HINTS AND SOLUTIONS **TEST CODE: NIMCET – 107**

1. (b) $A=\frac{a+b}{2}$, $G=\sqrt{ab}$ and $H=\frac{2ab}{a+b}$

$\therefore\ A-G=\frac{a+b}{2}-\sqrt{ab}=\frac{(\sqrt{a}-\sqrt{b})^2}{2}\ge 0 \Rightarrow A\ge G$

$G-H=\sqrt{ab}-\frac{2ab}{a+b}=\sqrt{ab}\left(\frac{a+b-2\sqrt{ab}}{a+b}\right)$

$=\frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b}\ge 0\Rightarrow G\ge H$

Hence $A\ge G\ge H$

2. (b) Total no. of subsets of a given set with x elements = 2^x.

So, $2^6-2^3=64-8=56$

3. (b) $AB=A\Rightarrow (AB)A=A.A\Rightarrow A(BA)=A^2$

$\Rightarrow AB=A^2\ (\because BA=B)\Rightarrow A=A^2\ (\because AB=A)$

Again $BA=B\Rightarrow (BA)B=B^2\Rightarrow B(AB)=B^2$

$\Rightarrow BA=B^2\Rightarrow B=B^2$

4. (c) Injective means 1 - 1.

5. (b) $\frac{x^{7/2}.y^{3/2}}{x^{5/2}.y^{1/2}}=x^{\left(\frac{7}{2}-\frac{5}{2}\right)}.y^{\left(\frac{3}{2}-\frac{1}{2}\right)}=xy$

6. (b) **7. (a)** **8. (c)**

9. (c) $\sec\theta$ does not lie between -1 and 1

10. (a) $y=\sin^{-1}\left[\frac{1-x^2}{1+x^2}\right]=\frac{\pi}{2}-\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$=\frac{\pi}{2}-2\tan^{-1}x \qquad \Rightarrow \frac{dy}{dx}=\frac{-2}{1+x^2}$

11. (d)

12. (b) $P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{1}{4}$ (given)

$\Rightarrow P(A\cap B)=\frac{1}{4}P(B)$ ……(i)

and $P(B\mid A)=\frac{P(A\cap B)}{P(A)}=\frac{1}{2}$ (given)

$\Rightarrow P(A\cap B)=\frac{1}{2}P(A)=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ ……(ii)

From (i) and (ii) $\frac{1}{8}=\frac{1}{4}P(B)\Rightarrow P(B)=\frac{1}{2}$

Now, we have $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$, $P(A\cap B)=\frac{1}{8}$ **i.e.**

$P(A\cap B)=P(A)P(B)$ $\Rightarrow$ **events are independent**

13. (c) Result

14. (a) If M be the mean, the

$\frac{\sigma}{M}\times 100=5 \Rightarrow$ **M = 20σ =** $20\times 2=40$.

15. (c) $D=\sqrt{x^2+(y-c)^2}$ is shortest when

$E=x^2+(y-c)^2$ is shortest Now

$E=x^2+(y-c)^2=y+(y-c)^2=y^2+y-2cy+c^2$

$\frac{dE}{dy}=0\Rightarrow y=c-\frac{1}{2}$. Also $\frac{d^2E}{dy^2}=2>0$

$\therefore$ Distance D is minimum when $y=c-\frac{1}{2}$ and it is

$\sqrt{\left(c-\frac{1}{2}\right)+\left(c-\frac{1}{2}-c\right)^2}=\frac{\sqrt{4c-1}}{2}$

16. (d) Collinearity means dependence and hence determinant is zero.

17. (a) For only zero solution $\Delta\ne 0$

$$\begin{vmatrix} a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}\ne 0\Rightarrow (a-b)(b-c)(c-1)\ne 0$$

$\Rightarrow (a-ar)(ar-ar^2)(ar^2-a)\ne 0$

$\left(\because b = ar, c = ar^2; a, b, c \text{ in G.P.}\right)$

$\Rightarrow a^3 r(1-r)^3(1+r) \neq 0 \Rightarrow r \neq 0, 1, -1$

18. (c) A is a triangular matrix.

19. (d) Coefficient of $a^3.b^6.c^8.d^9$ e.f. = 0

$(\because 3+6+8+9+1+1 = 28 > 20)$

20. (b) $f(x) = x^2 + x + 1$

$= x^2 + \frac{1}{2}(2x) + \frac{1}{4} + \frac{3}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

Max. value of $f(x)$ in [–1, 1] is at $x = 1$.

$\Rightarrow$ Max. value = $\left(1 + \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3$

21. (c) **22. (c)**

23. (d) Req. area $= \int_{-1}^{1}\left(\frac{2}{1+x^2} - x^2\right)dx = \pi - \frac{2}{3}$

24. (b) -1

Now $A = \left[\frac{1.2.3......n}{n....n}\right]^{1/n}$

$\log A = \lim_{n\to\infty}\frac{1}{n}\left[\log\frac{1}{n} + \log\frac{2}{n} + ... + \log\frac{n}{n}\right]$

$= \lim_{n\to\infty}\sum r = 1\frac{1}{n}\log\left(\frac{r}{n}\right) = \int_0^1 \log x dx = \left[x\log x - x\right]_1^0$

25. (c) $\frac{2b^2}{a} = \frac{1}{2}.2b \Rightarrow a = 2b$

or $4b^2 = a^2 \Rightarrow 4a^2(1-e^2) = a^2 \Rightarrow e^2 = \frac{3}{4}$ or $e = \frac{\sqrt{3}}{2}$

26. (c)

27. (b) $\int \frac{\text{dx}}{\sqrt{\sin^3 \text{x}.\cos \text{x}}} = \int \frac{\sec^2 \text{xdx}}{\sqrt{\tan^3 \text{x}}} = 2\tan^{1/2} \text{x} + \text{c}$

28. (c) Let $I = \int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x}dx = \int_0^{\sqrt{3}/2}\frac{dt}{3+4t}$, putting $\sin x = \text{t}$

$= \frac{1}{4}\left[\log(3+4t)\right]_0^{\sqrt{3}/2} = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right) \quad \therefore k = \frac{1}{4}$

29. (b) **30. (a)** Done in class room.

31. (d) $\int_0^{\pi/2}\frac{dx}{1+\cos x} = \int_0^{\pi/2}\frac{1-\cos x}{\sin^2 x}dx$

$= \int_0^{\pi/2}\text{cosec}^2 x dx - \int_0^{\pi/2}\text{cosec} x \cot x dx = \infty$

32. (c) 1^∞, take log and use L'Hospital.

33. (a) $\overline{a} + \overline{b} + \overline{c} = \overline{0} \Rightarrow \overline{a} + \overline{b} = -\overline{c}$

$\Rightarrow (\overline{a} + \overline{b}) \times \overline{b} = -\overline{c} \times \overline{b} = \overline{b} \times \overline{c}$

$\Rightarrow \overline{a} \times \overline{b} = \overline{b} \times \overline{c}$ …... (i)

Similarly $\overline{a} + \overline{c} = -\overline{b} \Rightarrow \overline{c} \times \overline{a} = \overline{a} \times \overline{b}$ ..….(ii)

From (i) and (ii) $\overline{a} \times \overline{b} = \overline{b} \times \overline{c} = \overline{c} \times \overline{a}$

34. (d) $\overline{a} \times \overline{b}$ perpendicular to the plane π containing the vectors $\overline{a}$ and $\overline{b}$ and $\overline{a} + \lambda\overline{b}$ is a vector along the plane π.

$\therefore$ $\overline{a} \times \overline{b}$ can't be equal to $\vec{a} + \lambda\vec{b}$ for any value of λ.

35. (c) $y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

subtangent = $\frac{y}{dy/dx} = \frac{y^2}{2a} = 2x$

$\therefore$ subtangent : absicissa = 2 : 1

36. (d) Period = L.C.M. $\left(\frac{2\pi}{1/n!}, \frac{2\pi}{1/(n+1)!}\right)$

= L.C.M. $\left(2\pi n!, 2\pi(n+1)!\right) = 2\pi(n+1)!$

37. (c) because $a\sin x + b\cos x \leq \sqrt{a^2+b^2}$ $\therefore$

$c \leq \sqrt{a^2+b^2}$ (because $a\sin x + b\cos x = c$)

$\Rightarrow |c| > \sqrt{a^2+b^2}$ is not possible.

38. (a) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

39. (d) $f(x) = \frac{\sin 8x \log\cot x}{\cos 2x}$

$f\left(\frac{\pi}{2} - x\right) = \frac{\sin 8(\pi/2 - x)\log\tan x}{\cos 2(\pi/2 - x)}$

$= \frac{\sin(4\pi - 8x) - (\log\cot x)}{\cos(\pi - x)} = -f(x)$

$= \int_0^{\pi/2} f(x)dx = 0$

40. (a) $|x| = 3 \Rightarrow x = 3$ for $x \in N$

$|x| = 3 \Rightarrow x = \pm 3, x \in Z$

$x^2 + 2x + 1 = 0 \Rightarrow x = -1, -1 \notin N$

$x^2 = 5 \Rightarrow x = \pm\sqrt{5} \notin N$

41. (b) **42. (c)** **43. (b)** **44. (d)** **45. (a)** **46. (c)**

47. (c) Swapping is the process of bringing the part of program from main memory to backing store i.e. secondary memory & sending back to main memory when required. So

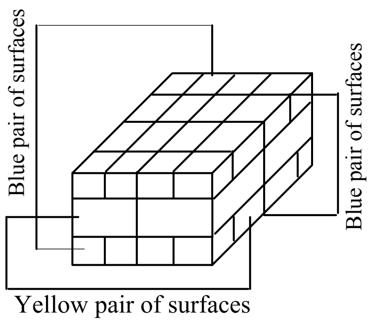
Swap space is the space on hard disk which will be occupied by the program coming from main memory.

48. (d) **49. (a)** **50. (d)** **51. (c)** **52. (b)**
53. (b) **54. (c)** **55. (a)**

56. (a) (i) 4 + 3 + 2 + 2 (5 + 5) = 29
(ii) 7 + 6 + 5 + 2 (4 + 4) = 34
(iii) 7 + 6 + 5 + 2 (3 + 3) = 18

57. (b) (2 + 7) (1 + 5) (2 + 7) (1 + 5) = 9696
(7 + 5) (3 + 4) (7 + 5) (3 + 4) = 127127
(6 + 8) (4 + 3) (6 + 8) (4 + 3) = 137137

Directions (Q. 58– 59): Figure -



58. (a) 4 bigger cubes only

59. (b) Cubes with two or more faces painted = cubes with three face painted + cubes with two face painted = 8 + 20.

60. (b) The inside element moves to diagonally opposite quadrant and out of two similar figures, the inner one is removed and the outer one is made dotted.

61. (d) The LHS figure rotates 90° ACW (Anti Clock Wise). The arrow gets detached from it and gets inverted while the remaining part of the figure gets laterally inverted.

62. (a) Let x min. be the time required for going to school from home. $\therefore$ Time required for returning home from school = x min.
Car leaves home by 4 p.m. – x min. and children be back home by 4p.m. + x min. On that day, children reached home 20 min. earlier.
= (4p.m. +x) – 20. $\therefore$ Total time for which car was in use = [(4p.m. +x) –20] – [4p.m. – x)] = 2 (x–10)
$\Rightarrow$ The car was used for upward and downward travel
= (x – 10) min. each. Let children be walking for y min.
$\therefore$ 3p.m. + y + (x –10) = (4p.m. + x) – 20 $\Rightarrow y$ = 50min.

Directions (Q. 63 – 64):
Order of sitting arrangement of 10 student is
E G F D J A H/C B I H/C
Exact position of C and H can't be determined. However, they will occupy either first or fourth place from the right end.

63. (d) **64. (c)**

Directions (Q. 65 – 66):
The given set of statements can be written in set notations as:
(i) $S \subset P$ (ii) $T \subset S$ (iii) $R \subset P \cap Q$

65. (d) $R \subset P \cap Q \subset P \Rightarrow R \subset P$ (From (III)
and $T \subset S,\ S \subset P \Rightarrow T \subset P$ (From (I) and (II))
It may be possible that $R \subset T$ therefore (I) is incorrect.
Now $T \subset P \Rightarrow$ (III) is true

66. (c) $Q \subset R$ and $R \subset P \cap Q \subset P \Rightarrow Q \subset P$

67. (d)

68. (d) $5000\left(1+\frac{r}{100}\right)^2 = 6050$

$\left(1+\frac{r}{100}\right)^2 = \frac{6050}{5000} = 1.21 \therefore r = 10\%.$

69. (d) Quantity of water
= $2000 \times \frac{5}{100} = 100\,ml$. Let x ml pure milk be added. Then
$(2000 + x)\frac{4}{100} = 100 \quad \therefore x = 500$ ml.

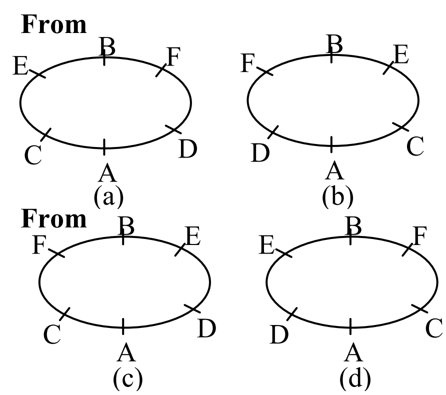
70. (c)

71. (a) Statement (a) + statement (b) gives conclusion I [$\because$ A +A =A]. Hence, conclusion I follows. Conclusion II does not follow because conversion of statement (a) gives the conclusion "Some ducks are parrots".

72. (a) Conclusion I can be obtained by conversion of statement (a). Hence, conclusion I follows. Conclusion II contradicts conclusion I. Hence, conclusion II does not follow.

73. (d) $\because$ A + I = no conclusion Hence, neither conclusion I nor conclusion II follows

74. (c)



75. (a) Required least number = (LCM of 13, 15, 19) - 11 = 3705 – 11 = 3694

76. (a) 17 lts milk and 3 lts water. Let x is water added
$\frac{17}{20+x} = 0.75 \Rightarrow x = 2\frac{2}{3}\,lts$

Directions (Q. 77 – 80):
D C E A B ➔ left
left ← G J I H F

77. (a) **78. (c)** B is the left most

79. (d) **80. (c)** **81. (a)**

82. (b) Let cost price of Rs. C

$$\therefore\ (C+50)\left(1+\frac{15}{100}\right)=1150$$

$$\Rightarrow\ C=\frac{1150\times 100}{115}-50=950$$

83. (d)

84. (a) $a^{\log_a x}=x$.

Therefore series is $1+\ldots+99=\dfrac{99\times 100}{2}$

85. (b) Area of given rectangle = $l \times w = lw$ cm^2
when length is made 3 cm longer,
Area of new rectangle = $(l + 3)w = lw+3w$
Hence increase in area = $(lw+3w)–lw = 3w$.

86. (b) B>R (i), L = B (ii), O < R (iii) L ≤ K (iv) from (ii) and (iv) B = L ≤ K ⇒ B ≤ K ⇒ I may be true but not necessary so. From (i) and (ii) L = B > R ⇒ L > R.

87. (d) A=J (i), C < D (ii), J ≥ G (iii) D > A (iv) from (i) and (iii) A = J ≥ G ⇒ A ≥ G ⇒ I may or may not be true. From (ii) and (iv) C < D > A ⇒ no definite relationship between C and A can be established.

88. (c) P ≥ M (i) Q = N (ii), P < M (iii) M = Q (iv) from (i), (iv), (iii) ⇒ P ≥ M = Q = N ⇒ either P > N or P = N.

89. (a) Z ≤ Y (i) K = L (ii), Y < X (iii) Z > K (iv) From (ii), (iv), (i) ⇒ L = K < Z ≤ Y ⇒ L < Y or Y > L.

90. (b) T < I (i), S = C (ii), S ≥ I (iii), C > O (Iv)(ii), (iii) ⇒ C = S ≥ I ⇒ C ≥ I. Hence I is not true. from (i) and (iii) ⇒ T < 1 ≤ S ⇒ T < S or S > T.

91. (c) r = 0.5 metres; circumference = 2πr metres
= 2π (0.5) = π metres ∴ no. of revolutions per km = 1000/π

92. (d) **93. (b)** **94. (b)** **95. (c)** **96. (d)**

97. (a) Let A and B together complete the work in 'x' days.

Then, $x=\dfrac{(x+6)(2x+3)}{4x+5}\Rightarrow x = 3$ hrs.

98. (d)

P(–)
|
Q(+) ⇔ K(–)
| |
(+)R – T(–)

(Clearly, 'P' is mother-in-law of 'K')

99. (b) In each step the upper two interchange places with lower two elements. The upper left element is replaced by a new one in the first two steps while the lower right is replaced by a new one in the next two steps.

100. (d) The whole figure rotates by 45° ACW in each step. One and two arcs forming petals are added in alternate steps. One arc is added on the ACW side.

101. (a) The half shaded squares move one step ACW and rotate by 90° ACW. A new square on the ACW side gets shaded and its shading is 90° ACW to its counterpart on the CW side.

102. (c) The CE element moves 2, 1½, 1, ½, 0 sides CW and order of the remaining element is reversed. A new element appears at the CW end.

103. (a) The weight may be in fraction as 91.3, 91.4 etc.

104. (c)	**105. (a)**	**106. (c)**	**107. (c)**
108. (c)	**109. (d)**	**110. (d)**	**111. (a)**
112. (b)	**113. (b)**	**114. (c)**	**115. (b)**
116. (b)	**117. (b)**	**118. (a)**	**119. (b)**
120. (c)			

Note: There may be some comprehension passage in English Section as in I.P. Mock. So it is advisable to practice reading comprehension from I.P. Mock also.