

# A Novel Mating Approach for Genetic Algorithms

Severino F. Galán\*

Dpto. de Inteligencia Artificial, UNED

Juan del Rosal, 16

28040 Madrid, Spain

E-mail: [seve@dia.uned.es](mailto:seve@dia.uned.es)

Phone: +34 91 3987300

Fax: +34 91 3988895

Ole J. Mengshoel

RIACS, NASA Ames Research Center

Mail Stop 269-3

Moffett Field, CA 94035, USA

E-mail: [omengshoel@riacs.edu](mailto:omengshoel@riacs.edu)

Phone: (650) 604-4199

## Abstract

Genetic algorithms typically use crossover, which relies on pairing or mating a set of selected parents. As part of crossover, random mating is often carried out in which the mate for a parent is chosen uniformly at random from the set of remaining parents. The present work focuses on this mating process in genetic algorithms. A novel approach to parent mating is presented which uses a criterion for mating individuals, constituting the basis for a wide range of different mating strategies. These strategies can be applied in combination with the traditional similarity-based criterion between individuals or with a fitness-based criterion. The novel mating algorithm created in this work uses a parameter called *mating index* that allows different mating strategies to be developed within a uniform framework: from an exploitative strategy called BEST-FIRST to an explorative one called BEST-LAST. SELF-ADAPTIVE mating is a novel mating strategy defined in the context of the novel algorithm, which allows a balance between exploitation and exploration to be obtained in a domain-independent manner. The present work consists of formally defining and implementing the novel mating approach, analyzing its behavior from a theoretical point of view, and developing an extensive experimental study to quantitatively determine the benefits of the different novel mating strategies compared to the traditional random mating approach.

---

\*Corresponding author

In the domain of real function optimization, the experiments show that BEST-FIRST strategy is the best option for unimodal problems, BEST-LAST strategy is the best option for multimodal problems, and SELF-ADAPTIVE strategy produces robust results for both types of problems.

**Keywords:** Genetic algorithms, premature convergence, mating strategies, mating index, self-adaptive mating.

## 1 Introduction

Genetic algorithms (GAs) [10, 9] use stochastic search methods based on natural evolution in order to solve adaptation problems in fields like optimization, design, learning, or scheduling, among others. A complete review of successful GA applications can be found in [1, Chapter 2] and [3, Chapter 6].

In a GA, a set of candidate solutions is created each generation. The quality of a solution, its fitness, determines its chance to survive and reproduce. Two processes form the basis of genetic algorithms: variation (recombination and mutation) and selection. While the former facilitates diversity and novelty, the latter favours quality. Ideally, at the end of a running GA, a solution with optimal or near-optimal fitness is found.

Premature convergence to local optima is one of the most frequent difficulties that arise when applying GAs to complex problems. It occurs when genetic operators can no longer generate offspring that are fitter than their suboptimal parents. Premature convergence is associated with the loss of diversity in the population. However, too much population diversity can lead to a dramatic deterioration of GA efficiency. Therefore, an important issue in the design and application of GAs is the trade-off between exploitation of the best individuals and exploration of alternative regions of the search space.

By focusing on the mating phase of GAs, the present work deals with achieving a proper balance between exploitation and exploration. Traditionally, mating takes place after parent selection and prior to recombination. Normally, parents are mated in pairs so that each pair of parents can subsequently be recombined. A key question is how mating should be carried out in order to improve GA performance. The traditional mating approach consists of selecting a parent's mate uniformly at random from the set of remaining parents. In addition to the traditional random mating approach, other approaches exist that apply mating restriction techniques based on similarity relations between parents [5, 7]. Although these methods have been shown to benefit GA performance, they are costly in computational terms. This disadvantage is due to the fact that similarity comparisons between two parents' chromosomes normally take place at a gene level. Furthermore, these methods were designed for rather specific contexts like *fitness sharing* [5] and *incest prevention* [7] and, therefore, their impact has been quite limited.

The goal of this work is to develop, analyze, and evaluate a novel and general approach to mating in GAs. By using fitness-based comparisons between

parents, rather than just similarity-based comparisons, the problem of complexity is overcome. Furthermore, the novel approach lends itself to a self-adaptive algorithm which gives rise to a simple and general mating strategy.

The rest of this paper is structured as follows. Section 2 reviews previous work on restricted mating in GAs. Section 3 introduces our novel approach to mating, along with the different mating strategies derived from it. The novel approach is illustrated in Section 4 through an example. Section 5 contains a formal analysis of the mating approach introduced in this work. Section 6 includes an extensive empirical evaluation of the novel mating strategies. A discussion of our approach to mating in GAs is made in Section 7. Finally, Section 8 contains the main conclusions derived from the present work.

## 2 Restricted Mating in Genetic Algorithms

The usual way of mating parents in GAs consists of taking a parent from the mating pool and selecting its mate by choosing uniformly at random one of the remaining parents. The mated parents are then removed from the mating pool, and the same process is repeated until all the individuals have been mated. Restricted mating techniques, which do not select a mate uniformly at random, have been successfully developed for specific contexts such as *fitness sharing* and *incest prevention*. Other approaches that incorporate mating preferences into evolutionary systems are: *assortative mating genetic algorithms* [8, 11, 15], *correlative tournament selection* [14], *seduction* [16], *tabu genetic algorithm* [21], and *evolving agents* [19, 18, 22].

Fitness sharing [5] is a method that forces the population to maintain different niches. In multimodal optimization problems, where a number of high-fitness individuals corresponding to various local optima are identified, niches are search space regions of high fitness. Fitness sharing adjusts fitnesses of individuals prior to parent selection, so that individuals are allocated to niches in proportion to the niches fitness. In order to improve the efficiency of fitness sharing, Deb and Goldberg [5] used a restricted mating approach whose goal was to avoid the creation of lethal (low fitness) individuals. Once niches are formed in the population, the recombination of two parents from different niches is likely to form lethal offspring. Therefore, restricted mating among individuals of the same niche is promoted. This is achieved by following the same scheme as random mating but, given a parent, a candidate mate is accepted only if the phenotype/genotype distance between them is smaller than a given threshold. Otherwise, another candidate is sought. If no candidate is accepted, one is chosen uniformly at random as in random mating. In the case of real functions optimization, the phenotype space corresponds to the real values of the variables, while the genotype space uses a binary representation for them. If similarity is measured within the phenotypic space, Euclidean distance is used. Hamming distance is employed when similarity between individuals is measured within the genotypic space. Other GA approaches applying restricted mating in the specific context of multimodal optimization problems are: *island models*

[13], *diffusion models* [12], and *automatic speciation models* [20].

In contrast to fitness sharing, incest prevention [7] was defined in the context of global optimization rather than niching. Incest prevention promotes restricted mating between dissimilar enough individuals. In general, when two similar individuals are mated, their offspring may not introduce significant new information about the search space, which provokes a reduction in the performance of the GA. Incest prevention follows a dual scheme to that used in fitness sharing: A candidate mate is accepted only if its phenotype/genotype distance to the current parent is greater than a given threshold. Usually, this threshold is reduced when better offspring is not obtained during the search process.

In comparison to random mating, similarity-based restricted mating was shown to produce a more effective exploration of the search space both in fitness sharing [5] and in incest prevention[7]. However, the time cost associated with measuring the distances between individuals is an important disadvantage of both these similarity-based restricted mating techniques. This work explores fitness-based mating as an alternative for establishing mating preferences with a lower computational cost. Although fitness-based restricted mating was addressed in [4, 2], this technique has not been sufficiently investigated in the past, due to the widespread use of similarity in the definition of mating approaches. One of the goals of this work is to thoroughly compare fitness-based mating strategies with their similarity-based counterparts.

The present work aims at formalizing a general mating approach which allows a wide range of mating strategies to be defined and effectively applied to the task of global optimization in GAs. Finally, a self-adaptive mating method is developed.

### 3 The New Mating Approach

This section introduces a novel approach to mating in GAs. Our novel approach has two main characteristics. Firstly, it allows mating preferences to be defined either in terms of similarity between individuals or in terms of fitness of individuals, in contrast to most of the mating strategies reviewed in Section 2, which are typically based on similarity between individuals. Secondly, the novel approach lends itself to a self-adaptive implementation in which each individual in the population has its own mating preference. In this way, different mating strategies can be applied depending on the hardness of the fitness function and the current state of the search process.

The novel approach is defined by the following algorithm, which constitutes a GA mating phase, taking place between parent selection and parent recombination.

**Algorithm 1** (*Mating step*). *Novel mating approach for GAs:*

*Input:*

$P_s$     *Population of selected parents*  
 $\gamma$     *Mating Size: number of eligible parents for next mating*

- $cr$  Criterion used for defining mating preferences  
( $cr \in \{\textit{similarity}, \textit{fitness}\}$ )
- $\alpha$  Mating Index: integer used for defining a mating preference  
( $2 \leq \alpha \leq \gamma$ )

Output:

$P_m$  Population of mated parents

1. Choose  $\gamma$  parents uniformly at random from  $P_s$  without replacement. The set of chosen parents is denoted by  $Ch$ .
2. Let  $p_1$  be the parent in  $Ch$  with highest fitness. Remove  $p_1$  from  $P_s$  and  $Ch$ , and include it in  $P_m$ . Set  $Ch$  now includes the candidate mates for  $p_1$ .
3. Order the  $\alpha - 1$  best candidates in  $Ch$  under criterion  $cr$ . When  $cr = \textit{similarity}$ , candidates are ranked according to phenotype similarity with  $p_1$ . When  $cr = \textit{fitness}$ , candidates with higher fitness are ranked first. Let  $B_\alpha(p_1) = \{b_2, b_3, \dots, b_{\alpha-1}, b_\alpha\}$  be the set of ordered best candidates obtained under criterion  $cr$ .
4. Choose  $p_2 = b_\alpha$  as mate for  $p_1$ , remove  $p_2$  from  $P_s$ , and include  $p_2$  in  $P_m$ . It should be noted that  $p_1$  and  $p_2$  are placed in contiguous positions in  $P_m$ .
5. Go back to step 1 if  $P_s$  is not empty yet.

In Algorithm 1,  $\gamma$  (mating size) different parents are randomly chosen for the next round of mating, and the fittest of them is mated with another individual as determined by  $cr$  (mating criterion) and  $\alpha$  (mating index). Similarity in the phenotype space is the traditional criterion used to establish mating preferences in GAs [5, 7]. This is why *similarity* has been included in the domain of  $cr$  in Algorithm 1. Due to the computational complexity of similarity comparisons, a new fitness-based criterion for establishing mating preferences is also introduced. While determining the similarity of two individuals requires examining their chromosomes gene by gene, comparing their fitnesses involves examining only two numbers.

Note that the novel mating approach defined by Algorithm 1 becomes the traditional mating approach when  $\gamma = 2$ , since parents are then mated uniformly at random. Therefore, the novel mating approach is a generalization of the traditional approach. The values of mating index  $\alpha$  induce different mating strategies corresponding to a wide range of degrees of exploitation versus exploration.

Several variants of the general scheme in Algorithm 1 are possible. This work focuses on three of them:

1. When  $\alpha = 2$ , the best parent is mated with the *first* mating candidate under criterion  $cr$ . Thus, the resulting scheme is called BEST-FIRST mating strategy.

2. When  $\alpha = \gamma$ , the best parent is mated with the *last* mating candidate under criterion  $cr$ . This strategy is called BEST-LAST mating.
3. When the parameter  $\alpha$  is made local to each individual, encoded into the chromosome, and subjected to recombination and mutation, a SELF-ADAPTIVE mating strategy results.

In the rest of this section, these variants are discussed in more detail.

### 3.1 Best-First Mating

Exploitation of the best solutions in the current population can be achieved by setting  $\alpha = 2$  in Algorithm 1. In this way, the fittest of the chosen parents,  $p_1$ , is mated with the first of the candidates under criterion  $cr$ . If a fitness-based criterion is used,  $p_1$ 's mating preference is clearly an exploitative strategy, since fitter candidates are preferred over the rest. If a similarity-based criterion is used,  $p_1$ 's mating preference is exploitative as well, since it is implicitly assumed that fitter candidates are more similar to  $p_1$  than the rest.

In step 3 of Algorithm 1, an ordering of the first  $\alpha - 1$  candidates in  $Ch$  under criterion  $cr$  is in fact not necessary in BEST-FIRST mating. Just the first of the candidates under criterion  $cr$  is sought. Thus, only a variable storing the currently first candidate is needed to implement BEST-FIRST.

BEST-FIRST mating with a similarity-based criterion is inspired by the mating strategy used by Deb and Goldberg [5] in the context of fitness sharing. Whereas Deb and Goldberg used a similarity threshold to guide the mating process within niches, BEST-FIRST mating employs a mating size parameter in order to obtain a certain degree of exploitation. Similarity-based BEST-FIRST mating is also similar to *positive assortative mating* [8, 11, 15], which chooses the most similar candidate as mate for the current individual. On the other hand, BEST-FIRST mating with a fitness-based criterion has common characteristics with some of the mating methods developed in [4, 2].

### 3.2 Best-Last Mating

Exploration of alternative solutions to the best ones in the current population can be performed by setting  $\alpha = \gamma$  in Algorithm 1. By doing that, the fittest of the chosen parents,  $p_1$ , is mated with the last of the candidates under criterion  $cr$ . If a fitness-based criterion is used,  $p_1$ 's mating preference is clearly an explorative strategy, since the fittest parent prefers less fit candidates over the rest. If a similarity-based criterion is used,  $p_1$ 's mating preference is explorative as well, since the most distant candidate in the phenotype space is chosen for mating.

In step 3 of Algorithm 1, an ordering of the first  $\alpha - 1$  candidates in  $Ch$  under criterion  $cr$  is not necessary in BEST-LAST mating. Only the last of the candidates under such a criterion is sought. Therefore, a unique variable storing the currently last candidate is needed to implement BEST-LAST.

BEST-LAST mating with a similarity-based criterion is inspired by the mating strategy used by Eshelman and Schaffer [7] for incest prevention. While Eshelman and Schachter used a similarity threshold to prevent incest, BEST-LAST mating achieves a particular degree of exploration by setting the mating size value. Similarity-based BEST-LAST mating is also similar to *negative assortative mating* [8, 11, 15], which chooses the most dissimilar candidate as mate for the current individual. On the other hand, to the best of our knowledge BEST-LAST mating with a fitness-based criterion has not yet been investigated in the literature.

### 3.3 Self-Adaptive Mating

Properly setting the parameters values has a great influence on GA performance. Parameters can be either manually tuned in advance or automatically controlled during execution. While manual parameter tuning is usually a time-consuming task, automatic parameter control has the advantage that parameters can be adapted to the state of the search process. A classification of parameter setting techniques for evolutionary algorithms can be found in [6]. This section deals with self-adaptive control of mating parameters. Self-adaptive parameter control consists of encoding the parameters into the chromosomes and performing recombination and mutation on them. In this way, the values of the parameters leading to better individuals will have a greater chance to survive.

If an individual  $j$  is represented as  $\langle x_{j,1}, \dots, x_{j,n} \rangle$ , its extended representation under SELF-ADAPTIVE mating would be  $\langle x_{j,1}, \dots, x_{j,n}, x_{j,n+1} \rangle$ , where  $x_{j,n+1} = \alpha_j$  is the mating index for individual  $j$ . In other words, the mating index is now a local parameter, and each individual has an independent mating preference. The algorithm performing SELF-ADAPTIVE mating can easily be obtained from Algorithm 1 by removing  $\alpha$  from the input and substituting  $\alpha - 1$  with  $x_{p_1,n+1} - 1$  in step 3.

It remains to consider how mating indexes are initialized, recombined, and mutated. As far as initialization is concerned, each mating index is assigned an integer generated uniformly at random from range  $[2, \gamma]$ . Recombination of the mating indexes of two parents can be carried out in several ways: by assigning to the two children the mean of the parents' mating indexes, or by letting the two children inherit the parents' mating indexes, among other possibilities. This work uses the latter method, since we have found it to produce better experimental results. Mutation of mating indexes is implemented by setting a probability  $p_+$  that mating index is incremented by one, a probability  $p_-$  that mating index is decremented by one, and a probability  $1 - p_+ - p_-$  that mating index is changed uniformly at random. Values  $p_+ = p_- \lesssim 0.5$  were employed, since they led to better performance in the experiments.

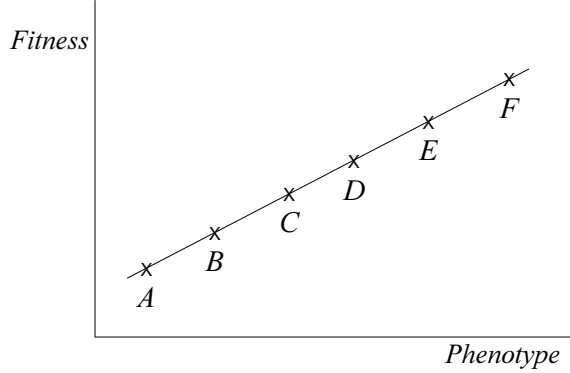


Figure 1: Population of six parents  $\{A, B, C, D, E, F\}$  to be mated.

## 4 An Example of the Novel Mating Strategies

This section presents an example illustrating the novel mating strategies described in Section 3: BEST-FIRST mating, BEST-LAST mating, and SELF-ADAPTIVE mating.

Consider a population of six selected parents,  $P_s = \{A, B, C, D, E, F\}$ , resulting after parent selection in a GA. The six parents have to be mated before recombination. Figure 1 depicts the parents according to their phenotype ( $x$ -axis) and their fitness ( $y$ -axis), where it is assumed that there is a bijection between phenotypes of individuals and a certain discretized interval of real numbers. In order to make the example independent of the criterion used for defining mating preferences (similarity-based or fitness-based), a linear fitness function is assumed in Figure 1 for parents  $\{A, B, C, D, E, F\}$ .

The random mating strategy mates parents by choosing a mate uniformly at random among the remaining parents. A possible mating resulting from this strategy is  $\{CF, EA, BD\}$ . It is important to note that fitness or similarity information is not used at any step of random mating.

If a mating size  $\gamma = 6$  is assumed for simplicity, the BEST-FIRST, BEST-LAST, and SELF-ADAPTIVE mating strategies create  $Ch = \{A, B, C, D, E, F\}$ . The first mate,  $p_1$ , is the parent with highest fitness in  $Ch$ ;  $p_1 = F$  in this case. In BEST-FIRST, the second mate,  $p_2$ , is the first of the candidates in  $\{A, B, C, D, E\}$  under criterion  $cr$ ;  $p_2 = E$  in this case. As a result,  $F$  and  $E$  are mated, and the same process continues until all of the parents have been mated. A new set  $Ch$  would be formed prior to each pairing between  $p_1$  and  $p_2$ . Ultimately, the mating resulting from this strategy is  $\{FE, DC, BA\}$ .

The BEST-LAST mating strategy works analogously to BEST-FIRST, but now the last of the candidates under criterion  $cr$  is assigned to  $p_2$  at each iteration of the mating algorithm. In this way,  $F$  and  $A$  are first mated. In the end, the mating resulting from this strategy is  $\{FA, EB, DC\}$ .



The SELF-ADAPTIVE mating is now considered. The following mating indexes will be assumed:  $\{\alpha_A = 5, \alpha_B = 2, \alpha_C = 3, \alpha_D = 3, \alpha_E = 4, \alpha_F = 2\}$ . The first mate,  $p_1$ , is again the parent with highest fitness in  $Ch$ ;  $p_1 = F$  in this case. The second mate,  $p_2$ , is the  $(\alpha_F - 1)$ -th candidate in  $\{A, B, C, D, E\}$  under criterion  $cr$ ;  $p_2 = E$  in this case. As a result,  $F$  and  $E$  are mated, and the same process continues until all of the parents have been mated. A new set  $Ch$  is formed prior to each pairing between  $p_1$  and  $p_2$ . When the number of candidates for  $p_1$  in  $Ch$  is smaller than  $\alpha_{p_1}$ , the last element in  $Ch$  under criterion  $cr$  is selected as mate for  $p_1$ . The mating resulting from this strategy is  $\{FE, DB, CA\}$ .

It should be noted that BEST-FIRST produces the best potential mating for the simple fitness function in Figure 1, since mating parents with high fitness will favor the creation of children with high fitness with higher probability. However, in more realistic and interesting fitness functions, it is clear that BEST-FIRST is not always optimal, as Section 6 shows.

## 5 Analysis

This section analyzes the novel mating approach introduced in this work by making use of an idealized model. The analysis is developed for two main types of domains: on the one hand, unimodal fitness functions with a unique optimum and, on the other hand, multimodal fitness functions with many local optima. In both cases, the influence of mating on the effectiveness to reach the global optimum is studied. For simplicity, this section only deals with similarity-based mating. As in Section 4, it will be assumed that there is a bijection between phenotypes of individuals and a certain discretized interval of real numbers.

### 5.1 Analysis for Unimodal Fitness Functions

Consider the maximization problem of the linear fitness function depicted in Figure 2,  $f(x) = x$ , where  $x$  represents individuals' phenotype defined in the range  $[x_1, x_2]$  with  $x_1 < x_2$ . For any other unimodal fitness function, an analogous argument to the following one could be made.

Let  $x' \in (x_1, x_2)$  (see Figure 2) denote the fittest individual within set  $Ch$  of Algorithm 1; therefore,  $x'$  is the next individual to be assigned a mate  $x''$ . Candidate mates for  $x'$  can only belong to range  $[x_1, x']$ , since  $f(x') \geq f(x'')$  from Algorithm 1.

Assume that the recombination of two parents,  $x_{p_1} \in [x_1, x_2]$  and  $x_{p_2} \in [x_1, x_2]$  with  $x_{p_1} < x_{p_2}$ , is performed in the phenotype space and produces children,  $x_{\text{child}(x_{p_1}, x_{p_2})}$ , uniformly distributed over  $[x_{p_1}, x_{p_2}]$ . Consequently, the expected fitness of a child of  $x_{p_1}$  and  $x_{p_2}$  can be calculated as:

$$\bar{f}(x_{\text{child}(x_{p_1}, x_{p_2})}) = \frac{f(x_{p_1}) + f(x_{p_2})}{2} = \frac{x_{p_1} + x_{p_2}}{2}.$$

Likewise, the expected fitness of a child resulting from the recombination of  $x'$

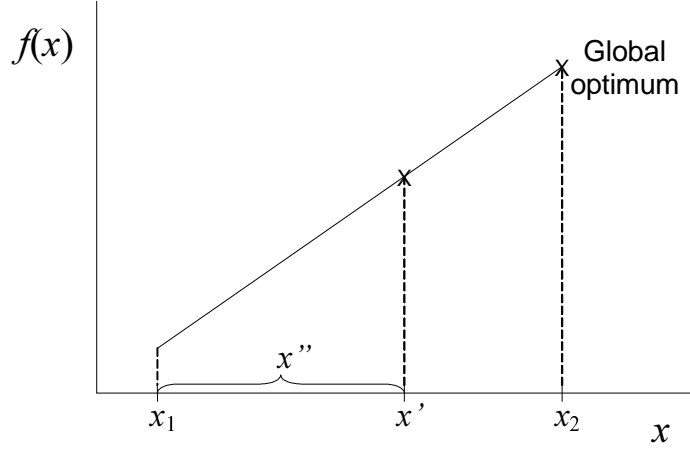


Figure 2: A linear unimodal fitness function with no local optima except for the global optimum.

and  $x''$ , with  $x'' \in [x_1, x']$ , is:

$$\bar{f}(x_{\text{child}(x', x'')}) = \frac{x' + x''}{2},$$

which increases as  $x''$  approaches  $x'$  and has a maximum at  $x'' = x'$ .

It can be concluded from the previous result that, in the case of unimodal fitness functions, those mating strategies under the novel approach favoring recombination with similar individuals produce a more effective search for the global optimum.

## 5.2 Analysis for Multimodal Fitness Functions

Consider the maximization problem of a multimodal fitness function,  $g(x)$ , where  $x$  represents individuals' phenotype defined in the range  $[x_1, x_2]$  with  $x_1 < x_2$ , as depicted in Figure 3. Function  $g(x)$  has a global optimum at  $x_{\text{op}}$ , whose basin of attraction lies in the range  $[x_{\text{op}} - b, x_{\text{op}} + b]$ .

Without loss of generality, consider that individual  $x' = x_1$  is to be assigned a mate  $x''$ . Function  $g(x)$  reaches several local optima between  $x'$  and  $x_{\text{op}} - b$ . Contrarily to Section 5.1, individual  $x'$  has been chosen outside the basin of attraction of the global optimum, which is the usual situation for the population individuals when a multimodal fitness function is optimized. The argument in Section 5.1 can be applied to individuals only in the basin of attraction of the global optimum.

As in Section 5.1, assume that the recombination of two parents,  $x_{p_1} \in [x_1, x_2]$  and  $x_{p_2} \in [x_1, x_2]$  with  $x_{p_1} < x_{p_2}$ , is performed in the phenotype space

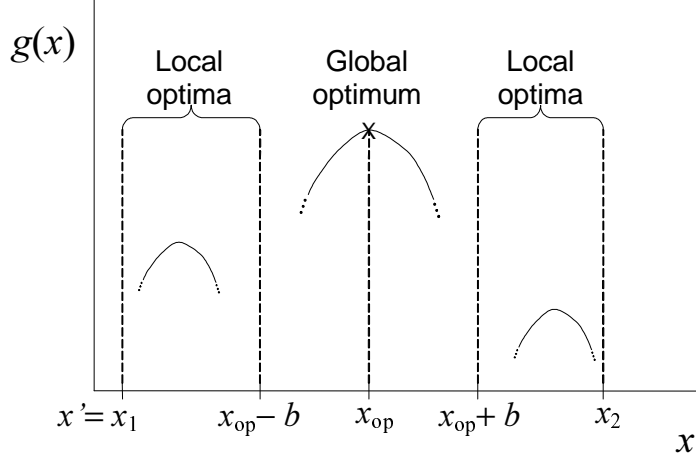


Figure 3: A multimodal fitness function.

and produces children,  $x_{\text{child}(x_{p1}, x_{p2})}$ , uniformly distributed over  $[x_{p1}, x_{p2}]$ . Since the problem of global optimization is to be solved, it is convenient to calculate the probability of reaching the basin of attraction of the global optimum by recombining  $x'$  with a candidate mate  $x'' \in [x_1, x_2]$ . Such a probability is zero for  $x'' \in [x_1, x_{\text{op}} - b)$ , since  $[x', x''] \cap [x_{\text{op}} - b, x_{\text{op}} + b] = \emptyset$ . For  $x'' \in [x_{\text{op}} - b, x_{\text{op}} + b]$ , the probability that the child reaches  $[x_{\text{op}} - b, x_{\text{op}} + b]$  is equal to  $\frac{x'' - (x_{\text{op}} - b)}{x'' - x'}$ . Finally, for  $x'' \in (x_{\text{op}} + b, x_2]$ , the probability is equal to  $\frac{2b}{x'' - x'}$ , since  $[x', x''] \cap [x_{\text{op}} - b, x_{\text{op}} + b] = [x_{\text{op}} - b, x_{\text{op}} + b]$ . In summary, as shown in Figure 4:

$$Pr(x = x'') = \begin{cases} 0 & \text{if } x'' \in [x_1, x_{\text{op}} - b) \\ \frac{x'' - (x_{\text{op}} - b)}{x'' - x'} & \text{if } x'' \in [x_{\text{op}} - b, x_{\text{op}} + b] \\ \frac{2b}{x'' - x'} & \text{if } x'' \in (x_{\text{op}} + b, x_2] \end{cases} .$$

It can be concluded from Figure 4 that, in the case of multimodal fitness functions and individuals outside the basin of attraction of the global optimum, those mating strategies under the novel approach favoring recombination with dissimilar individuals produce a more effective search for the global optimum. While candidate mates in  $[x_1, x_{\text{op}} - b)$  lead to local optima, there is always a probability greater than zero that the basin of attraction of the global optimum is reached if candidate mates are taken from  $[x_{\text{op}} - b, x_2]$ .

## 6 Experiments

This section contains a comparative evaluation of the following mating strategies: random mating, BEST-FIRST mating, BEST-LAST mating, and SELF-

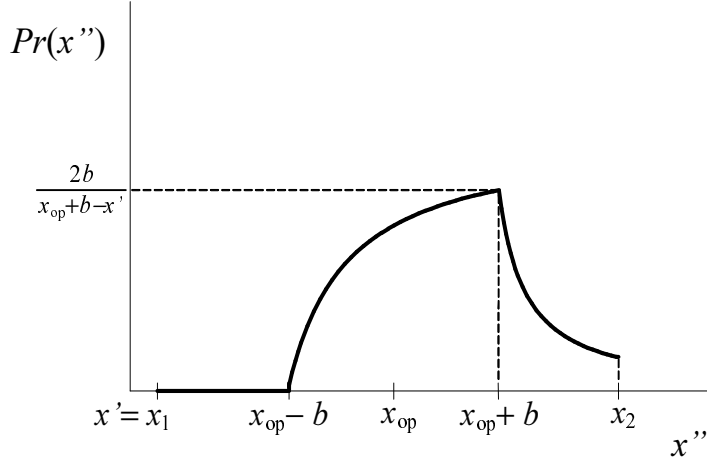


Figure 4: Probability that a child of  $x'$  and  $x''$  reaches the basin of attraction of the global optimum  $[x_{\text{op}} - b, x_{\text{op}} + b]$ .

ADAPTIVE mating. In general, BEST-FIRST mating strategy produces exploitation of the best solutions in the current population, BEST-LAST mating produces exploration of alternative solutions to the best ones in the current population, and SELF-ADAPTIVE mating produces a combination of exploration and exploitation that depends on the shape of the fitness function and the state of the search process.

The domain studied in the experiments is optimization of real functions. Given an  $n$ -dimensional function,  $f: \mathbb{R}^n \rightarrow \mathbb{R}$ , global optimization consists of determining  $x^* \in \mathbb{R}^n$  such that  $f(x^*) \geq f(x) \forall x \neq x^*$  with  $x \in \mathbb{R}^n$ . This definition corresponds to a maximization problem. In the case of minimization, the inequality to be considered is  $f(x^*) \leq f(x)$ . In this section, a discretized real interval is considered for each dimension of the function domain. Each interval point is encoded as a binary string by using a Gray code.

The experiments were performed by means of a simple GA using tournament parent selection with tournament size equal to two, one-point crossover with crossover probability equal to one, bit-flip mutation with mutation probability equal to the inverse of the chromosome length, generational survivor selection, and elitism for the best individual. Different seeds for the random number generator were used for each run of the simple GA. All of the experiments were carried out on a Pentium M processor 760 (2 GHz) running Windows.

The rest of this section is structured so that the following comparisons are progressively made:

- (a) *unimodal* vs. *multimodal* fitness functions: Two different types of functions were tested, namely Sphere function in Section 6.1 and Schwefel 7 function in Section 6.2. While Sphere is a unimodal function with just one

local optimum (the global optimum), Schwefel 7 is a multimodal function that contains a high number of local optima.

- (b) *fitness-based* vs. *similarity-based* mating preferences: Both cases are explored in Section 6.1 and Section 6.2.
- (c) *traditional* (random) vs. *advanced* (BEST-FIRST, BEST-LAST, and SELF-ADAPTIVE) mating strategies
- (d) varying *mating sizes*: The range of explored values is  $\gamma \in \{3, 5, 10, 20, 30\}$ .

## 6.1 Experiments for Sphere function

The Sphere function is defined as follows:

$$f_1(x) = \sum_{i=1}^n x_i^2.$$

This function has a minimum at  $(x_1 = 0, \dots, x_n = 0)$  whose value is 0. The experiments were designed for  $n = 20$ ,  $-10 \leq x_i \leq 10 \forall i \in \{1, \dots, 20\}$ , and 10 bits were used to represent each variable; as a result, chromosomes with 200 genes were employed. The population size selected was 100 individuals.

Figure 5 represents the evolution, generation by generation, of the mean best fitness for Sphere function when fitness-based mating strategies are used. One hundred runs were carried out for each experiment. Whereas the random mating strategy is depicted in the three graphs of Figure 5 for illustrative purposes, the rest of the mating strategies (BEST-FIRST, BEST-LAST, and SELF-ADAPTIVE) are depicted in just one graph. For these three advanced strategies, experiments were performed for different mating size values:  $\gamma \in \{3, 5, 10, 20, 30\}$ .

The BEST-FIRST mating strategy performs better than the random strategy, as shown in Figure 5(a). In general, the performance improvement obtained by BEST-FIRST increases with the mating size. This behavior can also be observed in Figure 5(c) for the SELF-ADAPTIVE strategy, although it takes mating size  $\gamma = 10$  to begin to see an improvement over the random strategy. From Figure 5(b), it is clear that the BEST-LAST mating strategy performs worse than the traditional random strategy in the case of Sphere function. This behavior gets worse as mating size increases.

Figure 6 shows the evolution of the mean best fitness for the Sphere function when similarity-based mating strategies are utilized. In general, the results for similarity-based mating follow a similar pattern to those in Figure 5 for fitness-based mating. However, in the case of similarity-based BEST-FIRST strategy, more generations are needed in order to outperform random mating, as shown in Figure 6(a). This is a disadvantage of similarity-based BEST-FIRST compared to fitness-based BEST-FIRST in the case of Sphere function.

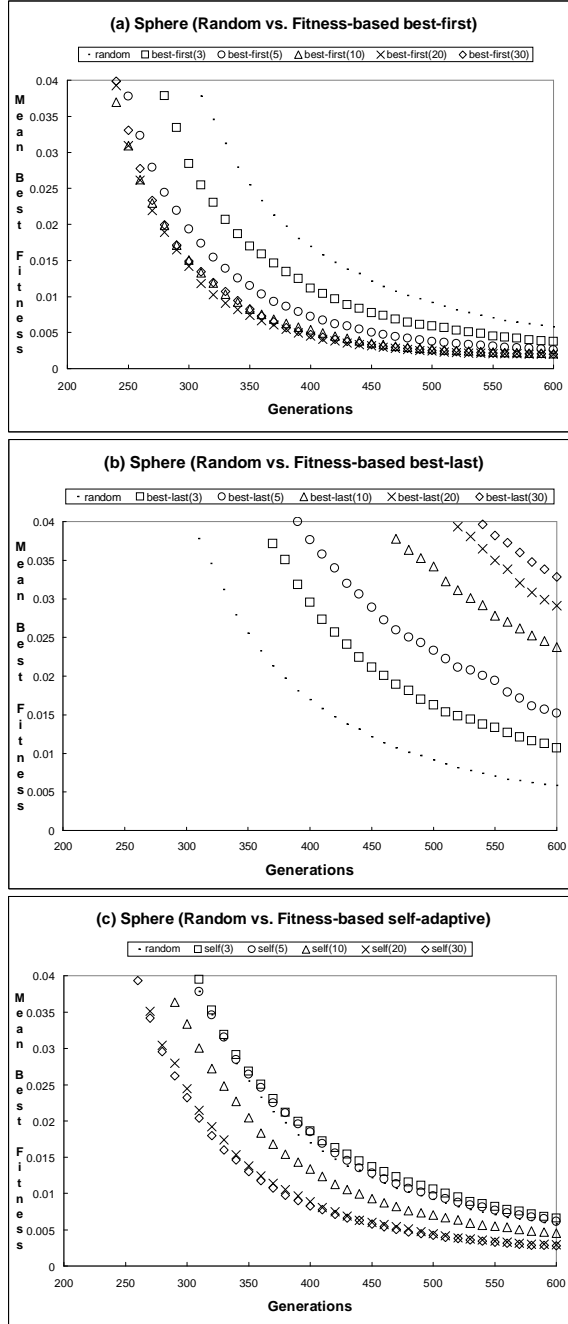


Figure 5: Mean best fitness results for the Sphere minimization problem under fitness-based random, BEST-FIRST (a), BEST-LAST (b), and SELF-ADAPTIVE (c) mating strategies. For the advanced strategies, mating size is varied from 3 to 30.

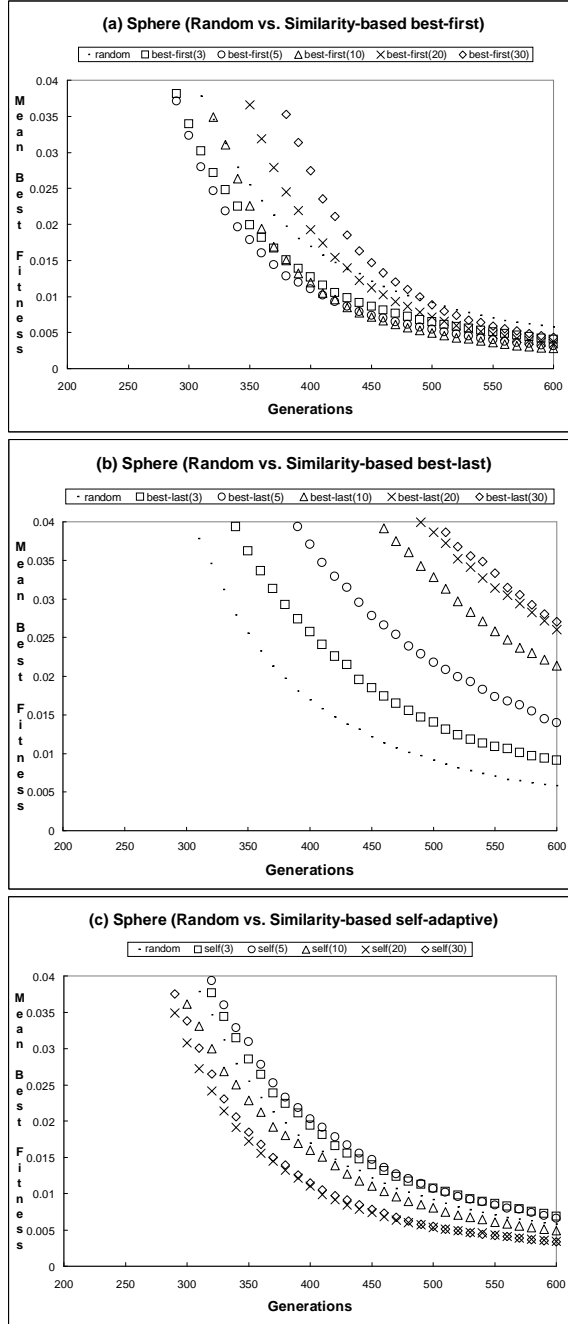


Figure 6: Mean best fitness results for the Sphere minimization problem under similarity-based random, BEST-FIRST (a), BEST-LAST (b), and SELF-ADAPTIVE (c) mating strategies. For the advanced strategies, mating size is varied from 3 to 30.

## 6.2 Experiments for Schwefel 7 Function

The Schwefel 7 function [17, pages 292-293] is defined in the following way:

$$f_2(x) = \sum_{i=1}^n x_i \cdot \sin\left(\sqrt{|x_i|}\right).$$

This function has a maximum at  $(x_1 = 420.9687, \dots, x_n = 420.9687)$  whose value is  $n \cdot 418.9829$ . The experiments were designed for  $n = 10$ ,  $-500 \leq x_i \leq 500 \forall i \in \{1, \dots, 10\}$ , and 100 bits were used to represent each variable; consequently, chromosomes with 1000 genes were created. The population size was 100 individuals. Due to the complexity of Schwefel 7 function, five hundred runs were performed for each experiment.

Figure 7 depicts the evolution of the mean best fitness for the Schwefel 7 function and fitness-based mating strategies. The opposite performance to that of the Sphere function is to some extent obtained for BEST-LAST and BEST-FIRST versus random mating. Firstly, Figure 7(b) shows that BEST-LAST mating strategy performs better than the random strategy in the case of Schwefel 7 function. In general, the performance improvement obtained by BEST-LAST increases with the mating size. Secondly, BEST-FIRST mating strategy performs worse than the random strategy, as shown in Figure 7(a). This behavior gets worse as mating size increases. On the other hand, contrarily to the case of Sphere function shown in Figure 5(c), fitness-based SELF-ADAPTIVE mating for Schwefel 7 function behaves worse as mating size grows. As depicted in Figure 7(c), although  $\gamma \in \{3, 5\}$  outperforms random mating, that is not the case for  $\gamma \in \{10, 20, 30\}$ .

Figure 8 contains the evolution of the mean best fitness for Schwefel 7 function when similarity-based mating strategies are used. In general, the results for similarity-based mating follow a similar pattern to those in Figure 7 for fitness-based mating. However, in the case of similarity-based strategies, the outperformances of both BEST-LAST and SELF-ADAPTIVE strategies with respect to random strategy are superior to those obtained from their fitness-based counterparts. This represents an advantage of similarity-based strategies over fitness-based strategies in the case of Schwefel 7 function.

## 7 Discussion

The experimental results obtained in Section 6 suggest that BEST-FIRST mating is the best option for unimodal problems, while BEST-LAST mating is the best option for highly multimodal problems. When the degree of multimodality is unknown, the SELF-ADAPTIVE mating approach is a good option offering robust results. In unimodal problems, SELF-ADAPTIVE mating clearly outperforms random mating as  $\gamma$  increases. In multimodal problems, SELF-ADAPTIVE mating produces better results than random mating for middle and low  $\gamma$  values. For mating size values in the range  $\gamma \in [3, 10]$ , SELF-ADAPTIVE mating performs



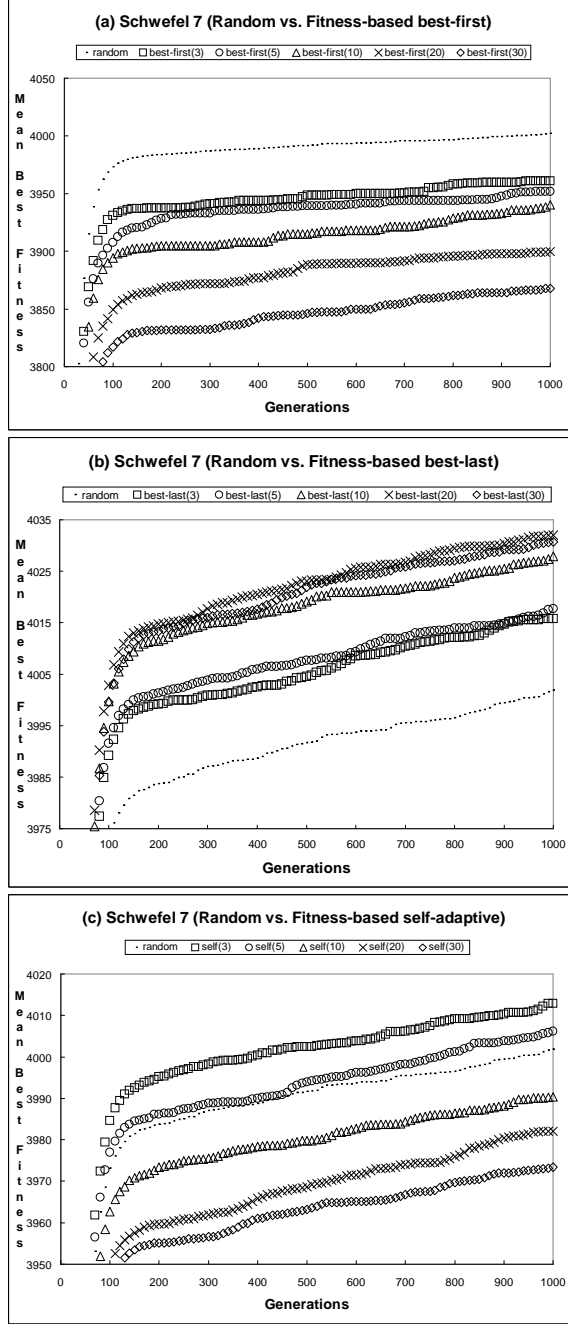


Figure 7: Mean best fitness results for the Schwefel 7 maximization problem under fitness-based random, BEST-FIRST (a), BEST-LAST (b), and SELF-ADAPTIVE (c) mating strategies. For the advanced strategies, mating size is varied from 3 to 30.

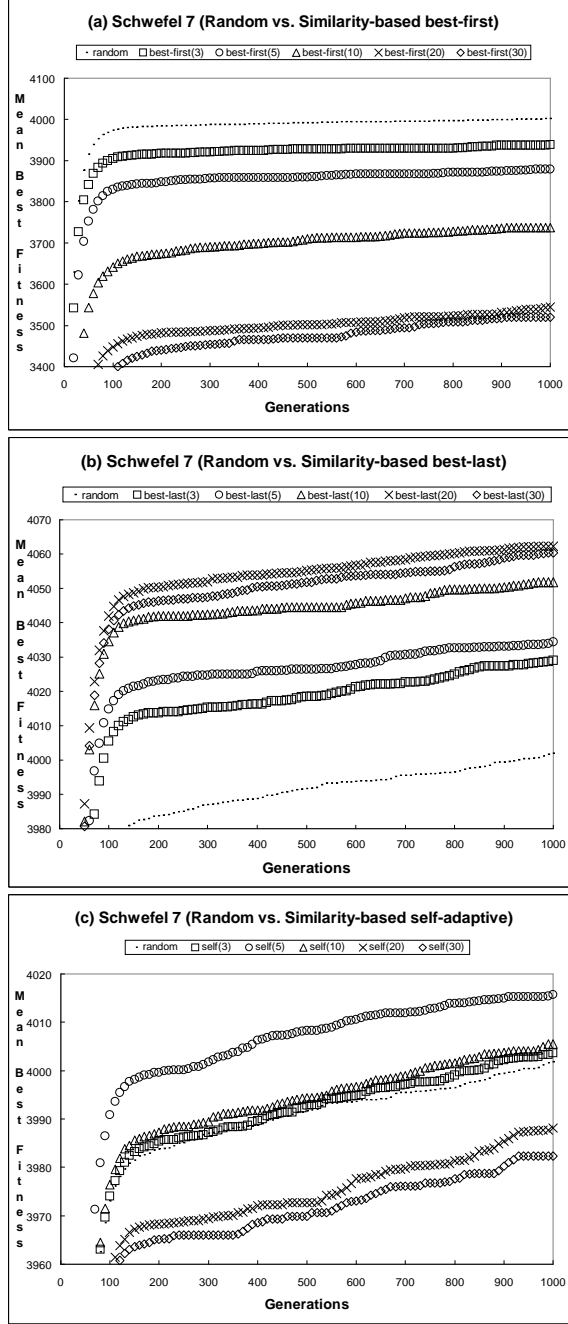


Figure 8: Mean best fitness results for the Schwefel 7 maximization problem under similarity-based random, BEST-FIRST (a), BEST-LAST (b), and SELF-ADAPTIVE (c) mating strategies. For the advanced strategies, mating size is varied from 3 to 30.

better than (or at least comparably to) random mating over the experiments in Section 6.

An example of how SELF-ADAPTIVE mating works is shown in Figure 9. This figure depicts the mean  $\alpha$  value in the population, generation by generation, for the two cases in which SELF-ADAPTIVE strategy had the best behavior in Section 6: fitness-based mating for Sphere function and similarity-based mating for Schwefel 7 function. The results are averaged over one hundred runs for Sphere function and over five hundred runs for Schwefel 7 function. In both cases, the mating size was assigned value  $\gamma = 20$ . Only the results for the first one hundred generations are depicted, since this is the range in which significant changes are obtained for the mean mating index. From Figure 9, the population mean mating index value in the initial population (approximately  $\alpha = 11$ ) decreases rapidly for Sphere function. This is in accordance with the experimental results obtained in Section 6.1, which showed that BEST-FIRST (where  $\alpha$  is small) produces the best results for Sphere function. On the other hand, the mean mating index for Schwefel 7 function is greater than that for Sphere function throughout the generations. This is what should be expected taking into account that, from Section 6.2, BEST-LAST (where  $\alpha$  is large) produces the best results for Schwefel 7 function. However, due to the fact that individuals with high mating index may produce lethal individuals after recombination in the case of Schwefel 7 function, the graph for this function is not as close to  $\alpha = 20$  as the graph for Sphere function is to  $\alpha = 2$ . This explains the results obtained in Section 6, in which SELF-ADAPTIVE mating applied to Sphere function produced similar results to those obtained through BEST-FIRST strategy, while SELF-ADAPTIVE mating applied to Schwefel 7 function could not reach the good results produced by BEST-LAST strategy.

While fitness-based strategies produce better results than similarity-based strategies for unimodal problems like Sphere function optimization, similarity-based strategies outperform fitness-based strategies in the case of multimodal problems like Schwefel 7 function optimization. However, an important advantage of fitness-based strategies is that they lead to substantial computation time savings as shown in Table 1.

## 8 Conclusion

Most of the existing approaches to mating in GAs apply restrictions based on similarity between individuals. The novel mating approach introduced in this work considers also an alternative fitness-based criterion for defining mating strategies, which is compared to the widespread similarity-based criterion. The fitness-based criterion offers important advantages regarding computation time savings and, in cases like unimodal function optimization, greater efficiency to approach the optimum in fewer generations.

An important group of mating methods for GAs, for instance *assortative mating* [8, 11, 15], use mating strategies that select just the most similar or the most dissimilar individual from a set of candidates. In our novel approach, a

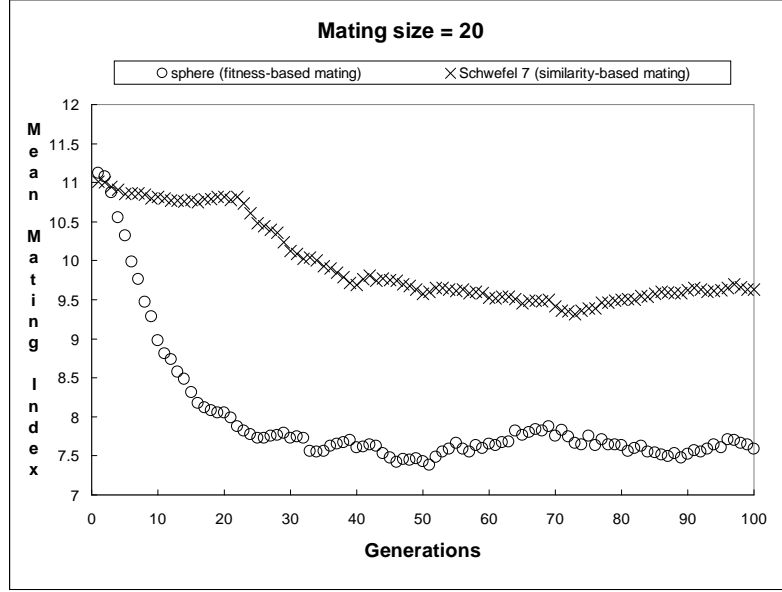


Figure 9: Mean  $\alpha$  value in the population for Sphere and Schwefel 7 functions under SELF-ADAPTIVE strategy with  $\gamma = 20$ .

		Sphere		Schwefel 7	
		Fitness	Similarity	Fitness	Similarity
Best-First	$\gamma = 3$	7.37	22.05	37.94	111.32
	$\gamma = 5$	7.23	36.85	38.39	198.82
	$\gamma = 10$	7.46	67.15	38.79	369.9
	$\gamma = 20$	7.62	115.94	38.74	680.56
	$\gamma = 30$	7.45	154.87	38.29	944.99
Best-Last	$\gamma = 3$	7.5	21	38.24	123.79
	$\gamma = 5$	7.42	36.86	37.98	203.7
	$\gamma = 10$	7.57	66.16	37.86	389.39
	$\gamma = 20$	7.63	124.88	35.88	687.93
	$\gamma = 30$	7.87	166.38	36.33	956.08
Self-Adaptive	$\gamma = 3$	8.81	26.86	45.32	129.42
	$\gamma = 5$	8.2	57.82	43.8	298.66
	$\gamma = 10$	8.89	203.54	43.05	1056.43
	$\gamma = 20$	9.06	675.39	43.91	3636.85
	$\gamma = 30$	9.38	1304.7	44.31	7235.56

Table 1: Mean computation times (in seconds) for each run of the experiments in Section 6.

parameter called *mating index* (see Section 3) allows any of the candidates to be chosen. In this way, if a similarity-based criterion is considered, a candidate with an arbitrary degree of similarity can be obtained or, if a fitness-based criterion is considered, a candidate with an arbitrary fitness can be selected. Therefore, a wide spectrum of mating strategies can be investigated by varying the mating index.

The novel mating approach facilitates the definition of a SELF-ADAPTIVE mating strategy in which each individual has its own mating preference (or mating index). In this way, the fittest individuals determine the most successful mating strategies generation by generation. SELF-ADAPTIVE mating turns out to be a robust strategy that performs well on a variety of different problems. That is not the case for strategies like BEST-FIRST and BEST-LAST, since they only outperform random mating in specific types of problems.

A future research topic derived from the present work is the definition of a mating strategy that deterministically controls mating index parameter throughout the GA generations. In this way, although the mating index would be the same for every individual in the population, it would change from generation to generation. A possible scheme would consist of assigning  $\alpha = \gamma$  at GA initialization and decreasing  $\alpha$  by one every a given number of generations. Other non-linear reduction schemes could also be possible. This deterministic scheme applies more exploration in the initial generations of the GA, when promising search areas are sought, and applies more exploitation in the final generations, when population diversity has decreased.

Another future research topic is the inclusion of mating size parameter  $\gamma$  as a local parameter in the chromosome of each individual. The performance of the new SELF-ADAPTIVE strategy, resulting from including mating size  $\gamma$  along with mating index  $\alpha$  as local parameters, should be compared to the strategies defined in this work.

## Acknowledgements

Severino F. Galán was supported by the Spanish Ministry of Science and Innovation through grant JC2007-00110 from its 2007 José Castillejo Program. This material is based upon work by Ole J. Mengshoel supported by NASA under awards NCC2-1426 and NNA07BB97C.

## References

- [1] T. Bäck, D. B. Fogel, and Z. Michalewicz, editors. *Evolutionary Computation 1: Basic Algorithms and Operators*. Institute of Physics Publishing, Bristol, UK, 2000.
- [2] G. Chakraborty and B. Chakraborty. Ideal marriage for fine tuning in GA. In *Proceedings of the 1999 IEEE International Conference on Systems, Man, and Cybernetics (IEEE SMC'99)*, volume 1, pages 631–636, Tokyo, Japan, 1999.

- [3] C. A. Coello, D. A. van Veldhuizen, and G. B. Lamont. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Kluwer Academic Publishers, Boston, MA, 2002.
- [4] S. De, S. K. Pal, and A. Ghosh. Genotypic and phenotypic assortative mating in genetic algorithm. *Informatin Sciences: an International Journal*, 105(1-4):209–226, 1998.
- [5] K. Deb and D. E. Goldberg. An investigation of niche and species formation in genetic function optimization. In *Proceedings of the 3rd International Conference on Genetic Algorithms*, pages 42–50, George Mason University, Fairfax, VA, 1989. Morgan Kaufmann, San Francisco, CA.
- [6] A. E. Eiben and J. E. Smith. *Introduction to Evolutionary Computing*. Springer, Berlin, 2003.
- [7] L. J. Eshelman and J. D. Schaffer. Preventing premature convergence in genetic algorithms by preventing incest. In *Proceedings of the 4th International Conference on Genetic Algorithms*, pages 115–122, San Diego, CA, 1991. Morgan Kaufmann, San Francisco, CA.
- [8] C. Fernandes, R. Tavares, C. Munteanu, and A. Rosa. Using assortative mating in genetic algorithms for vector quantization problems. In *Proceedings of the 2001 ACM Symposium on Applied Computing (ACM SAC'2001)*, pages 361–365, Las Vegas, NV, 2001. ACM, New York.
- [9] D. E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, Reading, MA, 1989.
- [10] J. H. Holland. *Adaptation in Natural and Artificial Systems*. The University of Michigan Press, Ann Arbor, MI, 1975. Second edition, The MIT Press, Boston, MA, 1992.
- [11] C. F. Huang. An analysis of mate selection in genetic algorithms. Technical Report CSCS-2001-002, Center for the Study of Complex Systems, University of Michigan, 2001.
- [12] B. Manderick and P. Spiessens. Fine-grained parallel genetic algorithms. In *Proceedings of the 3rd International Conference on Genetic Algorithms*, pages 428–433, George Mason University, Fairfax, VA, 1989. Morgan Kaufmann, San Francisco, CA.
- [13] W. N. Martin, J. Lienig, and J. P. Cohoon. Island (migration) models: evolutionary algorithms based on punctuated equilibria. In T. Bäck, D. B. Fogel, and Z. Michalewicz, editors, *The Handbook of Evolutionary Computation*, pages C6.3:1–16. Institute of Physics Publishing and Oxford University Press, 1997.
- [14] K. Matsui. New selection method to improve the population diversity in genetic algorithms. In *Proceedings of the 1999 IEEE International Conference on Systems, Man, and Cybernetics (IEEE SMC'99)*, volume 1, pages 625–630, Tokyo, Japan, 1999.
- [15] G. Ochoa, C. Mädler-Kron, R. Rodriguez, and K. Jaffe. Assortative mating in genetic algorithms for dynamic problems. In F. Rothlauf, J. Branke, S. Cagnoni, D. W. Corne, R. Drechsler, Y. Jin, P. Machado, E. Marchiori, J. Romero, G. D. Smith, and G. Squillero, editors, *Applications of Evolutionary Computing*, volume 3449, pages 617–622. Springer, 2005.

- [16] E. Ronald. When selection meets seduction. In *Proceedings of the 6th International Conference on Genetic Algorithms (ICGA'95)*, pages 167–173, Pittsburgh, PA, 1995. Morgan Kaufmann, San Francisco, CA.
- [17] H. P. Schwefel. *Numerical Optimization of Computer Models*, volume 26. Wiley, Chichester, UK, 1981.
- [18] R. E. Smith and C. Bonacina. Mating restriction and niching pressure: results from agents and implications for general evolutionary computation. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2003)*, pages 1382–1393, Chicago, IL, 2003. Springer.
- [19] R. E. Smith, C. Bonacina, P. Kearney, and W. Merlat. Embodiment of evolutionary computation in general agents. *Evolutionary Computation*, 8(4):475–493, 2000.
- [20] W. M. Spears. Simple subpopulation schemes. In *Proceedings of the 3rd Annual Evolutionary Programming Conference*, pages 296–307, San Diego, CA, 1994. World Scientific.
- [21] C. K. Ting, S. T. Li, and C. Lee. On the harmonious mating strategy through tabu search. *Information Sciences: an International Journal*, 156(3-4):189–214, 2003.
- [22] T. Unemi and M. Nagayoshi. Evolution of learning robot team via local mating strategy. In *Fourth European Conference on Artificial Life*, Brighton, UK, 1997. Poster Session.