

CPU-GPU Collaborative Acceleration of Bulletproofs - A Zero-Knowledge Proof Algorithm

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Abstract—Zero-knowledge proofs help to protect the privacy and security of blockchains by keeping the transaction information private. Bulletproofs are the state-of-the-art zero-knowledge technologies that perform confidential transactions without a trusted setup. However, Bulletproofs are still computationally inefficient to be applied in blockchain applications, and it is of great significance to parallelize Bulletproofs on GPUs. In this paper, we present a CPU-GPU collaborative framework to accelerate the inner-product arguments of Bulletproofs. To our best knowledge, it is the first time that Bulletproofs are implemented in a CPU-GPU hybrid system. The experiments show that our implementation achieves an average speedup ratio of 3.7x. The results also demonstrate that the CPU-GPU collaborative acceleration of Bulletproofs has properties of small size, high efficiency, and high scalability.

Index Terms—zero-knowledge proof, blockchain, parallel acceleration, CPU-GPU co-acceleration, inner-product arguments.

I. Introduction

Blockchain technology [1] has attracted great attention both from the industry and academia due to its unique feature of data immutability. It enables a community of users to record transactions in a shared ledger in a decentralized fashion without an authority, which cannot be changed easily after they are created [2] [3]. By recording transactions in a shared ledger, it allows everybody to see what happens on the blockchain. Thus an unfortunate side effect is that all the transaction data are public, which is at odds with the normal expectation of privacy [4]. Insufficient financial privacy [5] can have serious security and privacy implications for both commercial and personal transactions [6].

A confidential transaction (CT) is a cryptographic method of increasing the privacy and security of blockchains by keeping the transaction information private while preserving the ability of the public network to verify the ledger entries. It conducts a zero-knowledge proof for each piece of information that needs to be verified in the transaction to ensure that the information is legal and can be verified while not disclosing the content of the information [7].

For instance, assume that the two parties to the transaction are A and B, and A has two accounts at the beginning

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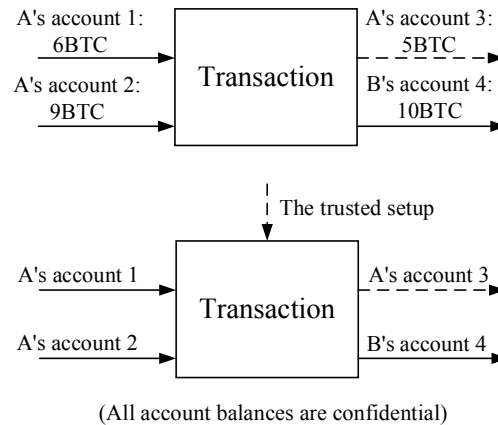


Fig. 1: The transaction without (top) or with (bottom) the confidential mechanism.

while B has one account with no balance. If A wants to transfer 10-BTC to B now, the transaction without the confidential mechanism is shown at the top of Fig. 1. It can be seen that after A transfers money from account 1, 2 to B, the balance of the original accounts 1, 2 is cleared, and then there is a new account 3 to store the balance after the transaction. As there is no confidential mechanism applied, all balance details are public.

Counter intuitively, the zero-knowledge proof technology can ensure that a valid transaction has taken place without disclosing any information about the account balance. The bottom of Fig. 1 shows the transaction flow after the zero-knowledge proof technology is applied, where it verifies that

- Proof 1: the balance of accounts 3 is no less than 0;
- Proof 2: the balance of accounts 4 is no less than 0;
- Proof 3: the sum of the balances of accounts 1 and 2 is no less than the sum of the balances of accounts 3 and 4.

With the above three proofs, the BTC transferring transaction can be committed without revealing the details of the remaining balance.

At present, ZK-SNARKs (Zero-Knowledge Succinct Non-interactive Argument of Knowledge) [8], ZK-STARKs (Zero-Knowledge Scalable Transparent Arguments of Knowledge) and Bulletproofs [9] are the state-of-the-art

zero-knowledge proof algorithms. ZK-SNARKs require a trusted setup ceremony to ensure that the two parties to the transaction will not collude. ZK-SNARKs are computationally efficient but the requirement of a trusted third party limits the applications in blockchains. ZK-STARKs improve upon ZK-SNARKs by removing the need for a trusted setup at the cost of computational inefficiency. Bulletproofs further improve ZK-STARKs by transforming interval proofs into vector inner product computation, which greatly reduces the complexity. However, verifying a Bulletproof is still more time-consuming than verifying a ZK-SNARK proof.

Fortunately, the inner product computation can be highly parallel computed in GPU. While inner product computation is readily parallelized, it is not trivial to apply CPU-GPU systems to boost the performance of inner product computation.

We propose to accelerate the verifying a Bulletproofs by fully unlocking the power of CPU-GPU systems to parallelize inner product computation. And to our best knowledge, it is the first time that the Bulletproofs are implemented in a CPU-GPU system. We summarize our contribution as follows.

- Accelerate the verifying a Bulletproof by fully parallelizing inner product computation in CPU-GPU systems and scales down the execution time by 3-4x.
- Applying data grouping and pipeline methods to process high-level input data to improve acceleration efficiency.
- Implement Bulletproofs on different CPU-GPU platforms and demonstrate the high efficiency and scalability of Bulletproofs.

The remainder of this paper is structured as follows: Section II provides basic concepts and general information about inner-product argument and elliptic curve point arithmetic in Bulletproofs while the designed CPU-GPU collaborative algorithm is described in Section III. The experiments and results based on the algorithm are presented in Section IV. Finally, in Section V, the conclusion and future work are presented.

II. Background

The current acceleration schemes in the field of zero-knowledge proofs are all accelerating the zk-snark algorithm and the zk-stark algorithm [10], and most of them are based on FPGA to achieve acceleration [11] or by changing the hardware structure, such as adding pipeline accelerators to achieve hardware acceleration [12]. As for GPU acceleration, the blockchain project Filecoin uses GPU acceleration to shorten the PoST proof time in zk-snark, and Long et al. [13] proposed a CPU-GPU systems to accelerate the computation tasks of IPR solution in matrix factorization. Privacy protection of cryptography is currently a hot research topic. Qiu et al. [14] proposed a novel privacy-preserving method using bipartite matching

and attracted much follow-up work. Dai et al. [15] presented some important research on privacy protection in smartphones.

Bulletproofs present a new zero-knowledge argument to prove that a secret committed value lies in a given interval relying only on the discrete logarithm assumption. An argument is a proof that holds only if the prover is computationally bounded and certain computational hardness assumptions hold. An inner-product argument proves that an inner product relation holds between committed vectors [16]. Bulletproofs build on the communication efficient inner-product argument proposed by Bootle et al. [17]. The argument proves that the prover knows the openings of two binding Pedersen vector commitments that satisfy a given inner product relation [9].

A. Inner-product argument

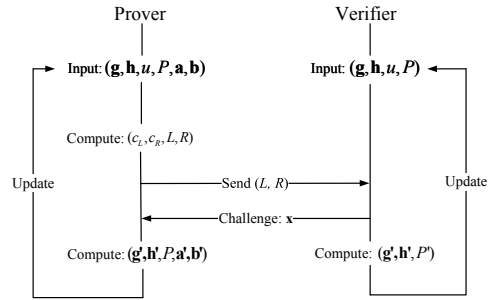


Fig. 2: The procedure of inner-product argument in Bulletproofs.

Fig. 2 describes the overall interactive procedure of inner-product argument in Bulletproofs. There is a prover and a verifier in Bulletproofs. As shown in Fig. 2, the verifier repeatedly sends a random challenge x to the prover and the prover will compute two variables L, R according to the received x and feedback L, R to the verifier.

We first introduce the notations used in the interaction between the prover and the verifier. Let $\mathbf{g}, \mathbf{h} \in \mathbb{G}^n$ denote two independent generators, and $u, P \in \mathbb{G}$ denote the binding vector commitments. The variable $c \in \mathbb{Z}_p$ is a scalar. In the inner-product argument, the prover proves that there exists two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_p^n$ that satisfy $P = \mathbf{g}^{\mathbf{a}} \mathbf{h}^{\mathbf{b}}$ and $c = \langle \mathbf{a}, \mathbf{b} \rangle$, i.e.,

$$\begin{aligned} &(\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, P \in \mathbb{G}, c \in \mathbb{Z}_p; \mathbf{a}, \mathbf{b} \in \mathbb{Z}_p^n) : \\ &P = \mathbf{g}^{\mathbf{a}} \mathbf{h}^{\mathbf{b}} \wedge c = \langle \mathbf{a}, \mathbf{b} \rangle \end{aligned} \quad (1)$$

Let us further restrict P to be a Pedersen's commitment of the vector $(\mathbf{a}, \mathbf{b}, c)$. The formula (1) is modified as follows.

$$\begin{aligned} &(\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, P \in \mathbb{G}, c \in \mathbb{Z}_p; \mathbf{a}, \mathbf{b} \in \mathbb{Z}_p^n) : \\ &P = \mathbf{g}^{\mathbf{a}} \mathbf{h}^{\mathbf{b}} u^{c \langle \mathbf{a}, \mathbf{b} \rangle} \end{aligned} \quad (2)$$

Initially, when $n > 1$, the input parameters of the prover are $(\mathbf{g}, \mathbf{h}, u, P, \mathbf{a}, \mathbf{b})$ and set $n' = \frac{n}{2}$, then P can

be rewritten as $P = H(\mathbf{a}_{[n']}, \mathbf{a}_{[n']}, \mathbf{b}_{[n']}, \mathbf{b}_{[n']}, c)$. In order to ensure that the verifier can calculate a new Pedersen commitment, the prover needs to provide the verifier with the L and R , which are defined as follows.

$$\begin{aligned} c_L &= \langle \mathbf{a}_{[n']}, \mathbf{b}_{[n']} \rangle \in \mathbb{Z}_p \\ c_R &= \langle \mathbf{a}_{[n']}, \mathbf{b}_{[n']} \rangle \in \mathbb{Z}_p \\ L &= \mathbf{g}_{[n']}^{\mathbf{a}_{[n']}} \mathbf{h}_{[n']}^{\mathbf{b}_{[n']}} u^{c_L} \in \mathbb{G} \\ R &= \mathbf{g}_{[n']}^{\mathbf{a}_{[n']}} \mathbf{h}_{[n']}^{\mathbf{b}_{[n']}} u^{c_R} \in \mathbb{G} \end{aligned} \quad (3)$$

Subsequently, the verifier generates and sends a random challenge $x \xleftarrow{\$} \mathbb{Z}_p^*$ to the prover, and the prover need to compute

$$\begin{aligned} \mathbf{a}' &= \mathbf{a}_{[n']} \cdot x + \mathbf{a}_{[n']} \cdot x^{-1} \in \mathbb{Z}_{n'}^p \\ \mathbf{b}' &= \mathbf{b}_{[n']} \cdot x^{-1} + \mathbf{b}_{[n']} \cdot x \in \mathbb{Z}_{n'}^p \\ \mathbf{g}' &= \mathbf{g}_{[n']}^{x^{-1}} \circ \mathbf{g}_{[n']}^x \in \mathbb{G}^{n'} \\ \mathbf{h}' &= \mathbf{h}_{[n']}^x \circ \mathbf{h}_{[n']}^{x^{-1}} \in \mathbb{G}^{n'} \\ P' &= L^{x^2} P R^{x^{-2}} \in \mathbb{G} \end{aligned} \quad (4)$$

and update the input $(\mathbf{g}', \mathbf{h}', P', \mathbf{a}', \mathbf{b}')$. In the meanwhile, after receiving (L, R) , the verifier computes $\mathbf{g}', \mathbf{h}', P'$ and updates the input. The interactive procedure is repeated until $n' = 1$. Finally, the verifier compares P' with $H(x^{-1}\mathbf{a}', x\mathbf{a}', x\mathbf{b}', x^{-1}\mathbf{b}', \langle \mathbf{a}', \mathbf{b}' \rangle)$. If the two values are equal, the verification is considered to be successful.

There are many computation steps of the inner-product argument as in equations (3) - (4) that can be performed in parallel. Furthermore, as Bulletproofs usually require a large number of inner-product arguments, the performance of Bulletproofs will be greatly improved if the inner-product argument is effectively accelerated.

B. Elliptic curve point arithmetic

In Bulletproofs, the implementation of the inner-product argument uses secure Elliptic Curve Cryptography (ECC), which is based on the algebraic structure of elliptic curves over finite (prime) fields. Thus, the elliptic curve points instead of numbers are used as basic mathematical objects such as addition and multiplication.

An elliptic curve is a set of points defined by $y^2 = x^3 + ax + b$ and a, b satisfying $4a^3 + 27b^2 \neq 0$, which ensures the curve does not contain singularities. Note that the elliptic curve is symmetric with respect to the x -axis. The elliptic curve point addition is shown in Fig. 3 and Fig. 4.

Let us assume a straight line and the elliptic curve intersect at points (P, Q, R) , and define $P + Q + R = 0$ (0 represents the identity element, which is a point at infinity). The three points (P, Q, R) form an elliptic curve group. The elliptic curve point addition is regarded as solving R given the two points P and Q , i.e., $P + Q = -R$.

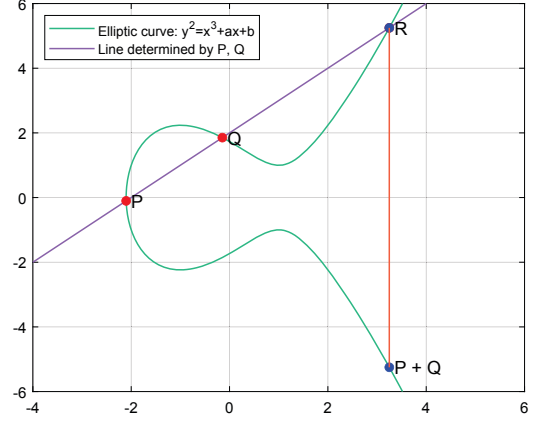


Fig. 3: Elliptic curve point addition ($P \neq Q$).

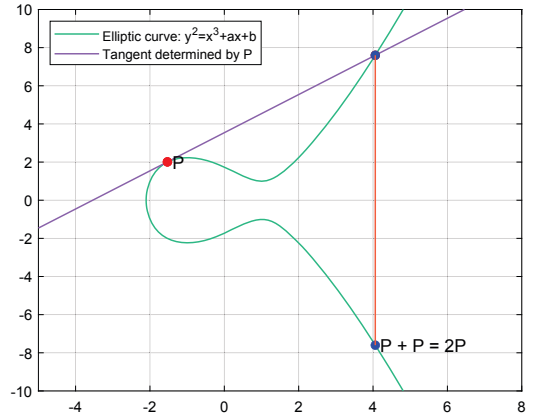


Fig. 4: Elliptic curve point addition ($P = Q$).

Let $\{(x_i, y_i), i \in P, Q, R\}$ represent the coordinates of P, Q, R , respectively. Assume a real field, when $P \neq Q$, the slope of the line λ is as follows.

$$\lambda = \frac{y_Q - y_P}{x_Q - x_P}, P \neq Q. \quad (5)$$

When $P = Q$, the slope λ is calculated as

$$\lambda = \frac{3x_P^2 + a}{2y_P}, P = Q. \quad (6)$$

Then the coordinates of R can be calculated as

$$x_R = \lambda^2 - x_P - x_Q, y_R = \lambda(x_P - x_R) - y_P. \quad (7)$$

However, the elliptic curve is defined in a finite field F_p instead of the real field in encryption. The finite field arithmetic follows the modulus rules, thus the slope of the line and the coordinates of R are calculated as follows,

where p is a prime number.

$$\begin{aligned}\lambda &= \frac{y_Q - y_P}{x_Q - x_P} \pmod{p}, P \neq Q \\ \lambda &= \frac{3x_P^2 + a}{2y_P} \pmod{p}, P = Q \\ \begin{cases} x_R = \lambda^2 - x_P - x_Q \pmod{p} \\ y_R = \lambda(x_P - x_R) - y_P \pmod{p} \end{cases}\end{aligned}\quad (8)$$

Finally, given the coordinates of R , the elliptic curve point addition operation $P + Q = -R$ can be obtained by finding the symmetric point of R , i.e., $(x_R, -y_R)$.

The elliptic curve point multiplication can be implemented by repeatedly applying addition as $nP = P + P + \dots + P$.

In the inner product proof and verification of Bulletproofs, there is a large amount of elliptic curve point addition and multiplication operations that can be parallelized. It will effectively improve the speed of inner-product arguments in Bulletproofs.

III. Algorithm Design

According to the algorithm level division implemented on GPU, parallelization can be divided into coarse-grained parallelization and fine-grained parallelization [18] [19]. Taking the parallelization in the elliptic curve as an example, coarse-grained parallelization refers to the use of serialization in the underlying algorithm, that is, a single thread calculates a finite field, but parallelism is used at the modular exponentiation level. The fine-grained parallelization refers to the split parallelization started by the underlying modular multiplication algorithm, and the finite field algorithm is calculated by multiple threads [20]. Although the coarse-grained parallel scheme combines the advantages of full serial and fine-grained parallel to a certain extent, due to the asymmetric characteristics of the elliptic curve algorithm, the design of coarse-grained parallel algorithms is much more difficult, and it is also easy to cause waste of computing resources, so the fine-grained parallelization scheme is selected here.

In Bulletproofs, the most significant parts are the circuit proof and rangeproof, which are both implemented based on inner product arguments. The procedure of the circuit proof and rangeproof contains a large number of vector inner product operations and modular operations based on elliptic curves in finite fields, that can be accelerated by parallel computing. The framework of the entire CPU-GPU collaborative acceleration is shown in Fig. 5.

In Fig. 5, there are two computing components, i.e., the CPU host and the GPU device. The Bulletproofs circuit and rangeproof module are initialized in the host, which includes data input, memory allocation, and parameter setup for the proof and verification stages. When the Bulletproofs algorithm steps into the parallel calculation part of the inner-product and elliptic modular operations, the host copies and transmits the data and kernels that need to be calculated to the GPU device for parallel

execution based on the CUDA platform [21] [22]. After the calculation is completed, the result will not be transmitted back to the host immediately but temporarily stored in the global memory of the GPU until the result needs to be used by the host.

Overall, Algorithm 1 is a specific collaborative acceleration scheme in the process of the inner-product argument. The communication volume of the argument was reduced by $\frac{1}{3}$. The prover and verifier calculate according to the formula 1-4. When it involves parallel inner product calculation and elliptic curve calculation, the data is sent to the GPU for parallel calculation, and the result is temporarily stored until it is used.

Algorithm 1 Inner-product arguments acceleration

Input: $\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, u, P \in \mathbb{G}, \mathbf{a}, \mathbf{b} \in \mathbb{Z}_p^n$;
Prover's input: $(\mathbf{g}, \mathbf{h}, u, P, \mathbf{a}, \mathbf{b})$;
Verifier's input: $(\mathbf{g}, \mathbf{h}, u, P)$;
Output: Verifier accepts or rejects;
if $n = 1$ then
Prover \rightarrow Verifier: $a, b \in \mathbb{Z}_p$;
Verifier: computes $c = a \cdot b$;
Verifier checks:
if $P = g^a h^b u^c$ then
return accepts;
else
return rejects;
end if
else if $n > 1$ then
Prover computes:
 $n' = \frac{n}{2}$
Host \rightarrow Device: $\mathbf{a}, \mathbf{b}, n'$, Device computes:
 c_L, c_R ;
Host \rightarrow Device: $\mathbf{g}, \mathbf{h}, u$, Device computes: L, R ;
Device \rightarrow Host: L, R ;
Prover \rightarrow Verifier: L, R ;
Verifier \rightarrow Prover: $x \xleftarrow{\$} \mathbb{Z}_p^*$;
Prover computes:
Host \rightarrow Device: P , Device computes:
 $\mathbf{a}', \mathbf{b}', \mathbf{g}', \mathbf{h}', P'$
Verifier computes:
Host \rightarrow Device: P , Device computes: $\mathbf{g}', \mathbf{h}', P'$;
Prover updates:
Device \rightarrow Host: $\mathbf{g}', \mathbf{h}', P', \mathbf{a}', \mathbf{b}'$;
 $(\mathbf{g}', \mathbf{h}', P', \mathbf{a}', \mathbf{b}') \rightarrow (\mathbf{g}, \mathbf{h}, P, \mathbf{a}, \mathbf{b})$
Verifier updates:
Device \rightarrow Host: $\mathbf{g}', \mathbf{h}', P'$;
 $(\mathbf{g}', \mathbf{h}', P') \rightarrow (\mathbf{g}, \mathbf{h}, P)$
Recursively run algorithm 1.
end if

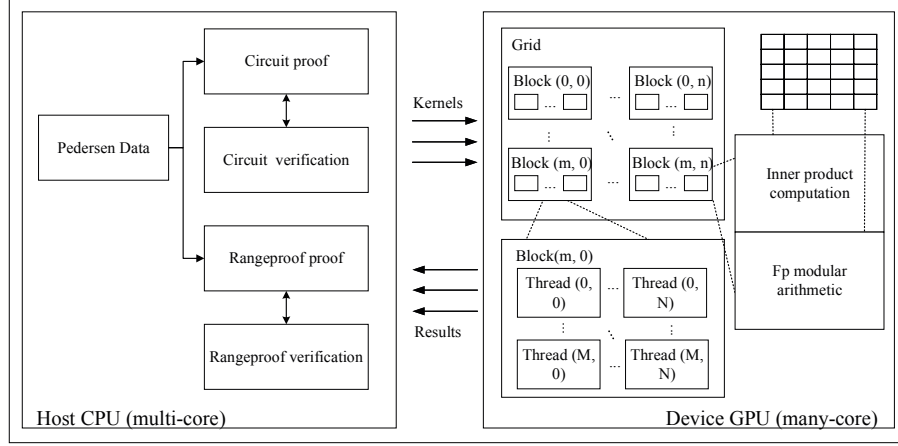


Fig. 5: The CPU-GPU collaborative operation framework.

IV. Experiments

The experiments are conducted on a Docker image of version 20.10.3 on Intel® Xeon® Gold 6230 CPU @ 2.10GHz, with the GPU devices of NVIDIA® Cuda release 10.1 on NVIDIA® GeForce® RTX 2080 Ti/NVIDIA® Tesla® V100 to evaluate the proposed algorithm with heterogeneous devices.

Fig. 6 reports the running time of proof and verification in Bulletproofs' rangeproof procedure on the RTX 2080 Ti platform. We gradually increase the input size from (1-bit, 1-commit) to (512-bit, 256-commit). Fig. 6 shows that as the input bit-width increases, the speedup ratio of Algorithm 1 increases with both the proof and verification procedures. The speedup ratio of the verification procedure is slightly larger than that of the proof procedure. It is because the verification contains more elliptic curve point arithmetic operations that can be parallelized.

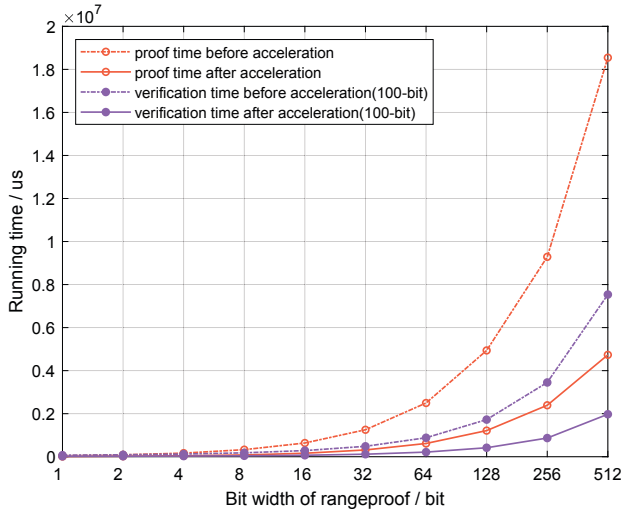


Fig. 6: Accelerated result comparison on RTX 2080 Ti.

In order to demonstrate the universal applicability of Algorithm 1 and explore the acceleration ability of different GPU platforms, the experiments are also tested on the V100 platform. The input data are Pedersen Hash varying from 8-bit (Pedersen-3) to 8192-bit (Pedersen-3072).

Table I presents the running time of the proof procedure in the circuit module of Bulletproofs with and without acceleration on different GPU platforms. It shows that there is no significant difference in the speedup ratios between different GPU devices. The more powerful Tesla V100 does not necessarily achieve a much higher speedup compared with the RTX 2080. We conclude that Algorithm 1 has the ability to perform well on a wide spectrum of GPU devices with sufficient cores.

TABLE I: Comparison of proof time on different GPUs

	Time on 2080 Ti / us		Time on V100 / us	
	without acc	with acc	without acc	with acc
Pedersen-3	9947	3761	10036	2695
Pedersen-6	17917	4840	18032	4829
Pedersen-12	34469	9264	34617	9286
Pedersen-24	67600	18017	67497	18062
Pedersen-48	133705	35475	132979	35568
Pedersen-96	261177	69462	260316	69627
Pedersen-192	508701	136028	509822	136318
Pedersen-384	999127	266884	1000085	267433
Pedersen-768	1986601	526438	1971723	527467
Pedersen-1536	3940234	1038547	3892699	1041097
Pedersen-3072	7894753	2062248	7727506	2066544
Avg-speedup		3.657		3.736

In addition, Fig 7 presents the speedup ratio of the proof procedure in the circuit module of Bulletproofs on different GPU platforms. It shows that the speed-up ratio of the proof process may be slightly larger as the number of verification bits increases on 2080 Ti while the acceleration ratio on the V100 is more stable.

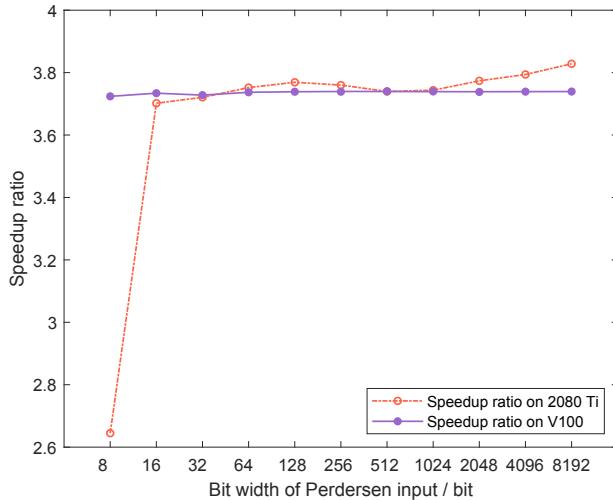


Fig. 7: Speedup ratio of proof procedure in circuit module on different GPUs.

V. Conclusion

This paper proposed a CPU-GPU cooperative acceleration design for the Bulletproofs algorithm. The running time of Bulletproofs was scaled down by 3-4x by accelerating inner product computation and finite field modular arithmetic in inner product arguments. In the meanwhile, by testing Algorithm 1 on different GPU devices, it demonstrated that the CPU-GPU cooperative acceleration design has the features of universality, high efficiency, and high scalability. It was noticed that the speedup ratio of the verification procedure depends on the proof and verification bit-width which has an impact on the GPU I/O efficiency. A verification bit-width that is a multiple of the GPU's I/O bit-width usually achieves a higher speedup ratio [23].

The future work includes data grouping and pipelining for high-bit data, and loop unrolling or software pipelining to reduce the data transmission consumption in the CPU-GPU systems.

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