

Assignment - 6

Q1) Single link proximity function - The minimum distance between any two points in the different clusters and that distance is the height at which they are joined into one cluster in the dendrogram.

→ So, from the table given in the question 1

$$= \text{dist}(P_3, P_6) = 0.1100$$

$$= \text{dist}(P_2, P_5) = 0.1388$$

$$= \text{dist}(\{P_3, P_6\}, \{P_2, P_5\})$$

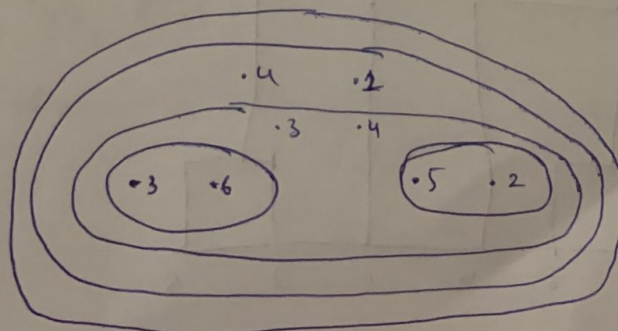
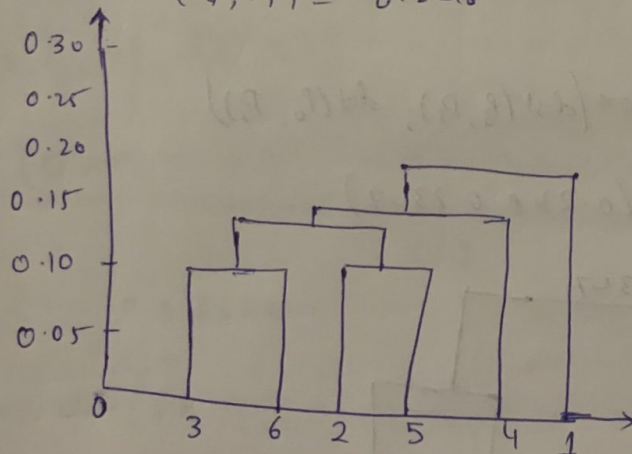
$$= \min(\text{dist}(P_3, P_2); \text{dist}(P_6, P_2); \text{dist}(P_3, P_5); \text{dist}(P_6, P_5))$$

$$= \min(0.1483, 0.2540, 0.2593, 0.3928)$$

$$= 0.1483$$

$$= \text{dist}(P_3, P_4) = 0.1513$$

$$\text{dist}(P_4, P_1) = 0.2218$$



Complete link proximity function:

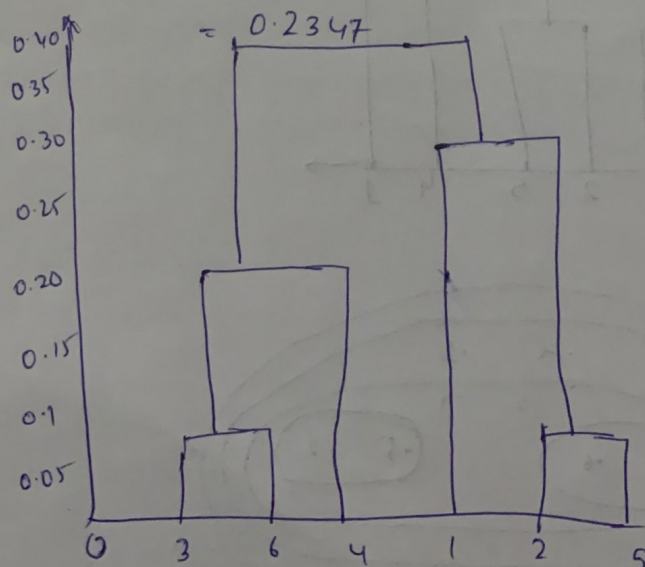
In here the proximity of two clusters is defined to be the maximum of the distance b/w any two points in different clusters.

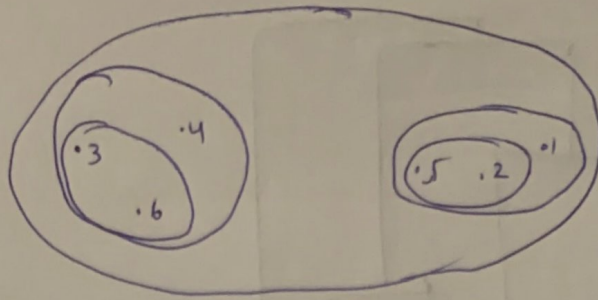
Points (P_3, P_6) are merged first and then it is merged with P_4 .

$$\begin{aligned} \text{dist}(\{P_3, P_6\}, \{P_4\}) &= \max(\text{dist}(P_3, P_4); \text{dist}(P_6, P_4)) \\ &= \max(0.1513, 0.2216) \\ &= 0.2216 \end{aligned}$$

$$\begin{aligned} \text{dist}(\{P_3, P_6\}, \{P_2, P_5\}) &= \max(\text{dist}(P_3, P_2); \text{dist}(P_3, P_5); \\ &\quad \text{dist}(P_6, P_2); \text{dist}(P_6, P_5)) \\ &= \max(0.1483, 0.2540; 0.2843, 0.3921) \\ &= 0.3921 \end{aligned}$$

$$\begin{aligned} \text{dist}(\{P_3, P_6\}, \{P_1\}) &= \max(\text{dist}(P_3, P_1); \text{dist}(P_6, P_1)) \\ &= \max(0.2318, 0.2347) \end{aligned}$$





Average Link proximity function

The average of the pairwise proximities between all pairs of points in the different clusters. This is the intermediate approach b/w single and complete link.

$$\Rightarrow \text{proximity}(\text{cluster}_1, \text{cluster}_2) = \sum_{\substack{P_i \in \text{cluster}_1 \\ P_j \in \text{cluster}_2}} \frac{\text{proximity}(P_i, P_j)}{\text{size}(\text{cluster}_1) \times \text{size}(\text{cluster}_2)}$$

$$\begin{aligned} \text{dist}(\{P_3, P_6, P_4\}, \{P_1, P_5\}) &= (0.2218 + 0.20347 + 0.3688) / 3 \times 1 \\ &= 0.2751 \end{aligned}$$

$$\begin{aligned} &= \text{dist}(\{P_3, P_6\}, \{P_1\}) = (0.2357 + 0.3421) / 2 \times 1 \\ &= 0.2889 \end{aligned}$$

$$\begin{aligned} \text{dist}(\{P_3, P_6, P_4\}, \{P_2, P_5\}) &= \\ &= \frac{(0.1483 + 0.2843 + 0.2540 + 0.3421 + 0.2047 + 0.2932)}{6 \times 2} \\ &= 0.2637 \end{aligned}$$

Because $\text{dist}(\{P_3, P_6, P_4\}, \{P_2, P_5\})$ is smaller than

$\text{dist}(\{P_3, P_6, P_4\}, \{P_1\})$ and $\text{dist}(\{P_2, P_5\}, \{P_1\})$

These two clusters are merged at the fourth stage.

