 0.315$$

$\therefore$ a is more similar to b than c.

→ Why use cosine similarity when you don't get a sense of direction?

→ Angle b/w two vectors & magnitude of individual vectors.

eg →



high dot product
low cosine similarity.

Vectors are less similar

average dot product.
high cosine similarity.

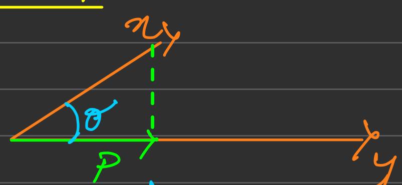
Vectors are actually closer.

3) **Unit vector** → Let $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

$$\text{unit vector}(w) = \hat{w} = \frac{w}{\|w\|}$$

Unit vector of a vector x = Vector in the direction of x but with a magnitude 1.

→ **Vector Projection** →



Projection of $\vec{x}$ of $\vec{y}$ given θ angle b/w them is the amount of $\vec{x}$ that lies on $\vec{y}$.

$$\cos\theta = \frac{|P|}{|x|} \quad \text{--- (1)}$$

$$\cos\theta = \frac{x^T y}{|x| \cdot |y|} \quad \text{--- (2)}$$

Comparing (1) & (2) →

$$\frac{|P|}{\cancel{|x|}} = \frac{x^T y}{\cancel{|x|}\,|y|} = \hat{y} = \frac{\vec{y}}{|y|}$$

$$\Rightarrow \boxed{|P| = x^T \hat{y}}$$

$$\text{Proj}(x, y) = \vec{x} \cdot \hat{y}.$$

→ Distance of a point from a line →

let line be : $m_1 x + m_2 y + m_3 = 0$
let the plane be : (x_0, y_0)

⊥ distance from point to line :

$$\frac{|m_1 x_0 + m_2 y_0 + m_3|}{\sqrt{m_1^2 + m_2^2}}$$

$$\frac{w^T x + w_0}{|w|}$$

$$w = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

→ Distance b/w a plane & point →

let the plane be $m_1 x + m_2 y + m_3 z + m_4 = 0$
let the point be (x_0, y_0, z_0)

$$\perp \text{ dist} = \frac{|m_1 x_0 + m_2 y_0 + m_3 z_0 + m_4|}{\sqrt{m_1^2 + m_2^2 + m_3^2}}$$

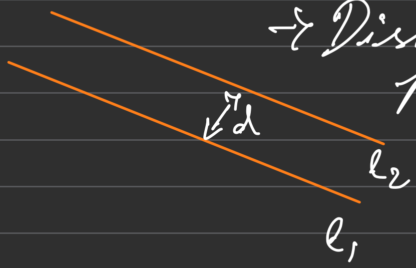
→ Towards & Away from normal →

let line be $m_1 x + m_2 y + m_3 = 0$
let a point be (x_0, y_0)

if $\quad m_1 x_0 + m_2 y_0 + m_3 > 0 \longrightarrow$ Point lies towards the normal.

if $\quad m_1 x_0 + m_2 y_0 + m_3 < 0 \longrightarrow$ Point lies away from normal.

→ Distance b/w two planes/lines →

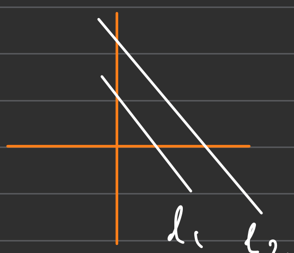


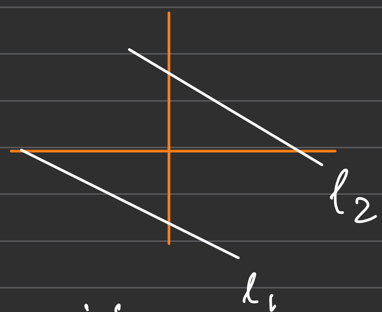
→ Distance can be found out only if two planes/lines are parallel to each-other.

$l_1 \to w^T x + k_1 \qquad l_2 \to w^T x + k_2$

$$d = \frac{|k_2 - k_1|}{\|w\|}$$

Q → How to know if two lines are in the same side of origin or different ?





let $d_i \to$ distance of l_i from origin.

→ if $|l_2 - l_1| == d(l_1, l_2) \longrightarrow$ Lines lie on same side of origin.

if $l_2 + l_1 == d(l_1, l_2) \to$ Lines lie on opposite side of origin.

+ve/-ve halfspace →

let line $\to w^T x + b$

let point be (x_i, y_i)

if $\quad w^T x_i + b > 0 \longrightarrow$ Point lies in the half.
Point lies in the direction of normal.