Example of defeasible logic using the Spindle reasoner

Let us take a simple modelling example from an overseas student in the university.

– *r1: student(X), overseas(X) ⇒payFPOS(X)*

– *r2: student(X), overseas(X), exchange(X) ⇒~ payFPOS(X)*

– *r3: student(X) ⇒ payHECS(X)*

– *r4: student(X), payFPOS(X) ⇒~payHECS(X)*

– *r4 > r3*

Now consider the following synopsis regarding the overseas student. An overseas student generally pays overseas student fees (FPOS) **unless** they come from an exchange background. The rule r1 and r2 clearly indicate that if the student is overseas then he/she has to pay FPOS and the rule r2 states that if that student is an exchange student then there is no need to pay the FPOS.

The rule r3 states that all the students must pay the Higher Education contribution scheme (HECS). However, the students which are paying FPOS does not require to pay the HECS. Since the rule r4 is more specific than rule r3 then we have a superiority relation between these two rules. Hence r4 >r3.

Now suppose we have an overseas student named as Sofia and we want to query upon whether she must pay HECS or not. If we look at this inconsistency as an intuitive reasoning, then in our knowledge base we not that Sofia does not have to pay the HECS. But let us reason it with our defeasible theory reasoner spindle.

- student(Sofia)
- overseas(Sofia)

These are two facts hence their provability is undisputable. Let us see the reasoner result from the spindle: -

+D	overseas(sofia)
+D	student(sofia)
-D	exchange(sofia)
-D	payFPOS(sofia)
-D	-payFPOS(sofia)
-D	payHECS(sofia)
-D	-payHECS(sofia)
+d	overseas(sofia)
+d	payFPOS(sofia)
+d	-payHECS(sofia)

+d	student(sofia)
-d	exchange(sofia)
-d	-payFPOS(sofia)
-d	payHECS(sofia)

Now let us see how the above literals are proved in the defeasible logic. We will be accessing the provability conditions from representation results. To prove the literal, it is definitely provable we have the following condition

$+\Delta$: If $P(i+1) = +\Delta q$ then either

$$q \in F \text{ or}$$
$$\exists r \in R_s[q] \forall a \in A(r) : +\Delta a \in P(1..i)$$

1) Literal +D overseas(student)

Now according to the above condition, a literal is definitely provable if it is a fact or there exist a strict rule, where the literal head is applicable. Since we know that the literal overseas(student) is a fact hence it is definitely provable.

2) Literal +D student(sofia)

Similar to the above condition because the literal student(Sofia) is a fact, and therefore facts are definitely provable.

3) Literal -D exchange(Sofia)

Let us see the proof condition of not definitely provable.

$-\Delta$: If $P(i+1) = -\Delta q$ then

$$q \notin F \text{ and}$$
$$\forall r \in R_s[q] \exists a \in A(r) : -\Delta a \in P(1..i)$$

The condition states that in order for a literal to be not definitely provable we need to show that the literal is not a fact **and** we need to establish that every strict rule with head q is known to be inapplicable. Thus, for every such rule r there must be at least one antecedent **a** for which a is not definitely provable.

Now in our case the literal exchange(Sofia) is not a fact and since there are no strict rules in our knowledge base, we conclude that this literal is not definitely provable (there are no strict applicable rules).

Similarly, for the following literals they are not definitely provable i.e. -D payFPOS(Sofia)

-D -payFPOS(Sofia)

-D payHECS(Sofia)

-D -payHECS(Sofia)

As these all literals are not facts and there are no strict rules where with head (this literal) is applicable.

Now we move on to the next set of literals

4) +d overseas(Sofia)

According to the prove condition, a literal is defeasibly proved if it is definitely proved. Since the overseas(Sofia) is already definitely proved (because it is a fact) so it is defeasibly proved.

5) +d student(Sofia)

The provability is same as for the overseas(Sofia) both are facts.

6) +d **FPOS(Sofia)**

Now let us look the provability conditions for the defeasibly proved literal

$+\partial$: If $P(i+1) = +\partial q$ then either
$(1) + \Delta q \in P(1..i)$ or
$(2) \quad (2.1) \exists r \in R_{nd}[q] \forall a \in A(r) : +\partial a \in P(1..i)$ and
$(2.2) - \Delta \sim q \in P(1..i)$ and
$(2.3) \forall s \in R[\sim q]$ either
$(2.3.1) \exists a \in A(s) : -\partial a \in P(1..i)$ or
$(2.3.2) \exists t \in R_{sd}[q]$ such that
$\forall a \in A(t) : +\partial a \in P(1..i)$ and $t > s$

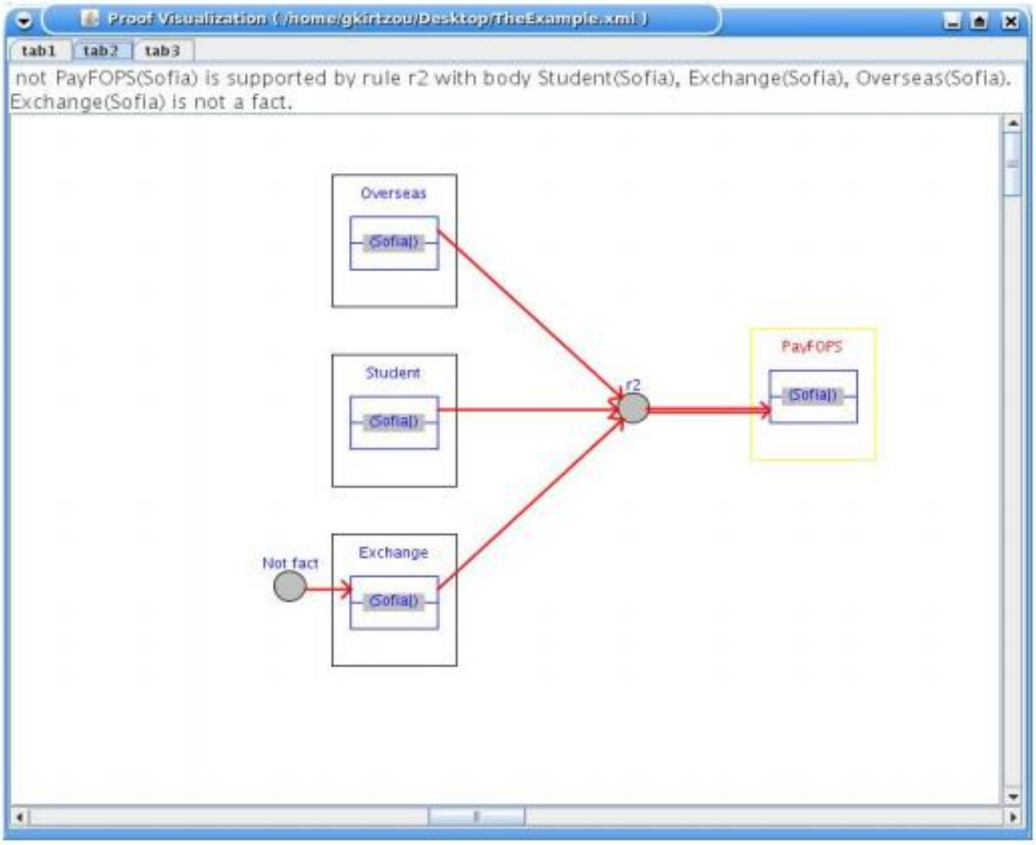
So let us prove the defeasibility of the above literal by considering all the steps, so a literals is defeasibly provable if it is definitely provable (1) and then we have a **or set of conditions.** Since our literal FPOS(Sofia) is not definitely provable (as explained above) then we move on to the set of conditions. The (2.1) states that there must exist a strict or defeasible rule with head as FPOS(Sofia), so we have a defeasible rule i.e. rule r1 with head FPOS(Sofia)

R1: student(Sofia), overseas(Sofia) ⇒payFPOS(Sofia)

Now moving on to the next set of condition that must be true, so (2.1 **and** 2.2 **and** 2.3), then we can prove its defeasibility. But now we need to consider attacks by the negation of this literal i.e. **-payFPOS(SOFIA),** so 2.2 states that in order to prove payFPOS(Sofia) we need to ensure that -payFPOS(SOFIA) is not definitely provable. Since we have already seen that -D of (-payFPOS(Sofia)) is definitely provable, then this condition is satisfied. Now 2.3 states that we must consider the set of all rules which are not known to be inapplicable and which have head -payFPOS(Sofia).

i.e. R2: student(sofia),overseas(sofia),exchange(sofia) => -payFPOS(sofia)

Essentially each such rule s attacks the conclusion q. For q to be provable, each such rule s must be counterattacked by a rule t with head q with the following properties: (i) t must be applicable at this point, and (ii) t must be stronger than s. Thus, each attack on the conclusion q must be counterattacked by a stronger rule. Now we know that rule R1 with head (payFPOS) is applicable and the rule R1 is stronger than the rule R2 because it's head payFPOS is obtained by two facts while -payFPOS is obtained by two facts and one non fact. We can also visualize this using block diagram



Henceforth, from above derivation we can conclude that the literal payFPOS(Sofia) is defeasibly provable.

7) +d -**payHECS(Sofia)**

At the beginning of the formal reasoning we wanted to see that Sofia which is an overseas student would need to pay for HECS fees. Moreover, after the spindle result, we can see that our intuition result that Sofia should not pay HECS aligned with our reasoning result. Now let us take a look at the provability of this literal.

$+\partial:$ If $P(i+1) = +\partial - payHECS(Sofia)$ then either

$(1) + \Delta - payHECS(Sofia) \in P(1\ldots i)$ or

Since the above rule is not applicable because -payHECS(Sofia) is not definitely provable we will move on to the other set of conclusions.

$(2) \quad (2.1) \exists r \in R_{nd}[-payHECS(Sofia)] \forall a \in A(r) : +\partial - payHECS(Sofia) \in P(1..i)$ and

Now 2.1 states that there must exist a defeasible rule with head -payHECS(Sofia) and it is applicable. Henceforth, we have the rule as follows

R4: student(Sofia), payFPOS(Sofia) ⇒~payHECS(Sofia)

Now we need to consider the reasoning in support of –(-payHECS(Sofia)) i.e. payHECS(Sofia)

$(2.2) - \Delta payHECS(Sofia) \in P(1..i)$ and

2.2 states that in order to prove -payHECS(Sofia) we need to show that payHECS(Sofia) is not definitely provable, which we already proved in earlier cases. Now moving on to next set of literals that define this and clause.

$$
\begin{aligned}
&(2.3)\forall s \in R[payHECS(Sofia)] \text{ either} \\
&(2.3.1)\exists a \in A(s) : -\partial a \in P(1..i) \text{ or} \\
&(2.3.2)\exists t \in R_{sd}[q] \text{ such that} \\
&\quad \forall a \in A(t) : +\partial a \in P(1..i) \text{ and } t > s
\end{aligned}
$$

Now 2.3 states that we need to consider the set of rules s with head payHECS(Sofia), i.e. rule
R3: student(Sofia) ⇒ payHECS(Sofia)

Now this rule attacks our original rule r4 i.e. student(Sofia),payFPOS(Sofia) => -payHECS(Sofia). Now r4 is our rule T (r4) is also applicable at this point and r4 is stronger than r3. (We have already defined the rule superiority).

Hence **+d -payHECS(Sofia) is provable** and **-d payHECS(Sofia)** is also provable. We cannot defeasibly prove -payHECS(Sofia) but in contrary we can prove payHECS(Sofia).Here is the visualization of the proof:-