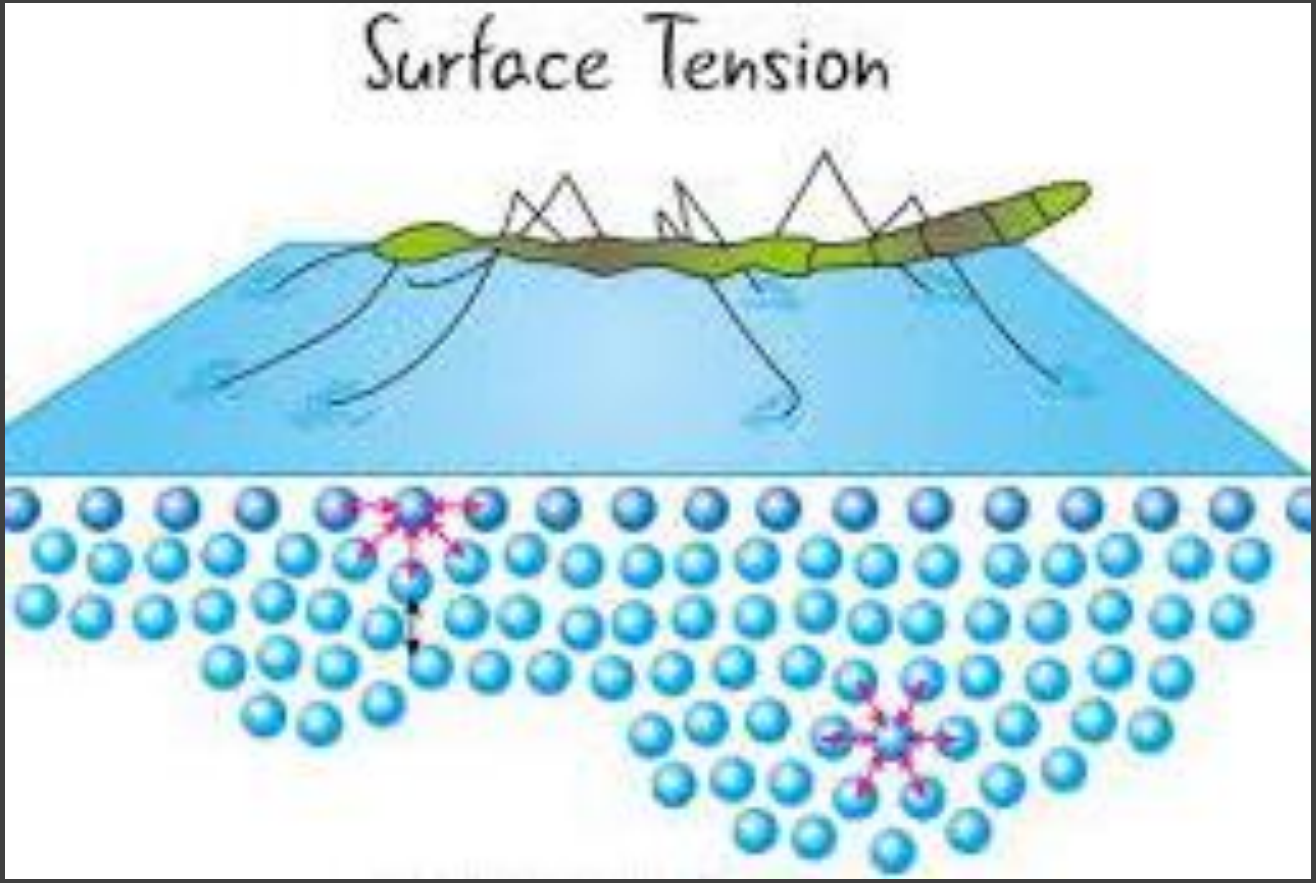
PHY 101

GENERAL PHYSICS I

Surface Tension & Capillarity

Course Content



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Cohesion and Adhesion in Liquids

Cohesive forces are attractive forces between molecules of the same type.

For example liquids can be held in open containers because cohesive forces hold the molecules together.

Adhesive forces are attractive forces between molecules of different types.

Such forces cause liquid drops to cling to window panes.

One of the effects directly attributable to cohesive and adhesive forces in liquids is surface tension.

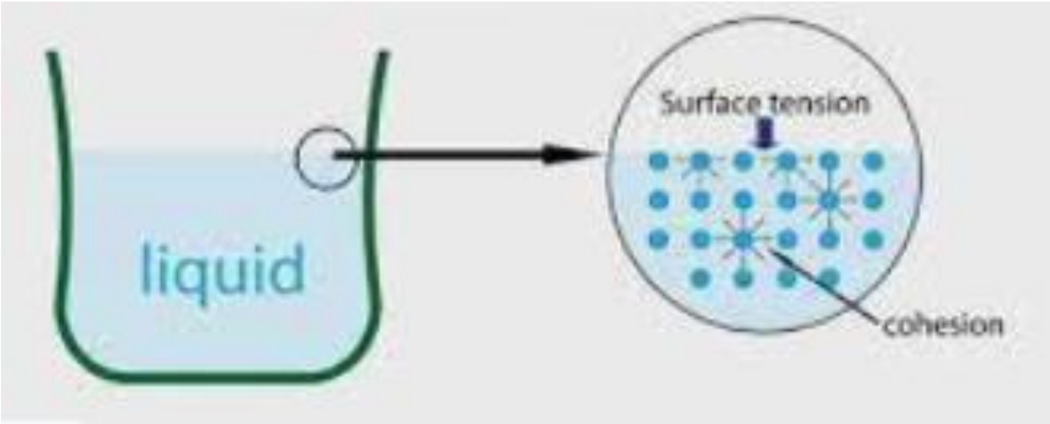
What is surface tension?

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension.**

Surface tension is sometimes called capillarity.

It is the force such as Van der Waals forces at the interphase between either a liquid and a gas or between two liquids which are immiscible i.e., they don't mix.

This is due to the unbalance force of cohesion between molecules near the interface.



Molecules that are embedded in the liquid which are far from the surface are subject to forces due to the molecules that surrounds them.

These forces are pulling on the molecules on all directions; thus, all the forces cancels out. Hence the net force on that molecule is zero.

However, molecules near the surface are subject to cohesive forces pulling it inward because no external force to counteract it at the surface, thus reducing the surface area.

Therefore, those inward net forces causes the surface to contract and these unbalanced force is what gives rise to the surface tension force.

This is what makes the surface act like a stretched membrane.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid.

Surface tension σ is defined to be the surface force F per unit length L exerted by a stretched liquid membrane along which the force acts.

It is given as

$$\sigma = F/L \ (N/m)$$

where

- F is the force per unit length
- L is the length in which force act
- σ is the surface tension of the liquid

Table 1: Surface Tension of Some Liquids

Liquid	Surface tension γ(N/m)
Water at 0°C	0.0756
Water at 20°C	0.0728
Water at 100°C	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37°C	0.058
Blood plasma at 37°C	0.073
Gold at 1070°C	1.000
Oxygen at −193°C	0.0157
Helium at −269°C	0.00012

Example 1

Compute the surface tension of a given liquid whose dragging force is 7 N and length in which the force acts is 2 m?

- *Solution*

Given,

$$\boldsymbol{F} = 7\,N$$
$$\boldsymbol{L} = 2\,m$$

According to the formula,

$$\boldsymbol{\sigma} = \boldsymbol{F}/\boldsymbol{L}$$

$$\Rightarrow \sigma = 7/2$$

$$\Rightarrow \sigma = 3.5\,N/m$$

Examples of Surface Tension

Water striders, which are small insects, can walk on water as their weight is considerably less to penetrate the water surface.

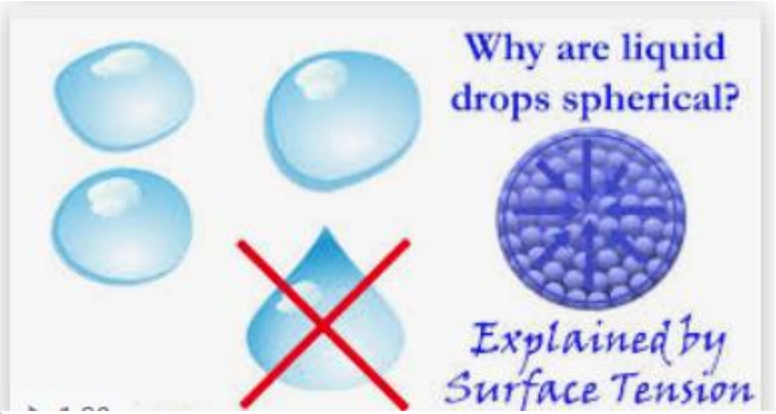
Like this, there are various examples of surface tension which are found in nature.

Some cases are provided below:

- Insects walking on water
- Floating a needle on the surface of the water.
- Rainproof tent materials where the surface tension of water will bridge the pores in the tent material
- Clinical test for jaundice

Examples continues

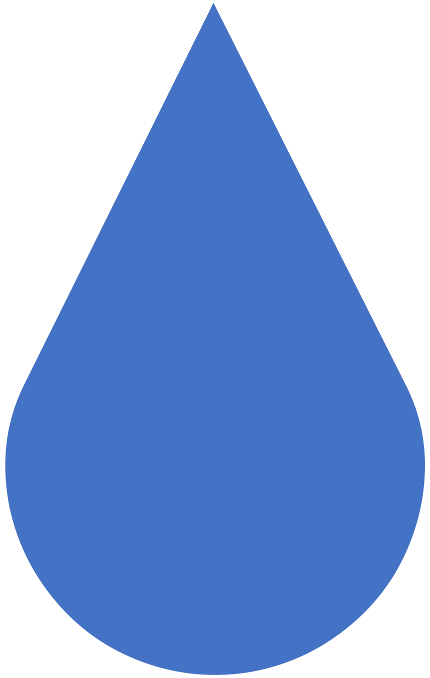
- Surface tension disinfectants (disinfectants are solutions of low surface tension).
- Cleaning of clothes by soaps and detergents which lowers the surface tension of the water.
- Washing with cold water.
- Round bubbles where the surface tension of water provides the wall tension for the formation of water bubbles.
- This phenomenon is also responsible for the shape of liquid droplets.



Methods of Measurement

Some methods of measurement of surface tension are given below:

- Spinning drop method
- Pendant drop method
- Du Noüy–Padday method
- Du Noüy ring method
- Wilhelmy plate method
- Pendant drop method
- Stalagmometric method
- Capillary rise method
- Bubble pressure method
- Resonant oscillations of a spherical and hemispherical liquid drop
- The vibrational frequency of levitated drops



Excess Pressure Inside the Water Bubble

Surface tension is the reason why liquids form bubbles and droplets.

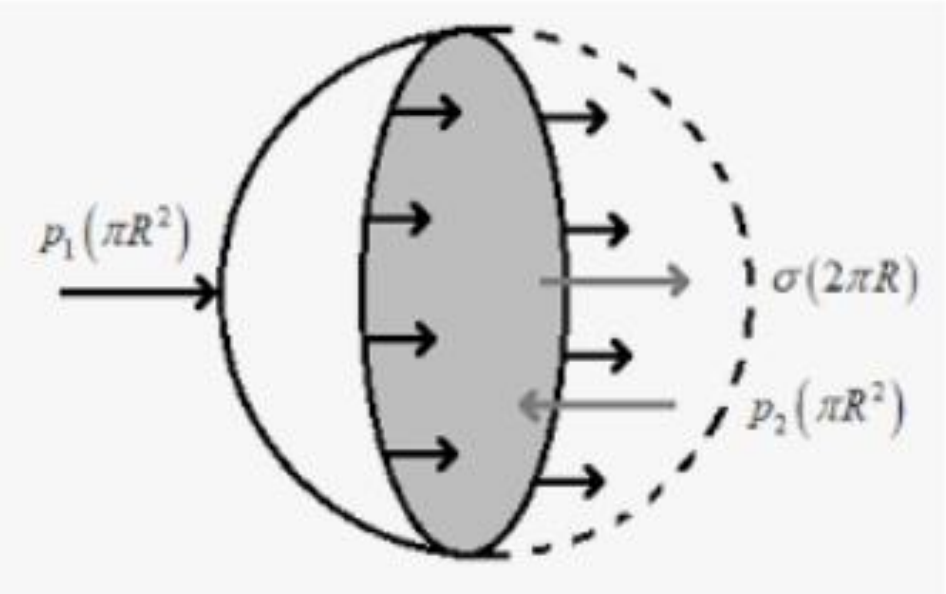
The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside.

Hence, to balance the surface tension force, there must be an excess pressure inside **the liquid drop with only one surface** e.g., a droplet of water in air cut into two hemispheres.

The forces acting on the hemisphere is shown in the Figures.

Suppose the pressure outside is P_1

And the pressure inside is $P_2 = P_1 + \Delta p$



The pressure difference is necessary to counterbalance the surface tension force.

The surface tension σ, acts around the perimeter of the circle.

The magnitude of the surface tension force is $\sigma 2\pi R$ (where $\sigma 2\pi R$ is the circumference of the bubble)

Therefore, force balance is $force\ inside\ = force\ outside$

$$(P_1 + \Delta p)\pi R^2 = P_1 \pi R^2 + \sigma 2\pi R$$

$$\Rightarrow \quad \Delta p = \frac{2\sigma}{R}$$

Surface tension forces are often small compared to other forces.

What about a hollow spherical soap bubble in air? In this case, the soap has two surfaces - inside and out.
The excess pressure inside a **hollow spherical soap bubble** is

$$\Delta p = \frac{4\,\sigma}{R}$$

Both these formulae show that the excess pressure within a small bubble is greater than that within a larger bubble.

Also, the excess pressure inside a **non-spherical drop** is given as

$$P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Example 2

Question

Calculate the gauge pressure inside a soap bubble $2.00 \times 10^{-4}\ m$ in radius using the surface tension for soapy water in Table 1. Convert this pressure to mm Hg.

Solution

The radius is given, and the surface tension can be found in Table 1, and so P can be found directly from the equation

$$\Delta p = \frac{4\sigma}{R}$$

$$= \frac{4(0.0370\ N/m)}{2.00 \times 10^{-4}\ m}$$

$$= 740\ N/m^2 = 740\ Pa$$

Convert to mm Hg:

$$P = 740\ N/m^2 \left(\frac{1.00\ mm\ Hg}{133\ N/m^2}\right)$$

$$= 5.56\ mm\ Hg$$

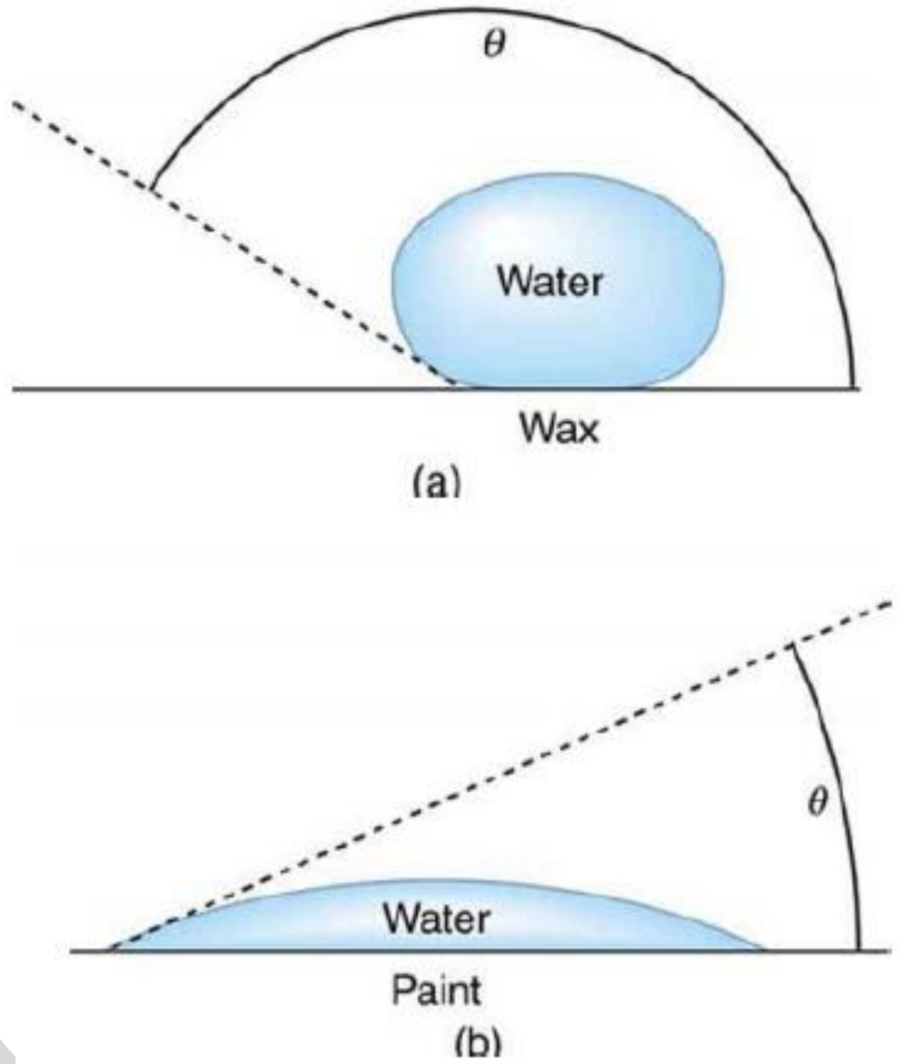
Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint?

The answer is that the adhesive forces between water and wax are much smaller than those between water and paint.

Competition between the forces of adhesion and cohesion are important in the macroscopic behaviour of liquids.

An important factor in studying the roles of these two forces is the contact angle θ between the tangent to the liquid surface and the surface.



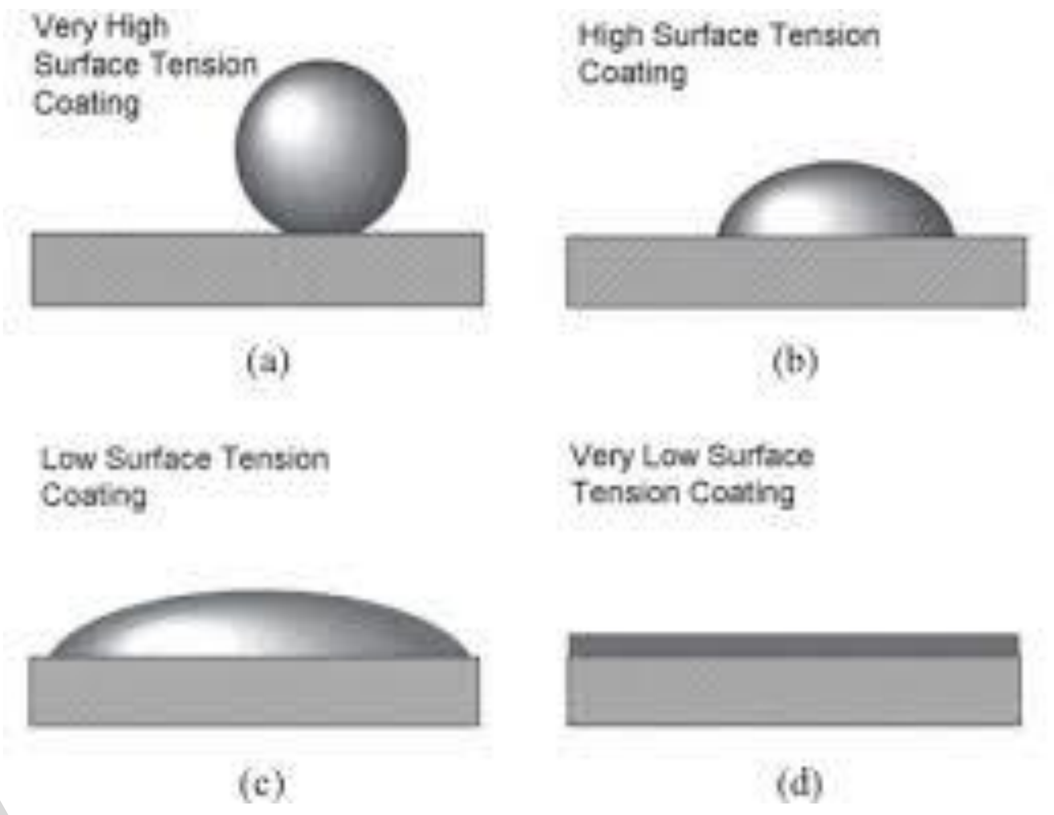
The Contact Angle

The contact angle θ is the angle between the tangent to the liquid surface and the surface.

It is directly related to the relative strength of the cohesive and adhesive forces.

The larger the strength of the cohesive force relative to the adhesive force, the larger θ is, and the more the liquid tends to form a droplet (Fig. a & b).

The smaller θ is, the smaller the relative strength, so that the adhesive force is able to flatten the drop (Fig. c & d).



θ = contact angle

If $\theta < 90^\circ$; liquid wets the surface

If $\theta > 90^\circ$; liquid is non – wetting

e.g., Mercury – Air – Glass; $\theta = 140^\circ$

Table 2: Contact angle of some substances

Interface	Contact angle Θ
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

Capillary Action

Capillary Action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube.

This is a phenomenon of surface tension.

The rise and fall of a liquid surface in a small narrow tube, when it is immersed vertically in a liquid is called capillary action (capillarity) depending on the combination of substances.

The rise of the liquid surface in the tube is called **capillary rise**.

The fall of the liquid surface is called **capillary depression**. The capillary rise and fall of the liquid is expressed in terms of cm or mm of liquid.

The value of capillary rise and fall depends upon the following:

- specific weight of the liquid,
- diameter of the tube,
- surface tension of the liquid,
- relative strength of the cohesive, and adhesive forces and,
- the contact angle θ.

Note

If θ is less than $90°$, then the fluid will be raised;

if θ is greater than $90°$, it will be suppressed.

For example, mercury has a very large surface tension, hence, a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop.

The curved surface of a fluid in a tube is called a meniscus.

The tendency of surface tension is always to reduce the surface area.

Surface tension thus flattens the curved liquid surface in a capillary tube.

This results in a downward force in mercury and an upward force in water.

Application of Capillarity

- In plants, the rise of water from the roots to all its parts takes place because of capillary action.
- The capillary action draws ink to the tips of a fountain pen from cartridge (reservoir) inside the pen.
- The towels that we use after taking bath, absorb water from our body because of capillary action.
- Sponge which has larger number of small pores acts as small capillaries and absorbs a large amount of water.
- The cotton clothes that we wear in hot summer day shows capillary action and absorbs all our body sweat and maintains the temperature of the body to normal.

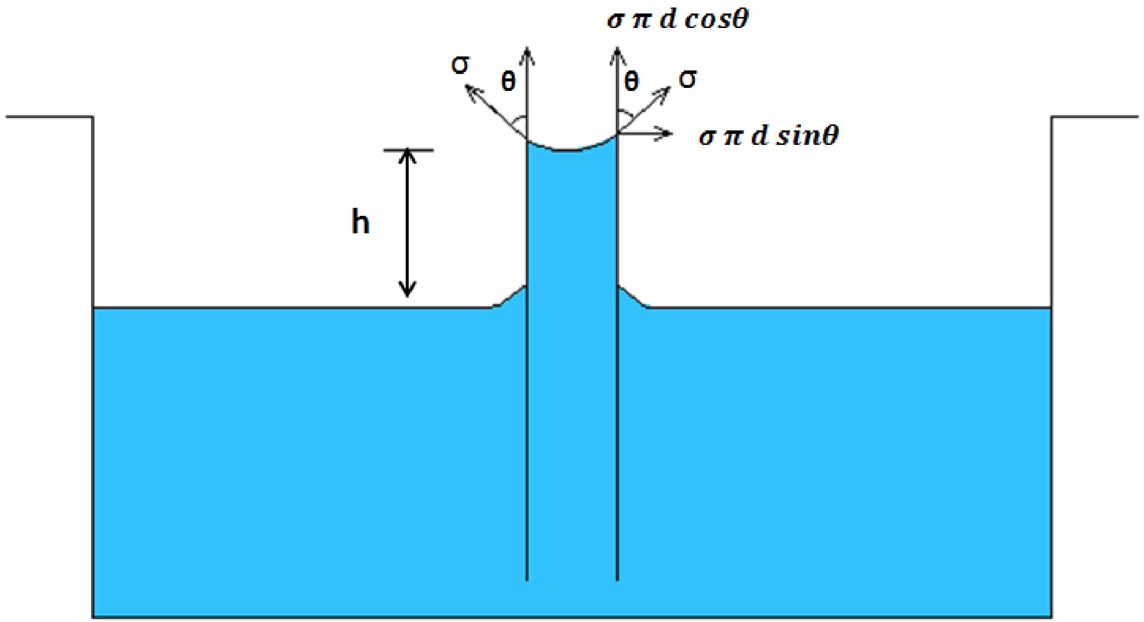
Capillary Rise

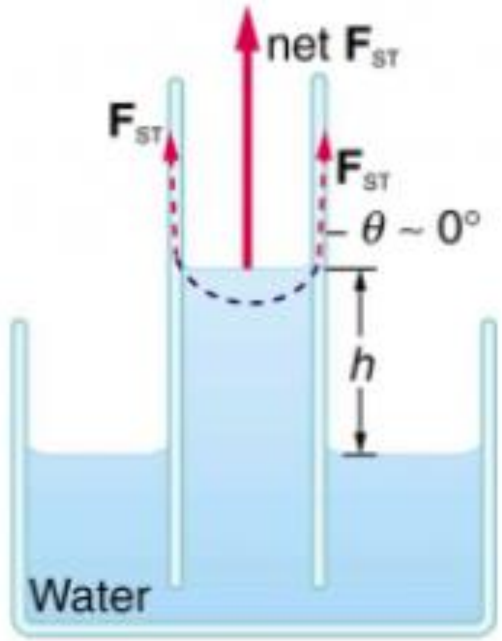
Take a small tube of diameter 'd' opened at both ends inserted in a liquid say water. Gradually the rise of the liquid takes above the level of the liquid.

Let h be the height of the liquid that rises in the tube.

Under equilibrium condition, the weight of the liquid of height h is balanced by the forces at the surface of the liquid.

But the force that acts at the surface of the liquid is due to surface tension of the liquid.





Let

$$\sigma = Surface\ tension\ of\ the\ liquid.$$
$$\theta = Angle\ of\ contact\ between\ liquid\ and\ glass\ tube.$$

The weight of liquid of height h in the tube

$$= mg = \rho vg = \rho(area\ of\ tube\ \times h)g$$
$$= \rho(\pi r^2 \times h)g = \rho\left(\frac{\pi}{4}d^2 h\right)g \quad (where\ r = {}^{d}/_{2})$$

where

m is the mass of liquid in tube

v is the volume of liquid in the tube

The vertical component of the surface tensile force

$$= (\sigma \times circumference)\cos\theta = (\sigma \times 2\pi r)\cos\theta = \sigma\pi d\cos\theta$$

Under equilibrium condition, the weight of the liquid in the tube is balanced by the vertical component of the surface tensile force. i.e.

$$\mathit{The\ weight\ of\ liquid\ in\ the\ tube} \\ = \mathit{vertical\ component\ of\ the\ surface\ tensile\ force}$$

$$\rho\left(\frac{\pi}{4}d^2h\right)g = \sigma\pi d\cos\theta$$

$$h = \frac{4\,\sigma\cos\theta}{\rho g d}$$

The value of θ in between clean glass tube and water is nearly equal to zero.

Capillary fall

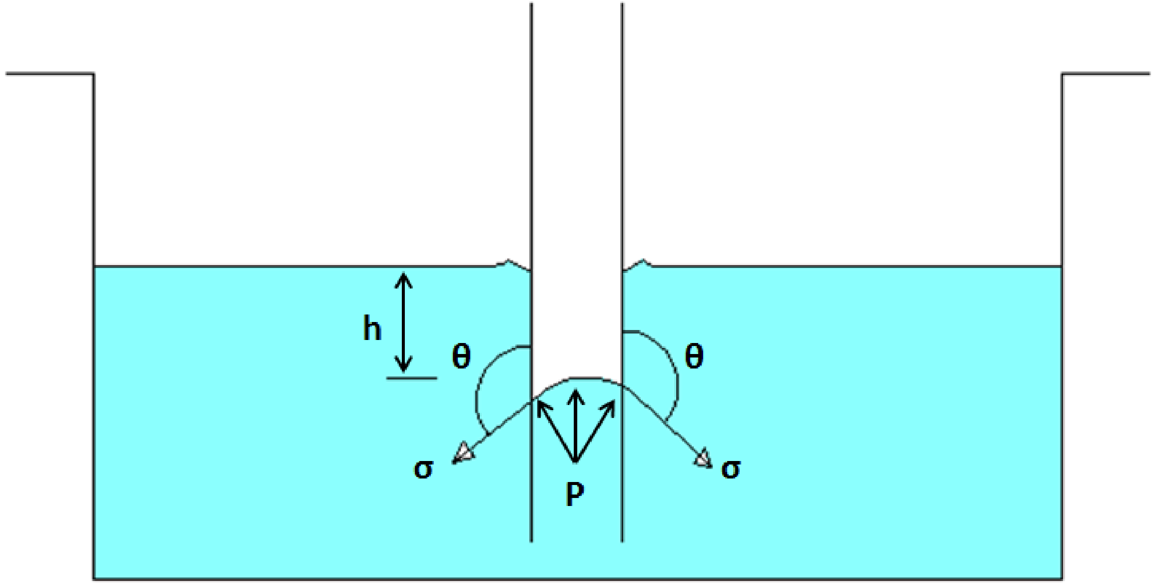
When the glass tube is dipped in mercury than instead of rising, the level of mercury in the tube goes down than the normal level of the outside liquid.

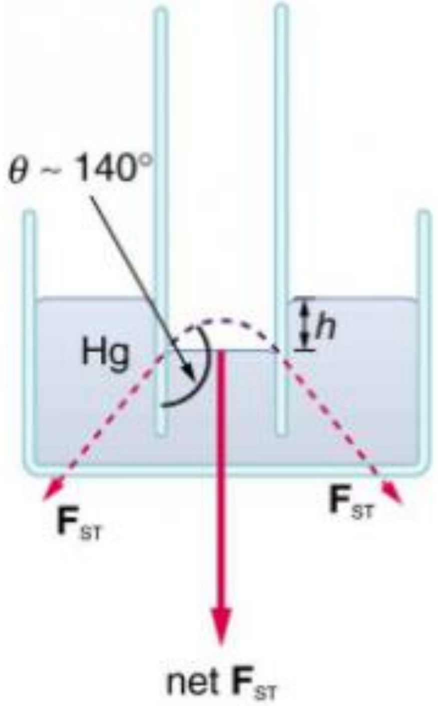
Let the 'h' be the height of depression of liquid in the tube.

Under equilibrium condition two forces are acting on the mercury inside the tube.

The first force is due to surface tension acting in the downward direction and

The second force is because of hydrostatic force acting in upward direction.





The force due to surface tension acting downward direction is given by

$$\sigma \pi d \cos\theta$$

And the force due to hydrostatic force acting downward is given by

$$= intensity\ of\ pressure\ at\ depth\ of\ h \times area$$

$$= \rho g h \times \frac{\pi}{4} d^2$$

Under equilibrium condition the two forces i.e., the force due to surface tension is equal to the hydrostatic force.

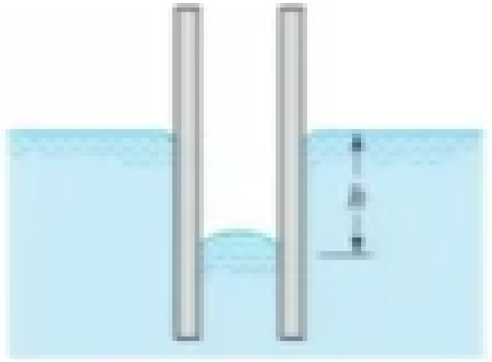
i.e.,

$$\sigma \pi d \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$h = \frac{4\,\sigma \cos\theta}{\rho g d}$$

The value of θ for the mercury & glass tube is 128°.

Example 3



Question

A glass tube contains mercury. The surface tension is $0.52\ N/m$, contact angle is 140° and density is $13{,}600\ kg/m^3$. What is the minimum tube diameter required to maintain the depression to less than $10\ mm$.

Solution

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$d = \frac{4\sigma\cos\theta}{\rho g h}$$

$$= \frac{4 \times 0.52 \times \cos 140^{\circ}}{13600 \times 9.81 \times (-0.010)}$$

$$= 1.19 \times 10^{-3}\ m$$

$$= 1.19\ mm$$

Questions

Question 1

Calculate the excess pressure within a bubble of air of radius $0.1\ mm$ in water. (Given that $\sigma = 72.7 \times 10^{-3}\ N/m$)

Answer: $1454\ Pa$

Question 2

Calculate the radius of a capillary tube that would raise sap $100\ m$ to the top of a giant redwood, assuming that sap's density is $1050\ kg/m^3$, its contact angle is zero, and its surface tension is the same as that of water at $20.0°\ C$.

Answer: $1.41 \times 10^{-7}\ m$