BCA

(SEM. IV) EXAMINATION, 2022-23
BCA – 4005 : MATHEMATICS – III

Time : 1 :30 Hours *Maximum Marks : 75*

1. Locus of $|z| > 1$ is :
   (A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 > 1$
   (C) $x^2 + y^2 < 1$ (D) $x^2 + y^2 \geq 1$

Ans. (C) $x^2 + y^2 > 1$

Explanation :

$$|z| > 1 \Rightarrow \sqrt{x^2 + y^2} > 1$$

$$\Rightarrow x^2 + y^2 > 1$$

2. Argument of $\left(\frac{z-1}{z+1}\right)$ is equal to :
   (A) $\tan^{-1}\left(\frac{2y}{x^2 + y^2 - 1}\right)$ (B) $\tan^{-1}\left(\frac{2x}{x^2 + y^2 - 1}\right)$
   (C) $\tan\left(\frac{2y}{x^2 + y^2 - 1}\right)$ (D) None of the above

Ans. (A) $\tan^{-1}\left(\frac{2y}{x^2 + y^2 - 1}\right)$

Explanation :

$$\text{Arg}\left(\frac{z-1}{z+1}\right) = \text{Arg}(z-1) - \text{Arg}(z+1)$$

$$= \tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1}\frac{y}{x+1}}\right)$$

$$= \tan^{-1}\left(\frac{2y}{x^2 + y^2 - 1}\right)$$

3. If $z = 3 + 4i$, then modulus of z is :
   (A) 5 (B) 7
   (C) 25 (D) 34

Ans. (A) 5

Explanation :

$$|z| = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5$$

9. If $f(z) = \log z$, , then real of $f(z)$ is :

(A) $\log|z|$ (B) $|z|$

(C) θ (D) r

Ans. (A) $\log|z|$

10. Particular integral of ODE $(D^2+1)y = e^{2x}$ is

(A) $\frac{1}{5}e^{x}$ (B) $\frac{1}{4}e^{2x}$

(C) $\frac{1}{5}e^{2x}$ (D) $\frac{1}{5}$

Ans. (A) $\frac{1}{5}e^{2x}$

Explanation :

$$[f(D)]^{-1}e^{ax} = (f(a))^{-1}e^{ax}$$

$$\Rightarrow \frac{1}{D^2+1}e^{2x} = \frac{1}{2^2+1}e^{2x} = \frac{e^{2x}}{5}$$

11. If $f(x) = \sin x$ is periodic function, then period of $f(x)$ is :

(A) π (B) $-\pi$

(C) 2π (D) $\frac{\pi}{2}$

Ans. (C) 2π

12. $\int_{c}^{c+2\pi} \cos nx\,dx$ is equal to :

(A) c (B) 0

(C) $2\pi c$ (D) πc

Ans. (B) 0

Explanation :

$$\int_{c}^{c+2\pi} \cos(nx)dx = \left[\frac{\sin(nx)}{n}\right]_{c}^{c+2\pi}$$

$$= \frac{\sin(nc+2n\pi) - \sin(nc)}{n}$$

$$= \frac{\sin(nc) - \sin(nc)}{n} = 0$$

13. Degree of ODE $\log\left(\frac{d^2y}{dx^2}\right) = 5x$ is :

(A) 2 (B) 3

(C) 0 (D) 1

Ans. No option is correct

4. ODE $Mdx + Ndy$ is exact ODE, if :

(A) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(C) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (D) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Ans. (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

5. ODE $\frac{dy}{dx} + Py = Qy^n$ is linear, if :

(A) $n = 1$ (B) $n = 2$

(C) $n = 3$ (D) $n = 0$

Ans. (D) $n = 0$

6. Conjugate of complex number $z = -3 + i$ is :

(A) $-3 - i$ (B) $3 - i$

(C) $-3 + i$ (D) $3 + i$

Ans. (A) $-3 - i$

Explanation :

$z = x + iy$

$\bar{z} = x - iy$

7. Roots of $z^4 = 1$ are :

(A) 1,1, i, i (B) 1, -1, i, - i

(C) i, i, i, i (D) 1, 1, 1, 1

Ans. (B) 1, -1, i, - i

8. Amplitude of $z = \frac{5}{12} i$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $-\frac{\pi}{2}$ (D) $-\frac{\pi}{4}$

Ans. (B) $\frac{\pi}{2}$

Explanation :

$$\arg(z) = \text{amp}(z) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{5/12}{0}\right)$$

$$= \frac{\pi}{2}, \text{ as } y = \frac{5}{12} > 0$$

Explanation : Degree of differential equation is highest degr... highest order derivative.

In $\log\left(\frac{d^2y}{dx^2}\right) = 5x$ the highest degree of $\frac{d^2y}{dx^2}$ goes upto ... so ...

defined while $\frac{d^2y}{dx^2} = e^{5x}$ the degree of differential equation is 1

14. Integrating factor of $\frac{dx}{dy} + Px = Q$ is :

(A) $e^{\int Pdy}$ (B) $e^{\int Pdx}$

(C) $e^{\int Pdx + c}$ (D) None of the above

Ans. (B) $e^{\int Pdx}$

15. Cube roots of unity are :

(A) $1, \omega, \omega^2$, Where $\omega = \frac{-1}{3} + \frac{\sqrt{3}}{2}i$

(B) 1, 1, 1

(C) 1, 2, 3

(D) $1, \omega, \omega^2$, Where $\omega = \frac{\sqrt{3}}{2} + \frac{1}{2}$

Ans. (A) $1, \omega, \omega^2$, Where $\omega = \frac{-1}{3} + \frac{\sqrt{3}}{2}i$

16. ODE $y + x\frac{dy}{sx} = x - y\frac{dy}{sx}$ is :

(A) Homogeneous ODE of degree 2

(B) Homogeneous ODE of degree 0

(C) Homogeneous ODE of degree 1

(D) None of the above

Ans. (C) Homogeneous ODE of degree 1

17. Divergence of a vector point function is :

(A) Scalar (B) Vector

(C) Both (A) and (B) (D) None of the above

Ans. (A) Scalar

18. Complementary function of ODE for 3 repeated real roots :

(A) $(c_1 + c_2 + c_3)e^{mx}$ (B) $(c_1 + c_2x + c_3x^2)e^{mx}$

(C) $(c_1 + c_2x)e^{mx}$ (D) $c_1 e^{m_2x} + c_2^{m_2x} + c_3 e^{m_3x}$

Ans. (B) $(c_1 + c_2x + c_3x^2)e^{mx}$

19. Degree of ODE $(y''')^3 + (y'')^2 + y = 0$ is :
(A) 3 (B) 2
(C) 1 (D) 0
Ans. (A) 3

20. Order of ODE $\frac{d^3y}{dx^3} + y = \sin x$ is :
(A) 1 (B) 2
(C) 3 (D) None of the above
Ans. (C) 3

21. Gradient of function $f(r)$ is :
(A) $f'(r)$ (B) $f'(r)\frac{\partial r}{\partial x}$
(C) $f''(r)$ (D) $f'(r)\nabla r$
Ans. (D) $f'(r)\nabla r$

Explanation :

$\nabla f(r) =$ Gradient of $f(r) = \sum i \frac{\partial}{\partial x} f(r)$

$= f'(r) \sum i \frac{\partial r}{\partial x}$

$= f'(r)\nabla r$

22. If $\vec{F} = xi + yj + zk$, then curl $\bar{F}$ is :
(A) 1 (B) 0
(C) $\vec{r}$ (D) $i + j + k$
Ans. (B) 0

23. If $\vec{F} = 2xy\hat{i} + x^2\hat{j} + 2yz\hat{k}$, then divergence of $\vec{F}$ is :
(A) 4xy (B) 4x
(C) 4 (D) 4y
Ans. (D) 4y

Explanation :

$\nabla.\vec{F} = \sum \frac{\partial}{\partial x} F_1 = \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial z}(2yz)$

$= 2y + 0 + 2y = 4y$

24. A vector field $\vec{F}$ is irrotational, if :
(A) Grad f = 0 (B) V = grad f
(C) $\nabla \times \vec{F} = 0$ (D) $\nabla.\vec{F} = 0$
Ans. (C) $\nabla \times \vec{F} = 0$

25. If $f(x) = x$ is periodic in $[0, 2\pi]$ then Fourier coefficient a_0 is

(A) 3π (B) $\frac{3\pi}{2}$

(C) 2π (D) $\frac{\pi}{4}$

Ans. (C) 2π

Explanation :

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x dx = \frac{1}{2\pi}\left[x^2\right]_0^{2\pi} = 2\pi$$

26. ODE $\frac{dy}{dx} + Qy = Py^n$ is Bernoulli equation, if :

(A) $n = 1$ (B) $n = 0$

(C) $n \neq 0$ (D) $n = 3$

Ans. (C) $n \neq 0$

27. Conjugate of $z = -3 + 2i$ is :

(A) -3-2i (B) 3-2i

(C) 3 + 2i (D) None of the above

Ans. (A) -3-2i

28. What is polar form of $z = 1 + i$?

(A) $\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ (B) $2\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

(C) $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$ (D) $\sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]$

Ans. (A) $\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

Explanation :

$x = r\cos\theta,\ y = r\sin\theta$

$$\Rightarrow \begin{cases} r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4} \end{cases}$$

29. Real part of e^{iz} is :

(A) $e^{y}\cos x$ (B) $e^{y}\sin x$

(C) $e^{-y}\cos x$ (D) $e^{-y}\sin x$

Ans. (C) $e^{-y}\cos x$

Explanation :

$$e^{iz} = e^{i(x+iy)} = e^{-y}e^{ix} = e^{-y}(\cos(x) + i\sin(x))$$

30. If $f(x)=\begin{cases} x+1, & -1<x<0 \\ x-1, & 0<x<1 \end{cases}$ in $(-1,1)$, then find Fourier coefficient a_0 :

(A) $a_0=-1$ (B) $a_0=1$

(C) $a_0=0$ (D) $a_0=2$

Ans. (C) $a_0=0$

Explanation :

$$a_0=\frac{1}{T}\int_a^b f(x)dx=\frac{1}{1}\left[\int_{-1}^{0}(x+1)dx+\int_0^1(x-1)dx\right]$$

$$=\left[\frac{(x+1)^2}{2}\right]_{-1}^{0}+\left[\frac{(x-1)^2}{2}\right]_0^{-1}=\frac{1}{2}-\frac{1}{2}=0$$

31. Which of the following is well known first order ODE?

(A) Euler's equation (B) Bernoulli's equation

(C) Laplace's equation (D) Poisson's equation

Ans. (B) Bernoulli's equation

32. Which of the following is equation of first order ODE?

(A) $\frac{dy}{dx}=y^2$ (B) $y\frac{dy}{dx}=\sin x$

(C) $\frac{dy}{dx}+x^2y=x$ (D) $\frac{d^2y}{dx^2}=x^2$

Ans. (C) (A), (B), (C) all are correct

Explanation :

(A) $\frac{dy}{dx}=y^2$ First order, first degree,

(B) $y\frac{dy}{dx}=\sin x$ First order, first degree, Linear in y

(C) $\frac{dy}{dx}+x^2y=x$ First order, first degree, Linear in y

(D) $\frac{d^2y}{dx^2}=x^2$ Second order, first degree

33. What is general solution of first order linear ODE?

(A) $\frac{dy}{dx}+Py^2=Q$ (B) $\frac{dy}{dx}+Py=Q$

(C) $\frac{dy}{dx}+Py=Qy^n$ (D) None of the above

Ans. (D) None of the above

Explanation :

(A) $\frac{dy}{dx}+Py^2=Q$ not linear differential equation

(B) $\frac{dy}{dx}+Py=Q$ Linear differential equation but not solution

(C) $\frac{dy}{dx}+Py=Qy^n$ Bernoulli as differential equation

34. if $\left|\frac{z-1}{z+1}\right|=2$, then locus of z is :

(A) Circle (B) Parabola

(C) Ellipse (D) Hyperbola

Ans. (A) Circle

Explanation :

$$\left|\frac{z-1}{z+1}\right|=2 \Rightarrow |z-1|=2|z+1|$$

$$\Rightarrow (x-1)^2+y^2=4\left[(x+1)^2+y^2\right]$$

$$\Rightarrow 3x^2+10x+3y^2+3=0$$

$$\left(x-\frac{5}{3}\right)^2+y^2=\frac{16}{9}$$

Equation of circle centered at $\left(\frac{5}{3},0\right)$ and radius $\left(\frac{4}{3}\right)$

35 Which of the following is true?

(A) $\arg(z_1z_2)=\arg z_1+\arg z_2$

(B) $\arg(z_1z_2)=\arg z_1-\arg z_2$

(C) $\arg(z_1z_2)=(\arg z_1)(\arg z_2)$

(D) $\arg\left(\frac{z_1}{z_2}\right)=\arg z_1+\arg z_2$

Ans (A) $\arg(z_1z_2)=\arg z_1+\arg z_2$

36. A vector field $\vec{F}$ is conservative if for scalar potential ϕ :

(A) $\vec{F}=\nabla\phi$ (B) $\vec{F}=\nabla^2\phi$.

(C) $\vec{F}=\nabla.\vec{F}$ (D) $\vec{F}=\nabla\times\vec{F}$

Ans (A) $\vec{F}=\nabla\phi$

37. If $f(x,y,z) = x^2yz + 4xz^2$ at (1, 2, - 1) then ∇f is :

(A) $8i + j - 10k$ (B) $8i - j - 10k$

(C) $8i + j + 10k$ (D) $-8i + k - 10k$

Ans. No option is correct

Explanation :

$$\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$= \left(2xyz + 4z^2\right)\hat{i} + x^2z\,\hat{j} + \left(x^2y + 8xz\right)\hat{k}$$

$$\nabla f_{(1,2,-1)} = 0\,\hat{i} - \hat{j} - 6\hat{k}$$

38. Degree of ODE $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$ is :

(A) 3 (B) 2

(C) 1 (D) 0

Ans. (B) 2

Explanation :

$$\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} \Rightarrow \left(\frac{d^2y}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

Def : Degree of differential equation is degree of highest order derivative after clearing radicals and fractions.

39. General solution of $\frac{dy}{dx} = y$ is :

(A) $y = ce^x$ (B) $y = ce^{2x}$

(C) $y = \log x$ (D) $y = x^2$

Ans. (C) $y = ce^x$

No option is correct after correction

Explanation :

$$\frac{dy}{dx} = y \Rightarrow \frac{dy}{dx} = dx \Rightarrow f_n y = x + A$$

$$\Rightarrow y = e^{x+c} = ce^x$$

40. If $2x\frac{dy}{dx} = y + \tan x$, then integrating factor is :

(A) $\tan x$ (B) $\sqrt{\tan x}$

(C) $\frac{1}{\tan x}$ (D) $\frac{1}{\sqrt{\tan x}}$

Ans. No option is correct

Explanation :

$$2x\frac{dy}{dx} = y + \tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2x}y = \frac{\tan x}{2x}$$

$$\text{IF} = e^{\int P dx} = \exp\left[\int\left(\frac{-1}{2x}\right)dx\right]$$

$$= \exp\left(-\frac{1}{2}\log x\right) = \exp\left(\log\frac{1}{\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{x}}$$

41. Complementary function for ODE $(D^2 - 4D + 4)y = 0$ is :

(A) $c_1 + c_2xe^{2x}$ (B) $(c_1 + c_2x)e^{2x}$

(C) $(c_1 + c_2)e^{2x}$ (D) ce^{2x}

Ans. (B) $(c_1 + c_2x)e^{2x}$

Explanation :

Auxiliary equation $m^2 - 4m + 4 = 0 \Rightarrow m = 2, 2$

$CF = (c_1 + c_2x)e^{2x}$

42. Integrating factor of $\frac{dx}{dy} + 4Px = Q$ is :

(A) $e^{\int 4P dx}$ (B) $e^{\int 4Q dx}$

(C) $e^{\int 4P dy}$ (D) $e^{\int 4Q dy}$

Ans. (A) $e^{\int 4P dx}$

43. Curl of a vector field represents :

(A) Irrotational field (B) Magnetic field

(C) Rotational field (D) Laurent field

Ans. (C) Rotational field

44. If divergence of vector field $\vec{F}$ is positive, then :

(A) $\vec{F}$ is not converging. (B) $\vec{F}$ is spreading out.

(C) $\vec{F}$ is not spreading out (D) None of the above

Ans. (C) $\vec{F}$ is spreading out.

45. What is divergence of vector field $\vec{F}$?

(A) Rate at which $\vec{F}$ is spreading out or converging.

(B) Rate of rotation of $\vec{F}$.

(C) Flow of $\vec{F}$ along its streamline.

(D) $\vec{F}$ is changing w.r.t. time.

Ans. (A) Rate at which $\vec{F}$ is spreading out or converging.

46. What is curl of $\vec{F}$?

(A) A scalar representing rate at which $\vec{F}$ is converging.

(B) A vector representing flow of $\vec{F}$ along its streamlines.

(C) Both (A) and (B)

(D) A vector representing the rotational behaviour of $\vec{F}$.

Ans. (D) A vector representing the rotational behaviour of $\vec{F}$

47. Which of the following can be represented using Fourier series?

(A) Only periodic functions

(B) Only non – periodic functions.

(C) Both periodic and non – periodic

(D) Only continuous functions

Ans. (A) Only periodic functions

48. What is modulus of a complex number z in polar form?

(A) The real part of complex number

(B) The magnitude on absolute real value of z

(C) The angle of z

(D) Imaginary part of z

Ans. (B) The magnitude on absolute real value of z

49. Which trigonometric function is used to convert a complex number from Cartesian to polar :

(A) Sine (B) Cosine

(C) Tangent (D) Arc tangent

Ans. (D) Arc tangent

50. What is gradient of scalar field?

(A) A vector representing divergence of scalar

(B) A vector perpendicular to level curves of scalar field

(C) A vector pointing maximum rate of increase

(D) None of the above

Ans. (B) A vector perpendicular to level curves of scalar field

51. For given ODE $x\frac{dy}{dx}+y=(\log x)y^2$ suitable transformation to it into linear is :

(A) $\frac{1}{y}=t$ (B) $\frac{1}{y^2}=t^3$

(C) $y^3=t$ (D) $y^2=t$

Ans. (A) $\frac{1}{y}=t$

Explanation :

$$x\frac{dy}{dx}+y=(\log x)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{x}\frac{1}{y}=\frac{\log x}{x}$$

Let $\frac{1}{y}=t$

$$-\frac{dt}{dx}+\frac{t}{x}=\frac{\log x}{x}$$

$$\Rightarrow\frac{dt}{dx}-\frac{t}{x}=\frac{\log x}{x}$$

52. Divergence of $\vec{a}\times\vec{r}$ is :

(A) $\vec{a}$ (B) $\vec{r}$

(C) 0 (D) 1

Ans. (C) 0

Explanation :

$$\vec{a}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ x & y & z\end{vmatrix}=\sum\hat{i}(a_2z-a_3y)$$

$$\text{div}\left(\vec{a}\times\vec{r}\right)=\sum\frac{\partial}{\partial x}(a_2z-a_3y)=0$$

53. If $\vec{F}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$

is solenoidal vector, then the value of $\vec{a}$ is :

(A) a = 1 (B) a = - 1

(C) a= - 2 (D) a = 2

Ans. (C) a= - 2

Explanation :

$$\nabla.\vec{F} = \frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+az)$$

$$= 1+1+a = 0 \Rightarrow a = -2$$

54. What is curl of $\hat{r}$?
(A) 0 (B) $\vec{r}$ (C) $\hat{r}$ (D) 0

Ans. (D) 0

55. If $f(x) = \sin x$ in [-l, l], then Fourier coefficient a_n is :
(A) $a_n = 0$ (B) $a_n = 2l$
(C) $a_n = \frac{1}{l}$ (D) $a_n = \frac{1}{2l}$

Ans. (A) $a_n = 0$

Explanation :

$$a_n = \frac{1}{l}\int_{-l}^{l} \sin x.\cos\left(\frac{n\pi x}{l}\right) dx = 0$$

As sin x. cos $\frac{n\pi x}{l}$ Is odd function integrated between (-l) to (+l).

56. Euler's coefficients in Fourier series are obtained by :
(A) Laplace form (B) Jacobian formula
(C) Euler's formula (D) Maclaurin's formula

Ans. (C) Euler's formula

57. If $f(x) = \sin^2 x \cos x$ in [-l, l], then $f(x)$ is :
(A) Odd function (B) Even function
(C) Neither odd nor even (D) Both odd and even

Ans. (B) Even function

58. Fourier series' is defined for :
(A) Continuous functions only
(B) Discontinuous functions only
(C) Both continuous and discontinuous functions
(D) None of the above

Ans. (C) Both continuous and discontinuous functions

59. What are the conditions for Fourier series expansion?
(A) Leibnitz condition (B) Abel's condition.
(C) Dirichlet condition (D) Fourier condition

Ans. (C) Dirichlet condition

60. What is the value of $(i)^{100}$?
(A) 1 (B) I (C) -i (D) -1

Ans. (A) 1

61. What is the value of $(1+i)^4$?

(A) -4 (B) 4i

(C) -4 + 4i (D) 4

Ans. (A) -4

Explanation :

$$(1+i)^4 = \left[\sqrt{2}e^{i\pi/4}\right]^4$$

$$= 4e^{-\pi i} = -4$$

62. $y^2\dfrac{dy}{dx} = \sin x$ is :

(A) Linear in x (B) Linear in y

(C) Non – linear in x (D) None of the above

Ans. (C) Non – linear in x

63. If f(x) is odd function in [-a, a], then $a_n (n \neq 1)$ is :

(A) Non-zero (B) Zero

(C) May or may not be zero (D) Never be zero

Ans. (B) Zero

64. If $f(x)$ is even function in $[-\pi, \pi]$, then b_n is :

(A) Always 0 (B) May be 0

(C) Never be 0 (D) None of the above

Ans. (A) Always 0

65. If $\phi(x, y, z) = x + z$, then directional derivative in the direc

$\vec{a} = i + j$ is :

(A) 1 (B) $\sqrt{2}$

(C) $\dfrac{1}{\sqrt{2}}$ (D) $\dfrac{1}{2}$

Ans. (C) $\dfrac{1}{\sqrt{2}}$

Explanation :

$$DD_a \to \phi = \text{grad}\,\phi.\hat{a}$$

$$\left(\hat{i}+\hat{k}\right).\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

66. A vector field is solenoidal, if :

(A) curl $\vec{F} = 0$ (B) grad $\vec{F} = 0$

(C) div $\vec{F} = 0$ (D) None of the above

Ans. (C) div $\vec{F} = 0$

67. If $\vec{v} = \vec{\omega} \times \vec{r}$, then curl $\vec{v}$ is :

(A) $\vec{\omega}$ (B) $\vec{\omega} \times \vec{r}$

(C) $2\vec{\omega}$ (D) None of the above

Ans. (C) $2\vec{\omega}$

Explanation :

$$\vec{v} = \vec{\omega} \times \vec{r} = \Sigma \hat{i}(\omega_2 z - \omega_3 y)$$

$$\text{curl } \vec{v} = \Sigma \hat{i}\left[\frac{\partial}{\partial y}(\omega_1 z - \omega_2 x) - \frac{\partial}{\partial z}(\omega_3 x - \omega_1 z)\right]$$

$$\text{curl } \vec{v} = = \Sigma \hat{i}\ 2\omega_1 = 2\vec{\omega}$$

68. Particular integral of $(D^2 + 1)y = \sin 2x$ is

(A) $\frac{1}{3}\sin 2x$ (B) $-\frac{1}{3}\sin 2x$

(C) $\frac{1}{2}\sin 2x$ (D) $-\frac{1}{2}\sin 2x$

Ans. (B) $-\frac{1}{3}\sin 2x$

Explanation :

$$PI = \frac{1}{D^2 + 1}\sin 2x$$

$$= \frac{1}{-2^2 + 1}\sin 2x,\ D^2 \to -2^2$$

$$= -\frac{1}{3}\sin 2x$$

69. ODE $\frac{dy}{dx} + Qy = Py^n$ is Bernoulli, if :

(A) $n \neq 1$ (B) $n = 0,1$

(C) $n = 8$ (D) $n \neq 0$

Ans. (D) $n \neq 0$

70. Particular integral of $\frac{1}{D - 1}e^x$ is :

(A) $\frac{1}{2}e^x$ (B) xe^x

(C) x^2e^x (D) None of the above

Ans (B) xe^x

Explanation :

$$PI = \frac{1}{D-1}e^x = e^x\frac{1}{(D+1)-1}; \quad \frac{1}{f(D)}e^{ax}\phi = e^{ax}\frac{1}{f(D+a)}\phi$$

$$= e^x\frac{1}{D}1$$

$$= e^x\int x\,dx = xe^x$$

71. Complementary function of $(D^2+1)y = 0$ is :

(A) $c_1\cos x + c_2\sin x$ (B) $c\sin x$

(C) $c_1\cos x$ (D) $c_1e^x + c_2e^{-x}$

Ans. (A) $c_1\cos x + c_2\sin x$

Explanation :

Auxiliary equation $m^2+1=0 \Rightarrow m = \pm i$

$CF = c_1\cos x + c_2\sin x$

72. ODE $y\frac{dy}{dx} = x$ is :

(A) Non – linear in y (B) Linear in x

(C) Order = 2 (D) Degree = 2

Ans. (A) Non – linear in y

73. Degree of homogeneity of ODE :

$$\left(x^{\frac{3}{2}} - y^{\frac{3}{2}}\right)dx + 3x^{\frac{1}{2}}y\,dy = 0$$

is :

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$

(C) 3 (D) $\frac{1}{2}$

Ans. (A) $\frac{3}{2}$

74. $(1+2x+3y^2)dx + (3+4xy+5x^2)dy = 0$ is

(A) linear in x (B) Linear in y

(C) Both non – linear in x and y (D) Both linear in x and y

Ans. (C) Both non – linear in x and y

75. Solution of $\frac{dy}{dx} = \frac{y}{x}$ is :

(A) $\frac{y}{x} = c$ (B) $xy = c$

(C) $\log y = \log x + c$ (D) Both (A) and (C)

Ans. (D) Both (A) and (C)

76. ODE $\frac{dy}{dx}=\frac{x^3+y^3}{x^2y}$ is :

(A) Homogeneous ODE of degree 3
(B) Non – homogeneous ODE
(C) Linear in y
(D) Linear in x

Ans. (A) Homogeneous ODE of degree 3

77. $\frac{dy}{dx}+Py=Qy^n$ is variable separable for :

(A) $n=2$ (B) $n=1$
(C) $n=-1$ (D) $n=3$

Ans. (B) $n=1$

Explanation :

$$\frac{dy}{dx}+Py=Qy \Rightarrow \frac{dy}{y}=\frac{dx}{Q-P}$$

78. $\frac{dy}{dx}=\frac{2x+3y+5}{3x+5y+7}$ can be transformed to homogeneous by :

(A) $x=X-4,\ y=Y+1$ (B) $x=X+4,\ y=Y-1$
(C) $x=X-4,\ y=Y-1$ (D) $x=X+4,\ y=Y+1$

Ans. (A) $x=X-4,\ y=Y+1$

Explanation : Under above transform it will become

$2(-4)+3(1)+5=0\ \&\ 3(-4)+5(1)+7=0$

79. Imaginary part of $f(z)=z\bar{z}$ is :

(A) x^2+y (B) x^2+y^2
(C) x^2-y^2 (D) 0

Ans. (D) 0

Explanation :

$$z\bar{z}=x^2+y^2,\quad \operatorname{Re}(z\,\bar{z})=x^2+y^2$$

$$\operatorname{Im}(z\,\bar{z})=0$$

80. Locus of $|z-1|\geq 1$ is :

(A) $(x-1)^2+y^2>1$ (B) $(x-1)^2+y^2\geq 1$
(C) $(x-1)^2+y^2=1$ (D) $(x-1)^2+y^2\leq 1$

Ans. (B) $(x-1)^2+y^2\geq 1$

Explanation :

$$|z-1|\geq 1 \Rightarrow \sqrt{(x-1)^2+y^2}\geq 1 \Rightarrow (x-1)^2+y^2\geq 1$$

81 Polar form of $z = -1$ is :

(A) $z = e^{i\pi}$ (B) $z = e^{-i\pi}$

(C) $z = e^{\frac{i\pi}{2}}$ (D) Both (A) and (B)

Ans. (D) Both (A) and (B)

Explanation :

$z = re^{i\theta}$, where $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 0^2} = 1$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0}{-1}\right) = \pm\pi$$

$z = e^{\pm i\pi}$

82. If $x + iy = \frac{2-3i}{4+7i}$, then :

(A) $y = \frac{2}{5}$ (B) $y = -\frac{2}{5}$

(C) $y = -\frac{1}{5}$ (D) $y = \frac{1}{5}$

Ans. (B) $y = -\frac{2}{5}$

Explanation :

$$x + iy = \frac{2-3i}{4+7i} = \frac{(2-3i)(4-7i)}{4^2+7^2}$$

$$= \frac{(8-21)-i(12+14)}{65} = -\frac{1+2i}{5}$$

$$x = -\frac{1}{5},\ y = -\frac{2}{5}$$

83. $\text{Arg}\left(\frac{z-1}{z+1}\right)$ is equal to :

(A) $\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) + 2n\pi,\ n \in Z$

(B) $\tan^{-1}\left(\frac{2x}{3}\right)$

(C) $\tan^{-1}(x^2 + y^2 - 1)$

(D) None of the above

Ans. (A) $\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) + 2n\pi,\ n \in Z$

Explanation :

$$\text{Arg}\left(\frac{z-1}{z+1}\right)=\text{Arg}(z-1)-\text{Arg}(z+1)$$

$$=\tan^{-1}\frac{y}{x-1}-\tan^{-1}\frac{y}{x+1}$$

$$=\tan^{-1}\left(\frac{\frac{y}{x-1}-\frac{y}{x+1}}{1+\frac{y}{x-1}\frac{y}{x+1}}\right)=\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)+2n\pi$$

34. If $z=\frac{a+ib}{c+id}$, then real (z) is :

(A) $\frac{ca+bd}{c^2-d^2}$ (B) $\frac{ca+ab}{c^2+d^2}$ (C) $\frac{ca+bd}{c^2+d^2}$ (D) $\frac{ca+bd}{a^2+b^2}$

Ans. (C) $\frac{ca+bd}{c^2+d^2}$

Explanation :

$$\frac{a+ib}{c+id}=\frac{(a+ib)(c-ib)}{(c+id)(c-id)}=\frac{ac+bd+i(bc-ad)}{c^2+d^2}$$

85. If $z_1=2+i$ and $z_2=3+2i$, then :

(A) $z_1z_2=13$ (B) $z_1z_2=13i$

(C) $z_1z_2=12+13i$ (D) $-13i=z_1z_2$

Anc. Question need correction

$$z_1=2+i,\ z_2=3+2i$$

$$z_1, z_2=(2+i)(3+2i)=(6-2)+i(3+2)$$

$$=4+5i$$

86. Modulus amplitude form of $z=-\sqrt{3}\,i$ is :

(A) $\sqrt{3}e^{\frac{\pi}{2}}$ (B) $\sqrt{3}e^{\frac{i\pi}{2}}$

(C) $\sqrt{3}e^{-\left(\frac{i\pi}{2}\right)}$ (D) $\sqrt{3}e^{\frac{i\pi}{4}}$

Ans. (C) $\sqrt{3}e^{-\left(\frac{i\pi}{2}\right)}$

Explanation :

$$z=-\sqrt{3}\,i=\left(e^{\pm\pi i}\right)\sqrt{3}\left(e^{\frac{\pi i}{2}}\right)=\sqrt{3}\ e^{\pi\left(\frac{1}{2}+1\right)i}$$

$$=\sqrt{3}\ e^{\frac{-\pi i}{2}},\quad \sqrt{3}\ e^{\frac{3\pi i}{2}}$$

87 $\frac{e^z + e^{-z}}{2}$ is equal to :

(A) $\cos z$ (B) $\cosh z$
(C) $\sin z$ (D) $\sin h\, z$

Ans. (B) $\cosh z$

88. Locus of $|z| = 1$ is :

(A) $|z| > 1$ (B) $|z| < 1$
(C) $x^2 + y^2 = 1$ (D) $x^2 - y^2 = 1$

Ans. (C) $x^2 + y^2 = 1$

89. If $f = \sin r$, $r = \sqrt{x^2 + y^2 + z^2}$, then grad (r) is :

(A) $\cos r$ (B) $\cos \hat{r}$
(C) $\sin r \nabla r$ (D) $\cos r \nabla r$

Ans. (D) $\cos r \nabla r$

Explanation :

$$\nabla f(r) = f'(r)(\nabla r)$$
$$\nabla \sin r = (\cos r)(\nabla r)$$

90. If $\vec{a}$ is constant vector and $\vec{r} = xi + yi + zk$, then $\nabla\left(\vec{a}\,.\,\vec{r}\right)$ is:

(A) 0 (B) $\vec{r}$
(C) $\vec{a}$ (D) $2\vec{a}$

Ans. (C) $\vec{a}$

Explanation :

$$\vec{a}\,.\,\vec{r} = \Sigma a, x$$
$$\nabla\left(\vec{a}\,.\,\vec{r}\right) = \sum i \frac{\partial}{\partial x} a, x = \sum \hat{i}\, a, = \vec{a}$$

91. If $\phi = x^2 y + y^2 x + z^2$, then $\nabla\phi$ at $(1, 1, 1)$ is :

(A) $i + j + k$ (B) $i - j + k$
(C) $3i + 3j + 3k$ (D) $3i + j + k$

Ans. No option is correct

Explanation :

$$\nabla\phi = \sum \hat{i} \frac{\partial\phi}{\partial x} = \left(2xy + y^2\right)\hat{i} + \left(x^2 + 2yx\right)\hat{j} + 2z\hat{k}$$
$$\nabla\phi = (1, 1, 1) = 3\hat{i} + 3\hat{j} + 2\hat{k}$$

92. $f(x) = \cos x$ is periodic with period T is :

(A) $T = 2\pi$ (B) $T = \pi$ (C) $T = \frac{\pi}{2}$ (D) $T = -\pi$

Ans. (A) $T = 2\pi$

93. If $z = -1 + i\sqrt{3}$, then $|z|$ is :

(A) $\sqrt{2}$ (B) 2 (C) -2 (D) $\frac{1}{\sqrt{2}}$

Ans. (B) 2

Explanation :

$$|z| = \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

94. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ if and only if :

(A) $z_1\bar{z}_2$ Is purely real. (B) $z_1\bar{z}_2$ is zero.

(C) $z_1\bar{z}_2$ is purely imaginary. (D) None of the above

Ans. (C) $z_1\bar{z}_2$ is purely imaginary.

Explanation :

$$|z_1 + z_2|^2 = (z_1 + z_2) = |z_1|^2 + |z_2|^2 + z_1, z_2 + z_2, z_1$$

$$= |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1, \overline{z_2})$$

$$= |z_1|^2 + |z_2|^2, \text{ if } z_1, z_2 \text{ is imaginary}$$

95. If $|z - 3i| = 5$, then center of circle is :

(A) $-3i$ (B) 5 (C) $\frac{3}{2}i$ (D) (0, -3)

Ans. No option is correct

Explanation :

$|z - a| = r$ is equation of circle with centre at $z = a = a_1 + ia_2$ or (a_1, a_2) with radius r.

96. If $f(x) = |\cos x|$ in $(-\pi, \pi)$ then Fourier series is :

(A) $f(x) = a_0 + \Sigma a_n \operatorname{con} nx$ (B) $f(x) = a_0 + \Sigma b_n \sin nx$

(C) $f(x) = a_0$ (D) None of the above

Ans. (D) None of the above

Explanation :

For even function $b_n = 0$, so

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \text{ where } f(x) \text{ is defined in } (-\pi, \pi).$$

97. Solution of ODE $\frac{dy}{dx}+\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}=0$ is :

(A) $\sin x - \sin^{-1} y = c$ (B) $\sin^{-1} x \sin^{-1} y = c$

(C) $\sin^{-1} x + \sin^{-1} y = c$ (D) None of the above

Ans. (C) $\sin^{-1} x + \sin^{-1} y = c$

Explanation :

$$\frac{dy}{dx}+\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}=0$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$$

$$\Rightarrow \sin^{-1} y + \sin^{-1} x = c$$

98. Complementary function of $(D^2-1)y - x^2 \cos x$ is :

(A) $c_1 + c_2 e^x$ (B) $c_1 e^x$

(C) $c_1 \cos x + c_2 \sin x$ (D) $c_1 e^x + c_2 e^{-x}$

Ans. (D) $c_1 e^x + c_2 e^{-x}$

Explanation :

Auxiliary equation $m^2 - 1 = 0 \Rightarrow m = \pm 1$

$CF = c_1 e^x + c_2 e^{-x}$

99. Integrating factor in $\frac{dv}{dx}+\frac{2}{x+1}v = x^3$ is :

(A) $(x+1)^2$ (B) $(x-1)^2$ (C) x^2+1 (D) x^2-1

Ans. (A) $(x+1)^2$

Explanation :

$$\text{If} = e^{\int P dx} = e^{\int \frac{2}{x+1} dx} = e^{2\log(x+1)} = e^{\log(x+1)^2}$$
$$= (x+1)^2$$

100. If curve $x^2 = c^2(1+y^2)$ passes through (1, 0), then the value of c is

(A) $c = \pm 1$ (B) $c = 2$ (C) $c = 3$ (D) $c = 4$

Ans. (A) $c = \pm 1$

Explanation :

$x^2 = c^2(1+y^2)$

Put $x = 1, y = 0 \Rightarrow 1 = c^2 \Rightarrow c = \pm 1$

BCA

(SEM. II) EXAMINATION, 2021-22
BCA – 4005 : MATHEMATICS - III

Time : 1 :30 Hours Maximum Marks : 100

1. Roots of equation $z^2+1=0$ are :
(A) i, - i (B) 1, i (C) 1, -1 (D) -1, i

Ans. (A) i, - i

Explanation :

$$z^2+1=0$$
$$z^2=-1=i^2$$
$$z=\pm i$$

2. Real part of $f(z)=4+i$ is :
(A) 1 (B) 4 (C) 4 + i (D) i

Ans. (B) 4

Explanation :

$$\text{Re} f(z)=4$$
$$\text{Im} f(z)=1$$

3. $\overline{z_1+z_2}$ is equal to :
(A) $\bar{z}_1-\bar{z}_2$ (B) $\bar{z}_1+\bar{z}_2$
(C) z_1-z_2 (D) NOT

Ans. (B) $\bar{z}_1+\bar{z}_2$

Explanation :

$$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$$

4. Value of $\sqrt{-100}$ is :
(A) 10 (B) -10 (C) 10i (D) -10i

Ans. Both (C) and (D) are correct

Explanation :

$$\sqrt{-100}=\sqrt{-100i^2}=\pm 10i$$

5. In Fourier series of f(x), coefficients a_0, a_n, b_n are called :
(A) Laplace coefficients (B) Fourier coefficients
(C) Taylor's coefficients (D) Not

Ans. (B) Fourier coefficients

Explanation :

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{T}\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{T}\right)$$

Where a_0, a_n, b_n are Fourier coefficients

6. If $f(x) = \sin x$ is given in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, then the value of a_n in series :

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (A) 0

Explanation :

$$f(x) = \sin x, \quad x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{T}\right)$$

$$a_n = \frac{1}{T}\int_a^b f(x)\cos\left(\frac{n\pi x}{T}\right)dx, \quad b_n = \frac{1}{T}\int_a^b f(x)\sin\left(\frac{n\pi x}{T}\right)dx$$

$$T = \frac{b-a}{2} = \frac{\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)}{2} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi/2}\int_{-\pi}^{\pi/2} \sin x . \cos\left(\frac{n\pi x}{T}\right)dx = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} \sin x . \cos(2nx)dx$$

$= 0$ as $\sin x . \cos(2n\pi)$ is an odd function integrated between from $\underline{-a}$ to $\underline{+a}$; $a = \frac{\pi}{2}$

7. Differential equation $\frac{3dy}{dx} + \frac{2}{x+1}y = \frac{x^3}{y^2}$ is reduced to linear transformation :

(A) $y = x^3$ (B) $y^2 = t$

(C) $y^3 = t$ (D) NOT

Ans. (C) $y^3 = t$

Explanation :

$$3y^2\frac{dy}{dx} + \frac{2}{x+1}y^3 = x^3$$

Let $y^3 = t \Rightarrow 3y^2 dy = dt$

$$\frac{dt}{dx} + \frac{2}{x+1}t = x^3$$

Which is linear.

8. $\int_c^{c+2\pi} \sin nx \, dx$ is :

(A) π (B) c (C) 0 (D) 2π

Ans. (C) 0

Explanation :

$$\int_c^{c+2\pi} \sin nx\, dx = \left[\frac{\cos(nx)}{n}\right]_c^{c+2\pi}$$

$$= -\left[\frac{\cos(n(c+2\pi)) - \cos(nc)}{n}\right]$$

$$= -\left[\frac{\cos(nc+2n\pi) - \cos(nc)}{n}\right]$$

$$= 0$$

As $\boxed{\cos(\theta+2\pi) = \cos\theta = \cos(\theta+2n\pi)}$

Where $n \in I$

9. If $f(x) = \cos x$ in $[-l, l]$ then Fourier coefficient b_n is :

(A) 1 (B) l (C) -l (D) 0

Ans. (D) 0

Explanation :

$f(x) = \cos x\ ;\ x \in [-l, l]$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{T}\right)$$

$$a_n = \frac{1}{T}\int_a^b f(x)\cos\left(\frac{n\pi x}{T}\right)dx$$

$$b_n = \frac{1}{T}\int_a^b f(x)\sin\left(\frac{n\pi x}{T}\right)dx$$

$$T = \frac{b-a}{2} = \frac{l-(-l)}{2} = l$$

$$b_n = \frac{1}{l}\int_{-l}^{l} \cos x.\sin\left(\frac{n\pi x}{l}\right)dx = 0$$

as $\cos x \sin\left(\frac{n\pi x}{l}\right)$ is odd function which is integrated between $-l$, l

so by property of definite integral.

10. Fourier coefficients a_0, a_n, b_n are obtained by :

(A) Taylor's Formula (B) Maclaurin's Formula
(C) Euler's Formula (D) Laplace Formula

Ans. (C) Euler's Formula

11. $\frac{1}{D^2}\sin 2x$ is equal to :

(A) $\frac{-1}{2}\sin 2x$ (B) $\frac{-1}{16}\sin 2x$ (C) $\frac{1}{4}\sin 2x$ (D) $\frac{-1}{4}\sin 2x$

Ans. (D) $\frac{-1}{4}\sin 2x$

Explanation :

$$\frac{1}{D^2}\sin 2x = -\frac{1}{4}\sin 2x$$

12. $\frac{1}{D+1}e^x$ is :

(A) e^x (B) $\frac{1}{2}e^x$

(C) $\frac{1}{4}e^x$ (D) $\frac{-1}{4}e^x$

Ans. (B) $\frac{1}{2}e^x$

Explanation :

$D \to 1 \quad e^{ax}$

13. If f(x) =x is periodic in $[0, 2\pi]$ then Fourier coefficient a_0 is :

(A) $a_0 = \pi$ (B) $a_0 = 2\pi$

(C) $a_0 = \pi/2$ (D) NOT

Ans. (B) $a_0 = 2\pi$

Explanation :

$$f(x) = x, [0, 2\pi], T = \frac{b-a}{2} = \frac{2\pi - 0}{2} = \pi$$

$$a_0 = \frac{1}{T}\int_a^b f(x)dx = \frac{1}{\pi}\int_0^{2\pi} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= 2\pi$$

14. Modulus value of $z = x + iy$ is :

(A) $\sqrt{x^2 + y}$ (B) $\sqrt{x^2 + y^2}$

(C) $\tan^{-1}\left(\frac{y}{x}\right)$ (D) $\sqrt{x^2 - y^2}$

Ans. (B) $\sqrt{x^2 + y^2}$

Explanation :

$z = x + iy$

$$|z| = \sqrt{(\text{Re}\, z)^2 + (\text{Im}\, z)^2} = \sqrt{x^2 + y^2}$$

15. Cube roots of unity are :

(A) $1, \omega, \omega^2$ (B) 1, - 1, 0

(C) 1, 1, 1 (D) Not

Ans. (A) $1, \omega, \omega^2$

Explanation :

$z^3 - 1 = 0 \Rightarrow z = 1^{1/3} = e^{2n\pi/3}$

$z = 1, \omega, \omega^2$ for $n = 0, 1, 2$

$= 1, \frac{-1 \pm \sqrt{3}i}{2}$

16. Argument value of $z = 1 + i$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 0 (D) $\frac{-\pi}{4}$

Ans. (B) $\frac{\pi}{4}$

Explanation :

$$\text{Arg } z = \tan^{-1}\left(\frac{\text{Im } z}{\text{Re } z}\right) = \tan^{-1}\left(\frac{1}{1}\right)$$

$= \frac{\pi}{4}$; as $z = 1 + i$ lies in first quadrant

17. If $(x+5)+i(y+3)=0$ then :

(A) $x = 5, y = 3$ (B) $x = -5, y = 3$

(C) $x = 5, y = -3$ (D) $x = -5, y = -3$

Ans. (D) $x = -5, y = -3$

Explanation :

$$(x+5)+i(y+3)=0$$

$$\Rightarrow \begin{cases} x+5=0 \\ y+3=0 \end{cases}$$

$$\Rightarrow \begin{cases} x=-5 \\ y=-3 \end{cases}$$

18. Differential equation $(1+4xy+2y^2)dx+(1+4xy+2x^2)dy=0$ is :

(A) Linear in x (B) Linear in y

(C) Exact ODE (D) Homogeneous ODE

Ans. (C) Exact ODE

Explanation :

$$(1+4xy+2y^2)dx+(1+4xy+2x^2)y=0$$

$$M = 1+4xy+2y^2, N = 1+4xy+2x^2$$

$$\frac{\partial M}{\partial y} = 4x+4y = \frac{\partial n}{\partial x}$$

So Exact ODE

19. Order of ODE $\left(\frac{dy}{dx}\right)^2 + y = 0$ is :

(A) 1 (B) 2 (C) Not defined (D) -1

Ans. (A) 1

Explanation :

Order = order of highest order derivative

Degree = degree of highest order derivative after clearing radi... fractions

20. Degree of ODE $\sin\left(\frac{dy}{dx}\right) = 1$ is :

(A) 1 (B) 0 (C) 2 (D) Does not exist

Ans. (D) Does not exist

Explanation : Does not exist because of $\sin\left(\frac{dy}{dx}\right)$

21. Maximum rate of increase of scalar point function $\phi(x, y, z)$ is:

(A) $\nabla\phi$ (B) $-|\nabla\phi|$ (C) $|\nabla\phi|$ (D) $-\nabla\phi$

Ans. (C) $|\nabla\phi|$

22. If $\phi(x,y,z) = x + y$, then directional derivative of $\phi(x,y,z)$ in ... direction of $\vec{a} = i$ is :

(A) 0 (B) 1 (C) -1 (D) i

Ans. (B) 1

Explanation :

$\phi(x,y,z) = x + y$

$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z} = \hat{i} + \hat{j}$

$\vec{a} = i \Rightarrow \hat{a} = \frac{\vec{a}}{|a|} = \frac{\hat{i}}{|\hat{i}|} = \hat{i}$

Directional derivative of ϕ in direction of $\vec{a}$

$= \text{grad}\,\phi.\hat{a}$

$= (\hat{i} + \hat{j}).\hat{i} \Rightarrow 1$

23. If $\vec{v} = \vec{w} \times \vec{r}$ then curl $\vec{v}$ is :

(A) $\vec{\omega}$ (B) $\vec{\omega} \times \vec{r}$ (C) $2\vec{\omega}$ (D) NOT

Ans. (C) $2\vec{\omega}$

Explanation :

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_1 & \omega_2 & \omega_3 \\ x & y & z \end{vmatrix} = \sum (\omega_2 z - \omega_3 y)\,\hat{i}$$

$$\text{curl}\,\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \omega_2 z - \omega_3 y & \omega_3 x - \omega_1 z & \omega_1 z - \omega_3 x \end{vmatrix}$$

$$\sum \hat{i}(\omega_1 + \omega_2) = 2\sum \hat{i}\,\omega_1 = 2\vec{\omega}$$

24. If $\vec{r} = xi + yj + zk$ then curl $\bar{r}$ equal to :

(A) 1 (B) $\vec{r}$ (C) 0 (D) 1

Ans. (C) 0

Explanation :

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{curl}\,\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = 0$$

25. If $\phi(x,y,z) = x + y + z$ then gradient of ϕ is :

(A) $i + j + k$ (B) $i + j$

(C) $x + y + z$ (D) 0

Ans. (A) $i + j + k$

Explanation :

$$\phi(x,y,z) = x + y + z$$

$$\text{grad}\,\phi = \hat{i}\frac{\partial}{\partial x}\phi + \hat{j}\frac{\partial}{\partial y}\phi + \hat{k}\frac{\partial}{\partial z}\phi$$

$$= \hat{i} + \hat{j} + \hat{k}$$

26. Which of the following is true?

(A) $|z_1 + z_2| \geq |z_1| + |z_2|$ (B) $|z_1 + z_2| = |z_1| + |z_2|$

(C) $|z_1 + z_2| \leq |z_1| + |z_2|$ (D) NOT

Ans. (C) $|z_1 + z_2| \leq |z_1| + |z_2|$

27. If $(2x + 2) + i(y - 5) = 0$ then :

(A) $x = 1, y = 5$ (B) $x = -1, y = -5$

(C) $x = 1, y = -5$ (D) $x = -1, y =$ ‾

Ans. (D) $x = -1, y = 5$

Explanation :

$(2x+2)+i(y-5)=0$

$$\begin{cases} 2x+2=0 \\ y-5=0 \end{cases}$$

$$\begin{cases} x=-1 \\ y=5 \end{cases}$$

28. If $f(x)=\sin x\cos 2x$ then f(x) is :
(A) Even function (B) Odd function
(C) Neither even nor odd (D) NOT

Ans. (B) Odd function

Explanation :

$f(x)=\sin x\cos 2x$

f(x) is odd function as product of one odd and one even function is odd

29. Infinite series $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{c}\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{c}\right)$ is:
(A) Fourier series (B) Laplace series
(C) Taylor's series (D) Not

Ans. (A) Fourier series

30. If $f(x)=\cos x$ is defined in [-1, 1] then Fourier coefficient b_n is:
(A) 4 (B) -1 (C) +1 (D) 0

Ans. (D) 0

Explanation :

$f(x)=\cos x,\ x\in[-1,1]$

$$b_n=\frac{1}{1}\int_{-1}^{1}\cos x\sin(n\pi x)dx=0$$

As $\cos x.\ \sin(nx\pi)$ is odd function

31. Real part of $f(z)=\sin z$ is :
(A) sin x cos y (B) sin x cos hy
(C) sin hx cos y (D) sin x sin hy

Ans. (B) sin x cos hy

Explanation :

$$\begin{aligned} f(z)=\sin z &= \sin(x+iy) \\ &= \sin x\cos(iy)+\cos x.\sin(iy) \\ &= \sin x\cosh y+i\cos x.\sinh y \end{aligned}$$

$\operatorname{Re}f(z)=\sin x\cosh y$

$\operatorname{Im}f(z)=\cos x\sinh y$

. Euler's Formula is :

(A) $e^{i\theta} = \cos\theta + i\sin\theta$ (B) $e^{-i\theta} = \cos\theta + i\sin\theta$

(C) $e^{i\theta} = 2\cos\theta + \sin\theta$ (D) $e^{-i\theta} = \cos\theta - \sin\theta$

ns. (A) $e^{i\theta} = \cos\theta + i\sin\theta$

3. A vector which is perpendicular to both vectors $\vec{a}$ and $\vec{b}$ is :

(A) $\left|\vec{a} \times \vec{b}\right|$ (B) $\vec{a} \times \vec{b}$ (C) $\left|2\vec{a} \times \vec{b}\right|$ (D) $\left|\vec{b} \times \vec{a}\right|$

Ans. (B) $\vec{a} \times \vec{b}$

Explanation : Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$ or $\vec{b} \times \vec{a}$

34. $\nabla f(r)$ is :

(A) $f'(r)$ (B) $f'(r)\nabla r$ (C) $f'(r)\vec{r}$ (D) $\frac{f'(r)}{r}$

Ans. (b) $f'(r)\nabla r$

Explanation :

$$\nabla f(r) = \Sigma \hat{i}\frac{\partial}{\partial x}f(r) = \Sigma \hat{i}f'(r)\frac{\partial r}{\partial x}$$

$$= f'(r)\Sigma \hat{i}\frac{\partial r}{\partial x} = f'(r)\nabla r$$

35. Gradient of $\left|\vec{r}\right|$ is :

(A) $\frac{\bar{r}}{r}$ (B) $\bar{r}$ (C) $\frac{\bar{r}}{r^2}$ (D) r

Ans. (A) $\frac{\bar{r}}{r}$

Explanation :

$$\text{grad}\left|\vec{r}\right| = \text{grad}|r| = \Sigma i\frac{\partial r}{\partial x} = \Sigma i\left(\frac{x}{r}\right)$$

$$= \frac{\hat{i}x + \hat{j}y + \hat{k}z}{r} = \frac{\vec{r}}{r} = \hat{r}$$

36. For solenoidal field vector $\vec{v}$, divergence of $\vec{v}$ is :

(A) Non zero (B) Zero (C) Never zero (D) NOT

Ans. (B) Zero

37. $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right) =$

(A) $\vec{a} \times \frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt} \times \vec{b}$ (B) $\bar{a} \times \frac{d\bar{b}}{dt} + \frac{d\vec{a}}{dt} \times \bar{b}$

(C) $\vec{a} \times \frac{d\vec{b}}{dt} + \vec{b} \times \frac{d\bar{a}}{dt}$ (D) NOT

Ans. (A) $\vec{a} \times \frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt} \times \vec{b}$

38. Curl of a vector field is :

(A) Scalar (B) Vector

(C) Never be a vector (D) NOT

Ans. (B) Vector

39. $\tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{3}{11}\right)$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) 1 (D) NOT

Ans. (A) $\frac{\pi}{4}$

Explanation :

$$\tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{3}{11}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} \times \frac{3}{11}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{44+21}{77}}{\frac{77-12}{77}}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right) = \tan^{-1} 1 = \frac{\pi}{4}$$

40. Imaginary part of $\frac{3-2i}{7+4i}$ is :

(A) $\frac{2}{5}$ (B) $\frac{-2}{5}$ (C) $\frac{1}{5}$ (D) NOT

Ans. (B) $\frac{-2}{5}$

Explanation :

$$\text{Im}\frac{3-2i}{7+4i}$$

$$=\text{Im}\left[\frac{(3-2i)(7-4i)}{7^2+4^2}\right]=\text{Im}\left[\frac{21-8+i(-12-14)}{49+16}\right]$$

$$=\text{Im}\left(\frac{13-i26}{65}\right)=\text{Im}\left(\frac{1-2i}{65}\right)$$

$$=\frac{-2}{5}$$

41. Locus of $|z|<1$ is :

(A) $x^2+y^2=1$ (B) $x^2+y^2>1$

(C) $x^2+y^2<1$ (D) $x+y<1$

Ans. (C) $x^2+y^2<1$

Explanation :

Locus of $|z|<1 \Rightarrow \sqrt{x^2+y^2}<1$

$\Rightarrow x^2+y^2<1$

42. $\frac{e^{iz}-e^{-iz}}{2i}$ is equal to :

(A) cos z (B) cos hz

(C) sin z (D) sin hz

Ans. (C) sin z

Explanation :

$$\frac{e^{iz}-e^{-iz}}{2i}=\sin z$$

43. If f(x) is periodic function with period 2c then Fourier coefficient a_0 is :

(A) $\frac{1}{4c}\int_a^b f(x)\,dx$ (B) $\frac{1}{3c}\int_a^b f(x)\,dx$

(C) $\frac{1}{c}\int_a^b f(x)\,dx$ (D) NOT

Ans. (B) $\frac{1}{c}\int_a^b f(x)\,dx$

Explanation :

$$a_0=\frac{1}{c}\int_a^b f(x)\,dx$$

44. ODE $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$ is :

(A) Homogeneous ODE (B) Non homogeneous ODE
(C) Linear in x (D) Linear in y

Ans. (A) Homogeneous ODE

Explanation :

$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$ is homogeneous

As order of $\frac{x^2 - y^2}{xy}$ is zero and order of homogeneous ODF is

45. Sequence $< \frac{1}{n} >$ is :

(A) Bounded above
(B) Bounded below
(C) Neither bounded above nor bounded below
(D) Option (A) and (B) both

Ans. (D) Option (A) and (B) both

Explanation :

$< \frac{1}{n} > = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots \right\}$

$= \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots \right\}$

Is monotonic decreasing sequence of positive number and boun above so it is bounded below also

$\lim_{n \to \infty} \frac{1}{n} = 0$

46. Sequence $< a_n > = < n >$ has :

(A) Infinitely many terms (B) Finitely many terms
(C) n terms (D) NOT

Ans. (A) Infinitely many terms

Explanation :

$< a_n > = < n > = \{1, 2, 3, 4, \cdots\}$

Has infinitely many terms

47. Sequence $< \frac{1}{n+1} >$ converges to :

(A) 1 (B) 0 (C) ∞ (D) -1

Ans. (B) 0

Explanation :

$\lim_{n \to \infty} \frac{1}{n+1} = 0$

48. Sequence $<a_n> = \{5, 5, 5, \ldots\ldots\}$ i.e. $a_n = 5 \forall n \in N$ is :

(A) Constant sequence (B) Not bounded above
(C) Not bounded below (D) Not bounded

Ans. (A) Constant sequence

Explanation :

$<a_n> = \{5, 5, \ldots\ldots\}$ is constant sequence

49. Complementary function for ODE $(D^2-1)y=0$, where $D \equiv \frac{d}{dx}$ is :

(A) $(c_1+c_2x)e^x$ (B) $c_1e^x+c_2e^{-x}$
(C) e^x (D) e^{-x}

Ans. (B) $c_1e^x+c_2e^{-x}$

Explanation :

$(D^2-1)y=0$

Auxiliary equation $m^2-1=0 \Rightarrow m=\pm i$

$CF = c_1e^x+c_2e^{-x}$

50. Complementary function for $(D^2+1)y=0$ is :

(A) $c_1\cos x$ (B) $c_2\sin x$
(C) $c_1+c_2\cos x$ (D) $c_1\cos x+c_2\sin x$

Ans. (D) $c_1\cos x+c_2\sin x$

Explanation :

$(D^2+1)y=0$

$m^2+1=0 \Rightarrow m=\pm 1$

$CF = c_1\cos x+c_2\sin x$

51. In sequence $<a_n>$, where $a_n=\frac{1}{n}$ least upper bound is :

(A) 0 (B) 1 (C) 2 (D) -1

Ans. (B) 1

Explanation : $\langle a_n\rangle = \left\langle \frac{1}{n}\right\rangle = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

Least upper bound $1 \geq a_n \ \forall n$

52. Sequence $<1+\frac{1}{n}>$ is :

(A) Bounded (B) Not bounded
(C) Bounded above only (D) Bounded below only

Ans. (A) Bounded

Explanation :

$$\left\langle 1+\frac{1}{n}\right\rangle = \left\{2,\frac{3}{2},\frac{4}{3},\frac{5}{4},\dots\right\}$$

$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)=1$ It is decreasing sequence

$2 \le 1+\frac{1}{n} < 1$ So bounded

53. Sequence $< 2+\frac{7}{n+5} >$ converges to :

(A) 7 (B) 0 (C) 5 (D) 2

Ans. (D) 2

Explanation :

$$\lim_{n\to\infty}\left(2+\frac{7}{n+5}\right) = 2+\lim_{n\to\infty}\frac{7}{n}=2$$

54. If sequence, $< a_n >$ is bounded above there exist a real number s.t. :

(A) $a_n \le k, \forall n \in N$ (B) $a_n \ge k, \forall n \in N$

(C) $a_n < -k, \forall n \in N$ (D) NOT

Ans. (A) $a_n \le k, \forall n \in N$

Explanation : For bounded above sequence

55. Series $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is :

(A) Convergent (B) Divergent

(C) Oscillatory (D) NOT

Ans. (B) Divergent

Explanation :

By p-test $\frac{1}{n^p}=\frac{1}{n} \Rightarrow p=1$

So divergent

56. If series $\sum\limits_{n=1}^{\infty}a_n$ is convergent then :

(A) $\lim\limits_{n\to\infty} a_n = 0$ (B) $\lim\limits_{n\to\infty} a_n \ne 0$

(C) $\lim\limits_{n\to\infty} a_n = 1$ (D) $\lim\limits_{n\to\infty} a_n =$ does not exist

Ans. (A) $\lim\limits_{n\to\infty} a_n = 0$

Explanation : For convergence the necessary condition is $\lim\limits_{n\to\infty} a_n = 0$

But it is not sufficient

57. Series $\sum_{n=1}^{\infty} \cos\frac{1}{n}$ is :

(A) Convergent (B) Divergent
(C) Oscillatory (D) NOT

Ans. (B) Divergent

Explanation :

$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1$ So divergent

58. Series $\sum_{n=0}^{\infty} x^n$ is convergent if :

(A) $|x| = 1$ (B) $|x| > 1$ (C) $|x| < 1$ (D) $x = 1$

Ans. (C) $|x| < 1$

Explanation : $\sum x^n$ is convergent if $|x| < 1$

{ by ratio test and Divergent Test}

And divergent $|x| \geq 1$

59. p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent :

(A) $p = 2$ (B) $p = 3$ (C) $p = 4$ (D) $p = 0$

Ans. (D) $p = 0$

Explanation : p- series is convergent for p>1 and divergent If $p \leq 1$ so for $p = 0$

60. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ is :

(A) Divergent series (B) Oscillatory series
(C) Convergent series (D) NOT

Ans. (C) Convergent series

Explanation : If $\sum_{n=1}^{\infty} \frac{1}{n^4}$ here $p = 4$ convergent

61. Field $\vec{F} = 2xy\,\hat{i} + (x^2 + z^2)\hat{j} + 2zy\hat{k}$ is :

(A) Solenoidal (B) Rotational
(C) Irrotational (D) NOT

Ans. (C) Irrotational

Explanation :

$$\vec{F} = 2xy\,\hat{i} + (x^2 + z^2)\hat{j} + 2zy\hat{k}$$

$$\text{div}\,\vec{F} = \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(x^2 + z^2) + \frac{\partial}{\partial z}(2zy)$$

$$= 2y + 0 + 2y + = 4y \neq 0 \quad \text{Not divergent}$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2 + z^2 & 2yz \end{vmatrix}$$

$$= \hat{i}(2z - 2z) + \hat{j}(0 - 0) + \hat{k}(2x - 2x)$$

$$\text{Curl}\ \vec{F} = = \vec{0} \quad \text{irrotational}$$

62. Curl of a vector field $\vec{f}(x,y,z) = x^2\hat{i} + 2z\hat{j} - y\hat{k}$ is :

(A) $-3\hat{i}$ (B) $-3\hat{j}$ (C) $-3\hat{k}$ (D) 0

Ans. (A) $-3\hat{i}$

Explanation :

$$\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2z & -y \end{vmatrix}$$

$$= \hat{i}(-1-2) + \hat{j}(0-0) + \hat{k}(0-0)$$

$$= -3\hat{i}$$

63. If curl $\left(\vec{v}\right) = 0$ then field $\vec{v}$ is :

(A) Solenoidal (B) Rotational
(C) Irrotational (D) NOT

Ans. (C) Irrotational

Explanation : curl $\vec{F} = 0 \Rightarrow$ Irrotational

64. Particular integral for $(D^2 + 1)y = e^{2x}$ is :

(A) $\frac{1}{4}e^{2x}$ (B) $\frac{1}{5}e^{2x}$ (C) $\frac{1}{2}e^{2x}$ (D) $\frac{1}{6}e^{2x}$

Ans. (B) $\frac{1}{5}e^{2x}$

Explanation : $PI = \frac{1}{D^2+1}e^{2x} = \frac{1}{2^2+1}e^{2x} = \frac{e^{2x}}{5}$

$D \to 2$

65. Particular Integral for $(D^2 + 2D + 1)y = 0$ is :

(A) $\frac{1}{2}$ (B) 0 (C) 2 (D) NOT

Ans. (B) 0

Explanation :

$$PI = \frac{1}{D^2 + 2D + 1} 0 = 0$$

66. Solution of $(D^2 + 5D + 4)y = 0$ is :

(A) $c_1 e^{x} + c_2 e^{-4x}$ (B) $c_1 e^{-x} + c_2 e^{4x}$

(C) $c_1 e^{-x} + c_2 e^{-4x}$ (D) NOT

Ans. (C) $c_1 e^{-x} + c_2 e^{-4x}$

Explanation :

$(D^2 + 5D + 4)y = 0$

Auxiliary equation $m^2 + 5m + 4 = 0$ $m = -1, -4$

$y = c_1 e^{-x} + c_2 e^{-4x}$

67. $\frac{dy}{dx} = \frac{ax + by + c}{dx + ey + f}$ is transformed into linear ODE by taking $x = X + h, y = Y + K$ if :

(A) $ae - bd = 0$ (B) $ae - bd \neq 0$

(C) $ad - be \neq 0$ (D) $ad - be = 0$

Ans. (B) $ae - bd \neq 0$

Explanation :

$$\begin{vmatrix} a & b \\ d & e \end{vmatrix} \neq 0 \Rightarrow ae - bd \neq 0$$

68. Solution of $\frac{dy}{dx} = \frac{x}{y}$ is :

(A) $y^2 = x^2 + c$ (B) $y^2 = -x^2 + c$

(C) $y = x + c$ (D) $y = x - c$

Ans. (A) $y^2 = x^2 + c$

Explanation :

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow ydy = xdx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + \frac{c}{2}$$

$$\Rightarrow y^2 = x^2 + c$$

69. $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$ is :

(A) Homogeneous ODE of degree 2

(B) Homogeneous ODE of degree 0

(C) Homogeneous ODE of degree 1

(D) Non homogeneous ODE

Ans. (C) Homogeneous ODE of degree 1

Explanation :

$$y - x\frac{dy}{dx} = x + y\frac{dy}{dx} \Rightarrow y - x = (x+y)\frac{dy}{dx}$$

$\frac{dy}{dx} = \frac{y-x}{y+x}$ homogeneous ODE of degree 1

70. $\frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$ reduces to variable separable form if :

(A) $x - y = v$ (B) $x + y = v$

(C) $x^2 + y^2 = v$ (D) $xy = v$

Ans. (B) $x + y = v$

Explanation :

$\frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$ Reduces to variable separable

By taking $x + y = v$

71. Integrating factor in ODE $\frac{dy}{dx} + ay = e^{mx}$ is :

(A) e^{mx} (B) e^{x}

(C) e^{a} (D) e^{ax}

Ans. (D) e^{ax}

Explanation :

$$\frac{dy}{dx} + ay = e^{mx} \Rightarrow IF = e^{\int a dx} = e^{ax}$$

72. Particular integral for $F(D)y = f(x)$ is :

(A) $\frac{1}{F(D)}f(x)$ (B) $\frac{1}{F(D)}f(x) + c(c \neq 0)$

(C) $\frac{1}{F(D)}f(x) + 1$ (D) NOT

Ans. (A) $\frac{1}{F(D)}f(x)$

Explanation :

$F(D)y = f(x)$

PI $\frac{1}{F(D)}f(x)$

73. Solution of $\frac{d^2y}{dx^2} - y = 0$ is :

(A) $y = c_1e^x + c_2e^{-x}$ (B) $y = c_1e^x + c_2e^x$

(C) $y = c_1e^x$ (D) $(c_1 + c_2x)e^x$

Ans. (A) $y = c_1e^x + c_2e^{-x}$

Explanation :

$(D^2 - 1)y = 0 \Rightarrow m^2 - 1 = 0 \Rightarrow m = \pm 1$

$y = cf = c_1e^xc_2e^{-x}$

74. Maximum value of directional derivative of $\phi = 2x^2 + 3y^2 + 5z^2$ of (1, 1, -1) is :

(A) 10 (B) -4 (C) $\sqrt{152}$ (D) 152

Ans. (C) $\sqrt{152}$

Explanation : Maximum value of directional derivative

$|\text{grad}\phi|$

$\text{grad}\phi = \hat{i}\frac{\partial}{\partial x}\phi + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z} = 4x\hat{i} + 6y\hat{j} + 10z\hat{k}$

$\text{grad}\phi_{(1,1,-1)} = 4\hat{i} + 6\hat{j} - 10\hat{k}$

$|\text{grad}\phi|_{(1,1,-1)} = \sqrt{4^2 + 6^2 + (-10)^2}$

$= \sqrt{16 + 36 + 100} = \sqrt{152}$

75. Divergence of $\vec{r}$ is :

(A) 1 (B) 0 (C) 3 (D) 4

Ans. (C) 3

Explanation :

$\text{div}\,\vec{r} = \text{div}(x\hat{i} + y\hat{j} + z\hat{k})$

$= \sum\frac{\partial}{\partial x}(x) = 3$

76. $\sin^{-1}x + \cos^{-1}x$ is:

(A) $\frac{\pi}{2}$ (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) NOT

Ans. (A) $\frac{\pi}{2}$

Explanation :

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

77. Solution of $x\,dx - y\,dy = 0$ is :

(A) $x = y + c$ (B) $x^2 - y^2 = c$

(C) $x - y = c$ (D) NOT

Ans. (B) $x^2 - y^2 = c$

Explanation :

$$x\,dx - y\,dy = 0 \Rightarrow \frac{x^2}{2} - \frac{y^2}{2} = \frac{c}{2}$$

$$x^2 - y^2 = c$$

78. $\frac{dy}{dx} + 2y = x^2$ is :

(A) Linear ODE in x (B) Linear in y

(C) Linear in both variable x and y (D) NOT

Ans. (B) Linear in y

Explanation :

$$\frac{dy}{dx} + 2y = x^2$$

$$\frac{dy}{dx} + P(x)y = \phi(x) \text{is linear in y}$$

79. Degree of homogeneous ODE $(x^2 - y^2)dx + 3xy\,dy = 0$ is :

(A) 1 (B) 2 (C) 0 (D) NOT

Ans. (B) 2

Explanation : Degree of homogeneous differential

$$(x^2 - y^2)dx + 3xy\,d = 0 \Rightarrow \frac{dy}{dx} = \frac{3xy}{x^2 - y^2}$$

Of degree 2

80. ODE $y\frac{dy}{dx} = x$ is :

(A) Linear in x (B) Non Linear in y

(C) Order of ODE is 2 (D) Degree of ODE is 2

Ans. (B) Non Linear in y

Explanation :

$$y\frac{dy}{dx} = x \Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

Non linear in x & y but homogeneous

81. Order of ODE $\left(\frac{dy}{dx}\right)^2 + y = 0$ is :

(A) 1 (B) 2 (C) Not defined (D) NOT

Ans. (A) 1

Explanation :

$$\left(\frac{dy}{dx}\right)^2 + y = 0$$

Order = 1 because of $\frac{dy}{dx}$

82. Integrating factor in linear ODE $\frac{dy}{dx} + Py = Q$ is :

(A) $e^{\int Pdx}$ (B) $e^{\int Pdy}$ (C) $e^{\int Pdx+C}$ (D) $e^{\int Pdy+C}$

Ans. (A) $e^{\int Pdx}$

Explanation :

$IF = e^{\int Pdx}$

83. Degree of ODE $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{d^2y}{dx^2} + y = 0$ is :

(A) 1 (B) 2 (C) 4 (D) NOT

Ans. (B) 2

Explanation :

Degree of differential equation is highest power of highest order derivative

84. $f(x, y) = x^2 + y^2 + xy$ is homogeneous function of degree :

(A) 1 (B) 0 (C) 2 (D) NOT

Ans. (C) 2

Explanation :

$f(x, y) = x^2 + y^2 + xy$

$= x^2[1 + (y/x)^2 + (y/x)]$

$= x^n \phi(y/x)$

Order of homogeneous function is 2

85. $\frac{dy}{dx} + Py = Qy^n, (n \neq 0, 1)$ is :

(A) Linear in y (B) Non Linear in x

(C) Bernoulli equation (D) NOT

Ans. (C) Bernoulli equation

86. ODE $M\,dx + N\,dy = 0$ is exact if :

(A $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (B $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (C $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (D $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

Ans. (A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Explanation :

M dx + N dy = 0 is exact

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

87. $\frac{dy}{dx} + Py = Qy^n$ is linear ODE if :

(A) n = 1 (B) n = 2 (C) n = 0 (D) n = -1

Ans. (C) n = 0

Explanation : for linear ODE $\frac{dy}{dx} + P(x)y = Q(x)$

So n = 0

88. In Fourier series of f(x), f(x) is :

(A) Periodic (B) Non periodic

(C) Discontinuous (D) Non differentiable

Ans. (A) Periodic

Explanation : In Fouries series f(x) is periodic peicewise contin function

89. In Fourier series, a_0, a_n, b_n are obtained by :

(A) Euler's formula (B) Laplace formula

(C) Taylor's Formula (D) Maclaurin's formula

Ans. (A) Euler's formula

Explanation : a_0, a_n, b_n are obtain by Euler's formula

90. If f(x) is periodic with period w =2l then Fourier coefficient :

(A) $a_0 = \frac{1}{4l}\int_a^b f(x)dx$ (B) $a_0 = \frac{1}{l}\int_a^b f(x)dx$

(C) $a_0 = \frac{4}{l}\int_a^b f(x)dx$ (D) NOT

Ans. (B) $a_0 = \frac{1}{l}\int_a^b f(x)dx$

91. Argument value of $z = -1 - i$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) $\frac{-3\pi}{4}$ (D) $\frac{3\pi}{4}$

Ans. (C) $\frac{-3\pi}{4}$

Explanation :

$$\text{Arg} z = \tan^{-1}\left(\frac{\text{Im} z}{\text{Re} z}\right) = \tan^{-1}\left(\frac{-1}{-1}\right)$$

$$= \frac{5\pi}{4}, \frac{-3\pi}{8} \text{(Third qudrant)}$$

92. Modulus value of $z = a + ib$ is :

(A) $a^2 + b^2$ (B) $\sqrt{a^2 + b^2}$ (C) $\sqrt{a^2 - b^2}$ (D) $a^2 - b^2$

Ans. (B) $\sqrt{a^2 + b^2}$

Explanation :

$$|z| = \sqrt{a^2 + b^2}$$
$$= \sqrt{(\text{Re}\,z)^2 + (\text{Im}\,z)^2}$$

93. Sequence {3, 3, 3, 3,...............} is :

(A) Bounded above only

(B) Bounded below only

(C) Bounded above and bounded below

(D) Unbounded

Ans. (C) Bounded above and bounded below

Explanation :

{3, 3, 3 ...}

is constant sequence so bounded

so bounded above and bounded below

$a_n = 3 \; \forall n \in N$

94. If sequence $< a_n >$ is convergent then $\lim\limits_{n \to \infty} a_n$ is :

(A) unique (B) Not unique

(C) Always more than one limits (D) NOT

Ans. (A) unique

Explanation :

$\lim\limits_{n \to \infty} a_n =$ unique, finite

$\Rightarrow \langle a_n \rangle$ is convergent

95. n^{th} partial sum in the series $\sum\limits_{n=1}^{\infty} a_n$ is :

(A) $a_1 + a_2 + a_3 + \ldots\ldots + a_{n-1}$

(B) $a_1 + a_2 + a_3 + \ldots\ldots + a_n$

(C) $a_1 + a_2 + a_3 + \ldots\ldots + a_{n+1}$

(D) $a_0 + a_1 + a_2 + \ldots\ldots + a_n$

Ans. (B) $a_1 + a_2 + a_3 + \ldots\ldots + a_n$

Explanation :

n^{th} partial sum $= \sum\limits_{n=1}^{n} a_n$

96. Sequence $\{1, -1, 1, -1, 1, -1 \ldots\ldots\}$ is

(A) Convergent (B) Divergent
(C) Oscillatory (D) Not convergent

Ans. (C) Oscillatory

Explanation :

$\{1, -1, 1, -1, 1, -1......\} = \langle(-1)^{n+1}\rangle$

Is oscillatory

97. Series $\sum_{n=0}^{\infty} x^n$ is oscillatory if :

(A) $x = 1$ (B) $x = -1$ (C) $x = 2$ (D) $x = 0$

Ans. (B) $x = -1$

98. Series $\sum_{n=1}^{\infty}(-1)^n$ is :

(A) Convergent (B) Divergent (C) Oscillatory (D)

Ans. (C) Oscillatory

99. Series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^{2022}$ is :

(A) Convergent (B) Divergent
(C) Oscillatory (D) Oscillatory finitely

Ans. (A) Convergent

Explanation : $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^{2022}$ is convergent by p-series tes

$p = 2022 > 1$

100. If $z = -1+i$ then modulus amplitude form of z is :

(A) $z = \sqrt{2}\, e^{\left(\frac{5\pi}{4}\right)i}$ (B) $z = \sqrt{2}\, e^{\left(\frac{-5\pi}{4}\right)i}$

(C) $z = \sqrt{2}\, e^{\left(\frac{\pi}{4}\right)i}$ (D) $z = \sqrt{2}\, e^{\left(\frac{-\pi}{4}\right)i}$

Ans. (B) $z = \sqrt{2}\, e^{\left(\frac{-5\pi}{4}\right)i}$

Explanation :

$z = -1+i$

$|z| = \sqrt{(-1)^2 + (1)^2} = 2$

$\theta = \text{Arg } z = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{3\pi}{4}, \frac{-5\pi}{4}$ (second quadrant)

$z = \sqrt{2}\, e^{\frac{-5\pi}{4} i}$

$z = |z| e^{i\theta}$

BCA

(SEM. II) MODEL PAPER – I

BCA – 4005 : MATHEMATICS - III

Time : 1 :30 Hours *Maximum Marks : 75*

Q.1. The n^{th} roots of any number are in ____________

(A) Arithmetic progression
(B) Geometric progression
(C) Harmonic progression
(D) No specific pattern

Ans. (B) Geometric progression

Q.2. Find the cube root of 8i lying in the first quadrant of the complex plane.

(A) $i-\sqrt{3}$ (B) $2i+\sqrt{3}$

(C) $i+2\sqrt{3}$ (D) $i+\sqrt{3}$

Ans. (D) $i+\sqrt{3}$

Q.3. What does $e^{2+\pi i}$ equal?

(A) ie^2 (B) 1

(C) e^2 (D) $-e^2$

Ans. (D) $-e^2$

Q.4. Which axis is known as real axis in argand plane?

(A) x-axis (B) y-axis
(C) z-axis (D) any axis

Ans. (A) x-axis

Q.5. Which axis is known as imaginary axis in argand plane?

(A) x-axis (B) y-axis
(C) z-axis (D) any axis

Ans. (B) y-axis

Q.6. Convert -1-i into polar form.

(A) $2-\sqrt{5},\pi/4$ (B) $2-\sqrt{3},\pi/4$

(C) $2-\sqrt{-3},\pi/4$ (D) $2-\sqrt{\pi/4}$

Ans. (C) $2-\sqrt{-3},\pi/4$

Q.7. Convert (8, 2π/3) into Argand plane representation.

(A) $\left(-4,\ 4\sqrt{3}\right)$ (B) $(4,\ 4\sqrt{3})$

(C) $\left(4\sqrt{3},\ 4\right)$ (D) $\left(-4\sqrt{3},\ 4\right)$

Ans. (A) $\left(-4,\ 4\sqrt{3}\right)$

Q.8. What will be the value of $x+y+z$ if $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$?

(A) -1/3 (B) 1

(C) 3 (D) -3

Ans. (D) -3

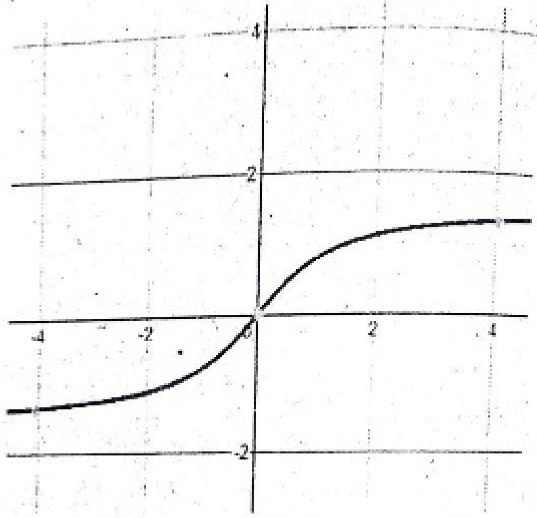
Q.9. What is the value of $\cos^{-1}(-x)$ for all x belon[g] [-1, 1] ?

(A) $\cos^{-1}(-x)$ (B) $\pi-\cos^{-1}(x)$

(C) $\pi-\cos^{-1}(-x)$ (D) $\pi+\cos^{-1}(x)$

Ans. (B) $\pi-\cos^{-1}(x)$

Q.10. The given graph is for which equation?

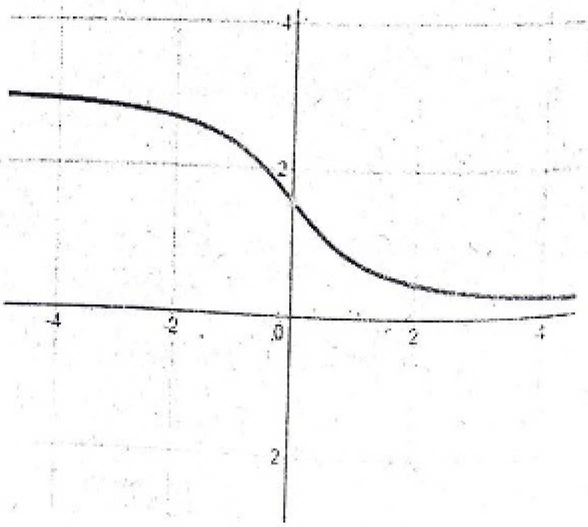


(A) $y=\cos^{-1}x$ (B) $y=\cot^{-1}x$

(C) $y=\operatorname{cosec}^{-1}x$ (D) $y=\tan^{-1}x$

Ans. (D) $y=\tan^{-1}x$

Q.11. The given graph is for which equation?



(A) $y = \cot^{-1} x$ (B) $y = \tan^{-1} x$

(C) $y = \cot x$ (D) $y = \operatorname{cosec}^{-1} x$

Ans. (A) $y = \cot^{-1} x$

Q.12. What is the principle value of $\sec^{-1}(2/\sqrt{3})$.

(A) $\pi/6$ (B) $\pi/3$

(C) $\pi/4$ (D) $\pi/2$

Ans. (A) $\pi/6$

Q.13. [-1, 1] is the domain for which of the following inverse trigonometric functions?

(A) $\sin^{-1} x$ (B) $\cot^{-1} x$

(C) $\tan^{-1} x$ (D) $\sec^{-1} x$

Ans. (A) $\sin^{-1} x$

Q.14. The domain of $\sin^{-1}(3x)$ is equal to ______

(A) [-1, 1] (B) [−1/3, 1/3]

(C) [-3, 3] (D) $[-3\pi, 3\pi]$

Ans. (B) [−1/3, 1/3]

Q.15. What is $\sec^{-1} x$ in terms of $\tan^{-1}$?

(A) $\tan^{-1}\sqrt{1} + x^2$ (B) $\tan^{-1} 1 + x^2$

(C) $\tan^{-1} x$ (D) $\tan^{-1}\sqrt{x^2} - 1$

Ans. (D) $\tan^{-1}\sqrt{x^2} - 1$

Q.16. What is the value of $\cos(\tan^{-1}(4/5))$?

(A) 5/4 (B) $5/\sqrt{41}$

(C) $\sqrt{41}/5$ (D) 4/5

Ans. (B) $5/\sqrt{41}$

Q.17. if $z = \cos 3\theta + i \sin 3\theta$, then $z^5 = \ldots$

(A) $\cos 15\theta + i \sin 15\theta$ (B) $\cos 15\theta - i \sin 15\theta$

(C) $\cos 8\theta + i \sin 8\theta$ (D) $\cos 8\theta - i \sin 8\theta$

Ans. (A) $\cos 15\theta + i \sin 15\theta$

Q.18. A number of the type $z = x + iy$ is called as

(A) Real number (B) Complex number

(C) Integer (D) Irrational number

Ans (B) Complex number

Q.19 In a complex number $z-7-3i$ the imaginary part of z is

(A) -3 (B) $3i$

(C) 7 (D) 7

Ans. (A) -3

Q.20. The conjugate of complex number $1+i$ is :

(A) $1-i$ (B) $1+i$

(C) 0 (D) None of these

Ans. (A) $1-i$

Q.21. If $z=1-\sqrt{3}i$ then modulus of z is equal to :

(A) 1 (B) 2

(C) 3 (D) 4

Ans. (B) 2

Q.22. If $z=i+i^2+i^3$ then real part of z is :

(A) -1 (B) 0

(C) 3 (D) None of these

Ans. (A) -1

Q.23. If z is complex number then $e^z-e^{-z}/2=\cdots$:

(A) $\sinh z$ (B) $i\sinh z$

(C) $\tanh z$ (D) None of these

Ans. (A) $\sinh z$

Q.24. The value of $i+i^2+i^3+i^4$ is :

(A) -1 (B) i

(C) 1 (D) 0

Ans. (D) 0

Q.25. If z= 1+ i then $\arg(z)=\ldots$:

(A) $\pi/4$ (B) $\pi/2$

(C) $\pi/3$ (D) None of these

Ans. (A) $\pi/4$

Q.26. Product of two roots of unity is a :

(A) Root of unity (B) 1

(C) -1 (D) None of these

Ans. (A) Root of unity

Q.27. If $\omega^3=1$ then $1+\omega+\omega^2=\ldots$:

(A) N (B) 1

(C) 0 (D) -1

Ans. (C) 0

Q.28. Let and v are real valued function of variables x, and $(z) = u + iv$ is analytic function. if $u = x^2 + y^2$ then find value of u_x :

(A) 2y (B) 2x

(C) -2x (D) -2y

Ans. (B) 2x

Q.29. A pole of order two is called

(A) Simple Pole (B) Double Pole

(C) Triple Pole (D) None of these

Ans. (B) Double Pole

Q.30. If $f(z) = u + iv$ is analytic function of Z, then $f(z)$ not is independent of Z

(A) True (B) False

(C) 1 and 2 (D) None of these

Ans. (B) False

Q.31. If $u = x$, hen an analytic function $f(z) = u + iv$ is

(A) $x + iy + c$ (B) $X + iy$

(C) $X - iy$ (D) None of these

Ans. (A) $x + iy + c$

Q.32. If $|z| < 1$ then $1 - z + z^2 - z^3 + \cdots$ is exp an ________.

(A) $\sin z$ (B) $1/1 + z$

(C) $1/1 - z$ (D) None of these

Ans. (B) $1/1 + z$

Q.33. What is the divergence of the vector field $\vec{f} = 3x2\hat{i} + 5xy2\hat{j} + xyz3\hat{k}$ at the point (1, 2, 3) :

(A) 89 (B) 80

(C) 124 (D) 100

Ans. (A) 89

Q.34. Divergence of $\vec{f}(x, y, z) = (x\hat{i} + y\hat{j} + z\hat{k})/(x2 + y2 + z2)^{3/2}$ $(x, y, z) \neq (0, 0, 0)$:

(A) 0 (B) 1

(C) 2 (D) 3

Ans. (A) 0

Q.35. Chose the curl of $\vec{f}(x, y, z) = x2\hat{i} + xyz\hat{j} - z\hat{k}$ at the point (2, 1, -2).:

(A) $2\hat{i} + 2\hat{k}$ (B) $-2\hat{i} - 2\hat{j}$

(C) $4\hat{i} - 4\hat{j} + 2\hat{k}$ (D) $-2\hat{i} - 2\hat{k}$

Ans. (D) $-2\hat{i} - 2\hat{k}$

Q 36. A vector field which has a vanishing divergence is called as ________

(A) Solenoidal field
(B) Rotational field
(C) Hemispheroidal field
(D) Irrotational field

Ans. (A) Solenoidal field

Q.37. Divergence and Curl of a vector field are ________

(A) Scalar & Scalar (B) Scalar & Vector
(C) Vector & Vector (D) Vector & Scalar

Ans. (B) Scalar & Vector

Q.38. The curl of vector field $\vec{f}(x,y,z) = x2\hat{i} + 2z\hat{j} - y\hat{k}$ is ________

(A) $-3\hat{i}$ (B) $-3\hat{j}$
(C) $-3\hat{k}$ (D) 0

Ans. (A) $-3\hat{i}$

Q.39. Del operator is also known as ________

(A) Divergence operator
(B) Gradient operator
(C) Curl operator
(D) Laplacian operator

Ans. (B) Gradient operator

Q.40. If $W = x^2y^2 + xz$, the directional derivative dW/dl in the direction $3a_x + 4a_y + 6a_z$ at (1,2,0). :

(A) 5 (B) 6
(C) 7 (D) 8

Ans. (B) 6

Q.41. If $W = xy + yz + z$, find directional derivative of W at (1,-2,0) in the direction towards the point (3,6,9).:

(A) -0.6 (B) -0.7
(C) -0.8 (D) -0.9

Ans. (C) -0.8

Q.42. A polar vector is one which?

(A) Gives the position of an object
(B) Tells how much and in which direction an object has changed its position
(C) Represents rotational effect
(D) Has a starting point of application

Ans. (D) Has a starting point of application

Q.43.Find the divergence of this given vector $\vec{F} = x^3y\hat{i} + 3xy^2z\hat{j} + 3zx\hat{k}$.

(A) $3x^2y + 6xyz + x$

(B) $2x^2y + 6xyz + 3x$

(C) $3x^2y + 3xyz + 3x$

(D) $3x^2y + 6xyz + 3x$

Ans. (D) $3x^2y + 6xyz + 3x$

Q.44.The Fourier cosine transform of the function $f(x)$ is

(A) $F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\, du$

(B) $F_c(\lambda) = \int_0^\infty f(u)\cos u\, du$

(C) $F_c(\lambda) = \int_0^\infty f(\lambda u)\cos u\, du$

(D) None

Ans. (A) $F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\, du$

Q.45.For the Fourier sine integral representation

$$\frac{2}{\pi}\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin\lambda x\, d\lambda = \begin{cases} 1, & 0 < x < \pi \\ 0, & x > \pi \end{cases} \quad F_s(\lambda) \text{ is :}$$

(A) $\frac{1+\cos\pi\lambda}{\lambda}$ (B) $\frac{1-\sin\pi\lambda}{\lambda}$ (C) $\frac{\cos\pi\lambda}{\lambda}$ (D) $\frac{1-\cos\pi\lambda}{\lambda}$

Ans. (D) $\frac{1-\cos\pi\lambda}{\lambda}$

Q.46.For the Fourier sine integral representation

$$\frac{2}{\pi}\int_0^\infty \frac{\sin\pi\lambda}{1-\lambda^2}\sin\lambda x\, d\lambda = \begin{cases} \sin x, & |x| \le \pi \\ 0, & |x| > \pi \end{cases}, F_s(\lambda) \text{ is :}$$

(A) $\frac{\sin\pi\lambda}{1-\lambda^2}$ (B) $\frac{\sin\pi\lambda}{1+\lambda^2}$ (C) $\frac{\cos\pi\lambda}{1-\lambda^2}$ (D) $\frac{1}{1-\lambda^2}$

Ans. (A) $\frac{\sin\pi\lambda}{1-\lambda^2}$

Q.47.If the Fourier integral representation $f(x)\frac{2}{\pi}\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ then value of integral $\int_0^\infty \frac{\sin\lambda}{\lambda}d\lambda$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) 0 (D) 1

Ans. (B) $\frac{\pi}{2}$

Q.48. In the Fourier integral representation of $\frac{1}{2\pi}\int_{-\infty}^{\infty}\pi\left(\frac{e^{-i\lambda\pi}+1}{1-\lambda^2}\right)e^{i\lambda x}d\lambda=\begin{cases}0, & x<0\\ e^{-x}, & x\geq 0,\end{cases}$ $F(\lambda)$ is :

(A) $\frac{1+\lambda^2}{1-i\lambda}$ (B) $\frac{\sin\lambda}{1+\lambda^2}$

(C) $\pi\frac{1-i\lambda}{1+\lambda^2}$ (D) $\frac{\cos\lambda}{1+\lambda^2}$

Ans. (C) $\pi\frac{1-i\lambda}{1+\lambda^2}$

Q.49. Fourier cosine transform of e^{-ax} is :

(A) s/s^2+a^2 (B) s/s^2-a^2

(C) a/s^2+a^2 (D) None

Ans. (C) a/s^2+a^2

Q.50. If (f) is even function then its Fourier transform (s) is ________

(A) real and odd (B) real and even

(C) Imaginary and even (D) Imaginary and odd

Ans. (B) real and even

Q.51. The Fourier sine transform of $\frac{e^{-ax}}{x}$ is

(A) $\tan^{-1}(s)$ (B) $\tan^{-1}\left(\frac{a}{s}\right)$

(C) $\tan^{-1}\left(\frac{s}{a}\right)$ (D) None

Ans. (C) $\tan^{-1}\left(\frac{s}{a}\right)$

Q.52. The Fourier transform $F(\lambda)$ of $f(x)=e^{-|x|}$ is given by

(A) $\frac{1-\lambda}{1+\lambda^2}$ (B) $\frac{1}{1+\lambda^2}$ (C) $\frac{1}{1-\lambda^2}$ (D) $\frac{2}{1+\lambda^2}$

Ans. (D) $\frac{2}{1+\lambda^2}$

Q.53. If $U(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$ then Z – Transform of $U(k)$ is given by

(A) $\frac{z}{z-1}, |z| > 1$ (B) $\frac{1}{z-1}, |z| > 1$

(C) $\frac{z}{z-1}, |z| > 1$ (D) $\frac{2}{z-1}, |z| > 1$

Ans. (C) $\frac{z}{z-1}, |z| > 1$

Q.54. If $\delta(k) = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{if } k \neq 0 \end{cases}$ then $Z\{\delta(k)\} =$ ________ :

(A) $\frac{1}{z}$ (B) $\frac{1}{z-1}$

(C) z (D) 1

Ans. (A) $\frac{1}{z}$

Q.55. $Z\{a^k\}$ for $k \geq 0$ is equal to ________

(A) $\frac{z}{z-a}$ (B) $\frac{a}{z-a}$ (C) $\frac{1}{z-a}$ (D) $\frac{z}{a-z}$

Ans. (A) $\frac{z}{z-a}$

Q.56. What is the set of all values of z for which $f(z)$ attains a finite value?

(A) Feasible region
(B) region of convergence
(C) Region of divergence
(D) None of these

Ans. (B) region of convergence

Q.57. If $|z| < |a|$ inverse of Z-transform of $\frac{z}{z-a}$ is given by

(A) $a^k, k \geq 0$ (B) $a^k, k < 0$

(C) $a^{k-1}, k \geq 0$ (D) $-a^k, k < 0$

Ans. (D) $-a^k, k < 0$

Q.58. Binomial expansion of $\frac{1}{1-y}, |y| < 1$ is ________

(A) $1 - y + y^2 - y^3 + \cdots \ldots \ldots$

(B) $-y - y^2 - y^3 - \cdots \ldots \ldots$

(C) $1+y+y^2+y^3+\cdots\cdots\cdots$

(D) None

Ans. (C) $1+y+y^2+y^3+\cdots\cdots\cdots$

Q.59. If $Z\{f(k)\}=F(Z)$ and $Z\{g(k)\}=G(z)$ then convolution $\{f(k)*g(k)\}=$ ________

(A) $\sum_{m=-\infty}^{\infty} f(m)g(k-m)$

(B) $\sum_{m=-\infty}^{\infty} f(m)g(m)$

(C) $\sum_{m=0}^{\infty} f(m)g(k-m)$

(D) $\sum_{m=0}^{\infty} f(k-m)g(m)$

Ans. (A) $\sum_{m=-\infty}^{\infty} f(m)g(k-m)$

Q.60. The function $F(z)=\dfrac{z}{(z-1)(z-2)^2}$ has a simple pole at point ________:

(A) $z=0$ (B) $z=1$

(C) $z=2$ (D) $z=\infty$

Ans. (B) $z=1$

Q.61. The function $F(z)=\dfrac{z}{z^2+1}$ has a poles :

(A) $z=1,-1$ (B) $z=i,i$

(C) $z=i,-i$ (D) None

Ans. (C) $z=i,-i$

Q.62. Find $\partial z/\partial x$ where $z=ax^2+2by^2+2bxy$:

(A) 3by (B) 2ax

(C) $3(ax+by)$ (D) $2(ax+by)$

Ans. (D) $2(ax+by)$

Q.63. Solution of the differential equation $dy/dx=y(x-y\ln y)/x(x\ln x-y)$ is ________

(A) $x\ln x+y\ln y/xy=c$

(B) $x\ln x-y\ln y/xy=c$

(C) $\ln x/x+\ln y/y=c$

(D) $\ln x/x-\ln y/y=c$

Ans. (A) $x\ln x+y\ln y/xy=c$

Q.64.Solution of the differential equation $dy/dx = e^{3x-2y} + x^2e^{-2y}$ is ____

(A) $e^2y^3 = e^{3x}/3 + x^2/2 + c$

(B) $e^{3y}(e^{2x} + x^3)/6 + c$

(C) $e^{2y}(e^{3x} + x^3)/6 + c$

(D) $e^{2y}/2 = e^{3x}/3 + x^3/3 + c$

Ans. (D) $e^{2y}/2 = e^{3x}/3 + x^3/3 + c$

Q.65.Solution of the differential equation $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ is ____

(A) $\frac{e^{2y}}{3} = \frac{e^{3y}}{3} + \frac{x^2}{2} + c$ (B) $\frac{e^{3y}(e^{2x} + x^3)}{6} + c$

(C) $\frac{e^{2y}(e^{3x} + x^3)}{6} + c$ (D) $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$

Ans. (D) $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$

Q.66.Solution of the differential equation $dy/dx = (4x + 2y + 1)^2$ is ____

(A) $1/2\sqrt{2}\tan^{-1}(4x + 2y + 1/\sqrt{2}) = x + c$

(B) $1/\sqrt{2}\cot^{-1}(4x + 2y + 1) = x + c$

(C) $1/\sqrt{2}\tan^{-1}(4x + 2y + 1/\sqrt{2}) = c$

(D) $\cot^{-1}(4x + 2y + 1) = x + c$

Ans. (A) $1/2\sqrt{2}\tan^{-1}(4x + 2y + 1/\sqrt{2}) = x + c$

Q.67.Solution of the differential equation $dy/dx + y \cot x = \cos x$ is ____

(A) $y\cos x = \sin^2 x/2 + c$ (B) $y\sin x = \sin^2 x/2 + c$

(C) $y\sin x = \cos^2 x/4 + c$ (D) $y\cos x = \sin^2 x/4 + c$

Ans. (B) $y\sin x = \sin^2 x/2 + c$

Q.68.For the differential equation dy/dx − 3y cotx = sin2x; y=2 when $x = \pi/2$, its particular solution is ____

(A) $y = 2\cos^2 x + 4\cos^3 x$

(B) $y = -2\sin^3 x + 4\sin^2 x$

(C) $y = -2\sin^2 x + 4\sin^3 x$

(D) $y = 4\cos^2 x + 2\sin^3 x$

Ans. (C) $y = -2\sin^2 x + 4\sin^3 x$

Q.69. A rectangular frame is to be made of 240 cm long. Determine the value of the length of the rectangle required to maximize the area :

(A) 24 cm (B) 60 cm

(C) 240 cm (D) 120 cm

Ans. (B) 60 cm

Q.70. Which of the following is one of the criterions for linearity of an equation?

(A) The dependent variable and its derivatives should be of second order

(B) The dependent variable and its derivatives should not be of same order

(C) Each coefficient does not depend on the independent variable

(D) Each coefficient depends only on the i

Ans. (D) Each coefficient depends only on the i

Q.71 Beta function is not a symmetric function :

(A) True (B) False

Ans. (B) False

Q.72. y = mx where m is arbitrary constant is the general solution of the differential equation is, ________

(A) $\frac{dy}{dx}=\frac{y}{x}$ (B) $\frac{dy}{dx}=\frac{x}{y}$

(C) $\frac{dy}{dx}=m$ (D) $\frac{dy}{dx}=-\frac{y}{x}$

Ans. (A) $\frac{dy}{dx}=\frac{y}{x}$

Q.73. The differential equation $\frac{dy}{dx}=\frac{x+2y-3}{3x+6y-1}$ is of the from ________

(A) variable separable

(B) exact

(C) non – homogeneous

(D) homogeneous

Ans. (C) non – homogeneous

Q.74. The differential equation satisfied by general solution $y = Ae^{x} + Be^{-x}$ where A and B are arbitrary constants, is

(A) $y^2 - y = 0$

(B) $y^2 + y = 0$

(C) $y^2 + y = Ae^{x} - Be^{-x}$

(D) $y^2 - y = 2Ae^{x}$

Ans. (A) $y^2 - y = 0$

Q.75. The solution of differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ is

(A) $\sec^2 x \tan y = C$

(B) $\sec^2 y \tan x = C$

(C) $\tan x \tan y = C$

(D) $\sec^2 x \sec^2 y = C$

Ans. (C) $\tan x \tan y = C$

Q.76. Integrating factor for differential equation $(x^2 + y^2 + 1) - 2xydy = 0$ is :

(A) $1/x$ (B) $1/x^3$

(C) $1/x^2$ (D) $1/xy$

Ans. (C) $1/x^2$

Q.77. The differential equation $dy/dx + x/1 + y^2 = y^2$ has integrating factor

(A) $e^{1/1+y2}$ (B) $e^{\tan-1x}$ (C) $e^{1/1+x2}$ (D) $e^{\tan-1y}$

Ans. (D) $e^{\tan-1y}$

Q.78. The differential equation $\cos x dy/dx + y = \sin x$ has integrating factor

(A) $e^{\sec x}$ (B) $(\operatorname{cosec} x - \cot x)$

(C) $(\sec x + \tan x)$ (D) $(\sec x - \tan x)$

Ans. (C) $(\sec x + \tan x)$

Q.79. The differential equation $dy/dx + \sqrt{xy} = x^3$ has integrating factor :

(A) $e^{2/3x\sqrt{x}}$ (B) $e^{1/3x\sqrt{x}}$

(C) $e^{\sqrt{x}}$ (D) e^{x}

Ans. (A) $e^{2/3x\sqrt{x}}$

Q.80. If the complex roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ of auxiliary equation of fourth order DE $\phi(D)y = 0$ are repeated twice then it's solution is _______

(A) $e^{\beta x}[c_1 \cos\alpha x + c_2 \sin\alpha x]$

(B) $e^{\alpha x}[(c_1 x + c_2)\cos\beta x + (c_3 x + c_4)\sin\beta x]$

(C) $(c_1 x + c_2)e^{\alpha x} + (c_3 x + c_4)e^{\beta x}$

(D) $e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$

Ans. (B) $e^{\alpha x}[(c_1 x + c_2)\cos\beta x + (c_3 x + c_4)\sin\beta x]$

Q.81. The solution of differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$ is ________.

(A) $c_1e^{2x}+c_2e^{-3x}$ (B) $c_1e^{-2x}+c_2e^{3x}$

(C) $c_1e^{-2x}+c_2e^{-3x}$ (D) $c_1e^{2x}+c_2e^{3x}$

Ans. (D) $c_1e^{2x}+c_2e^{3x}$

Q.82. The solution of differential equation $\frac{d^2y}{dx^2}+y=0$ is ________

(A) $c_1e^{x}+c_2e^{-x}$

(B) $(c_1x+c_2)e^{-x}$

(C) $c_1\cos x+c_2\sin x$

(D) $e^{x}(c_1\cos x+c_2\sin x)$

Ans. (C) $c_1\cos x+c_2\sin x$

Q.83. The solution of differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ is ________.

(A) $e^{x}(c_1\cos x+c_2\sin x)$

(B) $e^{x/2}\left[c_1\cos\left(\frac{3}{2}\right)x+c_2\sin\left(\frac{3}{2}\right)x\right]$

(C) $e^{-\frac{1}{2}x}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\right)x+c_2\sin\left(\frac{\sqrt{3}}{2}\right)x\right]$

(D) $c_1e^{x}+c_2e^{-x}$

Ans. (C) $e^{-\frac{1}{2}x}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\right)x+c_2\sin\left(\frac{\sqrt{3}}{2}\right)x\right]$

Q.84. The solution of differential equation $\frac{d^3y}{dx^3}+3\frac{dy}{dx}=0$ is ________

(A) $c_1+c_2\cos x+c_3\sin x$

(B) $c_1+c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x$

(C) $c_1+c_2e^{\sqrt{3}x}+c_3e^{-\sqrt{3}x}$

(D) $c_1\cos x+c_3\sin x$

Ans. (B) $c_1+c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x$

Q.85. The solution of differential equation $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-2\frac{dy}{dx}+12y=0$ is ________

(A) $c_1e^{-3x}+e^{x}\left(c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x\right)$

(B) $c_1e^{-3x}+\left(c_2\cos 3x+c_3\sin 3x\right)$

(C) $c_1e^{3x}+e^{-x}\left(c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x\right)$

(D) $c_1e^{-x}+c_2e^{-\sqrt{3}x}+c_3e^{\sqrt{3}x}$

Ans. (A) $c_1e^{-3x}+e^{x}\left(c_2\cos\sqrt{3}x+c_3\sin\sqrt{3}x\right)$

Q.86. Particular Integral of differential equation $\left(D^2-4D+4\right)y=\sin 2x$ is ________:

(A) $-\frac{\cos 2x}{8}$ (B) $\frac{\cos 2x}{8}$

(C) $\frac{\sin 2x}{8}$ (D) $x\frac{\cos 2x}{8}$

Ans. (B) $\frac{\cos 2x}{8}$

Q.87. Particular Integral of differential equation $\left(D^3+D\right)y=\cos x$ is ________:

(A) $-\frac{x}{2}\sin x$ (B) $\frac{x}{4}\cos x$

(C) $-\frac{1}{2}\cos x$ (D) $-\frac{x}{2}\cos x$

Ans. (D) $-\frac{x}{2}\cos x$

Q.88. Particular Integral of differential equation $\left(D^4+10D^2+9\right)y=\sin 2x+\cos 4x$ is ________

(A) $-\frac{1}{23}\sin 2x-\frac{1}{105}\cos 4x$

(B) $\frac{1}{15}\sin 2x+\cos 4x$

(C) $-\frac{1}{15}\sin 2x+\frac{1}{105}\cos 4x$

(D) $-\frac{1}{15}\sin 2x+\frac{1}{87}\cos 4x$

Ans. (C) $-\frac{1}{15}\sin 2x+\frac{1}{105}\cos 4x$

Q.89. Particular Integral of differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=10\sin x$ is __________:

(A) $-\frac{8}{3}\sin x$ (B) $\sin x-2\cos x$

(C) $4\sin x+2\cos x$ (D) $2\sin x+\cos x$

Ans. (D) $2\sin x+\cos x$

Q.90. Particular Integral of differential equation $\frac{d^3y}{dx^3}-4\frac{dy}{dx}=2\cosh 2x$ is __________:

(A) $\frac{1}{4}\cosh 2x$ (B) $\frac{x}{8}\cosh 2x$

(C) $\frac{x}{4}\cosh 2x$ (D) $\frac{x}{[illegible]}\sinh 2x$

Ans. (C) $\frac{x}{4}\cosh 2x$

Q.91. Particular Integral of differential equation $(D^2+6D-9)y=\sinh 3x$ is __________:

(A) $\frac{1}{18}\cosh 3x$ (B) $\frac{1}{2}\cosh 3x$

(C) $\frac{1}{18}\sinh 3x$ (D) $-\frac{1}{18}\cosh 3x$

Ans. (A) $\frac{1}{18}\cosh 3x$

Q.92. Particular Integral of differential equation $(D^4+D^2+1)y=53x^2+17$ is __________:

(A) $53x^2+17$ (B) $53x^2-89$

(C) $53x^2+113$ (D) $3x^2-17$

Ans. (B) $53x^2-89$

Q.93. Particular Integral of differential equation $(D^2-D+1)y=3x^2-1$ is __________:

(A) $3x^2+6x+5$ (B) x^2-6x+1

(C) $3x^2+6x-1$ (D) $x^2+18x-11$

Ans. (C) $3x^2+6x-1$

Q.94. Particular Integral of differential equation $(D^4+25)y=x^4+x^2+1$ is __________

(A) $\left(x^4 + x^2 - \frac{1}{25}\right)$

(B) $\left(x^4 + x^2 + \frac{49}{25}\right)$

(C) $\frac{1}{25}\left(x^4 + x^2 + 24x + 1\right)$

(D) $\frac{1}{25}\left(x^4 + x^2 + \frac{1}{25}\right)$

Ans. (D) $\frac{1}{25}\left(x^4 + x^2 + \frac{1}{25}\right)$

Q.95.Particular Integral of differential equation $\left(D^2 - 4D + 4\right)y = e^{2x}x^4$ is ________:

(A) $\frac{x^6}{120}e^{2x}$ (B) $\frac{x^6}{60}e^{2x}$

(C) $\frac{x^6}{30}e^{2x}$ (D) $\frac{x^5}{20}e^{2x}$

Ans. (C) $\frac{x^6}{30}e^{2x}$

Q.96.Solution of differential equation $\left(D^2 + 1\right)y = x$ is ________

(A) $c_1 \cos x + c_2 \sin x - x$

(B) $c_1 \cos x + c_2 \sin x + x$

(C) $c_1 \cos x + c_2 \sin x + 2x$

(D) $c_1 \cos x + c_2 \sin x - 2x$

Ans. (B) $c_1 \cos x + c_2 \sin x + x$

Q.97.On putting $x = e^z$ and using $d \equiv \frac{d}{dz}$ the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$ is transformed into ________:

(A) $\left(D^2 - 1\right)y = e^z$ (B) $\left(D^2 + 1\right)y = e^z$

(C) $\left(D^2 + 1\right)y = x$ (D) $\left(D^2 + D + 1\right)y = e^z$

Ans. (B) $\left(D^2 + 1\right)y = e^z$

Q.98.Classify the following differential equation $x\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

(A) 3 rd – order, linear.

(B) 3 rd – order non – linear
(C) 4 th – order linear
(D) 4 th – order non- linear

Ans. (B) 3 rd – order non – linear

Q.99. Consider the 2nd-order non-homogeneous differential equation

$$Y'' - 4y' + 3y = e^t + t^2$$

What is the complementary (or homogeneous) solution?

(A) $y_c = c_1e^t + c_2t^2$ (B) $y_c = c_1e^{-t} + c_2e^{-3t}$

(C) $y_c = c_1e^t + c_2e^3t$ (D) $y_c = c_1e^t + c_2e^{-3t}$

Ans. (B) $y_c = c_1e^{-t} + c_2e^{-3t}$

Q.100. The differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = A + B\log x$, complimentary function is given by________:

(A) $c_1x + c_2$ (B) $c_1x^2 + c_2$

(C) $c_1\log x + c_2$ (D) $\frac{c_1}{x} + c_2$

Ans. (C) $c_1\log x + c_2$.

BCA

(SEM. II) MODEL PAPER – II

BCA – 4005 : MATHEMATICS - III

Time : 1 :30 Hours

Maximum Marks : 75

Q.1. There is a point at which a function ceases to be analytic is said to be?

(A) Non-Singular point (B) Non-regular point
(C) Singular point (D) Regular point

Ans. (C) Singular point

Q.2. $A^2 + b^2 = ?$

(A) $(a+ib)(a-ib)$ (B) $(a+b)(a-b)$
(C) $(a+ib)(a-b)$ (D) $(a+b)(a-ib)$

Ans. (A) $(a+ib)(a-ib)$

Q.3. $|z1-z2| = ?$

(A) $\geq Z1+Z2$ (B) $> Z1+Z2$
(C) $>|Z1|+|Z2|$ (D) $\leq|Z1|+|Z2|$

Ans. (D) $\leq|Z1|+|Z2|$

Q.4. $|Z1|+|Z2| =$

(A) $> Z1+Z2$ (B) $\leq Z1+Z2$
(C) $\leq|Z1|+|Z2|$ (D) $>|Z1|+|Z2|$

Ans. (C) $\leq|Z1|+|Z2|$

Q.5. What is the polar form of a complex number?

(A) $r(\sec\theta + i\operatorname{cosec}\theta)$ (B) $r(\backslash\tan\theta + i\cot\theta)$
(C) $r(\sec\theta + i\cos\theta)$ (D) $r(\cos\theta + i\sin\theta)$

Ans. (D) $r(\cos\theta + i\sin\theta)$

Q.6. Which one result is obtained by the addition of a step?

(A) Ramp Function shifted by an amount equal to step
(B) Step Function shifted by an amount equal to ramp
(C) Step function of zero slope
(D) Ramp function of zero slope

Ans. (A) Ramp Function shifted by an amount equal to step

Q.7. Which block of the discrete time systems requires memory?

(A) Signal Multiplier (B) Unit Advance
(C) Unit Delay (D) Adder

Ans. (C) Unit Delay

Q.8. Which one belongs to the category of non-recursive systems?

(A) Non-causal FIR Systems
(B) Causal FIR Systems

(C) Non-causal IIR Systems
(D) Causal IIR Systems
Ans. (B) Causal FIR Systems

Q.9. An LTI system is called initially relaxed system only when______?
(A) zero input produces an output equal to unity
(B) zero input produces a non-zero output
(C) zero input produces zero output
(D) none of the above
Ans. (C) zero input produces zero output

Q.10. There is the number of samples present in an impulse response is said to be?
(A) element (B) length (C) array (D) string
Ans. (B) length

Q.11. Find mirror image of point representing $x+iy$ on imaginary axis.
(A) (x, y) (B) $(-x, -y)$ (C) $(-x, y)$ (D) $(x, -y)$
Ans. (C) $(-x, y)$

Q.12. Two vectors having the same initial points are called as ______________
(A) collinear vectors (B) unit vectors
(C) coinitial vectors (D) equal vectors
Ans. (C) coinitial vectors

Q.13. Which value is similar to $\sin^{-1}\sin(6\pi/7)$?
(A) $\sin^{-1}(\pi/7)$ (B) $\cos^{-1}(\pi/7)$
(C) $\sin^{-1}(2\pi/7)$ (D) $\operatorname{coses}^{-1}(\pi/7)$
Ans. (A) $\sin^{-1}(\pi/7)$

Q.14. If $i=\sqrt{-1}$ then $e^{ix}-e^{-ix}$
(A) $2i\sin x$ (B) $2\cos x$
(C) $2i\cos x$ (D) None of these
Ans. (A) $2i\sin x$

Q.15. If $i=\sqrt{-1}$ then $\cos ix$ is
(A) $I\cosh x$ (B) $\cosh x$
(C) A and B (D) None of these
Ans. (B) $\cosh x$

Q.16. The differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x^2+x^{-2}$ complimentary function is given by______________
(A) c_1x+c_2 (B) $c_1\log x+c_2$
(C) $c_1\cos x+c_2\sin x$ (D) $c_1\cos(\log x)+c_2\sin(\log x)$
Ans. (D) $c_1\cos(\log x)+c_2\sin(\log x)$

Q.17. The differential equation $(1+x)^2\frac{d^2y}{dx^2}+3(1+x)\frac{dy}{dx}-36y$ $=4\cos[\log(1+x)]$ on putting $1+x=e^z$ and using $D\equiv\frac{d}{dz}$ is transformed into____

(A) $(D^2+2D-36)y=4\cos[\log(1+x)]$

(B) $(D^2+2D-36)y=4\cos z$

(C) $(D^2+3D-36)y=4\cos z$

(D) $(D^2-2D-36)y=4\cos(\log z)$

Ans. (B) $(D^2+2D-36)y=4\cos z$

Q.18. For any two complex numbers z_1 and z_2

(A) $|z_1-z_2|\geq|z_1-z_2|$

(B) $|z_1-z_2|=|z_1-z_2|$

(C) $|z_1-z_2|>|z_1-z_2|$

(D) None of these

Ans. (A) $|z_1-z_2|\geq|z_1-z_2|$

Q.19. If $f(z)$ analytic function with constant modulus, hen

(A) (z) zero (B) (z) non – zero

(C) (z) constant (D) none of these

Ans. (C) (z) constant

Q.20. The differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\frac{x^3}{1+x^2}$ on putting $x=e^z$ and using $D\equiv\frac{d}{dz}$ is transformed into____________

(A) $(D^2-1)y=\frac{x^3}{1+x^2}$

(B) $(D^2-2D-1)y=\frac{e^{3z}}{1+e^{2z}}$

(C) $(D^2-1)y=\frac{e^{3z}}{1+e^{2z}}$

(D) $(D^2-1)y=\frac{e^{z^3}}{1+e^{z^2}}$

Ans. (C) $(D^2-1)y=\frac{e^{3z}}{1+e^{2z}}$

Q.22. The Value of $\lim(z\to i)(z^5-i)/(z+1)$ is :

(A) 1 (B) 5 (C) 4 (D) 0

Ans. (D) 0

Q.23. If $z = x + iy$ is any complex number then value of $|z^2| =$

(A) $x^2 - y^2$ (B) $x^2 + y^2$

(C) A and B (D) None of these

Ans. (B) $x^2 + y^2$

Q.24. What are the kinds of discontinuity?

(A) Minor and major kinds

(B) Increment and decrement kinds

(C) First and second kinds

(D) Zero and one kinds

Ans. (C) First and second kinds

Q.25. Modulus of complex number $i^7 + i^B$

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) None of these

Ans. (A) $\sqrt{2}$

Q.26. If z = 1 + I then arg (z) = ...

(A) $\pi/4$ (B) $\pi/2$ (C) $\pi/3$ (D) None of these

Ans. (A) $\pi/4$

Q.27. For any two complex numbers z1 and z2

(A) $|z_1 - z_2| \geq |z_1 - z_2|$ (B) $|z_1 - z_2| = |z_1 - z_2|$

(C) $|z_1 - z_2| > |z_1 - z_2|$ (D) None of these

Ans. (A) $|z_1 - z_2| \geq |z_1 - z_2|$

Q.28. Let $\phi = \phi(x, y)$ be a function of two real variables x and y then the Laplace differential equation is given by

(A) $\frac{a_2\phi}{ax^2} - \frac{a^z\phi}{ax^2} = 0$ (B) $\frac{a^2\phi}{ax^2} + 3\frac{a^z\phi}{ax^2} = 0$

(C) $\frac{a^2\phi}{ax^2} - 2\frac{a^z\phi}{ax^2} = 0$ (D) $\frac{a^2\phi}{ax^2} + \frac{a^z\phi}{ax^2} = 0$

Ans. (D) $\frac{a^2\phi}{ax^2} + \frac{a^z\phi}{ax^2} = 0$

Q.29. If f(z) analytic function with constant modulus, then

(A) (z)s zero (B) (z)s non – zero

(C) (z)s constant (D) None of these

Ans. (C) (z)s constant

Q.31. Let u and v are real valued function of variables x and y, then Cauchy-Riemann equations are represented as

(A) $u_x = v_y$ and $u_y = v_x$ (B) $u_x = v_y$ and $u_y = -v_x$

(C) $u_x = v_y$ and $u_y = -v_x$ (D) $u_x = v_y$ and $uy = 2v_x$

Ans. (C) $u_x = v_y$ and $u_y = -v_x$

Q.32.If $z = c + iy$ is any complex number and $a > 0$ be any real number, then the equation $|z| = a$ represents

(A) Circle (B) Half-circle
(C) Parabola (D) None of these

Ans. (A) Circle

Q.33.If z_1 and z_2 are any two complex number then $\arg(z_1/z_1) = \therefore$

(A) $\arg(z_1) + \arg(z_2)$ (B) $a(z_1) - \arg(z_2)$
(C) $\arg z_1 / \arg z_2$ (D) None of these

Ans. (B) $a(z_1) - \arg(z_2)$

Q.35.If n is the rational number then $(\cos\theta + i\sin)$

(A) $(\cos\theta - i\sin\theta)$ (B) $(\cos n\theta + i\sin)$
(C) A and B (D) None of these

Ans. (B) $(\cos n\theta + i\sin)$

Q.36.Product of two roots of unity is a

(A) Root of unity (B) 1
(C) -1 (D) None of these

Ans. (A) Root of unity

Q.37.If a complex number z lies in the interior or on the boundary of a circle of radius 3 units and centre (– 4, 0), the greatest value of |z +1| is

(A) 4 (B) 6
(C) 3 (D) 10

Ans. (B) 6

Q.38.The sum of all the $n - n^{th}$ roots of unity is ________

(A) 1 (B) 0
(C) 2 (D) None of these

Ans. (B) 0

Q.39.If $i = \sqrt{-1}$ then $e^{ix} - e^{-ix}$ is

(A) $2i\sin x$ (B) $2\cos x$
(C) $2i\cos x$ (D) None of these

Ans. (A) $2i\sin x$

Q.40.If z is complex number then $e^{z} - e^{-z} =$

(A) 2 cos hz (B) 2sin hz
(C) 2i sin z (D) None of these

Ans. (A) 2 cos hz

Q.41.Integrating factor for differential equation $(x^2 + y^2 + 1) - 2xydy = 0$ is

(A) $1/x$ (B) $1/x^3$
(C) $1/x^2$ (D) $1/xy$

Ans. (C) $1/x^2$

Q.42. Integrating factor for differential equation $y\log y dx + (x - \log y) = 0$ is

(A) $1/x$ (B) $1/y$ (C) $1/x^2$ (D) $1/y^2$

Ans. (B) $1/y$

Q.43. Integrating factor for differential equation $(x^2 + y^2 + x) + (xy)dy = 0$ is

(A) $1/x$ (B) $1/x^2$ (C) x^2 (D) x

Ans. (D) x

Q.44. The order of differential equation is always

(A) Positive integer (B) Negative integer
(C) Rational number (D) Whole number

Ans. (A) Positive integer

Q.45. An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called a Bernoulli differential equation

(A) True (B) False

Ans. (A) True

Q.46. The value of the Wronskian of two solutions f_1 and f_2 of differential equation either is zero for all x no $a \le x \le b$ or is zero for no x non $a \le x \le b$

(A) True (B) False

Ans. (A) True

Q.47. The integrating factor of x log x dy/dx + y = 2logx

(A) log x (B) log 2x (C) log 3x (D) log 4x

Ans. (A) log x

Q.48. The order of the differential equation $2x^2 d^2y/dx^2 - 3dy/dx + y = 0$

(A) 2 (B) 1 (C) 0 (D) Not defined

Ans. (A) 2

Q.49. The differential equation $1 + dy/dx - (d^2y/dx^2)^{3/2} = 0$ is of..

(A) Order 1and degree 2 (B) Order 2 and degree 3
(C) Order 3 and degree 6 (D) Order 3 and degree 3

Ans. (B) Order 2 and degree 3

Q.50. The solution of differential equation $ydx + xdy = 0$

(A) $x^2y = c$ (B) $xy = c$
(C) $xy^2 = c$ (D) $xy + 1 = c$

Ans. (B) $xy = c$

Q.51. The solution of differential equation dy/dx + tan x =0

(A) $y + \log\sin x = 0$ (B) $y + \sec^2 x = C$
(C) $y - \log\cos x = C$ (D) $y + \log\cot x = C$

Ans. (C) $y - \log\cos x = C$

Q.52. The solution of differential equation $dy/dx + x = 0$

(A) $x + y^2 = C$ (B) $x + y = C$

(C) $x^2 + y = C$ (D) $x^2 + 2y = C$

Ans. (D) $x^2 + 2y = C$

Q.53. The solution of differential equation $dy/dx + y = 0$

(A) $y = Ae^{-x}$ (B) $y = Ae^{x}$

(C) $y = Ae^{-y}$ (D) $y = Ae^{y}$

Ans. (A) $y = Ae^{-x}$

Q.54. The total derivative of xdy-ydx with I.F $1/x^2$ is.

(A) d(x/y) (B) d(y/x)

(C) d(logx/y) (D) d(x-y)

Ans. (B) d(y/x)

Q.55. The Integrating factor differential equation (2xlogx-xy)dy +(2y)dx =0

(A) $1/x^2 y^2$ (B) $1/x$

(C) $1/x^2$ (D) $1/y$

Ans. (B) $1/x$

Q.56. y = mx where m is arbitrary constant is the general solution of the differential equation is, ______

(A) $\frac{dy}{dx} = \frac{y}{x}$ (B) $\frac{dy}{dx} = \frac{x}{y}$

(C) $\frac{dy}{dx} = m$ (D) $\frac{dy}{dx} = -\frac{y}{x}$

Ans. (A) $\frac{dy}{dx} = \frac{y}{x}$

Q.57. What is the solution of the given equation $x^6y^6dy + (x^7y^5 + 1)dx = 0$?

(A) $(xy)^6/6 + \ln x = c$ (B) $(xy)^5/6 + \ln x = c$

(C) $(xy)^5/5 + \ln x = c$ (D) $(xy)^6/6 + \ln y = c$

Ans. (A) $(xy)^6/6 + \ln x = c$

Q.58. $xy^3(dy/dx)^2 + yx^2 + dydx = 0$ is a ______

(A) Second order, third degree, linear differential equation

(B) First order, third degree, non-linear differential equation

(C) First order, third degree, linear differential equation

(D) Second order, third degree, non-linear differential equation

Ans. (B) First order, third degree, non-linear differential equation

Q.59. Which of the following is one of the criterions for linearity of an equation?

(A) The dependent variable and its derivatives should be of second order

(B) The dependent variable and its derivatives should not be of same order

(C) Each coefficient does not depend on the independent variable

(D) Each coefficient depends only on the i

Ans. (D) Each coefficient depends only on the i

Q.60. Beta function is not a symmetric function.

(A) True (B) False

Ans. (B) False

Q.61. Solution of the differential equation $dy/dx = (4x+2y+1)^2$ is ____

(A) $1/2\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right) = x+c$

(B) $1/\sqrt{2}\cot^{-1}(4x+2y+1) = x+c$

(C) $1/\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right) = c$

(D) $\cot^{-1}(4x+2y+1) = x+c$

Ans. (A) $1/2\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right) = x+c$

Q.62. Solution of the differential equation $xy\,dy/dx = 1+x+y+xy$ is ________

(A) $(y-x)-\log(x(1+y)) = c$

(B) $(y-x)-\log(x(1+y)) = c$

(C) $(y+x)-\log(x) = c$

(D) $(y-x)-\log(y(1+x)) = c$

Ans. (A) $(y-x)-\log(x(1+y)) = c$

Q.63. Solve the differential equation $dy/dx = x^2+y^2/3yx$ is ________

(A) $xp = (x^2+2y^2)^{-3}$ (B) $x^2p = (x^2-2y^2)^3$

(C) $x^4p = (x^2-2y^2)^{-3}$ (D) $x^6p = (x^2+2y^2)^3$

Ans. (2) $x^2p = (x^2-2y^2)^3$

Q.64. If f(z) is an analytic function in and on closed contour C then $\int_C f(z)dz$ is :

(A) Zero (B) Non zero

(C) One (D) Two

Ans. (A) Zero

Q.65. If $C : |z| = 1$ is circle traced in anticlockwise direction then $\int_C z\,dz =$

(A) 10 (B) 0

(C) -1 (D) None of these

Ans. (B) 0

Q.66. The region of validity for Taylor's series about $Z = 0$ of $f(z) = e^z$ is ______

(A) $|z| = 0$ (B) $|z| < 1$

(C) $|z| < \infty$ (D) None of these

Ans. (B) $|z| < 1$

Q.67. A fly in the hall of dimensions $10\text{ m} \times 12\text{ m} \times 14\text{ m}$ starts to fly from one corner and ends up at the diametrically opposite corner. What is the magnitude of its flight?

(A) 17 m (B) 21 m (C) 26 m (D) 36 m

Ans. (B) 21 m

Q.68. If f(z) be analytic in simply connected region bounded by closed curve, C, then $\int_C f(z)dz =$

(A) -1 (B) 1

(C) 0 (D) None of these

Ans. (C) 0

Q.69. If $f(z) = z+1$ then $\int_C f(z)dz = \ldots\ldots$ where $C: |z| = 1$

(A) 0 (B) 1 (C) -1 (D) None of these

Ans. (A) 0

Q.70. Let $C: |z-a| = 2$, the value of integral $\int_C 1/(z-a)dz =$

(A) $2\pi i$ (B) 2π (C) 2 (D) 0

Ans. (A) $2\pi i$

Q.71. Caushy's integral formula for f(a) =

(A) $\frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$ (B) $\frac{1}{2\pi i}\int_C \frac{f(z)}{z}$

(C) $\frac{1}{2\pi i}\int_C \frac{f(a)}{z-a}dz$ (D) All of the above

Ans. (A) $\frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$

Q.72. The Value of $\int_{|z|=1} e^z dz = \ldots$

(A) -1 (B) 1 (C) 0 (D) None of these

Ans. (C) 0

Q.73. The Cauchy's Integral formula of $f^n(a) =$

(A) $\frac{ni}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}}dz, n \in N$ (B) $\frac{ni}{2\pi i}\int_C \frac{f(z)}{(z-a)^n}dz, n \in N$

(C) A and B (D) None of these

Ans. (A) $\frac{ni}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}}dz, n \in N$

Q.74. The geometric series $1+z+z^2+z^3+\ldots=$

(A) $(1/1+z),|z|<1$ (B) $(1/1-z),|z|<1$

(C) A and B (D) None of these

Ans. (B) $(1/1-z),|z|<1$

Q.75. The Series $1+\left(\frac{z}{1!}\right)+\left(\frac{z^2}{2!}\right)+\left(\frac{z^3}{3!}\right)+\ldots$

(A) e^z (B) e^{2z}

(C) e^{3z} (D) 1

Ans. (A) e^z

Q.77. Solve the differential equation $dy/dx=x^2+y^2/3yx$ is ______

(A) $xp=\left(x^2+2y^2\right)^{-3}$ (B) $x^2p=\left(x^2-2y^2\right)^3$

(C) $x^4p=\left(x^2-2y^2\right)^{-3}$ (D) $x^6p=\left(x^2+2y^2\right)^3$

Ans. (2) $x^2p=\left(x^2-2y^2\right)^3$

Q.78. The solution of differential equation $dy/dx=y/x+\tan y/x$ is ______

(A) $\cot(y/x)=xc$ (B) $\cos(y/x)=xc$

(C) $\sec^2(y/x)=xc$ (D) $\sin(y/x)=xc$

Ans. (D) $\sin(y/x)=xc$

Q.79. Particular solution of the differential equation $dy/dx=y^2-2xy-x^2/y^2+2xy-x^2$ $y=-1$ at $x=1$.

(A) $y=x$ (B) $y+x=2$

(C) $y=-x$ (D) $y-x=2$

Ans. (1) $y=-x$

Q.80. Solution of the differential equation $dy/dx=3x-6y+7/x-2y+4$ is ______

(A) $\log(x-2y+4)^2=c$

(B) $6x-2y+\log(x-2y+2)^2=c$

(C) $x-2y+\log(x-2y+6)^2=c$

(D) $-5x+\log(x-2y+3)^2=c$

Ans. (B) $6x-2y+\log(x-2y+2)^2=c$

Q.81. Solution of the differential equation (3y-7x+7)dx+(7y-3x+3)dy = 0 is ________

(A) $p=(y+x)^5(y-x)^2$

(B) $p=(y+x+2)^5(y-x+2)^2$

(C) $p=(y+x)^2(y-x)^5$

(D) $p=(y+x-1)^5(y-x+1)^2$

Ans. (D) $p=(y+x-1)^5(y-x+1)^2$

Q.82. Given the Z – transform $F(z)=\dfrac{z(8z-7)}{4z^2-7z+3}$ the limit $F(\infty)$ is

(A) 1 (B) 2 (C) ∞ (D) 0

Ans. (2) 2

Q.83. Relationship between Z – transform with Fourier transform is $f(n)=$ ________

(A) $\int_{-\pi}^{\pi} F\left(e^{i\theta}\right)e^{in\theta}d\theta$ (B) $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} F\left(e^{i\theta}\right)e^{in\theta}d\theta$

(C) $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} F\left(e^{i\theta}\right)d\theta$ (D) None

Ans. (B) $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} F\left(e^{i\theta}\right)e^{in\theta}d\theta$

Q.84. If $|z|<|a|$ inverse of Z-transform of $\dfrac{z}{z-a}$ is given by

(A) $a^k, k\geq 0$ (B) $a^k, k<0$

(C) $a^{k-1}, k\geq 0$ (D) $-a^k, k<0$

Ans. (4) $-a^k, k<0$

Q.85. For $|z|>1$ $Z^{-1}\left[\dfrac{z^2}{z^2+1}\right]=$ ________

(A) $\sin k\dfrac{\pi}{2}$ (B) $\cos k\dfrac{\pi}{2}$

(C) $\sin k\pi$ (D) $\cos k\pi$

Ans. (B) $\cos k\dfrac{\pi}{2}$

Q.86. Fourier cosine integral is defined as $f(x)=$ ________

(A) $\dfrac{1}{\pi}\int_0^\infty \int_0^\infty f(u)\cos\lambda u\cos\lambda x\, du d\lambda$

(B) $\dfrac{2}{\pi}\int_{-\infty}^\infty \int_{-\infty}^\infty f(u)\cos\lambda u\cos\lambda x\, du d\lambda$

(C) $\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\cos\lambda u\cos\lambda x\, du d\lambda$

(D) $\frac{2}{\pi}\int_{0}^{\infty}\int_{0}^{\infty} f(u)\cos\lambda u\cos\lambda x\, du d\lambda$

Ans. (D) $\frac{2}{\pi}\int_{0}^{\infty}\int_{0}^{\infty} f(u)\cos\lambda u\cos\lambda x\, du d\lambda$

Q.87. The Fourier transform of the function f(x) is

(A) $\int_{0}^{\infty} f(u)e^{-i\lambda u}du = F(\lambda)$ (B) $\int_{-\infty}^{\infty} f(u)e^{-i\lambda u}du = F(\lambda)$

(C) $\int_{1}^{\infty} f(u)e^{-i\lambda u}du = F(\lambda)$ (D) None

Ans. (B) $\int_{-\infty}^{\infty} f(u)e^{-i\lambda u}du = F(\lambda)$

Q.88. Find the divergence of this given vector $\vec{F} = x^3y\vec{i} + 3xy^2z\vec{j} + 3zx\vec{k}$.

(A) $3x^2y + 6xyz + x$

(B) $2x^2y + 6xyz + 3x$

(C) $3x^2y + 3xyz + 3x$

(D) $3x^2y + 6xyz + 3x$

Ans. (D) $3x^2y + 6xyz + 3x$

Q.89. If f is analytic inside and on closed contour C, Except at finite number of singular points $\sum R$ denotes sum of residues at its poles inside C then $\int_C f(z)dz =$

(A) $2\pi!\sum R$ (B) $\sum R$

(C) $2\sum R$ (D) $\pi\sum R$

Ans. (A) $2\pi!\sum R$

Q.90. The value of integral $\int_{|z|=2} dz/z = \dots$

(A) 2π (B) $\pi!$ (C) $\frac{2}{\pi}$ (D) $\frac{1}{\pi}$

Ans. (A) 2π

Q. By Cauchy's integral formula, $\int_C \frac{f(z)}{((z-a)^2)}dz =$

(A) $2\pi i f(a)$ (B) $2\pi i f'(a)$

(C) $\frac{2\pi}{2!}f''(a)$ (D) $\frac{2\pi i}{2!}f^n(a)$

Ans. (B) $2\pi i f'(a)$

Q.92.A zero of an analytic function f(z) is the value of z such that f(z) is equal to ______

(A) 1 (B) 2

(C) 0 (D) 3

Ans. (C) 0

Q.93.If $f(z) = z - 2/z^2(z-1)$, then the order of pole z = 0 is ______

(A) 0 (B) 1

(C) 2 (D) 3

Ans. (C) 2

Q.94.The poles of $f(z) = e^z / z^2 + a^2$ are :

(A) $\pm 2i$ (B) $\pm 3i$

(C) $\pm ai$ (D) $\pm 2ai$

Ans. (C) $\pm ai$

Q.95.If $|z| = 1$ then $2\cos\theta =$

(A) $z - 1/z$ (B) $z + 1/z$

(C) A and B (D) None of these

Ans. (B) $z + 1/z$

Q.96.If $f(z) = z^2 / \{(z-1)(z-2)(z-3)\}$ then poles of f(z) are

(A) Z = 1,2,3 (B) Z = 4,2,3

(C) Z = 1,4,3 (D) Z = 4,5,3

Ans. (A) Z = 1,2,3

Q.97.The singular points of $f(z) = 1/(z-2)(z-3)$ are ______

(A) 2, 4 (B) -2, - 3

(C) 2, 5 (D) 2, 3

Ans. (D) 2, 3

Q.98.The value of integral $\int_0^\infty d(x^2+1) =$

(A) 3 (B) $\pi/2$

(C) -1 (D) 2

Ans. (B) $\pi/2$

Q.99.The real and imaginary part of an analytic function... Laplace differential equation.

(A) Satisfy (B) Does not satisfy

(C) May or may not be satisfy (D) None of these

Ans. (A) Satisfy

Q.100.The simple poles of $f(z) = z^2 - 4/(z^2 + 5z + 4)$ are :

(A) 1, 4 (B) -1, 4

(C) -1, -4 (D) 2, 3

Ans. (C) -1, -4

BCA
(SEM. II) –MODEL PAPER–III
BCA – 4005 : MATHEMATICS – III

Time : 1 :30 Hours *Maximum Marks : 75*

Q.1. If z = $z = \cos 3\theta + i\sin 3\theta$ then $z^5 = \dots$

(A) $\cos 15\theta + i\sin 15\theta$ (B) $\cos 15\theta - i\sin 15\theta$

(C) $\cos 8\theta + i\sin 8\theta$ (D) $\cos 8\theta - i\sin 8\theta$

Ans. (A) $\cos 15\theta + i\sin 15\theta$

Q.2. A Complex number whose real part is zero, is called as ______

(A) Real number (B) Complex number

(C) Purely imaginary number (D) Purely real number

Ans. (C) Purely imaginary number

Q.3. Two complex numbers $x_1 = iy_1 + z_2$ and $x_2 = iy_2 + x_2 + iy_2$ are equal if

(A) $x_1 \le x_2$ (B) $x_1 = x_2$ and $y_1 = y_2$.

(C) $x_1 = x_2$ (D) None of these

Ans. (B) $x_1 = x_2$ and $y_1 = y_2$

Q.4. A number of the type $z = x + iy$ is called as

(A) Real number (B) Complex number

(C) Integer (D) Irrational number

Ans. (B) Complex number

Q.5. In a complex number $z - 7 - 3i$ the imaginary part of z is ...

(A) -3 (B) 3i

(C) 7 (D) 7

Ans. (A) -3

Q.6. The conjugate of complex number $1 + i$ is :

(A) 1-i (B) 1+i

(C) 0 (D) None of these

Ans. (a) 1-i

Q.7. If $z = 1 - \sqrt{3i}$ then modulus of z is equal to

(A) 1 (B) 2

(C) 3 (D) 4

Ans. (B) 2

Q.8. If $z = i + i^2 + i^3$, then real part of z is

(A) -1 (B) 0

(C) 3 (D) None of these

Ans. (D) None of these

Q.9. If z is complex number then $\left(e^{z}-e^{-z}\right)/2 =$ ______

(A) sin hz (B) i sin hz
(C) tan hz (D) None of these

Ans. (A) sin hz

Q.10. If $x=\cos\theta+i\sin\theta$ then $x-1/x=\ldots$

(A) $2i\sin\theta$ (B) $2\cos\theta$ (C) 2 (D) 2

Ans. (A) $2i\sin\theta$

Q.11. The differential equation $x^2\frac{d^2y}{dx^2}-5x\frac{dy}{dx}+5y=x^2\log x$ on putting $x=e^z$ and using $D\equiv\frac{d}{dz}$ is transformed into ______

(A) $\left(D^2-5D+5\right)y=ze^{z^2}$

(B) $\left(D^2-5D-5\right)y=e^{2z}z$

(C) $\left(D^2-6D+5\right)y=x^2\log x$

(D) $\left(D^2-6D+5\right)y=ze^{2z}$

Ans. (4) $\left(D^2-6D+5\right)y=ze^{2z}$

Q.12. The differential equation $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=6x$ on putting $2x+1=e^z$ and using $D\equiv\frac{d}{dz}$ is transformed into ______

(A) $\left(D^2-2D-3\right)y=\frac{3}{4}\left(e^z-1\right)$

(B) $\left(D^2+2D+3\right)y=3\left(e^z-1\right)$

(C) $\left(D^2+2D-12\right)y=\frac{3}{4}\left(e^z-1\right)$

(D) $\left(D^2-2D-3\right)y=6x$

Ans. (1) $\left(D^2-2D-3\right)y=\frac{3}{4}\left(e^z-1\right)$

Q.13. To reduce the differential equation $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^4$ to linear differential equation with constant coefficients, substitutions is ______

(A) $x=z^2+1$ (B) $x=e^z$
(C) $x=\log z$ (D) $x^2=\log z$

Ans. (B) $x=e^z$

Q.14. To reduce the differential equation $(x+2)^2\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+y=4x+7$ to linear differential equation with constant coefficients, substitutions is ______

(A) $x+2=e^{-z}$ (B) $x=z+1$

(C) $x+2=e^z$ (D) $x+2=\log z$

Ans. (C) $x+2=e^z$

Q.15. To reduce the differential equation $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=x^2+3x+1$ to linear differential equation with constant coefficients, substitutions is ______

(A) $3x+2=e^z$ (B) $3x+2=z$

(C) $x=e^z$ (D) $3x+2=\log z$

Ans. (A) $3x+2=e^z$

Q.16. Particular Integral of differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=e^{2x}$ is ______

(A) $e^{x}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)-\frac{1}{7}e^{2x}$

(B) $e^{\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)+\frac{1}{5}e^{2x}$

(C) $e^{-\frac{1}{2}x}\left(c_1\cos\frac{1}{2}x+c_2\sin\frac{1}{2}x\right)+\frac{1}{7}e^{x}$

(D) $e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)+\frac{1}{7}e^{2x}$

Ans. (D) $e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)+\frac{1}{7}e^{2x}$

Q.17. Particular Integral of differential equation $(D^2+6D+9)y=e^{-3x}x^{-3}$ is ______

(A) $\frac{e^{-3x}}{2x}$ (B) $e^{-3x}x$ (C) $\frac{e^{-3x}}{12x}$ (D) $(c_1x+c_2)e^{-3x}$

Ans. (A) $\frac{e^{-3x}}{2x}$

Q.18. Particular Integral of differential equation $(D^2 + 2D + 1)y = e^{-x}(1 + x^2)$ is ________

(A) $e^{-x}\left(\frac{x^2}{2} - \frac{x^4}{12}\right)$ (B) $e^{-x}\left(x + \frac{x^3}{3}\right)$

(C) $e^{-x}\left(\frac{x^2}{2} + \frac{x^4}{12}\right)$ (D) $\left(\frac{x^2}{2} + \frac{x^4}{12}\right)$

Ans. (C) $e^{-x}\left(\frac{x^2}{2} + \frac{x^4}{12}\right)$

Q.19. Particular Integral of differential equation $(D - 1)^3 y = e^x \sqrt{x}$ is ________

(A) $\frac{4}{15}e^x x^{5/2}$ (B) $\frac{8}{105}e^x x^{7/2}$

(C) $e^x x^{7/2}$ (D) $\frac{3}{8}e^x x^{-5/2}$

Ans. (2) $\frac{8}{105}e^x x^{7/2}$

Q.20. Particular Integral of differential equation $(D^2 - 4D + 4)y = \sin 2x$ is ________

(A) $-\frac{\cos 2x}{8}$ (B) $\frac{\cos 2x}{8}$ (C) $\frac{\sin 2x}{8}$ (D) $x\frac{\cos 2x}{8}$

Ans. (B) $\frac{\cos 2x}{8}$

Q.21. Particular Integral of differential equation $(D^3 + D)y = \cos x$ is ________

(A) $-\frac{x}{2}\sin x$ (B) $\frac{x}{4}\cos x$ (C) $-\frac{1}{2}\cos x$ (D) $-\frac{x}{2}\cos x$

Ans. (D) $-\frac{x}{2}\cos x$

Q.22. Particular Integral of differential equation $(D^2 + 1)y = \sin x$ is ________

(A) $-\frac{x}{2}\cos x$ (B) $-\frac{x}{4}\cos x$ (C) $-\frac{x}{2}\sin x$ (D) $-\frac{1}{2}\cos x$

Ans. (A) $-\frac{x}{2}\cos x$

Q.23. Particular Integral $\frac{1}{D+1}e^{e^x}$ where $D \equiv \frac{d}{dx}$ is ______

(A) $e^{-x}e^{e^x}$ (B) e^{e^x} (C) $e^x e^{e^x}$ (D) $e^{-2x}e^{e^x}$

Ans. (A) $e^{-x}e^{e^x}$

Q.24. Particular Integral $\frac{1}{D+1}e^{e^x}$ where $D \equiv \frac{d}{dx}$ is ______

(A) $e^{2x}e^{e^x}$ (B) $e^{-2x}e^{e^x}$ (C) e^{e^x} (D) $e^{-x}e^{e^x}$

Ans. (B) $e^{-2x}e^{e^x}$

Q.25. The solution of differential equation $4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$ is ______

(A) $e^{-x}(c_1\cos 2x+c_2\sin 2x)$

(B) $e^{-x/2}[c_1\cos x+c_2\sin x]$

(C) $e^{-2x}(c_1\cos x+c_2\sin x)$

(D) $c_1e^{-4x}+c_2e^{-5x}$

Ans. (B) $e^{-x/2}[c_1\cos x+c_2\sin x]$

Q.26. The solution of differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$ is ______

(A) $c_1e^{-6x}+c_2e^{-9x}$ (B) $(c_1x+c_2)e^{-3x}$

(C) $(c_1x+c_2)e^{3x}$ (D) $c_1e^{3x}+c_2e^{2x}$

Ans. (B) $(c_1x+c_2)e^{-3x}$

Q.27. The solution of differential equation $xdy-ydx=0$ is

(A) $y=x+c$ (B) $x^2-y^2=c$

(C) $xy=c$ (D) $y=cx$

Ans. (D) $y=cx$

Q.28. The solution of differential equation $(e^x+1)y=(y+1)e^x dx$ is

(A) $y-\log(1-y)=\log(e^x+1)+\log C$

(B) $y-\log(1+y)=\log(e^x+1)+\log C$

(C) $y+\log(1-y)=\log(e^x+1)+\log C$

(D) $y-\log(1+y)=\log(e^x-1)+\log C$

Ans. (B) $y-\log(1+y)=\log(e^x+1)+\log C$

Q.29. Angular momentum is:

(A) A scalar vector (B) An axial-vector

(C) A polar vector (D) A displacement vector

Ans. (B) An axial-vector

Q.30. The differential equation of the form $dy/dx + Py = Qyn, n \neq 1$ where P and Q are functions of x or constants, is

(A) Bernoulli's differential equation

(B) Exact differential equation

(C) Symmetric differential equation

(D) Linear differential equation

Ans. (A) Bernoulli's differential equation

Q.31. The general solution of linear differential equation $dy/dx + py = Q$ where P and Q are functions of x or constants, is

(A) $xe^{\int Pdy} = \int Qe^{\int Pdy} dy + C$

(B) $y = \int Qe^{\int Pdx} dx + C$

(C) $ye^{\int Pdx} = \int Q\,dx + C$

(D) $ye^{\int Pdx} = \int Qe^{\int Pdx} dx + C$

Ans. (D) $ye^{\int Pdx} = \int Qe^{\int Pdx} dx + C$

Q.32. The differential equation of the form $f'(x)dy/dx + pf(x) = Q$ where P and Q are functions of y or constants, can be reduced to linear differential equation by the substitution

(A) $f'(x) = u$ (B) $(x) = u$

(C) $P = u$ (D) $Q = u$

Ans. (B) $(x) = u$

Q.33. The differential equation satisfied by general solution $y = Ax^2 + Bx + C$ where A, B, C are arbitrary constants is

(A) ${}^3y/dx^3 = 0$ (B) ${}^2y/dx^2 = 2A$

(C) ${}^3y/dx^3 = A$ (D) ${}^4y/dx^4 = 0$

Ans. (A) ${}^3y/dx^3 = 0$

Q.34. The solution of differential equation $dy/dx + y = 0$ is

(A) $y = Ae^{-x}$ (B) $y = Ae^{x}$

(C) $x = Ae^{-y}$ (D) $x = Ae^{y}$

Ans. (A) $y = Ae^{-x}$

Q.35. The solution of differential equation $dy/dx + 1 + y^2/1 + x^2 = 0$ is

(A) $\tan^{-1}y - \tan^{-1}x = C$ (B) $\tan^{-1}y + \tan^{-1}x = C$

(C) $\tan y + \tan x = C$ (D) $\cos y + \cos x = C$

Ans. (D) $\tan^{-1}y + \tan^{-1}x = C$

Q.36. The solution of differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ is

(A) $\sec^2 x \tan y = C$ (B) $\sec^2 y \tan x = C$

(C) $\tan x \tan y = C$ (D) $\sec^2 x \sec^2 y = C$

Ans. (C) $\tan x \tan y = C$

Q.37. The solution of differential equation $(1+y^2)x + y(1+x^2)dy = 0$ is

(A) $(1-x^2)(1+y^2) = C$

(B) $\tan^{-1}x + \tan^{-1}y = C$

(C) $(1+x^2) = (1+y^2)$

(D) $(1+x^2)(1+y^2) = C$

Ans. (D) $(1+x^2)(1+y^2) = C$

Q.38. The differential equation $(2x/y^3)dx + (y^2 + ax^2/y4)dy = 0$ is exact if

(A) $a = -3$ (B) $a = 3$ (C) $a = -2$ (D) $a = 6$

Ans. (A) $a = -3$

Q.39. Integrating factor of homogeneous differential equation $(xy - 2y^2) + (3xy - x^2)dy = 0$ is

(A) $1/xy$ (B) $1/x^2y^2$ (C) $1/x^2y$ (D) $1/xy^2$

Ans. (D) $1/xy^2$

Q.40. Integrating factor for differential equation $(x^2y^2 + xy + 1)dx + (x^2y^2 - xy + 1)dy = 0$ is

(A) $1/2x^3y^3$ (B) $1/xy$ (C) $1/2x^2y^2$ (D) $1/x^2y$

Ans. (C) $1/2x^2y^2$

Q.41. If z is any complex number then sin iz) =...

(A) tan hz (B) cos hz (C) sin hz (D) None of these

Ans. (C) sin hz

Q.42. If $z = x + iy$ then $\cos z = i$

(A) Cos x cos hy + i sin x sin hy (B) Cos x cos hy – i sin x sin hy

(C) A and B (D) None of these

Ans. (B) Cos x cos hy – i sin x sin hy

Q.43.If $i=\sqrt{-1}$ then cosix is

(A) i cosh x (B) cosh x
(C) A and B (D) None of these

Ans. (B) cosh x

Q.44.Which of the following is a scalar quantity?

(A) Acceleration (B) Electric Field
(C) Work (D) Displacement

Ans. (C) Work

Q.45.A function which is differential at every point of region is said to be ________ in that region.

(A) Analytic (B) Not analytic
(C) Harmonic (D) None of these

Ans. (A) Analytic

Q.46.If $\lim_{(z\to a)} f(z)=u+iv$ then $\lim_{(z\to a)} f(z)=$

(A) u + iv (B) u – iv
(C) A and B (D) None of these

Ans. (B) u – iv

Q.47.Let u and v are real valued function of variables x, y and (z) = u + iv is analytic function. If $u=x^2+y$ then find value of vy.

(A) 2y (B) 2x (C) x (D) y

Ans. (B) 2x

Q.47.Let u and v are real valued function of variables x, y and (z) = u + iv is analytic function. If $u=x^2+y^2$ then find value of u_x.

(A) 2y (B) 2x (C) -2x (D) -2y

Ans. (B) 2x

Q.49.If f(z) is continuous at z_0 then it is not differentiable at z_0

(A) True Statement (B) False Statement
(C) A and B (D) None of these

Ans. (A) True Statement

Q.50.If f is analytic at z_0 then is not differential at z_0

(A) False Statement (B) True statement
(C) A and B (D) None of these

Ans. (A) False Statement

Q.51.If $(z)=u+iv$ is function of complex variable the Cauchy – Riemann equations are

(A) ux = -vy and uy = vx (B) uy = vx
(C) ux = -vy and uy = - vx (D) None of these

Ans. (C) ux = -vy and uy = - vx

Q.52. The function f(z) = z not is analytic function.

(A) May of may not true (B) True Statement
(C) False Statement (D) None, of these

Ans. (B) True Statement

Q.53. The $\lim_{(z\to i)} \frac{z+i}{z^3} =$

(A) -2 (B) 2 (C) i (D) -i

Ans. (A) -2

Q.54. The function $(z) = e^z$ is

(A) Analytic for all Z (B) Not analytic
(C) Not continuous (D) None of these

Ans. (A) Analytic for all Z

Q.55. If $\phi = \phi(x, y)$ then $\frac{a^2\phi}{ax^2} + \frac{a^2\phi}{ay^2} = 0$ is called

(A) Laplace differential equation
(B) C – R equation
(C) C-R Linear
(D) None of these

Ans. (A) Laplace differential equation

Q.56. Example of harmonic function is f(z) =

(A) e^z (B) z
(C) Conjugate of z (D) None of these

Ans. (D) None of these

Q.57. If $f(z) = u + iv$ is analytic function of Z, then f(z) not is independent of Z

(A) True (B) False
(C) A and B (D) None of these

Ans. (B) False

Q.58. A boy walks uniformly along the sides of a rectangular park with dimensions 400 m×300 m, starting from one corner to the other corner diagonally opposite. Which of the following statements is false?

(A) His displacement is 700 m
(B) His displacement is 500 m
(C) He has travelled a distance of 700 m
(D) His velocity is not uniform throughout the walk

Ans. (A) His displacement is 700 m

Q.59. If $\phi(x, y) = x + y$ then $\frac{a^2\phi}{ax^2} - \frac{a^2\phi}{ay^2} = 0$

(A) True Statement (B) No
(C) A and B (D) None of these

Ans. (A) True Statement

Q.60. The imaginary part of e^z is:

(A) $e^z \cos y$ (B) $e^z \sin y$

(C) $e^z \cos x$ (D) None of these

Ans. (B) $e^z \sin y$

Q.61. If real part u of analytic function $(z) = u + iv$ is given then $f(z) =$

(A) $(x, y) + iu_y(x, y)$ (B) $(x, y) - iu_y(x, y)$

(C) $(x, y) - u_y(x, y)$ (D) None of these

Ans. (B) $(x, y) - iu_y(x, y)$

Q.62. If c is a st. line segment from 0 to 1 then $\int_c x^2\,dx =$

(A) 1 (B) 0 (C) 1/3 (D) -1

Ans. (C) 1/3

Q.63. The line segment z = 0 to z = 1 +1 joins points

(A) (0,0) and (1,-1) (B) (0,0) and (1,1)

(C) (0,0) and (1,1) (D) None of these

Ans. (A) (0,0) and (1,-1)

Q.64. The solution of differential equation $dy/dx = y/x + \tan y/x$ is ________

(A) $\cot(y/x) = xc$ (B) $\cos(y/x) = xc$

(C) $\sec^2(y/x) = xc$ (D) $\sin(y/x) = xc$

Ans. (D) $\sin(y/x) = xc$

Q.65. Particular solution of the differential equation $dy/dx = y^2 - 2xy - x^2/y^2 + 2xy - x^2$ $y = -1$ at $x = 1$.

(A) $y = x$ (B) $y + x = 2$

(C) $y = -x$ (D) $y - x = 2$

Ans. (A) $y = -x$

Q.66. Solution of the differential equation $dy/dx = 3x - 6y + 7/x - 2y + 4$ is ________

(A) $\log(x - 2y + 4)^2 = c$

(B) $6x - 2y + \log(x - 2y + 2)^2 = c$

(C) $x - 2y + \log(x - 2y + 6)^2 = c$

(D) $-5x + \log(x - 2y + 3)^2 = c$

Ans. (2) $6x - 2y + \log(x - 2y + 2)^2 = c$

Q.67.Solution of the differential equation (3y-7x+7)dx+(7y-3x+3)dy=0 is ______

(A) $p=(y+x)^5(y-x)^2$

(B) $p=(y+x+2)^5(y-x+2)^2$

(C) $p=(y+x)^2(y-x)^5$

(D) $p=(y+x-1)^5(y-x+1)^2$

Ans. (D) $p=(y+x-1)^5(y-x+1)^2$

Q.68.Solution of the differential equation $dy/dx=(4x+2y+1)^2$ is ______

(A) $1/2\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right)=x+c$

(B) $1/\sqrt{2}\cot^{-1}(4x+2y+1)=x+c$

(C) $1/\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right)=c$

(D) $\cot^{-1}(4x+2y+1)=x+c$

Ans. (A) $1/2\sqrt{2}\tan^{-1}\left(4x+2y+1/\sqrt{2}\right)=x+c$

Q.69.Solution of the differential equation $xydy/dx=1+x+y+xy$ is ______

(A) $(y-x)-\log(x(1+y))=c$

(B) $(y-x)-\log(x(1+y))=c$

(C) $(y+x)-\log(x)=c$

(D) $(y-x)-\log(y(1+x))=c$

Ans. (A) $(y-x)-\log(x(1+y))=c$

Q.70.Solve the differential equation $dy/dx=x^2+y^2/3yx$ is ______

(A) $xp=\left(x^2+2y^2\right)^{-3}$ (B) $x^2p=\left(x^2-2y^2\right)^3$

(C) $x^4p=\left(x^2-2y^2\right)^{-3}$ (D) $x^6p=\left(x^2+2y^2\right)^3$

Ans. (B) $x^2p=\left(x^2-2y^2\right)^3$

Q.71.The solution of differential equation $dy/dx=y/x+\tan y/x$ is ______

(A) $\cot(y/x)=xc$ (B) $\cos(y/x)=xc$

(C) $\sec^2(y/x)=xc$ (D) $\sin(y/x)=xc$

Ans. (D) $\sin(y/x)=xc$

Q.72.Particular solution of the differential equation $dy/dx=y^2-2xy-x^2/y^2+2xy-x^2$ $y=-1$ at $x=1$.

(A) $y=x$ (B) $y+x=2$ (C) $y=-x$ (D) $y-x=2$

Ans. (A) $y=-x$

Q.73. Solution of the differential equation $dy/dx = 3x - 6y + 7/x - 2y + 4$ is ______

(A) $\log(x-2y+4)^2 = c$

(B) $6x - 2y + \log(x-2y+2)^2 = c$

(C) $x - 2y + \log(x-2y+6)^2 = c$

(D) $-5x + \log(x-2y+3)^2 = c$

Ans. (B) $6x - 2y + \log(x-2y+2)^2 = c$

Q.74. Solution of the differential equation (3y-7x+7)dx+(7y-3x+3)dy=0 is ______

(A) $p = (y+x)^5(y-x)^2$

(B) $p = (y+x+2)^5(y-x+2)^2$

(C) $p = (y+x)^2(y-x)^5$

(D) $p = (y+x-1)^5(y-x+1)^2$

Ans. (D) $p = (y+x-1)^5(y-x+1)^2$

Q.75. Which of the following is a vector?

(A) Surface Tension (B) Moment of Inertia
(C) Pressure (D) None of the above

Ans. (D) None of the above

Q.76. What is the resultant of two vectors with magnitudes 3 and 4 units, making an angle of 60 degrees between them?

(A) 7 units (B) 5 units
(C) 6 units (D) 2 units

Ans. (A) 7 units

Q.78. In Laurent's series, Coefficient $a_n = \cdots (n = 0, 1, 2, 3..)$

(A) $\frac{n!}{2\pi!}\int_c \frac{f(z)}{(z-a)^{n+1}}$ (B) $\frac{n!}{2\pi!}\int_c \frac{f(z)}{(z-a)^{n}}$

(C) A and B (D) None of these.

Ans. (A) $\frac{n!}{2\pi!}\int_c \frac{f(z)}{(z-a)^{n+1}}$

Q.77. The series $1 - z^2/2! + z^4/4! - \cdots \therefore = \cdots$

(A) cos z (B) sin z (C) tan z (D) -1

Ans. (A) cos z

Q.78. The series $1/z + 1/1! + z/2! + \cdots$. is the Laurent^' s expansion of function $f(z) =$

(A) e^z/z (B) $e^z/2z$ (C) 2 (D) 0

Ans. (A) e^z/z

Q.79. $1+z+z^2+\cdots z^{n-1}=$

(A) $1+z^n/1+z$ (B) $1-z^n/1-z$

(C) $1+z^n/1-z$ (D) None of these

Ans. (B) $1-z^n/1-z$

Q.80. If function f of complex variable is not analytic at z = a, then the point z=a is called.

(A) Singular point (B) Limit point

(C) Boundary point (D) None of these

Ans. (A) Singular point

Q.81. The function $f(z) = e^{(1/z)}$ has essential singularity at Z =

(A) 2 (B) 1

(C) -1 (D) 0

Ans. (D) 0

Q.82. A pole of order one is called

(A) Simple pole (B) Double pole

(C) Triple pole (D) None of these

Ans. (A) Simple pole

Q.83. If vector A = 3i + 4j and vector B = 2i - j, what is the magnitude of the resultant vector A + B?

(A) $\sqrt{5}$ (B) $\sqrt{13}$

(C) 5 (D) 13

Ans. (B) $\sqrt{13}$

Q.84. If z = a is a simple pole of f(z) then the residue at pole z = a of (z) is given by

(A) $\lim_{z\to}(z-a/2)f(z)$ (B) $\lim_{z\to}(z-a)f(z)$

(C) A and B (D) None of these

Ans. (B) $\lim_{z\to}(z-a)f(z)$

Q.85. If f (z) =1/z (z-1) is function of complex of complex variable then poles of f(z) are at

(A) 0,1 (B) 0, 2

(C) 0,5 (D) None of these

Ans. (A) 0,1

Q.86. If $(z) = 1/((z-1)^2)$ If is function of complex variable then doubles poles of $f(z)$ is at

(A) -1 (B) 1

(C) 0 (D) None of these

Ans. (B) 1

Q.87. Residue of function f(z) = 1/z at pole z = 0 is

(A) 1 (B) 2 (C) 2π (D) 0

Ans. (A) 1

88. If $(z)=(ze^z/z-1)$, then r residue of (z) at $z=1$ is

(A) e (B) e^2 (C) 2 (D) 0

Ans. (A) e

89. Find the divergence of this given vector $\vec{r}=12x^6y^6\hat{i}+3x^3y^3z\vec{j}+3x^2yz^2\vec{k}$.

(A) $12x^5y^6+2x^3yz+6x^2yz$

(B) $72x^5y^6+2x^3yz+3x^2yz$

(C) $72x^5y^6+2x^3yz+6x^2yz$

(D) $6x^5y^6+2x^3yz+6x^2yz$

Ans. (C) $72x^5y^6+2x^3yz+6x^2yz$

90. Find the curl for $\vec{r}=x^2yz\vec{i}+(3x+2y)z\vec{j}+21z^2x\vec{k}$.

(A) $\vec{i}(3x+2y)-\vec{j}(11z^2-x^2y)+\vec{k}(3z-x^2z)$

(B) $\vec{i}(x+2y)-\vec{j}(21z^2-x^2y)+\vec{k}(3z-x^2z)$

(C) $-\vec{i}(3x+2y)-\vec{j}(21z^2-x^2y)+\vec{k}(3z-x^2z)$

(D) $\vec{i}(3x+2y)-\vec{j}(21z^2-x^2y)+\vec{k}(3z-x^2z)$

Ans. (C) $-\vec{i}(3x+2y)-\vec{j}(21z^2-x^2y)+\vec{k}(3z-x^2z)$

Q.91. If $\lim_{z\to}(z^2+1)/(z+i)=a$ then the value of a is

(A) i (B) 0 (C) -i (D) 1

Ans. (B) 0

Q.92. If $u=u(x,y)$ satisfy Laplace equation $u_{xx}+u_{yy}=0$, then u is called as

(A) Analytic (B) Non analytic

(C) Harmonic (D) None of these

Ans. (C) Harmonic

Q.93. The integral $i=\int_{2\pi}^{0}\frac{d\theta}{5+3\cos\theta}$ is evaluated by substitution

(A) 2 (B) $z=e^{i\theta}$

(C) $Z=e^{i\theta}$ (D) $Z=0$

Ans. (B) $z=e^{i\theta}$

Q.94. The region of validity of $1/(1+z)$ for its Taylor series expansion about $z=0$

(A) $|z|<1$ (B) $|z|>1$

(C) $|z|=1$ (D) None of these

Ans. (A) $|z|<1$

Q.95 Which of the following operations results in a scalar quantity?

(A) Dot product (B) Cross product

(C) Addition of vectors (D) Subtraction of vectors

Answer: A) Dot product

Q.96 If vector A = 2i - j and vector B = 3i + 4j, what is the magnitude of the cross product of A and B?

(A) $\sqrt{29}$ (B) 29

(C) 7 (D) 5

Ans. (B) 29

Q.97 An analytic function f(z) = u + iv be such that u and v must satisfy Laplace differential equation then u and v are ________

(A) Analytic (B) Non analytic

(C) Harmonic (D) None of these

Ans. (C) Harmonic

Q.98. If $\lim\limits_{z \to 1+i} \frac{z^4 - 4}{z^2 + 2i} = A$ then the value of A is

(A) 4i (B) 0

(C) -4i (D) 1

Ans. (A) 4i

Q.99. If (z) = z + 1 and C is unit circle $|z| = 1$ then $\int (z) dz =$

(A) 1 (B) 0

(C) -1 (D) None of these

Ans. (B) 0

Q.100 Let $f(z) = \frac{p(z)}{Q(z)}$ such that P(z) and Q(z) are polynomials in z having no common factor deg Q(z) – deg $P(z) \geq 2$, and Q (z) = 0 has no real roots then

(A) $\pi i \sum R^+$ (B) $2\pi i \sum R^+$

(C) A and B (D) None of these

Ans. (B) $2\pi i \sum R^+$

□□