Example of defeasible logic using the Spindle reasoner

Let us take a simple modelling example from an overseas student in the university.

– r1: student(X), overseas(X) ⇒payFPOS(X)

– r2: student(X), overseas(X), exchange(X) ⇒∼ payFPOS(X)

– r3: student(X) ⇒ payHECS(X)

– r4: student(X), payFPOS(X) ⇒∼payHECS(X)

– r4 > r3

The predicates student(x) and overseas(x) are supposed to be a whole literal. The X can be replaced as an example instance of a student, such as a student name.

Now consider the following synopsis regarding the overseas student. An overseas student generally pays overseas student fees (FPOS) **unless** they come from an exchange background. The rule r1 and r2 clearly indicate that if the student is overseas then he/she has to pay FPOS and the rule r2 states that if that student is an exchange student then there is no need to pay the FPOS.

The rule r3 states that all the students must pay the Higher Education contribution scheme (HECS). However, the students which are paying FPOS does not require to pay the HECS. Since the rule r4 is more specific than rule r3 then we have a superiority relation between these two rules. Hence r4 >r3.

Now suppose we have an overseas student named as Sofia and we want to query upon whether she must pay HECS or not. If we look at this inconsistency as an intuitive reasoning, then in our knowledge base we not that Sofia does not have to pay the HECS. But let us reason it with our defeasible theory reasoner spindle.

These are two facts hence their provability is undisputable. Let us see the reasoner result from the spindle: -

Facts

- student(Sofia)
- overseas(Sofia)

1. +D overseas(sofia)

$+\Delta$: If $P(1) = +\Delta overseas(Sofia)$ then either

$overseas(Sofia) \in F$ or
$\exists r \in R_s[overseas(Sofia)] \forall a \in A(r) : +\Delta a \in P(1..1)$

The literal +D overseas(Sofia) is proved in line 1, i.e. P(1), then its provability depends on two factors

1.1 1.The literal must belong to the set of facts i.e. overseas(Sofia)∈ $(Facts)$ or 2.There must exist a strict rule with head overseas(Sofia) and for that such rule we need to show that its antecedent is definitely provable in previous layers. Here (R_s denotes the strict rules) and A(r) denotes the antecedent of each such rule.

1.2 From the above set of conditions, we can conclude the first condition is satisfied. Hence the literal overseas(Sofia) is definitely provable.

2. +D student(sofia)

$+\Delta$: If $P(2) = +\Delta student(Sofia)$ then either

$student(Sofia) \in F$ or

$\exists r \in R_s[student(Sofia)] \forall a \in A(r) : +\Delta a \in P(1..1)$

The literal +D student(Sofia) is proved in line 2, i.e. P(2), then its provability depends on two factors

2.1 1.The literal must belong to the set of facts i.e. student(Sofia)∈ $(Facts)$ or 2.There must exist a strict rule with head student(Sofia) and for that such rule we need to show that its antecedent is definitely provable in previous layers. Here (R_s denotes the strict rules) and A(r) denotes the antecedent of each such rule.

2.2 From the above set of conditions, we can conclude the first condition is satisfied. Hence the literal student(Sofia) is definitely provable.

3. -D exchange(sofia)

$$
\begin{aligned}
-\Delta: \ & \text{If } P(2+1) = -\Delta exchange(sofia) \text{ then} \\
& exchange(sofia) \notin F \text{ and} \\
& \forall r \in R_x[q] \exists a \in A(r) : -\Delta a \in P(1 \ldots 2)
\end{aligned}
$$

The literal -D student(Sofia) is proved in line 3, i.e. P(2), then its provability depends on two factors

3.1 1.The literal must **not** belong to the set of facts and 2. For every rule belonging to the strict rule (here R_x[q] denotes strict rule with head exchange(sofia)) with head exchange(sofia) is known to be inapplicable. Thus, for every such rule r there must be at least one antecedent exchange(sofia) which is not definitely provable in previous layers i.e. P(1…2)

3.2 From the above set of conditions, we can conclude that both conditions are satisfied. Hence the literal exchange(sofia) is not definitely provable.

Similarly, for the following literals they are also not definitely provable by above scenarios and we can also not definitely prove

4. -D payFPOS(sofia)

5. -D -payFPOS(sofia)

6. -D payHECS(sofia)

7. -D -payHECS(sofia)

8. +d overseas(sofia)

Since form P(1…7) prove lines, we can see that the literal overseas(sofia) is definitely provable, and the literal is a fact. Hence the literal is defeasibly provable.

9. +d payFPOS(sofia)

$$
\begin{aligned}
&+\partial: \text{ If } P(8+1) = +\partial payFPOS(sofia) \text{ then either}\\
&\qquad (1) + \Delta payFPOS(sofia) \in P(1..8) \text{ or}\\
&(2) \quad (2.1)\exists r \in R_{sd}[payFPOS(sofia)] \forall a \in A(r): +\partial a \in P(1..8) \text{ and}\\
&(2.2) - \Delta \sim payFPOS(sofia) \in P(1\ldots 8) \text{ and}\\
&(2.3)\forall s \in R[\sim payFPOS(sofia)] \text{ either}\\
&(2.3.1)\exists a \in A(s): -\partial a \in P(1..8) \text{ or}\\
&(2.3.2)\exists t \in R_{sd}[payFPOS(sofia)] \text{ such that}\\
&\forall a \in A(t): +\partial a \in P(1\ldots 8) \text{ and } t > s
\end{aligned}
$$

So, let us prove the defeasibility of the above literal by considering all the steps, so a literal is defeasibly provable if it is definitely provable (1) and then we have a **or set of conditions**. Since our literal FPOS(Sofia) is not definitely provable (as explained above) then we move on to the set of conditions. The (2.1) states that there must exist a strict or defeasible rule with head as FPOS(Sofia), so we have a defeasible rule i.e. rule r1 with head FPOS(Sofia) (here the R_{sd}[payFPOS(Sofia)] states that there must exist a rule (either strict or defeasible) with head FPOS(Sofia))

R1: student(Sofia), overseas(Sofia) ⇒payFPOS(Sofia)

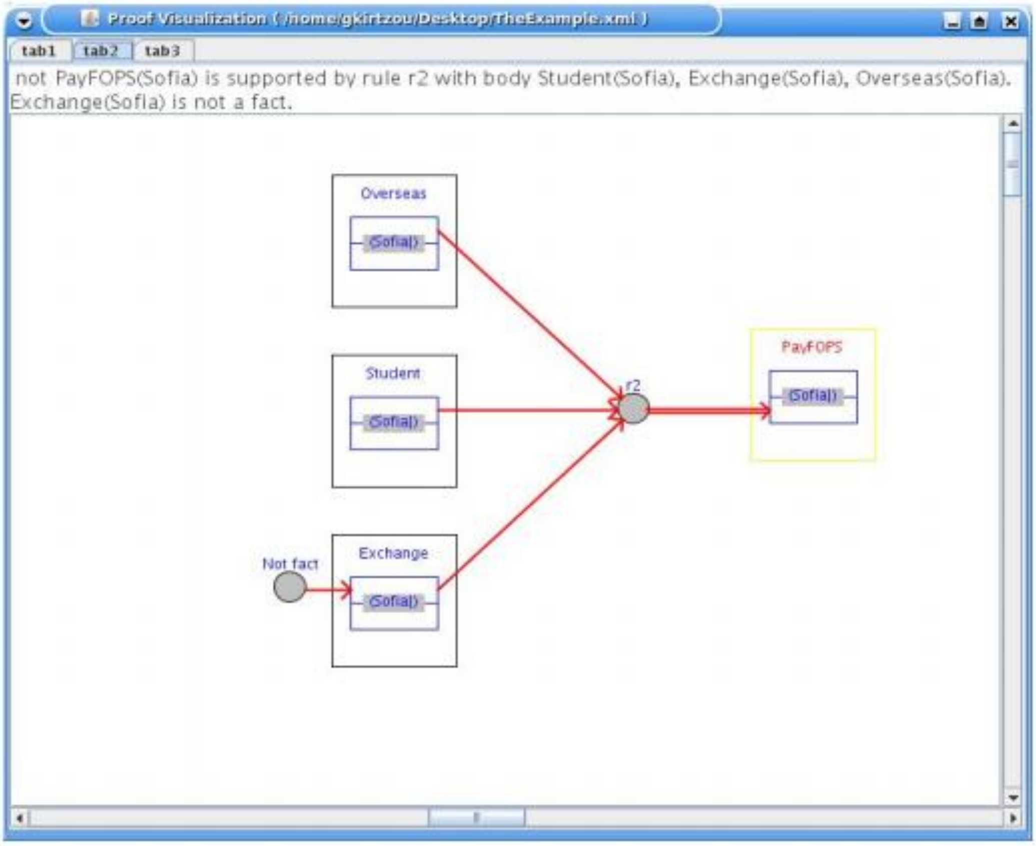
Now moving on to the next set of condition that must be true, so (**2.1 and 2.2 and 2.3**), then we can prove its defeasibility. But now we need to consider attacks by the negation of this literal i.e. -payFPOS(SOFIA), so **2.2** states that in order to prove payFPOS(Sofia) we need to ensure that -payFPOS(SOFIA) is not definitely provable in the prove lines(1 to 8). Since we have already seen that -D of (-payFPOS(Sofia)) is definitely provable, then this condition is satisfied. Now **2.3** states that we must consider the set of all rules which are not known to be inapplicable and which have head -payFPOS(Sofia). i.e.

R2: student(Sofia),overseas(Sofia),exchange(Sofia) => -payFPOS(Sofia)

Essentially each such rule s (here R2) attacks the conclusion q. For q to be provable, each such rule s must be counterattacked by a rule t with head q with the following properties: (i) t must be applicable at this point, and (ii) t must be stronger than s. Thus, each attack on the conclusion q must be counterattacked by a stronger rule. Now we know that rule R1 with head (payFPOS) is applicable and the rule R1 is stronger than the rule R2 because it's head payFPOS is obtained by two facts while -payFPOS is obtained by two facts and one non fact.

Now according to the semantic inheritance network in the defeasible logic, if antecedent of one rule is defeasibly linked with antecedent of the second rule, then automatically the first rule gains priority of the second rule. Here in our case rule r1 antecedent is defeasibly linked to rule r2 so rule r1 gains priority.

We can also visualize this using block diagram



10. +d -payHECS(sofia)

$-\partial$: If $P(9+1) = -\partial payHECS(Sofia)$ then
(1) $-\Delta payHECS(Sofia) \in P(1..9)$ and
(2) $(2.1)\forall r \in R_{sd}[-payHECS(Sofia)]\exists a \in A(r) : -\partial a \in P(1..9)$ or
$(2.2) + \Delta payHECS(Sofia) \in P(1..9)$ or
(2.3) $\exists s \in R[payHECS(Sofia)]$ such that
$(2.3.1)\forall a \in A(s) : +\partial a \in P(1..9)$ and
$(2.3.2)\forall t \in R_{sd}[payHECS(Sofia]$ either
$\exists a \in A(t) : -\partial a \in P(1..9)$ or $t \not> s$

So, let us prove the defeasibility of the above literal by considering all the steps, so a literal is defeasibly provable if it is definitely provable (1) and then we have a **or set of conditions**. Since our literal -payHECS(Sofia) is not definitely provable (as explained above) then we move on to the set of conditions. The (2.1) states that there must exist a strict or defeasible rule with head as -payHECS(Sofia), so we have a defeasible rule i.e. rule r4 with head -payHECS(Sofia) (here the R_{sd}[-payHECS(Sofia)] states that there must exist a rule (either strict or defeasible) with head -payHECS(Sofia) and it is applicable in previous prove lines).

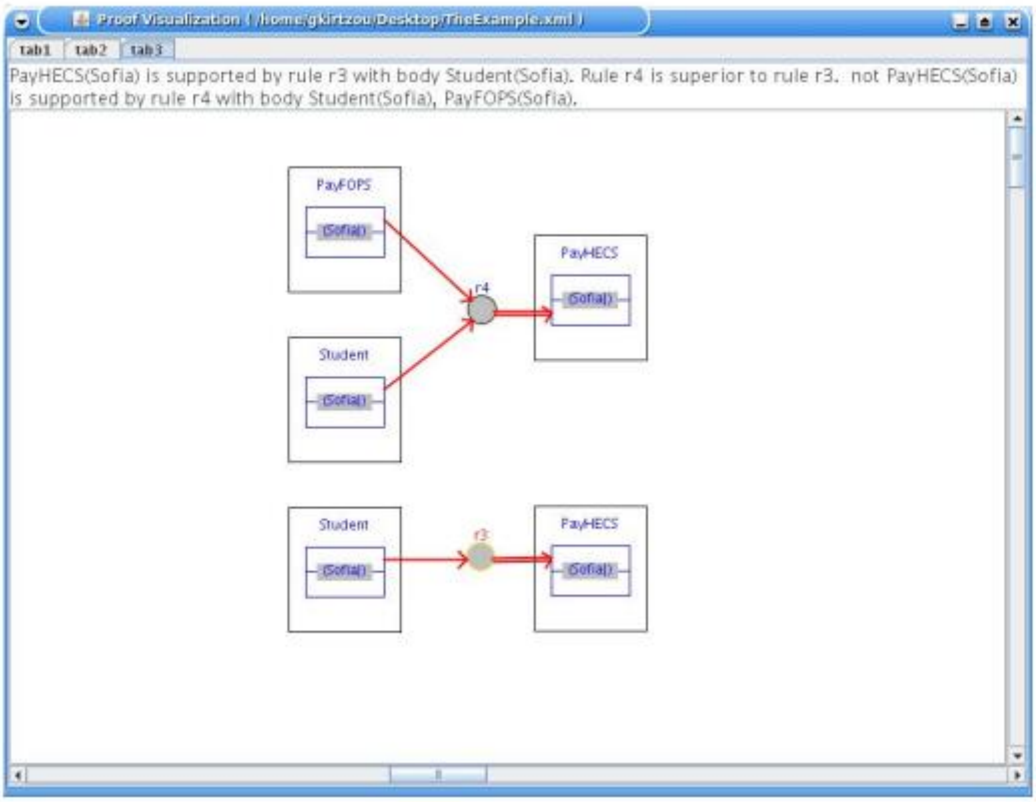
R4: student(Sofia), payFPOS(Sofia) ⇒ -payHECS(Sofia)

Now moving on to the next set of condition that must be true, so (**2.1 and 2.2 and 2.3**), then we can prove its defeasibility. But now we need to consider attacks by the negation of this literal i.e. ¬(¬payHECS(Sofia)) = payHECS(Sofia) , so **2.2** states that in order to prove ¬payHECS(Sofia) we need to ensure that payHECS(Sofia) is not definitely provable in the prove lines(1 to 9). Since we have already seen that (payHECS(Sofia)) is not definitely provable, then this condition is satisfied. Now **2.3** states that we must consider the set of all rules which are not known to be inapplicable and which have head payHECS(Sofia). i.e.

R3: student(Sofia) ⇒ payHECS(Sofia)

Essentially each such rule S (here R3) attacks the conclusion -payHECS(Sofia). For -payHECS(Sofia) to be provable, each such rule S must be counterattacked by a rule T (here Rule R4) with head -payHECS(Sofia) with the following properties: (i) T (here R4) must be applicable at this point, and (ii) T must be stronger than S. Thus, each attack on the conclusion must be counterattacked by a stronger rule. Now we know that rule R4 with head (-payHECS) is applicable and the rule R4 is stronger than the rule R3 because it is already being given to our knowledge base. So, the literal -payHECS(Sofia) is defeasibly proved.

The proof can be visualized with the flow chart given below



11. +d student(sofia)

The literal student(sofia) is a fact; hence it is definitely provable. If a literal is definitely provable then it is defeasibly provable.