2.) Let X be a $n \times d$ matrix (n is samples & d is the no. of features)

Assuming, mean is subtracted from X.

Covariance matrix $= C = X^T X / n$.

Now, it is a positive definite matrix and hence diagonalizable.

$C = VLV^T$ where V is matrix of eigenvectors & L is diagonal matrix of eigenvalues.

Here, XV is the new compressed data.

Now, using SVD, $X = UDV^T$, $C = VDU^TUDV^T/n$

$$= \frac{VD^2V^2}{n}$$

Now, comparing it with VLV^T, we can intuitively find that V is the matrix of principal components / eigen ~~values~~ vectors.

& $L = \frac{D^2}{n}$ and compressed data is $XV = UD$.

So, this is how we can use the SVD of the data matrix to do dimensionality reduction.