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IA 5

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An electric field in a region of vacuum is described by $\vec{E} = \hat{x} 1000 \cos(2\pi \times 20 \times 10^6 t - kz)$

a.) no, the wave is not uniform because it varies by a value of k in the positive z direction.

b.) it is a plane wave on the xy plane, but it travels on the xz plane.

c.) it travels in the $+z$ direction, and since it is in a vacuum it travels at the speed of light $3 \cdot 10^8$ m/s

d.) $k = 2\pi \cdot 20 \cdot 10^6 (3 \cdot 10^8) = 3.77$ (chat says 4.19)

$$\lambda = 2\pi / 3.77 = 1.6$$

e.) $\vec{E}(z) = \hat{x} 1000 e^{-jkz}$ ← incorrect after checking the book
chat said $\vec{E}(z) = \hat{x} 1000 e^{-jkz}$ but i'm also not sure that is correct.

f.) $B = \mu_0 H$ so in this case $B = \mu_0 H$

According to the RHR the Magnetic field H is in the \hat{y} direction.

so

$$\vec{H} = \hat{y} 1000 \cos(2\pi \cdot 20 \cdot 10^6 t - kz) \text{ A/m}$$
$$\vec{B} = \hat{y} 1000 \cos(2\pi \cdot 20 \cdot 10^6 t - kz) \text{ T}$$

Chat Conversation:

dont answer anything yet but this is the equation you will use for the quesions



Got it! Feel free to share the equation whenever you're ready.

An electric field in a region of vacuum is described by $E = \hat{x} * 1000 * \cos(2\pi * (20 * 10^6) t - kz)$ (V/m)



Thanks for sharing the equation! This is the expression for an electric field, \mathbf{E} , in a vacuum. It is described as a wave propagating in the z -direction with time t , and the field is oscillating in the x -direction.

The given equation is:

$$\mathbf{E}(t, z) = \hat{x} 1000 \cos(2\pi(20 \times 10^6)t - kz) \text{ V/m}$$

where:

- \hat{x} is the unit vector in the x -direction.
- 1000 is the amplitude of the electric field in volts per meter (V/m).
- 20×10^6 Hz is the frequency of the wave (20 MHz).
- t is time.
- k is the wavenumber (which relates to the wavelength).
- z is the position in the direction of wave propagation.

Please go ahead with your questions related to this equation!

The first 3 questions were simple and the answer was basically given in lecture so we did not feel the need to ask chat about them.

is the value of k 3.77?



Let's check whether the wavenumber k is 3.77 rad/m .

The wavenumber k is related to the frequency f and the speed of light c in vacuum by the following formula:

$$k = \frac{2\pi f}{v_p}$$

Where:

- $f = 20 \times 10^6 \text{ Hz}$
- $v_p = c = 3 \times 10^8 \text{ m/s}$ (speed of light in vacuum)

Now, let's calculate k :

$$k = \frac{2\pi(20 \times 10^6)}{3 \times 10^8}$$

Simplifying:

$$k = \frac{2\pi \times 20 \times 10^6}{3 \times 10^8} = \frac{40\pi}{3 \times 10}$$
$$k \approx \frac{125.66}{30} \approx 4.19 \text{ rad/m}$$

Conclusion:

The value of k is approximately 4.19 rad/m , not 3.77 rad/m .



is the wavelength 1.6



Let's calculate the wavelength λ of the wave using the relationship between the wavenumber k and the wavelength:

$$k = \frac{2\pi}{\lambda}$$

We already calculated $k \approx 4.19 \text{ rad/m}$. Now let's find the wavelength λ :

$$\lambda = \frac{2\pi}{k}$$

Substituting $k = 4.19 \text{ rad/m}$:

$$\lambda = \frac{2\pi}{4.19} \approx \frac{6.283}{4.19} \approx 1.5 \text{ m}$$

Conclusion:

The wavelength λ is approximately **1.5 meters**, not **1.6 meters**. However, 1.6 meters is a close approximation.

how would you describe the phasor of the electric wave



The **phasor** representation of a wave provides a way to describe its time-varying nature as a complex exponential, focusing on its amplitude and phase. It simplifies the analysis of oscillating fields like the electric field in the given equation.

Given Wave Equation:

$$\mathbf{E}(t, z) = \hat{x} 1000 \cos(2\pi(20 \times 10^6)t - kz) \text{ V/m}$$

To convert this into its **phasor** form, we follow these steps:

Phasor Representation:

The electric field in the time domain is expressed as a real cosine function. In phasor form, we use the complex exponential representation:

$$\cos(\omega t - kz) = \Re \left(e^{j(\omega t - kz)} \right)$$

where:

- $\omega = 2\pi f$ is the angular frequency,
- k is the wavenumber,
- j is the imaginary unit, and
- $\Re(\cdot)$ denotes the real part.

Thus, the time-dependent electric field can be expressed as:

$$\mathbf{E}(t, z) = \Re \left(\hat{x} 1000 e^{j(2\pi(20 \times 10^6)t - kz)} \right)$$

Phasor Form:

The phasor form represents only the spatial and amplitude dependence (removing the time factor), so the phasor $\tilde{\mathbf{E}}(z)$ is:

$$\tilde{\mathbf{E}}(z) = \hat{x} 1000 e^{-jkz}$$

- The amplitude of the electric field is 1000 V/m .
- The phase shift is given by $-kz$, which describes how the wave varies as it propagates in the z -direction.

Summary of the Phasor:

- **Amplitude:** 1000 V/m
- **Phase shift:** $-kz$
- **Direction:** The phasor is aligned with the x -axis (\hat{x}) and describes a wave propagating along the z -axis with a phase shift of $-kz$.

In phasor notation, the wave is simply $\tilde{\mathbf{E}}(z) = \hat{x} 1000 e^{-jkz}$, and the time-varying part can be introduced back by multiplying with $e^{j\omega t}$.

since this is in a vacuum does $H = B$?

In a vacuum, the magnetic field \mathbf{H} and the magnetic flux density \mathbf{B} are related by the permeability of free space μ_0 , and they are **not equal**. The relationship between \mathbf{H} and \mathbf{B} is:

$$\mathbf{B} = \mu_0 \mathbf{H}$$

Where:

- \mathbf{B} is the magnetic flux density in **teslas (T)**,
- \mathbf{H} is the magnetic field intensity in **amperes per meter (A/m)**,
- $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ is the permeability of free space.

Since μ_0 is a constant with a value $4\pi \times 10^{-7} \text{ H/m}$, the magnetic field intensity \mathbf{H} and the magnetic flux density \mathbf{B} are proportional but not equal.

Relation:

$$\mathbf{B} = (4\pi \times 10^{-7}) \mathbf{H}$$

So, for any given \mathbf{H} , you would multiply by μ_0 to get \mathbf{B} .

Thus, $\mathbf{H} \neq \mathbf{B}$ in a vacuum, but they are related by the permeability of free space μ_0 .

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We think that is wrong here, we know they are not technically equal and are different vectors, but the permeability of free space does not matter in a vacuum so they should both just be the original formula in the \hat{y} direction, with H being in A/m and B being in T as units. Chat seems to struggle with the vacuum concept. We will try arguing with it.

does permeability matter in a vacuum? it shouldnt



Permeability does indeed matter in a vacuum, but it plays a specific role in understanding the relationship between the magnetic field intensity \mathbf{H} and the magnetic flux density \mathbf{B} .

Key Points:

1. Vacuum Properties:

- In a vacuum, the permeability μ_0 is a constant value given by $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$.
- This constant characterizes how the magnetic field interacts with the vacuum itself, but it does not mean that magnetic fields do not exist in a vacuum.

2. Relationship Between \mathbf{H} and \mathbf{B} :

- The relationship $\mathbf{B} = \mu_0 \mathbf{H}$ indicates that the magnetic flux density \mathbf{B} in a vacuum is proportional to the magnetic field intensity \mathbf{H} .
- Thus, \mathbf{B} and \mathbf{H} are not the same, but they are directly related through the permeability constant.

3. Importance of μ_0 :

- Even in a vacuum, the value of μ_0 is essential for calculations involving electromagnetic waves, especially in determining how electric and magnetic fields relate to one another in wave propagation.
- It also plays a crucial role in determining the speed of electromagnetic waves in a vacuum, which is given by the relation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, where ϵ_0 is the permittivity of free space.

Maybe we were wrong, the free space implies a vacuum, I had a misunderstanding about that concept. The fields given by chat were:

- The magnetic field intensity \mathbf{H} is:

$$\mathbf{H}(t, z) \approx \hat{y} 2.65 \cos(2\pi(20 \times 10^6)t - kz) \text{ A/m}$$

- The magnetic flux density \mathbf{B} is:

$$\mathbf{B}(t, z) \approx \hat{y} 3.33 \times 10^{-6} \cos(2\pi(20 \times 10^6)t - kz) \text{ T}$$

Which seem correct now in hindsight. We will keep our above answers and the whole discussion in place though to show our process.