Hints and Solutions **MTD - II**

1. (b) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, which is 2.71828 ...

2. (c) 2nd degree term = – (sum of roots) = $-\left(\sum\alpha-\frac{\sum\alpha}{\alpha\,\beta\,\gamma}\right)$

3. (a)

4. (a) Put $x = \sin\theta$. Integral reduces to $\int_0^{\pi/2}\sin^6\theta\cos^2\theta\,d\theta$

5. (a) Since we have a homogeneous equation. So put $Z=\log x$ i.e. $x=e^Z$. Then if $D=\frac{d}{dx}$,

$xD=P$ where $P=\frac{d}{dz}, x^2D^2=P(P-1)$......

(Discussed in class):

Therefore $(x^2D^2+xD-1)y=0$ becomes

$(P(P-1)+P-1)y=0\Rightarrow(P^2-1)Y=0$. So Aux. equation is ,

$\lambda^2-1=0\Rightarrow\lambda=\pm1$. Hence L.I. solutions are $y=e^z$ and $y=e^{-z}$, i.e. $y=x$ and $y=\frac{1}{x}$.

6. (b) $\lim\limits_{n\to\infty}\frac{1}{n^2}[1+2+...+n]=\lim\frac{1}{n^2}\frac{n(n+1)}{2}=\lim\frac{1}{2}\left(1+\frac{1}{n}\right)$

$=\frac{1}{2}$ Alternate Method $\lim\sum\limits_{r=1}^{n}\frac{r}{n^2}=\int_0^1 x\,dx$

7. (c) For quadratic $\theta\to 1/2$

8. (b) $a=b=1$, $h=0$ and radius < 0.

9. (b) Asymptotes parallel to y axis is $1+x=0$. Also $\phi_3(m)=m^2+1=0$ is having imaginary roots. Therefore, no oblique asymptotes.

10. (a) $\log_x a,\ \log_y a,\ \log_z a$ are in H. P.

Therefore $\frac{2}{\log_y a}=\frac{1}{\log_x a}+\frac{1}{\log_z a}$

Changing base, and canceling $\log_e$ a, we get $2\log y=\log x+\log z\Rightarrow y^2=xz$

11. (c)

12. (d) (i) $Lim\frac{a_n}{b_n}=0$ and (ii) $\lim\frac{a_n}{c_n}=\infty$

Now (ii) imply if $\sum c_n$ is dgt. Then $\sum a_n$ is also dgt. and hence $\sum c_n$ can not diverge. (because $\sum a_n$ is given to be cgt.)

13. (b) **14. (a)**

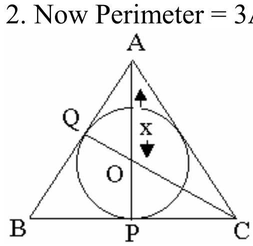
15. (d) (a) R.H.L. = 0, L.H.L. := 1 (b) same as (a), but origin shifted to a. (c) RH.L. = ∞, L.H.L. = 0.

16. (c) $\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}^2=\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}$

17. (c) $-1\le\log\frac{x^2}{3}\le1\Rightarrow\frac{1}{3}\le\frac{x^2}{3}\le3$ because $-1\le\sin x\le1$

18. (c)

19. (c) For least perimeter, triangle should be equilateral. O will divide AP in the ratio 2:1, but OP is given to be 1. Therefore AO = x = 2. Now Perimeter = 3AB = 3(AQ+QB).



$=3\left[\sqrt{x^2-1}+\frac{(x+1)}{(x^2-1)^{1/2}}\right]=6\sqrt{3}$

20. (c) Every group of order 1, 2 and 3 being cyclic. (An infinite cyclic group has exactly 2 generators).

21. (b)

22. (c) $P(x<1\cap y<3)=\int\limits_{-\infty}^{1}\int\limits_{-\infty}^{3}f(x,y)\,dx\,dy$

$=\int\limits_0^1\int\limits_2^3 f(x,y)\,dxdy=\int\limits_0^1\left[\int\limits_2^3\frac{1}{8}(6-x-y)dy\right]dx$

$=\int\limits_0^1\frac{1}{8}\left(\frac{7}{2}-x\right)dx=\frac{3}{8}$

23. (b) **24. (c)**

25. (b) None of the series is absolutely convergent. Therefore, their product is divergent.

26. (a) Apply Roll's theorem on

$f(x) = \frac{a_0 x^{n+1}}{n+1} + \frac{a_1 x^n}{n} + ... + a_n x = 0$ on the interval [0, 1]

27. (a) (Z + .) is not a field.

28. (c) A set A is s.t.b. denumerable (or countably infinite) if there exists a 1–1 correspondence between the set A and the set N of natural numbers. e.g. the set A = {1/2, 2/3 , 3/4 , ... } is denumerable because the mapping f : N → A defined by $f(n) = \frac{n}{n+1}$, n ∈ A is bijective. A set which is either finite or denumerable is called a countable set. [0, 1] ,R Q^c (irrationals) are not countable but N × N, Z, Q, Q+ are countable.

29. (c) **30. (a)** **31. (c)** **32. (d)**

33. (c) Six faces of the cube are $x = \pm a$, $y = \pm a$, $z = \pm a$ where a is the length of a side.

34. (b) First eliminate c and then replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$.

35. (d) On rotation by θ, the relation connecting x,y with x',y' is $x = x'\cos\theta - y'\sin\theta$ and $y = x'\sin\theta + y'\cos\theta$ Substitute this in $ax^2 + 2hxy + by^2 = 0$ and then equate the coefficients of x', y' (in the resulting equation) to 0 to get your answer.

36. (b) Let $f(x)$ = sin$x$ and $g(x)$ = cos$x$, for $x \in [\alpha, \beta]$. By Cauchy's mean value theorem, ∃ θ ∈ (α, β) s.t.

$$\frac{f(\beta)-f(\alpha)}{g(\beta)-g(\alpha)} = \frac{f'(\theta)}{g'(\theta)} \Rightarrow \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$$

37. (a) A finite set has no limit point.

38. (b) Compare with $y = mx - 2am - am^3$.

39. (d) Injection means 1–1 mapping. $x \to x^2$ is not 1–1 because $f(1) = 1 = f(-1)$.

40. (a)

41. (c) $r^2 = x^2 + y^2 \Rightarrow 2r\frac{\delta r}{\delta x} = 2x, 2r\frac{\delta r}{\delta y} = 2y$

$\Rightarrow \frac{\delta r}{\delta x} = \frac{x}{r}, \frac{\delta r}{\delta y} = \frac{y}{r}$ Now $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta r}.\frac{\delta r}{\delta x} = f'(r)\frac{x}{r}$

and $\frac{\delta^2 u}{\delta x^2} = \frac{\delta}{\delta x}\left(\frac{\delta u}{\delta x}\right) = \frac{\delta}{\delta x}\left(\frac{xf'(r)}{r}\right)$

$= \frac{1}{r^2}\left\{rf'(r) + x^2 f''(r) - \frac{x^2}{r}f'(r)\right\}$.

42. (b) In polar $x = a\cos^3\theta$, $y = a\sin^3\theta$.

Use $L = 4\int_0^{\pi/2}\left(\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right)^{1/2} d\theta$

43. (d)

44. (b) Domain of definition is $[0,\infty)$. $f'(0+) = \lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h((h)^{1/2} - (h+1)^{1/2})}{h} = -1$

45. (c)

46. (b) A case of Binomial distribution

47. (b) Use $AA^T = I$ which implies $x = \pm\frac{1}{3}$, $y = \pm\frac{2}{3}$. But $\frac{2}{3}x + \frac{2}{9} + \frac{2}{3}y = 0$ gives $x = 1/3$, $y = -2/3$.

48. (b) Compare given equation with cardon's equation $z^3 + 3Hz + G = 0$

49. (a) [ABA] + [ACA] = 0.

50. (c)

51. (a) C ≡ F, D ≡ G B ≡ H, A ≡ E (≡ : marriage)

52. (d) B E A D C

53. (b) Let O = a, × = b, □ = c then from 1st col. and 3rd row 2a + 2b = 6, a + 3b= 5 imply b = 1, a = 2. Now from first row C = 9. Compute

X + Y = (2c + 2a) + (2b + a + c).

54. (a) **55. (b)** **56. (a)** **57. (a)**

58. (d) **59. (c)**

60. (d) $A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C$

61. (c) $2^{10} = 10000000000 = (400)_{16}$

62. (b) **63. (c)**

64. (c) C ≠ 4 because $4^2 - (A + B)^2 = 231 \Rightarrow (A + D)^2 .< 0$. Similarly C ≠ 11, C = 20 satisfies all the equations.

<u>**Directions (Q. 65 – Q. 66)**</u>**:** So B = 1 where $x = 0$, $y = 1$ imply B = $x'y$ and $D = x \oplus y$.

65. (a)

66. (c)

x	y	B	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

So B = 1 where x = 0, y = 1 imply B = x'y and D = x ⊕ y.

67. (a) 0 3 8 15 …

+3 +5 +7

68. (c) Given number in hexadecimal system is CDE5 Find binary representation.

69. (b) The octal is 740263. Now find binary representation and count zeros. .

70. (d) If 3 comes uppermost then the face not visible is 4. Ans. is 1 + 2 + 3 + 5 + 6 = 17.

71. (d) Opposite faces are (4,6), (2,1) and (3,5) .

72. (c)

73. (d) NAND (↑) and NOR (↓) are not associative.

74. (d) 1996, 2000, and 2004 are leap years. 13th Jan, in 1996 will be sat. and in 1997, it will be Mon. (due to leap:year). Continue this way.

75. (a) $\frac{E + L + K}{G + N + U} = \frac{5 + 12 + 11}{7 + 14 + 21} = \frac{28}{48} = \frac{2}{3}$

76. (a) **77. (c)** **78 .(b)**

79. (c) **80. (c)** **81. (b)**

82. (b) **83. (a)**

84. (d) Equations are not consistent.

85. (b) **86. (d)**

87. (a)

N	D	Q	R
20	3	6	2
3	2	1	1
2	1	2	0

88. (c) To answer this use choices.

89. (b)

90. (a)

$$\begin{array}{r} 101\ \ 110\ \ 010 \\ -\,011\ \ 111\ \ 001 \\ \hline 001\ \ 111\ \ 001 \end{array} \equiv (1\ 7\ 1)8$$

91. (c) 0 ⊙ 0 = 1, 0 ⊙ 0 ⊙ 0 = 0

92. (d) **93. (a)** **94. (a)**

95. (d) **96. (d)** **97. (a)**

98. (b) Since code is formed by permuting the 26 letters of the alphabet, therefore there are at a time one alphabet representing another alphabet. ALPHA begins and ends with the same. Therefore (a) is eliminated. In ALPHA, LPH are different letters so their codes should also be different. Hence (b).

99. (c) NATION In code (3) ATION is ut snr so NATION is rutsnr (*) (use the choices in this way for 100 also). (*In code (1) A TION is nesbr, which is not the last five letters of any choices. Similarly last 5 letters of (2) and (4) are not there in the choices.

100. (a)

Additional Problems

1. Suppose that the actual amount of instant coffee which a filling machine puts into Jars is a random variable having normal distribution with $\sigma = 0.05$ ounce. If only 3 percent of the Jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars ?

(Given $\int_0^{1.88} \phi(Z)\,dZ = .47$)

Solution. Let X denotes the amount of coffee put in one jar. We are given P [X < 6] = .03

so $P\left[\frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}\right] = .03$ so $P\left[Z < \frac{6-\mu}{\sigma}\right] = .03$

$P[Z < -Z_1] = .03$ (because Z_1 is on the left of mean).

So $P[-Z_1 < Z < 0] = .47$ Since $\int_0^{1.88} \phi(z)\,dz = .47$ s given

So $Z_1 = 1.88 \Rightarrow \frac{6-\mu}{\sigma} = -1.88$ $6 - \mu = -1.88 \times 0.05$

$\mu = 6 + 1.88 \times 0.05 = 6 + .0940 = 6.094$.

2. A certain kind of appliance requires repairs on the average once every 2 years. Assuming that the times between repairs are exponentially distributed, what is the probability that such an appliance will work at least 3 years without requiring repairs?

(Given $\frac{1}{e^{1.5}} = .223$)

Solution. Given Av. no of arrivals = 1 in 2 years

Therefore Av. no. of arrivals in one year = $\frac{1}{2}$

$\Rightarrow \alpha = \frac{1}{2}$

Req. Prob. = P [No failure in 3 years] = P [repair time between two failures is greater than 3 years]

= P[T > 3] = 1 – P [T < 3]

$= 1 - \int_0^3 \alpha e^{-\alpha t}\,dt = 1 - \left[-e^{-\alpha t}\Big|_0^3\right]$

$= 1 - \left[1 - e^{-3\alpha}\right]$

$= e^{-3\alpha} = e^{-1.5} = .223$

3. In a given city, 4 percent of all licensed drivers will be involved in at least one car accident in any given year. What is the probability that among 150 licensed drivers randomly selected, at most three will he involved in at least one accident in any given year. Given $e^{-6} = .0025$)

Solution. Here p = 0.04 and n = 150

where p: Prob. of having an accident (or success)

n : total no. of trials

$\lambda = np = 6$ (average no. of success)

req. probability = $P[X \le 3]$

= P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3]

using Poisson dist. $P[X \le 3]$

$= e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2}{2!} e^{-\lambda} + \frac{\lambda^3}{3!} e^{-\lambda}$

$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{3}\right] = e^{-\lambda} \times 61 = e^{-6} \times 61 = .1525$