Summary - Stochastic Modelling

By: Sanne Jamila Razmara Olsen

Preliminaries

Regular Matrix: If there exists an integer $k > 0$ such that all of the elements in P^k are greater or equal to zero ($P_{ij}^k \geq 0$), we call P and the Markov chain $\{X_n\}$ regular.

Discrete Markov Chains

Limiting Distributions: In a regular Markov chain, the limiting distribution $\vec{\pi} = (\pi_0, \pi_1, ...\pi_N)$ gives the long-run mean fraction of time spent in each state. We have

$$P\vec{\pi} = \vec{\pi} \qquad\qquad \sum_{i=0}^{N} \pi_i = 1$$

Communication: If states i and j are accessible from each other, they are said to communicate.

Reducible/Irreducible: A Markov chain is **irreducible** if communication induces exactly one equivalence class. If not, its is **reducible**.

Period: The period of state i is $d(i) = \gcd(n \geq 1 : P_{ij}^{(n)} \geq 0)$. If $d(i) = 1$, we say that state i is **aperiodic**.

If all states communicates with each other, all states have the same period.

Recurrent or Transient: State i is **recurrent** if the probability of returning to state i in a finite number of time steps is one. A state that is not recurrent, is **transient**.

Theorem 4.4: In a positive recurrent aperiodic equivalence class with states $j = 0, 1...$ we have

$$1.\ \lim_{n\to\infty} P_{ij}^{(n)} = \vec{\pi}_j = \sum_{i=0}^{n} \pi_i P_{ij}$$

$$2.\ \sum_{i=0}^{n} \pi_i = 1$$

Poisson Processes

Birth-death Processes

Guassian Processes