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megaleap
Comprehensive MCA Entrance Coaching

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M. C. A. ENTRANCE **SOLUTION (CODE: III)**

1. (a) Multiplying the three equation $(z+x)(x-y)(y-z)=\pm 40$ dividing each equation with the above equation we get the solution .

2. (c) Given equations can be written as
$(x+1)(y+1)=30,\ (y+1)(z+1)=24$
$(z+1)(x+1)=20$
Multiplying all these and then dividing each equation with that, we get the solution (as in problem 1)

3. (b) Simplify $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$

4. (c) **5.** (c) **6.** (a) **7.** (b)
8. (d)

9. (c) Computer $\sum \lambda_i$ *and* $\prod \lambda i$ and compare with choices

10. (a) From (2) and (3) $x-2y+z=0$. Using (1) & (4), we get $x+2z=7$. Now use choice

11. (c) $|A|=0$. Infinitely may solution

12. (d) $\Delta=\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix}=0 \Rightarrow \lambda=-5.$

13. (d) $\int \frac{dx}{1+\sin x}=\tan x-\sec x+c=-\frac{1-\sin x}{\cos x}+c$

$$=\frac{-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}+c=-\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}+c=\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+c$$

14. (c) $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$

$$=2\left[\left(\sqrt{2}\right)^6+{}^6C_2\left(\sqrt{2}\right)^4+{}^6C_4\left(\sqrt{2}\right)^2+{}^6C_6\right]$$

15. (b) Put $x=1 \therefore (1+1-3)^{2143}=(-1)^{2143}=-1$

16. (a) $\frac{1}{n!}\left[\frac{n!}{1!(n-1)!}+\frac{n!}{3!(n-3)!}+...\right]$

$$\frac{1}{n!}\left[{}^nC_1+{}^nC_3+{}^nC_5+...\right]=\frac{2^{n-1}}{n!}$$

17. (d) **18.** (d)

19. (c) On expansion of both terms, we get only even binomial coefficients $\therefore =5$

20. (a) $T_{2+1}={}^5C_2x^{5-2}\left(x^{\log_{10}x}\right)^2=10,00,000$

$x^3\left(x^{\log_{10}x}\right)^2=10^5$

the log on both sides, we get

$3\log_{10}x+2\left(\log_{10}x\right)^2=5$

$\left(\log_{10}x-1\right)\left(2\log_{10}x+5\right)=0$

$x=10, x=10^{-5/2}$

21. (d) Given $y^2=4ax$

$\therefore \frac{dy}{dx}=\frac{4a}{2y}=\frac{2a}{y}=\frac{2a}{2at_1}=\frac{1}{t_1}$ normal at $\left(at_1^2, 2at_1\right)$ is

$y+t_1x=2at_1+at_1^3$ it meet the parabola again at $\theta\left(at_2^2, 2at_2\right)$

$\therefore 2at_2+t_1at_2^2=2at_1+at_1^3$

$\left(t_2-t_1\right)\left(2a+at_1\left(t_2+t_1\right)\right)=0$

$\frac{2}{t_1}+t_1+t_2=0, \qquad t_2=-t_1-\frac{2}{t_1}$

22. (a) Here a = 3 and therefore the extremities are point (6, 3) & (-6, 3) and vertex is V (0, 0),

So Area $=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 3 & 1 \\ -6 & 3 & 1 \end{vmatrix}=18$ Sq. units

23. (c) V (2, 0) and Directrix is y –axis
Distance between vertex & Directrix =2 =a (say)

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Distance of focus from Y –axis = 2a = 4
Coordinates of focus = (4, 0)

24. (d) $2a = 6, 2b = 4, \therefore e^2 = 1 - \frac{b^2}{a^2} = \frac{5}{9}$

Distance between pins = 2ae = $2\sqrt{5}$ cm.

25. (c) Let $SP = r$ and $S'P = r'$

$\angle ISC = \alpha, \angle IS'C = \beta$ where $I(x_1, y_1)$ is the incenter of the $\Delta PSS'$ then

$$\tan\alpha = \frac{y_1}{x_1 + ae}, \tan\beta = \frac{y_1}{ae - x_1} \qquad (1)$$

Let 2S be perimeter of $\Delta PSS'$ then

$$r + r' + 2ae = 2S \Rightarrow S = a(1+e) \qquad (II)$$

from $\Delta SPS', we\, have$

$$\tan\alpha = \sqrt{\frac{(S-r)(S-2ae)}{S(S-r')}}$$

$$\tan\beta = \sqrt{\frac{(S-r')(S-2ae)}{S(s-r)}}$$

$$\therefore \tan\alpha \text{ and } \beta = \frac{S-2ae}{S} = \frac{1-e}{1+e} \qquad (III)$$

from (I) & (III),

$$\frac{y^2}{a^2e^2 - x_1^2} = \frac{1-e}{1+e} \quad \therefore \frac{x_1^2}{1+e} + \frac{y_1^2}{1-e} = \frac{a^2e^2}{1+e}.$$

The locus therefore is an ellipse whose eccentricity is

$$\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{(1+e)-(1-e)}{(1+e)}} = \sqrt{\frac{2e}{1+e}}$$

26. (d) **27.** (b) **28.** (a)

29. (c) The three vertices by solving in pairs are A(0, 6), B$(2\sqrt{3}, 0)$ and C $C(-2\sqrt{3}, 0)$. So ABC is an equilateral triangle. G is the centroid on median OA such that $OG = \frac{1}{3}OA = 2$. $\therefore centre$ is (0, 2) and radius $\sqrt{16}$.

30. (a) Centres of given circles are $C_1(4,-1), C_2(1,3),\ r_1 = 3,\ r_2 = r.$ We know that circles touch internally if $|r_1 - r_2| = C_1C_2$ and externally if $|r_1 + r_2| = C_1C_2$. Thus the two circles will cut at two distinct points if

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$r - 3 < 5 < r + 3 \Rightarrow r < 8\, and\, r > 8 \therefore 2 < r < 8.$

31. (c) If y =mx be tangent from origin to circle then $\frac{7m-(-1)}{\sqrt{m^2+1}} = \pm 5,$ which on squaring gives

$$12m^2 + 7m - 12 = 0 \therefore m_1m_2 = -1 \Rightarrow \theta = \frac{\pi}{2}.$$

32. (d) **33.** (c) **34.** (b) **35.** (d)

36. (a) $\frac{\partial M}{\partial y} = 2xy, \frac{\partial N}{\partial x} = 6xy^2 + 2xy - 6x^2$

$\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = 6.$ $\quad I.F. = e^{6y}$. After multiplication by I. F. given equation becomes exact and can be solved to give

$$e^{6y}\left[\frac{x^2y^2}{2} - \frac{x^3}{3} + \frac{y^2}{6} - \frac{y}{18} + \frac{1}{108}\right] = c$$

37. (a) The equation is homogenous therefore the I.F. is $\frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$. On multiplying the given equation by I. F. we get

$\left(\frac{1}{y} - \frac{2}{x}\right)dx + \left(\frac{3}{y} - \frac{x}{y^2}\right)dy = 0$ which on solving this

$$\frac{x}{y} + \log\frac{y^3}{x^2} = c$$

38. (c) $e^y\frac{dy}{dx} + e^ye^x = e^{2x}. Put\, e^y = V$

$$\Rightarrow e^y\frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dv}{dx} + ve^x = e^{2x}.I.F.e^{e^x}.$$

Solution is $ve^{e^x} = \int e^{2x}e^{e^x}dx\left(Put\ e^x = t\right).$

39. (a) **40.** (b)

41. (b) $$\frac{d}{dx}\Delta_1 = \begin{vmatrix} 1 & b & b \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & 1 & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2$$

42. (b) $$\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} = 2\left(1 + \sin^2\theta\right),$$

$0 \le \sin^2\theta \le 1 \therefore\ 2 \le 2\left(1+\sin^2\theta\right) \le 4, \therefore [2,4]$

43. (c)

44. (a) $\log\left(1+x+x^2\right) = \log\left(\frac{1-x^3}{1-x}\right)$

$$= \log\left(1-x^3\right) - \log(1-x)$$

$$= \left[-x^3 - \frac{x^6}{2} - \frac{x^9}{3} + ...\right] + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + ...\right]$$

If n is a multiple of 3, say $n = 3m, m \in Z$ then coefficient of

$$x^{3m} = \frac{-1}{m} + \frac{1}{3m} = \frac{-3}{n} + \frac{1}{n} = \frac{-2}{n}$$

45. (c) **46.** (a)

47. (d) It is an increasing function so the minima occurs at x =0 only.

48. (d) If $x = t \, then \, y = t^2/2$, Let the distance of $\left(t, t^2/2\right)$ from (0, 5) be D, then Z = $D^2 = t^2 + \left(t^2/2 - 5\right)^2$

Therefore $\frac{dZ}{dt} = 0 \Rightarrow t = 0, \pm 2\sqrt{2}$ and

$\frac{d^2Z}{dt^2} = \frac{1}{2}\left(3t^2 - 8\right) > 0 \; for \; t = \pm 2\sqrt{2}, \therefore$ point closest is $\left(\pm 2\sqrt{2}, 4\right)$.

49. (b) The given equation is $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, which is an ellipse and any point on it can be taken as $\left(a\cos\phi, 2\sin\phi\right)$

If d be distance from (0, -2) then

$z = d^2 = a^2\cos^2\phi + 4\left(1 + \sin\phi\right)^2$

$\frac{dz}{d\phi} = -2a^2\cos\phi\sin\phi + 8\left(1 + \sin\phi\right)\cos\phi$

$= \left(4 - a^2\right)\sin 2\phi + 8\cos\phi = 0 \Rightarrow$

$\cos\phi = 0 \, or \, \sin\phi = \frac{4}{a^2 - 4} = \frac{1}{\frac{a^2}{4} - 1} > 1$, by given condition and hence rejected.

$\therefore$ we choose cos $\phi = 0 \Rightarrow \phi = \pi/2$, so the points becomes (0, 2)

Also $\frac{d^2z}{d\phi^2} = 2\left(a^2 - 8\right)$ is negative as $4 < a^2 < 8$ .Hence $z = d^2$ is maximum.

50. (c)

51. (a) $\int x^{-3} 5^{1/x^2} dx = k 5^{1/x^2}$. Let

$\frac{1}{x^2} = t \Rightarrow -2x^{-3}dx = dt. \therefore I = -\frac{1}{2}\int 5^t dt$

52. (c) Given $f\left(x\right) = f\left(x\right) = 1 \Rightarrow f'\left(x\right)dx = dt$

$I = \int t dt = \frac{t^2}{2} + c = \frac{1}{2}\left[f\left(x\right)\right]^2 + c$

53. (a) Let $x + \log x = t \Rightarrow \frac{1}{x}\left(x + 1\right)dx = dt$

54. (a) $\int e^{2x}.e^{\log x}dx = \int e^{2x}.x dx$. Integrate by parts.

55. (d) $I = \int \log\left(x^2 + x\right)dx = \int \log x\left(x + 1\right)dx$

$= \int\left(\log x + \log\left(x + 1\right)\right)dx$

56. (a) $F_1\left(x\right) = x^2 - 5x + 6$

$F_2\left(x\right) = x^2, Equating \; F_1\left(x\right) \; and \; F_2\left(x\right) \Rightarrow x = 6/5$

57. (d) Function is odd

58. (d) $\int_0^1 \frac{dx}{\left[x\left(a-b\right)+b\right]^2} = \left[\frac{-1}{\left(a-b\right)\left[x\left(a-b\right)+b\right]}\right]_0^1 = \frac{1}{ab}$

59. (a) $I = \int_0^a f\left(x\right)g\left(x\right)dx = \int_0^a f\left(a-x\right)g\left(a-x\right)dx$

$= \int_0^a f\left(x\right)\left[2 - g\left(x\right)\right]dx = 2\int_0^a f\left(x\right)dx - 1$.

60. (a)

61. (c) Put x $= \sin\theta$ and use Gamma formula

62. (b) $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2 - r^2}} = \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1/n}{\sqrt{1 - \left(\frac{r}{n}\right)^2}}$

$\int_0^1 \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x\Big]_0^1 = \frac{\pi}{2}$

63. (b)

64. (c) $P\left(x \le 5\right) = \sum_{i=0}^{5}\frac{e^{-4}4^i}{i!} = e^{-4}\sum\frac{4^i}{i!}$

(because $\lambda = 200 \times \frac{2}{100} = 4$)

65. (c)

66. (b) $\frac{dy}{dx} = \frac{2}{\sqrt{a^2 - b^2}}.\frac{1}{1 + \frac{a-b}{a+b}\tan^2\frac{x}{2}} \times \frac{1}{2}\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}$

$= \left(a + b\cos x\right)^{-1}$ (after simplification)

$\frac{d^2y}{dx^2} = \frac{b\sin x}{\left(a + b\cos x\right)^2}$

67. (b) **68.** (a) **69.** (c) **70.** (d)

71. (d)

72. (a) ${}^{12}C_2\left(0.1\right)^2\left(0.9\right)^8 = 0.2841$

73. (a) $P\left(X = 0\right) = e^{-\lambda} = 0.10 \quad where \, \lambda \; is \; mean$

$e^{-\lambda} = 0.1 \Rightarrow -\lambda = \log\frac{1}{10}$ λ = log 10 = 2.3026

74. (b)

75. (b) $\sqrt{2}e^{-\lambda}.\lambda = e^{-\lambda}\frac{\lambda^2}{2} \Rightarrow \lambda = 2\sqrt{2}$

76. (b) **77.** (c)

78. (b) $\int \sec^{P-1}x.\sec x\tan x \, dx, let \sec x = t,$

then $I = \int t^{p-1}dt = \frac{t^p}{P} + c$.

79. (a) $\int\frac{dx}{\left(x^2+1\right)\left(x+1\right)}$ Using Partial fraction

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$$\frac{1}{2}\int\frac{1}{x+1}dx+\int\frac{\frac{-1}{2}x+\frac{1}{2}}{\left(x^2+1\right)}dx$$

80. (b) $\int\frac{e^x.e^x}{\left(e^x+1\right)^{1/4}}dx$

Let $e^x+1=t^4 \therefore e^x dx=4t^3dt$

$I=\int\frac{\left(t^4-1\right)}{t}4t^3dt$,

$4\int\left(t^6-t^2\right)dt=4\left(\frac{t^7}{7}-\frac{t^3}{3}\right)$

$\frac{4}{21}t^3\left(3t^4-7\right)=\frac{4}{21}\left(e^x+1\right)^{3/4}\left(3e^x-4\right)$

81. (d) **82.** (c) **83.** (a) **84.** (a)
85. (c) **86.** (b) **87.** (b) **88.** (d)
89. (d) **90.** (d)

91. (a) Let the cost price of article be Rs. x

$\therefore x-19=\frac{x}{20}\Rightarrow x=20$

Hence S.P. at 5% profit = $\frac{105\times 20}{100}=Rs.21$

92. (a) $\frac{9\times 9+5(3)^2}{3\times 6-12}=\frac{81+45}{18-12}=\frac{126}{6}=21$

93. (c) LCM $14\times$ HCF and LCM +HCF = 600
$\therefore$ 14 $\times$ HCF + HCF = 600
$\Rightarrow HCF=40$

Second number= $\frac{14\times 40\times 40}{280}=80$

94. (c) Capitals of A, B, C are 3x, 5x, 7x
After one year their capitals are
3x – 45600, 5x, 7x + 337600
The ratio of investment is
3x – 45600 : 50 : 7x +337600 :: 24: 56: 167

95. (d) $\frac{35}{100}\times\frac{9}{21}=0.15$

96. (c) A : B = 4: 5 and B : C = 2: 3
A = 800 $\Rightarrow$ B = 1000 $\Rightarrow$ C = 1500

97. (b) $\frac{6x+4y}{10}=11.20$ *and* $y=x-2$ where x = wage of skilled man
and y = wage of unskilled man

98. (c) A : B = 3: 4, B : C = 6 : 9
$\therefore$ B : C = 4: 6
As C : D = 12 : 19 = 6 : 19/2
$\therefore$ A : B : C : D = 3 : 4: 6: 19/2 = 6: 8 : 12 : 19

99. (d) $\frac{4\times x\times 1}{100}+\frac{y\times 4\times 1}{100}=230$

$\Rightarrow 4x+5y=23000$ (1)

and $\frac{x\times 5\times 1}{100}+\frac{y\times 4\times 1}{100}=220$

$\Rightarrow 5x+4y=22000$ (2)

solving we get x = 2000 and y = 3000.

Therefore $\frac{x}{y}=\frac{2}{3}$.

100. (a) Shaded area = area of Δ ABC
- 3 (area of one sector)

$=\frac{\sqrt{3}}{4}a^2-3\left(\frac{\pi r^2\theta^0}{360°}\right)$

Putting a = 2, r =1, $\theta=60°$

$=\frac{\sqrt{3}}{4}\times 4-3\times\frac{60}{360}\times\pi\times 1=\sqrt{3}-\frac{\pi}{2}$

$=1.732-\frac{3.1428}{2}=.16cm^2$