1. (d) $y = \tan^{-1} x$, $y_1 = \dfrac{1}{1+x^2} \Rightarrow (1+x^2)y_1 = 1$

$\Rightarrow (y_{n+1})(1+x^2) + ({}^nC_1)(y_n)(2x) + ({}^nC_2)(y_{n-1})(2) = 0$

(Using Leibnitz theorem)

Putting $x = 0$, we get

$(y_{n+1}(0))(1) + {}^nC_1(y_n(0))(2\times 0) + {}^nC_2(y_{n-1}(0))(2) = 0$

$\Rightarrow y_{n+1}(0) = -n(n-1)y_{n-1}$

$\Rightarrow y(0) = 0,\ y_1(0) = 1,\ y_2(0) = 0,\ y_3(0) = -2,$

$y_4(0) = 0$ i.e. $y_n(0) = 0$ if n is even and is $(-1)^{\frac{n-1}{2}}(n-1)!$ if n is odd

$\therefore y_{101}(0) - y_{100}(0) = (100)! - 0 = 100!$

2. (a) $f_y(0,0) = \underset{k\to 0}{Lt} \dfrac{f(0,k) - f(0,0)}{k} = 0$

$f_y(h, 0) = \underset{k\to 0}{Lt} \dfrac{f(h,k) - f(h,0)}{k} = h$

$f_x(0, 0) = \underset{h\to 0}{Lt} \dfrac{f(h,0) - f(0,0)}{h} = 0$

$f_x(0, k) = \underset{h\to 0}{Lt} \dfrac{f(h,k) - f(0,k)}{h} = -k$

$f_{xy}(0, 0) = \underset{h\to 0}{Lt} \dfrac{f_y(h,0) - f_y(0,0)}{h} = 1$

$f_{yx}(0, 0) = \underset{k\to 0}{Lt} \dfrac{f_x(0,k) - f_x(0,0)}{k} = -1$ **3. (d)**

4. (c) The dimension of polynomials of degree ≤ 2 is 3. So basis must have 3 elements. **5. (c)** $[\bar{a}\ \bar{a}\ \bar{b}]$

6. (c) No common solution space **7. (c)**

8. (b) $f(x) = \sin^p x \cos^q x$

$f'(x) = [p\sin^{p-1} x \cos^{q+1} x - q\sin^{p+1} x \cos^{q-1} x] = 0$

$\Rightarrow \sin^{p-1} x\cos^{q-1} x[p\cos^2 x - q\sin^2 x] = 0$

$\Rightarrow \tan^2 x = \dfrac{p}{q} \Rightarrow x = \tan^{-1}\left(\sqrt{\dfrac{p}{q}}\right)$

Also $f''(x) < 0$ for $x = \tan^{-1}\left(\sqrt{\dfrac{p}{q}}\right)$

9. (a) $\phi(x) = \int_{5\pi/4}^{x} [3\sin t + 4\cos t]dt$

$\therefore \phi'(x) = 3\sin x + 4\cos x$, $\dfrac{5\pi}{4} \le x \le \dfrac{4\pi}{3}$

$\Rightarrow \phi'(x) < 0$ (Since $\sin x$ and $\cos x$ are both negative in third quadrant)

$\Rightarrow \phi(x)$ is monotonically decreasing, so the least value is attained at $4\pi/3$.

$\therefore l = \phi\left(\dfrac{4\pi}{3}\right) = [-3\cos t + 4\sin t]_{5\pi/4}^{4\pi/3} = \dfrac{3}{2} - 2\sqrt{3} + \dfrac{1}{\sqrt{2}}$

10. (c) For differentiable, it should be continuous **11. (d)**

12. (c) Let us find out the nature of diagonal elements and the relationship between a_{ij} and a_{ji}

Let $X = \begin{pmatrix} 0 \\ 0 \\ .. \\ i \\ 0 \\ 0 \end{pmatrix}$ ith place, $A = (a_{ij})_{n\times n}$ then $X'AX = 0$ gives $(a_{i1}, a_{i2}, ..., a_{in})\begin{pmatrix} 0 \\ 0 \\ .. \\ 1 \\ 0 \\ 0 \end{pmatrix} = 0$ ith place

$\Rightarrow a_{ii} = 0.\ \forall i$

Now let '1' is at both ith and jth place then $X'AX = 0$ gives

$$\left(a_{i1}+a_{j1}, a_{i2}+a_{j2}, \ldots a_{in}+a_{jn}\right)\begin{pmatrix}0\\ \vdots\\ 1\\ \vdots\\ 1\\ \vdots\\ 0\end{pmatrix}\begin{array}{l}\leftarrow \text{ith}\\ \\ \leftarrow \text{jth}\end{array} = 0$$

$a_{ji}+a_{ij}=0 \Rightarrow a_{ij}=-a_{ji}$

13. (c) $\mathrm{I}=\int_0^{\pi/2}(2\log\sin x-\log 2\sin x\cos x)dx$

$=\int_0^{\pi/2}(2\log\sin x-\log 2-\log\sin x-\log\cos x)dx$

$=-(\pi/2)\log 2\quad\left(\because \int_0^{\pi/2}\log\sin x dx=\int_0^{\pi/2}\log\cos x dx\right)$

14. (c) Given limit is

$$\underset{n\to\infty}{Lt}\frac{1}{n}\sum\left(\frac{r}{n}\right)^m=\int_0^1 x^m\,dx=\left[\frac{x^{m+1}}{m+1}\right]_0^1$$

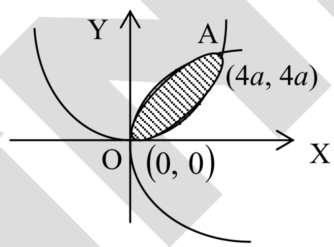
Case I: If $m+1$ is >0 then required limit $=\frac{1}{m+1}(1-0)=\frac{1}{m+1}$

Case II: If $m+1$ is <0 then let $m+1=-k$, $k>0$

$\therefore$ required limit $=\left[\frac{x^{-k}}{-k}\right]_0^1=\frac{-1}{k}\left[\frac{1}{x^k}\right]_0^1=\frac{-1}{k}[1-\infty]=\infty$

15. (b)

16. (a)



Area $=\int_0^{4a}\left(\sqrt{4ax}-\frac{x^2}{4a}\right)$

$=\frac{16}{3}a^2$

17. (c) Put $x^2=z$ so that $2xdx=dz$ or $dx=\frac{dz}{2\sqrt{2}}$

Now as $x=0$, $z=0$ and as $x\to\infty$, $z\to\infty$

$\therefore \int_0^\infty e^{-x^2}dx=\int_0^\infty e^{-z}\frac{dz}{2\sqrt{z}}=\frac{1}{2}\int_0^\infty e^{-z}z^{-\frac{1}{2}}dz$

$=\frac{1}{2}\int_0^\infty e^{-z}z^{\frac{1}{2}-1}dz=\frac{1}{2}\Gamma\left(\frac{1}{2}\right)\quad\therefore\left(\Gamma(x)=\int_0^\infty e^{-x}x^{n-1}dx\right)$

$=\frac{1}{2}\sqrt{\pi}$

18. (a) $\int_{-1}^2\int_{x^2}^{x+2}dydx=\int_{-1}^2\left[\int_{x^2}^{x+2}dy\right]dx=\int_{-1}^2[y]_{x^2}^{x+2}dx$

$=\int_{-1}^2\left(x+2-x^2\right)dx=\frac{9}{2}$

[**Note**: Integration is to be performed w.r.t. that variable having variable limits first and finally w.r.t. the variable with constant limits. If all the limits are constant, then the order of integration is immaterial provided the limits of integration are changed accordingly]

19. (a) The region R bounded by the circle $x^2+y^2=a^2$ can be expressed as

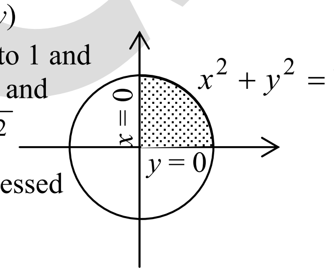
$-a\le x\le a$, $-\sqrt{a^2-x^2}\le y\le\sqrt{a^2-x^2}$

$\therefore \int\int_R x^2y^3dxdy=\int_{-a}^a\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}x^2y^3dydx=0$

($\because y^3$ is an odd function of y)

20. (c) Here x varies from 0 to 1 and for any value of y between 0 and 1, x varies from 0 to $\sqrt{1-y^2}$

$\therefore$ The region D can be expressed as



$0\le x\le\sqrt{1-y^2}$, $0\le y\le 1$

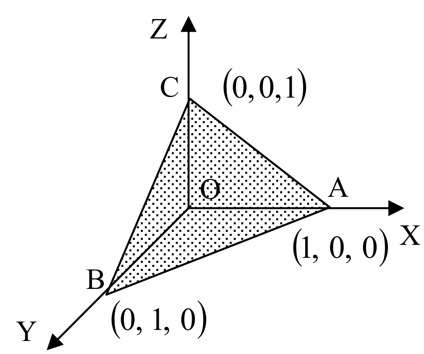
$\therefore \int\int_D x^2y^2dxdy=\int_0^1\int_0^{\sqrt{1-y^2}}x^2y^2dxdy$

$=\int_0^1\left[y^2\cdot\frac{x^3}{3}\right]_0^{\sqrt{1-y^2}}dy=\int_0^1\frac{1}{3}y^2\left(1-y^2\right)^{3/2}dy$

Now put $y=\sin\theta$ and evaluate the above integral using gamma function to give $\frac{\pi}{96}$.

21. (d) (c) is Simpson's $\frac{1}{3}$rd rule

22. (c)



The region R is bounded by the co-ordinate planes $(x=0, y=0, z=0)$ and the plane $x+y+z=1$. The values of x, y lie within the triangle OAB of the xy-plane bounded by the x-axis, the y-axis and the line $(x+y=1, z=0)$.

$\therefore$ For any point (x,y) within this triangle, z varies from the xy - plane to the plane ABC i.e. z varies from $z=0$ to $z=1-x-y$. When the integration w.r.t. z has been performed between these limits, the resulting integral in x and y is extended over the triangle OAB. Here x varies from 0 to 1 and for any intermediary value of x, y varies from 0 to $1-x$. Hence the region R can be expressed as

$0 \le x \le 1, \quad 0 \le y \le 1-x, \quad 0 \le z \le 1-x-y$

$\therefore$ Volume of tetrahedron $= \iiint_R dxdydz$

$$= \int_0^1\int_0^{1-x}\int_0^{1-x-y} dzdydx = \int_0^1\int_0^{1-x}[z]_0^{1-x-y}dydx$$

$$= \int_0^1\int_0^{1-x}(1-x-y)dydx = \int_0^1\left[(1-x)y-\frac{y^2}{2}\right]_0^{1-x}dx$$

$$= \int_0^1\left[(1-x)^2-\frac{(1-x)^2}{2}\right]dx = \int_0^1\frac{1}{2}(1-x)^2\,dx$$

$$= \frac{1}{2}.\left[\frac{(1-x)^3}{3(-1)}\right]_0^1 = -\frac{1}{6}[0-1] = \frac{1}{6}$$

23. (c) Changing to polar co-ordinates by putting $x = r\cos\theta$, $y = r\sin\theta$ We have $J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta}\end{vmatrix} = r$

When r varies from 0 to 1 and θ varies from 0 to 2π, (x, y) covers the region $x^2+y^2 \le 1$

$$\therefore \iint_{x^2+y^2\le 1}\left(x^2+y^2\right)^{7/2}dxdy = \int_0^{2\pi}\int_0^1\left(r^2\right)^{7/2}|J|drd\theta$$

$$= \int_0^{2\pi}\int_0^1 r^8\,dr\,d\theta = \int_0^{2\pi}\frac{1}{9}d\theta = \frac{2\pi}{9}$$

24. (a) Changing to cylindrical polar co-ordinates by the relations

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$

the region $\{(x, y, z): x^2+y^2 \le 1, 2 \le z \le 3\}$ is transformed to the region

$\{(r, \theta, z): 0 \le r \le 1,\ \ 0 \le \theta \le 2\pi,\ 2 \le z \le 3\}$ and

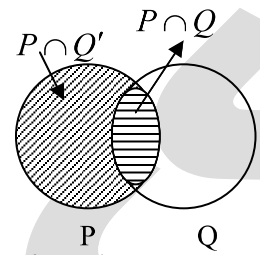
$dxdydz$ is to be replaced by $|J|dr\,d\theta\,dz = rdr\,d\theta\,dz$

$$\therefore \underset{\substack{x^2+y^2\le 1\\ 2\le z\le 3}}{\iiint} z\left(x^2+y^2\right)dx\,dy\,dz = \int_2^3\int_0^{2\pi}\int_0^1 z.r^2.rdr\,d\theta\,dz$$

$$= \int_2^3\int_0^{2\pi}\left[\frac{zr^4}{4}\right]_0^1 d\theta\ dz$$

$$= \int_2^3\int_0^{2\pi}\frac{1}{4}z\,d\theta\,dz = \frac{5\pi}{4}$$

25. (d)



26. (d) By Enler's formulae

E = V +F – 2

$\Rightarrow$ E = 9 + 11 – 2 = 18

27. (a) $\left((A \cup B') \cap B\right)' = (A \cup B')' \cup B'$

$= (A' \cap B) \cup B' = (A' \cup B') \cap (B \cup B') = A' \cup B'$

28. (d) Put $f(x) = t \Rightarrow f'(x)dx = dt$

$\Rightarrow g(x)dx = dt$ **29. (d)** **30. (c)**

31. (a) The auxiliary equation is

$m^2 + k^2 = 0 \Rightarrow m = \pm ik$

$\therefore$ Solution is $y = e^{0x}[c\cos kx + c'\sin kx]$

Now Choice (a) is $y = c_1e^{ikx} + c_2e^{-ikx}$

$= c_1[\cos kx + i\sin kx] + c_2[\cos kx - i\sin kx]$

$= (c_1+c_2)\cos kx + i(c_1-c_2)\sin kx$

$= c\cos kx + c'\sin kx$

32. (a) $v = \pi r^2 h$, $dv = \frac{\partial v}{\partial r}dr + \frac{\partial v}{\partial h}dh$

$\Rightarrow dv = 2\pi rhdr + \pi r^2 dh$

Now $r = 4$, $h = 8$, $dr = .02$ and $dh = -.01$

$\Rightarrow dv = 1.12\pi$

33. (c) For inconsistency, $\rho(A) < \rho(A\,|\,b)$, i..e.

$$\begin{vmatrix} 3 & 2 & -5 \\ 2 & -6 & k \\ 5 & -4 & -1 \end{vmatrix} = 0$$

performing $c_1 \to c_1 + c_2$, $c_1 \to -c_1$, we get

$\begin{vmatrix} -5 & 2 & -5 \\ 4 & -6 & k \\ -1 & -4 & -1 \end{vmatrix} = 0$ This will be true if third column is the first column $\Rightarrow k = 4$

34. (b) Let $N = .1525252.....$

$1000N = 152.525252....$

$10N = 1.525252$

Subtract $990N = 151 \Rightarrow N = \dfrac{151}{990}$

35. (c) **36. (d)**

37. (d) Let $x =$ number of weeks in which to sell.

Then amount $A = (100 + 10x)(150 - 5x)$

$\Rightarrow A = 15000 - 500x + 1500x - 50x^2$

$= 15000 + 1000x - 50x^2$

$\dfrac{dA}{dx} = 1000 - 100x = 0 \Rightarrow x = 10$

38. (c) 'Law of mean' means "First mean value theorem of differential calculus" i.e. 'Language's Mean Value theorem'. For $y = f(x)$ in an interval $a < x < b$, there is a value c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow f'(c) = \frac{-3/10}{3}$$

39. (d) $e^{-x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + ...$

$e^{-1/5} = 1 - \dfrac{1}{5} + \dfrac{1}{50} - \dfrac{1}{750} +$ (very much smaller term)

$= 1 - .20 + .02 - .00133..... = .82$

40. (d) $\dfrac{\partial \phi}{\partial x} = \cos x + \cos y + \tan z$

$\dfrac{\partial \phi}{\partial y} = -x \sin y, \; \dfrac{\partial \phi}{\partial z} = x \sec^2 z$

$\dfrac{\partial^2 \phi}{\partial y^2} = -x \cos y, \; \dfrac{\partial^2 \phi}{\partial z^2} = 2x \sec^2 z \tan z$

$\dfrac{\partial^2 \phi}{\partial x \partial y} = -\sin y, \; \dfrac{\partial^2 \phi}{\partial y \partial z} = 0$

41. (b) **42. (c)** **43. (b)** Put $x = \tan \theta$

44. (a) For a non zero solution, $|A| = 0$

i.e. $\begin{vmatrix} 1 & 1 & -1 & K \\ 1 & -1 & -3 & 2 \\ 1 & 2 & -2 & -4 \\ 1 & -3 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1 & -K/2 \\ 1 & -1 & -3 & -1 \\ 1 & 2 & -2 & 2 \\ 1 & -3 & 4 & -3 \end{vmatrix} (-2)$

Which vanishes if $K = -2$

45. (b) Triangular matrix **46. (a)**

47. (c) $x = r\cos\theta$, $y = r \sin \theta \Rightarrow r^2 = x^2 + y^2$

$\therefore$ The function e^{-r^2} is being integrated over the first quadrant area ($\because 0 < x < \infty, 0 < y < \infty$ means x and y are positive i.e. θ is in first quadrant). In polar coordinate, this area would be $\int_0^{\pi/2} \int_0^{\infty} r \, dr \, d\theta$

$\Rightarrow$ Original double integral

$= \int_0^{\pi/2} \int_0^{\infty} e^{-r^2} r \, dr \, d\theta$

48. (a) When r varies from 0 to a, θ varies from 0 to 2π, (x, y) covers the region $x^2 + y^2 \le a^2$ **49. (a)**

50. (b) $A'.A = I \Rightarrow A'$ is inverse of A i.e. $A' = A^{-1}$

Now $A^{-1} = \dfrac{1}{|A|} \, Adj \; A = \dfrac{1}{\mathrm{D}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

i.e. $\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \dfrac{1}{\mathrm{D}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \Rightarrow \dfrac{d}{\mathrm{D}} = e \Rightarrow \mathrm{D} = d/e$

51. (b) $\phi = 4xz^3 - 3x^2y^2z$

$\left(\dfrac{\partial \phi}{\partial x}\right)_{(2,-1,2)} = \left(4z^3 - 6xy^2z\right)_{(2,-1,2)} = 8$

$\left(\dfrac{\partial \phi}{\partial y}\right)_{(2,-1,2)} = \left(-6x^2yz\right)_{(2,-1,2)} = 48$

$\left(\dfrac{\partial \phi}{\partial z}\right)_{(2,-1,2)} = \left(12xz^2 - 3x^2y^2\right)_{(2,-1,2)} = 84$

Also $l = \dfrac{2}{7}$, $m = \dfrac{-3}{7}$, $n = \dfrac{6}{7}$

Hence directional derivative $= 8 \times \dfrac{2}{7} + 48 \times \dfrac{-3}{7} + 84 \times \dfrac{6}{7}$

$= \dfrac{376}{7}$

52. (d) Trapezoid approximates the polynomial by a straight line (1st degree curve). Simpson's $\dfrac{1}{3}$rd interpolates it by a parabola (i.e. a 2nd degree curve). Simpson's $\dfrac{3}{8}$th interpolates it by a cubic.

53. (b) $h = \dfrac{b - a}{\text{No. of intervals}} = \dfrac{4 - 0}{4} = 1$

(No. of ordinates is given to be 5, as we are given 5 values, $\therefore$ No. of subintervals = 4).

$x = 0$	1	2	3	4
$y = e^0$	e^1	e^2	e^3	e^4
$= 1$	2.72	7.39	20.09	54.60
y_1	y_2	y_3	y_4	y_5

$\therefore \int_0^4 e^x \, dx = \dfrac{1}{3}\left[(1 + 54.60) + 2(7.39) + 4(2.72 + 20.09)\right]$

= 53.87 (approximate)

54. (a) Probability of hitting target of first person = 3/4. Missing target = 1–3/4 = 1 /4.
Probability of hitting target of 2nd person = 2/3, missing target = 1 – 2/3 = 1/3, Probability of missing target when both persons try = 1/4 × 1/3 Required probability of hitting target = 1-1/12 = 11/12.

55. (b) $^6C_4\,(1/2)^6 + {}^6C_5\,(1/2)^6 + {}^6C_6\,(1/2)^6$.
$= (1/2)^6\,[(6 \times 5 / 2 \times 1) + 6 + 1]$
$= (1/2)^6\,[15 + 6 + 1] = (1/2^6) \times 22 = 11/32$.

56. (a) Given np = 6 and npq = 4.
Solving we get q = 2/3, n = 18

57. (d) Since P(x = 1) = P(x = 2)
we have $\lambda = 2$.

Now $P(x=4) = \dfrac{e^{-\lambda}\lambda^x}{x!} = \dfrac{e^{-2}2^4}{4!} = \dfrac{2}{3}e^{-2}$.

58. (b) Total determinant of second order with elements 0 or 1 = 2^4 = 16. The determinant will have the positive value in the following three cases:

$$\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}$$

. Required probability = 3/16

59. (a) This is a binomial problem with n = 5, p = 1/3
We have to find P(x = 4) + P(x = 5).

60. (a) P = .001, n = 3000, λ = nP = 300 × .001 = 3.

$$P(x=3) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-\lambda}3^x}{3!}$$

61. (d)	**62. (d)**	**63. (a)**	**64. (a)**	**65. (c)**
66. (c)	**67. (b)**	**68. (d)**	**69. (a)**	**70. (c)**
71. (d)	**72. (a)**	**73. (b)**	**74. (b)**	**75. (a)**
76. (d)	**77. (b)**	**78. (c)**	**79. (c)**	**80. (d)**
81. (c)	**82. (b)**	**83. (c)**	**84. (a)**	

85. (c) (! is complement) so
$(!\,(!\,x)) \rightarrow (!\text{false}) \rightarrow \text{True}\ \&\&\ \text{True} \rightarrow \text{True}$

86. (d) **87. (c)** *i* is increment by 4

88. (c) condition is false so %c and 107 is ASCII value of *k*.

89. (d) If any, one element is initialized in array all other elements of integer array is initialized by zero by default in static initialization.

90. (a) Call by address so value is Swap.

91. (c) **92. (a)** **93. (d)** **94. (a)** **95. (b)**

<u>**Solutions (96 – 102)**</u>**:** Use the following formulae to answer these problems.
A cube is a geometrical figure, which has size equal sides or surface.

<u>**Rules for counting the no. of painted or unpainted faces**</u>**:**

(1) The no. of 3 faces painted cubes = 8

(2) The no. of 2 face painted cubes = $(N-2)\times 12$ (*N* is no. of equal cubical blocks on each of the columns of the faces)

(3) The no. of 1 face painted cubes = $(N-2)^2 \times 6$

(4) No. of no face painted = $(N-2)^3$

(5) The no. of cubes having 3 faces painted with 3 different colors = 8

(6) The no. of cubes having 2 faces painted with 2 different colors and all other faces unpainted = <u>2</u> ×2 = <u>4</u> in the case of <u>3</u> cubic blocks; <u>4</u> × 2 = <u>8</u> in the case of <u>4</u> cubical blocks. <u>6</u> × 2 = <u>12</u> in the case of 5 cubical blocks and so on.

(7) The no. of cubes having only one face painted with one specific color and all other faces unpainted = <u>1</u>× = <u>2</u> in the case of <u>3</u> cubical blocks, <u>4</u> × 2 = <u>8</u> in the case of <u>4</u> cubical blocks, <u>9</u>×2 = <u>18</u> in the case of <u>5</u> cubical blocks and so on.

(8) The no. of cubes having 2 faces painted with one specific color = none.

(9) The no. of cubes having 2 faces painted with 2 different colors and all other faces may be painted or unpainted = <u>6</u> × 2 = <u>12</u> in the case of <u>3</u> cubical blocks, <u>8</u>×2 = <u>16</u> in the case of <u>4</u> cubical blocks, <u>10</u>×2 = <u>20</u> in the case of <u>5</u> cubical blocks and so on

(10) The total no. of cubes = $(N)^3$
where *N* is the no. of equal cubical blocks on each of the columns of the faces.

96. (b) $(N-2)^3 = (3-2)^3 = 1$

97. (c) $(N-2)^2 \times 6 = (3-2)^2 \times 6 = 6$

98. (d) $(N-2)\times 12 = (3-2)\times 12 = 12$ **99. (c)**

100. (b) (Use formula 6) = 4

101. (c) (Use formula 9) = 12

102. (b) (Use formula 5) = 8

103. (c) Divisible by 33 but not by 30.

Divisible by 33 : $\dfrac{3001}{33} - \dfrac{1001}{33} \equiv 90 - 30 = 60$

Divisible by both 33 and $30 = \dfrac{3001}{330} - \dfrac{1001}{330} \equiv 9 - 3 = 6$

[$\because$ LCM of 33 and 30 = 330]
Hence divisible by 33 but not 30 = $60 - 6 = 54$.

104. (b) $S_1 = \dfrac{n}{2}\{2\times 1 + (n-1)\times 1\} = \dfrac{n(n+1)}{2} = \dfrac{n^2+n}{2}$

$$S_2 = \frac{n}{2}\{2\times 1 + (n-1)\times 2\} = \frac{n(2n)}{2} = n^2$$

$$S_3 = \frac{n}{2}\{2\times 1 + (n-1)\times 3\} = \frac{n(3n-1)}{2} = \frac{3n^2-n}{2}$$

From the above it is clear that $S_1 + S_3 = 2S_2$

105. (d) The series is $\times 1+(7\times 1);\ \times 2+(6\times 2);$ $\times 3+(5\times 3);\ \times 4+(4\times 4);\ \times 5+(3\times 5)$.....

Suggestion

In MCA 2003 exam conducted by IIT Roorkee, there were 60 objective questions to be done in 2 hrs. **There was negative working and use of calculator was permitted**.

MCA Entrance 2004 Exam merged with AIMCET and hence no separate exam. was being conducted by IIT Roorkee. This year the time of the exam has been increased from 2 hrs to 3 hrs. and the ratio has been changed from 3:1:1 to 4:2:1. This year since the time has been increased by 1 hr., the number of questions ought to be more. We have 105 questions in this mock paper, but the actual number of questions may differ. It is suggested to go through the directions given on the first sheet of the actual test booklet to get yourself acquainted with the policies to be adopted by the examining body this year.

All the Best!

Additional Problem

1. If a coin is tossed twice what is the probability that it will land either heads both times or tails both times?

(a) 1/8 (b) 1/6 (c) 1/4 (d) 1/2

Sol. (d) We use the rule P (A or B) = P (A) + P (B). The individual probabilities are: 1/4 +1/4 = 2/4 = 1/2.

2. If someone draws a card at random from a deck and then, without replacing the first card, draws a second card, what is the probability that both cards will be aces?

(a) 1/3136 (b) 1/3631 (c) 1/227 (d) 1/221

Sol. (d) Event A is that the first card is an ace. Since 4 of the 52 cards are aces, P(A) = 4/52 = 1/13. Given that the first card is an ace, what is the probability that the second card will be an ace as well? Of the 51 remaining cards, 3 are aces. Therefore, P(B|A) = 3/51 = 1/17, and the probability of A and B is 1/13 × 1/17 = 1/221.

3. For Poisson a variate x if P(x = 1) = P(x = 2). Then mean of x is:

(a) 1/2 (b) 1 (c) 2 (d) 4

Sol. (c) By using

formula $P(x=1)=\dfrac{e^{-\lambda}\lambda^{x}}{x!}=\dfrac{e^{-\lambda}1^{x}}{1!}$....(1)

$P(x=2)=\dfrac{e^{-\lambda}2^{x}}{2!}$....(2) from equation (1) and (2)

$\dfrac{e^{-\lambda}2^{x}}{2!}.=\dfrac{e^{-\lambda}1^{x}}{1!}\quad \lambda=2.$

4. A box contains 9 tickets numbered from 1 to 9. If 3 tickets are drawn from the box, one by one then probability that the numbers on these are alternately either even, odd, even or odd; even, odd is:

(a) 4/9 (b) 5/18 (c) 20/63 (d) 5/42

Sol. (b) There are 9 tickets numbered from 1 to 9. There are 4 even numbers, i.e., 2, 4, 6, 8 and 5 odd numbers, i.e., 1, 3, 5, 7, 9. Let the first ticket drawn has even number. Required probability for this = 4/9.

For II tickets to be odd, probability = 5/8 and for III to be even, probability = 3/7.

Required probability = (4/9) × (5/8) × (3/7) = 5/42. For the tickets to be numbered odd, even, odd, the probabilities are 5/9, 1/2 and 4/7 respectively.

Required probability = (5/9) × (1/2) × (4/7) = 10/63. Therefore the probability that the numbers on the tickets are alternately odd and even = 5/42 + 10/63 = 35/126 = 5/18.

5. A purse contains two 20 paise coins and four 10 paise coins. A second purse contains four 20 paise coins and three 10 paise coins. If a coin is selected at random from one of the two purses, then the probability that it is a 20 paise coin is:

(a) 6/13 (b) 3/13 (c) 19/42 (d) 19/21

Sol. (c) There are two purses A and B (say). To select a purse, the probabilities are P (A) = 1/2 and P (B) = 1/2 respectively.

Now probability of getting 20 paise coin from purse A= 2/6 = 1/3

Probability of getting 20 paise coin from purse B = 4/7.

Using the law of total probability, we have

P (20 paise coin) = (1/2) × (1/3) + (1/2) × (4/7) = 1/6 + 2/7 = 19/42.

6. The two faces of five coins are marked with digits 2 and 3. These are tossed together. The probability of getting sum of digits as 12 is:

(a) 3/16 (b) 5/16 (c) 3/32 (d) 5/32

Sol. (b) The number of ways of getting 2, 2, 2, 3 and 3 is $\dfrac{5!}{2!3!}$ = 10. Total cases 2^5 = 32.

Hence probability = 10/32 = 5/16.

7. The standard deviation of a binomial distribution is

(a) npq (b) $(npq)^{1/2}$ (c) np/q (d) np

Sol. (b)

8. On throwing a coin thrice, the probability of getting all heads is:

(a) 0 (b) 1 (c) 1/8 (d) 3/8

Sol. (c) Probability of getting head in one throw = 1/2

In 3 throws for all heads, probability

$=1/2\times 1/2\times 1/2=1/8$

9. Probability of A solving the problem is 3/7 & of B solving the problem is 2/7. The probability of problem to be solved is:

(a) 25/49 (b) 49/29 (c) 20/49 (d) 29/49

Sol. (d) $P(A)=\frac{3}{7} \Rightarrow P(\overline{A})=\frac{4}{7}$ and $P(B)=\frac{2}{7} \Rightarrow P(\overline{B})=\frac{5}{7}$

Required probability $=1-P(\overline{A})P(\overline{B})=1-\frac{4x5}{7x7}=\frac{29}{49}$

10. A fair coin is thrown in the air four times. If the coin lands with the head up on the first three tosses, what is the probability that the coin will land with the head up on the fourth toss?

(a) 0 (b) 1/16 (c) 1/8 (d) 1/2

Sol. (d) The fourth toss is independent of the first three tosses - it has nothing to do with the first three tosses. Therefore, the probability remains one out of two.