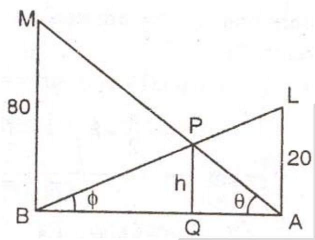
HINTS AND SOLUTIONS **TEST CODE: MTJNU - I**

1. (b) For $r = a\cos k\theta$ or $r = a\sin k\theta$, k is even, then the number of leaves be $2k$. k is odd, then the number of leaves be k. Here u = 4 and v = 3

2. (c)



$$\tan\theta = \frac{80}{AB} = \frac{h}{AQ}$$

$$\therefore AQ = h.\frac{AB}{80}$$

$$\tan\phi = \frac{20}{AB} = \frac{h}{BQ}$$

$$\therefore BQ = h.\frac{AB}{20}$$

Now, $AB = AQ + BQ$

3. (d) $\dfrac{dy}{dx} = \dfrac{3(x+\alpha)^2}{2\beta y}$

As, Subtangent $= \dfrac{y}{dy/dx} = \dfrac{2}{3}\beta^{1/3}y^{2/3}$

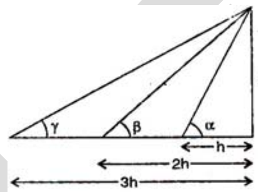
$$(\text{Subtangent})^2 = (\frac{4}{9}\beta^{2/3})y^{4/3}$$

and, Subnormal $= y.\dfrac{dy}{dx} = (\dfrac{3}{2\beta^{1/3}})y^{4/3}$

4. (b) Let A has $r(0 \le r \le n)$ elements. B is constructed by selecting some elements from remaining $(n - r)$ elements. Clearly, A can be chosen in n_{C_r} ways and B in $(n-r_{C_0+}n-r_{C_1+\ldots\ldots n-r_{C_{n-r}}}) = 2^{n-r}$. Total number of ways of choosing A and B is $n_{C_r \times 2^{n-r}}, 0 \le r \le n$

So, required number of ways $= \sum_{r=0}^{n} n_{C_r} \times 2^{n-r} = (1+2)^n$

5. (a) Clearly $\tan\alpha = \dfrac{h}{h} = 1$, $\tan\beta = \dfrac{1}{2}$, $\tan\gamma = \dfrac{1}{3}$



$$\tan(\alpha+\beta+\gamma)$$

$$= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\gamma\tan\alpha} = \infty = \tan\frac{\pi}{2}$$

6. (b) Let $u = \cos\theta\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\}$

$\therefore (u - \sin\theta\cos\theta)^2 = \cos^2\theta\{\sin^2\theta + \sin^2\alpha)$

$\Rightarrow u^2\tan^2\theta - 2u\tan\theta + u^2 - \sin^2\alpha = 0$ since $\tan\theta$ is real

$\therefore$ discriminant ≥ 0 $\therefore 4u^2 - 4u^2(u^2 - \sin^2\alpha) \ge 0$

$\Rightarrow u^2 \le 1 + \sin^2\alpha \Rightarrow |u| \le \sqrt{1+\sin^2\alpha}$

7. (d)

$$\cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) + 2\cos^4 x + \cos^2 x - 2$$

$$= \cos^6 x(\cos^2 x + 1)^3 + 2\cos^4 x + \cos^2 x - 2$$
$$= \sin^3 x(\sin x + 1)^3 + 2\sin^2 x + \cos^2 x - 2$$
$$\left[\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x\right]$$
$$= (\sin x + \sin^2 x)^3 + \sin^2 x + (\sin^2 x + \cos^2 x) - 2$$
$$= 1^3 + \sin^2 x + 1 - 2 = 1 + \sin^2 x - 1 = \sin^2 x$$

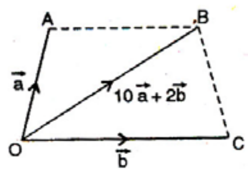
8. (c) $ax^2 + bx + c = a\left[\left(x^2 + \dfrac{b}{a}x\right) + \dfrac{c}{a}\right]$

$$= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right] = a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right]$$

whose least value = $\frac{4ac-b^2}{4a}$, when $x+\frac{b}{2a}=0$

9. (c) When values are defined for an enum, they are interpreted as integer values 0, 1, 2, etc by default. Thus, add is assigned 0, delete is assigned 1 and so on.

10. (b)



q = $\left|\vec{a}\times\vec{b}\right|$, p = $\frac{1}{2}\left|\vec{b}\times(10\vec{a}+2\vec{b})\right|+\frac{1}{2}\left|\vec{a}\times\left(10\vec{a}+2\vec{b}\right)\right|$

$\therefore$ p = $5\left|\vec{b}\times\vec{a}\right|+\left|\vec{a}\times\vec{b}=p\right| = 6\,|\,\vec{a}\times\vec{b}\,|=6q$

11. (d)

12. (a)

13. (b) $f(x)=\sin(\log x)$

Then $f\left(xy\right)+f\left(x/y\right)-2f\left(x\right)\cos\left(\log y\right)$

= $\sin\left(\log xy\right)+\sin\left(\log x/y\right)-2\sin\left(\log x\right)\cos\left(\log y\right)$

=

$\left[\sin\left(\log x+\log y\right)+\sin\left(\log x-\log y\right)\right]-2\sin\left(\log x\right)\cos\left(\log y\right)$

= $2\sin\left(\log x\right)\cos\left(\log y\right)-2\sin\left(\log x\right)\cos\left(\log y\right)=0$

14. (a) $y=\int_{\cos x}^{x}(u+u^2)du$

Here $\phi_1(x)=\cos x, \phi_2(x)=x$ and $f(u)=u+u^2$

since $\frac{dy}{dx}=f(\phi_2(x))\phi_2{}'-f(\phi_1(x))\phi_1{}'(x)$

$=(x+x^2)1-(\cos x+\cos^2 x)(-\sin x)$

$=(x+x^2)+\cos x.\sin x+\cos^2 x\sin x$

15. (d)

Here, $\vec{r}_1-\vec{r}_2=(9\hat{i}-\hat{j}+2\hat{k})-(3\hat{i}-2\hat{j}+\hat{k})=6\hat{i}+\hat{j}+\hat{k}$

Also $\vec{F}=5\hat{i}+\hat{k}$ and moment of the couple = $(\vec{r}_1-\vec{r}_2)\times\vec{F}$

16. (d) Since the least positive prime is 2;

$\therefore\ P_1\ge 2, P_2>2, P_3>2,........P_k>2$

$\therefore\ n\ge 2^{\alpha_1}.2^{\alpha_2}.2^{\alpha_3}.....2^{\alpha_k}$ (as given)

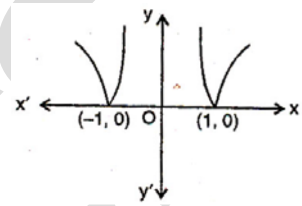
or $n\ge 2^{\alpha_1+\alpha_2+\alpha_3.....\alpha_k}$ Also, $\alpha_1+\alpha_2+\alpha_3.....\alpha_k\ge k$

$\therefore\ n\ge 2^k$

Taking log on both sides, $\log_e n\ge k\log_e 2$

17. (b)

Clearly $f(x)$ is everywhere continuous but not differentiable at $x=\pm 1$



18. (c) $a*(b*c)=a*(b+c-bc)$

$=a+(b+c-bc)-a(b+c-bc)$

19. (a)

20. (b)

21. (b) Round off error = True value - Rounded value = 9.089 - 9.09

22. (b) Since 10 > 3 > e > 2 required order is $\log_{10}\alpha<\log_3\alpha<\log_e\alpha<\log_2\alpha$.

23. (d) Bounded medium : signals are confined to the medium specified and don't leave it (except for smaller leakage amounts) Examples of Bounded medium: co-axial cable, optical fiber and wave guide.

24. (b)

25. (b) $r=a.\frac{1+e^{-\theta}}{1-e^{-\theta}}$ Then $\lim_{\theta\to\infty}a.\frac{1+e^{-\theta}}{1-e^{-\theta}}=a$

$\therefore$ The circular asymptote is $r = a$

26. (a)

27. (d) As n is not a multiple of 3, then $z^n+z^{2n}=z+z^2$ -1

28. (b)

29. (c)

30. (c)

31. (c) $\vec{r} = 3\vec{i} - \vec{j} + t(\vec{i} + 2\vec{j} + 3\vec{k})$. So a vector parallel to the line $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$

$\therefore$ Unit vector along the line is $\dfrac{\vec{i} + 2\vec{j} + 3\vec{k}}{\sqrt{1^2 + 2^2 + 3^2}}$

$\therefore$ the projection $= (\vec{i} + \vec{j} + \vec{k}).\dfrac{\vec{i} + 2\vec{j} + 3\vec{k}}{\sqrt{14}}$

32. (c) Putting $a = r\cos\alpha, b = r\sin\alpha,$ we get
$\sin x\cos\alpha + \cos x\sin\alpha = c/r$
or, $\sin(x+\alpha) = c/\sqrt{a^2+b^2}$ but $\because$ $|c| > \sqrt{a^2+b^2}$
$\therefore$ We have $|\sin(x+\alpha)| > 1$, not possible.

33. (c) Since imaginary roots occur in pairs
So, $(x-1)(x-3+2i)(x-3-2i) = 0$

34. (a) Circle that passes through origin and whose center lies on y-axis: $x^2 + y^2 + 2fy = 0$ where f is a parameter.

$\therefore$ $2x + 2y\dfrac{dy}{dx} + 2f\dfrac{dy}{dx} = 0 \Rightarrow$

$2x + 2y\dfrac{dy}{dx} - \dfrac{x^2+y^2}{y}.\dfrac{dy}{dx} = 0$

$\Rightarrow 2xy + (y^2 - x^2)\dfrac{dy}{dx} = 0$

35. (b) There are two ways in which one rupee coin is still in first purse.
(1) It did not went to 2nd bag at all.
(2) It goes to 2nd bag and then put back.

$P(1) = \dfrac{{}^9C_9}{{}^{10}C_9} \times \dfrac{{}^{19}C_9}{{}^{19}C_9} = \dfrac{1}{10}$

$p(2) = \dfrac{{}^9C_8 \times {}^1C_1}{{}^{10}C_9} \times \dfrac{{}^{18}C_8 \times {}^1C_1}{{}^{19}C_9} = \dfrac{81}{190}$

Required Probability $= \dfrac{1}{10} + \dfrac{81}{190} = \dfrac{10}{19}$

36. (d)

37. (a)

38. (d) In a singly linked list only one pointer holds the address of next node and since the pointer in each node stores the address on next node. So the nth node can be accessed only after accessing (n - 1) nodes before this.

39. (c) Total 46 characters are given. and since $2^5 = 32 < 46 < 2^6 = 64$ $\therefore$ 6 bits are required to encode them.

40. (c) 32×1024

41. (b) $j = i <= 5$; $i++$;

42. (d) If the first game is won by A, then its probability of alternately winning $= \dfrac{1}{2} \times \dfrac{1}{3} \times \dfrac{1}{2} = \dfrac{1}{12}$
If the first game is won by B, then its probability of alternatively winning $= \dfrac{1}{3} \times \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{18}$ Let $E: A$ and B win alternatively $\therefore$ P (E) $= 1/18 + 1/12 = 5/36$

43. (c) $4 + 5\left(\dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{334} + 3\left(\dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{365}$

$= 4 + 5\omega^{334} + 3\omega^{365}$ $\quad = 4 + 5\omega + 3\omega^2$

$= 4 + 5\left(\dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}\right) + 3\left(\dfrac{-1}{2} - \dfrac{i\sqrt{3}}{2}\right) = i\sqrt{3}$

44. (a) $(x+a)p^2 + (x-y)p - y = 0$
or $p(p+1)x - (p+1)y + ap^2 = 0$
or $y = px + \dfrac{ap^2}{p+1}$ (Clairaut's form) So, required solutions is

$y = cx + \dfrac{ac^2}{c+1}$ (Replacing p by c)

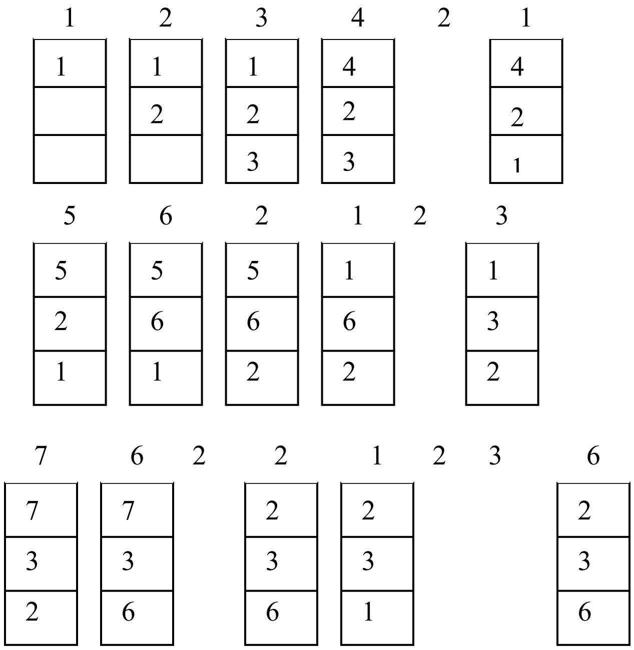
45. (d)

46. (c)

47. (a)

48. (d) As, standard deviation $\le$ Range $= b - a$ $\therefore$ variance $\le (b-a)^2$

49. (b) LRU : Replaces page that has not been used for largest period of time.



50. (b)

51. (b) Point $P(\alpha,\beta)$ on $x^2 - y^2 = a^2$ is nearest to the line $y = 2x$ if the tangent at P is parallel to the line $y = 2x$.

Now, $x^2 - y^2 = a^2 \Rightarrow 2x - 2y\dfrac{dy}{dx} = 0$ So, $\dfrac{dy}{dx} = \dfrac{\alpha}{\beta}$ at (α,β)

Also slope of the line $y = 2x$ is 2.

$\therefore \dfrac{\alpha}{\beta} = 2 \Rightarrow \alpha = 2\beta \;\therefore$ locus of P is $x - 2y = 0$

52. (a) Let number of present bacteria is x

$\therefore \dfrac{dx}{dt} \propto x \Rightarrow \dfrac{dx}{dt} = kx \qquad \therefore \log x = kt + c$

At time $t = 0, c = \log x$

$\log 2x = 2k + \log x \Rightarrow k = \dfrac{1}{2}\log 2$

Let at time T bacteria will be triple

$\therefore \log 3x = \dfrac{1}{2}\log 2 . T + \log x \Rightarrow T = 2\dfrac{\log 3}{\log 2}$

53. (d) Number of nodes in a heap with n leaf nodes = 2* n - 1

54. (d)

$$f^2 = \begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix} = \begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$$

$$f^3 = \begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix} = \begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$$

= I $\Rightarrow$ Order of f = 3

55. (d) $\tan\left(\dfrac{x^2+y^2}{x+y}\right)$ can't be put in the forms $x^n f\left(y/x\right)$ or $y^n\phi(x/y)$. Hence, not homogeneous.

56. (a) For a moderately skewed distribution
Mode = 3 median - 2 mean

57. (d) If X and Y are two independent variable, then r = 0. Putting r = 0 in equations of the two lines of regression, we get $Y = \bar{Y}$ and $X = \bar{X}$ which are of the type X = constant and Y= constant.

58. (d) The general solution of the differential equation is $y = c_1\cos x + c_2 \sin x$ $y(0) = c_1\cos 0 + c_2\sin 0 \Rightarrow c_1 = 0$ and $y(\pi) = -c_1 \Rightarrow c_1 = 0 \therefore c_2$ can have infinite values.

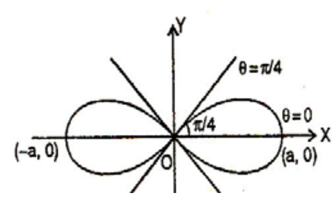
59. (b)

60. (c) $\log 2, \log(2^x - 1), \log(2^x + 3)$ are in A.P.

$\Rightarrow 2\log(2^x - 1) = \log 2 + \log(2^x + 3)$

$\Rightarrow (2^x - 1)^2 = 2(2^x + 3) \Rightarrow 2, 2^x - 1, 2^x + 3$ are in G.P.

61. (a)



the required area $A = 4\int_{\theta=0}^{\pi/4}\int_{r=0}^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta$

62. (a) $a_m = \dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \ldots + \dfrac{1}{2^m}$

$a_{m+1} = \dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \ldots + \dfrac{1}{2^m} + \dfrac{1}{2^{m+1}}$

$a_{m+1} - a_m = \dfrac{1}{2^{m+1}} > 0, \forall m > 0$

$\Rightarrow < a_m >$ is monotonically increasing .

63. (b)

64. (c)

65. (c) equating to zero the lowest degree term, we get

$3y(x^2-y^2)=0 \Rightarrow x+y=0,\ x-y=0, y=0$

$\Rightarrow$ There are three tangents at the origin

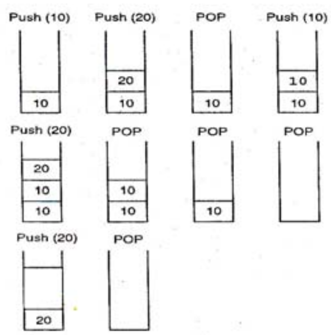
$\Rightarrow$ origin is a triple point

66. (b) Since the multiplication of a row of a matrix by a non-zero number does not alter its rank, so when each row is multiplied by 2, the rank will remain unchanged Hence, $\rho(2A)=\rho(A)=2$.

67. (a)

68. (b)

69. (b)



So sequence is 20, 20, 10, 10, 20.

70. (a)

71. (b) If n is a negative integer, then n = - m, where $m \in N$

$\therefore \lim\limits_{x\to\infty} \frac{x^n}{e^x} = \lim\limits_{x\to\infty} \frac{1}{e^x} = 0$ If $n \in N$

$\lim\limits_{x\to\infty} \frac{x^n}{e^x} = \lim\limits_{x\to\infty} n.\frac{x^{n-1}}{e^x}$ [L. Hospital's Rule]

Hence $\lim\limits_{x\to\infty} \frac{x^n}{e^x} = 0 \ \ \forall n$

72. (b) $p=0.001$ and $n=2000$

$\therefore \lambda = (\text{mean}) = np = 2000 \times 0.001 = 2$

$\Rightarrow$ Probability that exactly 3 persons will suffer

$$P(X=3)=\frac{e^{-\lambda}.\lambda^x}{x!}=\frac{e^{-2}.2^3}{3!}=\frac{4}{3}.e^{-2}$$

73. (c)

74. (a) We have , $\frac{\text{Total}}{n}=m$

$\Rightarrow$ Total population = $mn \Rightarrow$ New mean = $\frac{mn+x}{n+1}$

75. (b) The common chord of the given circles is

$6x+14y+c+d=0$ ....(i)

Since $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle $x^2+y^2-2x+8y-d=0$. So, (i) passes through the center of the second circle i.e. $(1,\ -4)$

$\therefore 6-56+c+d=0 \Rightarrow c+d=50$

76. (d) Line is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}=a$ (say) $\therefore$ Any point P on it is (a,a,a). D.R.'s of OP will be (a,a,a) or (1,1,1). Any plane through P (a,a,a) and at right angles to OP will have its equation as:

1. $(x-a+1).(y-a)+1.(z-a)=0$ or $x+y+z=3a$ Its intercepts on axes are $3a,\ 3a,\ 3a$.

Sum of their reciprocals is $3.\frac{1}{3a}=\frac{1}{a}$.

77. (a) Post order traversal yields 4, 5, 2, 6, 7, 3, 1.
Comparing with a, b, -, c, d, *, +, we get the labels of nodes 1, 2, 3, 4, 5, 6, 7 as +, -, *, a, b, c, d respectively.

78. (a) The given equation is

$x^2(\lambda+1)-x\{b(\lambda+1)+a(\lambda-1)\}+c(\lambda-1)=0$

According to question , sum of roots = 0

$\Rightarrow \frac{b(\lambda+1)+a(\lambda-1)}{\lambda+1}=0 \Rightarrow \lambda=\frac{a-b}{a+b}$

79. (a) $f'(x)=x\sin\frac{1}{x}.\ \forall x\in(0,\pi)$ As a finitely differentiable function is also continuous, so f(x) is continuous in (0, π).

80. (b) ${}^{n+1}C_{n-2}-{}^{n+1}C_{n-1}\le 100 \Rightarrow {}^{n+1}C_3-{}^{n+1}C_2\le 100$

$\Rightarrow \frac{(n+1)n(n-1)}{6}-\frac{(n+1)n}{2}\le 100$

$\Rightarrow (n+1)n(n-1-3)\le 600 \Rightarrow (n+1)(n)(n-4)\le 600$

The value of n satisfying this inequality are 2, 3, 4, 5, 6, 7, 8, 9

81.(c) $f(x+1)=\{x+1-[x+1]\}|\cos\pi(x+1)|$

$=\{x+1-[x]-1\}|-\cos\pi x|$

$=\{x-[x]\}|\cos\pi x|=f(x)$. So, $f(x)$ is periodic

82. (b) Let B: bowler, W : wicket-keeper and O: other. Thus we have B (4), W(2), O (10).

B(4)	W(2)	O(10)	
3	1	7	${}^4C_3.{}^2C_1.{}^{10}C_7 = 960$
3	2	6	${}^4C_3.{}^2C_2.{}^{10}C_6 = 840$
4	1	6	${}^4C_4.{}^2C_1.{}^{10}C_6 = 420$
4	2	5	${}^4C_4.{}^2C_2.{}^{10}C_5 = 252$

$\therefore$ Total selections on adding = 2472

83. (b) Number of persons who read at least two newspapers = Number of persons who read two newspapers and more = 12 + 2 + 8 + 5 = 27 it means out of 50 persons 27 read at least two newspapers. Number of such persons per 10,000 $= \left(\frac{27}{50}\times 10,000\right) = 5400$

84. (a)

85. (d)

86. (b) A^2=A.A=(AB). (AB) $[\because AB = A]$
= A (B.A)B
= A.BB $[\because BA = B]$
=(AB)B =AB=A $[\because AB = A]$
Similarly B^2=B $\therefore$ A and B are idempotent matrices

87. (b) Sparse graph is a graph, which has the number of edges much less than the possible number of vertices squared.

88. (b) Limit points other than the origin of the co-axial system of circles $x^2 + y^2 + 2gx + 2fy + c = 0$ are given by

$$\left(\frac{-gc}{g^2+f^{2'}}\frac{-fc}{g^2+f^2}\right) \textit{Here } g=-3, f=-4, c=1$$

Hence, other limiting point is $\left(\frac{3}{25},\frac{4}{25}\right)$

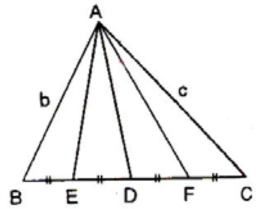
89. (c) The error is that the struct variable is declared twice. First time along with the type declaration, and then again in the initialisation statement.

90. (a)

91. (c) By given condition, $\phi(x) = f^{-1}(x)$
$\therefore \quad f[\phi(x)] = x.$ Differentianing w.r.t. x

$$f'[\varphi(x)]\phi'(x) = 1 \text{ or } \phi'(x) = \frac{1}{f'(\phi(x))} = 1 + [\phi(x)]^n$$

92. (a)



In $\Delta ABC, AD^2 = m_1^2 = \frac{c^2+b^2}{2} - \frac{a^2}{4}$

$[\because AB^2 + AC^2 = 2(AD^2 + BD)^2]$

$$In\ \Delta ABD, AE^2 = m_2^2 = \frac{c^2+AD^2}{2} - \frac{(a/2)^2}{4}$$

$$In\ \Delta ADC, AF^2 = m_3^2 = \frac{AD^2+b^2}{2} - \frac{(a/2)^2}{4}$$

$$\therefore m_2^2 + m_3^2 = AD^2 + \frac{b^2+c^2}{2} - \frac{a^2}{8} = 2m_1^2 + \frac{a^2}{8}$$

93. (b) $S = \cot^{-1} 2.1^2 + \cot^{-1} 2.2^2 + \cot^{-1} 2.3^2 + ...$ Thus,
$S = \tan^{-1} 3 - \tan^{-1} 1 + \tan^{-1} 5 - \tan^{-1} 3 + \tan^{-1} 7 - \tan^{-1} 5 + ...\infty$

$$= \tan^{-1}\infty - \tan^{-1} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

94. (a) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + ..\right)$$

$$-(\frac{1}{2^2} + \frac{2}{4^2} + \frac{2}{6^2} + ...)$$

$$= \frac{\pi^2}{6} - \frac{1}{4}(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...) = \frac{\pi^2}{6} - \frac{1}{4}.\frac{\pi^2}{6} = \frac{\pi^2}{8}$$

95. (d) $P((A_1 \cup A_2) \cap (\overline{A_1} \cap \overline{A_2}))$

$= P((A_1 \cup A_2) \cap (\overline{A_1 \cup A_2})) = 0 < \frac{1}{4}$

96. (c) Replacing x by $\frac{1}{y}$ in given equation

$cy^3 - 3by^2 + 3ay - 1 = 0$ has roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ in A.P.

$$\therefore \quad \frac{2}{\beta} = \frac{1}{\alpha} + \frac{1}{\gamma} \Rightarrow \frac{3}{\beta} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} \Rightarrow \frac{3}{\beta} = \frac{3b}{c} \Rightarrow \beta = \frac{c}{b}$$

97. (b) $42 = 2\times3\times7$ So, 2 can be assigned to either x or y or z in 3 ways. Similarly, each of 3 and 7 can be assigned in 3 ways. Thus the number of solutions is $3\times3\times3 = 27$

98. (a) **99. (c)** **100. (d)**

101. (c) $\left|\vec{a}\times\vec{b}.\vec{c}\right|$ = Volume of the parallelepiped

= $|\vec{a}|.|\vec{b}|.|\vec{c}|$ $\Rightarrow$ It is a rectangular parallelepiped i.e. concurrent edges are perpendicular to each other. So, $\vec{a}.\vec{b}=0,\ \vec{b}.\vec{c}=0,\ \vec{c}.\vec{a}=0.$

102. (c)

103. (a)

104. (b) Let (h,k) be the pole Then, its polar is

$kx+ky-3(x+h)-4(y+k)+5=0$

$\Rightarrow (h-3)x+(k-4)y-(3h+4k-5)=0$ .....(i)

$\Rightarrow$ Comparing (i) with $3x+4y-45=0$, we get

$\Rightarrow h-3=3$ and $k-4=4 \Rightarrow h=6$, $k=8$

105. (c) If $f(x)=\sin(1/x)$ and $g(x)=1/x$, then both $\lim\limits_{x\to0} f(x)$ and $\lim\limits_{x\to0} g(x)$ do not exist but $\lim\limits_{x\to0}\dfrac{f(x)}{g(x)}=0$ exist

106. (d) **107. (c)** **108. (b)**

109. (d) In the binary search tree the node with smaller value comes on the left side of nodes and the node with greater value comes on the right side of node. So the node which has minimum value is the node with no left child, because there will be no smaller value than that node.

110. (a)

111. (b) Subtangent = Subnormal$\Rightarrow \dfrac{y}{dy/dx}=y.\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}=\pm1$ So, length of tangent $=\dfrac{y\sqrt{1+\dfrac{dy}{dx}}}{\dfrac{dy}{dx}}=y\sqrt{2}$

112. (a) If the (1, 3) entry in M^3 is 2, it means there are 2 paths of a length 3, connecting nodes 1 and 3 In (a) there are 2 paths connecting 1 and 3.

$(1\to2\to3\to3\ and\ 1\to3\to3\to3)$.

113. (c)

114. (b) Bowley's coefficient of skewness

$=\dfrac{Q_1+Q_3-2Q_2}{Q_3-Q_1}=\dfrac{6+4-2\times4}{6-4}=1$, Where Q_1,Q_2,Q_3 are first, second and third quartiles respectively.

115. (a)

$$P(\bar{A}/\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\frac{P(\overline{A\cup B})}{P(\bar{B})}=\frac{1-P(A\cup B)}{1-P(B)}$$

116. (d) $\phi(x)=\phi'(x)\Rightarrow\dfrac{\phi'(x)}{\phi(x)}=1\Rightarrow\log\phi(x)$

$=x+\log c$ or $\phi(x)=ce^x. For\ x=1,\phi(1)=2;$ so $c=2/e$

$\therefore \phi(x)=2e^{x-1}\quad\Rightarrow\phi(3)=2e^2$

117. (a) If ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ and the straight line $y=mx+c$ intersect in real points, then the quadratic equation $\dfrac{x^2}{a^2}+\dfrac{(mx+c)^2}{b^2}=1$ has real roots.

$\therefore$ Discriminant $\ge0\Rightarrow c^2-b^2\le a^2m^2$

118. (a) Let $Sn=3+7+13+21+....+Tn$

Also, $Sn=\quad 3+7+13+...+Tn---1+Tn$

$0+3\dfrac{n-2}{2}[2.4+(n-2).2]=n^2+n+1$

Then S_n for the given series

$Sn=\sum\limits_{r=1}^{n}\cot^{-1}(r^2+r+1)=\sum\limits_{r=1}^{n}\tan^{-1}\left(\dfrac{1}{r^2+r+1}\right)$

$=\sum\limits_{r=1}^{n}\tan^{-1}\left(\dfrac{(r+1)-r}{1+r(r+1)}\right)=\sum\limits_{r=1}^{n}(\tan^{-1}(r+1)-\tan^{-1}r)$

$=\tan^{-1}(n+1)-\tan^{-1}=\tan^{-1}\dfrac{n+1-1}{1+(n+1).1}$

$=\tan^{-1}\left(\dfrac{n}{n+2}\right)$ for n =10 we have,

$=\tan^{-1}\left(\dfrac{10}{12}\right)=\tan^{-1}\left(\dfrac{5}{6}\right)$

119. (b) **120. (c)**

Additional Problems

1. Three urns contain 6 red and 4 black; 4 red and 6 black, and 5 red and 5 black balls respectively. Once of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that is drawn from the first urn.

(a) 2/5 (b) 4/9

(c) 20/49 (d) 9/43

Sol. (a) Let E_1, E_2, E_3 and A be the events defined as follows: E_1= urns first is chosen, E_2 = urn second is chosen, E_3=urn third is chosen, A = ball drawn is red. Since there are three urns and one of the three urns is chosen at random, therefore $P(E_1) = P(E_2) = P(E_3) = 1/3$. If E_1 has already occurred, then urn first has been chosen which contain 6 red and 4 black balls. The probability of drawing a red ball from it is 6/10.

So, P (A/E_1) = 6/10. Similarly, P (A/E_2) = 4/10 and P (A/E_3) = 5/10. We are required to find $P(E_1/A)$, i.e., given that the ball drawn is red, what is the probability that it is drawn from the first urn. By Baye's rule

$$= P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)(P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{(1/3)\times(6/10)}{(1/3)\times(6/10)+(1/3)\times(4/10)+(1/3)\times(5/10)} = \frac{6}{15}$$

2. Two persons A and B throw a die alternately till one of them gets a "three" and wins the game. Find their respective probability of winning, if A begins

(a) 7/11 (b) 3/23

(c) 5/11 (d) 9/43

Sol. (c) We define the following events:

E = Person A gets a three, F = Person B gets a three. Clearly $P(E) = 1/6$, $P(F) = 1/6$, $P(\bar{E}) = 1 - 1/6 = 5/6$ and $P(\bar{F}) = 1 - 1/6 = 5/6$. A wins if he throws a three in the first throw. His probability of throwing a 'three' in first throw = $P(E) = 1/6$. A will get third throw if he fails in first and B fails in second throw. Therefore Probability of winning of A in third throw

$= P(\bar{E}\cap\bar{F}\cap E) = P(\bar{E})P(\bar{F})P(E) = (5/6)(5/6)(1/6)$.

$= (5/6)^2 \times (1/6)$

Similarly probability of winning of A in fifth throw .

$= P(\bar{E}\cap\bar{F}\cap\bar{E}\cap\bar{F}\cap E)$

=

$P(\bar{E})P(\bar{F})P(E)P(\bar{F})P(E) = (P(\bar{E}))^2 P(E) = (5/6)^4(1/6)...$

and so on .Hence, probability of winning of

$A = P(E\cup(\bar{E}\cup\bar{F}\cup E)\cup(\bar{E}\cap\bar{F}\cap\bar{E}\cap\bar{F}\cup E)\cup...]$

$= \{P(E) + P(\bar{E}\cap\bar{F}\cap E) + P(\bar{E}\cap\bar{F}\cap\bar{E}\cap\bar{F}\cap E)...$

$= (1/6) + (5/6)^2 \times (1/6) + ... = 6/11$

Thus, probability of winning of B = 1 – Probability of winning of A = 1 – 6/11 = 5/11.

3. The probability of A, B, C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.

(a) 23/56 (b) 25/56

(c) 5/11 (d) 19/56

Sol. (b) Let E_1, E_2, E_3 be the events that the problem is solved by A, B, C respectively. Then $P(E_1) = 1/3$, $P(E_2) = 2/7$ and $P(E_3) = 3/8$.

Exactly one of A, B and C can solve the problem in the following mutually exclusive ways:

(i) A solves and B and C do not solve i.e. $E_1 \cap \bar{E}_2 \cap \bar{E}_3$

(ii) B solves and, A and C do not solve, i.e. $\bar{E}_1 \cap E_2 \cap \bar{E}_3$

(iii) C solves and A and B do not solve, i.e. $\bar{E}_1 \cap \bar{E}_2 \cap E_3$

Therefore, required probability = P(I or II or III).

$= P\left[(E_1 \cap \bar{E}_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap E_2 \cap \bar{E}_2) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3)\right]$

$= P(E_1)P(\bar{E}_2)P(\bar{E}_3) + P(\bar{E}_1)P(E_2)P(\bar{E}_3) + P(\bar{E}_1)P(\bar{E}_2)P(E_3)$

$= (1/3)\times(5/7)\times(5/8) + (2/3)\times(2/7)\times(5/8) + (2/3)\times(5/7)\times(3/8)$

$= 25/56$

4. A class consists of 80 students; 25 of them are girls and 55 boys; 10 of them are rich and the remaining poor; 20 of them are fair complexion. What is the probability of selecting a fair complexioned rich girls.

(a) 5/216 (b) 5/512

(c) 5/256 (d) 5/56

Sol. (b) Consider the events. A = selecting a fair complexioned student; B = Selecting a rich student; C = Selecting a Girl

We have P (A) = 20/80=1/4, P (B) = 10/80 = 1/8 and P(C) = 25/80=5/16.

Since A, B, C are independent events, therefore

Required Probability = P $(A \cap B \cap C)$ = P (A) P (B) P(C) = (1/4)× (1/8) × (5/16) = 5/512.

5. Six married couples are standing in a room. If 4 people are chosen at random, find the probability that two married couples are chosen.

(a) 1/33 (b) 1/65

(c) 10/23 (d) 23/24

Sol. (a) Total cases = ${}^{12}C_4$ and Fav. Cases = ${}^{6}C_2$

Req. Probability = 1/33

6. A box contains 10 radio tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. What is the probability that the process stops after the third test?

(a) 1/45 (b) 2/45
(c) 4/45 (d) 23/45

Sol. (b) For process to stop at third trial we must have two defective tubes in three with third being defective. This can happen in two ways:

The tubes drawn are defective, non defective and defective or non defective, defective and defective

Req. Probability = $(2/10)x(8/9)x(1/8)+(8/10)x(2/9)x(1/8) = 2/45$

7. There are four letters and four envelopes. The letters are placed into envelopes randomly. The probability that all letters are not placed in the correct envelopes is:

(a) 1/24 (b) 23/24 (c) 19/24 (d) 9/24

Sol. (b) The probability that all letters are placed in the correct envelopes is 1/24. The probability that all letters are not placed in the correct envelope is

$(1-1/24) = 23/24$