SANMACS INDIA

megaleap
Comprehensive MCA Entrance Coaching

309 & 313 PRATAP CHAMBERS, GURUDWARA ROAD, KAROL BAGH, DELHI - 110005 PH.: 28757911, 28757630, 32917966

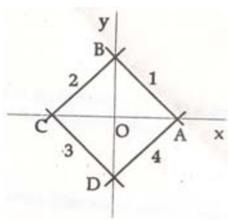
SAI SANMACS PLAZA, PLOT NO. - 6A, COMMUNITY CENTER, DDA, SECTOR - 8, ROHINI, DELHI - 110085 PH.: 32458354, 32522575

e-mail : info@sanmacs.com website: www.sanmacs.com

HINTS AND SOLUTIONS **TEST CODE: MTPO - III**

1. (a) Total out comes =36
Doublets = 6 i.e. (1, 1), (2, 2), --, (6, 6)
$\therefore$ No. of cases where one die > other die = 30
$\Rightarrow$ Prob. = $\frac{30}{36} = \frac{5}{6}$

2. (d) $|x| + |y| = 1$ has possible equations
(i) $x + y = 1$, $x > 0$, $y > 0$ (ii) $-x + y = 1$, $x < o$, $y > o$
(iii) $-x - y = 1$, $x < 0$, $y < 0$ (iv) $x - y = 1$, $x > 0$, $y < 0$
Area enclosed by these lines is



$\therefore$ A = 1/2 BD*AC = 2 sq. units

3. (b) Let's suppose first digit ='2'. So, remaining two digits on addition gives 4. Possibilities of those two digits are (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)
$\Rightarrow$ 5 possible numbers starting with '2' ----- (1)
Similarly, 5 numbers with '2' at second place ---- (2)
5 numbers with '2' at last place ------ (3)
$\therefore$ Total number of trials = $3 \times 5 = 15$

4. (b) Equation of tangent to $y^2 = 16x$ is $m^2 x - my + 4 = 0$
Passes through (- 4, 5) $\Rightarrow -4m^2 - 5m + 4 = 0$ $\therefore$ $m_1 + m_2 = \frac{-5}{4}$

5. (d) 26 ways of selecting a block of 5 days in a period of 30 days i.e. I block 1–5, II block 5-10, --------, 26th block 26-30 and ways of arranging films among themselves =5! $\therefore$ Total no. of arrangements = $26 \times 5!$
= 3120

6. (b) Let tan A = x, tan B = y, tan C = z $\therefore$ $x + y + z = 4$ and find minimum value of $x^2+y^2+z^2$ Let P(x, y, z) be any point. On the plane surface $x + y + z = 4$
$\therefore$ $OP^2 = x^2 + y^2 + z^2$ is least if OP is least i.e. If OP is the length of $\perp^r$ from origin O to the plane. Here OP= $\frac{4}{\sqrt{3}}$
$\therefore$ $OP^2 = \frac{16}{3}$ = minimum value of $x^2 + y^2 + z^2$.

7. (a) Let x, y be the number of Rs.5 and Rs.10 notes.
$\therefore$ $5x + 10y = 500 \Rightarrow y = \frac{100 - x}{2}$, we should have only even integral value of x, so that y too is integer .
$\therefore$ $x = 0, 2, 4,$ -------, 100 $\Rightarrow$ 51 possibilities.

8. (a) $|z|$ =5 is a circle with center (0, 0), radius 5
$|z - 10| = |z - 8|$ is a line $x = 9$ and so do not intersect. Hence no sol. satisfying equations.

9. (c) Because M and K are two sport athletes.
Te $\rightarrow$ G H J K L M N O
Fo $\rightarrow$ H M O
Ba $\rightarrow$ K N
$\Rightarrow [G, J, L]$ are only single game players .

10. (c) HK, KM can't be together $\therefore$ (a),(b),(d) violated

11. (a) all other choices violate that two sport athletes should not sit together

12. (b) N is a two sport athlete

13. (b) LOTUS

14. (b) By 2P.M. the angle between the hr. & min. hand = 60^o
The min. hand goes ahead of the hr. hand = 5.5^o per min.
Total degree by which min. hand has to go ahead of hr. hand $=90^o+60^o=150^o$ Thus, Time req. for it =150*(10/55) –

15. (c) $2x + 3 = 1$, $2x + 3 = 17$ $\Rightarrow x = -1$, $x = 7$

16. (c) $\underset{n \to 100}{lt} \frac{u_n + 1}{u_n} = |x - 3| \Rightarrow \sum u_n$ cgt. If $|x - 3| < 1$
$\Rightarrow 2 < x < 4$

17. (d) Ellipse is a curve , square is a polygon

18. (c) Order is stronger in implication than request Dictate is stronger in implication than suggest

19. (b) $a < b < c \Rightarrow \frac{1}{a} > \frac{1}{b} > \frac{1}{c}$
Multiply by $a\ b\ c \Rightarrow ab < ac < bc$

20. (c) log 2 = log k – log2 $\Rightarrow$ $k = 4$

21. (a) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 1$, $\forall\ \theta$
$\therefore$ No Solution except (0, 0)

22. (c) $|x - k| + |x - L|$, where $L > K$ has $L - K$ as least value
$\therefore$ $|x - 2| + |x - 3|$ has least value 3 – 2 = 1

23. (b) X ~ B (6, P) $\therefore$ P (X = r) = ${}^6C_r \quad q^{6-r} \quad p^r$

Now, 8 P (X = 2) = 3 P (X = 3) $\Rightarrow \frac{q}{p} = \frac{1}{2}$ Also, $p + q = 1$

$\therefore\ p = 2/3$

24. (d) $a\left(x^3 - y^3\right) + b\left(x^2 - y^2\right) + c(x - y) = 0$ where $x = \cos\theta$, $y = \sin\theta$ $\therefore (x - y)\ \left[a\left(x^2 + xy + y^2\right) + b(x + y) + c\right] = 0 \Rightarrow x = y$ or $\cos\theta = \sin\theta \Rightarrow \theta = \pi/4$

25. (b) **26. (c)**

27. (d) $\sin^{-1} y = A + x$

differentiating $\frac{dy}{dx} = \sqrt{1 - y^2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - y^2$

28. (a) 2! (n – 1)! = 5x (3! (n – 2)!) $\Rightarrow$ n = 16

29. (c) $\left(x^2 + 3\right)^{10} = a_0 x^{20} + a_1 x^{18} + ... + a_{10} x^2 + 3^{10}$

$x = 1 \Rightarrow 4^{10} = \left(a_0 + a_1 + ... + a_{10}\right) + 3^{10}$

$\Rightarrow a_0 + a_1 + ... + a_{10} = 4^{10} - 3^{10}$

30. (a) $(x - 2)^2 + (y - 2)^2 = 25$

$\therefore x = 2 + 5\cos\theta,\ \ y = 2 + 5\sin\theta$

$\Rightarrow x + y = 4 + 5(\cos\theta + \sin\theta) = 4 + 5\sqrt{2}\ \sin(\theta + 45°)$

Max. $x + y = 4 + 5\sqrt{2}$

31. (b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 3\vec{i} + \vec{j} + \vec{k}$ and

$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = 3\vec{i} + \vec{j} + \vec{k}$ $\therefore$ $\cos\theta = \frac{-9 - 1 + 1}{\sqrt{11}\sqrt{11}} = \frac{-9}{11}$ If θ were acute angle then $11\cos\theta = 9$

32. **(d)** Scalar triple product of $i + j,\ j + k,\ k + i = 2 \neq 0$ $\therefore$ vectors are non-planar, so not a Δ.

33. (d) $e^x > 0\ \forall x$ $\therefore$ $e^x + 1 > 1$, $\frac{1}{e^x + 1} < 1 \Rightarrow \frac{-2}{e^x + 1} > -2$

$\Rightarrow 1 - \frac{2}{e^x + 1} > -1 \Rightarrow \frac{e^x - 1}{e^x + 1} > -1$ $\therefore$ $(-1, \infty)$

34. (d) $1 + \cos 3x$ $\therefore D^5(y) = D^5(1) + D^5(\cos 3x)$

$= 0 + 3^5\ \cos\left(\frac{5\pi}{2} + 3x\right) = 3^5\ \cos(3\pi)$, at $x = \pi/6 = -3^5$

35. (c) $(\cos\theta - \lambda)^2 + \sin^2\theta = 0$

$\Rightarrow (\lambda - \cos\theta)^2 = -\sin^2\theta = i^2\sin^2\theta$

$\Rightarrow \lambda = \cos\theta + i\sin\theta$

36. (b) A. M > G.M

$\Rightarrow \left(\frac{\alpha}{\beta} + \frac{\beta}{\gamma} + \frac{\gamma}{\alpha}\right)\Big/3 > \sqrt[3]{\frac{\alpha}{\beta}.\frac{\beta}{\gamma} + \frac{\gamma}{\alpha}}$

37. (c) $f(n) = \sqrt{n+1} - \sqrt{n}$

$f'(n) = \frac{1}{2\sqrt{n+1}\sqrt{n}}\ \ \left(\sqrt{n} - \sqrt{n+1}\right) < 0$

$\Rightarrow$ $f'(n)$ is decreasing $f\ n$ of n hence $f(n)$ decreases as n increases.

38. (d) $p^2 = q^2 + r^2 \Rightarrow p, q, r$ are the sides of a rt. angled Δ. and in Δ. Sum of lengths of two sides > third side

39. (a) – 6x–1>27 $\Rightarrow x < \frac{-14}{3}$

$\therefore$ greatest integral value of x is –5

40. (b) 'Direct' is to Order while others involve giving a choice to act.

41. (d) $Un = \frac{1.3.5.----(2n-1)}{2.4.6.---(2n)}\left(\frac{1}{3}\right)^n$

$\therefore$ $\frac{Un+1}{Un} = \frac{2n+1}{(2n+2)}\frac{1}{3}$ $\underset{n\to\infty}{Lt}\frac{Un+1}{Un} = \frac{1}{3} < 1$

$\therefore$ series $\sum Un$ is cgt.

Hence its nth $\rightarrow 0$ as $n \to \infty$

42. (b) z = 7 $\because$ $x\ y$ multiplied by 7= $x\ y$ multiplied by z = 315 Also, 315 ÷ 7 = 45 = $x\ y$ $\therefore$ x = 4

43. (b) $\alpha + \beta = 5,\ \alpha - \beta = 3$ $\therefore$ $\alpha = 4,\ \beta = 1$

$\Rightarrow p = \alpha * \beta = 4$ **44. (a)**

45. (c) In backward sequence (Z to A)

G= 20th, R = 9th, I = 18th, D = 23th $\therefore$ avg. = 17.5 Similarly, F, U, N, D = 15.75 $\left(= \frac{21 + 6 + 13 + 23}{4}\right)$

46. (d) $e^{2x} - 2 = e^x = 2, -1$

$\therefore$ $e^x = 2$ ($\because$ - 1 not possible) $\Rightarrow x = \log_e 2$

47. (d) ao (e (x)) = a (x) = $\frac{1}{1-x} \neq e(x)$

48. **(b)** A = $\int_1^2 x dy = \int_1^2 \log y\ dy$ ($\because y = e^x$)

$= \log 4 - 1$

49. (a) S = 1 + x + x^2 +----- where x = (0.2) a converge if | x |<| i.e. (0.2) a <1 $\Rightarrow$ a < 5

50. **(d)** Let $f(x) = a\,x + b$ and $f(x + 1) = f(x - 1) \Rightarrow 2a = 0$ or a =0 $\therefore$ $f(x) = b$ = constant

51. (b) Let S = $1 + \cos\alpha + \cos 2\alpha + ... = \frac{1}{1 - \cos\alpha} = 2$

$\Rightarrow \cos\alpha = \frac{1}{2}$ $\therefore 1 + \sin^2\alpha + \sin^4\alpha + ... = \frac{1}{1 - \sin^2\alpha} = 4$

52. (a) P (B / $\overline{A}$) = $\frac{P(B \cap \overline{A})}{P(\overline{A})} = \frac{P(B) - P(A \cap B)}{1 - P(A)} = \frac{11}{20}$

$\therefore$ P (A$\cap$B) = $\frac{1}{30}$ $\Rightarrow$ A$\cap$B $\neq \phi \Rightarrow$ A and B not mutually exclusive. Also, P (A). P (B) = 2/15 $\neq$ P (A$\cap$B)

$\Rightarrow$ A and B are not independent

53. (d) $\bar{a} * \left(\vec{c} + \vec{b}\right) = \left(\vec{b} + \vec{c}\right) * \left(\vec{b} + \vec{c}\right) = 0$

54. (c) New A.M. = $2\bar{x} + \frac{20}{8} = 2 * 30 + 2.5 = 62.5$

55. (a) G.M < A.M. $\Rightarrow$ G.M < 4

56. (b) n! = $\left(\frac{n+1}{3!}\right) \Rightarrow n = 5$

57. (d) Let Z = $\dfrac{1+2\omega+3\omega^2}{\omega+2\omega^2+3}=\dfrac{1+2\omega+3\omega^2}{\omega+2\omega^2+3}=\dfrac{1}{\omega}=\omega^2$

58. (c) $\underset{x\to 2}{Lt}\dfrac{x^3+x^2-4x-4}{x-2}$

$\underset{x\to 2}{Lt}(x+1)(x+2)=12$

$\therefore f(x)$ is continuous at $x = 2$ provides $f(2) = 12$ i.e. $a = 12$

59. (b)

60. (c) Pulpit is a place where minister does his job also; Bench is a place where judge does his job.

61. (a)	**62. (c)**	**63. (b)**	**64. (b)**
65. (d)	**66. (c)**	**67. (d)**	**68. (a)**
69. (b)	**70. (c)** No use of at least		**71. (d)**

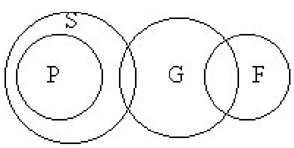
72. (a) Hardly had he finished

73. (a) the symbol inside circle changes alternatively. Arrow moves clock wise vert. to horz. and vice versa; vert. outside and horz. inside

74. (d) Cross moves clockwise, outer most object goes to inner most , second to outmost object goes to outermost, innermost to second most place. **75. (c)**

76. (a) Two and three elements disappear in alternate steps and the elements get inverted in each step.

77. (b)



78. (c) L (K J) I (H) G (FE) D (C) B (AZ) Y

i.e. Letters are in Z to A sequence and number of letters skipped between consecutive positions is alternatively two and one

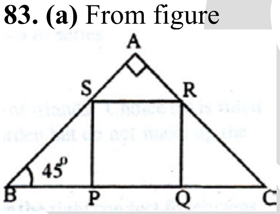
79. (b) Let 5 be one of two nos. The product end with zero if number is 2 or 4 or 6 or 8

$\Rightarrow$ 4 pairs of nos. Whose products end with zero.

80. (a) $\sin 4x$ has period $\dfrac{2\pi}{4}=\dfrac{\pi}{2}$

81. (c) A (B+I) = I $\Rightarrow$ A^{-1} = B+I **82. (d)**

83. (a) From figure



BP = QC = x $\therefore$ BC = $3x$ and x^2 + = 40. Now

$A=\dfrac{1}{2}3x\sin 45°\,3x\cos 45°\left(a=\dfrac{1}{2}bh\right)=\dfrac{9x^2}{4}=90cm^2$

84. (b) Z = 4+iy , | Z | = 5 $\Rightarrow$ $y = +3$

$\therefore\ \theta=\tan^{-1}\left(\dfrac{3}{4}\right)$ (only available choice)

85. (d) $\left(\dfrac{4}{9}\right)^x+4\left(\dfrac{6}{9}\right)^x=5\Rightarrow\left(\dfrac{2}{3}\right)^{2x}+4\left(\dfrac{2}{3}\right)^x=5$

$\Rightarrow p^2+4p=-5=0,\ p=\left(\dfrac{2}{3}\right)^x\ \Rightarrow p=1(-5\text{ not possible})\Rightarrow x=0$

86. (b) The center of the circle (2,0) & r = 3 units.

Thus, Volume generated by sphere of radius as 3 is $=\dfrac{4\pi}{3}(3)^3$

$= 36\pi$

87. (c) Let volume V of cube = a^3 , a is side.

If Volume is 8V = 8 a^3 $\Rightarrow$ side = $2a$

Surface areas of cubes = $6a^2$, $24a^2$ respectively

$\therefore$ Time req. to paint II cube = 4t hrs.

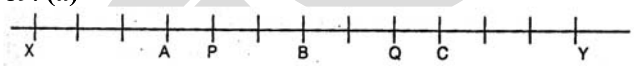
88. (d) Let C. P. of 1 mango = Rs.1

$\therefore$ Investment = Rs.100

For 20% profit, he has to sell = 90 mangoes for Rs.120

$\therefore$ C.P: S.P of 1 mango = 1: $\dfrac{120}{90}$ =3:4

89. (a)



PB + AQ = (XB – XP) + (XQ – XA) = 6 – 4 + 8 – 3 = 7

90-93:

Explanation: The given information can be condensed as follows:

[A, B, C, D are the men and R, S, T, U are the women]

(i) Mrs. C>S (ii) Mrs. D>U (iii) U is A's sister

(iv) R<S,T,U (v) C ✓was not present at U's wedding After using (i) to (v) the ✓ in CT is allowed.Thus,

DT is ✗. Hence ✓DS . Also, ✓A and so, ✓BU

✓: Maritial Relation WIFE ✗: Non existence

HUSBAND

	R	S	T	U
A	✓ By elemination			✗ By (iii)
B				✓ By elemina tion
C	✗ By (i) and (iv)	✗ By (i)	✓ By eleminati on	✗
D	✗ By (ii) and (iv)	✓ By elemination	✗	✗ By (ii)

✓: Existence of maritial Relation

✗: Non-Existence

90. (c) $\because$ Rekha is youngest women and wife of Arun.

91. (a) Uma is Arun's sister and Badri's wife. Thus Arun is Prem's Uncle and Rekha (Arun's wife) is Prem's aunt.

92. (d) R<U $\Rightarrow$ A<B $\therefore$ B $\nless$ A

while in (a) C>Mrs. C>S ; (b) R<S,T,U< husbands

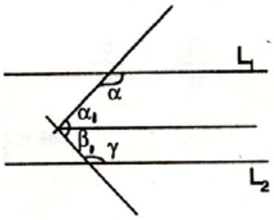
(c) R< S $\Rightarrow$ R!s husband < S's husband $\Rightarrow$ A<D

93. (b) A–27, B–29, C–31, D–33, R–28. The ages of other woman 30, 32, 34 Mrs. C>S $\Rightarrow$ T>S, Mrs. D>U $\Rightarrow$ S>U

$\Rightarrow$ T>S>U

94. (b) A person can't be frd. Of himself ($\neq$R)
If A is a frd. Of B $\Rightarrow$ B is a frd. Of A (=S) If A is frd. Of B, B is frd. Of C $\not\Rightarrow$ A is frd. of C ($\neq$T) **95. (c)**

96. (d)



Draw L3 parallel L1 & L2 $\therefore \alpha + \alpha_1 = 180°$ $\beta_1 + \gamma = 180°$
(sum of angles on the same side of transversal is 180)
$1 \Rightarrow \alpha + (\alpha_1 + \beta_1) + \gamma = 360°$
$\Rightarrow \alpha + \beta + \gamma = 360°$

97. (b) No. of man hrs. req. to complete the job = $10 \times 10 \times 10 = 1000$
No. of man hrs. made available = $2 \times 5 \times 8 = 80$
$\therefore$ % of work completed = $\frac{80}{1000} \times 100 = 8\%$

98. (b) n (T $\cap$ F) = x (say) $\therefore$ n (T $\cup$ F)' = x
Only T=20, only F=15
$\therefore$ $20 + x + 15 + x = 35 + 2x$, which is odd
$\therefore$ Ans. = 37, only odd choice available.

99. (c) Let external angle = x $\therefore$ internal = $4x$
$4x + x = 180° \Rightarrow x = 36° \therefore$ no. of sides = $\frac{360}{36} = 10$

100. (c) $\overrightarrow{n_1}, \overrightarrow{n_2}$ are normals to two given planes $\overrightarrow{n_1} \times \overrightarrow{n_2}$ is parallel to both planes
$\therefore$ $\overrightarrow{n_1} \times \overrightarrow{n_2} = \left(\vec{i} + 1\vec{j} + \vec{k}\right) * \left(2\vec{i} + \vec{j} + 2\vec{k}\right) = \overrightarrow{3i} - \overrightarrow{3k}$
$\therefore$ d.r's (1, 0, –1)