Model of Job Shop Scheduling Based on Graph Theory and Combination of Machines

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Abstract: The paper gives up some hypotheses of the Job Shop Scheduling problem. Firstly, the operations of the jobs are planed following the priority of the jobs. Secondly, one operation of the job can be done by the combination of machines not only by one machine. Finally, a model based on the graph theory is introduced, an algorithm based on the greedy algorithm is proposed and a case study was used to illustrate the feasibility and effectiveness of the proposed method.

Keywords: Job Shop Scheduling, combination of machines, graph theory

1 INTRODUCTION

Generally, there are many constraints in the problem of Job Shop Scheduling (JSS) which is the most complex and hardest of all of Scheduling problem. JSS comes directly from the practice of production and be considered to belong to the class of decision problems which are NP. It can be more easily formulated under some hypotheses [1]-[3], models and finds a solution. Some hypotheses appear in this kind of job-shop problems:

- A finite set of jobs ,A finite set of machines
- The priority of the jobs are the same
- Each job consists of a chain of operations
- Each machine can handle at most one operation at a time
- Each operation needs to be processed during an uninterrupted period of a given length on a given machine

Purpose is to find a schedule, that is, an allocation of the operations to time intervals to machines that has minimal cost.

Based on those hypotheses, JSS can be formulated to such an easy mathematical model that it may not use the most efficient machines, the cost may not be the lowest and not be planned following the priority of the jobs.

According to the analysis, we can consider giving up some hypotheses. First, all jobs are planned following the priority, the prior the earlier be planned. The production capacity of machines can be divided into a number of small parts according to the delivery of the jobs, and each operation of the job has the different small capacity done by different machine. Second, one operation of the given job can be done by several machines not only by one machine, that is, the operation can be done by the combination of the machines at the same time. Finally, a mathematical model is introduced and a graph model can be proposed.

On solving the problem, we can establish the operation processes for each job and the priority order of the jobs. In accordance with priority sequence for the organization of work production, high-priority job can be planed early. Each job is scheduled by one turn according to the Method (2). After the organizing of the previous job, we can increase or decrease the production capacity of machines according to the different delivery and establish the graph model for the next job as Method (1).

Method（1）In accordance with the operation processes of each jobs, we build a graph model. The model contains a number of nodes, each vertex set of the nodes represents the set of the machines which can do the given operation. The edges between nodes represent the adjacent relationship. For each vertex of each vertex set, we divide it into a number of small dots in accordance with its production capacity and production Lot. The dots represent the small production capacities of the machines, so that we can make the combination it with other machine.

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Method（**2**）Based on the graph coloring and the greedy algorithm theory, from the initial dot of the graph model , choose a suitable small dot and mark it as coloring according to the production cost or efficiency of the machine. When the given job's production output is finished, the job moves to the next node, until the final operation to be made. Thus we find a schedule of this job. We use this method of model and solution on an example to illustrate the feasibility and effectiveness of the proposed method.

2 MODEL OF JSS BASED ON THE COMBINATION OF MACHINES

2.1 Mathmatical model

First present the symbols in the mathematical model previously described problems.

$I=\{I^1,I^2,\cdots,I^n\}$ ——the set of jobs;

$I^i=\{I_1^i,I_2^i,\cdots,I_{hi}^i\}$ ——the operations of job i ;

$O^i=\{O_1^i,O_2^i,\cdots,O_{hi}^i\}$ ——the needed production output of the job i ;

$P=\{P^1,P^2,\cdots,P^n\}$ ——the set of machines;

Q_k ——the production capacity of the machine P_k ;

$P_j^i=\{P_{j,1}^i,P_{j,2}^i,\cdots,P_{j,k}^i\}$ ——the machine set which can do the j -th operation of the job i , the job can choose several machines in P_j^i ;

$Q_{j,k}^i$ ——the production capacity of the machine P_k when it do the j -th operation of the job i ;

$C_{j,k}^i$ ——the unit production cost of the machine P_k when it do the j -th operation of the job i ;

$t_{j,k}^i$ ——the finished time of the machine P_k when it do the j -th operation of the job i ;

$TC_{j,k}^i$ ——the unit transportation cost from the machine P_k to machine P_l when they do the job i ; if $k=l$, then $TC_{j,k}^i=0$;

t^i ——the delivery of job i ;

T ——the delivery of all the jobs;

$$\delta_{k,l}^i=\begin{cases}1,\text{if job i from }P_k\text{ to }P_l\\0,\text{others}\end{cases}$$

$S=\{S^1,S^2,\cdots,S^n\}$ ——the machine set of the job i chose, and $S^i=\{S_j^i\}$, $S_j^i\subset P_j^i$, S_j^i is the machine set of the j -th operation of the job i chose;

$$\gamma_{k,l}^i=\begin{cases}1,\text{if }P_k\in S_j^i,\\0,\text{other}\end{cases};$$

$q_{j,k}^i$ ——the production capacity of the machine P_k when it do the j -th operation of the job;

According to the above definition, the JSS problem can be changed to the following optimization problem, namely, select a machine set $S=\{S^1,S^2,\cdots,S^n\}$ that:

$$MinJ=\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{j=1}^{hi}\gamma_{j,k}^iC_{j,k}^iQ_{j,k}^i\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{l=1}^{m}\delta_{k,l}^iTC_{k,l}^i \quad (1)$$

$$\text{s. t.}\quad \sum_{k=1}^{m}\gamma_{j,k}^iQ_{j,k}^i\ge O_j^i \quad (2)$$

$$\sum_{k=1}^{m}\sum_{j=1}^{hi}t_{j,k}^i\gamma_{j,k}^i\le t^i \quad (3)$$

$$\sum_{i=1}^{n}t^i\le T \quad (4)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{hi}q_{j,k}^i\gamma_{j,k}^i\le Q_k \quad (5)$$

The formula (1) is the objective function of the schedule function, the first part of the production cost function, and the second part is the transportation cost; the formula (2) lets the production capacities of the chose combination machines must meet the given job's output; the formula (3) lets the production of given job be completed before the due date; the formula (4) lets all the jobs be completed before the total time; the formula (5) let the given machine's 'task' can not be greater than its production capacity (let the time as benchmarks).

2.2 Graph model

Construct a graph model of the JSS. As shown in Figure 1, a batch operation processes look like a tree. The circle of the graph represent one operation of given job, the edge with the arrow represent the constraints between the operations. The next operation can start only after the previous operation is finished.

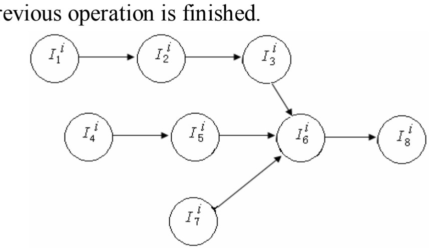

Fig 1. Operation Process Graph

In order to derivate the method, we improve the model to describe the problem. According to the operation process of a given job, let $I_j^i (j = 1,2,\cdots,hi)$ be the node set which contains many dots, with the big long-oval denoted; the vertices in them are the machine P_k which can take on the given operation, with the flat oval denote each machine .Select an appropriate number x_j^i as the smallest batch of the job. Let all the machine capacities $Q_{j,k}^i$ of that process can be divide exactly by this number (this number must exist, at least 1). Let the machine $P_{j,k}^i$ be divided into $a_{j,k}^i = Q_{j,k}^i / x_j^i$ parts in accordance with the production capacity, then some vertices of $P_{j,k}^i (j = 1,2,\cdots,hi)$ $P_{j,k}^i$ be divided into $a_{j,k}^i$ parts, denote as $\{P_{j,k1,}^i P_{j,k2}^i,\cdots,P_{j,ka_{j,k}^i}^i\}$. Each dot in this oval be endowed with the number $C_{j,k}^i$, denote as the unit production cost. Thus, we get a multi-part graph, each part contains a number of vertices (small oval) and each small oval contains a number of dots. Each dot stands for one part. Take the two operation processes I_1^i and I_2^i from Figure 1 as a case of the model's improvements, shown in Figure 2. For the sake of simplicity, the dots and endowed values are not marked.

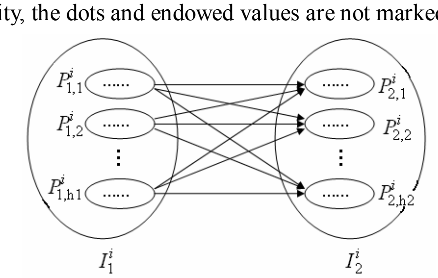

Fig 2. Job Subdivision Graph

In the actual process of mapping, we can not draw the small oval, directly draw a number of dots according to the production capacity, then I_j^i contains the vertex set

$$\{P_{j,11}^i, P_{j,12}^i,\cdots,P_{j,1a_{j,1}^i}^i, P_{j,21}^i, P_{j,22}^i,\cdots,P_{j,2a_{j,2}^i}^i,\cdots, P_{j,k1}^i, P_{j,k2}^i,\cdots,P_{j,ka_{j,k}^i}^i\}。$$

2.3 Algorithm

Algorithm design is based on the thinking of the greedy algorithm. Greedy algorithm is a stepwise method to find a optimal solution. At each step, find a decision-making (in certain criterion) which looks like optimal. The decision once has been made, it can not be changed. The criterion which is making the decision is the greedy criterion.

We can notice that the criterion used in the scheduling problem is the criterion in the actual environment of machines used by the scheduling rules. It does not guarantee to get the optimum value, but the results are usually very close to optimum value and can be used as a satisfying value.

According to the priority of the jobs, order all the jobs, get the set $I = I^1, I^2,..., I^n$, the previous job is prior to the next one in the set, such as I_1^i prior to I_2^i.

（1）Construct the function Check $i = i+1$ as follows:

Check $i = i+1$: the function is to select the machine which is alternative to the operation process when I_1^i converted to I_2^i. Those machines may already be assigned a number of jobs when it finished the previous job, and their capacity must be decreased. Select the remaining capacity of them (the un-coloring dots), plus the increasing capacity due to the extension of delivery (if there is a extension of delivery) or subtract the capacity due to the reduction of delivery (if it less than zero, then remove the machine as the alternative machine), and construct a new graph model for the job I^{i+1}.

（2）Algorithm

Step 0. Let $i = 0, j = 0, S^i = \Phi$, and all dots in I^i are not colored;

Step 1. If $i = n$, then stop. Output $J, S = \{S^1, S^2,..., S^n\}$; else let $i = i+1, j = 1$, call Check $i = i+1$, goto Step 3.

Step 2. If $j \le hi - 1$, the let $j = j+1$, goto Step 3, else goto Step 1;

Step 3. If $O_j^i > 0$, goto Step 4, else goto Step 2;

Step 4. Select the un-coloring dots in I_j^i which value is $C_{j,k}^i$, calculate

$$J = J + x_j^i C_{j,k}^i + \sum_{l=1}^{m} \delta_{l,k}^i TC_{l,k}^i + \sum_{g=1}^{m} \delta_{k,g}^i TC_{k,g}^i,$$

while $P_l, P_g (l = 1,2,...,m)$ is the P_k's all the dots and all the close previous or behind dots, if P_k is the initial dot, then let $\delta^i_{l,k} = 0$, if P_k is the final dot, then let $\delta^i_{k,g} = 0$. Select a dot $P^i_{l,ky}$ which let the J be minimum, let $S^i_j = S^i_j + \{P^i_{l,ky}\}, O^i_j = O^i_j - x^i_j$, goto Step 3.

3 CASE STUDY

3.1 Case

In the algorithm designing, each job is planned in accordance with its priority by the same steps. So we only take the first job for example to analysis.

Suppose the given job contains four operation processes $\{I_1, I_2, I_3, I_4\}$ and five workpieces, as shown in Figure 3.

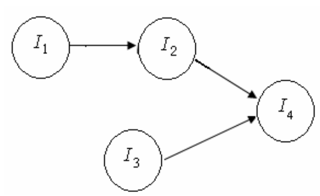

Fig 3. The operation processes graph of the case

Suppose the set $\{P_1, P_2, P_3, P_4, P_5, P_6\}$ are the six machines, the following two tables are the related data. Table 1 lists the distance between those machines. Table 2 lists the relationships between the machines and jobs, that the first number in the table is the production capacity of the given machine, the second number is the unit production cost.

Table1. Distance table

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	0	3	1. 5	5. 5	3	2
P_2	3	0	2	2	2	2
P_3	1. 5	2	0	2	1	3
P_4	5. 5	2	2	0	1	2
P_5	3	2	1	1	0	3
P_6	2	2	3	2	3	0

Table2. Production capacity table

	P_1	P_2	P_3	P_4	P_5	P_6
I_1	3, 3	4, 3	0	5, 4	0	0
I_2	0	4, 3	0	5, 2	0	6, 2
I_3	0	0	4, 5	0	8, 5. 5	6, 5. 5
I_4	0	0	5, 5	3, 5. 5	0	4, 6

3.2 Results of the Case

We use two methods to solve the above case, and compare their results.

(1) Using the general methods without the combination machines, as articles [2], [3], [4], the results for the case is that: I_1 can only choose; can choose P_4 ; I_2 can choose P_4 or P_5 ; I_3 can choose P_5 or P_6 ; I_4 can only choose P_3 . Thus minJ=97.5 according to the transportation cost.

(2) In accordance with the algorithm of this paper: I_1 can choose P_1, P_2, P_4 , I_2 can choose P_2, P_4, P_6 ; I_4 can choose P_4, P_5, P_6 . Using the algorithm in the paper, I_1 chose $P_1(3)$ and $P_2(2)$; I_2 chose $P_3(4)$ and $P_5(1)$; I_4 chose $P_3(5)$, the number in the bracket is the capacity of the machines did. Thus minJ=91.5.

From the above example, the proposed method obtains a better value than the general method. The cost of example dropped to 91.5 from 97.5, and all jobs were planned according to their priority.

4 CONCLUSION

Researches on the JSS problem mainly based on a number of assumptions, obtained some solutions. But they simplified the problem that model did not accord with the practice. This paper gives up some of the assumptions, build a model based on the graph theory and let the model more suitable for the problem. In accordance with model, propose an algorithm to solve the problem. A case study is used to illustrate the feasibility and effectiveness of the proposed method. How to improve the efficiency with intelligent algorithm to large-scale problem is the further study.

REFERENCES

[1] X. S. Gu, A Survey of Production Scheduling under Uncertainty, Journal of East China University of Science and Technology, Vol.26, No.5,441-446, 2000.

[2] D. Sun, Battar, Scheduling larger job shops: a decomposition approach, International Journal of Production Research, Vol.34, No. 7, 2019-2033, 1996.

[3] C. Chu, M. C. Portmann, J. M. Proth, A splitting-up approach to simplify job—shop scheduling problems, International Journal of Production Research, Vol. 30, No. 4, 859-870, 1992.

[4] J. F. Goncalves, G. C. Mauricio, A hybrid genetic algorithm for the job shop scheduling problem. AT&T Labs Research, 2002.

[5] W. Zhang，J. Lin, Multi- stage production mod el based on mass customization, Computer Integrated Manufacturing Systems, Vol. 13, No. 2, 246-250,280, 2007.

[6] J. Sun, J. Lin, Study of supply chain optimization scheduling in mass customization based on ant colony algorithm, Computer Applications, Vol. 26, No. 11, 2631-2634, 2683, 2006.