megaleap
Comprehensive MCA Entrance Coaching

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HINTS AND SOLUTIONS **TEST CODE: MTPO - IV**

1. (b) Let $x_1\ x_2, ..., x_{10}$ be the entries. By data,

$$\frac{x_1+x_2+------+x_5}{5}=\frac{2}{3}\ \frac{x_6+x_7+---x_{10}}{5}=k$$

$$\frac{x_1+x_2+------+x_{10}}{10}=\frac{2}{3}\ \frac{5k+\frac{15k}{2}}{10}$$

$$=\frac{25k}{20}\ \therefore \frac{25k}{20}=25 \text{ (given)} \Rightarrow k=20$$

2. (a) S.D. is independent of change of origin (but dependent on change of scale) $\therefore \sigma$

Alternate $\sigma^2=\dfrac{(a-x)^z+(b-x)^2+(c-x)^2+(d-x)^2}{4}$

$$\sigma^2{}_1=\frac{1}{4}\sum\left(a+k-\overline{x_1}\right)^2 \quad =\frac{1}{4}\sum(a-x)^2\ \left(\because \overline{x_1}=\bar{x}+k\right)$$

$$=\sigma^2 \Rightarrow \sigma_1=\sigma$$

3. (c) The roots are α, 2α (say) $\therefore 3\alpha=-k,\ 2\alpha^2=k+1$

Eliminating α, we get $2k^2=9(k+1)$

4. (d) The asymptotes intersect at the center of hyperbola. The point of intersection of $x+y=5,\ x-y=3$ is $(4, 1)$

5. (b) (n+1)! = (n+1) n! $\therefore$ L.C.M of n!, (n+1)! = (n+1)!

6. (b) Let the C.P. = x Rs. $\therefore$ S.P = $\dfrac{11x}{10}$ Rs.

Assumed C.P. (20% less than x Rs.) = $\dfrac{80}{100}x$ Rs. Assumed S.P. = $\left(\dfrac{11}{10}x+10\right)Rs.$ $\therefore$ Profit = S.P – C.P. = $\dfrac{3x}{10}+10$

$$\text{Profit \%}=\frac{\frac{3x}{10}+10}{\left(\frac{80}{100}x\right)}\times 100=40\Rightarrow x=500Rs.$$

7. (a) Let x, x+1 be the numbers P = $x(x+1)$, S = $x+x+1$

Now, P= $\dfrac{120}{100}S\Rightarrow 5P=6S$

$$\Rightarrow 5(x^2+x)=6(2x+1)\Rightarrow x=2,-3/5\Rightarrow x=2$$

$[\because x \text{ can't be non}-\text{integer}]$ $\therefore$ S = $2x$ + 1 = 2 (2) + 1 = 5

8. (c) $\left(2+\sqrt{3}\right)^3=2^3+3.2^2\sqrt{3}+3.2\left(\sqrt{3}\right)^2+\left(\sqrt{3}\right)^3$

$$=8+12\sqrt{3}+18+3\sqrt{3}=26+15\sqrt{3}$$

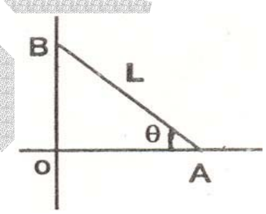
$$\therefore a+b\sqrt{3}=26+15\sqrt{3}\Rightarrow a=26,\ b=15$$

$$\therefore a+b=26+15=41$$

9. (d) I= $\displaystyle\int_3^4\frac{1}{x(x+1)}dx=\int_3^4\left(\frac{1}{x}-\frac{1}{(x+1)}\right)dx$ (by p.f.)

$$=\log(16/15)$$

10. (b)



Let $\angle OAB=\theta$ $\therefore$ OA=L $\cos\theta$, OB= L $\sin\theta$

$\Rightarrow$ A (L $\cos\theta$, 0), B (0, L $\sin\theta$)

$\therefore$ Midpoint M of AB = $\left(\dfrac{\angle\cos\theta}{2},\dfrac{\angle\sin\theta}{2}\right)$

$$\Rightarrow \text{OM}^2=\frac{L^2}{4}\left(\cos^2\theta+\sin^2\theta\right)\Rightarrow OM=\frac{L}{2}$$

11. (d) A way of defining 1-1 onto function from A to B is similar to a way of filling up 4 seats with 4 persons $\therefore$ No. of 1-1 onto functions = no. of ways of filling 4 places with 4 persons = ${}^4p_4$ = 24 ways

12. (c) Find highest power of 5 in 23! = $\left[\dfrac{23}{5}\right]$ = 4

13. (a) $\dfrac{10x}{100}+\dfrac{20}{100}x^2=21\ \therefore x=10$

Now, $\dfrac{20x}{100}+\dfrac{10}{100}x^2=2+10=12$

14. (b) $\dfrac{1}{n^2+n}=\dfrac{1}{n}-\dfrac{1}{n+1}$ (By partial fractions)

$$\therefore t_1=1-\frac{1}{2},t_2=\frac{1}{2}-\frac{1}{3},-----,t_n=\frac{1}{n}-\frac{1}{n+1}$$

$$\Rightarrow S=t_1+t_2+....=1-\frac{1}{n+1}\ n\to\infty\ \therefore S=1$$

15. (d) $\begin{vmatrix} 2 & a \\ a & 8 \end{vmatrix} = 0 \Rightarrow 16 - a^2 = 0$ $a^2 = 16$ $a = -4, 4$

16. (c) 4 lines in direction I
4 lines in direction II
For || gm. select 2 lines from I, 2 lines from II
$= {}^4c_2 \times {}^4c_2 = 36$

17. (a) $\vec{a}, -2\vec{b}$ are collinear $\Rightarrow \vec{a}, \vec{b}$ are also collinear and collinear vectors have same direction ratios.

18. (c) No. of single, double, three, four digit nos. formed by using 6 digits with repetition = $6^1+6^2+6^3+6^4 = 1554$

19. (d) By placing one coin in each pan of balance and other coin outside, we have : if the left or right pan lowers then the coin on the corresponding Pan is faulty. If pans are at same level then coin not put in pan is faulty. $\Rightarrow$ In one weighing we identify faulty coin out of 3. So, if three weighings are permitted we find faulty coin out of 3^3 i.e. 27 coins.

20. (d) Shaded area = Area of rectangle – area of Δ

$= 120 - \frac{1}{2} \times 12 \times 10 = 60$ sq. units

21. (d) $3\sin x + 4\cos x = r\cos\alpha \sin x + r\sin\alpha\cos x$
(assume $3 = r\cos\alpha,\ 4 = r\sin\alpha$)
$= r(\sin x + \alpha) \therefore -r \le r\sin(x+\alpha) \le r$

$-r \le 3\sin x + 4\cos x \le r$ and $r^2 = 3^2 + 4^2 = 5$
range = $(-5,\ 5)$

22. (d) p (α ,5- α) is the pt. on $x + y = 5$
p is equidistant from (0,0) and (6.6)
$\Rightarrow\ \alpha^2 + (5-\alpha)^2 = (\alpha-6)^2 + (5-\alpha-6)^2$
$\Rightarrow$ 37 = 35, not possible .

23. (a) Imaginary roots occur in pair
$\therefore$ roots are $3+i,\ 3-i,\ \alpha$ (say)
$6 + \alpha = -a,\ \left(9 - i^2\right)\alpha = -20 \Rightarrow \alpha = -2 \therefore a = -4$

24. (a) n = no. of subsets of A having 6 elements out of 11 =
${}^{11}c_6 = {}^{11}c_4 \frac{7}{5}$

m = no. of subsets of A having 4 elements of 11 = ${}^{11}c_4$
$\therefore \frac{n}{m} = \frac{7}{5} > 1 \Rightarrow n > m$

25. (a) A (m elements) , B (n elements) f: A $\to$ B
$\Rightarrow n^m = 1^4 = 1$

26. (b) $f(x) = x - [x]$ $f(n+h) = n + h - [n+h] = n + h - n = h$
where n is integer, $0 < h < 1$
$\therefore\ 0 < f(n+h) < 1 \Rightarrow f(x)$ is not constant
Also, $f(n+h) = f(n+1+h) = f(1+(n+h))$
$\Rightarrow f(x) = f(1+x) \Rightarrow f(x)$ is periodic of period 1.

27. (d) year is leap year $\Rightarrow$ 29 days in Feb. After 19 Feb. we have 10 more days in Feb.
$\therefore$ upto 28 June = 1 (for 19 Feb.) + 10 +31 days of March. + 30 days Apr. +31 days May +28 days June = 131 days = 18 weeks + 5 days (Friday)

28. (c)

Boxes	4	3 ↑
Packets	110	120 ↑
Biscuits	12	11 ↑
Time	12	X ↑

$4 \times 110 \times 12 \times 12 = 3 \times 120 \times 11 \times x$ $x = 16$ hrs.

29. **(b)**

30. (a) $I = \int_0^{\pi/2} \frac{\sin^3\theta - \cos^3\theta}{\sin^5\theta + \cos^5\theta} d\theta$

$= \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2}-\theta\right) - \cos^3\left(\frac{\pi}{2}-\theta\right)}{\sin^5\left(\frac{\pi}{2}-\theta\right) + \cos^5\left(\frac{\pi}{2}-\theta\right)}$

$= = -\int_0^{\pi/2} \frac{\sin^3\theta - \cos^3\theta}{\sin^5\theta + \cos^5\theta} = -I \Rightarrow I = 0$

31. (b)
$(1+\log_3 2)(1+\log_6 5) = (\log_3 3 + \log_3 2)(\log_6 6 + \log_6 5)$
$= \log_3 6 \,.\, \log_6 30 = \log_3 30 = \log_3 3 + \log_3 10 = 1 + \log_3 10$

32. (b)

33. (a) $1^2 + 2^2 + 3^2 + 4^2 + ... + (2n)^2 - 2^2 + 4^2 + .. + (2n)^2$
$= \frac{2n(2n+1)(4n+1)}{6} - \frac{2^2 n(n+1)(2n+1)}{6} = \frac{(2n+1)2n(2n-1)}{6}$
$= \underset{n\to\infty}{Lt} \frac{2n(2n-1)(2n+1)}{6n^3} = \frac{1}{3}\left(2 - \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) = \frac{4}{3}$

34. (c) $x|x| = \begin{Bmatrix} x^2, & x+ve \\ -x^2, & x-ve \end{Bmatrix}$
$\therefore \sum x|x| = -(25+16+9+4+1) + 0 + (25+16+9+4+1) = 0$

35. (d) $\sqrt{\frac{x-1}{x-2}} \ge 0$

if $\frac{x-1}{x-2} = \frac{(x-1)(x-2)}{(x-2)^2} > 0 \Rightarrow (x-1)\,(x-2) > 0$
$\Rightarrow\ x < 1,\ x > 2 \Rightarrow (-\infty, 1) \cup (2,\ \infty)$

36. (b) 300 votes divided equally among 5 contestants
$\Rightarrow$ each has 60 votes. To win by minimum votes one contestant has 1 more vote = 61 votes

37. (d) $(a-b)^2 \ge 0 \Rightarrow a^2 + b^2 \ge 2ab$
$\therefore a^2 + b^2 + c^2 + d^2 + p^2 + q^2 \ge 2(ab + cd + pq)$
$\Rightarrow\ \frac{3}{2} \ge ab + cd + pq$

38. (a) Points of intersection of graph of line $x = 1$, $x = 2$, $y = 3$ and $y = 5$ are A (1, 3), B (2, 3), C (1, 5), D (2, 5)
$\therefore$ Values of 2x+3y at A, B, C, D=11, 13, 17, 19 Then Bounds are end values 11, 19

39.

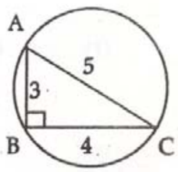
40. (c) $\sin x + \sin y = 2\sin\frac{x+y}{2}.\cos\frac{x-y}{2}$

$= \cos\frac{x-y}{2}$ ($\because x + y = 60°$) ≤ 1

41. (b) $|x-2| \leq 4 \Rightarrow -4 \leq x-2 \leq 4 \Rightarrow -2 \leq x \leq 6$

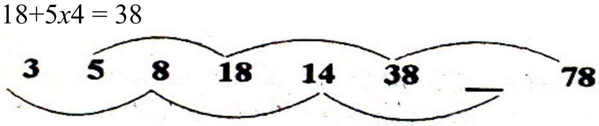
Also, $x^2 \leq 16 \Rightarrow -4 \leq x \leq 4$ $-2 \leq x \leq 4$

42. (c)



Circum circle of right Angled Δ = Circle on the hypotenuse of right. Angled Δ as diameter $\therefore$ r = 2.5

43. (b) odd sequence : 3
3+5x1 = 8
8+5x2 = 18
18+5x4 = 38



even sequence : 5
5+3x1 = 8
8+3x2 = 14
14+3x4 = 26

44. (a) $\alpha-\delta, \alpha, \alpha+\delta$ are roots $\therefore 3\alpha = 9 \Rightarrow \alpha = 3$

Hence $\alpha^2 - 9(3)^2 + a(3) + b = 0 \Rightarrow 3a + b = 54$

45. (d) $\frac{1}{2}+\frac{1}{3}+\frac{1}{N} < 1 \Rightarrow \frac{1}{N} < \frac{1}{6} \Rightarrow N > 6$, many natural nos. satisfying it.

46. (c) collinear if 1 + 2 + t = 0 $\Rightarrow$ t = –3

47. (d) G.C.D of 48, 72, 240 = 24
$\therefore x + 7 = 24 \Rightarrow x = 17$, not divisible by any of choices.

48. (d) α, β, φ be cube roots of unity

$\therefore\ \alpha+\beta+\varphi = 1+\omega+\omega^2 = 0$

$\alpha^5+\beta^5+\varphi^5 = 1+\omega^5+\omega^{10} = 0$

49. (b) $\frac{\delta u}{\delta x} - 2x\ f^{\ 1}\left(x^2+y^2\right), \frac{\delta u}{\delta y} = 2y\ f^{\ 1}\left(x^2+y^2\right)$

$x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 2\left(x^2+y^2\right)f^{\ 1}\left(x^2+y^2\right)$

50. (c) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...$

$\therefore \sin x + \cos x = 1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + ...$

51. (a)

52. (b) Telephone (electronic) mode of communication while Letter (non-electronic) mode of communication. Similarly, is phonograph (electronic), Manuscript (non electronic)

53. (c) Nose is a part of face and sticks out; Knob is a part of door and sticks out.

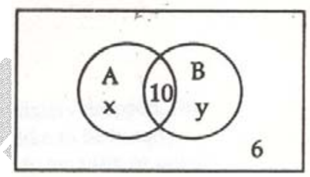
54. (b) Let perimeter of each fig. is 12 π units
$\therefore$ side of equilateral $\Delta = 4\ \pi$
side of square = 3 π and radius of circle = 6

$\therefore$ E $= \frac{4\sqrt{3}}{4}16\pi^2$, $S = 9\pi^2$, $C = 36\pi$

$\Rightarrow$ E : S : C = $\sqrt{3}\pi : 9\pi : 36$

Clearly, $4\sqrt{3}\pi < 9\pi < 36 \Rightarrow E < S < C$

55. (b)



Let Total = Z A $\Rightarrow x + 10$ B $\Rightarrow y + 10$

$z = x + 10 + y + 6 - (1).\ x + 10 = \frac{3}{2} - (2)\ \ y + 10 = \frac{2z}{3} - (3)$

$\Rightarrow x = 2,\ y = 6,\ z = 24$

56. (b) A vector along AB = $\overrightarrow{j}$ $\overrightarrow{OA} = \overrightarrow{ai}$

The vector eq. AB is $\overrightarrow{r} = \overrightarrow{ai} + \overrightarrow{tj}$

57. (c) III term = ${}^nc_2 a^{n-2}\ b^2\ x^2$ IV term = ${}^nc_3 a^{n-3} b^3 x^3$

${}^nc_2 a^{n-2} b^2 = {}^nc_3\ a^{n-3}\ b^3 \Rightarrow nb = 3a + 2b$

Directions (58 – 61): Explanation: Let us write the given conditions (i) to (iv) as follows: (i) D = A – 3 (ii) B is the middle term (iii) B – F = C – D > 0 (iv) G > F

Using (ii) we can write the position of B as --------- B ---------

Using (i) we infer that A>B. Otherwise there would not be any D such that D = A – 3. Further A cannot be the 7th number as in such a case A – 3 = B $\neq$ D contradicting (i) again.

_ _ _ BA _ _ (or) _ _ _ B _ A _ Using (i) we get the position of D also. $\underbrace{\text{\_D\_BA\_ \_}}_{\text{I say}}$ (or) $\underbrace{\text{\_ \_DB\_A\_}}_{\text{II say}}$

Using (iii) we infer the following in I and II:

(I) If C is in the 3rd position, then C – D = 1 but there would be no F such that B – F = 1.

If C is in 6th or 7th position then C – D = 4 or 5 again implying that there is no F such that B – F = 4 or 5. Obviously C cannot be in Ist position since C is greater than D. Thus, I is completely dropped.

(II) C must be in 5th or 7th position since C > D. If C is 7th, C – D = 4 but there would be no F such that B – F = 4. Therefore, C must be 5th and C – D = 2. $\Rightarrow$ F, must be 2nd as only then B – F = 2. We get __ FDBCA __. Using (iv) G must necessarily be the 7th and so E is the Ist.

Therefore, the numbers in increasing order are EFDBCAG

58. (d) **59. (a)** **60. (b)** **61. (c)**

62. (c) $9 < x^2 + y^2 < 25 \Rightarrow$ region between two concentric circle of radii 3, 5 units. **63. (a)**

64. (b) $\underset{n \to 2}{Lt} \dfrac{e^x - e^2}{x-2} = \underset{n \to 2}{Lt}\ e^x = e^2$

65. (c) $x^2 = 4 + 2\sqrt{3} = \left(1+\sqrt{3}\right)^2 \Rightarrow x = 1+\sqrt{3},\ \ x > 0$

$\therefore x^3 = \left(4+2\sqrt{3}\right)\left(1+\sqrt{3}\right) = 10 + 6\sqrt{3}$

66. (d) Total outcomes = 6^3 = 216
Non favourable events = (1, 1, 1), (6, 6, 6) = 2
$\therefore$ required probability = $\dfrac{214}{216} = \dfrac{107}{108}$

67. (b) **68. (c)** **69. (c)**

70. (d) Two horizontal arrows appear on different sides of vertical arrows in all other figures.

71. (a) **72. (c)** **73. (b)** **74. (b)**
75. (b) **76. (d)** **77. (c)** **78. (a)**
79. (b) **80. (c)** **81. (a)** **82. (b)**

83. (c) $\dfrac{\text{History}}{\text{Maths}} = \dfrac{60}{80} = \dfrac{3}{4}$ For Candidates C

84. (a) E highest in History , Economics , Sociology

85. (d) 340 (sociology)

86. (b) $\dfrac{80-75}{80} \times 100 = 6.25\%$ more

87. (a) average of A $\to$ 76 B $\to$ 57.8 C $\to$ 65.6, D $\to$ 58, 2, E $\to$ 69.4 $\therefore$ 76-57.8 = 18.2%

88. (d) Arrow moves clock wise 90°, circular moves diagonally.

89. (b) Outer block moves clock wise and then inner arrows interchanged .

90. (c) Figure moves clockwise and then semicircle is interchanged with adjacent triangle. **91. (c)**

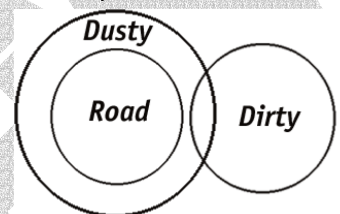
92. (a) I.F.= $e^{\int p dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

93. (a) $\left(D^2 - (2+3)D + 2 \times 3\right) y = 0 \ \ \left(D^2 - 5D + 6\right) y = 0$

94. (b) Let $x\,y = 1 \because \dfrac{x+y}{2} \ge \sqrt{xy} \Rightarrow x + y \ge 2$

95. (a) $1 + p + p^2 + ...$ converges if $|\,p\,| < 1 \Rightarrow |\,2x - 1\,| < 1$
$-1 < 2x - 1 < 1 \Rightarrow 0 < x < 1$

96. (b) $\log y = \lambda x \qquad \dfrac{1}{y}\dfrac{dy}{dx} = \lambda = \dfrac{\log y}{x} \Rightarrow x\left(\dfrac{dy}{dx}\right) = y(\log y)$



97. (d) **98. (d)** **99. (b)**

100. (a) x : car, y : motor cycle
$4x + 2y = 30 + 2\,(x + y)$
$2x = 30 \Rightarrow x = 15$