



IE 588

Project Report

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1. Introduction

As social networks of today taking on our lives, we acquire and diffuse information continuously throughout the day. All types of information go through our feed and we select either to ignore or diffuse it. By nature, not all of this information is true. A false information originates either on purpose or unwillingly, and people who encounter this can spread it to their network of friends, family. The power of the rumor should not be undermined since they led to false convictions and harmful consequences for people throughout history. The witch hunts in middle ages are very solid examples of how a mere accusation can cause people to be sentenced to die. A bizarre rumor originates, spreads like wildfire and creates social panic. A similar dynamic represents itself in modern times as lynching of people through social media. It is a very important issue thus, needs to be addressed and worked on.

Assume the spread of the false information is targeting a certain individual. When the rumor reaches to targeted person, target starts another spread; this time the correcting factor or the anti-rumor. Now there exists two opposing ideas spreading among people through the social exchanges of people and continues until everybody is convinced about one of the ideas, which in modern times, is usually the truth. (A major assumption of our model)

From the information above it can be said that how the rumor spread is highly dependent on the interaction of the people in a network, how they react to the information, how they decide which information to believe. This dependency

on the interactions make spread of a rumor a very interesting subject who emerges itself from simple human behavior.

2. Methodology

With the rumor spread model, in every iteration the number of people spreading the idea is increasing, the number of possible recipients (the people who receive the information) increasing; resulting in big increases in number of interactions to be evaluated and number of decisions to be made. When things get this complicated it gets very misguided to observe the system by using algorithms and tools of sort since modeling human behavior mathematically is not a passable way. Thus, as it is done with complicated systems like this, simulation is preferred to replicate the real-life system by taking individual traits and global properties into account. By using simulation, it will be possible to avoid the mathematical burden of trying to fit an algorithm that might capture the dynamic and will be possible to have conduct many experimentations on the model, testing the coherency and predict the possible outcomes for future cases.

For the reasons explained above simulation is chosen to be the approach for examination of the rumor spread. However, there are many types of simulation methods and it is crucial to find the one that will suit the characteristics of the problem at hand. Although the system dynamics modelling is a good way to model the interacting elements, it is not that practical to model a system where each individual element acts different in every situation under a uniformity assumption. Being highly dependent on the individuals' behavior and

interactions between individuals the dynamics of the rumor spread requires a modelling tool that can respond to these aspects. These aspects described are fundamental in agent-based modelling, making it the perfect tool to use for modelling rumor spread.

3. Literature Review

Competitive diffusion is a highly popular topic in the domain of Agent Based Modelling. Conceptually, studies in this topic are related to competing innovations, products or simply word of mouth information as in our case. There are several baseline approaches in modelling such systems which include SIR, game theory and information cascade models.

Game-theoretic approaches assume a two-sided control between competing parties in the diffusion model. (Tzoumas et al) In a product or innovation diffusion, this approach is quite valid but, in our case, we don't define a potent governor on rumor spread. Study of Weng et al. argues that information cascade models are more accurate in examining the spread of an information than classic epidemic modelling as diseases does not spread in the same way as behaviors because of differing complexity. (Weng et al) However, their study is limited to spread of one flow of behavior in the case of viral memes.

We have found SIR models very compatible to our subject. Rumor can be treated as a virus. If an agent is infected, it can infect neighboring agents with a probability. Correcting truth or anti-rumor which starts spreading after a time

lag is the cure for this virus which diffuses quite similar to the rumor, both following information spreads. One major difference of this case from the traditional SIR models is that cure or vaccine can also spread, ultimately converging the system to a consensus of truth in our case. In the literature, there are several differing applications of SIR models in competitive information diffusion. Influenced by the diffusion theory by Rogers, Fu et al incorporated different agent types based on their acceptance or reluctance to accept new information and classified them by three types: innovator, ordinary and laggard. (Rogers) (Fu et al.) By means of this, they tested how effective heterogeneous agent distribution is on convergence length in different network topologies.

Rate of information spread and switching sides is a big topic in network domain. In most of the studies, agents are assumed to adjust their information based on their present stance, acting more welcoming to information of the same side while being more reluctant to adopt opposing information. (Serrano et al.) Therefore, two rates are defined for acceptance of new information: spread rate (affirming stance) and switch rate (falsifying stance). (Fu) Serrano et al. also states that agents cured of rumor may not be as enthusiastic about spreading their faults with anti-rumors and defined "denier" agents who does not diffuse or accept new information.

4. Model Description

A Watts-Strogatz small world model is constructed to represent our small network of agents. The algorithm first creates a ring of n nodes (*NumberOfTurtles*), where each node is connected to a certain given number (*NeighborhoodSize*) of nodes on either side. Then, each link for a node is rewired with a distant node with a probability (*RewireProb*) Distance between linked nodes represents the closeness of the social relation between two agents, smaller the link length, closer the relationship.

During the initialization, a certain fraction of turtles (*InnovatorFraction*) are selected to be of Innovator type while the rest being Ordinary. Innovators are more open to new information while Ordinaries are more reluctant. We selected not to define a new agent breed, instead we made use of turtles-own parameters to distinguish two agent types.

Each agent holds a parameter called opinion which indicates its stance. If opinion is a negative number, agent is an endorser of the rumor; if positive, the truth. Each turn an agent acquires opinions in its neighborhood and updates its own opinion accordingly. Parameters called switch rate and spread rate were defined as information updating coefficients. Switch rate is used when the acquired opinion has less "magnitude" compared to self-opinion, while spread rate is used for more "magnitude" opinions. Larger the rate, more weigh is given on the opinions of others. Innovators are defined to have greater switch and spread rates than Ordinaries.

Rumor starts at the beginning of the simulation horizon. A randomly chosen individual is infected with a rumor targeted to a specific other individual. Rumor then diffuses through links in the network. When it reaches the target, target agent starts spreading the anti-rumor, the correcting factor. Since the defence of the target is assumed omnipotent, switch rate of the target is set to a very low number. (One cannot convince the target to rumor)

At each time period (tick), agents gather opinion of their neighbors through links and update their self-opinion using gathered opinion. It is worth mention that agents take a weighted average of their neighbors' opinion based on their proximity, or in this case, link-length. Updating of their own opinion is executed similar to exponential smoothing, using either the switch rate or the spread rate as the smoothing constant.

In short, the model takes following four parameters as inputs: number of turtles, neighborhood size, rewiring probability and innovator fraction. As the outcome of interest, there are several ways of measuring the performance of a spread. One way is to set a simulation limit and check the percentages of population believing in rumor and the truth. However, we decided to use the convergence length of the simulation as our outcome of interest. Since there is an agent (target) radiating truth with a very low switch rate, model definitely converges to a truth consensus. The length of this convergence is an accurate measure not only for the damage caused by the rumor but also for the dynamics of the network.

5. Experimentation and Results

After the construction of the model and taking a few trial runs, we have concluded that the model is yielding coherent outcomes. As described before, model has 4 input parameters. Default parameters of the model is set as the following:

1. number of turtles : 50
2. neighborhood size : 5
3. rewiring probability : 0.2
4. innovator fraction : 0.5

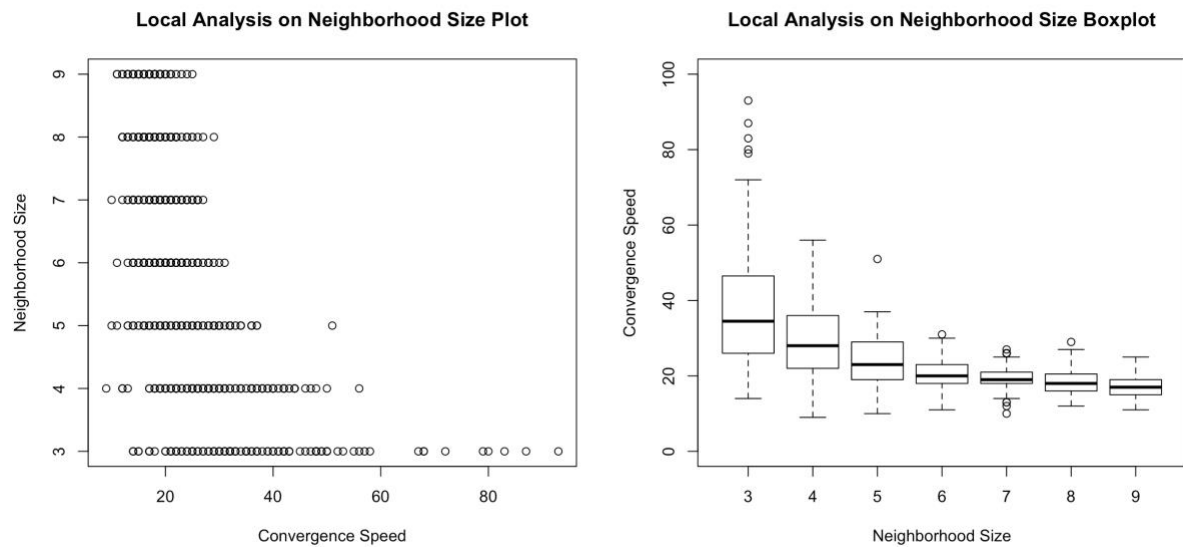
In order to test the effect of each and to conduct the model verification, local sensitivity analysis for all 4 parameters are conducted using BehaviorSpace tool of NetLogo. It should be noted that there are several other parameters that might need tuning and sensitivity checked such as spread rate, switch rate and initial pulse of the rumor. However, we chose to deal with them conceptually and assume their values via several test runs.

Paired t-tests were used as the main statistical tool to examine the factorial effects of input parameters. Type-1 error probability (α) is set to 0.05 for the tests and the results of some tests are tabulated. Refer to the RScript file for the rest. In addition to paired t-tests, linear regression models were also fitted using convergence speed as response and parameter under analysis as the explanatory variable. P-value of the parameter coefficients can also be used to

test the hypothesis of factorial effect. (Linear Models can also be found in RScript file)

❖ Local Sensitivity Analysis on Neighborhood Size

In order to test the effect of neighborhood size on convergence speed, we took sensitivity runs with different neighborhood sizes - rest of the parameters kept default. We have selected neighborhood sizes $\{3, 4, 5, 6, 7, 8, 9\}$ for the local sensitivity analysis, replicating each case for 100 runs.



Above, are the scatterplot and the boxplot for the sensitivity runs. We can observe a diminishing downward trend in convergence speed as neighborhood size increases. In order to statistically prove whether the effect is significant, we have conducted paired t-tests for all consecutive pairs of neighborhood sizes

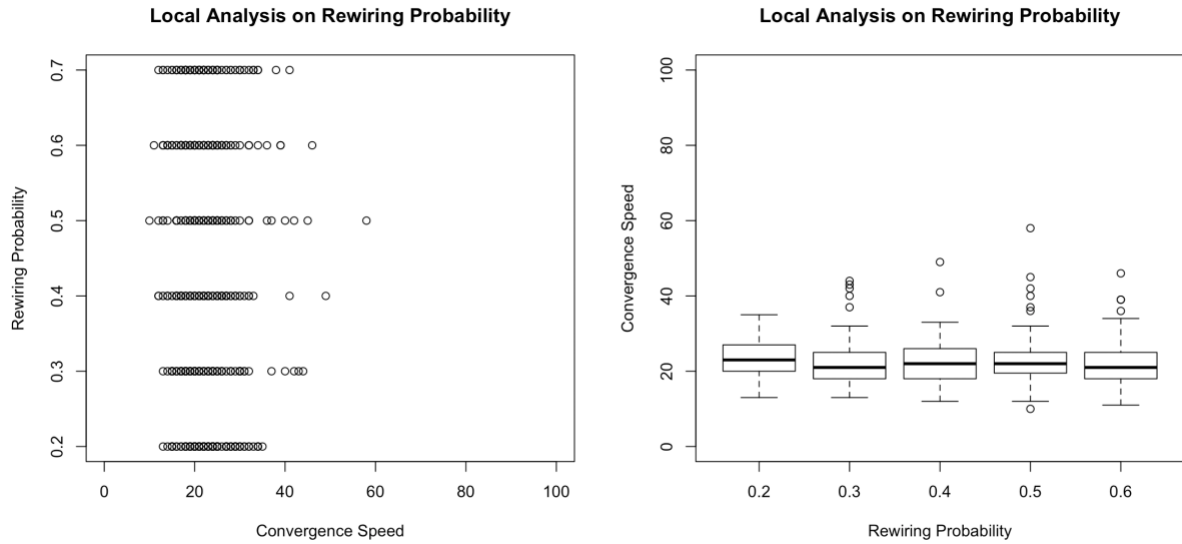
Paired T-tests for Convergence Length under different levels of Neighbourhood Size

| Neighbourhood Size Pairs | Mean of CL Differences | T-stat | df | P-value | Result |
|--------------------------|------------------------|--------|----|-----------|--------|
| 3 - 4 | 8.41 | 4.3857 | 99 | 2.888e-05 | Reject |
| 4 - 5 | 5.28 | 4.3926 | 99 | 2.813e-05 | Reject |
| 5 - 6 | 3.61 | 4.723 | 99 | 7.674e-06 | Reject |
| 6 - 7 | 0.99 | 2.0179 | 99 | 0.04631 | Reject |
| 7 - 8 | 1.11 | 2.1984 | 99 | 0.03025 | Reject |
| 8 - 9 | 1.36 | 3.0714 | 99 | 0.002751 | Reject |

With type-1 error probability (α) set to 0.05, we can safely reject all the tests above. There are undeniable statistical differences between the pairs of different sizes. This shows that neighborhood size has an effect on the convergence length, if rest of the parameters are in default settings.

❖ Local Sensitivity Analysis on Rewiring Probability

Rewiring is the process of an agent breaking a link with a closer agent to form a link with a more distant one. Being an intercross factor, rewiring probability highly affects the strength of linkages. In order to test the effect of rewiring probability on convergence speed, we took sensitivity runs with rewiring probabilities $\{0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$ under default settings, replicating each case for 100 runs.



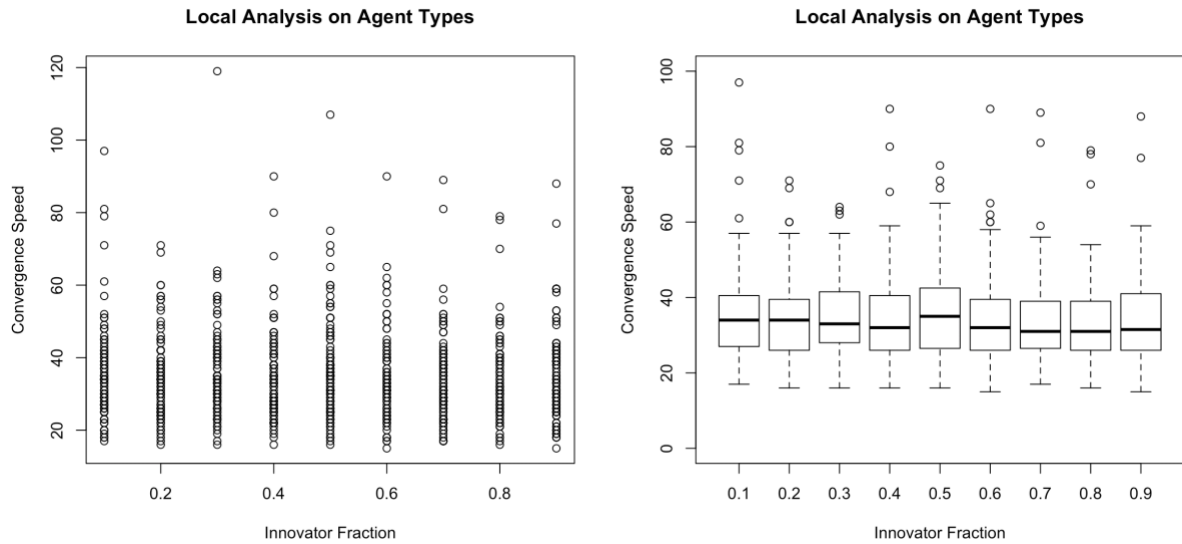
Paired T-tests for Convergence Length under different levels of Rewiring Probabilities

| Rewiring Probability Pairs | Estimate Mean of Differences | T-stat | df | P-value | Result |
|----------------------------|------------------------------|---------|----|---------|----------------|
| 0.2 - 0.3 | 0.8 | 1.03 | 99 | 0.3055 | Fail to reject |
| 0.3 - 0.4 | 0.24 | 0.27895 | 99 | 0.7809 | Fail to reject |
| 0.4 - 0.5 | -1.99 | -1.2447 | 99 | 0.2162 | Fail to reject |
| 0.5 - 0.6 | 2.44 | 1.5892 | 99 | 0.1152 | Fail to reject |

Plots do not show any observable difference on convergence length for differing rewiring probabilities. T-test also show no sign of significant effect on convergence length between any pairs. Thus we have enough evidence to argue that rewiring probability does not affect convergence length by itself under default settings. It is still too early to comment on its effect through interactions with other parameters.

❖ Local Sensitivity Analysis on Agent Types

A lot of studies in the related literature defined different types of agents to form a heterogeneous network. Our model too tests the effect of such individuality differences between agents. Input parameter innovator fraction is used to determine the percentages of innovator type agents and ordinary agents in our simulation. Sensitivity runs were taken using innovator fractions $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ under default settings, replicating each case for 100 runs.



Paired T-tests for Convergence Length under different levels of Innovation Fraction

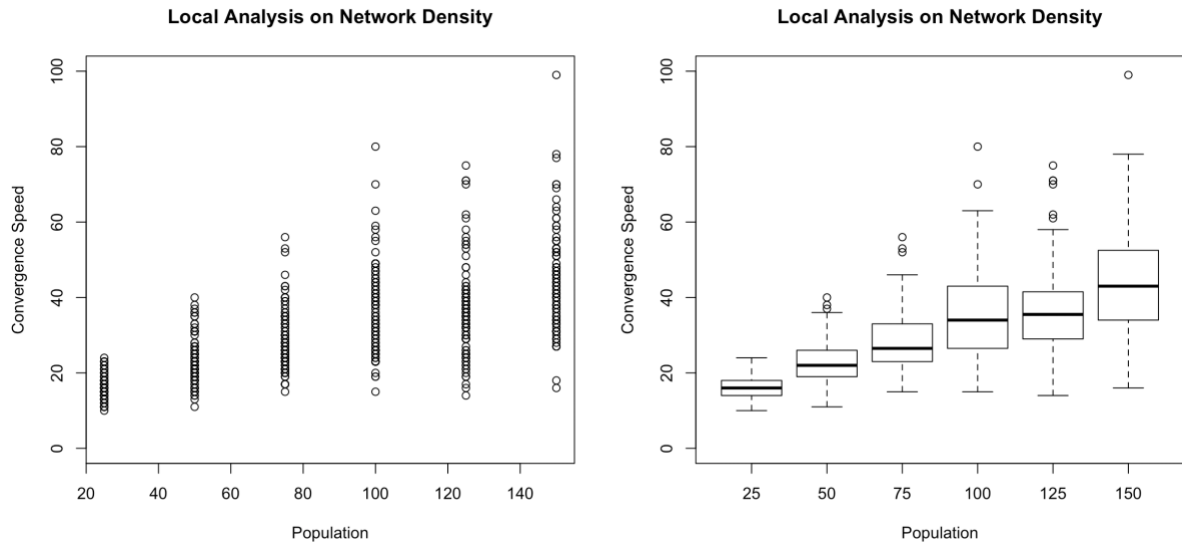
| Innovation Fraction Pairs | Estimate Mean of Differences | T-stat | df | P-value | Result |
|---------------------------|------------------------------|----------|----|---------|----------------|
| 0.1 - 0.3 | -0.39 | -0.20652 | 99 | 0.8368 | Fail to reject |
| 0.3 - 0.5 | -0.86 | -0.40694 | 99 | 0.6849 | Fail to reject |

| | | | | | |
|-----------|-------|----------|----|---------|----------------|
| 0.5 - 0.7 | 3.03 | 1.7217 | 99 | 0.08825 | Fail to reject |
| 0.7 - 0.9 | -0.96 | -0.54807 | 99 | 0.5849 | Fail to reject |

Intuitively, we would expect an increasing innovator fraction to accelerate the convergence process by providing faster information flows for both ends of competition. However, both the plots and statistical tests show there are no evidence that agent type distribution in a network change the convergence rate. Reasons to be revisited later in the global analysis section.

❖ Local Sensitivity Analysis on Number of Turtles

Another question about the nature of convergence is regarding the density of the network. Does convergence happen quicker in a more populated network than a less populated one? Sensitivity runs were taken using number of turtles $\{25, 50, 75, 100, 125, 150\}$ under default settings, replicating each case for 100 runs.



Paired T-tests for Convergence Length under different levels of Number of Turtles

| Number of Turtles Pairs | Estimate Mean of Differences | T-stat | df | P-value | Result |
|-------------------------|------------------------------|---------|----|-----------|----------------|
| 25 - 50 | -6.76 | -9.8507 | 99 | 2.316e-16 | Reject |
| 50 - 75 | -5.44 | -4.8808 | 99 | 4.052e-06 | Reject |
| 75 - 100 | -7.59 | -6.4142 | 99 | 4.877e-09 | Reject |
| 100 - 125 | -2.64 | -1.0965 | 99 | 0.2755 | Fail to reject |
| 125 - 150 | -8.45 | -2.5404 | 99 | 0.01263 | Reject |

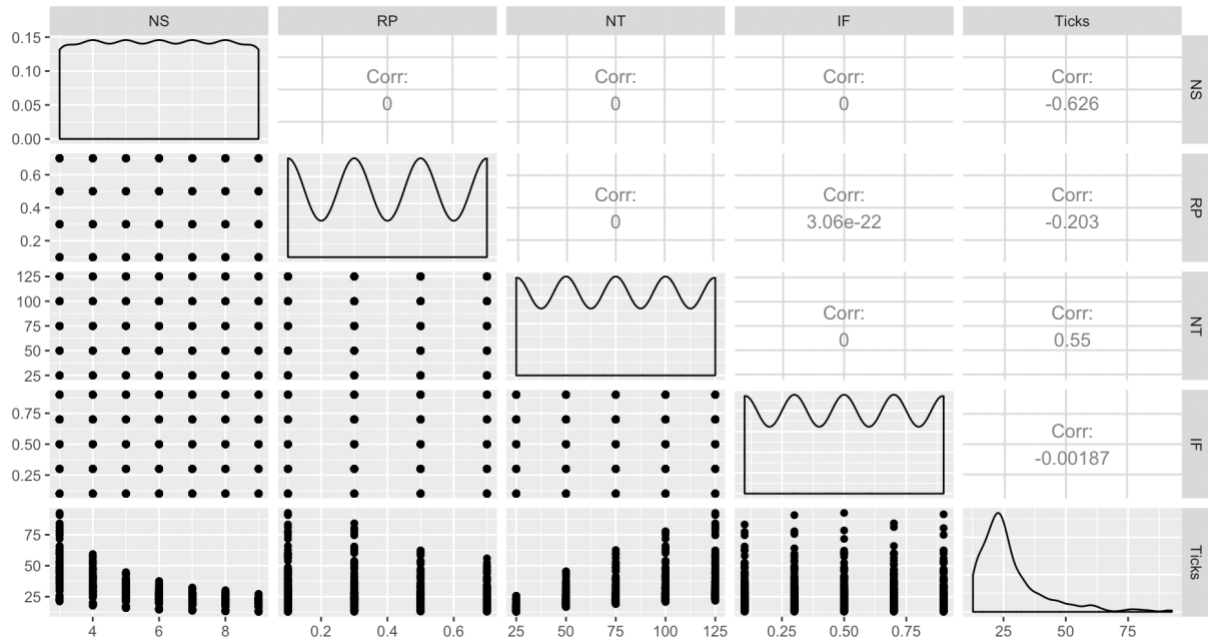
Plots show a very visible trend in convergence length with respect to number of turtles. We can also observe that as number of turtles increases; in addition to a positive mean shift, variance also increases. In paired t-tests, only one of the pairs of this test did not result in significant difference. But in general, it is safe to say the change in number of turtles has an effect on convergence length which can be observed in the summary of the linear model as well.

❖ Global Sensitivity Analysis

In the case of interdependence of input parameters, individual analysis for each can lead to misleading conclusions. Therefore, we have performed a global sensitivity analysis by selecting certain ranges for each variable and observing their joint impact on our outcome of interest, convergence length. Following levels are selected for experimentation.

- Neighborhood Size: {3, 4, 5, 6, 7, 8, 9}
- Rewiring Probability: {0.1, 0.3, 0.5, 0.7}
- Number of Turtles: {25, 50, 75, 100, 125}
- Innovator Fraction: {0.1, 0.3, 0.5, 0.7, 0.9}

By this setup, there are $7 * 4 * 5 * 5 = 700$ parameter combinations. Each parameter combination is replicated 100 times for a total 70000 runs. Means of the replications were taken for each combination and arranged in a tidy order. Correlation plots are useful in seeing the individual correlation of each variable to the convergence.



Correlation plot shows a strong correlation of neighborhood size (NS) and number of turtles (NT) to the convergence length (Ticks). Unlike the local sensitivity results, rewiring probability (RW) also has a moderate correlation with convergence length. Conclusion for innovator fraction still holds true, no apparent effect is present. In sum, number of turtles and neighborhood size seem

like the main driving forces behind convergence with a moderate effect of rewiring probability as well.

In order to conduct a neat examination on global analysis results, we have decided to pair our parameters under two classes: Population Characteristic (PC) and Network Linkage (NL). PC parameters are defined to be number of turtles and innovator fraction, representing the distribution and behavioral properties of the agents. NL parameters are defined to be the neighborhood size and rewiring probability, representing the characteristics of the network agents' interaction. Following table shows the combinations each class correspond to.

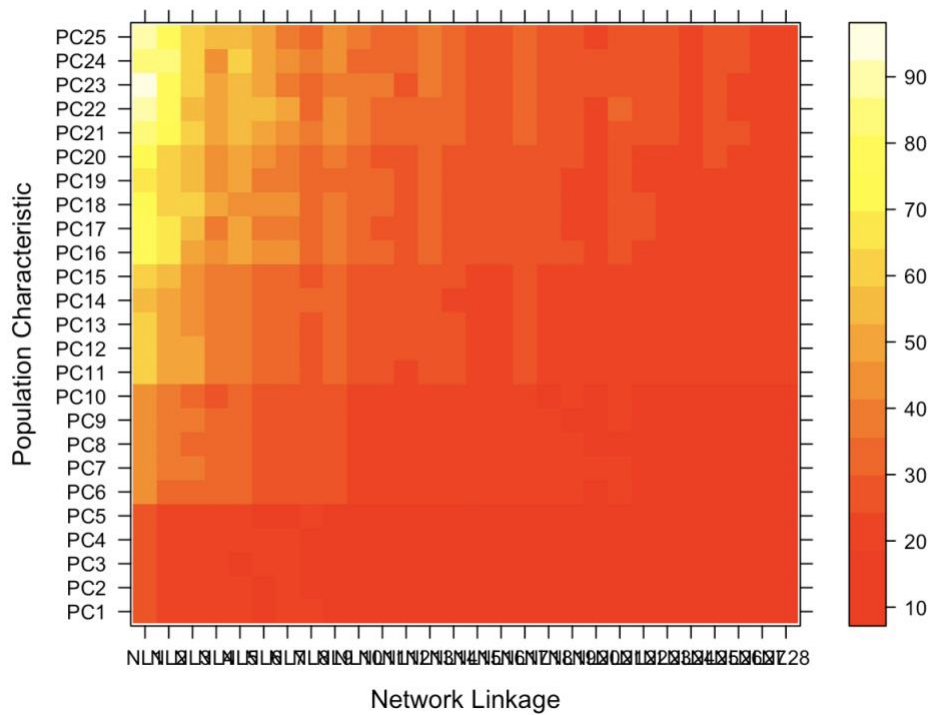
| | NL1 | NL2 | NL3 | NL4 | NL5 | NL6 | NL7 | NL8 | NL9 | NL10 | NL11 | NL12 | NL13 | NL14 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| NS | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 | 6 |
| RP | 0.1 | 0.3 | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.7 | 0.1 | 0.3 |
| | NL15 | NL16 | NL17 | NL18 | NL19 | NL20 | NL21 | NL22 | NL23 | NL24 | NL25 | NL26 | NL27 | NL28 |
| NS | 6 | 6 | 7 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | 9 | 9 |
| RP | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.7 |

| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 | PC10 | PC11 | PC12 | PC13 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| NT | 25 | 25 | 25 | 25 | 25 | 50 | 50 | 50 | 50 | 50 | 75 | 75 | 75 |
| IF | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 0.1 | 0.3 | 0.5 |
| | PC14 | PC15 | PC16 | PC17 | PC18 | PC19 | PC20 | PC21 | PC22 | PC23 | PC24 | PC25 | |
| NT | 75 | 75 | 100 | 100 | 100 | 100 | 100 | 125 | 125 | 125 | 125 | 125 | |
| IF | 0.7 | 0.9 | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | |

In total, we had 28 Network Linkage cases and 25 Population Characteristics to infer. Output data was rearranged and relabeled with respect

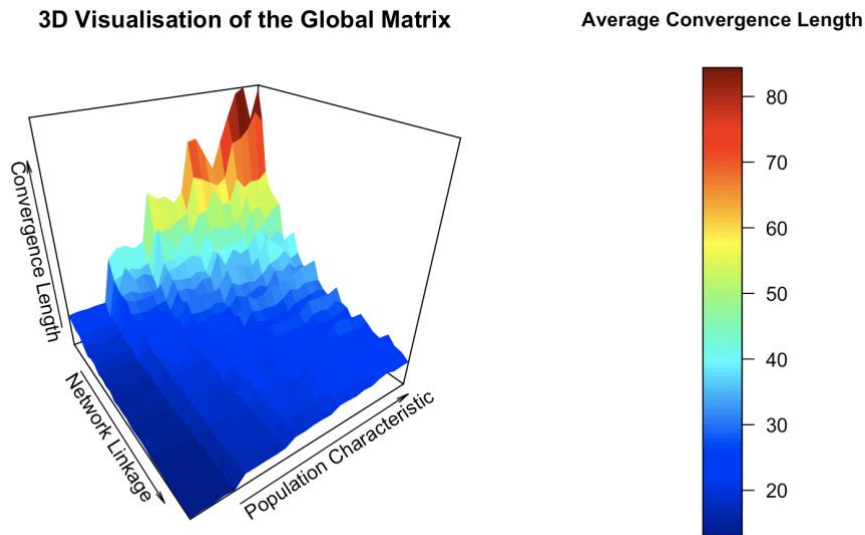
to those classifications. Following is the heat map of the output data from global sensitivity analysis. Rows are coded with PC and columns with NL cases. Color dimension indicates the level of average convergence speed of 100 replications in the corresponding parameter combination. Maximum convergence length is observed at the upper left corner of the map where the number of turtles is at its maximum level of 125 and neighborhood size is at its minimum, 3.

Convergence Heat Map of 25 Population Characteristics & 28 Network Linkage



Another visible pattern is that along the PC axis, convergence length level gains an increase after each 5 steps. This is solely caused by the number of turtles level which alters after 5 levels. The other PC parameter, innovator fraction causes small local deviations on convergence length. A more detailed analysis is required for the effect of innovation fraction to see whether it follows some significant period.

A trend is also visible along the NL axis as well. But this trend is not as periodic as in PC. We can infer that both parameters under the class of NL has contribution to the trend in convergence length with neighborhood size having the greater effect.



3D Perspective plots are perfect tools in observing such three-dimensional matrices. Below, both altitude and color dimensions show average convergence lengths under different cases of NL and PC. We see that there is a very significant peak at the corner of the plot, and it descends as it moves away from it.

Local deviations caused by innovator fraction is more visible in this graph. Along PC axis, convergence length reaches local peaks for innovation fraction of 0.5 and local pits for extreme levels such as 0.1 and 0.9. Rumor spreads and collapses faster in more homogenous populations with respect to reluctance/acceptance to new information.

Convergence length reaches its global maximum for the greatest level of number of turtles and lowest level of neighborhood size. It can be inferred that number of turtles and neighborhood size have a joint interaction effect on convergence length.

To be able to analyze the behavior of each class for different conditions, rows and columns are color coded among themselves. Appendix A shows the global analysis output matrix with rows color-coded by their convergence lengths. Inference regarding the number of turtles being a main driving factor holds true and more visible, every NL row undergoes a rough-cut increase after each 5 PC columns, in other words, after each level change in the number of turtles.

Appendix B is the column based color-coded global analysis output matrix. Just like the heat map, transition from fast convergence to slow convergence is smoother. Neighborhood size parameter is indeed effective on the convergence level, but it acts in unison with rewiring probability parameter.

6. Conclusion

In conclusion, number of agents in a network and neighborhood sizes per agent are the most impactful factors in convergence speed of a rumor.

Individually, they are found to be quite effective; jointly they are the main driving force which diminishes the effect of other parameters and carries the spreading phenomena to a whole different level.

In the local analysis section agent type distribution (innovator fraction) of the network was found to be statistically insignificant on convergence speed under default settings. However, by conducting global analysis we have found that heterogeneity of agents increases the convergence speed of the system. A network comprising uniformly of innovator or ordinary individuals is expected to converge faster than an equally distributed one. We also found that rewiring probability moves the system very similar to neighborhood size and found to have a moderate effect which was not apparent in local analysis.

Spread rate and switch rates highly effect the diffusion phenomena. By selecting moderate values for these, we managed to build a healthy model. But as a further extension, tuning and analysis of those model parameters is suggested. In addition to those, if real life data can be acquired, model validation analysis is highly encouraged.

7. References

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Appendix A

- Global Analysis Output Matrix row-wise color-coded with respect to average convergence length

| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 | PC10 | PC11 | PC12 | PC13 | PC14 | PC15 | PC16 | PC17 | PC18 | PC19 | PC20 | PC21 | PC22 | PC23 | PC24 | PC25 |
|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| NL1 | 25.41 | 25.74 | 25.49 | 25.61 | 25.5 | 42.21 | 45.37 | 45.39 | 43.33 | 45.19 | 62.65 | 59.82 | 59.74 | 58.32 | 59.91 | 77.19 | 77.79 | 71.67 | 65.92 | 74.92 | 83.25 | 90.83 | 92.59 | 81.18 | 91.48 |
| NL2 | 24.07 | 22.39 | 23.45 | 22.78 | 21.15 | 35.08 | 36.29 | 37.57 | 36.23 | 37.63 | 48.72 | 48.31 | 48.86 | 50.77 | 53.04 | 65.49 | 64.05 | 62.2 | 60.18 | 62.33 | 74.58 | 75.63 | 78.45 | 84.19 | 80.32 |
| NL3 | 22.57 | 23.75 | 23.32 | 23.21 | 22.35 | 32.8 | 40.53 | 34.53 | 35.95 | 33.64 | 51.28 | 47.05 | 41.35 | 46.35 | 45.5 | 52.25 | 53.36 | 60.78 | 54 | 53.69 | 62.26 | 58.3 | 59.84 | 59.57 | 62.27 |
| NL4 | 22.61 | 22.04 | 22.68 | 22.52 | 22.24 | 31.83 | 32.61 | 30.53 | 30.01 | 29.59 | 39.44 | 39.8 | 40.42 | 37.93 | 38.77 | 43.47 | 40.65 | 47.68 | 44.1 | 46.58 | 49.59 | 49.88 | 52.28 | 45.62 | 55.91 |
| NL5 | 20.06 | 19.65 | 18.3 | 18.7 | 19.93 | 31.88 | 29.93 | 30.03 | 30.24 | 33.2 | 37.3 | 38.06 | 39.46 | 40.58 | 38.72 | 47.98 | 48.76 | 46.87 | 47.08 | 47.29 | 58.05 | 56.36 | 57.44 | 59.39 | 53.96 |
| NL6 | 17.71 | 18.12 | 19.19 | 18.69 | 18.31 | 27.93 | 26.96 | 26.78 | 27.71 | 28.29 | 35.45 | 34.36 | 33.88 | 35.37 | 34.16 | 41.4 | 41.01 | 41.62 | 41.16 | 43.96 | 52.36 | 53.82 | 47.07 | 51.19 | 52.09 |
| NL7 | 18.88 | 19.11 | 18.59 | 19.76 | 17.94 | 27.49 | 25.56 | 24.76 | 26.96 | 25.67 | 32.46 | 32.34 | 30.39 | 33.52 | 33.28 | 42.24 | 36.1 | 41.45 | 38.63 | 39.78 | 44.56 | 47.41 | 40.16 | 43.18 | 40.59 |
| NL8 | 18.74 | 18.23 | 18.32 | 18.52 | 18.96 | 25.21 | 25.29 | 24.79 | 25.45 | 24.28 | 29.48 | 27.08 | 29.12 | 30.03 | 28.93 | 32.35 | 32.09 | 31.91 | 31.76 | 32.89 | 37.27 | 33.58 | 34.36 | 37.6 | 34.01 |
| NL9 | 16.52 | 17.04 | 16.58 | 16.66 | 16.19 | 26.49 | 25.8 | 25.09 | 25.59 | 25.72 | 31.45 | 31.77 | 32.04 | 30.68 | 32.03 | 38.05 | 38.49 | 37.85 | 35.48 | 38.63 | 44.8 | 43.32 | 40.46 | 42.84 | 44.1 |
| NL10 | 16.93 | 16.22 | 15.91 | 15.92 | 16.77 | 22.62 | 23.04 | 24.16 | 22.68 | 23.44 | 28.16 | 27.42 | 27.29 | 28.71 | 27.43 | 34.31 | 32.27 | 32.25 | 31.45 | 33.26 | 36.4 | 37.51 | 39.75 | 35.38 | 36.76 |
| NL11 | 16.01 | 16.58 | 16.17 | 17.22 | 15.81 | 22.5 | 23.2 | 22.47 | 22.8 | 22.47 | 25.4 | 25.74 | 27.4 | 26.27 | 27.36 | 30.66 | 28.93 | 29.94 | 31.21 | 29.4 | 35.22 | 33.58 | 35.92 | 32.81 | 33.2 |
| NL12 | 16.61 | 16.76 | 16.45 | 15.85 | 16.91 | 21.73 | 22.5 | 21.89 | 20.73 | 22.34 | 24.03 | 25.56 | 25.49 | 24.78 | 25.23 | 27.22 | 26.84 | 27.01 | 26.72 | 26.93 | 29.97 | 31.35 | 29.78 | 29.92 | 31.42 |
| NL13 | 14.58 | 15.08 | 14.74 | 14.62 | 14.62 | 22.42 | 22.5 | 22.87 | 23.33 | 22.38 | 28 | 27.54 | 27.92 | 28.63 | 26.87 | 33.68 | 31.17 | 31.78 | 34.57 | 32.65 | 33.96 | 35.07 | 36.67 | 36.95 | 37.64 |
| NL14 | 15.12 | 14.74 | 14.83 | 15.38 | 14.79 | 20.87 | 20.55 | 21.4 | 19.86 | 20.53 | 25.21 | 25.39 | 24.28 | 23.86 | 24.24 | 29.43 | 28.18 | 28.27 | 27.55 | 29.29 | 32 | 30.33 | 31.95 | 31.61 | 30.75 |
| NL15 | 14.91 | 15.38 | 15.13 | 15.42 | 15.06 | 20.07 | 20.79 | 20.25 | 20.34 | 20.6 | 23.95 | 23.34 | 23.76 | 23.85 | 23.57 | 27.1 | 25.6 | 27.03 | 26.68 | 25.5 | 27.68 | 28.23 | 28.8 | 29.34 | 28.28 |
| NL16 | 15.15 | 15.11 | 15.16 | 15.02 | 15.54 | 19.52 | 19.91 | 19.33 | 20.06 | 19.33 | 23.22 | 22.13 | 22.96 | 22.64 | 21.88 | 24.76 | 25.15 | 25.76 | 24.6 | 24.86 | 26.25 | 25.6 | 26.72 | 27.24 | 26.99 |
| NL17 | 14.08 | 13.98 | 14.12 | 13.6 | 13.84 | 20.88 | 19.87 | 20.62 | 20.57 | 20.79 | 25.96 | 24.37 | 25.4 | 24.35 | 24.43 | 27.84 | 27.86 | 27.62 | 28.23 | 27.96 | 31.73 | 31.4 | 31.26 | 31.03 | 32.4 |
| NL18 | 14.25 | 14.25 | 14.22 | 14.42 | 14.32 | 19.3 | 19.38 | 18.92 | 19.79 | 18.45 | 22.59 | 22.19 | 22.16 | 22.94 | 22.7 | 27.15 | 25.71 | 25.54 | 25.94 | 25.78 | 28.17 | 28.5 | 27.45 | 29.68 | 27.76 |
| NL19 | 14.47 | 14.45 | 13.84 | 14.52 | 14.99 | 19.09 | 19.08 | 19.45 | 18.47 | 18.71 | 21.84 | 22.59 | 22.17 | 21.97 | 21.48 | 25.32 | 24.2 | 23.53 | 24.2 | 24.62 | 26.48 | 25.23 | 25.99 | 26.46 | 27.01 |
| NL20 | 14.33 | 14.79 | 14.31 | 14.57 | 14.27 | 18.51 | 18.75 | 18.25 | 18.43 | 17.59 | 21.43 | 20.93 | 21.08 | 20.69 | 21.2 | 22.97 | 22.4 | 22.57 | 23.78 | 22.61 | 23.96 | 24.13 | 24.24 | 24.6 | 23.54 |
| NL21 | 13.59 | 13.58 | 13.34 | 13.41 | 13.34 | 19.17 | 18.93 | 18.47 | 19 | 19.7 | 23.87 | 23.02 | 22.32 | 22.36 | 23.06 | 26.26 | 25.06 | 25.52 | 25.02 | 26.98 | 28.99 | 30.24 | 29.27 | 28.24 | 28.06 |
| NL22 | 13.49 | 13.89 | 13.32 | 13.54 | 13.39 | 17.69 | 17.63 | 17.91 | 17.79 | 17.98 | 22.06 | 21.38 | 20.93 | 20.48 | 21.25 | 23.62 | 24.43 | 24.94 | 23.5 | 23.62 | 26.76 | 26.3 | 25.79 | 25.15 | 26.36 |
| NL23 | 13.6 | 13.51 | 13.3 | 13.53 | 13.82 | 17.31 | 18.25 | 18.01 | 18.36 | 18.3 | 20.45 | 20.53 | 20.95 | 21.4 | 20.87 | 23.78 | 22.12 | 22.06 | 23.58 | 22.56 | 24.81 | 24.83 | 24.24 | 25.06 | 24.41 |
| NL24 | 13.2 | 13.87 | 13.77 | 13.74 | 13.32 | 17.29 | 17.56 | 17.76 | 17.52 | 17.83 | 20.56 | 20.54 | 19.95 | 20.1 | 19.88 | 21.56 | 21.66 | 22.03 | 21.97 | 22.18 | 23.07 | 22.93 | 23.14 | 23.43 | 23.74 |
| NL25 | 13.33 | 13.06 | 13.01 | 12.93 | 12.75 | 17.21 | 17.61 | 17.52 | 17.87 | 17.65 | 21.5 | 21.57 | 21.59 | 21.45 | 21.05 | 23.79 | 24.15 | 23.97 | 23.32 | 24.38 | 26.17 | 27.11 | 26.24 | 27.13 | 27.03 |
| NL26 | 13.05 | 12.96 | 13.05 | 12.96 | 13.23 | 17.19 | 17.11 | 17.39 | 17.71 | 17.38 | 19.96 | 19.42 | 19.5 | 20.04 | 19.96 | 21.95 | 22.97 | 22.92 | 22.65 | 21.9 | 24.55 | 24.2 | 24.18 | 24.31 | 24.39 |
| NL27 | 13.27 | 13.16 | 13.37 | 13.52 | 13.04 | 17.08 | 17.15 | 17.15 | 16.64 | 17.52 | 19.37 | 19.16 | 19.57 | 20.25 | 19.55 | 21.37 | 22.22 | 21.87 | 21.94 | 21.55 | 23.79 | 23.61 | 23.5 | 22.78 | 23.78 |
| NL28 | 12.99 | 13.21 | 13.02 | 13.47 | 13.05 | 16.93 | 16.85 | 16.97 | 16.87 | 17.11 | 19.25 | 19.14 | 19.62 | 19.6 | 19.41 | 21.13 | 20.98 | 20.71 | 21.4 | 21.05 | 22.28 | 22.8 | 21.73 | 22.02 | 21.95 |
| Mean for PCs | 16.41 | 16.45 | 16.32 | 16.43 | 16.27 | 23.15 | 23.54 | 23.22 | 23.22 | 23.33 | 28.75 | 28.31 | 28.20 | 28.40 | 28.42 | 33.37 | 32.65 | 33.12 | 32.38 | 33.11 | 37.11 | 37.22 | 37.12 | 37.07 | 37.51 |

Appendix B

- Global Analysis Output Matrix column-wise color-coded with respect to average convergence length

| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 | PC10 | PC11 | PC12 | PC13 | PC14 | PC15 | PC16 | PC17 | PC18 | PC19 | PC20 | PC21 | PC22 | PC23 | Mean for NLS |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------|
| NL1 | 25.41 | 25.74 | 25.49 | 25.61 | 25.5 | 42.21 | 45.37 | 45.39 | 43.33 | 45.19 | 62.65 | 59.62 | 59.74 | 56.32 | 59.91 | 77.19 | 77.79 | 71.67 | 65.92 | 74.92 | 83.25 | 90.83 | 92.59 | 55.72 |
| NL2 | 24.07 | 22.39 | 23.45 | 22.78 | 21.15 | 35.08 | 36.29 | 37.57 | 36.23 | 37.63 | 48.72 | 48.31 | 48.86 | 50.77 | 53.04 | 65.49 | 64.05 | 62.2 | 60.18 | 62.33 | 74.58 | 75.63 | 78.45 | 47.36 |
| NL3 | 22.57 | 23.75 | 23.32 | 23.21 | 22.35 | 32.8 | 40.53 | 34.53 | 35.95 | 33.64 | 51.28 | 47.05 | 41.35 | 46.35 | 45.5 | 52.25 | 53.36 | 60.78 | 54 | 53.69 | 62.26 | 58.3 | 59.84 | 42.55 |
| NL4 | 22.61 | 22.04 | 22.68 | 22.52 | 22.24 | 31.83 | 32.61 | 30.53 | 30.01 | 29.59 | 39.44 | 39.8 | 40.42 | 37.93 | 38.77 | 43.47 | 40.65 | 47.68 | 44.1 | 46.58 | 49.59 | 49.88 | 52.28 | 36.40 |
| NL5 | 20.06 | 19.65 | 18.3 | 18.7 | 19.93 | 31.88 | 29.93 | 30.03 | 30.24 | 33.2 | 37.3 | 38.06 | 39.46 | 40.58 | 38.72 | 47.98 | 48.76 | 46.87 | 47.08 | 47.29 | 58.05 | 56.36 | 57.44 | 37.21 |
| NL6 | 17.71 | 18.12 | 19.19 | 18.69 | 18.31 | 27.93 | 26.96 | 26.78 | 27.71 | 28.29 | 35.45 | 34.36 | 33.88 | 35.37 | 34.16 | 41.4 | 41.01 | 41.62 | 41.16 | 43.96 | 52.36 | 53.82 | 47.07 | 33.27 |
| NL7 | 18.88 | 19.11 | 18.59 | 19.76 | 17.94 | 27.49 | 25.56 | 24.78 | 26.96 | 25.67 | 32.46 | 32.34 | 30.39 | 33.52 | 33.28 | 42.24 | 36.1 | 41.45 | 38.63 | 39.78 | 44.56 | 47.41 | 40.16 | 31.18 |
| NL8 | 18.74 | 18.23 | 18.32 | 18.52 | 18.96 | 25.21 | 25.29 | 24.79 | 25.45 | 24.28 | 29.48 | 27.08 | 29.12 | 30.03 | 28.93 | 32.35 | 32.09 | 31.91 | 31.76 | 32.89 | 37.27 | 33.58 | 34.36 | 27.33 |
| NL9 | 16.52 | 17.04 | 16.58 | 16.66 | 16.19 | 26.49 | 25.8 | 25.09 | 25.59 | 25.72 | 31.45 | 31.77 | 32.04 | 30.68 | 32.03 | 38.05 | 38.49 | 37.85 | 35.48 | 38.63 | 44.8 | 43.32 | 40.46 | 29.86 |
| NL10 | 16.93 | 16.22 | 15.91 | 15.92 | 16.77 | 22.62 | 23.04 | 24.16 | 22.68 | 23.44 | 28.16 | 27.42 | 27.29 | 28.71 | 27.43 | 34.31 | 32.27 | 32.25 | 31.45 | 33.26 | 36.4 | 37.51 | 39.75 | 26.69 |
| NL11 | 16.01 | 16.58 | 16.17 | 17.22 | 15.81 | 22.5 | 23.2 | 22.47 | 22.8 | 22.47 | 25.4 | 25.74 | 27.4 | 26.27 | 27.36 | 30.66 | 28.93 | 29.94 | 31.21 | 29.4 | 35.22 | 33.58 | 35.92 | 25.13 |
| NL12 | 16.61 | 16.76 | 16.45 | 15.83 | 16.91 | 21.73 | 22.5 | 21.89 | 20.73 | 22.34 | 24.03 | 25.56 | 25.49 | 24.78 | 25.23 | 27.22 | 26.84 | 27.03 | 26.72 | 26.93 | 29.97 | 31.35 | 29.78 | 23.29 |
| NL13 | 14.58 | 15.08 | 14.74 | 14.62 | 14.62 | 22.42 | 22.5 | 22.87 | 23.33 | 22.38 | 28 | 27.54 | 27.92 | 28.63 | 26.87 | 33.68 | 31.17 | 31.78 | 34.57 | 32.65 | 33.96 | 35.07 | 36.67 | 25.96 |
| NL14 | 15.12 | 14.74 | 14.83 | 15.38 | 14.75 | 20.87 | 20.55 | 21.4 | 19.86 | 20.53 | 25.21 | 25.39 | 24.28 | 23.86 | 24.24 | 29.43 | 28.18 | 28.27 | 27.55 | 29.29 | 32 | 30.33 | 31.95 | 23.43 |
| NL15 | 14.91 | 15.38 | 15.13 | 15.42 | 15.06 | 20.07 | 20.75 | 20.25 | 20.34 | 20.6 | 23.95 | 23.34 | 23.76 | 23.85 | 23.57 | 27.1 | 25.6 | 27.03 | 28.66 | 25.5 | 27.68 | 28.23 | 28.8 | 22.31 |
| NL16 | 15.15 | 15.11 | 15.16 | 15.02 | 15.54 | 19.52 | 19.91 | 19.33 | 20.06 | 19.33 | 23.22 | 22.13 | 22.96 | 22.04 | 21.88 | 24.76 | 25.15 | 25.76 | 24.6 | 24.86 | 26.25 | 25.6 | 26.72 | 21.31 |
| NL17 | 14.08 | 13.98 | 14.12 | 13.6 | 13.84 | 20.88 | 19.87 | 20.62 | 20.57 | 20.79 | 25.96 | 24.37 | 25.4 | 24.35 | 24.43 | 27.64 | 27.86 | 27.62 | 28.23 | 27.96 | 31.73 | 31.4 | 31.26 | 23.06 |
| NL18 | 14.25 | 14.25 | 14.22 | 14.42 | 14.32 | 19.3 | 19.38 | 18.92 | 18.79 | 18.45 | 22.59 | 22.19 | 22.16 | 22.94 | 22.7 | 27.15 | 25.71 | 25.54 | 25.94 | 25.78 | 28.17 | 28.5 | 27.45 | 21.48 |
| NL19 | 14.47 | 14.45 | 13.84 | 14.52 | 14.99 | 19.09 | 19.08 | 19.45 | 18.47 | 18.71 | 21.94 | 22.59 | 22.17 | 21.97 | 21.48 | 25.32 | 24.2 | 23.53 | 24.2 | 24.62 | 26.48 | 25.23 | 25.99 | 20.73 |
| NL20 | 14.33 | 14.79 | 14.31 | 14.57 | 14.27 | 18.51 | 18.75 | 18.25 | 18.43 | 17.59 | 21.43 | 20.93 | 21.06 | 20.69 | 21.2 | 22.97 | 22.4 | 22.57 | 23.78 | 22.61 | 23.96 | 24.13 | 24.24 | 19.82 |
| NL21 | 13.59 | 13.58 | 13.34 | 13.41 | 13.34 | 19.17 | 18.93 | 18.47 | 19 | 19.7 | 23.67 | 23.02 | 22.32 | 22.36 | 23.06 | 26.26 | 25.06 | 25.52 | 25.02 | 26.98 | 28.99 | 30.24 | 29.27 | 21.49 |
| NL22 | 13.49 | 13.89 | 13.32 | 13.54 | 13.39 | 17.69 | 17.63 | 17.91 | 17.79 | 17.98 | 22.06 | 21.38 | 20.93 | 20.48 | 21.25 | 23.62 | 24.43 | 24.94 | 23.5 | 23.62 | 26.76 | 26.3 | 25.79 | 20.07 |
| NL23 | 13.6 | 13.51 | 13.3 | 13.53 | 13.82 | 17.31 | 18.25 | 18.01 | 18.36 | 18.3 | 20.45 | 20.53 | 20.95 | 21.4 | 20.87 | 23.78 | 22.12 | 22.06 | 23.58 | 22.56 | 24.81 | 24.83 | 24.24 | 19.57 |
| NL24 | 13.2 | 13.87 | 13.77 | 13.74 | 13.32 | 17.29 | 17.56 | 17.76 | 17.52 | 17.83 | 20.56 | 20.54 | 19.95 | 20.1 | 19.88 | 21.56 | 21.66 | 22.03 | 21.97 | 22.18 | 23.07 | 22.93 | 23.14 | 18.93 |
| NL25 | 13.33 | 13.06 | 13.01 | 12.93 | 12.75 | 17.21 | 17.61 | 17.52 | 17.87 | 17.65 | 21.5 | 21.57 | 21.59 | 21.45 | 21.05 | 23.79 | 24.15 | 23.97 | 23.32 | 24.38 | 26.17 | 27.11 | 26.24 | 19.97 |
| NL26 | 13.05 | 12.96 | 13.05 | 12.96 | 13.23 | 17.19 | 17.11 | 17.39 | 17.71 | 17.38 | 19.96 | 19.42 | 19.5 | 20.04 | 19.96 | 21.95 | 22.97 | 22.92 | 22.65 | 21.9 | 24.55 | 24.2 | 24.18 | 18.97 |
| NL27 | 13.27 | 13.16 | 13.37 | 13.52 | 13.04 | 17.08 | 17.15 | 17.15 | 16.64 | 17.52 | 19.37 | 19.16 | 19.57 | 20.25 | 19.55 | 21.37 | 22.22 | 21.87 | 21.94 | 21.55 | 23.79 | 23.61 | 23.5 | 18.68 |
| NL28 | 12.99 | 13.21 | 13.02 | 13.47 | 13.05 | 16.93 | 16.85 | 16.97 | 16.87 | 17.11 | 19.25 | 19.14 | 19.62 | 19.6 | 19.41 | 21.13 | 20.98 | 20.71 | 21.4 | 21.05 | 22.28 | 22.8 | 21.73 | 18.24 |

Appendix C

- NetLogo Code

```
extensions [nw]

turtles-own[
  opinion
  acquiredopinion
  spreadrate
  switchrate]

globals[target]

to setup
  clear-all

  reset-ticks

  ask patches [set pcolor white]

  nw:generate-watts-strogatz turtles links NumberOfTurtles NeighborhoodSize
  RewireProb

  [ fd 10

    set color grey

    set shape "circle"

    ; set spreadrate (10 + random 90) / 100

    ; set switchrate (1 + random 99) / 100

    set spreadrate 0.4

    set switchrate 0.2

    set opinion 0]

  ask n-of (int InnovatorFraction * NumberOfTurtles) turtles [

    set spreadrate 0.6

    set switchrate 0.4

  ]

end

to go
```

```

diffusion
  if ticks = 0
    [rumorstart]
    if [opinion] of target < 0
      [correcting]
    color-map
  tick
  if [opinion] of min-one-of turtles [opinion] > 0 [stop]
  if [opinion] of max-one-of turtles [opinion] < 0 [stop]
  if ticks > 300 [stop]
end

to diffusion
  ask turtles[acquireopinion]
  ask turtles[updateopinion]
end

to acquireopinion
  let carry 0
  let me self
  let avglink mean [link-length] of my-links
  ask in-link-neighbors[
    let rate [link-length] of link-with me / avglink
    set carry carry + opinion * rate]
  set acquiredopinion (carry / count in-link-neighbors)
end

to updateopinion
  if abs acquiredopinion > abs opinion
    [set opinion (acquiredopinion * spreadrate + opinion * (1 - spreadrate))]
  if abs acquiredopinion <= abs opinion
    [set opinion (acquiredopinion * switchrate + opinion * (1 - switchrate))]

```

```

end

to rumorstart

  ask one-of turtles[

    set target one-of turtles with [who != [who] of myself]

    set opinion -100

    set shape "triangle"

  ]

  ask target [

    set color green

    set spreadrate 0.4

    set switchrate 0.01

    set shape "target"

  ]

end

to correcting

  ask target [set opinion 100]

end

to color-map

  ask turtles[

    if opinion < -0 [set color pink]

    if opinion < -5 [set color magenta + 1]

    if opinion < -10 [set color red]

    if opinion > 0 [set color cyan]

    if opinion > 5 [set color sky]

    if opinion > 10 [set color blue]

  ]

end

```