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Comprehensive MCA Entrance Coaching

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HINTS AND SOLUTIONS **TEST CODE: MTPO - II**

1. (b) $fog\ (x + n) = f\,[g\ (x + n)] = f\,[g\ (x)] = \text{fog}\ (x)$ (because g is of period n). Therefore period $\leq n$
[Result if $h\ (x + t) = h\ (x)$, then the period of $h\ (x) \leq t$]

2. (d) Let there be m lines in S_1 and n lines in S_2. The lines will intersect in mn points. Therefore $mn = 12$.
Case (1) $m = 1, n = 12$ (or) $m = 12, n = 1$
Case (2) $m = 6, n = 2$ (or) $n = 2, n = 6$
Case (3) $m = 4, n = 3$ (or) $m = 3, n = 4$.
In case (1), the lines cannot form a parallelogram. In case (2), the number of parallelograms that they form is ${}^6C_2 \times {}^2C_2 = 15$.
In case (3). the number of parallelograms that they can form is ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

3. (a) $T_{n+1} = {}^nC_r (x^2)^{n-r}\left(\frac{-1}{x}\right)^r$

A term independent of $x \Rightarrow 2n - 2r - r = 0$

$\Rightarrow \frac{2}{3}n = r$ For r to be integer, n should be a multiple of 3.

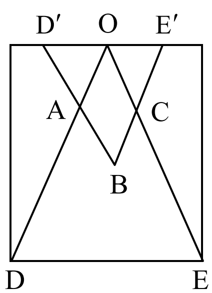
4. (d) Let $x + y + z = 2$ represents a plane and P (x, y, z) be any point on this. If O is (0,0,0) then $x^2 + y^2 + z^2$ is $(OP)^2$ which will be minimum if this is the $\perp$ distance of O from the plane

5. (b) $U_n = \frac{n^2}{n!} = \frac{n(n-1)+n}{n!} = \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$

$U_1 = \frac{1}{0!}, U_2 = \frac{1}{0!} + \frac{1}{1!}, U_3 = \frac{1}{1!} + \frac{1}{2!}$

$\sum U_n = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + ... = 2e$

6. (d)
Let cone (1) be ODE,



for which d=1, $l = \sqrt{\left(\frac{1}{2}\right)^2 + 1} = \frac{\sqrt{5}}{2}$

and surface area
$= \pi.\frac{\sqrt{5}}{2}.\frac{1}{2} = \frac{\sqrt{5}}{4}\pi$.

Let (2) be BD′ E′ for which, $d = \frac{1}{2}, l = \frac{\sqrt{5}}{4}\pi$ and surface area is $\pi\frac{\sqrt{5}}{4}\frac{1/2}{2} = \frac{\pi\sqrt{5}}{16}$. Therefore combined surface area $= \frac{\sqrt{5}}{4}\pi + \frac{\sqrt{5}}{16}\pi = \frac{5\sqrt{5}}{16}\pi$. We have to remove the surface area OABC which are two cones with base 1/4 and height1/4. Surface area of one of these cones is $\pi\frac{\sqrt{5}}{8}\frac{1}{8} = \frac{\sqrt{5}\pi}{64}$.

Therefore surface area of cavity $= \frac{5\sqrt{5}}{16}\pi - \frac{\sqrt{5}}{32}\pi = \frac{9\sqrt{5}}{32}\pi$

7. (c) $BA = I \Rightarrow A = B^{-1} \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{|B|}\begin{pmatrix} h & -f \\ -g & e \end{pmatrix}$

8. (b) Let x be the number of flowers that the priest brought. His actions are (1) offering some flowers to deity A (2) putting the remaining, flowers in the pond and getting them doubled (3) offering some flowers to deity B (4) putting the remaining flowers in the pond, getting doubled (5) offering some flowers to deity C. Since he offers the same number of flowers (say y) each time, the number of flowers that he had after the first, second, third, fourth and fifth action are
(1) $x - y$ (2) $2x - 2y$ (3) $2x - 3y$ (4) $4x - 6y$ (5) zero

By data $4x - 6y - y = 0 \Rightarrow x = \frac{7y}{4}$.

Since x, y are to be integers, the values of y are multiples of 4. Let $y = 4$. Therefore $x = 7$ (or) a multiple of 7. The priest brought 7 flowers is the available answer.
Note: If the priest brought 14, 21 .. (or) a multiple of 7 flowers, he may be able to offer the flowers as explained in the problem.

9. (b) We have $U_n = 1 + 2 + ... + n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$

$\therefore U_1 + U_2 + ... + U_n = \frac{1}{2}(1^2 + .. + 10^2 + 1 + 2 + ... + 10)$

10. (b) Since one root is common, we have $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$ (= k say). Now in a triangle

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = 0 \Rightarrow c = 90^0$

11. (b) $|z| = |z - 2 + 2i + 2 - 2i| \le |z - 2 + 2i| + |2 - 2i|$

$\le 1 + \sqrt{4+4}$

12. (c) Let $2x$ be the height of the cylinder and r is its radius, then $R^2 = x^2 + r^2 \Rightarrow r^2 = R^2 - x^2$.
Vol. of cylinder is $\pi r^2 h = 2\pi r^2 x = 2\pi R^2 x — 2\pi x^2$

$\dfrac{dV}{dx} = 0 \Rightarrow x^2 = \dfrac{R^2}{3}$ for which $\dfrac{d^2V}{dx^2} < 0$.

13. (a) It is a triangular matrix.

14. (b) $\int_{-1/2}^{0} (1+2y)dy + \int_{0}^{1/2} (1+2y)dy$

15. (a) $\int_0^b \pi x^2 dy = 2\pi \int (b^2 - y^2)\left(\dfrac{a^2}{b^2}\right)dy$

16.(d) The nos. in which 3 occurs only once = $^3C_1 \times 9 \times 9$.
The nos. in which 3 occurs twice $^3C_2 \times 9$
The nos. in which 3 occurs thrice =1
Therefore, total = $1 \times 243 + 2 \times 27 + 3 \times 1 = 300$

17. (d) Let z = x+iy. Therefore $z + \bar{z} = i(z - \bar{z})$

$\Rightarrow 2x = -2y$ i.e. x + y = 0, i.e. x and y are of opposite signs.

18. (a) Let x be the required number.
Therefore, x = (LCM of 2^5, 5^2, 7)+1 = $(2)^5 (5)^2 7 + 1$

19. (a) The vertices of the tetrahedron are O(0,0,0), A(6,0,0), B (0,12,0) and C (0,0,12), therefore the volume of the tetrahrdron $OABC = \dfrac{1}{6}\left[\overrightarrow{OA}.\left(\overrightarrow{OB} \times \overrightarrow{OBC}\right)\right] = \dfrac{1}{6}(6)(12)(12) = 144$ cubic units. Since OA, OB, OC are at right angles.

20. (b) Let us consider $at_1 + bt_2 + ct_3 = 0$
(i.e.) $a(x^2+2) + b(2x-6) + c(5) = 0$
(i.e.) $ax^2 + x(2b) + 2a - 6b + 5c = 0$.
The above equation should be true for all values of x. This will be possible if each of coefficient of x^2, coefficient of x and the constant term vanish.
(i.e.) a = 0 2b = 0 and 2a – 6b + 5c = 0, (i.e.) a = 0, b = 0 and c = 0. Therefore the polynomials $f_1(x)$, $f_2(x)$ and $f_3(x)$ are linearly independent.

21. (c) $(1+x)^{2n}$ contains five terms on expansion $\Rightarrow$ 2n+1 = 5
$\Rightarrow$ n=2, $\therefore$ the number of terms in $(1+x)^{4n}$ is 4(2)+1=9

22. (b) $\dfrac{ds}{dx} = -(2(x_1 - x) + 2(x_2 - x) + ... + 2(x_n - x)) = 0$

$\Rightarrow nx = x_1 + \ldots + x_n$ for which $\dfrac{d^2y}{dx^2} = 2n > 0$

23. (b) $y = a^x$ (1), $y = b^x$ (2) Therefore, $a^x = b^x \Rightarrow x = 0$, y=1;, therefore curves intersect at (0,1). From (1) $y' = y\log a$
From (2) $y' = y\log b$ Now use angle formula at (0,1).

24. (b) Divisible by 13 : $\dfrac{1000}{13} - \dfrac{300}{13} = 76 - 23 = 53$
Divisible by both 13 and 14

$\dfrac{1000}{182} - \dfrac{300}{182} = 5 - 1 = 4$. Hence divisible by 13 but not 14 = 53 – 4 = 49.

25. (b) $S = \sum_{x=1}^{10} f(x) = \sum_{x=1}^{10} x(x+1)(x+2)$

$= \sum_{x=1}^{10}(x^3 + 3x^2 + 2x) = \sum_{x=1}^{10} x^3 + \sum_{x=1}^{10} 3x^2 + \sum_{x=1}^{10} 2$

$= \left(\dfrac{10 \times 11}{2}\right)^2 + 3.\dfrac{(10)(11)(21)}{6} + 2\dfrac{(10)(11)}{2}$

26. (a) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the matrix. The entries are to be 1 or zero. Therefore there are 2×2×2×2=16 matrices of order 2×2 whose elements are either 1 or zero. Let us find the number of non singular matrices. The matrix A will be non singular, if ad-bc is not equal to zero. This will be possible if Case (1) ad=1 and bc=0 (or) Case (2) ad=0 and bc=l. Let us consider ad=1 and bc=0. This is possible if a=1, d=1, b=0, c=0, or ad=1, d=1, b=1, c=0 or a=1, d=1, b=0, c=1. There are 3 possibilities by considering ad=1 and bc=0. There are 3 more such possibilities for case(2) ad=0, bc=1. In all there will be six non-singular matrices and hence 16-6=10 singular matrices.

27. (c) Let b : boys and g : girls
boys' remark $\Rightarrow$ g = 2(b – 1)
girl's remark $\Rightarrow$ g – 1 = b $\Rightarrow$ g = 4, b = 3

28. (d) Consider 2sin20 cos20 cos40 cos80

= sin40 cos40 cos80 = $\dfrac{1}{2} 2\sin 40 \cos 40 \cos 80 = \dfrac{1}{2}\sin 80 \cos 80$

$= \dfrac{1}{4} 2\sin 80 \cos 80 = \dfrac{\sin 160}{4}$

Therefore cos20 cos40 cos80 $= \dfrac{\sin 160}{8.\sin 20} = \dfrac{\sin(180-20)}{8.\sin 20} = \dfrac{1}{8}$

since sin(180-20) = sin 20

29. (a) $T_{r+1} = {}^nC_r\left(\dfrac{3}{2}x^2\right)^{r-1}\left(\dfrac{-1}{3x}\right)^2$. A term independent of $x \Rightarrow 2n - 2r - r = 0 \Rightarrow \dfrac{2}{3}n = r$. For r to be integer, n should be a multiple of 3.

30. (c) $|z + iz| = 8 \Rightarrow |z| = 4\sqrt{2} \Rightarrow |z^2| = 32$

31. (c) There are 5 positive numbers and 4 negative numbers. If we select positive numbers (or) 1 positive number and 2 negative numbers, the product will be positive. This can be done in $^5C_3 + {}^5C_1 \times {}^4C_2$ = l0 + 30 = 40 ways

32. (c) Working rule for findings the nature of intersection of three planes $a_r x + b_r y + c_r z + d_r = 0 \quad r = 1, 2, 3,....$
Step–1 Write the coefficient matrix

$A = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix}$

Step–2 : Omit the fourth coloumn to get the determinant

$\Delta_4 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Now if $\Delta_4 \neq 0$ the planes intersect in a point. Step–3. But if $\Delta_1 = 0$ (Δ_i is obtained by omitting ith column from A) then find $\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$ and if $\Delta_3 \neq 0$ the planes form a triangular prism. If $\Delta_3 = 0$, the planes intersect in a line.

33. (c) Let $lx + my + n = 0$ be the equation of the line. l, m, n being real.

$\frac{l}{2}(z+\bar{z})+\frac{m}{2i}(z-\bar{z})+n=0$ (take $z = x+iy$)

$\Rightarrow \left(\frac{l-\text{mi}}{2}\right)\text{z}+\left(\frac{l+\text{mi}}{2}\right)\bar{\text{z}}+\text{n}=0$

34. (b) Area of the hexagon

$\frac{1}{2}[\text{PG}\times\text{AB}+\text{PH}\times\text{BC}+\text{PI}\times\text{CD}+\text{PJ}\times\text{DE}+\text{PK}\times\text{EF}+\text{PL}\times\text{FA}]$

$\frac{1}{2}a\left[\text{PG}+\text{PH}+\text{PI}+\text{PJ}+\text{PK}+\text{PL}\right]$, by data "$a$" is the side of the hexagon. Hence PG+PH+PI+PJ+PK+PL

$=\left(\frac{3\sqrt{3}}{2}a^2\right)\left(\frac{2}{a}\right)=3\sqrt{3}a$ **35. (d)** **36. (a)** **37. (b)**

38. (d) (11001) change the binary series to decimal and find the logic.

39. (a) Invertenate means habit that have existed for a long time and are likely to continue.

40. (a) Abysmal means massive or immeasurable.

41. (c) To ruminate is to contemplate.

42. (b) Frible with reference to land means that which is easily broken.

43. (c) Mandlin is to be sentimental or full of pity

44. (d) At time t = 0, p is at (0, 0) and Q is at (0, –50). At time t = t hrs, P will be at (50t, 0) and Q might have covered a distance of 25t towards 0. Therefore Q is at (0, –50 + 25t).

Therefore, $x = 50t$, $y = -50 + 25t$

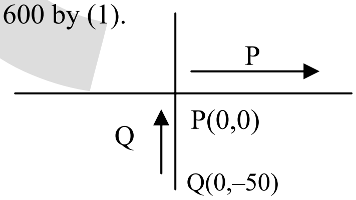
$PQ^2 = (50t)^2 + (25t - 50)^2$ (1)

$= 25^2 (4t^2 + t^2 - 4t + 4)$.

Therefore, PQ is minimum when $10t = 4 \Rightarrow t=2/5$.

Therefore, $PQ^2 = 400 + 1600$ by (1).

Therefore $PQ = 20\sqrt{5}$



45. (b) The equation of the transverse axis of $xy = 16$ is $x = y$. Therefore the ends of the transverse axis are at B(– 4, – 4) and A(4, 4). Hence $AB = \sqrt{128} = 8\sqrt{2}$. In the case of rectangular hyperbola, length of conjugate and transverse axis is the same.

46. (d)

Sides chosen	**No. of pts. chosen**	**No. of As**
AB, BC, CA	1 on each	${}^4C_1\times{}^4C_1\times{}^4C_1$
AB, BC	2 on AB, 1 on BC	${}^4C_2\times{}^4C_1=24$
AB, BC	1 on AB, 2 on BC	${}^4C_1\times{}^4C_2=24$
AC, BC	2 on AC, 1 on BC	24
AC, BC	1 on AC, 2 on BC	24
AB, AC	2 on AB 1 on AC	24
AB, AC	1 on AB, 2 on AC	24
Total 208		

47. (a)

48. (a) $629 \equiv -1 \bmod 21$ Therefore $(629)^{24} \equiv (-1)^{24} \bmod 21$

49. (c) Let us find the no. of squares of different dimensions in a chess board. They are 8 in number and their dimensions are 1×l, 2×2,… 8×8 (1). Let us find the rectangles whose sides are m,n (m ≠ n). Then we have to select two out of the numbers (1, 2, 3, ..., 8), as sides of the rectangle without allowing repetition. Therefore, there will be ${}^8C_2$ = 28 (2). Therefore, total is 8 + 28 = 36.

50. (c) $\sin^2 72 = \sin^2(90-18) = \cos^2 18$

Therefore $\sin^2\theta = 2-\left(\sin^2 18+\cos^2 18\right)=1$

51. (b) $O(A \cap B) = 2n \Rightarrow$ 1)$n^2 \geq 2n$ and 2) $6n \geq 2n$ Evidently $6n \geq 2$. But $n^2 \geq 2n$ for n(n-2) ≥ 0, i.e. n ≥ 2. Min. value of n is 2.

52. (c) The number (2n–1) appears (2n–1) times. Call this the nth block. Therefore number of numbers in the first n blocks is $1+3+5+...+(2n-1)=\frac{n}{2}\left[2+(n-1)2\right]=n^2$

$14^2 \leq n^2 < 15^2$. (for n=14, 2n-1=27) So the last 27 occurs in 196 position. After that 29 occurs (29 times)

53. (b) $x = \log_a bc, y = \log_b ca, z = \log_c ab$

$\sum\frac{1}{x+1}=\sum\frac{1}{\log_a bc+\log_a a}=\frac{1}{\log_a abc}$

54. (c) $\frac{\sqrt{3}\cos 20-\sin 20}{\sin 20\cos 20}=\frac{2\left[\sqrt{3}/2\cos 20-1/2\sin 20\right]}{1/2\sin 40}$

$=\frac{2[\sin 60\cos 20-\cos 60\sin 20]}{\frac{1}{2}\sin 40}=\frac{4\sin(60-20)}{\sin 40}=4$

55. (b) $\frac{1+i}{1-i}=i$

56. (c) $(1+x^2)^5(1+x)^4$

$=\left({}^5C_0+{}^5C_1x^2+{}^5C_2x^4+{}^5C_3x^6\right)\times\left({}^4C_0+{}^4C_1x+{}^4C_2x^2+....\right)$

Therefore, coeff. of $x^5 = {}^5C_1{}^4C_3 + {}^5C_2{}^4C_1$ **57. (b)** Cram this

58. (d) If θ were the central angle of a sector of a circle, then

Iength of arc AB = l = rθ … (1) and area OAB=A=1/2 l.r . (2)
By data l+2r = 40 …(3)
Therefore A=1/2 (40–2r)r = 20r – r^2. dA/dr = 20-2r and $d^2A/dr^2 = -2 < 0$. Hence dA/dr = 0 ⇒ 2r = 20 ⇒ r = 10.
Therefore A is maximum if r=10. Therefore l = 40 – 20 = 20cms. Therefore θ = 20/10 = 2 radians.

For Q59-62, use the following grid

H	C	A
B	G	E
F	I	D

59. (c) **60. (a)** **61. (a)** **62. (b)**

63. (b) One, Two, Three Four, Five, Six, Seven, Eight, Nine, Ten

64. (c)

9	4	2	2	5	6
↓	↓	↓	↓	↓	↓
6^{th}	3^{rd}	5^{th}	2^{nd}	4^{th}	1^{st}

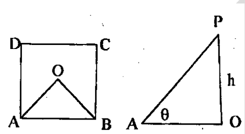
65. (c) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}(\text{given}) \Rightarrow r = \frac{1}{2\sqrt{\pi}}$

66. (d) A boolean algebra can be defined only if n is a power of 2 (i.e.) $n = 2^k$ for some $k \in N \Rightarrow n=8$.

67. (c) The no. of shake hands among n persons is
$^nC_2 = \frac{n(n-1)}{2} = 66 \Rightarrow n = 12$

68. (a) $OA = h\cot\alpha, OA^2 + OB^2 = a^2$

$2h^2\cot^2\alpha = a^2 \Rightarrow \tan^2\alpha = \frac{2h^2}{a^2}$



69. (b) The lateral surface area of cone = $\pi r l = \pi(5)(13) = 65\pi$
The surface area of the solid considered = surface area of the top of the cylinder + lateral surface area of the cylinder + lateral surface are of the cone = $\pi(25) + 2\pi(5)(20) + \pi(5)(13) = 290\pi\ cm^2$.

70. (a) $f'(x) = 0 \Rightarrow \frac{\cos x}{x} = 0 \Rightarrow \cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2},\ n \in I$ Also $f''(x) = \frac{-1}{x}\sin x - \frac{1}{x^2}\cos x$

$\therefore\ f''[(2x+1)\pi/2] = \frac{-2}{(2x+1)\pi}\sin\left((2x+1)\frac{\pi}{2}\right)$

Which is positive when 'x' is odd.

71. (c) $a^2 + b^2 + c^2 = 0$ is possible only when $a = b = c = 0$
$\Rightarrow a^n + b^n + c^n = 0$

72. (d) Since the box is rectangular, there are 4 sides which occur in 2 different pairs. Therefore, you can't determine the area of every side using statement (1) and (2)

73. (d) The number who subscribed to atleast one magazine is the sum of the numbers who subscribed to exactly one, two and three magazines. So 38 = N1 + N2 + N3, where N1, N2 and N3 are the number who subscribed to 1,2 and 3 magazines respectively. We need to find N2. Statement (1) is not sufficient since it tells the value of N3, but N1 and N2 are still both unknown. Even if we also use statement (1), we cannot find N2 since we have no information about the number of subscribers to magazines B and C.

74. (d) Statement (1) is insufficient since 9 (which is odd) and 6 (which is even) are both divisible by 3. Statement (2) is also insufficient since 81 is odd and 36 is even. 81 and 36 are also both divisible by 3, so (1) and (2) together are still insufficient.

75. (d) Statement (1) would be sufficient if there were information about the width of each book. Since statement (2) only gives information about the weight of each book, both statements together are not sufficient.

76. (d) If a=3, b=2, and c=1, then statements (1) and (2) are both true, but if a=2, b=3, and c=1, statements (1) and (2) are still true.

77. (d) In all the other the arrows are pointing in the same directions.

78. (d) There are only three lines in the other figures.

79. (c) The position of the small ring is on the base of the triangle except in (c).

80. (d) Retirement is the end (or interruption) of service. Similarly, graduation is end (or interruption) of studies.

81. (d) The relationship between 'Urge' and 'insist' is one of degree. Actually, urge is a weaker from of insist. Choices (b) and (c) having no degree idea are eliminated. Relating (d) to the question pair, 'urge' and 'request' are ideas that provide a choice to act while 'insist' and 'demand' do not permit as much choice. Choice (a) also cannot show an exactly similar relationship.

82. (c) Relate the question pair as follows

T	A	K	E
↓	↓	↓	
U	B	L	
F			

The string "PLAY" can be related similar as

P	L	A	Y
↓	↓	↓	
Q	M	B	
Z			

83. (c) Except Mercury all the others are solid metals.

84. (d) Except D, all the other letters are formed by using straight lines only.

85. (b) In all the others, the fifth letter is a small letter while in (b) it is capital. **86. (d)** **87. (d)** **88. (b)** **89. (c)**

90. (b) Here the rule followed is (use these for Q86-90)
From input to step I : Add $1^2, 2^2, 3^2$, … to the corresponding no of the input.
Step I to Step II : Arrange numbers in ascending order.
Step II to Step III : Square each number and subtract 4 from it.
Step Ill to Step IV : Subtract 20 from each number.
Step IV to Step V : Interchange the reverse order of second half numbers with the first half number.

87. Add 20 in step IV; This way third step can be determined. Now add 4 to each no. of Step III and find square root of them; this way step II can be determined. But from step II, step I can't be determined.