

M.C.A ENTRANCE

UP MOCK: HINTS AND SOLUTIONS

SECTION: A

1. (c) Peano's Axioms Italian mathematician G. Peano gave the following postulates regarding natural numbers in 1859.

P - 1. $1 \in \mathbb{N}$ i.e. 1 is a natural number

P - 2 For every $n \in \mathbb{N}$, \exists a unique natural number n^+ , called the successor of n , i.e. $n^+ = n + 1, n \in \mathbb{N}$

P - 3. 1 is not the successor of any natural number

P - 4 Two natural number having the same successors are identical, i.e.

if $m, n \in \mathbb{N}$, then $m^+ = n^+ \Rightarrow m = n$

P - 5. If $A \subset \mathbb{N}$ be such that

(i) $1 \in A$ and (ii) $m \in A \Rightarrow m^+ \in A$ for each $m \in A$, then

$A = \mathbb{N}$ (This axiom is known as the principle of finite induction).

2. (c) This relation is an equivalence relation. Consequently it decomposes the set into disjoint equivalence classes. Let us denote the quotient set of $\mathbb{N} \times \mathbb{N}$ by I , i.e., I is the union of all equivalence classes

of $\mathbb{N} \times \mathbb{N}$ relative to the relation \sim . Any element of I is called an integer. (This is the definition of integers through equivalence relation).

3. (c) 4. (a) The horizontal range $R = \frac{u^2 \sin 2\alpha}{g}$

is maximum when $\sin 2\alpha$ is maximum [$\because u$ and g are fixed quantities, only α varies]

i.e. $\sin 2\alpha = 1 \Rightarrow 2\alpha = \pi/2 \Rightarrow \alpha = \pi/4$

5. (d) Angular velocity $= \frac{2\pi}{T} \text{ rad/sec} = \frac{2\pi}{3600} \text{ rad/sec} = \frac{\pi}{1800} \text{ rad/sec}$.

6. (c) This is an equivalence relation. Therefore, the relation decomposes that $I \times I_0$ into mutually disjoint classes. Let us denote the quotient set by Q . Any element of Q is called a rational number (This is definition of rational number through equivalence relation).

7. (c) g.c.d (12, 16) = 4 which does not divide 5, hence no solution (The congruence $ax \equiv b \pmod{m}$ has a solution iff the g.c.d of a and m divides b)

8. (c) Consider $A = \{x : x \geq 1\}$ which is bdd. Below and $B =$

$\{x : x \leq 1\}$ which is bdd above. But $A \cup B = \mathbb{R}$ is neither bdd. above nor bdd. below. 9. (a) 10. (d)

11. (a) The rational effect of a force on a body about a point is called the moment of the force about the point. It may be clockwise or anticlockwise.

12. (a) The necc. And suff. Condition, that a system of forces acting at a point be in equilibrium, are that the algebraic sums of the resolved parts of the forces along three mutually perpendicular directions should be zero separately.

13. (a)

14. (c) Comparing $y = 5x + 1$ with $y = mx + \frac{r}{m}$.

15. (b) $P(A) = .4 \therefore$ Required Prob. $= 1 - [P(\bar{A})]^3$

16. (d) Given series is G. P. with $a = x, r$

$= -x$. SUM is $y = \frac{x}{1+x}$ solve for x .

17. (d) $f(y/x) = \cos[\log(y/x)] = \cos[\log y - \log x]$

$f(xy) = \cos[\log xy] = \cos[\log x + \log y]$

Now substitute and get the value 0.

18. (c) $p = 1/5, q = 4/5 \therefore$ Reqd.

Prob. $= \sum_{r=3}^5 {}^5C_r (1/5)^r (4/5)^{5-r}$ 19. (b) 20. (d)

21. (a) $\sigma^2 = \frac{\sum n^2}{n} - \left(\frac{\sum n}{n}\right)^2$ 23. (c) $\cos \theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| |\bar{b}|} = 60^\circ$

22. (a) 23. (c)

24. (c) $T_n = \frac{1+2+2^2+\dots+2^{n-1}}{n!} = \frac{1(2^n-1)}{(2-1)(n!)}$

$= \frac{2^n-1}{n!}$ [sum of n terms of a G. P.]

$\therefore T_n = \frac{2^n}{n!} - \frac{1}{n!}$ putting $n = 1, 2, 3, \dots$

$T_1 = \frac{2}{1!} - \frac{1}{1!}, T_2 = \frac{2^2}{2!} - \frac{1}{2!}, T_3 = \frac{2^3}{3!} - \frac{1}{3!}, \dots$

$\therefore S = \left(\frac{2}{1!} + \frac{2^2}{2!} + \dots\right) - \left(\frac{1}{1!} + \frac{1}{2!} + \dots\right)$

$= [(e^2 - 1) - (e - 1)] = e^2 - e$.

(Adding +ve and -ve terms separately) 25. (b)

26. (b) Put $\theta = \tan^{-1} x$

27. (c) $T_{r+1} = (-1)^r {}^{15}C_r x^{45-5r}$. This term is constant when

$$45 - 5r = 0 \Rightarrow r = 9 \therefore T_{10} = {}^{-15}C_9 = {}^{-15}C_6.$$

$$28. (d) \Delta = 0 \Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0.$$

Now apply $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ and Simplify.

29. (a) $np = 4, npq = 3 \Rightarrow q = 3/4, p = 1/4$. Now mode is an integer x , such that $np + p > x > np - q$ 30. (b)

31. (c) If n coins be tossed, then the prob. of n head $= \frac{1}{2^n}$ and prob. of

$$\text{getting only 1 head} = {}^nC_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} = \frac{n}{2^n}.$$

\therefore The Prob. of getting atleast 2 heads is

$$1 - \left(\frac{1}{2^n} + \frac{n}{2^n}\right) = 1 - \left(\frac{n+1}{2^n}\right) > .99 \Rightarrow .01 > \frac{n+1}{2^n}.$$

$$\text{Now if } n = 10 \text{ then } \frac{10+1}{2^{10}} = .0107... (> .01)$$

$$\text{If } n = 11 \text{ then } \frac{11+1}{2^{11}} = .0058... (< .01)$$

\therefore 11 coins atleast should be tossed.

32. (c) Let A, B, C be the 3 points. Then it can be shown that $AB + BC + CA = 0$

$$\Rightarrow AC = AB + BC$$

\therefore A, B, C are in a line or show that

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0 \Rightarrow \text{collinear.}$$

33. (c) 34. (c)

35. (d) The slopes of tangents from (x_1, y_1) are given by

$$m^2 (x_1^2 - a^2) - 2x_1 y_1 m + y_1^2 + b^2 = 0. \text{ Let the slops be } m_1 \text{ and}$$

$$m_2. \text{ Now given that } \theta_1 + \theta_2 = 90^\circ \Rightarrow \theta_1 = 90^\circ - \theta_2$$

$$\Rightarrow \tan \theta_1 = \cot \theta_2 \Rightarrow \tan \theta_1 \tan \theta_2 = 1$$

$$\Rightarrow \frac{y_1^2 + b^2}{x_1^2 - a^2} = 1$$

36. (c) Given fraction

$$\begin{aligned} &= \frac{15}{\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}} \\ &= \frac{15}{3\sqrt{10} - 3\sqrt{5}} \cdot \frac{3\sqrt{10} + 3\sqrt{5}}{3\sqrt{10} + 3\sqrt{5}} \\ &= \sqrt{10} + \sqrt{5} = \sqrt{5}(\sqrt{2} + 1) \end{aligned}$$

37. (c)

$$38. (c) I = \int_{0.5}^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx + \int_3^4 3 dx + \int_4^{4.5} 4 dx + 2$$

$$\int_0^1 |x| dx = 0 + 1 + 2 + 3 + 4(0.5) + 2 \int_0^1 x dx = 9.$$

39. (a) The given series can be written as $\tan^{-1} (1/3) +$

$$\tan^{-1} (1/7) + \dots = \tan^{-1} \frac{2-1}{1+2}$$

$$+ \tan^{-1} \frac{3-2}{1+3-2} + \dots + \tan^{-1} \frac{(n+1)-n}{1+n(n+1)}$$

$$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2)$$

$$+ \dots + (\tan^{-1} (n+1) - \tan^{-1} (n))$$

$$= \tan^{-1} (n+1) - \tan^{-1} n = \tan^{-1} \frac{n}{n+2}$$

40. (a) The no. of possible outcomes $= 5^4 = 625$.

$$41. (c) \text{LHL} = -\sqrt{2}, \text{RHL} = \sqrt{2}$$

$$\therefore \lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{2} |\sin (x-1)|}{(x-1)}$$

42. (b) x, y, z are in G.P. $\Rightarrow y^2 = xz \Rightarrow 2 \log y = \log x + \log z$

$$\Rightarrow 2(1 + \log y) = (1 + \log x) + (1 + \log z)$$

(adding 2) $\Rightarrow 1 + \log x, 1 + \log y, 1 + \log z$ are in A.P.

\therefore Their reciprocals are in H.P.

($\because x > 1 \Rightarrow 1 + \log x \neq 0$ etc.)

43. (a)

$$44. (c) \text{ Given parabola is } x^2 + y^2 = \left[\frac{x+y-1}{\sqrt{2}} \right]^2$$

$$\Rightarrow \sqrt{x^2 + y^2} = \frac{x+y-1}{\sqrt{2}} \text{ i.e. locus of a point whose distance from } (0,$$

0) is equal to the distance from the line $x+y-1=0$ \therefore focus is (0, 0)

45. (d) The log of a function $f(x)$ is defined only for positive values of $f(x)$. Therefore $\log f(x)$ is defined for the values of x given in (c) but for these value of x , $\sin x < 1$ $\therefore \log \sin x$ is < 0 . Thus the given function is defined for no real value of x .

46. (c) $\frac{(2n)!}{(n!)}$. The desired expression is the term independent of x in the

product $(1+x)^n (1+\frac{1}{x})^n$ or in $\frac{1}{x^n} (1+x)^{2n}$ which is nothing but the

term containing x^n in $(1+x)^{2n}$. The coeff. Of x^n in $(1+x)^{2n}$ is

$${}^{2n}C_n = \frac{(2n)!}{n!n!}.$$

47. (a) S.D. remains the same.

48. (c)

$$49. (c) A = 3 + \frac{4}{A} \Rightarrow A = -1 \text{ or } A = 4 \text{ But } A \text{ is not } < 0 \therefore A = 4 \text{ Also}$$

$$B = 3 + \frac{B}{4} \Rightarrow B = 4 \therefore A = B = 4.$$

50. (b) Squaring the given expression, we get $2 |\vec{OA}| |\vec{OB}| \cos \theta < 0$

$$\Rightarrow \cos \theta < 0 \Rightarrow \theta > 90^\circ$$

$$51. (a) \vec{a} \times \vec{b} = 5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}, |\vec{a} \times \vec{b}| = 5\sqrt{3}.$$

Unit vector along $\vec{a} \times \vec{b} = \frac{5(i+j+k)}{5\sqrt{3}} = \frac{\sqrt{3}(i+j+k)}{3}$. Hence desired

$$\text{vector} = \frac{5\sqrt{3}(i+j+k)}{3}.$$

52. (c) Equation of directrix.

53. (b) $f'(x) = \sin(5x^2 + 3x)^2 (10x + 3) - 0$

54. (d) Integrand $F(x) = f(x) - f(-x)$

$$\therefore F(-x) = [f(-x) - f(-(-x))] = -F(x) \Rightarrow F(x) \text{ is odd. } \pi/2$$

$$\therefore \int_{\pi/2}^{\pi/2} F(x) dx = 0$$

55. (a) $P(A \cap B^c) = P(A) - P(A \cap B)$

$$= P(A) - P(A) P(B) [\because A, B \text{ are independent}]$$

$$= P(A) [1 - P(B)] = P(A) P(B^c).$$

$\therefore A$ and B^c are independent.

56. (b)

57. (a) $\sigma_x^2 = \frac{\sum x^2}{n} = \frac{90}{n}, \quad \text{cov}(x, y) = \frac{\sum xy}{n} = \frac{60}{n}, \therefore r^2 = 0.64 =$

$$\left[\frac{\text{cov}(x, y)}{\sigma_x^2 \sigma_y^2} \right]^2 \Rightarrow n = 10.$$

58. (b) Put $x^2 = t$

59. (a) $T_n = \frac{n^2}{n!} = \frac{n}{(n-1)!} = \frac{(n-1)+1}{(n-1)!}$

$$= \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$$

$$\therefore \sum T_n = \sum \frac{1}{(n-2)!} + \sum \frac{1}{(n-1)!} = e + e = 2e$$

60. (c) Integrating by parts, taking θ as the first function, we have

$$\left[\theta \frac{\sin^3 \theta}{3} \right]_0^{\pi/2} - \int_0^{\pi/2} (1) \frac{\sin^3 \theta}{3} d\theta$$

$$= -\frac{2}{3} \int_0^{\pi/2} \sin^3 \theta d\theta = -\frac{4}{9}.$$

61. (c) The $(r+1)^{\text{th}}$ term in the expansion of

$$\left[\left(\frac{3}{2} \right) x^2 - \left(\frac{1}{3x} \right) \right]^9 \left(-\frac{1}{3x} \right)^r$$

$$= {}^9C_r (-1)^r \frac{3^{9-2r}}{2^{9-r}} x^{18-3r} \quad (1)$$

To find the coeff. Of the term independent of x in the expansion of $(1+x$

$$+ 2x^3) \left[\frac{3}{2} x^2 - \frac{1}{3x} \right]^9 \quad (2), \text{ we must get the coeff. of } x^0, x^{-1} \text{ and } x^{-3}$$

in (1). For x^0 , $r=6$, for x^{-1} , there is no value of r and for x^{-3} , $r=7$.

Therefore the desired coeff. is

$$1 \cdot {}^9C_6 (-1)^{-6} \frac{3^{9-12}}{2^{9-6}} + 2 \cdot {}^9C_7 (-1)^7 \frac{3^{9-14}}{2^{9-7}} = \frac{17}{54}$$

62. (d) Using $\alpha + \beta = -p$, $\alpha\beta = 1$,

$\gamma + \delta = -q$, $\gamma\delta = 1$, we have

$$(\alpha - \beta)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = (\gamma^2 + p\gamma + 1)(\delta^2 - p\delta + 1) \quad (1)$$

Now since γ and δ are roots of $x^2 + qx + 1 = 0$, we have $\gamma^2 + 1 = -q\gamma$,

$\delta^2 + 1 = -q\delta$, substitute in (1) and get the answer $q^2 - p^2$

63. (d) For inconsistency, $p(A) < p(A/B)$.

64. (a) Let $L_1 = a_1x + b_1y + c_1 = 0$ meets x axis in $P\left(-\frac{c_1}{a_1}, 0\right)$ and y

axis in $Q\left(0, -\frac{c_1}{b_1}\right)$ and $L_2 = a_2x + b_2y + c_2 = 0$ meets x axis in

$R\left(-\frac{c_2}{a_2}, 0\right)$ and y axis in $S\left(0, -\frac{c_2}{b_2}\right)$. Since P, Q, R, S are concyclic,

$$\therefore OP \cdot OR = OQ \cdot OS \Rightarrow a_1a_2 = b_1b_2.$$

65. (c) $\frac{bc}{ad} = \frac{b+c}{a+d} \Rightarrow \frac{1}{d} - \frac{1}{c} = \frac{1}{b} - \frac{1}{a} = A$ (say)

$$\text{Also } \frac{bc}{ad} = \frac{3(b-c)}{(a-d)} \Rightarrow \frac{1}{d} - \frac{1}{a} = 3\left[\frac{1}{c} - \frac{1}{b}\right]$$

$$\Rightarrow \frac{1}{c} + A - \frac{1}{a} = 3\left(\frac{1}{c} - \frac{1}{a} - A\right)$$

$$\Rightarrow A = \frac{1}{c} - \frac{1}{a} - A$$

$$\Rightarrow A = \frac{1}{c} - \frac{1}{a} - \left(\frac{1}{b} - \frac{1}{a}\right) = \frac{1}{c} - \frac{1}{b}$$

$$\therefore A = \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c} \therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$$

are in A.P. $\Rightarrow a, b, c, d$ in H.P.

66. (d) According to the given information

$$a + b + c + d = (\alpha + 1)d = (\beta + 1)a$$

$$\text{Now } a \neq -1, \text{ we have } d = \left(\frac{\beta + 1}{\alpha + 1}\right)a$$

$$\Rightarrow a + b + c = \alpha d = \alpha \left(\frac{\beta + 1}{\alpha + 1}\right)a$$

$$\Rightarrow \left[1 - \alpha \left(\frac{\beta + 1}{\alpha + 1}\right)\right]a + b + c = 0 \Rightarrow a, b, c \text{ are coplanar, a contradiction}$$

$$\therefore \alpha = -1 \therefore a + b + c + d = 0.$$

67. $U_n = \int_0^{\pi/2} \frac{1 - \cos 2nx}{1 - \cos 2x} dx$

Let $n = 1$, then $U_1 = \pi/2$. Now consider the given determinant.

$$\Delta = \begin{vmatrix} U_1 & U_2 & U_3 \\ U_4 & U_5 & U_6 \\ U_7 & U_8 & U_9 \end{vmatrix} \text{ Applying } C_1 \rightarrow C_1 - 2C_2 + C_3, \text{ we get}$$

$$\Delta = \begin{vmatrix} U_1 - 2U_2 + U_3 & U_2 & U_3 \\ U_4 - 2U_5 + U_6 & U_5 & U_6 \\ U_7 - 2U_8 + U_9 & U_8 & U_9 \end{vmatrix}$$

$$\text{Now } U_n + U_{n-2} - 2U_{n+1} =$$

$$\int_0^{\pi/2} \frac{2 \cos(2n+2)x - \cos(2n+4)x - \cos 2nx}{1 - \cos 2x} dx$$

$$= \int_0^{\pi/2} 2 \cos(2n+2)x dx = 0.$$

$$\therefore \Delta = \begin{vmatrix} 0 & U_2 & U_3 \\ 0 & U_5 & U_6 \\ 0 & U_8 & U_9 \end{vmatrix} = 0.$$

$$68. (c) \tan 89^\circ = \tan(90^\circ - 1^\circ) = \cot 1^\circ$$

$$\tan 88^\circ = \cot 2^\circ \dots$$

$$\therefore \tan 1^\circ \tan 2^\circ \dots \tan 88^\circ + \tan 89^\circ$$

$$= \tan 1^\circ \dots \tan 45^\circ \cot 44^\circ \dots \cot 1^\circ = 1.$$

$$69. (c) I = \int_{-2}^2 ax^3 dx + \int_{-2}^2 bx dx + \int_{-2}^2 c dx$$

$$= cx]_{-2}^2 = 4c [\because x^3 \text{ and } x \text{ are odd functions}]$$

$$70. (c) \text{ Let } I_n = \int_0^{\pi} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta$$

$$\therefore I_{n-1} = \int_0^{\pi} \frac{\sin^2(n-1)\theta}{\sin^2 \theta} d\theta$$

$$\therefore I_n - I_{n-1} = \int_0^{\pi} \frac{\sin(2n-1)\theta}{\sin \theta} d\theta$$

$$\text{Also } \int_0^{\pi} \frac{\sin n\theta}{\sin \theta} d\theta$$

$$= \pi, \text{ if } n \text{ is odd (standard result)}$$

$$= 0, \text{ if } n \text{ is even}$$

$$\therefore \text{ from (1) } I_n - I_{n-1} = \pi$$

$$\therefore I_n = \pi + I_{n-1}$$

$$= \pi + \pi + I_{n-2} = \dots = (n-1)\pi + I_1$$

$$\text{where } I_1 = \int_0^{\pi} \frac{\sin^2 \theta}{\sin^2 \theta} d\theta = \pi$$

$$\therefore I_n = (n-1)\pi + \pi = n\pi.$$

$$71. (c)$$

72. (b) functions which are not algebraic are called Transcendental functions. The exponential, logarithmic, trigonometric and inverse trigonometric are all transcendental.

$$73. (d) P(X+Y \geq 1) = 1 - P(X+Y < 1)$$

$$= 1 - P(X+Y = 0)$$

$$= 1 - P(X=0)P(Y=0)$$

$$= 1 - {}^5C_0 \left(\frac{1}{2}\right)^5 \cdot {}^7C_1 \left(\frac{1}{2}\right)^7 = \frac{4095}{4096}$$

$$74. (c) P(X=4) = P(X=2)$$

$$\therefore {}^6C_4 p^4 (1-p)^2 = {}^6C_2 p^2 (1-p)^4$$

$$\Rightarrow 9 = \left(\frac{1-p}{p}\right)^2 \Rightarrow p = \frac{1}{4}.$$

$$75. (b) \text{ Using } \frac{T_{r+1}}{T_r} = \frac{c_r}{c_{r-1}} = \frac{n-r+1}{r}, \text{ we have } r \frac{c_r}{c_{r-1}} = n-r+1$$

Putting $r = 1, 2, 3, \dots, n$ and adding, we get

$$\frac{c_1}{c_0} + 2 \frac{c_2}{c_1} + \dots + n \frac{c_n}{c_{n-1}} = \sum n = \frac{n(n+1)}{2}$$

$$76. (c) \frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz} = t \text{ (say)}$$

$$\Rightarrow p \cong \frac{a-x}{tx} = \frac{1}{t} \left(\frac{a}{x} - 1\right) \text{ Similarly } q = \frac{1}{t} \left(\frac{a}{y} - 1\right), r = \frac{1}{t} \left(\frac{a}{z} - 1\right)$$

$$\text{Since } q = \frac{1}{t} \left(\frac{a}{y} - 1\right), r = \frac{1}{t} \left(\frac{a}{z} - 1\right)$$

Since p, q, r are in A. P.

$$\therefore \frac{1}{t} \left(\frac{a}{x} - 1\right), \frac{1}{t} \left(\frac{a}{y} - 1\right), \frac{1}{t} \left(\frac{a}{z} - 1\right) \text{ are also in A.P.}$$

$$\Rightarrow \frac{a}{x} - 1, \frac{a}{y} - 1, \frac{a}{z} - 1 \text{ in A.P.} \Rightarrow \frac{a}{x}, \frac{a}{y}, \frac{a}{z} \text{ in A.P.} \Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ in A.P.}$$

$$\Rightarrow x, y, z \text{ in H.P.}$$

$$77. (a) \text{ Let } e^{sm \cdot x} = y. \text{ Then the given equation is}$$

$$y^2 - 4y - 1 = 0 \Rightarrow y = 2 \pm \sqrt{5} \Rightarrow e^{\sin x} = 2 \pm \sqrt{5} \Rightarrow e^{\sin x} = 2 + \sqrt{5}$$

$$[\because \exp. \text{ functions are } > 0] \Rightarrow \sin x = \log_e (2 + \sqrt{5}) > 1$$

$$[\because e > 1 \text{ and } 2 + \sqrt{5} > e]. \text{ Which is not possible?}$$

$$78. (a)$$

$$79. (b) P(B) = 1 - P(B) = \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 5/6 = P(A) + \frac{1}{2} - \frac{1}{3} \Rightarrow P(A) = 2/3$$

$$\therefore P(A)P(B) = P(A \cap B).$$

$$80. (c) \text{ Let } \left(t, \frac{1}{t}\right), t \neq 0 \text{ be any point on } xy = 1. \text{ Differentiating } xy = 1,$$

w.r.t. x we get

$$\left(\frac{dy}{dx}\right)_{(t, 1/t)} = -\frac{1}{t^2} \therefore \text{slope of the normal is } t^2. \text{ But it is given to be}$$

$$-b/a.$$

$$\therefore t^2 = -b/a \therefore a, b \text{ must be of opposite signs. } \therefore a < 0 \text{ and } b > 0.$$

$$81. (b) \text{ Number of terms in the expansion of } (x+y+z)^n, \text{ where}$$

$$n \in I^+ \text{ is } \frac{(n+1)(n+2)}{2}.$$

$$\therefore \frac{(n+1)(n+2)}{2} = 45 \Rightarrow n = 8.$$

$$82. (a) \quad 83. (b)$$

$$84. (b) \lim_{x \rightarrow \tan^{-1} 3} \frac{\tan^2 x - 2 \tan x - 3}{\tan^2 x - 4 \tan x + 3} =$$

$$\lim_{x \rightarrow \tan^{-1} 3} \frac{(\tan x - 3)(\tan x + 1)}{(\tan x - 3)(\tan x - 1)} = 2.$$

$$85. (a) \text{ Put } x = \tan \theta$$

86. (a) For the equations to be consistent, we must have

$$\rho(A) = \rho(A|B) = 2$$

$$\Rightarrow \lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda = 1 \text{ or } \lambda = 2$$

87. (b) 88. (b)

$$89. (d) \log y = \lim_{x \rightarrow 0} (\cot^2 x) (\log \cos x)$$

$$= \lim_{x \rightarrow 0} \frac{\log \cos x}{\tan^2 x} (0/0).$$

$$= \lim_{x \rightarrow 0} \frac{(1/\cos x)(-\sin x)}{2 \tan x \sec^2 x} = -\frac{1}{2}$$

$$\therefore y = e^{-1/2}$$

90. (b) 91. (a)

$$92. (c) \rho(A) = \rho(A|B) = 2 < 3.$$

93. (c) The equation of bisector of the given equation is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{(-h)}. \text{ Since } y = mx \text{ is a bisector } \therefore (x^2 - m^2 x^2)h =$$

$$(b - a)x.m \Rightarrow (1 - m^2)h = m(b - a).$$

94. (d) General equation of second degree represents two parallel straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0.$$

$$h^2 = ab.$$

$$95. (c) \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}}$$

$$= \sqrt{2 + \sqrt{2 + 2 \cos^2 2\theta}} = \sqrt{2 + 2 \cos 2\theta}$$

$$= \sqrt{4 \cos^2 \theta} = 2 \cos \theta$$

$$96. (d) \log_{10} x^2 = \log_{10} 10 + \log_{10} \sqrt{y} \Rightarrow x^2 = 10\sqrt{y} \Rightarrow y = \frac{1}{25}$$

97. (a) The given expression can be written as

$$\log_{10} \left[(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ) \dots (\tan 45^\circ) \right]$$

$$= \log_{10} (1.1 \dots 1) = \log_{10} 1 = 0.$$

98. (d) Given expression is equal to

$$(\cot 16^\circ \cot 44^\circ - 1) + (\cot 44^\circ \cot 76^\circ - 1)$$

$$- (\cot 76^\circ \cot 16^\circ + 1) + 3$$

$$= \cot(44^\circ + 16^\circ) [\cot 44^\circ + \cot 16^\circ]$$

$$+ \cot(44^\circ + 76^\circ) [\cot 76^\circ + \cot 44^\circ]$$

$$- \cot(76^\circ - 16^\circ) [\cot 76^\circ - \cot 16^\circ] + 3$$

$$= \cot 60^\circ [\cot 44^\circ + \cot 16^\circ] + \cot 120^\circ$$

$$[\cot 76^\circ + \cot 44^\circ] - \cot 60^\circ [\cot 16^\circ - \cot 76^\circ] + 3$$

$$= \frac{1}{\sqrt{3}} [\cot 44^\circ + \cot 16^\circ - \cot 76^\circ - \cot 44^\circ - \cot 16^\circ + \cot 76^\circ + 3] = 3$$

$$99. (d) a \cos A = b \cos B \Rightarrow \sin A \cos A =$$

$$\cos(A + B) = 0 \Rightarrow A + B = 90^\circ \therefore \Delta \text{ is right angled.}$$

$$\sin(A - B) = 0 \Rightarrow A = B \text{ so } \Delta \text{ is isosceles.}$$

100. (d) First law is called law of inertia.

101. (d) 102. (a) 103. (b) 104. (c) 105. (c)

SECTION: B SOLUTIONS:

106. (b) Paper cost = 10%, Printing cost = 35% their percentage ratio

$$= \frac{10}{35} \times 100 = 28.6 \text{ (app.)}$$

107. (c) Difference between Binding and cutting charges and Royalty = 18% - 15% = 3%. If 4% is equivalent to Rs. 6,000, then 3% is equivalent to

$$= \text{Rs. } \frac{6,000}{4} \times 3 = \text{Rs. } 4,500$$

108. (d) Central angle for paper Cost (10%)

$$= \frac{360^\circ \times 10}{100} = 36^\circ$$

109. (b) If Printing cost (35%) = Rs. 17,500. Then Royalty paid (15%)

$$\text{Rs. } = \frac{17,500}{35} \times 15 = \text{Rs. } 7,500$$

110. (b) Central angle of Printing cost

$$= \frac{360^\circ \times 18}{100} = 64.8^\circ$$

Hence, difference between their central angles

$$= 126^\circ - 64.8^\circ = 61.2^\circ$$

111. (b)

112. (c) "Night" is opposite of "Day". Similarly "Cruel" is the opposite of "Kind".

113. (c) 114. (b)

115. (d) First and third letters in a term are moved 2 steps backward whereas second and fourth are moved 2 steps forward.

116. (c) One is the outcomes of the other.

$$\sin B \cos B \left[\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \right]$$

$$\Rightarrow \sin 2A = \sin 2B \Rightarrow 2 \cos(A + B) \sin(A - B) = 0$$

$$\Rightarrow \cos(A + B) = 0 \text{ or } \sin(A - B) = 0$$

117. (a) 118. (c)

119. (d) Food relieves hunger as sleep relieves weariness.

120. (c) "Dark" and "Light" are antonyms so are "HOT" and "COLD".

121. (d) "Writer" creates a "book". "Composer" creates a "Song".

122. (d) The question pair is a noun (person) and descriptive adjective. The parts of speech of the correct answer pair must also be the same as that of the question pair. Hence the correct answer is (d).

Enemy : Bad
(Noun) (Adjective)

123. (a) A pair of scissors cuts clothes as an axe cuts wood.

124. (b) "Exempt" means free from obligation or liability. "Obligate", means the opposite. The relationship is opposite. "Immune" means protected from and "Susceptible" means the opposite

125. (c) Worker and Article relationship

126. (b) Soldiers form part of a Regiment as coil forms a part of motor. This is part and whole relationship.

127. (c) 128. (c)

129. (a) All other are internal organs.

130. (c) All except (c) have 31 days.

131. (d) All the other words are synonyms.

132. (d) All other are plants and challenger is space vehicle.

133. (d) Remaining forms part of a normal nuclear family

(134-137) Forward sequence pattern. 134. (b) 135. (a) 136. (b) 137. (d) 138-141

$$138. (a) 8 \times 3 = 24, 4^2 = 16, 4^2 + 3^2 = 25 \text{ (observing different figures)}$$

139. (c) 140. (c) 141. (a) 142. (a)

$$142. 2^2 - 1, 6^2 - 1, 10^2 - 1, 14^2 - 1, 18^2 - 1, \dots$$

143. (d) Each number is multiplied by 5 and then one is added to the result.

144. (b) The number are written according to the rule,

$$1 \times 3, 3 \times 5, 15 \times 7, 105 \times 9, 945 \times 11, \dots$$

146. (c) The last two colours combined make the first one.

147. (b) 148. (b) 149. (d) 150. (c)