Example Problems Week of 2014/2/3, last updated 2014/1/31/217pm
CSC 280: Computer Models & Limitations Spring 2014
Instructor: Lane A. Hemaspaandra TAs: Dan Hassin, Mark Mullock, Xing Yan

1 Easy Problems

1. Draw a DFA accepting the following language, over the alphabet $\{a, b\}$: $L = \{w \mid$ the number of symbols in w is odd, i.e., the string w is of odd length.)$\}$. Then specify the DFA formally, as a 5-tuple, giving each part of the 5-tuple.

2. Draw a DFA accepting the following language, over the alphabet $\{a, b\}$: $L = \{w \mid$ the number a's in w is at most two and the number of b's in w is at least two$\}$. Then specify the DFA formally, as a 5-tuple, giving each part of the 5-tuple.

3. Draw an NFA accepting the language of all strings, over the alphabet $\{a, b\}$, that end with aba or with bab. Then specify the NFA formally, as a 5-tuple, giving each part of the 5-tuple.

4. Draw an NFA accepting the language of all strings, over the alphabet $\{a, b\}$, that have abba as a substring.

2 Harder Problems

1. Draw a DFA accepting the following language, over the alphabet $\{0, 1\}$: All strings that when interpreted as binary integers are multiples of 7. (What is the integer related to a string for this problem? Ignore leading 0's. Treat the empty string as if it is interpreted as the value 0. So 00000 represents 0 and 0101 represents 5 and 101 represents 5.) (Hint: The best way to do this is to be inspired by long division. The reason I say that that is the best is the long-division inspired attack gives you a way to more generally solve division problems, across differs bases and across whatever positive integer you want to divide by.) Now, after solving this, do you see how to change your answer to capture exactly the strings that interpreted as binary integers are 3 greater than a multiple of 7 (i.e., the integer is congruent, modulo 7, to 3).

2. Draw an NFA accepting the following language, over the alphabet $\{0, 1\}$: All strings w such that w contains 011 as a substring and also contains 100 as a substring. (Warning: we have not said which comes first, and, also, it is legal for them to overlap.)

3. Let Σ be the standard 26-letter alphabet $\{a, b, \cdots, z\}$. $L = \{a_1a_2a_2a_1 \mid a_1 \in \Sigma \text{ and } a_2 \in \Sigma\}$. Describe an DFA for L (a picture-sketch is ok, as long as it is quite clear). Argue that each DFA accepting L must contain at least 26^2 states. (Do this by arguing that if any two of the 26^2 two-symbol strings lead to the same state as each other, the DFA will make an error on some four-symbol string.) (Note that by generalizing this, we can easily show, for example, natural lower bounds of, for example, 26^{1776} states for an appropriate similar language.)

3 Very Challenging Problems

1. Let $\Sigma = \{0, 1\}$. Let A be an arbitrary regular set over alphabet Σ. Consider the language of first halves of strings in A: $L = \{w \mid (\exists y)[|w| = |y| \wedge wy \in A\}$. Prove that L is regular. (Hint: Describe an NFA for L.) (Food for thought: What about first 1/3s? Middle thirds? Final thirds? Thirteenth seventeenths? Answer: They're all regular.)

2. Let $\Sigma = \{0, 1\}$. Let A be an arbitrary regular set over alphabet Σ. Let B be an arbitrary set over alphabet Σ (it might not be regular!). Consider the language $L = \{w \mid (\exists y \in B)[ywy \in A\}$. Prove that L is regular. (Hint: Describe an NFA for L. This is a rather hard problem inspired by the closure of regular sets under quotient.)