

# RATIO AND PROPORTION

## Ratio

a, b

a : b

a : b : a/b

a/b

$$a/b = \frac{a^x}{b^x} = a_b$$

a : b  
ax bx

$$a/b = \frac{a/x}{b/x} = a/b$$

1) Duplicate Ratio of  $a/b = a^2/b^2$

$$2/3 = \frac{2^2}{3^2} = \frac{4}{9}$$

2) Subduplicate ratio of  $a/b = \sqrt{a/b}$

$$\frac{16}{25} = \sqrt{\frac{16}{25}} = \sqrt{\frac{4^2}{5^2}} = \frac{4}{5}$$

3) Triplicate ratio of  $a/b = \frac{a^3}{b^3}$

$$2/3 = \frac{2^3}{3^3} = \frac{8}{27} \checkmark$$

4) Subtriplicate ratio or  $a/b = \sqrt[3]{a/b}$

$$\frac{64}{125} = \sqrt[3]{\frac{64}{125}} = \sqrt[3]{\frac{4^3}{5^3}}$$
$$= 4/5 \checkmark$$

5) compounded ratio of  $\underline{a:b}$  and  $\underline{c:d}$

$$= \underline{ac} : bd$$

If the compounded ratio of  $x:2$  and  $a:y$  is

3:4 find  $x:y$

a:b, c:d

ac:bd

$$ax:2y = 3:0$$

$$\frac{ax}{2y} = \frac{3}{x_2}$$

$$\frac{3x}{y} = 1/2 \quad \frac{x}{y} = 1/6 \checkmark$$

$$\frac{x}{y} = \frac{3}{a_2} \times \frac{2}{a_3} = 1/6 \quad \frac{y}{y} = 1/6$$

$$a:b = 2:3$$

$$b:c = 4:5$$

$$a:b:c = ?$$

$$\begin{array}{rcl} a:b & = & 2_u : 3_v \\ b:c & = & 3_x : 5_y \\ \hline a:b:c & = & 8 : 12 : 15 \end{array}$$

$$\begin{array}{rcl} a:b & = & 2:3 \\ b:c & = & 4:5 \\ \hline a:b:c & = & 8:12:15 \end{array}$$

$$a:b = 4:5 \quad , \quad b:c = 2:7 \quad a:b:c = ?$$

$$\begin{array}{rcl} a:b & = & |^4 \quad : \quad 5 | \\ b:c & = & | \downarrow 2 \quad : \quad 7 \downarrow \\ \hline \end{array}$$

$$a:b:c = 8:10:35$$

$$a:b = 4:5, \quad b:c = 2:3, \quad c:d = 1:2$$

$$a:b:c:d = ?$$

$$\begin{aligned} a:b &= 4 : 5 \\ b:c &= 2 : 3 \\ c:d &= 1 : 2 \end{aligned}$$

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$$a:b:c:d = 8:10:15:30$$

$a:b::2:3, b:c::4:5.$

find  $a:b:c.$

$$\begin{array}{rcl} a:b & = & \downarrow^{2:3} \\ \underline{b:c} & = & \downarrow_{4:5} \\ a:b:c & = & 8:12:15 \end{array}$$

$a:b::2:3$ ,  $b:c::4:1$ ,  $c:d::2:5$  find  $a:b:c:d$ .

$$\begin{aligned} a:b &= \boxed{2 : 3} \\ b:c &= \boxed{4 : 1} \\ c:d &= \boxed{2 : 5} \\ \hline \end{aligned}$$

$$a:b:c:d = 16:24:6:15 \checkmark$$

If  $a:b = 2:3$ ,  $a:d = 3:4$ , and  $e:d = 3:5$ , then  $e:b = ?$

1.  $10:9$

$$a:b = 2:3$$

2.  $8:15$

$$\begin{array}{rcl} b:a & = & 3:2 \\ a:d & = & 3:4 \\ \hline d:e & = & 5:3 \end{array}$$

3.  $9:10$

4.  $15:8$

$$\frac{b}{a} = \frac{3}{2} \quad \frac{a}{d} = \frac{3}{4} \quad \frac{d}{e} = \frac{5}{3}$$

$$\frac{b}{a} \times \frac{a}{d} \times \frac{d}{e} = \frac{3}{2} \times \frac{3}{4} \times \frac{5}{3}$$

$$\frac{b}{e} = \frac{15}{8} \quad \frac{e}{b} = \frac{8}{15}$$

If  $A : B = 2 : 3$ ,  $B : C = 4 : 5$  and  $C : D = 6 : 7$ , then the value of  $\frac{A+B+C}{D}$  is:

1. 3

2. 7

3. 5

4. 2 ✓

$$A : B = 2 : 3$$

$$B : C = 4 : 5$$

$$C : D = 6 : 7$$

$$= \frac{16+24+30}{35}$$

$$= \frac{70}{35} = 2$$

$$\underline{A : B : C : D = 2 \times u \times 6 : 3 \times u \times 6 : 3 \times 5 \times 6 : 3 \times 5 \times 7}$$

$$\underline{A : B : C : D = 16 : 24 : 30 : 35}$$

If  $l : m : n = 1 : 2 : 4$ , then  $\sqrt{5l^2 + m^2 + n^2}$  is equal to:

1.  $4n$

$l=x, \quad m=2x, \quad n=4x$

2.  $5l$

3.  $2m$

4. 5

$$\begin{aligned} &= \sqrt{5x^2 + 4x^2 + 16x^2} \\ &= \sqrt{x^2(25)} = 5x \\ &= 5l \end{aligned}$$

If  $a : b = 2 : 3$  and  $b : c = 2 : 3$ , then what is the value of  $(3a^2 + b^2 - c^2) : (a^2 + 2b^2 - c^2)$ ?

1.  $3 : 5$

$$a:b = 2:3$$

2.  $4 : 5$

$$b:c = 2:3$$

3.  $5 : 7$

$$\underline{b:c = 2:3}$$

4.  $3 : 7$

$$\underline{\underline{a:b:c = 4:6:9}}$$

$$\frac{3 \times 16 + 36 - 81}{16 + 2 \times 36 - 81} = \frac{48 + 36 - 81}{16 + 72 - 81} = \frac{84 - 81}{88 - 81} = \frac{3}{7}$$

If  $V_1 : V_2 = 1 : 2$  and  $V_1 + V_2 = 147$ , then what is the value of  $V_2 - V_1$ ?

X 1. 48

$$V_1 : V_2 = 1 : 2.$$

$$= 98 - 49$$

X 2. 56

$$V_1 = x, \quad V_2 = 2x$$

$$= 49$$

X 3. 98

✓ 4. 49 ✓  $x + 2x = 147$

$$V_1 = 49$$

$$V_2 = 98$$

$$3x = 147$$

$$x = \frac{147}{3}$$

$$x = 49$$

Three positive numbers are in the proportion 3 : 4 : 6. If the sum of their squares is 244, then what is the largest number?

1. 16

2. 8

3. 12

4. 6

$$a:b:c \quad \overline{a^2 + b^2 + c^2} = 244$$

$$a = 3x, \quad b = 4x, \quad c = 6x$$

$$a^2 + b^2 + c^2 = 244$$

$$9x^2 + 16x^2 + 36x^2 = 244$$

$$x^2 = \frac{244}{61}$$

$$x^2(9+16+36) = 244$$

$$x^2 = 4$$

$$x = 2$$

$$6 \cdot x^2 = 244$$

$$c = 6 \times 2 = 12$$

If  $X^2 + Y^2 = 100$  and  $X : Y = 4 : 3$ , then what is the value of  $X^2 - Y^2$ ?

1. 18

2. 28

3. 36

4. 24

$$X = 4a, \quad Y = 3a \quad X = 8, \quad Y = 6$$

$$X^2 + Y^2 = 100$$

$$X^2 - Y^2$$

$$16a^2 + 9a^2 = 100$$

$$-6u - 36 = 28$$

$$25a^2 = 100$$

$$a^2 = u = 2^2$$

$$a = 2$$

The ratio of two whole numbers is  $\underline{\underline{5 : 7}}$ . Which of the following can be their sum?

- 1. 54
- 2. 60
- 3. 46
- 4. 68

$$5 : 7 = \frac{5}{\cancel{1}} : \frac{7}{\cancel{1}} = 5 : 7$$
$$x : 1 \quad 5 + 7 = 12$$

$$x : 2 \quad 10 + 10 = 20$$
$$x : 4 \quad 20 + 28 = \frac{48}{12} \textcircled{4}$$

If  $X : Y = 13 : 12$  and  $X - Y = 2$ , then what is value of  $2X + 3Y$ ?

- 1. 144
  - 2. 120
  - 3. 124
  - 4. 136

$$\begin{array}{rcl} x = 13a & , & y = 12a \\ = \underline{26} & & = \underline{24} \\ 13a - 12a & = & 2 \\ a & = & 2 \end{array}$$

For three numbers, the ratio of the first and the second number is  $2 : 3$  and that of the second and the third number is  $4 : 5$ . If the sum of the three numbers is  $\underline{140}$ , then what is the second number?

1. 60

2. 48

3. 96

4. 32

$$\begin{aligned} a:b &= 2:3 \\ b:c &= 4:5 \end{aligned}$$
$$a:b:c = 8:12:15$$

$$a+b+c = 140$$

$$8k+12k+15k = 140$$

$$35k = 140$$

$$k = \frac{140}{35}$$

$$b = 12k$$

$$= 12 \times 4 = 48$$

If  $a:b = 2:5$  and  $b:c = 6:7$  then find  $a:b:c$ ?

- a) 30:12:35
- b) 12:30:25
- c) 30:35:12
- d) 12:35:30

If  $a:b=3:4$ ,  $b:c=6:11$  then Find  $a:c$

- a) 9:11
- b) 11:9
- c) 9:22
- d) 12:33

If  $a:b=2:3$ ,  $b:c=4:5$  and  $c:d=7:4$  then find  $a:b:c:d$ ?

a)  $56:84:105:60$

b)  $56:60:105:84$

c)  $84:60:135:56$

d)  $84:56:35:60$

$$a:b = 2:3$$

$$b:c = 4:5$$

$$c:d = 7:4$$

$$\underline{a:b:c:d = 2 \times 4 \times 7 : 3 \times 5 \times 7 : 3 \times 5 \times 4}$$

$$= 56 : 84$$

If  $\underline{a} : \underline{b} = 3 : 7$ ,  $\underline{b} : \underline{c} = 11 : 9$  and  $\underline{c} : \underline{d} = 14 : 13$  find  $\underline{a} : \underline{d}$

a)  $39 : 22$

b)  $33 : 63$

c)  $22 : 52$

d)  $22 : 39$

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{3}{7} \times \frac{11}{9} \times \frac{14}{13}^2$$
$$= \frac{22}{39}$$

$(a+b):c = (11:4)$  ,  $(b+c):a = 26/9$ ,  $(c+a):b = ?$

a) 13:4

b) 11:10

c) 19:16

d) 17:13

The ratio of A and B is 2 : 3 and the ratio of B and C is 4 : 5 . Then find  $A^2 : B^2 : BC$ .

A और B का अनुपात 2 : 3 है तथा B और C का अनुपात 4 : 5 है। तो  $A^2 : B^2 : BC$  का मान क्या होगा?

$$a : (b + c) :: 1 : 3$$

$$c : (a + b) :: 5 : 7$$

$$\text{find } b : (a + c) = ?$$

$a : (b+c) :: 2 : 3$  ↗↖  
 $b : (a+c) :: 4 : 7$  ↘↖  
find  $c : (a+b) = ?$

13: 42 ✓

$$\begin{array}{rcl} a+b+c & = & 5.. \\ a+b+c & = & 11.. \\ \hline \\ a : (b+c) & = & 22 : 33 - 55 \\ b : (a+c) & = & 20 : 35 - 55 \\ \hline \\ a & = & 22 \\ b & = & 20 \\ c & = & 13 \end{array}$$

$b+c = 33$   
 $20+13 = 33$

If  $(a + b) : (b + c) : (c + a) :$   
 $: 4 : 7 : 9$  &  $a + b + c = 30$   
then find  $c = ?$

If  $(a + b) : (b + c) : (c + a) : 4 : 5 : 7$  &  $a + b + c = 24$   
then find  $c = ?$

If (यदि)  $a:b :: 3:4$  &  $b:c :: 4:7$

then (तब)  $\frac{a+b+c}{c} = ?$

c

## Proposition

If  $a, b, c, d$  are in proportion

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b :: c:d$$

$$ad = bc$$

$$ad = bc$$

1) mean proportion

$$x = \sqrt{ab} \quad |$$

Or  $\frac{a}{c} = \frac{b}{x}$   $a:b::c:d$

$$c:x :: x:b$$

$$x^2 = ab$$

$$x = \sqrt{ab}$$

$\frac{a}{c}, \frac{b}{d}$ .

2) Third proportion of

$$x = \frac{b^2}{a} \quad |$$

$$\frac{b^2}{a}$$

$a, b, b, x$   
 $a:b :: b:x$

$$b^2 = ax$$

$$x = \frac{b^2}{a}$$

3) Fourth proportion of a, b, c, x

$$a : b :: c : x$$
$$x = \frac{bc}{a}$$

$$a, b$$

$$x = ab$$

$$x = \frac{b^2}{a}$$

$$x = \frac{bc}{a}$$

Find the mean proportion of 3.6 and 0.9?

3.6 और 0.9 का मध्य समानुपाती संख्या क्या है?

- (a) 0.6 (b) ~~1.8~~ (c) 1.6 (d) 2.7

$$3.6 = \frac{36}{10}$$

a, b

$$3.6 \times \frac{10}{10} = \frac{36}{10}$$

$$x = \sqrt{ab}$$

$$x = \sqrt{3.6 \times 0.9}$$

$$= \sqrt{\frac{36}{10} \times \frac{9}{10}} = \sqrt{\frac{36 \times 9}{100}}$$

$$\sqrt{10^2}$$

$$= \frac{18}{10} : 1.8$$

$$= \frac{6 \times 3}{10}$$

Find out fourth proportion of 9, 10, 18

- (a) 20 (b) 15 (c) 18 (d) 25

$\bar{a} \bar{b} \bar{c}$

$$x = \frac{bc}{a}$$

$$x = \frac{10 \times 18^2}{9} = 20$$

What is the ratio of the mean proportional between 4.5 and 0.5, and the third proportional to 4.5 and 9.0?

X 1.  $3 : 8$

X 2.  $2 : 9$

X 3.  $1 : 6$

✓ 4.  $1 : 12$  ✓

mean:  $\sqrt{ab}$

$= \sqrt{4.5 \times 0.5}$

$= \sqrt{\frac{45}{10} \times \frac{5}{10}}$

$= \sqrt{\frac{225}{10^2}}$

$= \frac{15}{10} = 1.5$

3<sup>rd</sup> prop:  $\frac{b^2}{a}$

$= \frac{9^2}{4.5}$

$= \frac{81}{4.5} \times 10^2$

$= \frac{81}{4.5} \times 100 = 1800 = 18$

$\frac{3}{2} : 1.8$

$\frac{6}{2} : 18 \times 2$

$1 : 12$

mean.P:  $\sqrt{\frac{15^2}{10^2}} = \frac{15}{10} = 3\frac{1}{2}$

What is the ratio of the mean proportional between 24 and 150 and the third proportional between 12 and  $6\sqrt{5}$  ?

1.  $2 : 1$

2.  $1 : 2$

3.  $1 : 4$

4.  $4 : 1$

mean P

$$\begin{aligned} \bar{a} &= \sqrt{ab} \\ &= \sqrt{24 \times 150} \end{aligned}$$

$$= 6 \times 10 \times 15$$

$$= 2 \times 2 \times 3 \times 5$$

$$\text{mean} = 60$$

$$\begin{array}{r} 2 \\ 2 \\ 2 \\ \hline 12 \\ 6 \\ 3 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \\ 5 \\ \hline 150 \\ 75 \\ 25 \\ 5 \end{array}$$

$$\frac{2 \times 2 \times 3 \times 5}{2 \times 2 \times 3 \times 5} = 1$$

$$\begin{aligned} 30d &= \frac{-a}{\bar{a}} \\ &= \frac{(6\sqrt{5})^2}{12} \end{aligned}$$

$$\begin{array}{r} 3 \\ 3 \\ 5 \times 5 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 60 \\ 60 : 15 \\ 1 : 1 \end{array} = 15$$

The fourth proportional to  
12, 21, 8 is:

- (a) 8.9 (b) 56 (c) 14 (d) 17

$$\text{4th prop} = \frac{bc}{a}$$
$$= \frac{21 \times 8^2}{12} = 14$$

What least number must be subtracted from each of the number 21, 38, 55 and 106 so that the new number may be proportional? 21, 38, 55 और 106 के प्रत्येक पद में से कम -से -कम क्या घटाया जाये कि वे समानुपाती हो जाये ?

- (a) 2 ~~X~~    (b) 4    (c) 6    (d) 8

$$\frac{21-x}{23x} = \frac{55-x}{106-2}$$

$$\frac{21-x}{38-x} = \frac{55-x}{106-x}$$

$$\frac{1}{2} = \frac{1}{2} \checkmark$$

What number should be added to each of  $\underline{6}$ ,  $\underline{14}$ ,  $\underline{18}$  and  $\underline{38}$ , so that the resulting numbers make a proportion?

- (a) 1 ✗
- (b) 2
- (c) 3
- (d) 4

$$a : b :: c : d$$
$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{6+x}{14+x} = \frac{18+x}{38+x}$$

$$\frac{8}{26} = \frac{20}{40}$$

$$\frac{1}{2} = \frac{1}{2}$$



The ratio of two number is  $1 : 2$ . If 7 is added in both number then ratio become  $3 : 5$ . Then find these number.

$$\begin{array}{rcl} S & \begin{array}{c} 1:2 \\ \diagup \quad \diagdown \\ 3:5 \end{array} & S \\ C & \hline 17 & +7 \end{array}$$

**Two number are in the ratio of 3 and 5. If 9 is substracted from both then ratio become 12 : 23. Then number is?**

**The income of A and B are in the ratio 5 : 3 and their expenses are in the ratio 9 : 5. If both saves respectively Rs. 1300 and Rs. 900, then find their income.**

The income of A and B are in the ratio 5 : 7 and their expenses are in the ratio 9 : 11. If A saves Rs. 1200 and B saves Rs. 2000 then find their income.

P and Q earn in the ratio 2 : 1. They spend in the ratio 5 : 3 and save in the ratio 4 : 1. Find the monthly income of each if the total monthly savings of both P and Q together is Rs. 5000?

If three numbers are in the ratio of 1:2:3 and half of the sum is 18, then the ratio of squares of the numbers is :

तीन संख्या 1:2:3 के अनुपात में हैं | और योग का आधा राशि 18 है ,तो संख्याओं के वर्गों का अनुपात निकाले ?

- (a) 6:12:13
- (b) 1:2:4
- (c) 36:144:324
- (d) 3:5:7