UN5390: Scientific Computing I

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Using fourth order Runge-Kutta adaptive time step method to simulate the celestial free return trajectory followed by a spacecraft in the earth-moon gravitational field

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Introduction

NASA [1], in its renewed efforts for lunar exploration, has resurrected the once famous Apollo missions [2]. However, as if bringing fiction to reality [3], Apollo 15 mission accidentally left one of the astronauts behind on the Moon. This project attempts to send a previously unplanned rescue mission, Apollo 16, to safely bring the astronaut back to earth. Owing to budget and other limitations, the mission can only afford one trans-lunar injection (TLI) with no further course corrections and the spacecraft must use the free return trajectory – a class of trajectories in which the trajectory of a spacecraft traveling away from a primary body (e.g., earth) is modified by the presence of a secondary body (e.g., moon) causing the spacecraft to return to the primary body.

Description

To simplify programming and get an initial estimate of the course, NASA has provided the following working assumptions and simulation parameters.

1. All motion happens in a 2D plane

2. Mass of the spacecraft is negligible compared to that of the earth and the moon. The earth stays fixed at the center of the coordinate system while the moon orbits the earth in a circular orbit. There are no other perturbations from other objects, celestial and/or otherwise. The earth and moon lie along the x-axis at $t = 0$

3. $v_\circ$ is the spacecraft's velocity and α is its angular position in the earth's parking orbit (EPO). Rockets fire for a very brief time and Trans-Lunar Injection (TLI) consumes negligible amount of time compared to the duration of voyage. At this instant, $t = 0$ (i.e., when TLI takes place), the spacecraft reaches the velocity $v = v_\circ + \Delta v$. There are no additional course corrections beyond the TLI

$G \; = 6.67428 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$	universal constant of gravitation
$M \; = 5.9736 \times 10^{24}\, kg$	mass of the earth
$m \; = 7.3477 \times 10^{22}\, kg$	mass of the moon
$R_{\mathrm{E}} = 6378.14 \times 10^3\, m$	radius of the earth
$R_{\mathrm{m}} = 1737.10 \times 10^3\, m$	radius of the moon
$r_{\mathrm{s}} \; = r_E + 185\, km$	radius of the parking orbit
$d \; = 3843.03 \times 10^5\, m$	distance between the earth and the moon
$T \; = 27.3216$ days	period of revolution of moon around the earth
$h \; = 0.10\, s$	suggested time step for RK4 method

The programming component of this project involves identifying the combination of angle, α, and velocity $v = v_\circ + \Delta v$, for the spacecraft – in earth's parking orbit – so that the fourth order Runge-Kutta adaptive time step method predicts the minimum travel time. The trajectory must take spacecraft within 1750 km of lunar center, loop around the moon and return it to within 100 km of earth's surface. Due to resource limitations for NASA as well as the astronaut on the moon, the spacecraft's journey (i.e., total travel time) must be completed within seven days.

Expected workflow

1. Starting from Newton's Law of Gravitation, write down the necessary theoretical/analytical steps
2. Programs must be aptly named and modularized with appropriate comments, and must look pretty – in compliance with programming etiquette discussion. Use the debugger and/or the profiler to improve the code, if necessary. If this is a solo project, incorporating parallelism is necessary
3. The program, in its entirety, can often take several hours to run – for a given pair of α and v. Does the solution approximately resemble physical reality? Keep track of the obstacles and how you solved them
4. For each combination of α and v that satisfies NASA's requirements (i.e., reach within 1750 km of lunar center, return within 100 km of earth's surface, and seven day limit on total travel time), save an aptly named data file with following fields as its columns

   ```
   Iteration number
   h
   Time (h:mm:ss)
   Spacecraft's x coordinate
   Spacecraft's y coordinate
   Spacecraft-Earth distance
   Spacecraft-Moon distance
   ```

5. Include a visualization of the most economical trajectory that satisfies all of NASA's requirements and its computational details (hostname, CPU, memory, compilation and execution instructions, software name and version, compiler name and version, and CPU and wall times)

References

[1] National Aeronautics and Space Administration.
`http://www.nasa.gov/`.

[2] The Apollo Missions.
`https://www.nasa.gov/mission_pages/apollo/missions/`.

[3] A. Weir. *The Martian*. Crown Publishing, 2011.