

## HINTS AND SOLUTIONS

TEST CODE: MTPO - I

1. (a) Let  $AB = 2x$ ,  $BC = 2y$ . Therefore sum  $S$  of the areas of semi circles =  $\frac{\pi \cdot (x^2) + \pi \cdot (y^2)}{2}$  But  $4x^2 + 4y^2 = AC^2 = 60$

$$\Rightarrow x^2 + y^2 = 15 \text{ Therefore } S = 7.5\pi \text{ cm}^2.$$

2. (b) Let Mr. X require  $x$  days and Mr. Y require  $y$  days to complete the job by working on it separately.

$$\text{Therefore } \frac{40}{x} + \frac{15}{y} = 1 \text{ and } \frac{25}{x} + \frac{25}{y} = 1$$

$$\Rightarrow \frac{40}{x} - \frac{25}{x} + \frac{15}{y} - \frac{25}{y} = 0 \Rightarrow \frac{15}{x} = \frac{10}{y} \text{ (i.e.) } \frac{y}{x} = \frac{2}{3}$$

3. (a) Let us suppose all the machines are making tablets of the same weight 4gms each. Then the total weight should be  $4(1+2+3+\dots+10) = 220$  gms since 1 tablet from the first machine, 2 tablets from the 2nd machine.... and 10 tablets from the 10th machine are picked [on the supposition that all the tablets are of the same weight]. But the total weight is given to be 224gms since one machine produces tablets of weight 5gms each. The difference is  $224 - 220 = 4$ gms. This means there are 4 tablets of weight 5 gms each among all the tablets picked. Therefore the 4th machine should be producing tablets of weight 5gms each.

4. (b) Note : The III dimension is height  $h$ .

Ram is able to paint  $2(h)$   $(a+b) = 2 \times 3 (6+4) = 60\text{m}^2$  in 2 days and Joseph is able to paint  $2(3) (5+6) = 66\text{m}^2$  in 3 days.

Therefore their efficiencies are in the ratio  $\frac{60}{2} : \frac{66}{3}$  (i.e.) 15:11.

5. (c) Let  $O$  be the centre of the small coin. Let  $B, D$  be the centers of the larger coins. Evidently angle  $BOD = 90^\circ$ . Let  $r, R$  be the radii of the smaller and the larger coins. From  $\triangle OBD$ ,  $OB = r + R$ ;  $OD = r + R$  and  $BD = R + R = 2R$ . Since  $\triangle OBD$  is a right angled triangle,  $4R^2 = 2(r+R)^2$  (i.e.)  $2R^2 = r^2 + 2Rr + R^2$  (i.e.)  $R^2 - 2Rr - r^2 = 0$  (i.e.)  $x^2 - 2x - 1 = 0$  where  $x = R/r$

$$\text{(i.e.) } (x-1)^2 = 2 \Rightarrow x = 1 + \sqrt{2} \Rightarrow \frac{R}{r} = \frac{1 + \sqrt{2}}{1} \text{ (i.e.) } r : R = 1 : 1 + \sqrt{2}$$

6. (d) Let there be  $m$  lines in  $S_1$  and  $n$  lines in  $S_2$ . The lines will intersect in  $mn$  points. Therefore  $mn = 12$ . Case (1)  $m = 1$ ,  $n = 12$  (or)  $m = 12$ ,  $n = 1$

Case (2)  $m = 6$ ,  $n = 2$  (or)  $m = 2$ ,  $n = 6$

Case (3)  $m = 4$ ,  $n = 3$  (or)  $m = 3$ ,  $n = 4$

In Case (1), the lines cannot form a parallelogram. In Case (2), the number of parallelograms that they form is  ${}^6C_2 \times {}^2C_2 = 15$ . In case (3), the number of parallelograms

that they can form is  ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

7. (d) The given equation is  $e^{2x} + e^x - 3 + e^{-x} + e^{-2x} = 0$  ... (1) by dividing the given equation by  $e^{2x}$ . Therefore (1) may be written as  $p^2 + \frac{1}{p^2} + p + \frac{1}{p} = 3$  where  $p = e^x$ . If  $p$  is positive

then  $p + \frac{1}{p}$  will be greater than 2. Similarly  $p^2 + \frac{1}{p^2}$  is

greater than 2. Therefore  $p^2 + \frac{1}{p^2} + p + \frac{1}{p} > 4$  and hence

$p^2 + \frac{1}{p^2} + p + \frac{1}{p}$  cannot become equal to 3. Therefore the

given equation has no solution.

8. (a) Let  $x$  kgs,  $y$  kgs and  $z$  kgs of the three varieties of A, B, C be mixed. Therefore  $60x + 70y + 80z = 75(x + y + z)$ . (i.e.)  $5z = 15x + 5y$  (i.e.)  $z = 3x + y$ . The equation  $z = 3x + y$  will be satisfied by many sets of values  $x, y, z$ . Of the given answers (a) is 1:1:4 and  $x = 1, y = 1, z = 4$  satisfy  $z = 3x + y$  Therefore one of the many possible ratios of mixing is 1:1:4.

9. (b) Let  $x$  boys and  $y$  girls be in the class. By data  $39x + 43y = 42(x + y)$ , therefore  $y = 3x$ . Number of boys: Number of girls =  $x : y = 1 : 3$ .

10. (a) Evidently  $DP = 14 - 9 = 5$  cms. From  $\triangle BPC$ ,  $BC^2 = 15^2 - 9^2 = 12^2 \Rightarrow BC = 12$  cms. From  $\triangle APD$ ,  $AP^2 = AD^2 + DP^2 = 12^2 - 5^2 = 169 \Rightarrow AP = 13$  cms. In  $\triangle ABP$ ,  $AP < AB < BP$ .  $\therefore \gamma < \beta < \alpha$  (i.e.)  $\alpha > \beta > \gamma$ .

11. (c) This problem is solved by using Bayes' theorem.

$$\text{Required Probability} = \frac{P(A \cap R)}{P(B \cap R) + P(A \cap R)}$$

$$= \frac{\frac{5}{16}}{\frac{5}{16} + \frac{2}{9}} = \frac{45}{45 + 32} = \frac{45}{77}$$

$$\therefore P(A \cap R) = 1/2 \times 5/8 = 5/16 \quad P(B \cap R) = 1/2 \times 4/9 = 2/9$$

12. (a)

13. (b)  $P(A \text{ winning}) =$

$$a + (1-a)(1-2a)a + (1-a)(1-2a)(1-a)(1-2a)a + \dots$$

$$= a(1+x+x^2+\dots) \text{ where } x = (1-a)(1-2a)$$

$$= \frac{a}{1-x} = \frac{a}{1-(1-a)(1-2a)} = \frac{a}{1-(1-a-2a+2a^2)}$$

$$= \frac{a}{3a-2a^2} = \frac{1}{3-2a}$$

By data  $P(A \text{ winning}) = 3/7$

$$\Rightarrow \frac{1}{3-2a} = \frac{3}{7} \Rightarrow 7 = 9 - 6a \Rightarrow a = \frac{1}{3}$$

14. (c) There are 5 positive numbers and 4 negative numbers. If we select positive numbers (or) 1 positive number and 2 negative numbers, the product will be positive. This can be done in  ${}^5C_3 + {}^5C_1 \times {}^4C_2 = 10 + 30 = 40$  ways.

15. (a) Let  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  be the matrix. The entries are to be 1

or zero. Therefore there are  $2 \times 2 \times 2 \times 2 = 16$  matrices of order  $2 \times 2$  whose elements are either 1 or zero. Let us find the number of non singular matrices. The matrix A will be non singular if  $ad - bc$  is not equal to zero. This will be possible if

**Case (1)**  $ad = 1$  and  $bc = 0$  (or) **Case (2)**  $ad = 0$  and  $bc = 1$ . Let us consider  $ad = 1$  and  $bc = 0$ . This is possible if  $a = 1, d = 1, b = 0, c = 0$ , or  $a = 1, d = 1, b = 1, c = 0$  or  $a = 1, d = 1, b = 0, c = 1$ . There are 3 possibilities by considering  $ad = 1$  and  $bc = 0$ . There are 3 more such possibilities for case (2)  $ad = 0, bc = 1$ . In all there will be six non-singular matrices and hence  $16 - 6 = 10$  singular matrices.

16. (d)

17. (d) The elements of R are such that 1)  $\underline{a}$  odd and  $\underline{b}$  even 2)  $\underline{a}$  even and  $\underline{b}$  odd 3)  $\underline{a}$  odd and  $\underline{b}$  odd. There will be  $3 \times 3 = 9$  entries in each category. Therefore there are 27 elements in R....

18. (d) This system of two linear equations will have many

$$\text{solutions if } \frac{2}{a} = \frac{3}{6} = \frac{a}{b} \quad (\text{i.e.}) \quad a = 4 \text{ and } \frac{a}{b} = \frac{1}{2}$$

$$\Rightarrow b = 2a = 8 \text{ Therefore } a + b = 12$$

19. (d) Let us find  $f_0 f(x)$ . By data,  $f_0 f(x) = 2f(x) + 3$

$$= 2(2x+3) + 3 = 4x + 9$$

$$\text{Therefore } f_0 f(x-1) = 4x - 4 + 9 = 4x + 5.$$

$$\text{By data } f_0 f(x-1) = 5 \Rightarrow 4x + 5 = 5 \Rightarrow x = 0$$

20. (d) Slope of the line joining (1, 2), (2, 3) =  $y'$  at that point.

21. (b)  $-3/2$  (Use differential calculus)

22. (d) If  $\theta$  were the central angle of a sector of a circle, then length of arc  $AB = l = r\theta$  ..... (1)

$$\text{and area OAB} = A = 1/2 l r \quad \dots\dots\dots (2)$$

$$\text{By data } l + 2r = 40 \quad \dots\dots\dots (3)$$

$$\text{Therefore } A = 1/2 (40 - 2r) r = 20r - r^2. \quad dA/dr = 20 - 2r \text{ and } d^2A/dr^2 = -2 < 0. \text{ Hence } dA/dr = 0 \Rightarrow 2r = 20 \Rightarrow r = 10.$$

$$\text{Therefore } A \text{ is a maximum if } r = 10. \text{ Therefore } l = 40 - 20$$

$$= 20 \text{ cms. Therefore } \theta = 20/10 = 2 \text{ radians}$$

$$23. (a) \text{ Simplify } y = 2 - \frac{1}{3x+2} \therefore y_3 = \frac{(-1)^3 3^3 3!}{(3x+2)^4}$$

$$24. (b) (abc)^{1/3} \left( l^\infty \text{ case} \right)$$

25. (d) Consider  $u = x^y$ . Therefore  $\log u = y \log x$

$$\therefore \frac{1}{u} \cdot \frac{\partial u}{\partial x} = \frac{y}{x} \Rightarrow \frac{\partial u}{\partial x} = \frac{u \cdot y}{x} \quad \& \quad \frac{1}{u} \cdot \frac{\partial u}{\partial y} = \log x \Rightarrow \frac{\partial u}{\partial y} = u \log x$$

$$\text{Hence } \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} \log x + \frac{u}{x} = \frac{u \cdot y}{x} \log x + \frac{u}{x}$$

$$\text{At } x = 1, y = k, \text{ we have } u = 1^k = 1$$

$$\therefore \frac{\partial^2 u}{\partial x \partial y} = \frac{1}{1} k \log(1) + \frac{1}{1} = 1 \text{ since } \log 1 = 0$$

26. (b) By problem  $e^{x+y} = x^2 \Rightarrow \log x^2 = 2 \log x$

$$\Rightarrow y = 2 \log x - x, \text{ Then integrate by parts.}$$

27. (a)  $4xy = x^4 + c$  (Linear differential equation)

28. (c) Let  $V_1$  be the volume of the cylindrical block without any cavity or hole inside it. Therefore  $V_1 = \pi (5)(5) 10 = 250\pi$ . Let  $V_2$  be the volume of the frustum. Its height = 3cms, bottom and top radii are 5cms & 3cms.

$$\text{Therefore } V_2 = \frac{\pi}{3} (3) (5^2 + (3) (5) + 3^2) = \pi (49). \text{ Let } V_3 \text{ be the}$$

$$\text{volume of the cylindrical bore. Therefore } V_3 = \pi (3)(3) 7$$

$$= 63\pi. \text{ The volume of the remaining material}$$

$$= 250\pi - (49 + 63)\pi = 138\pi \text{ cm}^3.$$

29. (d) We have

$$x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) = 0$$

$$\text{Consider } x^7 - 1 = 0 \text{ (i.e.) } x = 1^{1/7} = \cos\left(\frac{2k\pi}{7}\right) + i \sin\left(\frac{2k\pi}{7}\right)$$

where  $k = 0, 1, 2, 3, \dots, 6$ . Evidently  $x = 1^{1/7}$  will have 7 values of which  $x = 1$  is the only real value. The other 6 values are complex numbers. Therefore

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0 \text{ has no real solution.}$$

$$30. (d) \alpha + \beta + \gamma = -3 \Rightarrow f(\alpha + \beta + \gamma) = f(-3)$$

$$31. (c) U_n = \frac{3n+1}{n!} = \frac{3n}{n!} + \frac{1}{n!} = \frac{3}{(n-1)!} + \frac{1}{n!}$$

$$\therefore U_1 + U_2 + \dots = \left(\frac{3}{0!} + \frac{1}{1!}\right) + \left(\frac{3}{1!} + \frac{1}{2!}\right) + \dots$$

$$= 3\left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots\right) + \left(\frac{1}{1!} + \frac{1}{2!} + \dots\right) = 3e + e - 1$$

$$= 4e - 1 \left[ \text{since } e = \left[ \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots \right] \right]$$

32. (c) Since  $f(x) = x^2 + px + q = 0$  has exactly one root between  $x = 0$  and  $x = 1$  the values of  $f(0)$  and  $f(1)$  are of opposite signs.

$$\therefore f(0) \cdot f(1) \text{ is negative. (i.e.) } f(0) \cdot f(1) = q \cdot (1+p+q) < 0$$

since  $f(0)=q$  and  $f(1)=1+p+q$ .

**33. (c)** The values of 20% of 20 is 4 and the value of 50% of 20 is 10. Therefore 4 entries are increased by 3 each and 10 entries are increased by 8 each. The sum of the entries originally is  $20 \times 55 = 1100$ . The sum of the entries after the changes =  $1100 + 4 \times 3 + 10 \times 8 = 1192$ . The A.M. of new entries =  $1192/20 = 59.6$

**34. (b)**  $\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$  Here  $\sum x = 10 \times 5 + 15 \times 8 = 170$ .  $\sum x^2 = 10 \times 5^2 + 15 \times 8^2 = 250 + 960 = 1210$

$$\sigma^2 = \frac{1210}{25} - \left(\frac{170}{25}\right)^2 = \frac{4840}{100} - (6.8)^2 = 48.4 - 46.24 = 2.16$$

**35. (b)**  $U_n = \frac{n^2}{n!} = \frac{n(n-1) + n}{n!} = \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$

$$U_1 = \frac{1}{0!}, U_2 = \frac{1}{0!} + \frac{1}{1!}, U_3 = \frac{1}{1!} + \frac{1}{2!}$$

$$\sum U_n = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots = 2e$$

**36. (d)**  $r^2 = pr - q + 1$

**37. (a)** A is a singular matrix

**38. (a)**

**39. (a)** Let  $B = A - 3I$ . If  $\alpha, \beta, \gamma$  are the characteristic roots of A, by theorem, the characteristic roots of B will be  $\alpha-3, \beta-3, \gamma-3$ .

**40. (d)** Consider

$$2 \sin 20 \cos 20 \cos 40 \cos 80 = \sin 40 \cos 40 \cos 80$$

$$= \frac{1}{2} 2 \sin 40 \cos 40 \cos 80 = \frac{1}{2} \sin 80 \cos 80 = \frac{1}{4} 2 \sin 80 \cos 80$$

$$= \frac{\sin 160}{4} \text{ Therefore } \cos 20 \cos 40 \cos 80$$

$$= \frac{\sin 160}{8 \cdot \sin 20} = \frac{\sin(180-20)}{8 \cdot \sin 20} = \frac{1}{8} \text{ since } \sin(180-20) = \sin 20$$

**41. (d)** Evidently  $-1 < \cos x < 1$

Therefore  $\cos x + 2\cos 2x + 3\cos 3x + 4\cos 4x$  cannot become less than  $-(1+2+3+4)$  (i.e.)  $-10$ . Therefore  $\cos x + 2\cos 2x + 3\cos 3x + 4\cos 4x + 12 = 0$  has no solution.

**42. (b)**

**43. (b)** Let us consider  $at_1 + bt_2 + ct_3 = 0$

$$\text{(i.e.) } a(x^2 + 2) + b(2x - 6) + c(5) = 0$$

(i.e.)  $ax^2 + x(2b) + 2a - 6b + 5c = 0$ . The above equation should be true for all values of  $x$ . This will be possible if each of coefficient of  $x^2$ , coefficient of  $x$  and the constant term vanish.

(i.e.)  $a = 0, 2b = 0$  and  $2a - 6b + 5c = 0$ , (i.e.)  $a = 0, b = 0$  and  $c = 0$ . Therefore the polynomials  $f_1(x), f_2(x)$  and  $f_3(x)$  are linearly independent.

**44. (c)**  $[\vec{a} \times \vec{b} \vec{a} \vec{b}] = (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}|^2 > 0$ .

Therefore  $[\vec{a} \times \vec{b} \vec{a} \vec{b}]$  is positive unless  $\vec{a}$  is parallel to  $\vec{b}$ .

**45. (a)** By problem H is the ortho-centre of  $\Delta ABC$ . Therefore  $\vec{HA} \cdot \vec{HB} \cdot \vec{HC}$  will be perpendicular to  $\vec{BC}, \vec{CA}, \vec{AB}$  respectively.

Therefore  $\vec{HA} \cdot \vec{BC} = 0, \vec{HB} \cdot \vec{CA} = 0$  and  $\vec{HC} \cdot \vec{AB} = 0$ . Hence  $\vec{HA} \cdot \vec{BC} + \vec{HB} \cdot \vec{CA} + \vec{HC} \cdot \vec{AB} = 0$

**46. (b)** The normal to  $x + 2y + z = 8$  is  $\vec{i} + 2\vec{j} + \vec{k}$ . Required

$$\text{projection } \frac{(2\vec{i} + \vec{j} + \vec{k}) \cdot (\vec{i} + 2\vec{j} + \vec{k})}{|\vec{i} + 2\vec{j} + \vec{k}|} = \frac{5}{\sqrt{6}}$$

**47. (a)** The vertices of the tetrahedron are O (0,0,0), A (6,0,0), B (0,12,0) and C (0,0,12)

$\therefore$  The volume of the tetrahedron OABC

$$= \frac{1}{6} [\vec{OA} \cdot (\vec{OB} \times \vec{OC})] = \frac{1}{6} (6)(12)(12) = 144 \text{ cubic units.}$$

Since OA, OB, OC are at right angles.

**48. (a)** Evidently (15, 0, 0) is a point on  $2x+2y+z = 30$ .

Therefore the required distance = distance of (15, 0, 0) from

$$2x+2y+z+30=0 \text{ i.e. } \frac{30+30}{\sqrt{4+4+1}} = \frac{60}{3} = 20 \text{ units}$$

**49. (a)**  $2Z_1 = Z_1 + Z_2 + Z_1 - Z_2$

$$\therefore |2Z_1| < |Z_1 + Z_2| + |Z_1 - Z_2| \text{ (i.e.) } < 12 - 6 < 18$$

$$\therefore |Z_1| < 9 \text{ Again } 2Z_1 = Z_1 + Z_2 - (Z_2 - Z_1).$$

$$\text{Therefore } |2Z_1| > |Z_1 + Z_2| - |Z_2 - Z_1| \text{ (i.e.)}$$

$$|2Z_1| > 12 - 6 = 6 \text{ (i.e.) } |Z_1| > 3. \text{ Therefore } 3 < |Z_1| < 9.$$

**50. (d)** Evidently

$$i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = \cos \left( 2k\pi + \frac{\pi}{2} \right) + i \sin \left( 2k\pi + \frac{\pi}{2} \right)$$

$$\text{Therefore } \sqrt{i} = \cos \left( k\pi + \frac{\pi}{4} \right), k \text{ is an integer. When } k = 0,$$

$$\sqrt{i} = \cos \frac{\pi}{4} = \frac{(1+i)}{\sqrt{2}}.$$

**51. (d)**

**52. (d)** Let  $w = -5Z + 1$ . (i.e.)  $w - 1 = -5Z$ . Therefore  $|w - 1| = |-5Z| = 10$  since  $|Z| = 2$ . Therefore  $|w - 1| = 10$  represents a circle with its centre at (1, 0) and of radius 10 units.

**53. (c)**

**54. (a)** - 5050

$$\text{Let } S = 1^2 - 2^2 + 3^2 - 4^2 + \dots + 99^2 - 100^2$$

$$= (1-2)(1+2) + (3-4)(3+4) + \dots + (99-100)(99+100)$$

$$= (-1)(1+2+3+4+\dots+99+100) = (-1) \frac{(100) \cdot (101)}{2}$$

$$= -50(101) = -5050$$

**55. (b)** Area of the hexagon

$$\frac{1}{2} [PG \times AB + PH \times BC + PI \times CD + PJ \times DE + PK \times EF + PL \times FA]$$

$$= \frac{1}{2} a [PG + PH + PI + PJ + PK + PL], \text{ by data "a" is the side of the hexagon. Hence } PG + PH + PI + PJ + PK + PL$$

$$= \left( \frac{3\sqrt{3}}{2} \cdot a^2 \right) \left( \frac{2}{a} \right) = 3\sqrt{3}a$$

**56. (a)**

57. (b) We have  $U_n = 1 + 2 + \dots + n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$

$\therefore U_1 + U_2 + \dots + U_{10} = 1/2 (1^2 + 2^2 + \dots + 10^2 + 1 + 2 + \dots + 10)$

58. (b) ABC is given to be an equilateral triangle. The circle on BC as diameter will subtend  $90^\circ$  at D. Therefore CD is the altitude from C to AB. Therefore D will be the mid point of AB.

The area of  $\Delta BDC = \frac{1}{2} \Delta ABC = \frac{1}{2} \cdot \frac{\sqrt{3}}{4} \cdot (14)^2 = \frac{49\sqrt{3}}{2}$

Area of semicircle BDC  $= \frac{\pi}{2} r^2 = \frac{\pi}{2} (49) = \frac{49}{2} \cdot \frac{22}{7} = 77$

The shaded area  $= (77 - \left(\frac{49\sqrt{3}}{2}\right)) \text{cm}^2$ .

59. (a)

60. (a) Let  $\alpha, \beta, 3$  be the roots of the equation. Therefore  $\alpha + \beta + 3 = 5$  (i.e.)  $\alpha + \beta = 2 \dots (1)$ .

Again  $\alpha\beta(3) = b \dots (2)$  (i.e.)  $b = 3\alpha\beta = 3\alpha(2 - \alpha) = 6\alpha - 3\alpha^2$   
 $= -3(\alpha^2 - 2\alpha) = -3((\alpha - 1)^2 - 1) = 3(1 - (\alpha - 1)^2)$ .

The value of  $b$  will be greatest if  $\alpha = 1$  and the greatest value  $b = 3$ .

61. (c) have a common point of intersection

62. (b)  $6 \left(\frac{0}{0}\right)$  form) Apply L, Hospitals

63. (a) Let  $n$  be the number of vertices (sides) of the polygon. The sum of the internal angles of a polygon of  $n$  sides  $= (2n - 4)$  right angles  $= (14 - 4) \times 90^\circ = 900^\circ$

64. (a) Consider  $f_0(g(x)) = (x^2 + 4)^2 + (x^2 + 4) + 1$   
 Evidently  $x^2 + 4$  is positive and hence  $(x^2 + 4)^2 + (x^2 + 4) + 1$  cannot become zero for any real value of  $x$ .  $\therefore$  there is no real number  $x$  for which  $f_0(g(x)) = 0$  is possible.

65. (c) The equations  $x^2 + ax + b = 0$  and  $x^2 + bx + a = 0$  will have a common root  $\alpha$ . If  $\alpha^2 + a\alpha + b = 0$  &  $\alpha^2 + b\alpha + a = 0$  are true (i.e.) if  $(a - b)\alpha + (b - a) = 0$  (i.e.) if  $\alpha = 1$ . Since  $b$  and  $a$  are not equal, the common root is  $\alpha = 1$ . Now apply the L Hospital Rule.

66. (b)  $1 + 5 + 9 + \dots + x$  is an arithmetic progression

67. (b) Let angle LDC  $= x^\circ$  and angle LCD  $= y^\circ$  Therefore  $x + y = 180^\circ - 70^\circ = 110^\circ - (1)$ . Therefore, from  $\Delta ABC$ , angle A + angle B +  $y = 180^\circ - (2)$ . From  $\Delta DEF$ ,  $x + \text{angle E} + \text{angle F} = 180^\circ - (3)$  Hence sum of angles A, B, E and F  $= 360^\circ - (x + y) = 360^\circ - 110^\circ = 250^\circ$ .

68. (c) We have  $(b + c + a)(b + c - a) = kbc$   
 $\Rightarrow (b + c)^2 - a^2 = kbc \Rightarrow b^2 + c^2 - a^2 = (k - 2)bc$ .

In  $\Delta ABC$ ,  $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(K - 2)bc}{2bc} = \frac{K - 2}{2}$

Evidently  $-1 < \cos A < 1 \therefore -1 < \frac{K - 2}{2} < 1$

i.e.  $-2 < K - 2 < 2 \Rightarrow 0 < K < 4$

69. (b) Let  $x$  be the number of flowers that the priest brought. His actions are (1) offering some flowers to deity A (2) putting the remaining flowers in the pond and getting them doubled (3) offering some flowers to deity B (4) putting the remaining

flowers in the pond, getting doubled (5) offering some flowers to deity C.

Since he offers the same number of flowers (say  $y$ ) each time, the number of flowers that he had after the first, second, third, fourth and fifth actions are

(1)  $x - y$  (2)  $2x - 2y$  (3)  $2x - 3y$  (4)  $4x - 6y$  (5) zero

By data  $4x - 6y = y \Rightarrow 4x = 7y \Rightarrow x = 7y/4$

Since  $x, y$  are to be integers, the values of  $y$  are multiples of 4. Let  $y = 4$ . Therefore  $x = 7$  (or) a multiple of 7. The priest brought 7 flowers is the available answer. *Note: If the priest brought 14, 21 ... (or) a multiple of 7 flowers, he may be able to offer the flowers as explained in the problem.*

70. (d)  $U_n = \frac{(n+2)!}{n!(n+1)!} = \frac{(n+2)(n+1)n!}{n!(1+n+1)} = (n+1)$

$\therefore \sum U_n = \sum n + \sum 1 = \frac{(n)(n+1)}{2} + n = \frac{(n)(n+3)}{2}$

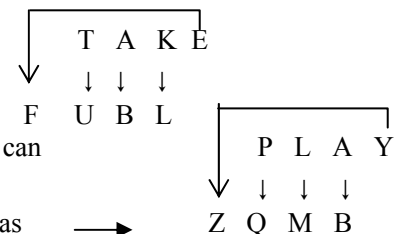
71. (a)

72. (d)

73. (d) Retirement is the end (or interruption) of service. Similarly, graduation is end (or interruption) of studies so, (d) is the answer. Other choices fail to satisfy this relationship.

74. (d) The relationship between 'Urge' and insist is one of degree. Actually, urge is a weaker form of insist. Choices (b) and (c) having no degree idea are eliminated. Relating (d) to the question pair, 'urge' and 'request' are ideas that provide a choice to act while 'insist' and 'demand' do not permit as much choice. Choice (a) also cannot show an exactly similar relationship.

75. (c) Relate the question pair as follows



The string 'PLAY' can

be related similarly as  $\rightarrow$

76. (d) We have a sequence of small letters followed by a sequence of corresponding capital letters. Further, the distinct letters are n, q, t, w, the intermediate letters op, rs, uv are skipped; the middle two letters q and t, repeated. Subsequently, we must skip two letters each time and allow for the middle two letters to repeat as follows:

$(xy) \leq (ab) \leq (cc) \leq (de) \leq (ff) \leq (gh) \leq (i)$  (i.e.)  $zccffi$  should be the next segment to be followed by the same capital letters

77. (c) Except Mercury all the others are solid metals

78. (c) Except D, all the other letters are formed by using straight lines only.

79. (b) In all the others, the fifth letter is a small letter while in (b) it is capital

**Question (80) to (83):**

80. (b) The digit 3 and the word TO are common to statements (i) and (iv). The digit 1 and FINISH are common to (iii) & (iv). Thus, 3 stands for TO and 1 for FINISH. So, 6 stands for BETTER.

**81. (d)** The digit 0 and TIME are common to (ii) and (iii). Already we have noted that 1 stands for FINISH. So, from (iii) we conclude 8 stands for ON.

**82. (d)** The digit 5 and STUDY are common to (i) and (ii). but, the words THIS and LESSON appear only in (i). Thus by elimination the codes 7 and 4 stand for THIS and LESSON in some order. So, we conclude that 7 or 4 is the code for THIS.

**83. (d)** All the four statements have been used to answer (80) to (82)

**84. (d)**

**85. (c)**

**86. (a)**

**87. (a)**

**88. (d)** The dots inside the figure reduce by half each time.

**89. (d)** The small ellipses inside the triangles are increasing by one each time.

**90. (c)** The number of small semi-circles are halved each time and the dots inside must remain as 8 with 3 dots on top and 1 dot below.

**91. (b)** An integer  $k$  is divisible by another integer  $m$  if  $k = mr$ , where  $r$  is an integer. So statement (1) implies  $k = 4r$  for some integer  $r$ . Statement (1) alone is insufficient because 12 is divisible by 4 ( $4 \times 3 = 12$ ) but 12 is not divisible by 8.

Statement (2) alone is sufficient. Statement (2) implies that  $k = 16r$  for some integer  $r$  but since  $16 = 8 \times 2$  that means  $k = 8 \times 2r$  and  $2r$  is an integer. So  $k$  is divisible by 8.

**92. (c)** Statement (2) alone is insufficient since you need to know what direction town B is from town C.

Statement (1) alone is insufficient, since you need to know how far it is from town B to town C.

Using both statements (1) and (2), A, B and C form a right triangle with legs of 9 miles and 12 miles. The distance from town A to town B is the hypotenuse of the triangle, so the distance from town A to town B is  $\sqrt{9^2 + 12^2} = 15$  miles.

**93. (c)** Statement (1) and (2) by themselves are insufficient since you need to know the area of the floor, and statement (1) only gives the length and statement (2) only gives the width. Using statement (1) and (2) together, the area of the floor is  $5 \times 10$  square feet. Since the area of each square is  $5^2 = 25$  square inches, each square has area  $25/144$  square feet. Therefore, the number of square is  $50 \div 25/144 = 288$ .

**94. (c)** To solve for two variables, you need two equations. Therefore statement (1) and (2) together are sufficient.

**95. (d)** if any value of  $a_n$  is an even integer, then all succeeding values are even. (Any even integer  $+ 2$  is an even integer). Since  $a_{10}$  appears after  $a_1$  and  $a_9$ , both statement (1) alone and statement (2) alone are sufficient.

**96. (a)** The distance between the point (2,4) and (5,10) is the square root of  $(2-5)^2 + (4-10)^2$  which is  $\sqrt{45}$ . So statement (1) alone is sufficient since  $\sqrt{45}$  is greater than 6. (The distance between any two points is less than or equal to the diameter which is  $2 \times \text{radius}$ ). Statement (2) alone is not sufficient since the distance between (2,4) and (4,1) is less than 6. So (a) is correct choice.

**97. (c)** Statement (1) reduces  $a^b$  to  $1^b = 1$ , and  $b^a = b$ .

Statement (2) then allows us to decide, since  $b > 2$  implies  $b > 1$ .

Statement (2) alone is not sufficient. If  $a = 1$  and  $b = 3$ , then  $a^b = 1$  is less than  $b^a = 3$ . However, if  $a = 3$  and

$b = 4$ , then  $a^b = 81$ , which is greater than  $b^a = 64$ .

**98. (b)** Statement (2) alone is obviously sufficient. Statement (1) is not sufficient since  $x = 2$  and  $y = 1$  and  $x = -2$  and  $y = -1$  both satisfy statement (1).

**99. (c)** Statement (1) is insufficient since any of the four numbers could be the largest.

Statement (2) alone is insufficient since  $z$  could be larger than 24 or it could be smaller than one of the numbers  $x$ ,  $y$  or  $w$ .

Statement (1) and statement (2) together are sufficient (1) implies that  $w + x + y + z = 100$  or  $z = 100 - w - x - y$ . Now using (2), we can see that  $100 - w - x - y$  is greater than  $100 - 24 - 24 - 24 = 28$ . So  $z$  must be the largest number.

**100. (c)** Statement (1) tells us only that  $x = 6 + y$ , so it is not sufficient. In the same way (2) alone will give only one of the unknowns in terms of the other. However, if we use both (1) and (2), we obtain a system of two equations which can be solved for  $x$  and  $y$ .