

Hints and Solution

MTD - I

1. (a) 0 and 1 are repeated an infinite number of times and hence are limiting points.

2. (b) 3. (a)

4. (b) For any normal distribution Mean Deviation $\cong \frac{4}{5}\sigma$

5. (b)	x	$p(x)$	x^2	$x^2 + 1$	$(x^2 + 1)p(x)$
	0	.1	0	1	.1
	1	.2	1	2	.4
	2	.3	4	5	1.5
	3	.4	9	10	$\frac{4}{5}$
					<u>6</u>

\therefore Expected value of $y = x^2 + 1$ is

$$E(Y) = E(x^2 + 1) = \sum (x^2 + 1)p(x) = 6$$

6. (c) use distance formula.

7. (c) Here let the 'x' denote the pocket money of the son.

Then x takes following values

X: 0, 5, 10

Now X = 0 means both coins show tails

$$\Rightarrow p(x=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^2}$$

X = 5 means one head, other tail

P(X = 5) = HT or TH

$$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = 2 \times \frac{1}{2^2}$$

$$X=10 \text{ means both heads } \therefore p(X=10) = \frac{1}{2^2}$$

\therefore Expected value of pocket money

$$= E(X) = \sum Xp(X) = 0 \times \frac{1}{2^2} + 5 \times 2 \times \frac{1}{2^2} + 10 \times \frac{1}{2^2} = \frac{20}{4} = 5$$

8. (b)

9. (d) If X_i are iid $N(\mu, \sigma^2)$ then

$\sum X_i$ has normal distribution with mean = $\sum_{i=1}^n \mu = n\mu$

$$\text{Var. } \sum_{i=1}^n \sigma^2 = n\sigma^2 \therefore \sum X_i \sim N(n\mu, n\sigma^2)$$

10. (b) The diff. amount of money means how many different totals of money he can get from these five diff. coins.

\Rightarrow how many diff. coins selection he can make.

he can make a selection of 1 coin and he gets one amount.

he can select 2 coins, add the value and gets another amount.

continuing in this way problem simplifies to make atleast 1 coin selection from 5 coins. which can be done in

$$2^5 - 1 = 31 \text{ way}$$

11. (b) $w! = x! + y! + z!$ without loss of generality, we can assume $x \leq y \leq z$. It is obvious that $w \geq z + 1$ so that

$$(z+1)! \leq w! = x! + y! + z! \leq 3z!$$

$$\Rightarrow z+1 \leq 3 \text{ or } z \leq 2 \text{ and } x \leq 2, y \leq 2$$

$\therefore x = y = z = 2$ and $w = 3$ gives the only soln.

12. (d) Done in the classroom. The largest number is $3^{1/3}$

13. (a) using Descartes's Rule the maximum number of real roots comes out to be 0.

14. (d) $cx + dy - ax - b = 0$. Comparing with the general

equation of second degree $h = \frac{c}{2}, a = 0, b = 0$, hence $h^2 > ab$.

15. (d) $z = 4x^2 - 2xy + 6y^2$ s.t. $x + y = 72 \therefore x = 72 - y$

$$\therefore z = 4(72 - y)^2 - 2(72 - y)y + 6y^2$$

$$= 4[5184 + y^2 - 144y] - 144y + 2y^2 + 6y^2$$

$$\Rightarrow \frac{dz}{dy} = 4[2y - 144] - 144 + 4y + 12y = 0$$

$$\Rightarrow 24y = 720 \Rightarrow y = 30 \therefore x = 42$$

$$\therefore z = 4 \times 1764 - 2(42 \times 30) + 6 \times 900 = 9936$$

16. (b) Plane cutting axes in A, B, C is $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$ (1)

Since it passes through (a, b, c) we have

$$\frac{a}{l} + \frac{b}{m} + \frac{c}{n} = 1 \quad (2)$$

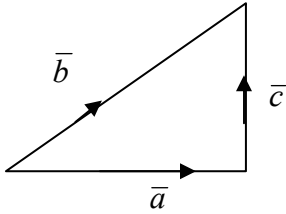
Now sphere OABC is $x^2 + y^2 + z^2 - lx - my - nz = 0$

centre is $\left(\frac{l}{2}, \frac{m}{2}, \frac{n}{2}\right) = (x_1, y_1, z_1) \therefore l = 2x_1, m = 2y_1, n = 2z_1$

$$\therefore \frac{a}{x_1} + \frac{b}{y_1} + \frac{c}{z_1} = 2$$

17. (c)

18. (b)



$$19. (a) \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 4 & -5 \\ 5 & 6 & -3 \end{vmatrix}$$

$$= i(-12+30) - j(-9+25) + k(18-20)$$

$$= 18i - 16j - 2k = \vec{b} \cdot \vec{c} = 10 + 12 - 6 = 16$$

$$\therefore (\vec{a} \times \vec{b}) \cdot (\vec{b} \cdot \vec{c}) = 32(9i - 8j - k)$$

$$20. (d) [a, b, c] = (\vec{a} \times \vec{b}) \cdot \vec{c} = (18i - 16j - 2k) \cdot (5i + 6j - 3k)$$

$$= 90 - 96 + 6 = 0$$

21. (b) **Circular asymptote:** if $r = f(\theta)$ be an equation of curve and if $\lim_{\theta \rightarrow \infty} f(\theta) = a$, then the circle $r = a$ is called circular asymptote of the curve $r = f(\theta)$

$$r = a \frac{e^\theta + 1}{e^\theta - 1} = a \frac{1 + e^{-\theta}}{1 - e^{-\theta}} \rightarrow a \text{ as } \theta \rightarrow \infty$$

\therefore circular asym. is $r = a$

22. (d) Asymptote parallel to y -axis is $1 - x = 0$

Asymptote parallel to x -axis is $y = 0$

Oblique asymptotes can be obtained using the usual method by computing m from $\phi_3(m) = 0$ and $c = \frac{-\phi_2(m)}{\phi_3(m)}$. The

oblique asymptote comes out to be $y = x + 2$.

$$23. (c) x^{243} + x^{81} + x^{27} + x^9 + x^3 + x = (x^2 - 1)\phi(x) + Rx + s \quad \dots (1)$$

where $\phi(x)$ is a poly. of degree 241 and R and S are constants. Since (1) is true for all x , in particular (1) is true for $x = 1, x = -1$

$$x = 1 \Rightarrow 1 + 1 + 1 + 1 + 1 + 1 = R + S$$

$$x = -1 \Rightarrow -1 - 1 - 1 - 1 - 1 - 1 = -R + S \quad \therefore R + S = 6 \text{ and}$$

$$-R + S = -6 \Rightarrow 2S = 0 \Rightarrow S = 0 \text{ and } R = 6$$

\therefore Remainder is $6x$ (Alt. Put $x^2 = 1$ in the given equation)

24. (b) Since $\lim_{n \rightarrow \infty} a_n = a \therefore \lim_{n \rightarrow \infty} a_{n+1} = a$ also

from $a_{n+1} = \cos a_n$ we get $\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} \cos a_n = \cos(\lim_{n \rightarrow \infty} a_n)$

$\Rightarrow a = \cos a$, so $a < 1$ if positive

$a = \cos a \Rightarrow (a, \cos a)$ lies on the line $y = x$. Now

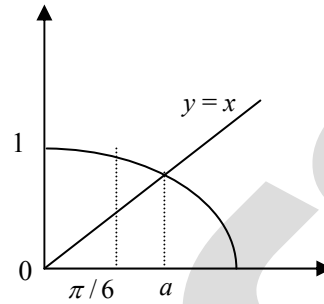
$$\cos(\pi/6) = \frac{\sqrt{3}}{2} = .8 \text{ and } \pi/6 = .52$$

i. e. $\cos \pi/6 > \pi/6$ i. e. $y \text{ coord.} > x \text{ coord.}$

Therefore value of $\cos x$ in $(0, \pi/6)$ is above the line $y = x$.

$$\text{Also } \cos(\pi/3) = 1/2 = .5 \text{ and } \pi/3 = \frac{22}{21} = 1.04,$$

i. e. $y \text{ coord.} < x \text{ coord.}$ i. e. value of $\cos x$ at $\pi/6$ lies below $y = x$. Hence value of ' a ' such that $a = \cos a$ must lie in $(\pi/6, \pi/3)$



$$25. (a) f(x) = (\sin x)(\sin 2x) \dots (\sin nx) \\ f'(x) = (\cos x)(\sin 2x) \dots (\sin nx) \\ + (\sin x)(2 \cos 2x) \dots (\sin nx) \\ + (\sin x) \dots (n \cos nx)$$

$$\therefore f'(x) = f(x)[\cot x + 2 \cot 2x + \dots + n \cot nx]$$

26. (b) Done in the class room.

27. (c) Draw the graph. The extreme points are $(0, 5)$, $(0, 5.5)$, $(7.3, 0)$ and $(5, 0)$. Also slope of objective function is same as one of the constraints, therefore case of alternate optimal solution.

$$28. (d) \text{ Let } f(x) = e^{-1/x^2}, x \neq 0, f(0) = 0$$

(It has a remarkable property $f^n(0) = 0 \forall n$)

$$\text{Maclaurin's exp. is } 0 + x \cdot 0 + \frac{x^2}{2} \cdot 0 + \dots$$

Which is certainly not equal to $f(x)$ for any value of (x) , other than 0. Hence (a) is true due to this example.

$$\text{For choice (b) } \lim_{x \rightarrow \infty} \frac{x^m}{e^{\alpha x}} \left(\frac{\infty}{\infty} \right) \rightarrow 0 \therefore \text{Demo is stronger}$$

$\therefore e^{\alpha x}$ tends to ∞ faster than x^m

choice (c) is the definition of a real number.

$$29. (a) i + \frac{1}{2}i^2 + \frac{1}{3}i^3 + \dots$$

$$\text{Real part } -\frac{1}{2} + \frac{1}{4} - \frac{1}{6} \dots$$

$$\text{and Imaginary part is } 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots$$

both are convergent \therefore series is also convergent

$$\sum \left| \frac{i^n}{n} \right| = 1 + 1/2 + \dots \text{ which is divergent.}$$

30. (b) Let $f(x) = 1 + 2x^3$ and $g(x) = 2 + x + 3x^2$ be two polynomials in $R[x]$ of degree 3 and 2 respectively where $R = \{0, 1, 2, .5\} + 6 \times 6$

Then $f(x)g(x) = 2 + x + 3x^2 + 4x^3 + 2x^4$ which is of degree 4.
(perform multiplication and addition over modulo 6)

31. (b) If u and v are two solutions of a homogeneous linear differential equation then so is $c_1u + c_2v$ and c_1 and c_2 are two constants. Put $c_1 = 1$ and $c_2 = -1$.

32. (c) Let $\cos \theta = t$ then $\sin^2 \theta = 1 - \cos^2 \theta = 1 - t^2$

$$\Rightarrow a \cos \theta + b \sin \theta = c \Rightarrow b \sin \theta = c - a \cos \theta$$

$$\Rightarrow b^2 \sin^2 \theta = c^2 + a^2 \cos^2 \theta - 2ac \cos \theta$$

$$\Rightarrow b^2(1 - t^2) = c^2 + a^2 t^2 - 2act$$

$$\Rightarrow (a^2 + b^2)t^2 - 2act + c^2 - b^2 = 0$$

Its roots are $\cos \alpha$ and $\cos \beta$. Hence sum of the roots is

$$\cos \alpha + \cos \beta = \frac{2ac}{a^2 + b^2}$$

33. (d) $m^2 - n$ (They just want different and not independent)

34. (c) Done in the class room. First series converges to \log_2

whereas second series converges to $\frac{1}{2} \log_2$.

Note: If the terms of the series (A) are so rearranged that the first p positive terms are followed by the first q negative terms, then the next p positive terms are followed by next q negative terms and so on, then the resulting series converges to $\log(2\sqrt{p/q})$.

35. (b) $E(x^2) = \sum x^2 f(x) = \frac{80}{16} = 5$ **36. (a)** **37. (d)**

38. (d) Let $A = \{-1, 1, 2\}$ then $B = \{1, 4\}$ hence choices (a) and (c) are wrong. Further Let $A = \{1, 2\}$ then $B = \{1, 4\}$, choice (b) is wrong.

39. (c) $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and

$$16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1, \quad a = 5, \quad b = 4$$

$$\text{Now } b^2 = a^2(1 - e^2) \Rightarrow e = 3/5$$

$$ae = 5(3/5) = 3$$

$\therefore F_1, F_2$ are Focii of the above ellipse and P is any pt. on it

\therefore sum of focal distances = $2a$

$$PF_1 + PF_2 = 2 \times 5 = 10$$

40. (b) Since $f(x)$ is a poly. fn, it is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Hence by Lagrange's M.V.T. $\exists c \in (0, 1)$ s. t.

$$\frac{f(b) - f(a)}{b - a} = f'(c) \Rightarrow \frac{2 - 3}{1} = f'(c) \Rightarrow f'(c) = -1$$

$$\text{Now, } f'(x) = 2(x - 1) \Rightarrow f'(c) = 2(c - 1)$$

$$\therefore 2c - 2 = -1 \Rightarrow c = 1/2$$

$\therefore f(c) = \frac{9}{4}$ so that the pt. of contact of tangent is $\left(\frac{1}{2}, \frac{9}{4}\right)$ and

slope is $f'(c) = -1$

$$\therefore \text{tangent is } y - \frac{9}{4} = -1(x - 1/2) \Rightarrow 4x + 4y = 11$$

41. (d) Result:

approx 95% confidence interval for population proportion

is $\bar{p} + 2\sqrt{\frac{pq}{n}}$ and exactly 95% confidence interval for pop.

proportion is $\bar{p} + 1.96\sqrt{\frac{pq}{n}}$

42. (d) 6

43. (c) $x^3 - 3x^2 + px + 1 = 0$ (1)

$a - d, a, a + d$ are roots $\therefore 3a = 3 \Rightarrow a = 1$

Sub. in (1) we get $1 - 3 + p + 1 = 0 \Rightarrow p = 1$

$$\therefore (1) \Rightarrow x^3 - 3x^2 + x + 1 = 0$$

$$\text{also } (a^2 - d^2)a = -1 \Rightarrow 1 - d^2 = -1 \Rightarrow d^2 = 2 \Rightarrow d = \pm\sqrt{2}$$

\therefore roots are $1 - \sqrt{2}, 1, 1 + \sqrt{2}$

$$\therefore (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 1 + 2 + 1 + 2 = 6$$

44. (b) If $AX = b$ is $m \times n$ system of equations and $\rho(A) = r$ then the number of linearly independent solutions is $n - r$.

45. (c)

46. (b) $y = \log_e e^x + \frac{3}{4}[\log(x - 2) - \log(x + 2)]$

$$= x + \frac{3}{4}[\log(x - 2) - \log(x + 2)]$$

$$y' = 1 + \frac{3}{4}\left[\frac{1}{x - 2} - \frac{1}{x + 2}\right] = 1 + \frac{3}{4}\left[\frac{x + 2 - x - 2}{x^2 - 4}\right] = \frac{x^2 - 1}{x^2 - 4}$$

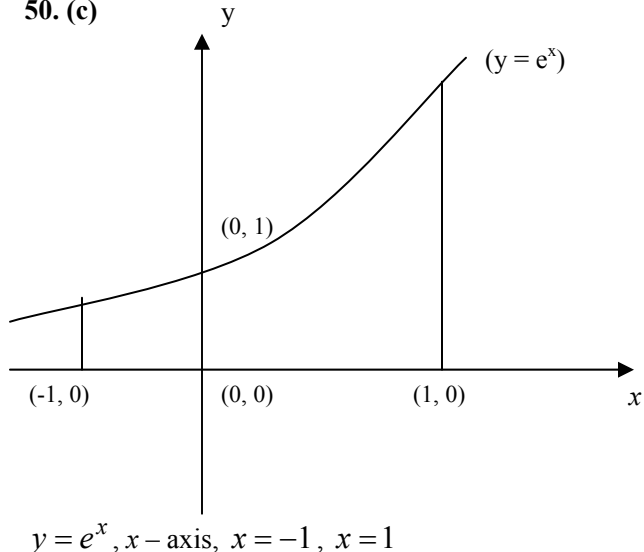
47. (c) Use Euler's theorem.

48. (b) $\int_0^\infty \frac{dx}{\sqrt{x}e^{\sqrt{x}}} \quad \text{Let } \sqrt{x} = t \Rightarrow \frac{1}{2}\sqrt{x}dx = dt$

$$\Rightarrow 2 \int_0^\infty \frac{dt}{e^t} = -2[e^{-t}]_0^\infty = -2[e^{-\infty} - e^{-0}] = 2$$

49. (d) Asymptotes are defined for a hyperbola whose eccentricity is more than 1.

50. (c)



$$\text{Area} = \int_{-1}^1 e^x dx = e^x \Big|_{-1}^1 = e - \frac{1}{e} = \frac{e^2 - 1}{e}$$

51. (d) \otimes represents X-NOR is true when both bits are identical.

52. (b) $(765)_8 = (111110101)_2$

$= (000001011)_2$ (2's complement) $= (013)_8$

53. (d) To convert to base 8, we group in 3 bits $\therefore 2^3 = 8$

To convert to base 16, we group in 4 bits $\therefore 2^4 = 16$

\therefore To convert to base 32, we require grouping of 5 bits

Here, 11001 10001 11011 11000

25 17 27 24

54. (c) $a\bar{b} + a\bar{b}c + \bar{a}\bar{b}d + \bar{d}\bar{b}e + \bar{a}bf$

$= a\bar{b}(1+c) + \bar{a}\bar{b}d + \bar{d}\bar{b}e + \bar{a}bf$

$= (a\bar{b} + \bar{a}\bar{b}d) + \bar{d}\bar{b}e + \bar{a}bf$ $[\because 1+c=1]$

$= (a\bar{b} + \bar{b}d + \bar{d}\bar{b}e + \bar{a}bf)$ $[\because a\bar{b} + \bar{a}\bar{b}d = \bar{a}\bar{b} + \bar{b}d]$

$= a\bar{b} + (\bar{b}d + \bar{d}\bar{b}e) + \bar{a}bf$

$= a\bar{b} + \bar{b}d + \bar{b}\bar{e} + \bar{a}bf$ $[\because \bar{b}d + \bar{d}\bar{b}e = \bar{b}d + \bar{b}\bar{e}]$

55. (b) BCD is a 4-bit code.

56. (d) Let $z = 4$

57. (d) Let $x = 12$

Then binary representation of 12 = 1 1 0 0

$a_3 a_2 a_1 a_0$

$(a_0 + a_2) - (a_1 + a_3) = (0+1) - (0+1) = 0$, divisible by 3

58. (b) Let $n = 3$ \checkmark : Valid entry

Sum = 0, $i = 1$ \times : Invalid entry

k	j	Fact	j=k	sum=sum+i/Fact	i=n	i=i+1
2	2	1		$\frac{1}{2!}$	\times	2
		2!	\checkmark			
3	2	1	\times	$\frac{1}{2!} + \frac{2}{3!}$	\times	3
	3	3!	\checkmark			
4	2	1!	\times			
		2!	\times			
	3	3!	\times			
	4	4!	\checkmark	$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!}$	\checkmark	

59. (b) (o) : old, (n) : new

a (o)	b (o)	a = a \oplus b (n) (o) (o)	b = a \oplus b (n) (n) (o)	a = b \oplus a (n) (n) (n)
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	1	1

Clearly, values of a and b we obtained are swapped values.

60. (b) Let the initial value of result is 10, which is the smallest two digit positive number, then $R = \sqrt{10+72} = \sqrt{82} > \sqrt{81} > 9$ At every step value of R reduces and when it becomes 9 then $R = \sqrt{9+72} = \sqrt{81} = 9$ and hence it finally terminates to 9.

61. (c)

A	B
12.1	14.4 B = B - A
12.1	2.3 A = A - B
	until A \leq B
.6	2.3
0.6	0.5
0.1	0.5
0.1	0.1

62. (a) Since number is odd \therefore the unit digit cannot be 6 \Rightarrow choices (b), (c) are eliminated. Now consider the square of the largest two digit odd number $(99)^2 = 9801 \Rightarrow$ two most significant digits can never be 99. Therefore, choice (a) is eliminated.

63. (a)

64. (a) Before solving this example, let us make certain observations. Divide this table into various diagonals as follows

1	3	6	10	15
2	5	9	14	20
4	8	13	19	26
7				

Ist diagonal is having 1 entry

2nd diagonal is having 2 entries

3rd diagonal is having 3 entries and so on.

Also the largest entry in any diagonal is its first entry and then its value keeps on reducing by 1.

Also we can find the total number of entries covered upto any diagonal, for example, the total number of entries upto 3rd diagonal = the sum of first three naturals = $1+2+3=6$. Also this 6 is the top most entry of the diagonal.

We can also find the position of a cell in terms of diagonal i.e. suppose we want to find that number 14 lies in which diagonal. Number 14 lies in 2nd row and 4th column i.e. its position is (2,4) then to find the diagonal in which it lies we add 2 and 4 and subtract 1 i.e. $2+4-1=5 \Rightarrow$ number 14 lies in 5th diagonal. Now choice (a) and (b) lies in 63rd diagonal whereas (c) lies in 62nd diagonal so we start with 63rd

diagonal. The total number of elements covered upto 63rd diagonal = $1 + \dots + 63 = 2016$.

\Rightarrow index of 2016 = (1, 63)

\therefore index of 2015 = (2, 62)

index of 2014 = (3, 61)

:

:

2001 = (16, 48)

:

1995 = (22, 42)

So answer is (22, 42).

65. (a)

66. (c) In the binary representation of $2^n - 1$ we have n 1's.

Therefore $2^{20} - 1$ has 20 1's and again 1 is subtracted from it, so the least significant Bit (LSB) becomes zero \Rightarrow the number of 1's left is 19.

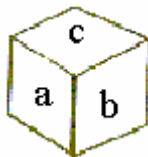
67. (b) $2^{15} - 1 = 11 \dots 11$

15 1's

$2^{15} - 3 = 11 \dots 101$

\therefore Only one zero is there.

68. (a)



$$c = (a + b)i$$

where $i = 2$ for 1st block

$i = 3$ for 2nd block

$i = 4$ for 3rd block and so on

69. (d) Since '.' stands for multiplication, we can perform cancellation on both the sides. After canceling common letters,

we get $\frac{E}{Y} = 0$. Since E is to be divisible by Y , It cannot be prime. So (a), (b), (c) choices are ruled out.

70. (c) Consider choice (a) which is $a * y = z$ and let $y = 3$ then prior to calculation $z = a * y = 3a$ but after that $z = 3a^2$ and $y = 2$ so the relation $a * y = z$ no longer holds.

Choice (b) which is $z = a^Y$

Initially $Z = a^3$ After algorithm $Z = a^4$, $Y = 2$

So relation no longer holds. Choice (c) which is $a^b = z * a^Y$ i.e. $Z = a^{b-Y} = a^{b-3}$ before application of algorithm. After algorithm it becomes $Z = a^{b-2}$ and $y = 2$ (i.e. $Z = a^{b-Y}$)

So the relation holds.

71. (c) Interchange of steps does not affect anything as steps are independent.

72. (d) No. of girls in XID = 25% of 40 = 10. Ratio of boys and girls in XIID is 5 : 1 and there are 40 boys in XIID. No. of girls in XIID = $\{1/5 \times 40 = 8\}$ $\therefore 10 - 8 = 2$ girls failed.

73. (d) Total no. of boys playing Chess and Badminton = $32 + 52 = 84$ $\therefore 4:1 \therefore 84:x \Rightarrow x = 21$, but 21 is 25% of 84. So, we

need a combination other than Chess and Badminton whose sum is 84.

74. (a) No. of boys of XIA + girls of XIB & XIC = $44 + 12 + 12 = 68$. Let x be the total no. of students.

Then 25% of $x = 68$, $\Rightarrow x = 272$

75. (d) 4 times the no. of girls in XIB & XIC = $4(12 + 12) = 96$. But none of the pairs given, has this total.

76. (b) No. of boys of XIE playing Chess + girls of XIB playing Table Tennis and girls of XIC Playing Hockey = $4 + 4 + 2 = 10$. Total no. of students $228 + 25\% \text{ of } 228 = 285$.

Let $x\% \text{ of } 285 = 10 \Rightarrow x = 3.51$

77. (d) Since no. of girls = 25% of the no. of boys, so only 25% of the boys can participating in social work.

78. (a) Learn as a result.

79. (b) 80. (d) 81. (c) 82. (b) 83. (b) 84. (a)

85. (a) Divide 1st term by 1, 2nd by 2, 3rd by 3 and so on.

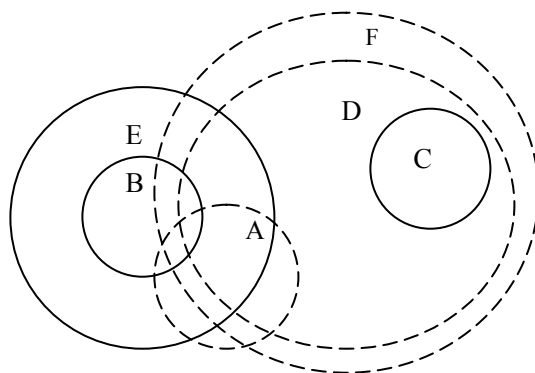
$$86. (b) \left[\frac{500}{5} \right] + \left[\frac{500}{5^2} \right] + \dots + \left[\frac{500}{5^4} \right]$$

$$87. (a) \text{ Use } \frac{1}{n+n^2} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\text{Therefore, given sum} = \frac{1}{100} - \frac{1}{1000}$$

88. (d) In all other, sum of digits is 12.

Directions (89 – 92): For a puzzle like this one, which relates various interlocking groups to one another, you'll need to draw a circle diagram showing the interrelationships of the groups named. Use solid lines to represent groups whose relationships are definitely established; use broken lines to represent groups about which some ambiguity exists. Your diagram should look more or less this one.



89. (a) If Miss Garfield is within the circle labeled D (lovers of Donne), she may or may not be within the circle labeled C (lovers of Coleridge). Each of the other statements presents as definite fact something which may or may not be true.

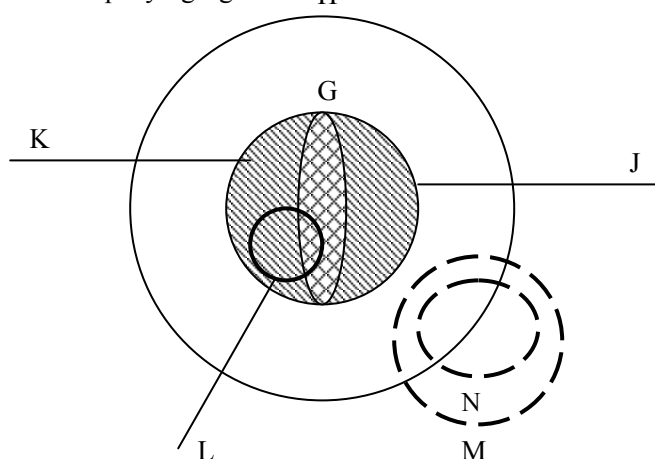
90. (b) As the diagram shows, the circle labeled B has no overlap with the circle labeled C. Therefore, there are no Browning-lovers who are also Coleridge-lovers. Browning lovers may also enjoy Auden, Donne or Frost they definitely enjoy Eliot, since the circle labeled B is entirely within the circle labeled E.

91. (d) All of the choices are possibly true, with the exception of choice E. Since the circle labeled C and the circle labeled E

have no overlap, Miss Iaaguchi definitely does not enjoy the poetry of Eliot.

92. (c) Since some Auden-lovers are Eliot lovers, and since all Eliot-lovers are Coleridge-despisers, there must be some Auden-lovers who are Coleridge-despisers.

Directions (93 – 96): The conditions can be diagrammed as in the accompanying figure.



Notice that the positions of two elements (M,N) and the existence of an overlap between J and K are uncertain; these uncertainties are indicated by broken lines.

93. (d) Choice A is not necessarily true, for the reasons given in the answer to number 3. Choices B and C are possible, but not necessarily true. Choice D fails to take the J's into account—it would be true only if the J's and K's overlapped completely. Choice E is correct: statement (2) means that any G that is not a K must be a J. Note that this choice is true even if J's and K's do overlap completely – in that case no P can be a G, but the “if then” statement in choice E remains true.

94. (d) From the diagram; statement (6) says that all M's must lie outside the G's; but all N's must lie inside the M's, according to statement (5). Since the only information about the M's is that they are not G's, none of the other statements, which attempt to put M's inside or outside the H's can be deduced.

95. (c) This statement is inconsistent with the condition that no M's are G's. Choice A is possible—we could draw the G circle so that it coincided with the H circle. Choices B,D, and E are all possible, since we can draw Many where outside G.

96. (d) If you thought that statement (2) meant that no J's are K's, you were wrong; it states that all G's are J's or K's, but doesn't exclude the possibility that some G's are both and that J and K overlap. (Consider an analogous statement like “All voters in my town voted for Republicans or Democrats.”) So statement (4) doesn't imply the statement given here, and I is out. It is possible that no L's are J's, however, even if J and K do overlap (II). Finally, the additional statement would mean that if L's were K's, they couldn't be J's (III).

97. (d) Event X (baldness) occurs after Event Y (practicing law). The author of the argument assumes that Event Y caused Event X, and vows to avoid Event X by avoiding Event Y. This is poor reasoning, especially since the author is overlooking at least one far more probable cause for Event X, i.e., heredity. The same kind of poor reasoning is used in

choice D, where Event X = injury, Event Y = eating at Rosie's, and the overlooked probable cause is unsafe working conditions. Choice B has the second closest resemblance, but here the reasoning is some what more plausible; speeding can lead to one's getting a speeding ticket. Choices A and C all differ from the original argument in the latter portion of their reasoning.

98. (b) The logical structure of the argument is : If P, then Q;Q, therefore P. This is fallacious reasoning, and we're asked to find the same fallacious reasoning in one of the choices. Choice B commits exactly the same error. Choice A says: If P, then Q;P, therefore Q (which is valid). Choice E says: If P, then Q; Not Q, therefore not P (also valid). Neither of these structures matches that of the original argument. Neither choice C nor choice D can be symbolized using just two letters; thus neither can match the original.

99. (b) Choice B states the hidden assumption on which the original, fallacious argument was based. If it were true, the original conclusion (“her theory must be correct”) would be valid.

100. (c) The argument presented here against proposed legislation is based upon unrelated, earlier legislation. Linking the water project and the highway proposal is similar in logic to linking the anti pornography proposal and the earlier women's rights legislation.

Additional Problem

Directions (Q. 1 – 4): Read the following: We assume that we have a checkerboard (assumed to be infinite plane in all four directions) of the form:

		1	2	3		
		8	A	4		
		7	6	5		

Each cell has eight neighbors as shown above. Each cell may be either empty denoting no living being over there or it is marked 'A' denoting one living being over there. In the latter case, the cell is called a counter. The rules of the game are:

I Survivals: Every counter with two or three neighbors alive, survives for next generation.(to go to next generation is called a move.)

II Deaths: Each counter with one alive neighbors or no alive neighbors dies from isolation in the next generation. Also, if a counter has four or more neighbors alive, then it dies in next generation due to suffocation.

III Births: If a cell is dead, then in the .next generation it will become alive if it have exactly three alive neighbors.

We explain a move by an example. The configuration

	1	2	3	4	5
a					
b	A		A		
c	A	A			
d			A		

becomes the following in the next generation

	1	2	3	4	5
--	---	---	---	---	---

a		.			
b	A				
c	A		A		
d		A			

1. The initial configuration

	A	A	A	A		A	A	A	A

- (a) dies out (i.e. no counter left) after two moves.
 (b) is periodic of period two (i.e. on alternate moves, it acquires earlier form)
 (c) stabilizes after almost three moves (i.e. after stabilization, the configuration does not change at all on subsequent moves)
 (d) becomes periodic, after three moves.

2. The initial configuration

		A	A	A		
		A		A		
		A	A	A		

- (a) dies out after two moves. (b) is periodic of period two
 (c) stabilizes after two moves (d) none of these

3. The initial configuration

	A	A				
	A	A				
			A	A		
			A	A		

- (a) dies after two moves (b) is periodic of period two
 (c) stabilizes after two moves
 (d) becomes periodic after three moves

4. The initial configuration:

		A		A		
		A	A	A		
		A		A		

- (a) dies out after two moves. (b) is periodic of period two.
 (c) stabilizes after two moves
 (d) Assumes following configuration after two moves.

		A	A		A	A
		A	A		A	A
		A	A		A	A

Answer to Additional Problem

1. (c) 1st Move (D: dies)

		A	A				A	A	
	D	A	A	D		D	A	A	D
		A	A				A	A	

2nd Move

		A	A				A	A	
	A	D	D	A		A	D	D	A
		A	A				A	A	

3rd Move

		A	A				A	A	
	A	D	D	A		A	D	D	A
		A	A				A	A	

In fact it stabilizes after two moves.

2. (c) Similarly as above 3. (b) Similarly as above
 4. (d) Similarly as above

Additional Results

1. Let $L = \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} [\cos(m!(\pi x))^{2n}]$ then $L = 1$ when $x \in \mathbb{Q}$ (rational) and $L = 0$, when $x \in \mathbb{Q}$ (irrationals).

2. If $f_n(x) = \frac{x^2}{x^2 + nx + 1}$, $0 < x < 1$, $n = 1, 2, 3, \dots$, then $\{f_n(x)\}$ is monotonically decreasing, $f_n(x) \rightarrow 0$ and $\{f_n(x)\}$ does not converge uniformly.

3. If $f_n(x) = \frac{x^2}{x^2 + (1+nx)^2}$, $0 \leq x \leq 1$, $n = 1, 2, \dots$ then $\{f_n(x)\}$ converges point wise to 0 but does not converge uniformly. Moreover it has no subsequence, which can converge uniformly to 1 on $[0, 1]$.

4. The sequence $\{f_n(x)\}$ where $f_n(x) = \frac{x}{n+x}$ is uniformly convergent in $[0, k]$, $k < \infty$ but only point wise convergent when interval extends to ∞ .

5. The series $\sum f_n(x)$, the sum of whose n terms is

$$S_n(x) = \frac{x}{1+nx^2} \text{ converges uniformly for all real } x.$$

$$6. \text{ Series } \frac{x}{x+1} + \frac{x}{(1+x)(1+2x)} + \frac{x}{(1+2x)(1+3x)} + \dots$$

uniformly converges on $[a, b]$, $a > 1$ but only point wise in $[0, b]$.

7. Sequence $\{f_n(x)\}$ where $f_n(x) = \frac{x}{1+nx^2}$ $x \in \mathbb{R}$ converges uniformly on any closed interval I .