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Comprehensive MCA Entrance Coaching

309 & 313 PRATAP CHAMBERS, GURUDWARA ROAD, KAROL BAGH, DELHI - 110005 PH.: 28757911, 28757630, 32917966

SAI SANMACS PLAZA, PLOT NO. - 6A, COMMUNITY CENTER, DDA, SECTOR - 8, ROHINI, DELHI - 110085 PH.: 32458354, 32522575

e-mail : info@sanmacs.com website: www.sanmacs.com

HINTS AND SOLUTIONS **TEST CODE: MTPO - IV**

1. (b) Let $x_1\ x_2, ..., x_{10}$ be the entries. By data,

$$\frac{x_1+x_2+-----+x_5}{5} = \frac{2}{3}\,\frac{x_6+x_7+---x_{10}}{5} = k$$

$$\frac{x_1+x_2+-----+x_{10}}{10} = \frac{2}{3}\,\frac{5k+\frac{15k}{2}}{10}$$

$$= \frac{25k}{20} \therefore \frac{25k}{20} = 25 \text{ (given)} \Rightarrow k = 20$$

2. (a) S.D. is independent of change of origin (but dependent on change of scale) $\therefore \sigma$

Alternate $\sigma^2 = \dfrac{(a-x)^z+(b-x)^2+(c-x)^2+(d-x)^2}{4}$

$$\sigma^2{}_1 = \frac{1}{4}\sum\left(a+k-\overline{x_1}\right)^2 \quad = \frac{1}{4}\sum(a-x)^2 \left(\because \overline{x_1} = \bar{x}+k\right)$$

$$= \sigma^2 \quad \Rightarrow \quad \sigma_1 = \sigma$$

3. (c) The roots are α , 2α (say) $\therefore 3\alpha = -k$, $2\alpha^2 = k+1$

Eliminating α , we get $2k^2 = 9(k+1)$

4. (d) The asymptotes intersect at the center of hyperbola. The point of intersection of $x+y=5,\ \ x-y=3$ is $(4,\ 1)$

5. (b) (n+1)! = (n+1) n! $\therefore$ L.C.M of n!, (n+1)! = (n+1)!

6. (b) Let the C.P. = x Rs. $\therefore$ S.P = $\dfrac{11x}{10}$ Rs.

Assumed C.P. (20% less than x Rs.) = $\dfrac{80}{100}x$ Rs. Assumed

S.P. = $\left(\dfrac{11}{10}x+10\right)Rs.$ $\therefore$ Profit = S.P – C.P. = $\dfrac{3x}{10}+10$

Profit %= $\dfrac{\frac{3x}{10}+10}{\left(\frac{80}{100}x\right)}\times 100 = 40 \Rightarrow x = 500Rs.$

7. (a) Let x, x+1 be the numbers P = $x\ (x+1)$, S = $x + x + 1$

Now, P= $\dfrac{120}{100}S \Rightarrow 5P = 6S$

$\Rightarrow 5(x^2+x) = 6(2x+1) \Rightarrow x = 2, -3/5 \Rightarrow x = 2$

$[\because x$ can't be non – integer] $\therefore$ S = $2x$ + 1 = 2 (2) + 1 = 5

8. (c) $\left(2+\sqrt{3}\right)^3 = 2^3 + 3.2^2\sqrt{3} + 3.2\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^3$

$= 8 + 12\sqrt{3} + 18 + 3\sqrt{3} = 26 + 15\sqrt{3}$

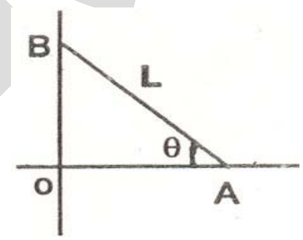
$\therefore a + b\sqrt{3} = 26 + 15\sqrt{3} \Rightarrow a = 26,\ b = 15$

$\therefore a + b = 26 + 15 = 41$

9. (d) I= $\displaystyle\int_3^4 \frac{1}{x(x+1)}dx = \int_3^4\left(\frac{1}{x} - \frac{1}{(x+1)}\right)dx$ (by p.f.)

$= \log(16/15)$

10. (b)



Let $\angle OAB = \theta$ $\therefore$ OA=L $\cos\theta$, OB= L $\sin\theta$

$\Rightarrow$ A (L $\cos\theta$, 0), B (0, L $\sin\theta$)

$\therefore$ Midpoint M of AB = $\left(\dfrac{\angle\cos\theta}{2}, \dfrac{\angle\sin\theta}{2}\right)$

$\Rightarrow$ OM^2 = $\dfrac{L^2}{4}\left(\cos^2\theta + \sin^2\theta\right) \Rightarrow OM = \dfrac{L}{2}$

11. (d) A way of defining 1-1 onto function from A to B is similar to a way of filling up 4 seats with 4 persons $\therefore$ No. of 1-1 onto functions = no. of ways of filling 4 places with 4 persons = ${}^4p_4$ = 24 ways

12. (c) Find highest power of 5 in 23! = $\left[\dfrac{23}{5}\right] = 4$

13. (a) $\dfrac{10x}{100} + \dfrac{20}{100}x^2 = 21 \therefore x = 10$

Now, $\dfrac{20x}{100} + \dfrac{10}{100}x^2 = 2 + 10 = 12$

14. (b) $\dfrac{1}{n^2+n} = \dfrac{1}{n} - \dfrac{1}{n+1}$ (By partial fractions)

$\therefore t_1 = 1 - \dfrac{1}{2}, t_2 = \dfrac{1}{2} - \dfrac{1}{3}, -----, t_n = \dfrac{1}{n} - \dfrac{1}{n+1}$

$\Rightarrow\ S = t_1 + t_2 + ... = 1 - \frac{1}{n+1}\ n \to \infty \therefore S = 1$

15. (d) $\begin{vmatrix} 2 & a \\ a & 8 \end{vmatrix} = 0 \Rightarrow 16 - a^2 = 0\ \ a^2 = 16\ a = -4, 4$

16. (c) 4 lines in direction I
4 lines in direction II
For || gm. select 2 lines from I, 2 lines from II
$= {}^4c_2 \times {}^4c_2 = 36$

17. (a) $\vec{a}, -2\vec{b}$ are collinear $\Rightarrow \vec{a}, \vec{b}$ are also collinear and collinear vectors have same direction ratios.

18. (c) No. of single, double, three, four digit nos. formed by using 6 digits with repetition = $6^1+6^2+6^3+6^4 = 1554$

19. (d) By placing one coin in each pan of balance and other coin outside, we have : if the left or right pan lowers then the coin on the corresponding Pan is faulty. If pans are at same level then coin not put in pan is faulty. $\Rightarrow$ In one weighing we identify faulty coin out of 3. So, if three weighings are permitted we find faulty coin out of 3^3 i.e. 27 coins.

20. (d) Shaded area = Area of rectangle – area of Δ

$= 120 - \frac{1}{2} \times 12 \times 10 = 60$ sq. units

21. (d) $3\sin x + 4\cos x = r\cos\alpha \sin x + r\sin\alpha\cos x$
(assume $3 = r\cos\alpha,\ 4 = r\sin\alpha$)
$= r(\sin x + \alpha) \therefore -r \le r\sin(x+\alpha) \le r$

$-r \le 3\sin x + 4\cos x \le r$ and $r^2 = 3^2 + 4^2 = 5$
range = $(-5, 5)$

22. (d) p (α ,5- α) is the pt. on $x + y = 5$
p is equidistant from (0,0) and (6.6)
$\Rightarrow\ \alpha^2 + (5-\alpha)^2 = (\alpha - 6)^2 + (5 - \alpha - 6)^2$
$\Rightarrow$ 37 = 35, not possible .

23. (a) Imaginary roots occur in pair
$\therefore$ roots are $3+i,\ 3-i,\ \alpha$ (say)
$6 + \alpha = -a,\ (9 - i^2)\alpha = -20 \Rightarrow \alpha = -2 \therefore a = -4$

24. (a) n = no. of subsets of A having 6 elements out of 11 =
${}^{11}c_6 = {}^{11}c_4\ \frac{7}{5}$

m = no. of subsets of A having 4 elements of 11 = ${}^{11}c_4$
$\therefore\ \frac{n}{m} = \frac{7}{5} > 1 \Rightarrow n > m$

25. (a) A (m elements) , B (n elements) f: A $\to$ B
$\Rightarrow\ n^m = 1^4 = 1$

26. (b) $f(x) = x - [x]\ f(n+h) = n + h - [n+h] = n + h - n = h$
where n is integer, $0 < h < 1$
$\therefore\ 0 < f(n+h) < 1 \Rightarrow f(x)$ is not constant
Also, $f(n+h) = f(n+1+h) = f(1+(n+h))$
$\Rightarrow f(x) = f(1+x) \Rightarrow f(x)$ is periodic of period 1.

27. (d) year is leap year $\Rightarrow$ 29 days in Feb. After 19 Feb. we have 10 more days in Feb.
$\therefore$ upto 28 June = 1 (for 19 Feb.) + 10 +31 days of March. + 30 days Apr. +31 days May +28 days June = 131 days = 18 weeks + 5 days (Friday)

28. (c)

Boxes	4	3 ↑
Packets	110	120 ↑
Biscuits	12	11 ↑
Time	12	X ↑

$4 \times 110 \times 12 \times 12 = 3 \times 120 \times 11 \times x\quad x = 16$ hrs.

29. **(b)**

30. (a) $I = \int_0^{\pi/2} \frac{\sin^3\theta - \cos^3\theta}{\sin^5\theta + \cos^5\theta} d\theta$

$$= \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - \theta\right) - \cos^3\left(\frac{\pi}{2} - \theta\right)}{\sin^5\left(\frac{\pi}{2} - \theta\right) + \cos^5\left(\frac{\pi}{2} - \theta\right)}$$

$$= = -\int_0^{\pi/2} \frac{\sin^3\theta - \cos^3\theta}{\sin^5\theta + \cos^5\theta} = -I\ \Rightarrow\ I = 0$$

31. (b)
$(1 + \log_3 2)(1 + \log_6 5) = (\log_3 3 + \log_3 2)(\log_6 6 + \log_6 5)$
$= \log_3 6\ .\log_6 30 = \log_3 30 = \log_3 3 + \log_3 10 = 1 + \log_3 10$

32. (b)

33. (a) $1^2 + 2^2 + 3^2 + 4^2 + ... + (2n)^2 - 2^2 + 4^2 + .. + (2n)^2$

$$= \frac{2n(2n+1)(4n+1)}{6} - \frac{2^2 n(n+1)(2n+1)}{6} = \frac{(2n+1)2n(2n-1)}{6}$$

$$= \underset{n\to\infty}{Lt} \frac{2n(2n-1)(2n+1)}{6n^3} = \frac{1}{3}\left(2 - \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) = \frac{4}{3}$$

34. (c) $x|x| = \begin{Bmatrix} x^2, & x+ve \\ -x^2, & x-ve \end{Bmatrix}$

$\therefore \sum x|x| = -(25+16+9+4+1) + 0 + (25+16+9+4+1) = 0$

35. (d) $\sqrt{\frac{x-1}{x-2}} \ge 0$

if $\frac{x-1}{x-2} = \frac{(x-1)(x-2)}{(x-2)^2} > 0 \Rightarrow (x-1)\ (x-2) > 0$

$\Rightarrow\ x < 1,\ x > 2 \Rightarrow (-\infty, 1) \cup (2, \infty)$

36. (b) 300 votes divided equally among 5 contestants
$\Rightarrow$ each has 60 votes. To win by minimum votes one contestant has 1 more vote = 61 votes

37. (d) $(a-b)^2 \ge 0 \Rightarrow a^2 + b^2 \ge 2ab$
$\therefore a^2 + b^2 + c^2 + d^2 + p^2 + q^2 \ge 2(ab + cd + pq)$
$\Rightarrow\ \frac{3}{2} \ge ab + cd + pq$

38. (a) Points of intersection of graph of line $x = 1$, $x = 2$, $y = 3$ and $y = 5$ are A (1, 3), B (2, 3), C (1, 5), D (2, 5)

$\therefore$ Values of 2x+3y at A, B, C, D=11, 13, 17, 19 Then Bounds are end values 11, 19

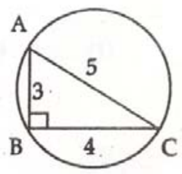
39. (c) $\sin x + \sin y = 2\sin\frac{x+y}{2}.\cos\frac{x-y}{2}$

$= \cos\frac{x-y}{2}$ $(\because x + y = 60°) \le 1$

40. (b) $|x-2| \le 4 \Rightarrow -4 \le x-2 \le 4 \Rightarrow -2 \le x \le 6$

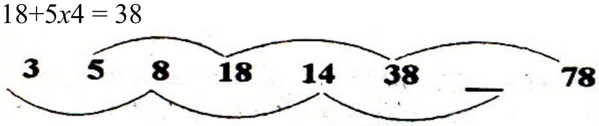
Also, $x^2 \le 16 \Rightarrow -4 \le x \le 4$ $-2 \le x \le 4$

41. (c)



Circum circle of right Angled Δ = Circle on the hypotenuse of right. Angled Δ as diameter $\therefore$ r = 2.5

42. (b) odd sequence : 3
3+5x1 = 8
8+5x2 = 18
18+5x4 = 38



even sequence : 5
5+3x1 = 8
8+3x2 = 14
14+3x4 = 26

43. (a) $\alpha-\delta, \alpha, \alpha+\delta$ are roots $\therefore 3\alpha = 9 \Rightarrow \alpha = 3$

Hence $\alpha^2 - 9(3)^2 + a(3) + b = 0 \Rightarrow 3a + b = 54$

44. (d) $\frac{1}{2}+\frac{1}{3}+\frac{1}{N} < 1 \Rightarrow \frac{1}{N} < \frac{1}{6} \Rightarrow N > 6$, many natural nos. satisfying it.

45. (c) collinear if 1 + 2 + t = 0 $\Rightarrow$ t = –3

46. (d) G.C.D of 48, 72, 240 = 24
$\therefore x + 7 = 24 \Rightarrow x = 17$, not divisible by any of choices.

47. (d) α, β, φ be cube roots of unity

$\therefore\ \alpha + \beta + \varphi = 1 + \omega + \omega^2 = 0$

$\alpha^5 + \beta^5 + \varphi^5 = 1 + \omega^5 + \omega^{10} = 0$

48. (b) $\frac{\delta u}{\delta x} - 2x\ f^{1}\left(x^2+y^2\right), \frac{\delta u}{\delta y} = 2y\ f^{1}\left(x^2+y^2\right)$

$x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 2\left(x^2+y^2\right)f^{1}\left(x^2+y^2\right)$

49. (c) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...$

$\therefore \sin x + \cos x = 1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + ...$

50. (a)

51. (b) Telephone (electronic) mode of communication while Letter (non-electronic) mode of communication. Similarly, is phonograph (electronic), Manuscript (non electronic)

52. (c) Nose is a part of face and sticks out; Knob is a part of door and sticks out.

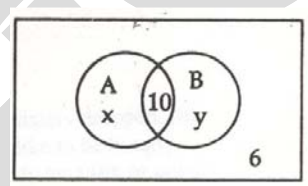
53. (b) Let perimeter of each fig. is 12 π units
$\therefore$ side of equilateral $\Delta = 4\ \pi$
side of square = 3 π and radius of circle = 6

$\therefore$ E $= \frac{4\sqrt{3}}{4}16\pi^2, S = 9\pi^2, C = 36\pi$

$\Rightarrow$ E : S : C = $\sqrt{3}\pi : 9\pi : 36$

Clearly, $4\sqrt{3}\pi < 9\pi < 36 \Rightarrow E < S < C$

54. (b)



Let Total = Z A $\Rightarrow x$ +10 B $\Rightarrow y$ + 10

$z = x+10+y+6-(1).\ \ x+10 = \frac{3}{2}-(2)\ \ \ y+10 = \frac{2z}{3}-(3)$

$\Rightarrow x = 2,\ y = 6,\ z = 24$

55. (b) A vector along AB = $\overrightarrow{j}$ $\overrightarrow{OA} = \overrightarrow{ai}$

The vector eq. AB is $\overrightarrow{r} = \overrightarrow{ai} + \overrightarrow{tj}$

56. (c) III term = ${}^{n}c_2 a^{n-2}\ b^2\ x^2$ IV term = ${}^{n}c_3 a^{n-3} b^3 x^3$

${}^{n}c_2 a^{n-2} b^2 = {}^{n}c_3\ a^{n-3}\ b^3 \Rightarrow nb = 3a + 2b$

Directions (57 – 60): Explanation: Let us write the given conditions (i) to (iv) as follows: (i) D = A – 3 (ii) B is the middle term (iii) B – F = C – D > 0 (iv) G > F
Using (ii) we can write the position of B as --------- B ---------
Using (i) we infer that A>B. Otherwise there would not be any D such that D = A – 3. Further A cannot be the 7th number as in such a case A – 3 = B $\ne$ D contradicting (i) again.
_ _ _ BA _ _ (or) _ _ _ B _ A _ Using (i) we get the position of D also. $\underbrace{\_D\_BA\_\_}_{\text{I say}}$ (or) $\underbrace{\_\_DB\_A\_}_{\text{II say}}$

Using (iii) we infer the following in I and II:
(I) If C is in the 3rd position, then C – D = 1 but there would be no F such that B – F = 1.
If C is in 6th or 7th position then C – D = 4 or 5 again implying that there is no F such that B – F = 4 or 5. Obviously C cannot be in Ist position since C is greater than D. Thus, I is completely dropped.
(II) C must be in 5th or 7th position since C > D. If C is 7th, C – D = 4 but there would be no F such that B – F = 4. Therefore, C must be 5th and C – D = 2. $\Rightarrow$ F, must be 2nd as only then B – F = 2. We get __ FDBCA __. Using

(iv) G must necessarily be the 7th and so E is the Ist.
Therefore, the numbers in increasing order are EFDBCAG

57. (d) **58. (a)** **59. (b)** **60. (c)**

61. (c) $9 < x^2 + y^2 < 25 \Rightarrow$ region between two concentric circle of radii 3, 5 units. **62. (a)**

63. (b) $\underset{n\to 2}{Lt} \dfrac{e^x - e^2}{x-2} = \underset{n\to 2}{Lt}\ e^x = e^2$

64. (c) $x^2 = 4 + 2\sqrt{3} = \left(1+\sqrt{3}\right)^2 \Rightarrow x = 1+\sqrt{3},\ \ x > 0$

$\therefore x^3 = \left(4+2\sqrt{3}\right)\left(1+\sqrt{3}\right) = 10 + 6\sqrt{3}$

65. (d) Total outcomes = $6^3 = 216$
Non favourable events = (1, 1, 1), (6, 6, 6) = 2
$\therefore$ required probability = $\dfrac{214}{216} = \dfrac{107}{108}$

66. (b) **67. (c)** **68. (c)**

69. (d) Two horizontal arrows appear on different sides of vertical arrows in all other figures.

70. (a) **71. (c)** **72. (b)** **73. (b)**
74. (b) **75. (d)** **76. (c)** **77. (a)**
78. (b) **79. (c)** **80. (a)** **81. (b)**

82. (c) $\dfrac{\text{History}}{\text{Maths}} = \dfrac{60}{80} = \dfrac{3}{4}$ For Candidates C

83. (a) E highest in History , Economics , Sociology

84. (d) 340 (sociology)

85. (b) $\dfrac{80-75}{80} \times 100 = 6.25\%$ more

86. (a) average of A $\to 76$ B $\to 57.8$ C $\to 65.6$, D $\to 58, 2,$
E $\to 69.4$ $\because$ 76-57.8 = 18.2%

87. (d) Arrow moves clock wise 90°, circular moves diagonally.

88. (b) Outer block moves clock wise and then inner arrows interchanged .

89. (c) Figure moves clockwise and then semicircle is interchanged with adjacent triangle. **90. (c)**

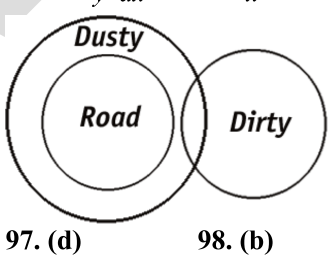
91. (a) I.F.$= e^{\int p dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

92. (a) $\left(D^2 - (2+3)D + 2\times 3\right)y = 0 \ \ \left(D^2 - 5D + 6\right)y = 0$

93. (b) Let $x\,y = 1 \because \dfrac{x+y}{2} \geq \sqrt{xy} \Rightarrow x + y \geq 2$

94. (a) $1 + p + p^2 + ...$ converges if $|\ p\ | < | \Rightarrow ||\ 2x-1\ | < |$
$-1 < 2x - 1 < 1 \Rightarrow 0 < x < 1$

95. (b) $\log y = \lambda x$ $\quad \dfrac{1}{y}\dfrac{dy}{dx} = \lambda = \dfrac{\log y}{x} \Rightarrow x\left(\dfrac{dy}{dx}\right) = y(\log y)$



96. (d) **97. (d)** **98. (b)**

99. (a) x : car, y : motor cycle
$4x + 2y = 30 + 2\,(x + y)$
$2x = 30 \Rightarrow x = 15$