



A REPORT ON THE SUDOKU SOLVER

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ABSTRACT

In the last decade, solving the Sudoku puzzle has become every one's passion. The simplicity of puzzle's structure and the low requirement of mathematical skills caused people to have enormous interest in accepting challenges to solve the puzzle. Therefore, developers have tried to find algorithms to generate the variety of puzzles for human players so that they could be even solved by computer programming. In this essay, we have presented an algorithm called pencil-and-paper using human strategies. The purpose is to implement a more efficient algorithm and then compare it with another Sudoku solver named brute force algorithm. This algorithm is a general algorithm that can be employed into any problems. The results have proved that the pencil-and-paper algorithm solves the puzzle faster and more effectively than the brute force algorithm.

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1 Introduction

Currently, Sudoku puzzles are becoming increasingly popular among people all over the world. The game has become popular now in many countries and many developers have tried to generate even more complicated and more interesting puzzles. Today, the game appears in almost every newspaper, in books and on many websites.

In this essay we present a Sudoku Solver named pencil-and-paper algorithm using simple rules to solve the puzzles. The pencil-and-paper algorithm is formulated based on human techniques. This means that the algorithm is implemented based on human perceptions. Therefore, the name of the solver is pencil-and-paper algorithm. The Brute force algorithm is then used to compare with this algorithm to evaluate the efficiency of the proposed algorithm. The brute force is a general algorithm that can be applied to any possible problem. This algorithm generates any possible solutions until the right answer is found.

The following subsections describe the problem statement, the purpose of this project, and the abbreviations and the definitions.

1.1 Problem Statement

Solving Sudoku has been a challenging problem in the last decade. The purpose has been to develop more effective algorithms to reduce the computing time and utilize lower memory space. This essay develops an algorithm for solving Sudoku puzzles by using a method, called pencil-and-paper algorithm. This algorithm resembles human methods, i.e. it describes how a person tries to solve the puzzle by using certain techniques. Our aim is to implement the pencil-and-paper algorithm by using these techniques.

There are currently different variants of Sudoku such as 4X4 grids, 9X9 grids and 16X16 grids. This work is focused on classic and regular Sudoku of 9X9 board, and then a comparison is performed between the paper-and-pencil method and Brute force algorithm. Hopefully, by doing this work we might be able to answer the following questions: How does the pencil-and-paper algorithm differ from the Brute force algorithm? Which one of them is more effective? Is it possible to make these algorithms more efficient?

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>
1			9				7	2	8
2	2	7	8			3		1	
3		9					6	4	
4		5			6		2		
5			6				3		
6		1			5				
7	1			7		6		3	4
8				5		4			
9	7		9	1			8		5

Fig.1. An example of a Sudoku puzzle.

1.2 Purpose

The aim of the essay is to investigate the brute force algorithm and the pencil-and-paper algorithm. Later these two methods are compared analytically. It is expected here to find an efficient method to solve the Sudoku puzzles. In this essay we have tried to implement the pencil-and-paper algorithm that simulates how human beings would solve the puzzle by using some simple strategies that can be employed to solve the majority of Sudoku.

1.3 Abbreviations and Definitions

In this essay we have tried to use the same terminology, which is commonly used in other journals and research papers. In the following paragraph, there is a brief description of some of the abbreviations and definitions that are used in the text.

Sudoku: is a logic-based, combinatorial number placement puzzle [4]. The word “Sudoku” is short for Su-ji wa dokushin ni kagiru (in Japanese), which means “the numbers must be single”, see Fig. 1.

Box (Region, Block): a region is a 3x3 box like the one shown in figure 1. There are 9 regions in a traditional Sudoku puzzle.

Cell (Square): is used to define the minimum unit of the Sudoku board.

Candidates: the number of possible values that can be placed into an empty square.

Clues: the given numbers in the grid at the beginning.

Grid (board): the Sudoku board consists of a form of matrix or windows.

2 Background

This section starts with an explanation about Sudoku. Then research is carried out on the previous works about this subject. Later, we discuss further evaluated algorithms and finally a description of how the work is being carried out is presented.

2.1 Short about Sudoku

Sudoku is a logic-based puzzle that is played by numbers from 1 to 9. The Puzzle first appeared in newspapers in November 1892 in France and then Howard Garns an American architect presented it in its modern form [1,5]. There are already many journals, papers and essays that researched about Sudoku Solvers and most of them present different types of algorithms. Sudoku's popularity is based on several reasons. First, it is fun and fascinating, and very easy to learn because of its simple rules. There are currently many different types of Sudoku puzzles, classic Sudoku that contains a 9X9 grid with given clues in various places, mini-Sudoku that consists of a grid with 4X4 or 6X6 sizes. The other type of Sudoku is Mega Sudoku that contains a grid with 12X12 or 16X16 sizes [2]. In this text, the focus is mostly on the classic Sudoku, i.e. 9X9 grid. Furthermore, Sudoku has become so popular, compared to other games, all over the world because its rules are easy to understand, and it can improve our brain and it is fun.

The structure of the puzzle is very simple, especially the classic puzzle. This essay is mainly focused on the classic puzzle of a 9X9 grid. There already exist several digits in the board that make the puzzle solvable. It means that some numbers are already placed in the Sudoku board before starting to play. The board consists of 81 cells, which are divided into nine 3X3 sub boards and each 3X3 sub board is called "box" or "region". The main concept of the game is to place numbers from 1 to 9 on a 9X9 board so that every row, column and box contains any numbers but once. This means that no number is repeated more than once. An example of this game is illustrated in Fig.1.

Generally, the puzzle has a unique solution. There are certain techniques to solve the puzzle by hand and these rules can be implemented into a computer program. These techniques are presented in more detail in 2.3.

2.2 Previous Research

We have noticed that there is a large volume of published studies describing Sudoku problems. Furthermore, several research have been made to solve Sudoku problems in a more

efficient way [4]. It has conclusively been shown that solving the puzzle, by using different algorithms, is possible but most developers seek for optimizations techniques such as genetic algorithms, simulated annealing, etc.

Different authors have made relative works already. *Nelishia Pillay* gives a solution for solving Sudoku by combining human intuition and optimization [5]. This author has investigated the use of genetic programming to improve a space of programs combined of the heuristics moves. However, we seek a solution to solve Sudoku puzzle based on human strategies, which uses techniques such as: naked single method, hidden single method etc.

J.F. Crook also discussed solving Sudoku and presented an algorithm on how to solve the puzzles of differing difficulty with pencil-and-paper algorithm. This method has not been implemented and therefore it is hard to discuss how the algorithm performs [3]. *Tom Davis* has done research about “The Mathematics of Sudoku”. Tom has described all techniques that people usually use to solve puzzles, but his major attempt is to describe these techniques from a mathematical perspective. However, all the strategies he mentions are not required to solve the puzzle. For instance, the easy puzzles can be solved using only one or two strategies [7].

2.3 Examined algorithms

2.3.1 Pencil-and-paper algorithm

In this work, we implement a solution based on some strategies used by humans when solving the puzzle, therefore, it is called pencil-and-paper algorithm.

The paper-and-pencil algorithm contains human strategies. These strategies have been examined below in more detail. These techniques are almost easy to understand for human players, but it might be hard to search in the puzzle, since there are several things to look for at the same time. As there are puzzles with different types of difficulty, the easy and medium puzzles can be solved using some simple techniques such as unique missing method, naked singles. However, to solve difficult problems we may examine other techniques as well (locked candidates, naked and hidden pairs, triplets etc.) [7].

Unique missing candidate

The unique missing candidate is used when any row, column or box is missing only one single digit.

Naked Singles

This method is useful when we find a square that can only take one single value, once the contents of other squares in the same row, column and box are considered. Additionally, this

is when the row, column and box hold 8 different numbers and one single number is left for that square, see Fig. 2.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>
1				1		4			
2			1				9		
3		9		7		3		6	
4	8		7				1		6
5							3		
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>
1				1		4			
2			1				9		
3		9		7		3		6	
4	8	2	7				1		6
5			6				3		
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

Fig. 2. A description of the naked single method. In the left figure square 4b can hold just one possible number, which is 2 as it is inserted in the right figure.

As we see in figure 2, it is possible to list all the candidates from 1 to 9 in each unfilled square, i.e. square 4b can only hold number 2 since it is the only candidate for this position. The most significant aspect is that when a candidate is found for a certain position then it can be removed from the list as a possible candidate in the row, column and box [7]. The reason that it is called the “naked single” method is that this kind of square contains only one possible candidate.

Hidden Singles

The hidden single method is like the naked single method but the way to find the way to find the empty square is different. When there is only one square in the row, column or box that can take a certain number, then the square must take that number. For example, in figure 3, we can see that both row2 and row3 contain the digit 9 so according to the rules, row1 must also hold number 9 (in the square 123def). In the right side of figure 3 below, number 9 is inserted by using the hidden singles method.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>
1				1		4			
2			1				9		
3		9		7		3		6	
4	8		7				1		6
5									
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>
1				1	9	4			
2			1				9		
3		9		7		3		6	
4	8		7				1		6
5									
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

Fig. 3. These two figures show how the hidden single technique can be used [8].

Locked candidate

Tom Davis has stated that it is possible to find a box where the only place for a candidate would be a row or column within that box [7]. If a candidate belongs to a row or column then we can remove this candidate as a possible one with other boxes that the row (or column) connected with them. This condition is shown in figure 3. The box $456abc$ must hold a 9 and the only positions the number can be placed are in squares $5a$ or $5c$ and both in the same row. This means that the number 9 is eliminated in row 5 from other boxes that include row 5.

Naked Pairs, Triplets

These techniques are very similar to the naked single technique, but in this method, we find the same two candidates in two squares. By using this information, we can find a possible candidate to other squares. For example, in figure 4, squares $9d$ and $9f$ can only contain values 2 and 7. By having this knowledge, it is obvious that square $9d$ and $9f$ cannot contain 1 or 6 so those candidates are removed. The only candidates are 2 and 7 in squares $9d$ and $9f$.

	a	b	c	d	e	f	g	h	i
1	9	6			1			3	
2	3		3				8		4
3		7						9	6
4				3		8			
5	6		9					8	5
6				4		9			
7		2		5	8	4		6	
8	5		8				2		7
9		4		2	7	9	3	5	

Fig. 4. An illustration of naked pair technique [8].

There are other techniques (e.g. x-wings, swordfish, etc.) the reader, therefore, is referred to [7,8].

2.3.2 Brute force algorithm

Kovacs describes some of the brute force methods used for solving Sudoku puzzles [9]. The simplest method randomly produces a solution to the puzzle called “unconstrained grid”, after that the program checks whether it is a valid solution. If not, the process is repeated until a solution is found. This algorithm can be applied simply and will find a valid solution for any problems because it will go through all possibility solutions. However, this method can be time consuming but according to Kovacs the algorithm can be optimized.

Generally, the brute force algorithm goes through the empty squares, filling in numbers from the existing choices, or removing failed choices if a “dead-end” is reached. For example, Brute force solves a puzzle by inserting the digit “1” in the first square. If the digit is allowed to be there by checking row, column and box then the program goes to the next square and put the digit “1” in that square. The program discovers that the “1” is not allowed, then the digit increments by one i.e. it has become 2. When a square is noticed where none of the digits (1 to 9) is permitted, then the program backtracks and comes back to the prior square. The value in those squares increases by 1. The process is repeated until the correct digits fill all 81 squares.

2.4 Approach

Here we carried out a study on how to solve Sudoku puzzles based on the pencil-and-paper algorithm. The concept of this work is to implement a solution to solve the puzzle based on

human strategies. The obtained results are compared with a well-known algorithm called Brute force algorithm that are presented in chapter 3. The result and conclusions are presented in chapter 4.

3 Analysis and Results

This section starts with analysis and discussions about two mentioned algorithms. A comparison is carried out between two algorithms to find out which algorithm is more efficient. At the end of the section, there are discussions on the difficulty level of the puzzles and time complexity.

3.1 Pencil-And-Paper Solver

There are several methods that are used by human players when playing Sudoku. However, it may be impossible to implement all these methods. It is found that the hidden single method or pair method are difficult to apply in computer programming, since a human player has a better overview over the whole Sudoku board than the computer programming does. This is since a human player can scan two rows or two columns to check whether a certain digit is allowed to be in an empty square in the box that is supposed to be filled up. Implementing the above task in computer programming causes significant time consumption.

The methods that are used in this algorithm are the following:

- Unique missing candidate
- Naked single method
- Backtracking

3.1.1 Unique missing candidate

This method is useful when there is just one empty square in a row, column or box. The digit that is missing can be placed in that empty square. A similar definition is that if eight of nine empty squares are filled in any row, column or box, then the digit that is missing can fill the only empty square. This method can be useful when most of the squares are filled, especially at the end of a solution. It can also be suitable when solving easy puzzles and this method is efficient to find solution in this case. In this algorithm, the method goes through all rows, columns and boxes separately. The method then checks if a single value is missed in any row, column or box and places the single digit in that specific square (see the appendix).

3.1.2 Naked single method

The second method that is used in the pencil-and-paper algorithm is the Naked single method. This method checks every empty square in the Sudoku board and finds the square that can only take one single digit and the missing digit then is assigned to that square. Note that once the squares are filled by naked single digits other naked singles will appear. This process is repeated until the method has found all empty squares with the needed corresponding one single value and complete the board [7].

This method is a useful method when a human player solves the game. However, if the corresponding method is combined with the unique missing candidate method, then both the methods can solve the puzzles both in easy and medium levels quickly and more efficiently.

3.1.3 Backtracking (guessing method)

The unique missing method and the naked single method can solve all puzzles with easy and medium level of difficulties. To solve puzzles with even more difficult levels such as hard and evil the backtracking method has been used to complete the algorithm. A human player solves the puzzle by using simple techniques. If the puzzle is not solvable by using the techniques the player, then tries to fill the rest of the empty squares by guessing.

The backtracking method, which is like the human strategy (guessing), is used as a help method to the pencil-and-paper algorithm. In other words, if the puzzle cannot be filled when using the unique missing method and the naked single method, the backtracking method will take the puzzle and fill the rest of empty squares. Generally, the backtracking method is to find an empty square and assign the lowest valid number in the square once the content of other squares in the same row, column and box are considered. However, if none of the numbers from 1 to 9 are valid in a certain square, the algorithm backtracks to the previous square, which was filled recently.

The above-mentioned methods are an appropriate combination to solve any Sudoku puzzles. The naked single method can find quickly single candidates to the empty squares that needed only one single value. Since the puzzle comes to its end solution the unique missing method can be used to fill rest of the puzzles. Finally, if either method fills the board the algorithm calls the backtracking method to fill the rest of the board.

Pencil-And-Paper Algorithm

Pencil-and-paper (puzzle [][]){
 puzzle [][]

Solve the puzzle{

For 1 to 10

Naked Single Method (puzzle[][])
 check all rows (puzzle[][]) //check if there is only one candidate that
 is missing, place that missing digit in the empty square
 check all columns (puzzle[][])
 check all boxes (puzzle[][])
 }

If the puzzle is solved: print

Otherwise: recursiveBacktracking (puzzle[][])
Print the solved puzzle
}

The Backtracking algorithm

recursiveBacktrackning(Puzzle[][]){

Puzzle [][] //global

solvePuzzle(row,col){

if (no more choices): the puzzle is solved!

If (puzzle[row][col]= notEmpty):

move to the next square.

for 1 to 9: if(checkRow(row,col,digit) & checkCol(row,col,digit) & checkBox(row,col,digit){

puzzle[row][col]= digit;

move to the next square

}

if not, valid number is found go the previous square that was recently filled

}

}

3.2 Brute Force Solver

The second algorithm that is examined in this work is Brute force algorithm. Usually, the brute force algorithm can be applied to any possible algorithm. For example, when finding a password, the algorithm generates any possible password until the right one is found. In this case the algorithm goes through every empty square and places a valid digit in that square. If no valid number is found the algorithm comes back to the previous square and changes the value in that square. The process is repeated until the board is filled with numbers from 1 to 9.

The advantage of the brute force algorithm is that the algorithm can guarantee a solution to any puzzle since it generates all possible answers until the right answer is found if the puzzles are valid [9]. Additionally, the running time can be unrelated to level of difficulty, because the algorithm searches for every possible solution.

To compare the pencil-and-paper algorithm with the brute force algorithm a (pre- implemented) brute force algorithm has been used during testing [10].

3.3 Comparing

In this section we present the result of the testing and examine the differences between the pencil-and-paper algorithm and the brute force algorithm.

The proposed algorithm has proved that is able to solve Sudoku puzzles with any levels of difficulty. We have assumed to have four levels of difficulty during testing. These levels are easy, medium, hard and evil (challenging). This algorithm can solve the easy and medium puzzle without using backtracking method (less than 20 ms). To solve the puzzles with more difficult levels such as hard or evil the algorithm uses the backtracking method as well. During testing we have noticed that the given algorithm performs better than the brute force algorithm in the term of the runtime (the time the algorithm takes to be executed). The diagram-1 below shows the differences between these two algorithms. Note that the puzzles, which are used in the testing, are taken from a valid webpage [11]. The webpage generates Sudoku puzzles with different ratings.

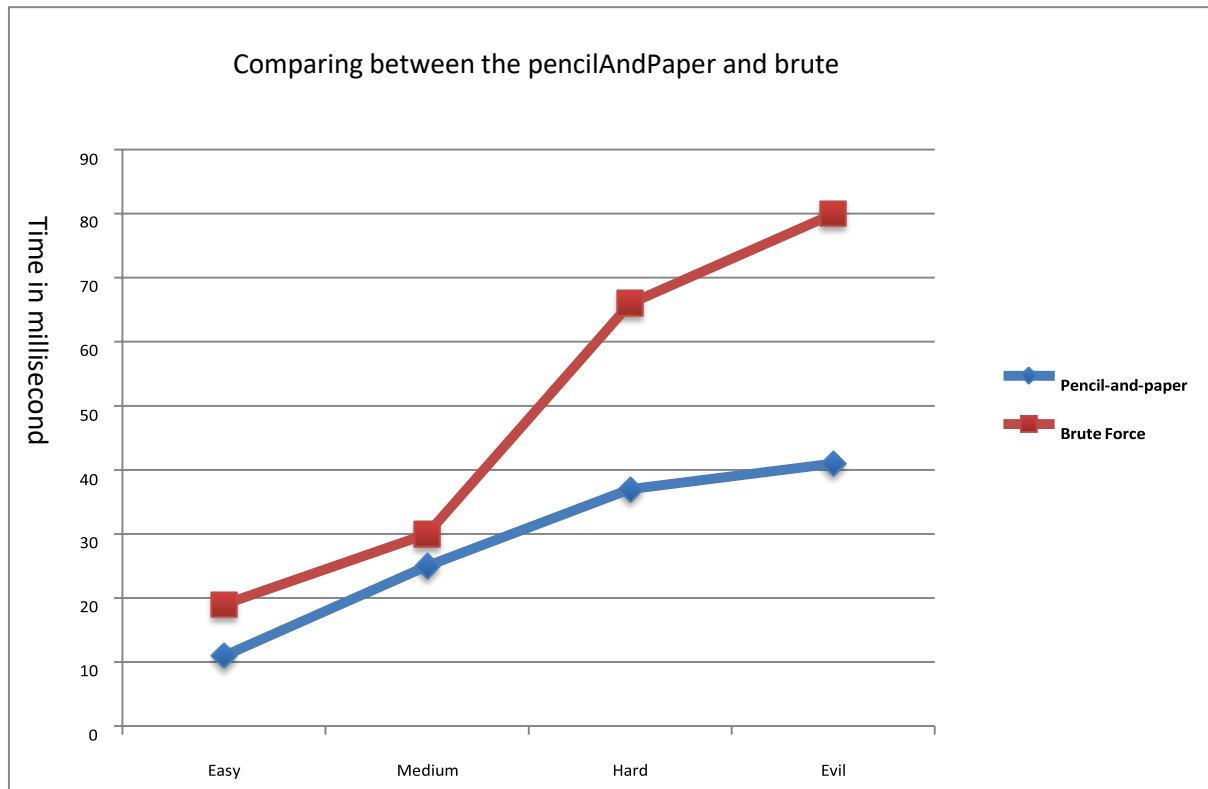


Diagram 1. A comparison between the pencil-and-paper algorithm and brute force algorithm.

This diagram depicts the differences between the pencil-and-paper algorithm and the brute force algorithm based on how long it takes to solve the puzzles by a computer. The pencil- and-paper algorithm solves the puzzle quicker than the brute force algorithm. The given data is based on the averaging of the computing time for several puzzles that have been tested with the same difficulty levels such as easy, medium, hard, and evil respectively. For instance, the time obtained for the easy level is the result of averaging several computing times with easy level. In the given diagram the vertical axis represents the running time of the puzzles in milliseconds and the horizontal axis the difficulty levels.

Generally, the backtracking method, which is like the brute force algorithm, can solve the puzzles quicker than the pencil-and-paper algorithm. The question now is why should we use backtracking and human methods together? There are three reasons to do so. Firstly, the purpose of this work is to implement an algorithm applying human strategies. Secondly, human players also use the backtracking method when they get stuck. It means that players check different alternatives and place the numbers in the empty squares by guessing when there are no options left. Finally, employing human strategies makes the algorithm more efficient based on the number of comparisons. In other words, the naked single method fills the empty squares by performing fewer comparisons in a short time as it uses a better technique. However, the backtracking method runs a greater number of analytical circulation while solving the puzzles resulting consumption of memory space. This can clearly be shown in diagram 2. The result shows that the number of comparisons in the backtracking method is higher than when the human strategies and backtracking methods are used together.

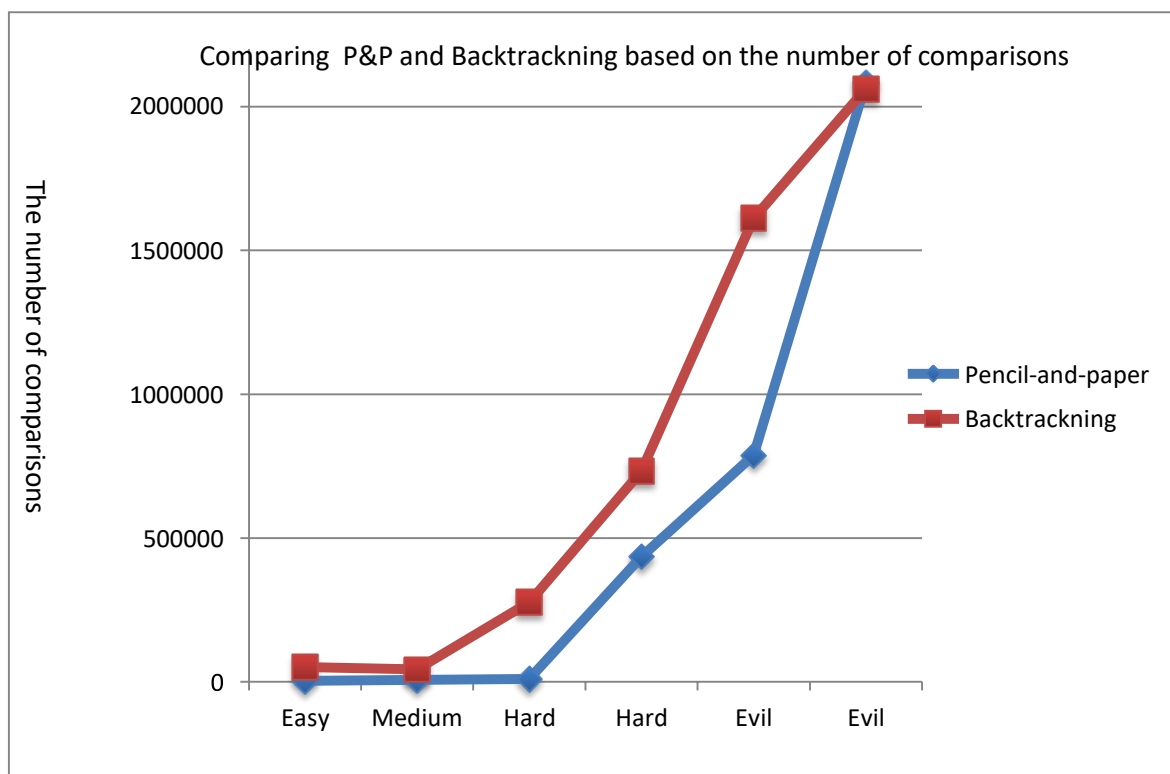


Diagram 2. Comparing the pencil-and-paper algorithm (human strategies and backtracking) and the backtracking method.

3.4 The Difficulty Level of Sudoku Puzzles

The difficulty level of Sudoku puzzles depends on how the given numbers are placed in the Sudoku board and how many numbers (clues) are given. Generally, the most significant aspect of difficulty ratings of Sudoku puzzles is which techniques are required to solve the puzzles. In other words, it is important where the given numbers are placed logically.

Puzzles, which need more techniques to solve, can be called as difficult one. On the other side there are puzzles that can be solved by using simple methods and this kind of puzzle can be defined as easy or medium level. As mentioned above, there are four difficulty levels that are used in testing (easy, medium, hard and evil). We have found during testing that the classifications of difficulty levels are not easy as it is stated. This is because there have been puzzles which were marked as hard level, but they had been solved using simple techniques and vice versa.

There are people who believe that the difficulty ratings have to do with the number of revealed numbers on the puzzle board. Generally, a Sudoku puzzle needs at least 17 clues to be solvable. It means that solving a Sudoku puzzle with 17-clues is more difficult than a puzzle with 30-clues. The more given numbers, the easier and quicker the solution is. This statement may not be always truth, since the testing has shown that the puzzles with fewer clues could be solved in shorter time than the puzzles with more clues. Diagram 3 below describes the relation between the number of clues in the puzzles and their run-time. It is

might be expected that if the number of clues become more the run-time of solving the puzzle would be shorter. For instance, when solving a puzzle with 28 clues the solving time increases rapidly. The reason is that the puzzle needs more techniques to solve it, or the algorithm needed to iterate as long as the solution is found.

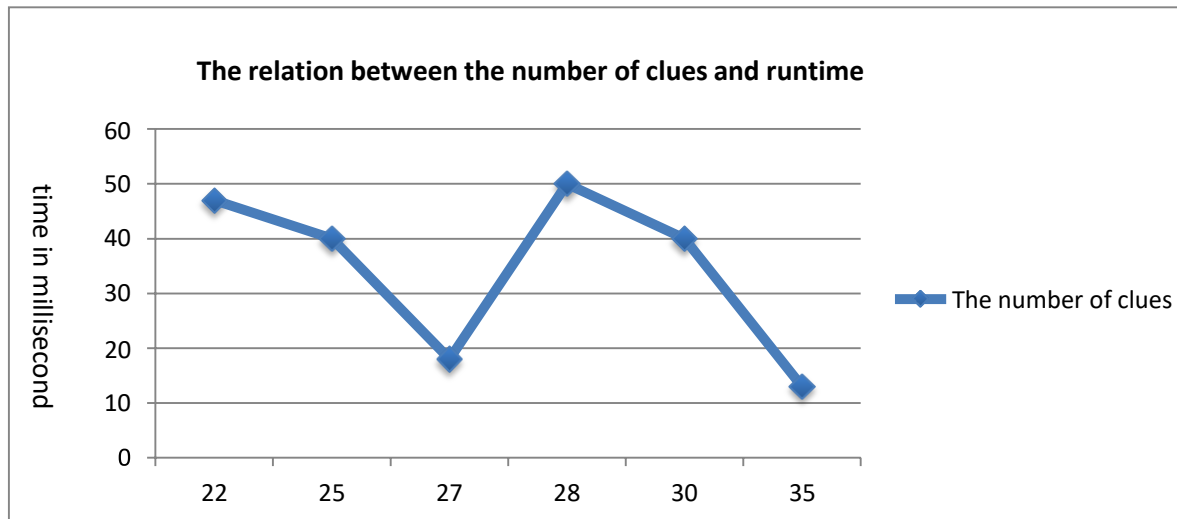


Diagram 3. This diagram shows the relationship between the number of clues (in the horizontal axis) and the run-time of the puzzles (in the vertical axis).

3.5 Time complexity

Time complexity of an algorithm describes the time that is needed to run on a computer, and it is commonly expressed using O notation. One way to estimate time complexity is to count the number of operations achieved by the algorithm. Since the performance time can be changed with different inputs of the same size we use the worst-case time of complexity, denoted as $T(n)$. A recent study by Kovacs showed that the worst-case complexity is related to the difficulty of the hardest puzzle.

We shall examine the time complexity of Sudoku solver. The input to a Sudoku solver is a Sudoku board. The standard board is a 9×9 grid and both smaller and larger boards are used. These types of puzzles with small sizes can be solved quickly and faster by computer, but only because this is small for a computer. Solving Sudoku is one of NP-complete problems and it says that Sudoku algorithms do not scale well to larger boards and puzzles, for example 10000×10000 grids is not feasible. If the size of input to Sudoku solver goes to infinity the time of complexity will increase exponentially. However, when solving puzzles with limited input size such as 9×9 grids it is feasible because they can be solved in polynomial time.

4 Conclusions

This study has shown that the pencil-and-paper algorithm is a feasible method to solve any Sudoku puzzles. The algorithm is also an appropriate method to find a solution faster and more efficiently compared to the brute force algorithm. The proposed algorithm can solve such puzzles with any level of difficulties in a brief period (less than one second).

The testing results have revealed that the performance of the pencil-and-paper algorithm is better than the brute force algorithm with respect to the computing time to solve any puzzle.

The brute force algorithm is a useful method to solve any Sudoku puzzles and it can guarantee to find at least one solution. However, this algorithm is not efficient because the level of difficulties is irrelevant to the algorithm. In other words, the algorithm does not adopt intelligent strategies to solve the puzzles. This algorithm checks all workable solutions to the puzzle until a valid solution is found, which is a time-consuming procedure resulting in an inefficient solver. As has already been stated, the main advantage of using the algorithm is the ability to solve any puzzles and a solution is certainly guaranteed.

Further research needs to be conducted to optimize the pencil-and-paper algorithm. A viable way could be implementing other human strategies (x-wings, swordfish, etc.). Other alternatives might be to establish whether it is feasible to implement an algorithm based only on human strategies so that no other algorithm participates in the pencil-and-paper algorithm and make sure that these strategies can solve any puzzles with any level of difficulty.

