HINTS AND SOLUTIONS **Test Code: MTJNU-II**

1. (a) Let $x='a'$ be an integer root of the given equation then $'a'$ must divide $1155 \Rightarrow a$ is odd. But if $'a'$ is odd then $a^8+39a^2+1155-(24a^7+18a^5)$ is also odd and hence not 0. Therefore given equation has no integral root.

2. (a) $\dfrac{\cos A}{a}=\dfrac{\cos B}{b}=\dfrac{\cos C}{c}$

$\Rightarrow \dfrac{\cos A}{K\sin A}=\dfrac{\cos B}{k\sin B}=\dfrac{\cos C}{K\sin C}$

$\Rightarrow \cot A=\cot B=\cot C$

$\Rightarrow A=B=C=60°$

3. (c) Let $\phi(x)=\int_\alpha^x f(t)dt$ be odd then $\phi(-x)=-\phi(x)$

$\Rightarrow \int_\alpha^{-x} f(t)dt=-\int_\alpha^x f(t)dt$

$\Rightarrow \int_\alpha^0 f(t)dt+\int_0^{-x} f(t)dt=-\int_a^0 f(t)dt-\int_0^x f(t)dt$

$\Rightarrow \int_a^0 f(t)dt-\int_0^x f(-y)dy=-\int_a^0 f(t)dt-\int_0^x f(t)dt$

(putting $t=-y$ in the second integral on L.H.S.)

$\Rightarrow 2\int_0^\alpha f(t)dt-\int_0^x f(y)dy=-\int_0^x f(t)dt$

$\Rightarrow 2\int_0^\alpha f(t)dt-\int_0^x f(t)dt=-\int_0^x f(t)dt$

$\Rightarrow 2\int_0^\alpha f(t)dt=0 \Rightarrow \int_0^\alpha f(t)dt=0$

4. (b) use Wilson theorem, which states
$(p-1)!+1\equiv 0(\text{mod } p)$ where p is a prime

5. (c) $7x+98y=1000$ is a Diophantine equation. The Diophantine equation $ax+by=c,\ a,\ b,\ c\in I$ have integral solutions iff g.c.d. of $'a'$ and $'b'$ divides $'c'$. Here $(7,\ 98)=7$ which does not divide 1000.

6. (c) $3^1=3,\ 3^2=9,\ 3^3=27,\ 3^4=81,\ 3^5=243$. The only possible last digits are $3,\ 9,\ 7$ and 1 and this pattern repeats.

Now $3^{4798}=\left(3^4\right)^{1199}.32$

Now the last digit of $(3^4)^{1199}$ is 1. Therefore, the last digit is 9.

7. (c) $2^{300}5^{600}4^{400}=2^{600}5^{600}2^{500}=10^{600}2^{500}$

8. (b) $30^4=2^4 3^4 5^4$.
There divisions are (4+1)(4+1) (4+1)

9. (d) From pages 1 to 9 there are 9. 1 = 9 digits
From pages 10 to 99 there are 90. 2 = 180 digits
From pages 100 to 599 there are 500. 3 = 1500 digits
From pages 600 to 659 there are 60. 3 = 180 digits
From pages 660 to 666 there are 7. 3 = 21 digits
Thus, Total 1890 digits
Therefore book has 666 pages.

10. (c) $6^6=2^6 3^6, 8^8=2^{24}, 12^{12}=2^{24}3^{12}$

Let $k=2^p 3^q$

given LCM of $2^6 3^6, 2^{24}, 2^p 3^q=2^{24}3^{12}$

$\therefore\ q=12,\quad p=0,\ 1,\ 2,......24$

$\therefore$ no. of values of $k=25$

11. (c)

12. (b) $2\sin\theta\cos\theta=\sin 2\theta$

13. (b)

14. (c) No. of diagonals = ${}^nC_2-n$ when $'n'$ is no. of sides.

15. (d) Let edge of the cube be l cm. Then its volume is $v = l^3 \Rightarrow \frac{dv}{dt} = 3l^2 \frac{dl}{dt}$. But $\frac{dl}{dt} = 5cm/\sec$.

when $l = 4cm, \frac{dv}{dt} = 3\times(4)^2 \times 5 = 240$

16. (d) $\left(aba^{-1}\right)^2 = aba^{-1}aba^{-1} = ab^2a^{-1}$

$= a\ aba^{-1}a^{-1} = a^2ba^{-2} \quad \therefore \left(aba^{-1}\right)^4 = \left(\left(aba^{-1}\right)^2\right)^2$

$= a^2b^2a^{-2} = a^2aba^{-1}a^{-2} = a^3ba^{-3}$

$\Rightarrow \left(aba^{-1}\right)^8 = \left(\left(aba^{-1}\right)^4\right)^2 = \left(a^3ba^{-3}\right)^2 = a^3b^2a^{-3}$

$= a^3aba^{-1}a^{-3} = a^4ba^{-4}$

Since, $\left(aba^{-1}\right)^{16} = \left(\left(aba^{-1}\right)^8\right)^2 = \left(a^4ba^{-4}\right)^2 = a^4b^2a^{-4}$

$= a^4aba^{-1}a^{-4} = a^5ba^{-5} = ebe^{-1} = b$

Therefore $\left(aba^{-1}\right)^{16} = b$

$\Rightarrow \left(b^2\right)^{16} = b \Rightarrow b^{32} = b$

$\Rightarrow b^{31} = e \therefore\ O(b)/31$. but since 31 is a prime.

O (b) = 31

17. (b)

18. (d)

19. (b) $\cos x + i\sin x = e^{ix}$

$\therefore\ f(x) = e^{in^2x}$

20. (d) It is a circle.

21. (c) Prob. that a point lies in $\left(\theta, \theta + d\theta\right) = \frac{d\theta}{2\pi}$

$P(x \le X \le x + dx) = \frac{2d\theta}{2\pi}$ where $x = \cos\theta$

$\therefore f(x) = \frac{1}{\pi}\left|\frac{d\theta}{dx}\right| = \frac{1}{\pi\sqrt{1-x^2}}, -1 \le x \le 1$

22. (a) Let $v_k = \log y_k$ Then given difference equation reduces to $V_{k+2} - 3V_{k+1} + 2V_k = 0$

$\Rightarrow V_1 = \log 2$ and $V_2 = 0$ Which can be rewritten as

$\left(E^2 - 3E + 2\right)V_k = 0$, which is a linear difference equation with constant coefficients. The auxiliary equation has roots 1 and 2, therefore the general solution is

$V_k = C_1.1^k + C_2 2^k$, using initial values.

$C_1 = 2\log 2$ and $C_2 = -1/2\ \log 2$

23. (b) $\log_2\left(\log_3\left(\log_5\left(\log_7 N\right)\right)\right) = 11 \Rightarrow N = 7^{5^{3^{2^{11}}}}$

Hence 7 is the only prime factor.

24. (b) $x = \sqrt{1+x}$ or $x^2 = 1 + x$ or $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

The sign is discarded as $x > 0$. Hence $x = \frac{1+\sqrt{5}}{2} \in (1,\ 2)$

25. (a) Equation of Bisector

26. (c) Z_1 and Z_2 are collinear

27. (a) Normal always passes through the center of the circle

28. (d)

29. (c)

30. (b) $U_n = \frac{2.5.8....(3n-1)}{6.12.18.....6n}$

$\therefore\ \frac{U_n}{U_{n+1}} = \frac{6n+6}{3n+2} \to 2$ as $n \to \infty$. Hence convergent.

31. (d) Let $f(x, y) = (x-2)^2 - y(y-1)^2$

$\therefore\ f_x = 2\left(x-2\right)$ and $f(y) = -2y(y-1) - (y-1)^2$

For double points $f_x = f_y = f = 0$

$f_x = 0 \Rightarrow x = 2, f_y = 0 \Rightarrow y = 1,\ 1/3$

Also $\left(2,\ 1\right)$ satisfies $f = 0$ and hence is a double point.

32. (b)

33. (b) for $x \ge 0$, equation is $3x^2 - 2x = 0$

for $x < 0$, it is $3x^2 + 2x = 0$

34. (b) $2x \equiv 3 \mod 20$

$\therefore$ we want integral x, so that $20/2x - 3$ but $2x - 3$ is an odd integer, not divisible by 20

35. (a)

36. (a)

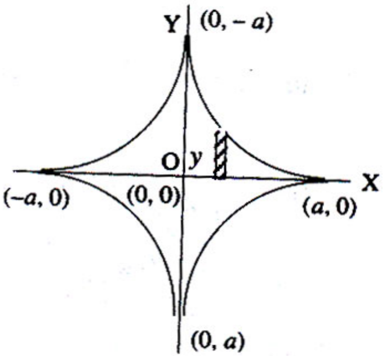
37. (b) use comparison least after rationalisation

38. (d) Given curve $x = a\cos^3 t$

$y = a\sin^3 t$ .... (1)

Eliminating t, we get $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$

or, $x^{2/3} + y^{2/3} = a^{2/3}$ (astroid) .... (2)



Required area = $4\int_0^a y dx$

$= -4\int_0^{\pi/2} a\sin^3 t.3a\cos^2 t\sin t dt$

$= -12a^2\int_0^{\pi/2} \sin^4 t\cos^2 t dt = -12a^2 \frac{3.1.1}{6.4.2}\frac{\pi}{2} = -\frac{3\pi a^2}{8}$

$= \frac{3\pi a^2}{8}$ (neglecting -ve sign for area)

39. (c) $y = 2\left\{\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right\}$

$= 2\left\{\sin x\frac{\cos\pi}{6} + \cos x\sin\frac{\pi}{8}\right\}$

$= 2\sin(x + \pi/6)$

y is max. when $\sin(x + \pi/6)$ is max. i.e. 1

$\Rightarrow x + \pi/6 = \pi/2 \Rightarrow x = \pi/3$

40. (a) Since $(1, 1)$ and $(0, 1)$ forms a basis, we can write

$(a, b) = x(1, 1) + y(0, 1) = (x, x + y)$

$\Rightarrow x = a, x + y = b \Rightarrow y = b - a$

$\therefore (a, b) = a(1, 1) + (b - a)(0, 1)$

$T(a, b) = aT(1, 1) + (b - a)T(0,1) = 3a + (b - a)(-2)$

41. (c)

42. (a) Let $x = (27 + \sqrt{756})^{1/3} + (27 - \sqrt{756})^{1/3}$

Then cubing both sides

$x^3 = 27 + \sqrt{756} + 27 - \sqrt{756}$

$+3(27 + \sqrt{756})^{1/3}(27 - \sqrt{756})^{1/3}x$

$\Rightarrow x^3 = 54 + 3(729 - 756)^{1/3}.x$

$\Rightarrow x^3 = 54 - 3.3x \qquad \Rightarrow x^3 + 9x - 54 = 0$

$\Rightarrow (x - 3)(x^2 + 3x + 18) = 0 \Rightarrow x = 3, x = -\frac{3}{2} \pm i\frac{\sqrt{63}}{2}$

$\Rightarrow x = 3$ (neglecting imaginary values)

43. (d) Write a polynomial whose roots are

$\sqrt{2} + \sqrt{3}, \sqrt{2} - \sqrt{3}, -\sqrt{2} + \sqrt{3}, -\sqrt{2} - \sqrt{3}$

44. (d) Given α, β, γ are the roots of equation

$x^3 + qx + r = 0 \;\therefore\; \sum\alpha = 0, \sum\alpha\beta = q, \alpha\beta\gamma = -r$

Taking

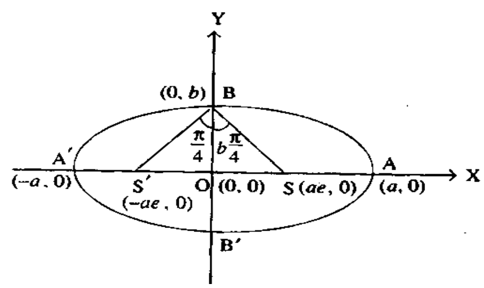
$\sum\frac{\alpha}{\beta + \gamma} = \sum\frac{\alpha}{(\alpha + \beta + \gamma) - \alpha} = \sum\frac{\alpha}{0 - \alpha} =$

$\sum(-1) = -3$

45. (b) $Adj(Adj\ A) = |A|^{n-2} A$

But A is $2 \times 2 \;\therefore\; Adj(AdjA) = A$

46. (b) Let the equation of an ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$\frac{OS}{OB} = \tan\frac{\pi}{4}$ or $\frac{ae}{b} = 1$

or, $ae = a\sqrt{1 - e^2} \because b = a\sqrt{1 - e^2}$

or, $e^2 = 1 - e^2$ or, $2e^2 = 1$ or, $e = \frac{1}{\sqrt{2}}$

47. (d) Given straight line

$2x + y - 9 = 0$ .... (1)

We know that equation of any normal to parabola $y^2 = 4ax$ is $y = mx - (2am + am^3)$ .... (2)

Equations (1) and (2) must be same, therefore comparing, we get $1=\dfrac{m}{-2}=\dfrac{2am+am^3}{-9}$

$\Rightarrow\ m=-2$ and , $2am+am^3=-9$

$\Rightarrow\ -4a-8a=-9\ \ \Rightarrow a=\dfrac{3}{4}$

Hence equation of parabola is $y^2=3x$

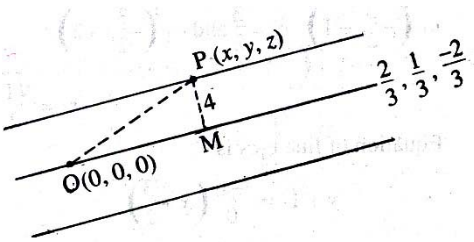
48. (c) Given radius 4 and axis of right circular cylinder is $x=2y=-z$

or $\qquad \dfrac{x-0}{2}=\dfrac{y-0}{1}=\dfrac{z-0}{-2}$

Prop. dc's of axis of cylinder are 2, 1, -2

$\therefore$ Dc's are $\dfrac{2}{3},\dfrac{1}{3},\dfrac{-2}{3}$

Let any point P(x, y, z) on the right circular cylinder, then its



Equation is given by

$OP^2-(OM)^2=(PM)^2$

$x^2+y^2+z^2-\left(\dfrac{2x+y-2z}{3}\right)^2=4^2$ or,

$9(x^2+y^2+z^2)-(4x^2+y^2+4z^2+4xy-4yz-8zx)=144$

or, $5x^2+8y^2+5z^2-4xy+4yz+8zx-144=0$

49. (b) $\lambda=0$, $\mu=1$

50. (a)

51. (d)

52. (c)

53. (a)

54. (a) LHS is $\cos(e^x)$ which is ≤ 1 always but the R.H.S. is $>1\ \forall\ x$

55. (b) Arg $\left(\dfrac{z-1}{z+1}\right)=\pi/3\Rightarrow\left|\dfrac{z-1}{z+1}\right|=\tan^{-1}\pi/3=\sqrt{3}$

Which is circle of Appolonius.

56. (c) Real part of $\exp(e^{i\theta})=e^{\cos\theta+i\sin\theta}$

$=e^{\cos\theta}e^{i\sin\theta}=e^{\cos\theta}\left(\cos(\sin\theta)+i\sin(\sin\theta)\right)$

57. (d) If ω is a complex root of unity then the determinants $\omega^3=1,\omega^2+\omega+1=0$

$$\Delta=\begin{vmatrix}1 & 1+i+\omega^2 & \omega^2\\ 1-i & -1 & \omega^2-1\\ -i & -i+\omega-1 & -1\end{vmatrix}=$$

$$\begin{vmatrix}2(1-i) & -2 & 2(\omega^2-1)\\ 1-i & -1 & \omega^2-1\\ -i & -i+\omega-1 & -1\end{vmatrix}$$

$R_1\to R_1+R_2+R_3$

$$=\frac{1}{2}\begin{vmatrix}2(1-i) & -2 & 2(\omega^2-1)\\ 2(1-i) & -2 & 2(\omega^2-1)\\ -i & -i+\omega-1 & -1\end{vmatrix}$$

$\because$ First two rows are identical $\therefore$ A = 0

58. (c) $\rho(A)<\rho(A\,|\,b)$

59. (d) $f(x)=\sin^{-1}\log_2\dfrac{x}{3}$

$\therefore$Domain of $\sin^{-1}x$ is $(-1,1)$

Clearly $\sin^{-1}\log_2\dfrac{x}{3}$ exist, if

$-1\le\log_2\dfrac{x}{3}\le 1$ or, $2^{-1}\le\dfrac{x}{3}\le 2^1$ or, $\dfrac{1}{2}\le\dfrac{x}{3}\le 2$ or,

$\dfrac{3}{2}\le x\le 6$

$\therefore$ Domain $\left[\dfrac{3}{2}\ ,6\right]$

60. (a) $\tan u=\dfrac{x^3+y^3+x^2y-xy^2}{x^2-xy+y^2}=v\,$(say)

which is homogeneous function of first degree By Euler's theorem $x\dfrac{\partial v}{\partial x}+y.\dfrac{\partial v}{\partial y}=1.u$

$$x\frac{\partial}{\partial x}(\tan u)+y\frac{\partial}{\partial y}(\tan u)=1.\tan u$$

$$x\sec^2 u\frac{\partial u}{\partial x}+y\sec^2 u\frac{\partial u}{\partial y}=1.\tan u$$

$$x.\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{\tan u}{\sec^2 u}=\sin u\cos u$$

$$=\frac{1}{2}2\sin u\cos u=\frac{1}{2}\sin 2u$$

61. (c) $I=\int 1.\cos(\log x)dx$

$$=x\cos(\log x)-\int x.[-\sin(\log x)].\frac{1}{x}dx$$

$$=x\cos(\log x)+\int \sin(\log x)dx$$

$$=x\cos(\log x)+x\sin(\log x)$$

$$-\int x\cos(\log x).\frac{1}{x}dx+c$$

$$I=x\cos(\log x)+x\sin(\log x)-I+c$$

$$\therefore\ I=\frac{x}{2}\{\cos(\log x)+\sin(\log x)\}+c$$

62. (c) $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist, so in the interval including '0' Lagrange's is not valid.

63. (a) According to problem $x^3=(1+i)=k(say)$ (say)

$\Rightarrow\ x^3-k=0\quad\therefore$ sum of roots $=\alpha+\beta+\gamma=0$

64. (c) $\phi(10)=\phi(2)\phi(5)=4$

65. (c) 1^∞ form. take log and then use L' Hospital

66. (c)

$$A=\lim_{n\to\infty}\left\{\sum_{r=1}^{2n}\left(\frac{r^2}{r^3+n^3}\right)\right\}=\lim_{n\to\infty}\left\{\frac{1}{n}\sum_{r=1}^{2n}\left(\frac{\frac{r^2}{n^2}}{1+\frac{r^3}{n^3}}\right)\right\}$$

$$=\int_0^2\frac{x}{1+x^3}dx=\int_1^9\frac{1}{t}.\frac{dt}{3}\qquad \text{Let } \begin{matrix}1+x^3=t\\ 3x^2dx=dt\end{matrix}$$

$$=\frac{1}{3}\left[\log_e t\right]_1^9=\frac{1}{3}[\log_e 9-\log_e 1]=\frac{2}{3}\log_e 3$$

67. (a) Homogeneous differential equation substitute $y=vx$

68. (d)

69. (a) $\begin{vmatrix}1&0&0\\2&a&a\\2&1&1\end{vmatrix}=\begin{vmatrix}a&a\\1&1\end{vmatrix}=a-a=0\ \ \forall\ a$

70. (d) $\begin{vmatrix}1&0&0\\0&1&1\\1&1&\lambda\end{vmatrix}=0\Rightarrow\begin{vmatrix}1&1\\1&\lambda\end{vmatrix}=0\Rightarrow\lambda-1=0$

71. (b)

72. (c)

73. (b) Binary Search Tree: value at every node is greater than every value in its left subtree and is lesser than every value in its right subtree.

74. (a)

75. (d) Method I: convert each digit of the octal number in 3-bit binary and then multiply
Method II: multiply both numbers as such, taking care that whenever product of its digits exceed 7 then reconvert them into base 8.

76. (d)

77. (b) Statements can be read as:
$n=5;\ x=n;\ n++;\ --x;\ y=x;$

78. (c)	**79. (a)**	**80. (c)**	**81. (a)**
82. (c)	**83. (c)**	**84. (c)**	**85. (d)**
86. (c)	**87. (d)**	**88. (d)**	

89. (a) $x.y'+x.z+x.y=x.(y'+y)+x.z$
$=x+x.z=x(1+z)=x$

90. (c)

91. (b) $\sigma^2=\frac{1}{n}\left[1^2+2^2+...+n^2\right]$

$$-\left[\frac{1}{n}(1+2+....+n)\right]^2$$

$$=\frac{1}{n}\left\{\frac{n(n+1)(2n+1)}{6}\right\}-\left(\frac{n+1}{2}\right)^2=\frac{n^2-1}{12}$$

92. (a) As, $P(x)=\dfrac{e^{-\lambda}\lambda^{x}}{x!}$

$\therefore \dfrac{e^{-\lambda}\lambda^{2}}{2!}=\dfrac{9e^{-\lambda}\lambda^{4}}{4!}+90\dfrac{e^{-\lambda}\lambda^{6}}{6!}$ (Given)

$\Rightarrow \lambda^{4}+3\lambda^{2}-4=0 \Rightarrow \lambda=\pm 1$

93. (a)

94. (c)

95. (b)

Column I, 14 + 12 = 26, 10 + 16 = 26
Column II, 9 + 7 = 16, 5 + 11 = 16
Column III, 4 + 2 = 6 , so O + ? =6 or ? = 6

96. (a)3 9 36 72 216 864 1728 5184

$\times 3$ $\times 4$ $\times 2$ $\times 3$ $\times 4$ $\times 2$ $\times 3$

So, number 3468 is wrong and should be replaced by 5184

97. (c)

895 870 821 740 619 450

$-(5)^{2}$ $-(7)^{2}$ $-(9)^{2}$ $-(11)^{2}$ $-(13)^{2}$

98. (c) $P(x)=\dfrac{e^{-m}m^{x}}{x!}$

Thus $P(0)+P(2)=e^{-m}+\dfrac{e^{-m}m^{2}}{2\,!}$

99. (a) p : Prob. of clearing a hurdle = $\dfrac{5}{6}$

q : Prob. of not clearing a hurdle = $\dfrac{1}{6}$

Since, knocks down a hurdle is same as not clearing a hurdle which is < 2, it means P (0 or 1 failure)
In other words, P (10 or 9 success)

Thus, $P(q \text{ or } 10)={}^{10}C_{9}\left(\dfrac{5}{6}\right)^{9}\left(\dfrac{1}{6}\right)+{}^{10}C_{10}\left(\dfrac{5}{6}\right)^{10}$

100. (c)