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Q.1

Ans. Let mean $= \theta_1$, var $= \theta_2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\wedge -\frac{(x-\mu)^2}{2\sigma^2}}$$

Joint density of $(x_1, x_2, x_3 \ldots)$ is

$$L(\theta_1, \theta_2; x_1, x_2, x_3 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

taking log on both sides,

$$\ln[L(\theta_1, \theta_2)] = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\ln[L(\theta_1, \theta_2)] = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\Sigma(x_i-\theta_1)^2$$

θ_1 Differentiating $\ln[L(\theta_1, \theta_2)]$ wrt θ_1

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}} \leftarrow \bar{x}$$

θ_2 $\ln[L(\theta_1, \theta_2)]$ wrt θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2 = 0$$

$$\boxed{\theta_2 = \frac{1}{n}\Sigma(x_i-\theta_1)^2} \leftarrow Var(X)$$

Q2

Ans

$B(m, \theta) \rightarrow$ binomial distribution

$m \rightarrow$ no. of trials $\qquad \theta = (0, 1)$

$f(x) = {}^{n}C_{x} \, p^{x} (1-p)^{m-x} \qquad p = \theta$

$\therefore \; L(\theta, x_1, x_2 \ldots x_m) = \prod_{i=1}^{n} P(x_i \mid m, p)$

$L(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$

taking log on both sides

$\ln(L(\theta)) = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \sum_{i=1}^{n} x_i \log\theta + \sum_{i=1}^{n} (m - x_i) \log(1-\theta)$

differentiate w.r.t. θ

$\frac{d \ln(L)}{d(\theta)} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)(-1)$

$\Rightarrow \; \frac{1}{\theta} \sum x_i = \frac{1}{1-\theta} \sum (m - x_i)$

$(1-\theta) \sum x_i = \theta \sum (m - x_i)$

$\Rightarrow \; \boxed{\theta = \frac{\sum x_i}{m}} \leftarrow \bar{x}$