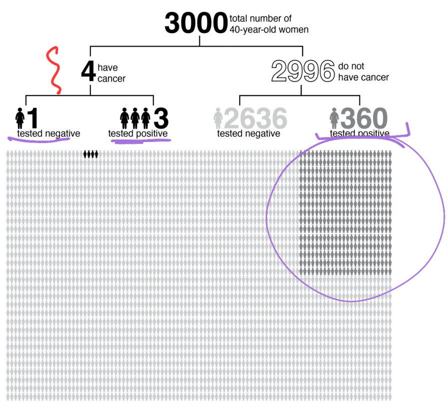
Forward. $P(\text{Evidence} \mid \text{Hyp})$.

Bayes Theorem

Reverse. $P(\text{Hyp} \mid \text{Evidence})$



Medical Diagnosis

- Forward probability:
  Probability of a positive test, given that one has the disease
  $P(\text{test} \mid \text{disease})$
  evidence? Hyp.
- Inverse probability:
  The probability that one has the disease, given that the test came out positive
  $P(\text{disease} \mid \text{test})$
  Hyp. observation

- Updated probability of the disease given the test outcome is positive
  $P(D \mid T) = \left(\frac{P(T \mid D)}{P(T)}\right) P(D)$ Prior $= \frac{1}{100}$
- $P(D) = \frac{1}{700}$
- $P(T \mid D)$ sensitivity of the test
  $P(T \mid D) = \frac{73}{100}$ True positive rate

- Probability of a positive test
  $P(T) = P(D)P(T \mid D) + P(\neg D)P(T \mid \neg D)$
  $P(T) = \frac{1}{700}\left(\frac{73}{100}\right) + \frac{699}{700}\left(\frac{12}{100}\right) \approx \frac{12.1}{100}$
- The likelihood ratio is

$$\frac{73}{12.1} \approx 6$$

- The updated probability is $\frac{P(T \mid D)}{P(T)} \times P(D)$ $P(D \mid T)$
  $= 6 \times \frac{1}{700} = \frac{1}{116}$

$\cancel{\frac{1}{20}}$

- But, if $P(D) = \frac{1}{20}$, then a positive test increases $P(D \mid T)$ to $\frac{1}{3}$

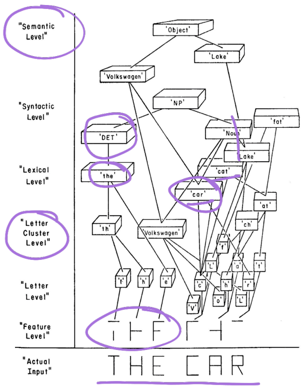
$6 \times \frac{1}{20} \approx \frac{1}{3}$

1950

1956.

1980

Methods to deal with uncertainty:
Fuzzy Logic,
Dempster-Shafer Theory,
Certainty Factors,
Belief Networks



Bayes rule (a two-node Bayesian Network)

Hypothesis

↓

Evidence

- The arrow merely signifies that we know the forward probability.
- In the Bayes Network, this arrow does not have any causal meaning.

$P(E|H)$

A three-node Bayesian Network (Junction)

A → B → C

Fire → Smoke → Alarm

B → D, B → C

- Chain
- B is a Mediator that transmits the effect of A to C
- The Mediator *screens off* information about A from C and vice versa.
- A and C are conditionally independent given the value of B

A three-node Bayesian Network (Junction)

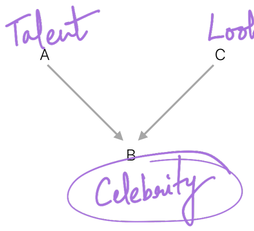
B → A, B → C

Child's Age → Shoe size, Child's Age → Reading ability

- Fork Common Cause.
- B is the confounder of A and C
- A confounder will make A and C statistically correlated
- Correlation between A and C is explained by B
- A and C are conditionally independent, given B.

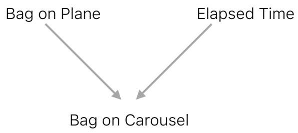
Spurious Correlation

A three-node Bayesian Network (Junction)



- Collider
- A and C may be unrelated in the general population
- However, conditioning on B will make A and C dependent (with a negative correlation)
    - Called as the Explain-Away effect or the collider bias

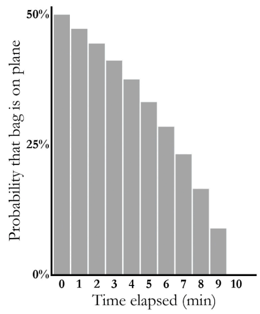
Example of Bayesian Network (1)



- Solve the inverse probability problem
    - If x minutes have passed and I still haven't gotten my bag, what is the probability that it was on the plane?

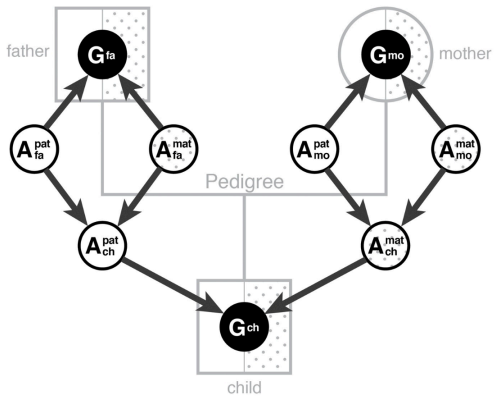
Conditional Probability Table

Probability of →, Given ↓		*carousel* = false	*carousel* = true
bag on plane	*time elapsed*		
False	0	100	0
False	1	100	0
False	2	100	0
False	3	100	0
False	4	100	0
False	5	100	0
False	6	100	0
False	7	100	0
False	8	100	0
False	9	100	0
False	10	100	0
True	0	100	0
True	1	90	10
True	2	80	20
True	3	70	30
True	4	60	40
True	5	50	50
True	6	40	60
True	7	30	70
True	8	20	80
True	9	10	90
True	10	0	100



Example of Bayesian Network (2)

- Bonaparte — DNA matching software used by Netherlands Forensic Institute
- Input to the Bayesian Network: Victim's DNA
- The Bayesian Network computes the likelihood that it fits into a specific slot in the pedigree



Example of Bayesian Network (2)

- Turbo Codes