SANMACS INDIA

megaleap
Comprehensive MCA
Entrance Coaching

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HINTS AND SOLUTIONS **TEST CODE: MTIP-I**

1. (b) $\frac{x^2}{16}+\frac{y^2}{16}=1$

Here $a^2 = 16$

Foci are $(\pm ae, 0)$ ...... (1)

Now, $\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y^2}{\left(\frac{9}{5}\right)^2}=1$

Then $b^2 = a^2\ (e^2 - 1)$

$\Rightarrow \frac{81}{25}=\frac{144}{5}(e^2-1)$

$\Rightarrow\ e^2=\frac{25}{16}\Rightarrow e=\frac{5}{4}$

$\therefore$ Its foci are $(\pm ae, 0)$ or $\left(\pm\frac{12}{5}\times\frac{5}{4}, 0\right)$ or $(\pm 3, 0)$

Since foci of ellipse and hyperbola coincide

$\therefore$ From (1), ae = 3

$\therefore b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 16 - 9 \Rightarrow b^2 = 7.$

2. (d) Apply $C1 \to C1 + C2 + C3$

$1+\omega^n+w^{2n}$ is a factor of Δ

For $n \neq 3,\ 1+\omega+\omega^{2n}=0 \Rightarrow \Delta=0$

For n = 3, $\Delta = 0$

3. (c) Let $\hat{n}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\therefore \bar{u}.\hat{n}=0 \Rightarrow a_1+a_2=0$

$\bar{v}.\hat{n}=0 \Rightarrow a_1-a_2=0$

$\Rightarrow a_1=0, a_2=0$

$\Rightarrow a_3=1$ $(\because \hat{n}$ is a unit vector)

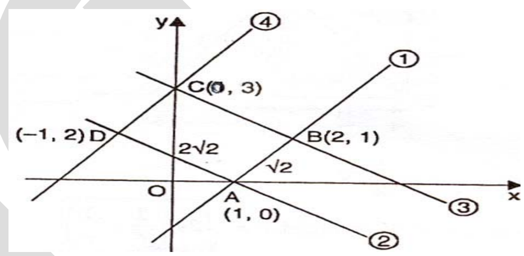
$\therefore |\bar{w}.\hat{n}|=3$

4. (b) Given lines are y = x - 1 ......(1)

y = - x + 1 ......(2)

y = 3 - x ......(3)

y = 3 + x ......(4)



ABCD is a rectangle with area

$=\sqrt{2}\times 2\sqrt{2}=4\, sq.units.$

5. (a) $\frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{1}{1+y^2}e^{\tan^{-1}y}$

$I.F.=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

$\therefore$ Solution is

$x.e^{\tan^{-1}y}=\int e^{\tan^{-1}}y.\frac{1}{1+y^2}e^{\tan^{-1}y}dy+k$

Put $\tan^{-1}y = z$

6. (b) Let $f'(x)=f(x)\Rightarrow f(x)=e^x.$

$\therefore\ g(x) = x^2 - f(x) = x^2 - e^x$

$\Rightarrow \int_0^1 f(x)g(x)dx=\int_0^1(x^2e^x-e^{2x})$

$=\left[x^2e^x\right]_0^1-\int_0^1 2xe^x dx-\left[\frac{e^{2x}}{2}\right]_0^1$

7. (a) $f(-x)=\log(-x+\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$

$=\log\left(\frac{(x^2+1)-x^2}{\sqrt{x^2+1}+x}\right)\log 1-\log(\sqrt{x^2+1}+x)$

$=-\log(\sqrt{x^2+1}+x)=-f(x)\Rightarrow f(x)\, is\, odd$

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8. (d) $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left(\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i}{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i}\right)^x = 1$

$\Rightarrow \left(\frac{e^{i\frac{\pi}{4}}}{e^{-i\frac{\pi}{4}}}\right)^x = 1 \Rightarrow \left(e^{i\frac{\pi}{2}}\right)^x = 1 \Rightarrow e^{i\frac{x\pi}{2}} = 1$

Which is satisfied for x = 4n, where n is any positive integer.

9. (c) $\left(1-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)$.....

$= \left(1-\frac{1}{2}+\frac{1}{3}-......\right)+\left(-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....\right)$

$= \log_e 2 + \log_e 2 - 1$

$= 2\log_e 2 - 1 = \log_e 4 - 1 = \log_e\left(\frac{4}{e}\right)$

10. (b) Given limit is $\frac{0}{0}$

$\therefore \lim_{x\to 0}\frac{\frac{1}{3+x}+\frac{1}{3-x}}{1} = k$

$\Rightarrow \lim_{x\to 0}\frac{6}{9-x^2} = k \Rightarrow \quad k = \frac{2}{3}$

11. (d) 6 men can be seated is 5!. Out of 6 places, 5 women can be seated in ${}^6P_5$ ways.

$\therefore$ Required number of ways = 5! * ${}^6P_5$

= 5! * 6 * 5 * 4 * 3 * 2 = 6! * 5!

12. (d) $np = 4, npq = 2 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X=1) = {}^8c_1 pq^7 = 8\left(\frac{1}{2}\right)^8 = \frac{1}{32}.$

13. (c) Let $z = e^{i\theta}$ and $\omega = e^{i\phi}$

$z\omega = e^{i(\theta+\phi)}$ and $|z\omega| = 1$

$Arg\, z - Arg\,\omega = \frac{\pi}{2} \Rightarrow \theta - \phi = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{2} + \phi$

Now, $\bar{z} = e^{-i\theta} = e^{-i\left(\frac{\pi}{2}+\phi\right)} = e^{-i\frac{\pi}{2}}.e^{-i\phi}$

and $\bar{z}\omega = e^{-i\frac{\pi}{2}}e^{-i\phi}e^{i\phi} = e^{-i\frac{\pi}{2}}$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$

14. (a) $T_{r+1} = {}^{256}c_r\left(3^{1/2}\right)^{256-r}\left(5^{1/8}\right)^r$

$\therefore T_{r+1}$ is integer if $\frac{256-r}{2}$ and $\frac{r}{8}$ are positive integers

$\therefore 256 - r \geq 0\, or\, 0 \leq r \leq 256$

$\therefore$ r can be 0, 8, 16, 24, ...,8 x 32

These are 33

15. (a) Given det. = (1+abc) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\Rightarrow$ 1 + abc = 0 $\Rightarrow$ abc = -1 $\left(\because \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0\right)$

16. (a) $f(x) = \begin{cases} xe^{-\left(-\frac{1}{x}+\frac{1}{x}\right)} & , x<0 \\ xe^{-\left(\frac{1}{x}+\frac{1}{x}\right)} & , x>0 \\ 0 & , x=0 \end{cases}$

| x | is not differentiable at x =0

| x | is continuous at x = 0

17. (a) Two quadratic equations

x^2 - 3x + 2 = 0 and x^2 + 3x + 2 = 0

which have 4 solutions.

18. (b) given limit = $\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\, dt}{x\sin x} = \lim_{x\to 0}\frac{[\tan t]_0^{x^2}}{x\sin x}$

$\lim_{x\to 0}\frac{\tan x^2}{x\sin x} = \lim_{x\to 0}\frac{\tan x^2}{x^2\frac{\sin x}{x}}$

$= \lim_{x\to 0}\frac{\tan x^2}{x^2}\lim_{x\to 0}\frac{1}{\frac{\sin x}{x}} = 1$

19. (c) $4 - x^2 \neq 0 \quad \Rightarrow \quad x \neq \pm 2$

$x^3 - x > 0 \Rightarrow x(x^2-1) > 0$

$\Rightarrow x < 0, x^2 - 1 < 0 \, or \, x > 0, x^2 - 1 > 0$

$\Rightarrow \quad -1 < x < 0 \quad and \quad x > 1$

$\therefore$ Domain is $(-1,\ 0) \cup (1,\quad 2) \cup (2, \alpha)$

20. (c) Let

$$\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x} \Rightarrow \int \frac{e^{\sin x}}{x}dx = F(x) \text{ ......(1)}$$

Now, $I = \int_1^4 \frac{3}{x}e^{\sin x^3}dx = \int_1^4 \frac{e^{\sin x^3}}{x^3}3x^2 dx$

Put $x^3 = t \Rightarrow 3x^2\,dx = dt$

x = 1, t = 1, x = 4, t = 64

$$\therefore\ I = \int_1^{64} \frac{e^{\sin t}}{t}dt = [F(t)]_1^{64} = F(64) - F(1)$$

Clearly, x=64.

21. (a) $f(x) = \sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$ exists if

$$-1 \le \log_3\left(\frac{x}{3}\right) \le 1$$

$$\Leftrightarrow\ 3^{-1} \le \frac{x}{3} \le 3^1 \Leftrightarrow 1 \le x \le 9 \Leftrightarrow [1,\ 9].$$

22. (b) Since $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$

$\therefore$ α, β are roots of the equation $x^2 - 5x + 3 = 0$

$\Rightarrow$ $\alpha + \beta = 5$, $\alpha\beta = 3$ ...... (1)

$\therefore$ $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of $x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}.\frac{\beta}{\alpha} = 0$

or $x^2 - \left(\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$

or $x^2 - \frac{25-6}{3}x + 1 = 0$......... [By (1)]

$\Rightarrow$ $3x^2 - 19x + 3 = 0$

23. (d) sin y = x sin (a + y) or $x = \frac{\sin y}{\sin(a+y)}$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

24. (c) $f(x) = \cot^{-1}x + x \Rightarrow f'(x) = -\frac{1}{1+x^2} + 1$

$= \frac{x^2}{1+x^2}$,

clearly, f '(x)>0 for all x So, f(x) increases in (-∞, ∞)

25. (a) $y = (x + \sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2}.2x\right)$$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$$

$\Rightarrow \sqrt{1+x^2}\,y_1 = ny, \quad where \quad y_1 = \frac{dy}{dx}$

$\Rightarrow (1+x^2)y_1^2 = n^2y^2$

$\Rightarrow (1+x^2)2y_1y_2 + y_1^2.2x = n^2 2yy_1$

$\Rightarrow (1+x^2)y_2 + xy_1 = n^2y, \quad where \quad y_2 = \frac{d^2y}{dx^2}.$

26. (a) $\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x \Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$\cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$ But $\operatorname{cosec} x = \sqrt{1+\cot^2 x}$

$$\Rightarrow \operatorname{cosec} x = \frac{1+\cos\alpha}{1-\cos\alpha}$$

$$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} \Rightarrow \sin x = \tan^2\frac{\alpha}{2}.$$

27. (a) $\int_0^{10\pi}|\sin x|dx = 10\int_0^{\pi}|\sin x|dx$

$$=10\left(\int_0^{\pi/2}\sin x\,dx + \int_{\pi/2}^{\pi}\sin x\,dx\right)$$

28. (b) $2^{1/4}.4^{1/8}.8^{1/16}...... \infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+.........\infty}$

S= $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... \Rightarrow \frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + ...$

Subtracting, $\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... = \frac{1}{2}$ (G. P.)

$\Rightarrow$ S = 1. Hence value is $2^1=2$.

29. (d) $x^y = e^{x-y}$

Taking the log on both sides

y log x= x - y or y(log x + 1) = x

$$\Rightarrow \quad y = \frac{x}{1+\log x}$$

$$\frac{dy}{dx} = \frac{(1+\log x) - x\left(\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

30. (b) $a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$

$=a^9 r^{36} = (ar^4)^9 = 2^9 = 512$.

31. (a) $\lim\limits_{x\to 0}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ or $\lim\limits_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$

$$= \lim_{x\to\infty}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+1}{4x+1}}\right]^{\frac{x(4x+1)}{x^2+x+1}}$$

$$= e^{\lim\limits_{x\to\infty}\frac{x(4x+1)}{x^2+x+2}} = e^4.$$

32. (d) $x+iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = -3i\begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$

$\Rightarrow$ x=0, y=0

33. (c) $\lim\limits_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$

$$= \lim_{x\to 2}\frac{xf(2)-2f(2)+2f(2)-2f(x)}{x-2}$$

$$= \lim_{x\to 2}\frac{(x-2)f(2)}{x-2} - 2\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$= f(2) - 2f'(2) = 4 - 2\times 4 = -4.$$

34. (a) Let q = 1 - p. Since, head appears first time in even throws 2 or 4 or 6 .....

$$\therefore \quad \frac{2}{5} = qp + q^3p + q^5p + ...$$

$$\therefore \quad \frac{2}{5} = \frac{qp}{1-q^2} \Rightarrow \frac{2}{5} = \frac{(1-p)p}{1-(1-p)^2} = \frac{(1-p)p}{(2-p)p}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{1-p}{2-p} \Rightarrow p = \frac{1}{3}.$$

35. (d)

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right) = \tan^{-1}\left(\frac{1}{2}\right).$$

36. (d) We have, $(1 + 2x + 3x^2 + ...)^{-3/2}$

$= [(1 - x)^{-2}]^{-3/2} = (1 - x)^3$

coefficient of $x^5 = 0$

37. (a) Applying $R_1 + R_2 + R_3$ we obtain

$$\begin{vmatrix} 1-i & \omega^2+\omega & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega & -1 \end{vmatrix} = 0$$

($\because \omega^2 + \omega = -1$ $\therefore$ R_1 and R_2 become identical)

38. (b) Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

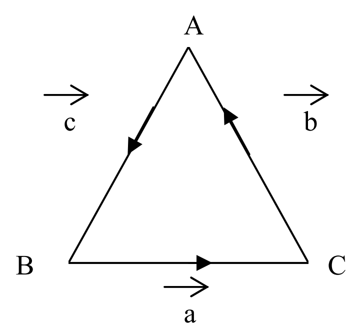
It is given that $e = \frac{1}{2}$ and ae = 2

Therefore, a = 4

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 12$

Thus the required ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

39. (b) We have $\vec{a} + \vec{b} + \vec{c} = 0$



$\vec{a} + \vec{b} = -\vec{c} \quad \Rightarrow \quad \vec{a}\times\vec{c} + \vec{b}\times\vec{c} = 0$

$\Rightarrow \quad \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

40. (b) We have $3^{2x^2-7x+7} = 3^2 \Rightarrow 2x^2 - 7x + 7 = 2$

$\Rightarrow \quad 2x^2 - 7x + 5 = 0$

The discriminant of this equation is positive. Hence, it has two real roots.

41. (c)	**42. (b)**	**43. (c)**
44. (d)	**45. (d)**	**46. (a)**
47. (c)	**48. (c)**	**49. (b)**
50. (b)	**51. (d)**	**52. (d)**
53. (a)	**54. (b)**	**55. (d)**

56. (c) Replace “of balancing” by “to balance”.

57. (c) Replace “would stay” by “would have stayed”.

58. (b) Verb should be “is” because “twenty miles” is used collectively.

59. (c) Replace “be” by “is”.

60. (c) Replace “did he” by “him”.

61. (a)	**62. (b)**	**63. (b)**
64. (c)		

65. (b) When she heard about th e death of her father, she collapsed *in a dead faint.*

66. (c) Let us *hold over* the discussion of this problem until the next meeting.

67. (d) Take this medicine and you will feel better *in no time.*

68. (c) I found Ram *in high feather* yesterday.

69. (a) Our economic policy *is in line with* the interests of all the sections of the society.

70. (b)	**71. (a)**	**72. (d)**
73. (a)	**74. (b)**	**75. (b)**
76. (a)	**77. (b)**	**78. (b)**
79. (b)	**80. (d)**	**81. (a)**
82. (c)		

83. (c) All other are output devices

84. (d) All other are optical scanner devices

85. (d)

86. (d) SSI has 1-11 gates, MSI has 12 to 99 gates, LSI has 100 to 10,000 gates, VLSI has more than 10,000 gates

87. (a)	**88. (d)**	**89. (d)**
90. (a)	**91. (a)**	**92. (c)**
93. (c)	**94. (b)**	

95. (b) All others are input devices

96. (b)	**97. (a)**	**98. (a)**
99. (b)	**100. (c)**	**101. (a)**
102. (b)		

103. (d) All others are components of a computer

104. (c)	**105. (a)**	**106. (a)**

107. (d) All others are impact printers

108. (d)	**109. (c)**	**110. (a)**
111. (b)	**112. (b)**	**113. (d)**
114. (b)	**115. (b)**	

116. (d) The statement only says that opening a library in Rambli would be a wastage. But doesn’t say why? May be opening a library in Rambli is a wastage since there is already a library there.

117. (b) Films have become indispensable but it does not imply that they are the only medium of entertainment, but it is certain that people enjoy films.

118. (b) Detergents are being recommended to clean clothes. i.e. They have ability to clean, or ability to dislodge grease and dirt, but doesn’t mean the ability to form more lather.

119. (b) If state is secular, it means it is against religious bias but not against religion as such.

120. (d)

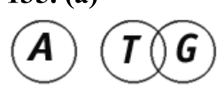
121. (c) Either increase of pollution at upper level is favourable or at the ground level.

122. (b)	**123. (d)**	**124. (a)**
125. (a)	**126. (b)**	**127. (c)**
128. (d)	**129. (c)**	**130. (b)**
131. (a)	**132. (b)**	

133. (c) change order of statements. Now A+A=A.
This thief is a man can be changed to some men are thieves.

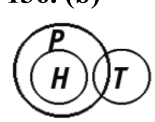
134. (a) change order of statements.Now A+A=A=(All) saints are sinners.

135. (a)



A:Almighty ,T:Takers, G:Givers

136. (b)



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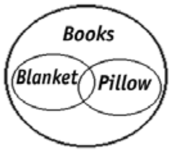
137. (d) I + E = O. Hence, 'Some copies are not pens'. Again, 'Some pens are not desks' follows from 'No desks are pens'.

138. (b)



Directions (Q. 139 - 143): ✓ Indicates corresponding condition is fulfilled by the candidate ✗ indicates corresponding condition is not fulfilled by the candidate

	I	II	III	IV	V
139. (a)	✓	✓	✓	✓	✓
140. (d)	✗	✗	✓	✓	✗
141. (d)	✓	✗ (ii)	✗ (iii)	✓	✓
142. (c)	✓	✓	✗ (iii)	✓	✓
143. (b)	✗ (i)	✓	✓	✓	✓

Directions (Q. 144 - 148):

	A	B	C	D	E	F
Historical	✗	✗		✗	✗	
Industrial				✗		
Hill-Station	✗					

A is neither historical nor hill-station, so it must be an industrial city. D is neither historical nor industrial, so must be hill-station. A&B are not alike, so B must be hill-station. Lastly since informations given are all negative, a lack of negative information about a city means that the city does have qualities whose absence is not mentioned. E.g. there is no negative information about C. Hence, we assume that C is a hill-station as well as historical, & industrial. Final table is :

	A	B	C	D	E	F
Historical	✗	✗	✓	✗	✗	✓
Industrial	✓	✗	✓	✗	✓	✓
Hill-Station	✗	✓	✓	✓	✓	✓

144. (b) **145. (c)** **146. (c)**
147. (a) **148. (d)** **149. (a)**
150. (d)