

HINTS AND SOLUTIONS

TEST CODE: MTPO - III

1. (a) Total outcomes = 36

Doublets = 6 i.e. (1, 1), (2, 2), ..., (6, 6)

∴ No. of cases where one die > other die = 30

$$\Rightarrow \text{Prob.} = \frac{30}{36} = \frac{5}{6}$$

2. (d) $|x| + |y| = 1$ has possible equations

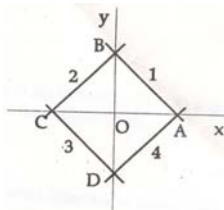
(i) $x + y = 1, x > 0, y > 0$

(ii) $-x + y = 1, x < 0, y > 0$

(iii) $-x - y = 1, x < 0, y < 0$

(iv) $x - y = 1, x > 0, y < 0$

Area enclosed by these lines is



$$\therefore A = \frac{1}{2} BD \cdot AC = 2 \text{ sq. units}$$

3. (b) Let's suppose first digit = '2'. So, remaining two digits on addition gives 4. Possibilities of those two digits are (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)

⇒ 5 possible numbers starting with '2' ----- (1)

Similarly, 5 numbers with '2' at second place ---- (2)

5 numbers with '2' at last place ----- (3)

$$\therefore \text{Total number of trials} = 3 \times 5 = 15$$

4. (b) Equation of tangent to $y^2 = 16x$ is $mx^2 - my + 4 = 0$

$$\text{Passes through } (-4, 5) \Rightarrow -4m^2 - 5m + 4 = 0 \therefore m_1 + m_2 = \frac{-5}{4}$$

5. (d) 26 ways of selecting a block of 5 days in a period of 30 days i.e. I block 1-5, II block 5-10, -----, 26th block 26-30 and ways of arranging films among themselves = 5! ∴ Total no. of arrangements = $26 \times 5!$
= 3120

6. (b) Let $\tan A = x, \tan B = y, \tan C = z \therefore x + y + z = 4$ and find minimum value of $x^2 + y^2 + z^2$ Let $P(x, y, z)$ be any point. On the plane surface $x + y + z = 4$

∴ $OP^2 = x^2 + y^2 + z^2$ is least if OP is least i.e. If OP is the length of \perp^r from origin O to the plane. Here $OP = \frac{4}{\sqrt{3}}$

$$\therefore OP^2 = \frac{16}{3} = \text{minimum value of } x^2 + y^2 + z^2.$$

7. (a) Let x, y be the number of Rs.5 and Rs.10 notes.

$$\therefore 5x + 10y = 500 \Rightarrow y = \frac{100 - x}{2}, \text{ we should have only even}$$

integral value of x , so that y too is integer.

$$\therefore x = 0, 2, 4, \dots, 100 \Rightarrow 51 \text{ possibilities.}$$

8. (a) $|z| = 5$ is a circle with center (0, 0), radius 5

$|z - 10| = |z - 8|$ is a line $x = 9$ and so do not intersect. Hence no sol. satisfying equations.

9. (c) Because M and K are two sport athletes.

$$\text{Te} \rightarrow G \text{ H J K L M N O}$$

$$\text{Fo} \rightarrow \text{H M O}$$

$$\text{Ba} \rightarrow \text{K N}$$

⇒ $[G, J, L]$ are only single game players.

10. (c) HK, KM can't be together ∴ (a), (b), (d) violated

11. (a) all other choices violate that two sport athletes should not sit together

12. (b) N is a two sport athlete

13. (b) LOTUS

14. (b) By 2 P.M. the angle between the hr. & min. hand = 60°

The min. hand goes ahead of the hr. hand = 5.5° per min.

Total degree by which min. hand has to go ahead of hr. hand

$$= 90^\circ + 60^\circ = 150^\circ \text{ Thus, Time req. for it} = 150 \times \frac{10}{55} =$$

$$15. (c) 2x + 3 = 1, 2x + 3 = 17 \Rightarrow x = -1, x = 7$$

$$16. (c) \lim_{n \rightarrow 100} \frac{u_n + 1}{u_n} = |x - 3| \Rightarrow \sum u_n \text{ cgt. If } |x - 3| < 1$$

$$\Rightarrow 2 < x < 4$$

17. (d) Ellipse is a curve, square is a polygon

18. (c) Order is stronger in implication than request Dictate is stronger in implication than suggest

$$19. (b) a < b < c \Rightarrow \frac{1}{a} > \frac{1}{b} > \frac{1}{c}$$

$$\text{Multiply by } a b c \Rightarrow ab < ac < bc$$

$$20. (c) \log 2 = \log k - \log 2 \Rightarrow k = 4$$

$$21. (a) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = 1, \forall \theta$$

∴ No Solution except (0, 0)

22. (c) $|x - k| + |x - L|$, where $L > K$ has $L - K$ as least value

$$\therefore |x - 2| + |x - 3| \text{ has least value } 3 - 2 = 1$$

$$23. (b) X \sim B(6, P) \therefore P(X = r) = {}^6C_r \cdot q^{6-r} \cdot p^r$$

Now, $8 P(X=2) = 3 P(X=3) \Rightarrow \frac{q}{p} = \frac{1}{2}$ Also, $p+q=1$

$$\therefore p = 2/3$$

24. (d) $a(x^3 - y^3) + b(x^2 - y^2) + c(x - y) = 0$ where $x = \cos \theta$,
 $y = \sin \theta \therefore (x - y) [a(x^2 + xy + y^2) + b(x + y) + c] = 0 \Rightarrow x = y$
or $\cos \theta = \sin \theta \Rightarrow \theta = \pi/4$

25. (b) 26. (c)

27. (d) $\sin^{-1} y = A + x$

$$\text{differentiating } \frac{dy}{dx} = \sqrt{1-y^2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = 1-y^2$$

28. (a) $2!(n-1)! = 5x(3!(n-2)!) \Rightarrow n = 16$

29. (c) $(x^2 + 3)^{10} = a_0 x^{20} + a_1 x^{18} + \dots + a_{10} x^2 + 3^{10}$

$$x=1 \Rightarrow 4^{10} = (a_0 + a_1 + \dots + a_{10}) + 3^{10}$$

$$\Rightarrow a_0 + a_1 + \dots + a_{10} = 4^{10} - 3^{10}$$

30. (a) $(x-2)^2 + (y-2)^2 = 25$

$$\therefore x = 2 + 5 \cos \theta, y = 2 + 5 \sin \theta$$

$$\Rightarrow x + y = 4 + 5(\cos \theta + \sin \theta) = 4 + 5\sqrt{2} \sin(\theta + 45^\circ)$$

$$\text{Max. } x + y = 4 + 5\sqrt{2}$$

31. (b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 3\vec{i} + \vec{j} + \vec{k}$ and

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = 3\vec{i} + \vec{j} + \vec{k} \therefore \cos \theta = \frac{-9-1+1}{\sqrt{11}\sqrt{11}} = \frac{-9}{11} \text{ If } \theta$$

were acute angle then $11 \cos \theta = 9$

32. (d) Scalar triple product of $i + j$, $j + k$, $k + i = 2 \neq 0$

\therefore vectors are non-planar, so not a Δ .

33. (d) $e^x > 0 \forall x \therefore e^x + 1 > 1, \frac{1}{e^x + 1} < 1 \Rightarrow \frac{-2}{e^x + 1} > -2$

$$\Rightarrow 1 - \frac{2}{e^x + 1} > -1 \Rightarrow \frac{e^x - 1}{e^x + 1} > -1 \therefore (-1, \infty)$$

34. (d) $1 + \cos 3x \therefore D^5(y) = D^5(1) + D^5(\cos 3x)$

$$= 0 + 3^5 \cos\left(\frac{5\pi}{2} + 3x\right) = 3^5 \cos(3\pi), \text{ at } x = \pi/6 = -3^5$$

35. (c) $(\cos \theta - \lambda)^2 + \sin^2 \theta = 0$

$$\Rightarrow (\lambda - \cos \theta)^2 = -\sin^2 \theta = i^2 \sin^2 \theta$$

$$\Rightarrow \lambda = \cos \theta + i \sin \theta$$

36. (b) A.M > G.M

$$\Rightarrow \left(\frac{\alpha}{\beta} + \frac{\beta}{\gamma} + \frac{\gamma}{\alpha}\right) / 3 > \sqrt[3]{\frac{\alpha}{\beta} \cdot \frac{\beta}{\gamma} \cdot \frac{\gamma}{\alpha}}$$

37. (c) $f(n) = \sqrt{n+1} - \sqrt{n}$

$$f'(n) = \frac{1}{2\sqrt{n+1}\sqrt{n}} (\sqrt{n} - \sqrt{n+1}) < 0$$

$\Rightarrow f'(n)$ is decreasing $f(n)$ of n hence $f(n)$ decreases as n increases.

38. (d) $p^2 = q^2 + r^2 \Rightarrow p, q, r$ are the sides of a rt. angled Δ . and in Δ . Sum of lengths of two sides > third side

39. (a) $-6x - 1 > 27 \Rightarrow x < \frac{-14}{3}$

\therefore greatest integral value of x is -5

40. (b) 'Direct' is to Order while others involve giving a choice to act.

$$\mathbf{41. (d)} \quad U_n = \frac{1.3.5. \dots (2n-1)}{2.4.6. \dots (2n)} \left(\frac{1}{3}\right)^n$$

$$\therefore \frac{U_{n+1}}{U_n} = \frac{2n+1}{(2n+2)} \cdot \frac{1}{3} \quad \text{Lt } \frac{U_{n+1}}{U_n} = \frac{1}{3} < 1$$

\therefore series $\sum U_n$ is cgt.

Hence its n th $\rightarrow 0$ as $n \rightarrow \infty$

42. (b) $z = 7 \therefore x y$ multiplied by 7 = $x y$ multiplied by $z = 315$
Also, $315 \div 7 = 45 = x y \therefore x = 4$

43. (b) $\alpha + \beta = 5, \alpha - \beta = 3 \therefore \alpha = 4, \beta = 1$

$$\Rightarrow p = \alpha * \beta = 4 \quad \mathbf{44. (a)}$$

45. (c) In backward sequence (Z to A)

G = 20th, R = 9th, I = 18th, D = 23th \therefore avg. = 17.5 Similarly,

$$F, U, N, D = 15.75 \left(= \frac{21+6+13+23}{4} \right)$$

46. (d) $e^{2x} - 2 = e^x = 2, -1$

$$\therefore e^x = 2 (\because -1 \text{ not possible}) \Rightarrow x = \log_e 2$$

47. (d) $ao(e(x)) = a(x) = \frac{1}{1-x} \neq e(x)$

48. (b) $A = \int_1^2 x dy = \int_1^2 \log y dy (\because y = e^x)$

$$= \log 4 - 1$$

49. (a) $S = 1 + x + x^2 + \dots$ where $x = (0.2) a$

converge if $|x| < 1$ i.e. $(0.2) a < 1 \Rightarrow a < 5$

50. (d) Let $f(x) = ax + b$ and $f(x+1) = f(x-1) \Rightarrow 2a = 0$
or $a = 0 \therefore f(x) = b = \text{constant}$

51. (b) Let $S = 1 + \cos \alpha + \cos 2\alpha + \dots = \frac{1}{1 - \cos \alpha} = 2$

$$\Rightarrow \cos \alpha = \frac{1}{2} \therefore 1 + \sin^2 \alpha + \sin^4 \alpha + \dots = \frac{1}{1 - \sin^2 \alpha} = 4$$

$$\mathbf{52. (a)} \quad P(B/\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})} = \frac{P(B) - P(A \cap B)}{1 - P(A)} = \frac{11}{20}$$

$$\therefore P(A \cap B) = \frac{1}{30} \Rightarrow A \cap B \neq \phi \Rightarrow A \text{ and } B \text{ not mutually}$$

exclusive. Also, $P(A), P(B) = 2/15 \neq P(A \cap B)$

$\Rightarrow A$ and B are not independent

$$\mathbf{53. (d)} \quad \vec{a} * (\vec{c} + \vec{b}) = (\vec{b} + \vec{c}) * (\vec{b} + \vec{c}) = 0$$

54. (c) New A.M. = $2\bar{x} + \frac{20}{8} = 2 * 30 + 2.5 = 62.5$

55. (a) G.M < A.M. \Rightarrow G.M < 4

$$\mathbf{56. (b)} \quad n! = \left(\frac{n+1}{3!}\right) \Rightarrow n = 5$$

57. (d) Let $Z = \frac{1+2\omega+3\omega^2}{\omega+2\omega^2+3} = \frac{1+2\omega+3\omega^2}{\omega+2\omega^2+3} = \frac{1}{\omega} = \omega^2$

58. (c) $\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 4x - 4}{x - 2}$

$\lim_{x \rightarrow 2} (x+1)(x+2) = 12$

$\therefore f(x)$ is continuous at $x = 2$ provides $f(2) = 12$ i.e. $a = 12$

59. (b)

60. (c) Pulpit is a place where minister does his job also; Bench is a place where judge does his job.

61. (a) 62. (c) 63. (b) 64. (b)

65. (d) 66. (c) 67. (d) 68. (a)

69. (b) 70. (c) No use of at least 71. (d)

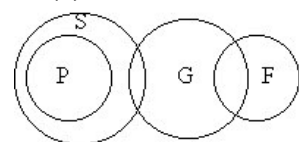
72. (a) Hardly had he finished

73. (a) the symbol inside circle changes alternatively. Arrow moves clock wise vert. to horz. and vice versa; vert. outside and horz. inside

74. (d) Cross moves clockwise, outer most object goes to inner most, second to outmost object goes to outermost, innermost to second most place. 75. (c)

76. (a) Two and three elements disappear in alternate steps and the elements get inverted in each step.

77. (b)



78. (c) L (K J) I (H) G (FE) D (C) B (AZ) Y
i.e. Letters are in Z to A sequence and number of letters skipped between consecutive positions is alternatively two and one

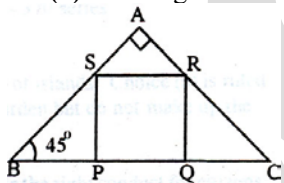
79. (b) Let 5 be one of two nos. The product end with zero if number is 2 or 4 or 6 or 8

\Rightarrow 4 pairs of nos. Whose products end with zero.

80. (a) $\sin 4x$ has period $\frac{2\pi}{4} = \frac{\pi}{2}$

81. (c) $A(B+I) = I \Rightarrow A^{-1} = B+I$ 82. (d)

83. (a) From figure



$BP = QC = x \therefore BC = 3x$ and $x^2 + = 40$. Now

$A = \frac{1}{2} 3x \sin 45^\circ 3x \cos 45^\circ \left(a = \frac{1}{2} bh \right) = \frac{9x^2}{4} = 90 \text{ cm}^2$

84. (b) $Z = 4 + iy, |Z| = 5 \Rightarrow y = +3$

$\therefore \theta = \tan^{-1} \left(\frac{3}{4} \right)$ (only available choice)

85. (d) $\left(\frac{4}{9} \right)^x + 4 \left(\frac{6}{9} \right)^x = 5 \Rightarrow \left(\frac{2}{3} \right)^{2x} + 4 \left(\frac{2}{3} \right)^x = 5$

$\Rightarrow p^2 + 4p = -5 = 0, p = \left(\frac{2}{3} \right)^x \Rightarrow p = 1 (-5 \text{ not possible}) \Rightarrow x = 0$

86. (b) The center of the circle (2,0) & $r = 3$ units.

Thus, Volume generated by sphere of radius as 3 is $= \frac{4\pi}{3} (3)^3 = 36\pi$

87. (c) Let volume V of cube $= a^3$, a is side.

If Volume is $8V = 8a^3 \Rightarrow \text{side} = 2a$

Surface areas of cubes $= 6a^2, 24a^2$ respectively

\therefore Time req. to paint II cube $= 4t$ hrs.

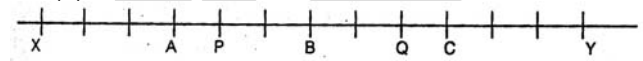
88. (d) Let C. P. of 1 mango = Rs.1

\therefore Investment = Rs.100

For 20% profit, he has to sell = 90 mangoes for Rs.120

\therefore C.P: S.P of 1 mango $= 1: \frac{120}{90} = 3:4$

89. (a)



$PB + AQ = (XB - XP) + (XQ - XA) = 6 - 4 + 8 - 3 = 7$

90-93:

Explanation: The given information can be condensed as follows:

[A, B, C, D are the men and R, S, T, U are the women]

(i) Mrs. C > S (ii) Mrs. D > U (iii) U is A's sister

(iv) $R < S, T, U$ (v) C was not present at U's

wedding After using (i) to (v) the \checkmark in CT is allowed. Thus,

DT is \times . Hence \checkmark , DS. Also, \checkmark and so, BU

\checkmark : Marital Relation WIFE \times : Non existence

H
U
S
B
A
N
D

	R	S	T	U
A	\checkmark By elimination			\times By (iii)
B				\checkmark By elimination
C	\times By (i) and (iv)	\times By (i)	\checkmark By elimination	\times
D	\times By (ii) and (iv)	\checkmark By elimination	\times	\times By (ii)

\checkmark : Existence of marital Relation

\times : Non-Existence

90. (c) \therefore Rekha is youngest women and wife of Arun.

91. (a) Uma is Arun's sister and Badri's wife. Thus Arun is Prem's Uncle and Rekha (Arun's wife) is Prem's aunt.

92. (d) $R < U \Rightarrow A < B \therefore B \not< A$

while in (a) $C > \text{Mrs. C} > S$; (b) $R < S, T, U < \text{husbands}$

(c) $R < S \Rightarrow R$'s husband $< S$'s husband $\Rightarrow A < D$

93. (b) A-27, B-29, C-31, D-33, R-28. The ages of other woman 30, 32, 34 Mrs. C > S $\Rightarrow T > S$, Mrs. D > U $\Rightarrow S > U$

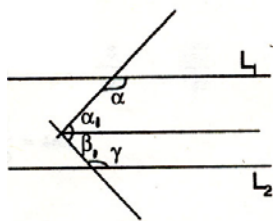
$\Rightarrow T > S > U$

94. (b) A person can't be frd. Of himself ($\neq R$)

If A is a frd. Of B \Rightarrow B is a frd. Of A ($=S$) If A is frd. Of B, B is frd. Of C \Rightarrow A is frd. of C ($\neq T$)

95. (c)

96. (d)



Draw L3 parallel L1 & L2 $\therefore \alpha + \alpha_1 = 180^\circ$ $\beta + \gamma = 180^\circ$

(sum of angles on the same side of transversal is 180)

$$1 \Rightarrow \alpha + (\alpha_1 + \beta) + \gamma = 360^\circ$$

$$\Rightarrow \alpha + \beta + \gamma = 360^\circ$$

97. (b) No. of man hrs. req. to complete the job =

$$10 \times 10 \times 10 = 1000$$

$$\text{No. of man hrs. made available} = 2 \times 5 \times 8 = 80$$

$$\therefore \% \text{ of work completed} = \frac{80}{1000} \times 100 = 8\%$$

98. (b) $n(T \cap F) = x$ (say) $\therefore n(T \cup F)' = x$

Only T=20, only F=15

$$\therefore 20 + x + 15 + x = 35 + 2x, \text{ which is odd}$$

$$\therefore \text{Ans.} = 37, \text{ only odd choice available.}$$

99. (c) Let external angle = x \therefore internal = $4x$

$$4x + x = 180^\circ \Rightarrow x = 36^\circ \therefore \text{no. of sides} = \frac{360}{36} = 10$$

100. (c) \vec{n}_1, \vec{n}_2 are normals to two given planes $\vec{n}_1 \times \vec{n}_2$ is parallel to both planes

$$\therefore \vec{n}_1 \times \vec{n}_2 = (\vec{i} + \vec{j} + \vec{k}) * (2\vec{i} + \vec{j} + 2\vec{k}) = 3\vec{i} - 3\vec{k}$$

$$\therefore \text{d.r's } (1, 0, -1)$$