Review of Optimization-Based Scheduling Method for Agile Earth-Observing Satellite Constellation

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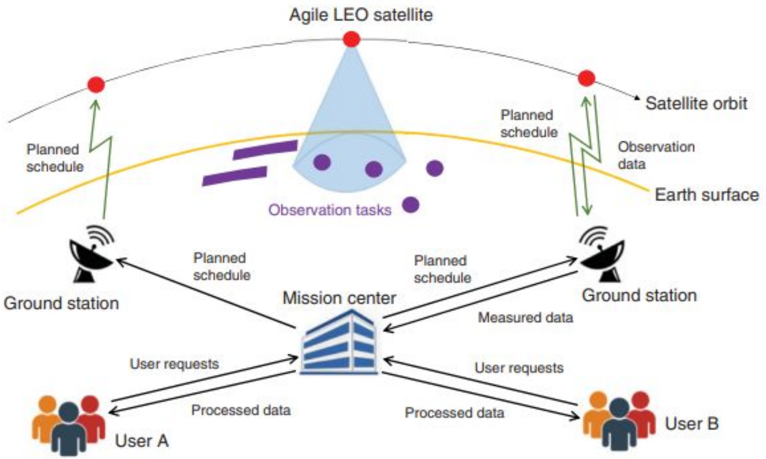
Preface

This presentation is a review of Optimization-Based Scheduling Method for Agile Earth-Observing Satellite Constellation which is co-authored by Doo-Hyun Cho, Jun-Hong Kim, Han-Lim Choi and Jaemyung Ahm from Korean Advance Institute of Science and Technology, Daejeon. It was published in Journal of Aerospace Information Systems in 2018. The nomenclature used in this ppt is same as used in publication to leave no scope for any confusion. Figures used are taken from journal itself. Any other publication is mentioned as and when referred. Initially problem statement is introduced, then literature survey is discussed leading to problem formulation and constraints assumed in the publication.

Problem Statement : Mission Overview

- The mission is formulation for task scheduling of a constellation of low Earth orbit Earth observing satellites
- The user requests for Earth observing data is defined by location, geometric configuration, resolution, priority etc
- Request can be of a single spot observation mode or wide area coverage mode
- Requests are weighted differently based on profits and scientific values
- Data is downloaded at each ground station from the satellite passing over it



Literature Review

- As part of literature review section of the paper, many scheduling optimization algorithms are discussed and mentioned. Discussing a few here:
- A few Heuristic approaches:

  Priority-based algorithm [1]   Rules-based algorithm[2]   Tabu search algorithm[3]

- Mixed Integer Programming[4]
- Mixed Integer Linear Programming[5]
- Successive Sublimation Dynamic Programming[6]
- Heuristic + MILP based algorithm[7]

[1] Wang, P., Reinelt, G., Gao, P., and Tan, Y., “A Model, a Heuristic and a Decision Support System to Solve the Scheduling Problem of an Earth Observing Satellite Constellation,”
[2] Wang, J.-M., Li, J.-F., and Tan, Y.-J., “Study on Heuristic Algorithm for Dynamic Scheduling Problem of Earth Observing Satellites,”
[3] Bianchessi, N., Cordeau, J.-F., Desrosiers, J., Laporte, G., and Raymond, V., “A Heuristic for the Multi-Satellite Multi-Orbit and Multi-User Management of Earth Observation Satellites,”
[4] Spangelo, S., Cutler, J., Gilson, K., and Cohn, A., “Optimization-Based Scheduling for the Single-Satellite, Multi-Ground Station Communication Problem,”
[5] google search majorly
[6]Tanaka, S., and Fujikuma, S., “A Dynamic-Programming-Based Exact Algorithm for General Single-Machine Scheduling with Machine Idle Time,”
[7] Kennedy, A. K., and Cahoy, K. L., “Performance Analysis of Algorithms for Coordination of Earth Observation by CubeSat Constellations,”

Literature Review

1. **Priority based Algorithms**: Priority-based heuristic algorithms can be used to determine the priority of different tasks or satellite passes that need to be scheduled. Tasks are assigned a priority value based on certain criteria such as their importance, urgency, or resource requirements
2. **Rules-based Algorithms:** These algorithms are based on a set of pre-defined rules or criteria that are used to determine the scheduling order of satellite passes. Various rules can be combined and weighted to create a more complex rule-based heuristic algorithm that considers multiple factors when scheduling satellite passes. In the paper referred a heuristic rule of max-contention for retraction and a heuristic rule of min-occupation for insertion is designed
3. **Tabu search Algorithms:** The basic idea behind tabu search is to maintain a set of tabu moves or forbidden solutions that have already been visited by the algorithm in the past. By avoiding these solutions, the algorithm can explore new solutions and avoid getting stuck in local optima. In the paper exploration, intensification and diversification are the steps taken for the same

Literature Review

1. **Mixed Integer Programming**: MIP stands for Mixed Integer Programming, and it is a mathematical optimization technique used to solve problems with both continuous and discrete decision variables. The MIP formulation for EOS scheduling involved modeling the problem as an objective function that seeks to maximize the total download data, subject to a set of constraints that reflect the physical and operational limitations of the power system.

2. **Mixed Integer Linear Programming:** MILP stands for Mixed Integer Linear Programming, which is a specific type of optimization problem that involves finding the optimal solution to a linear program with some of the decision variables restricted to take on only integer values. MILP problems are typically solved using branch-and-bound or branch-and-cut algorithms that iteratively search the space of feasible solutions and identify the optimal solution.

3. **Successive Sublimation Dynamic Programming:** To cope up with high space requirement problem of dynamic programming, SSP was introduced. It has sequential decision process

4. **Heuristic + MILP algorithm:** In this approach MILP based formulation is performed for onboard planning is restricted to single satellite and heuristic is used for crosslink between nearby satellites. Its an efficient exact algorithm for single machine problem expanded to multiple using heuristic

Problem Formulation

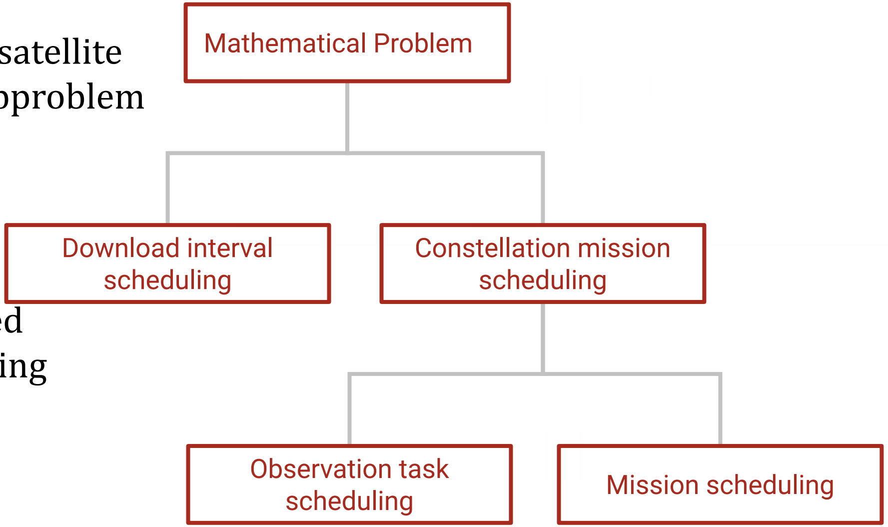
To reduce the complexity of the problem, it is divided into two subproblems and Two-step Binary Linear Programming approach is used:

- **Download Interval Scheduling:**
  The data download time intervals between each satellite and ground station are allocated through this subproblem

- **Constellation Mission Scheduling:**

  Observation Task Scheduling:
  Observation tasks of each satellite is scheduled and aim is to make maximum profit, considering unrestricted data and energy

  Mission Scheduling:
  This subproblems deals with observation scheduling problem and energy, data constraints (upper and lower limits)



Problem formulation - Download Interval Scheduling

Objective function:

$$\max\left[w_1\sum_{s\in S}\sum_{g\in G}\sum_{k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}} p^G_{s,g,k}+(1-w_1)Z\right]$$

$$Z\leq\sum_{g\in G}\sum_{k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}} p^G_{s,g,k}\quad\forall s\in S$$

Maximize the sum of all download time of all satellites and minimum of download time of each satellite with different weights whose sum is 1

Constraints:

$$x^G_{s,g,k}M+(r^G_{s,g,k}-t^G_{s,g,k})\leq M\quad\forall s\in S,\quad g\in G,\quad k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}$$

$$x^G_{s,g,k}M+(t^G_{s,g,k}-\delta^G_{s,g,k})\leq M\quad\forall s\in S,\quad g\in G,\quad k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}$$

Start processing time of each time window is bounded

$$(1-x^G_{s,g,k})M+p^G_{s,g,k}\leq M\quad\forall s\in S,\quad g\in G,\quad k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}$$

$$0\leq p^G_{s,g,k}\leq\delta^G_{s,g,k}-t^G_{s,g,k}\quad\forall s\in S,\quad g\in G,\quad k\in\{1,\ldots,|\mathrm{TW}^G_{s,g}|\}$$

Download processing time of each time window is bounded

$$\theta^{S2G}_{s,(g_1,k_1),(g_2,k_2)}M+(t^G_{s,g_1,k_1}+p^G_{s,g_1,k_1}+\tau^{S2G}_{s,(g_1,k_1),(g_2,k_2)}-t^G_{s,g_2,k_2})\leq M\ \forall s\in S,\quad g_1,g_2\in G,\quad g_1\neq g_2,\quad k_1\in\{1,\ldots,|\mathrm{TW}^G_{s,g_1}|\},k_2\in\{1,\ldots,|\mathrm{TW}^G_{s,g_2}|\}$$

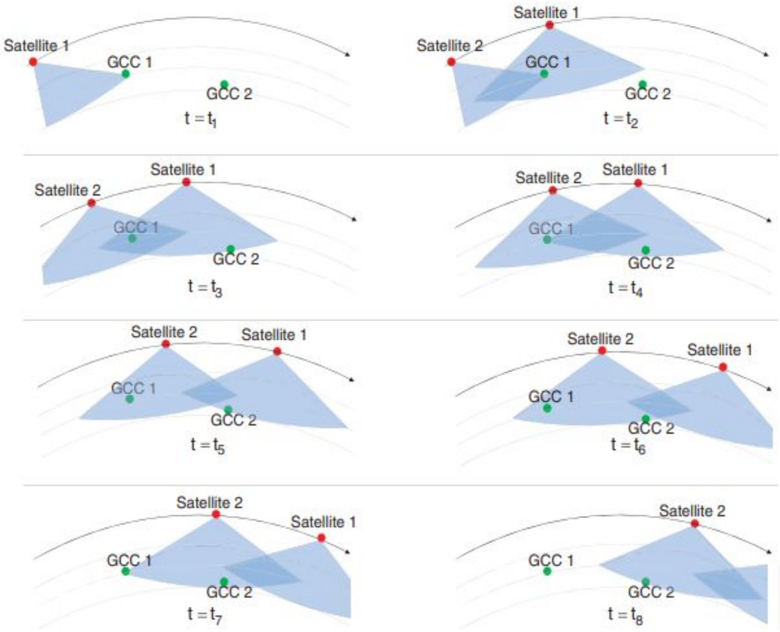
$$\theta^{S2G}_{s,(g_1,k_1),(g_2,k_2)}+\theta^{S2G}_{s,(g_2,k_2),(g_1,k_1)}=1\quad\forall s\in S,\quad g_1,g_2\in G,\quad g_1\neq g_2,\quad k_1\in\{1,\ldots,|\mathrm{TW}^G_{s,g_1}|\},\quad k_2\in\{1,\ldots,|\mathrm{TW}^G_{s,g_2}|\}$$

One satellite can perform only 1 download at a time and Each ground station can perform only on 1 download at a time

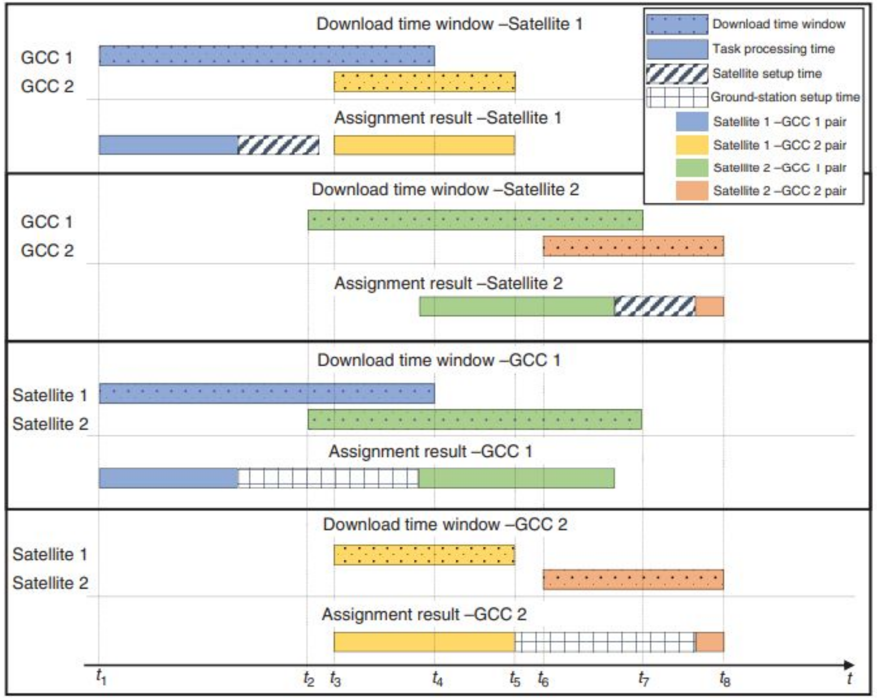
$$\theta^{G2S}_{g,(s_1,k_1),(s_2,k_2)}M+(t^G_{s_1,g,k_1}+p^G_{s_1,g,k_1}+\tau^{G2S}_{g,(s_1,k_1),(s_2,k_2)}-t^G_{s_2,g,k_2})\leq M\ \forall s_1,s_2\in S,\quad s_1\neq s_2,\quad g\in G,\quad k_1\in\{1,\ldots,|\mathrm{TW}^G_{s_1,g}|\},\quad k_2\in\{1,\ldots,|\mathrm{TW}^G_{s_2,g}|\}$$

$$\theta^{G2S}_{g,(s_1,k_1),(s_2,k_2)}+\theta^{G2S}_{g,(s_2,k_2),(s_1,k_1)}=1\quad\forall s_1,s_2\in S,\quad s_1\neq s_2,\quad g\in G,\quad k_1\in\{1,\ldots,|\mathrm{TW}^G_{s_1,g}|\},\quad k_2\in\{1,\ldots,|\mathrm{TW}^G_{s_2,g}|\}$$

Problem formulation - Download Interval Scheduling



Download task scheduling with two satellites and two ground station



The visibility time windows & assignment results are shown for each satellite

Problem formulation - Observation Task Scheduling

Objective function:

$$\max \sum_{j \in J} \sum_{s \in S} \sum_{k \in \{1,\ldots,|\mathrm{TW}^O_{j,s}|\}} w^O_j x^O_{j,s,k}$$

To maximize the sum of profits of every on time completed observation, weights denote profit and x are binary variable which is 1 for completion of that observation

Constraints:

$$\mathbb{P}_{j_1,j_2} M + \left(t^O_{j_1} + \sum_{k_1 \in \{1,\ldots,|\mathrm{TW}_{j_1,s}|\}} x_{j_1,s,k_1} \cdot p^O_{j_1,s} - t^O_{j_2} \right) \leq M \quad \forall\, j_1, j_2 \in J, \quad j_1 \neq j_2, \quad s \in S$$

Following task should be assigning after preceding task is completed, given precedence indicator is 1

$$(1 - \mathbb{C}_{j,s}) M + x_{j,s,k} \leq M \quad \forall\, j \in J, \quad s \in S, \quad k \in \{1, \ldots, |\mathrm{TW}^O_{j,s}|\}$$

If the capacity (data and energy) of a satellite isn't enough for a task, that task can't be done by the satellite

$$\theta^O_{j_1,j_2,s} M + \left(t^O_{j_1} + \left(\sum_{k_1 \in \{1,\ldots,|\mathrm{TW}_{j_1,s}|\}} \sum_{k_2 \in \{1,\ldots,|\mathrm{TW}_{j_2,s}|\}} x_{j_1,s,k_1} \cdot \left(p^O_{j_1,s} + \tau^{S2O}_{s,(j_1,k_1),(j_2,k_2)} \right) \right) - t^O_{j_2} \right) \leq M \ \forall\, j_1, j_2 \in J, \ \ j_1 \neq j_2, \ \ s \in S$$

$$\theta^O_{j_1,j_2,s} + \theta^O_{j_2,j_1,s} = 1 \quad \forall\, j_1, j_2 \in J, \quad j_1 \neq j_2, \quad s \in S$$

Only one observation can be done by a satellite at a time and one task has to follow the other, can't overlap

$$\theta^{wj}_{j,s,k} M + (r^O_{j,s,k} - t^O_j) \leq M \quad \forall\, j \in J, \quad s \in S, \quad k \in \{1, \ldots, |\mathrm{TW}^O_{j,s}|\}$$

The start time of the task should be after the start time of the window

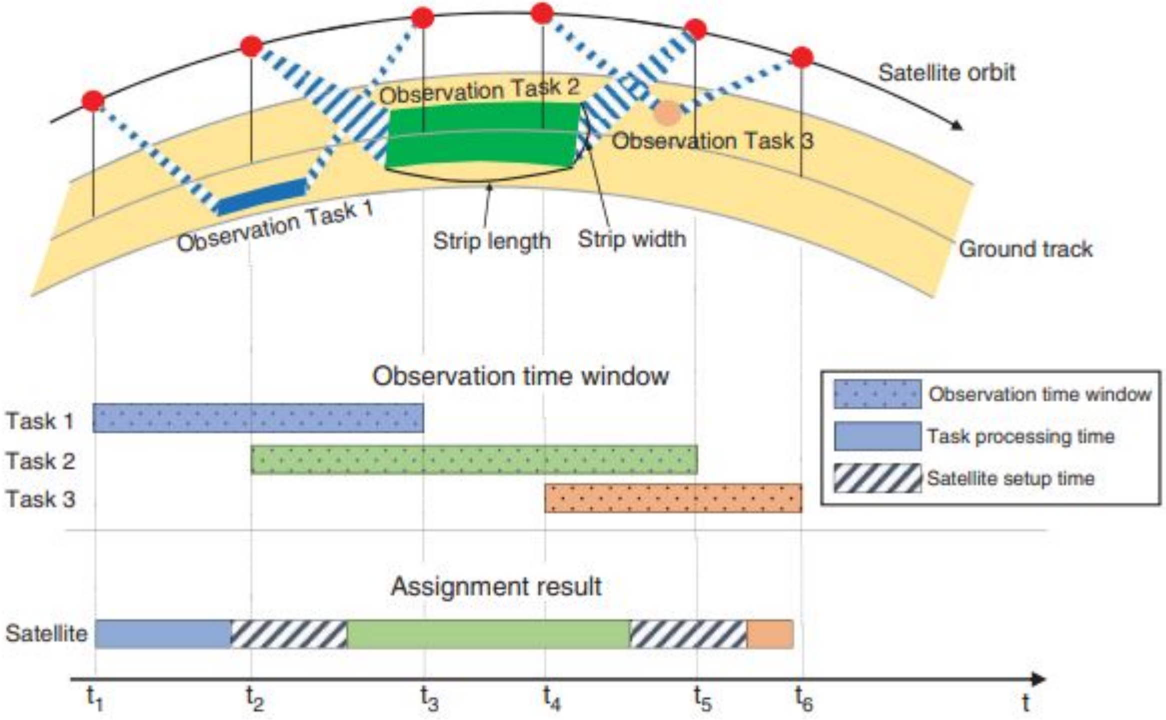
$$\theta^{jw}_{j,s,k} M + (t^O_j + p^O_{j,s} \cdot x^O_{j,s,k} - \delta^O_{j,s,k}) \leq M \quad \forall\, j \in J, \quad s \in S, \quad k \in \{1, \ldots, |\mathrm{TW}^O_{j,s}|\}$$

The end time of the task should be before the endtime of the window

$$M + (\theta^{wj}_{j,s,k} + \theta^{jw}_{j,s,k} - 2) \geq x^O_{j,s,k} M \quad \forall\, j \in J, \quad s \in S, \quad k \in \{1, \ldots, |\mathrm{TW}^O_{j,s}|\}$$

To make sure, thetas (support variables determining if the observation is true for the window) are true for x = 1

Problem formulation - Observation Task Scheduling



Observation scheduling problem

Problem formulation - Mission Scheduling

Objective function:

$$\max \left[w_2 \sum_{j\in J} \sum_{s\in S} \sum_{k\in\{1,\dots,|TW^O_{j,s}|\}} w^O_j x^O_{j,s,k} + (1-w_2) \sum_{s\in S} \sum_{i_s\in I_s} \Delta d^D_{i_s} \right]$$

Maximize sum of profits of every on time completed observation and the sum of all data changes when respective observation is completed (total data downloaded, which was excluded in Observation task scheduling part)

$$d_{0_s} = d^{\text{start}}_s \quad \forall\, s \in S$$

$$d^{\min}_s \le d_{i_s} \le d^{\max}_s \quad \forall\, s \in S, \qquad i_s \in I_s$$

$$d_{i_s+1} = d_{i_s} + \Delta d^O_{i_s} \cdot x^O_{j,s,k} - \Delta d^D_{i_s} \quad \forall\, s \in S, \quad i_s \in I_s$$

Initialising data stored for each satellite, data stored at all times should be within upper and lower bounds of the system and data stored after a task should be date at the time of previous task and new data acquired

$$e_{0_s} = e^{\text{start}}_s \quad \forall\, s \in S$$

$$e^{\min}_s \le e_{i_s} \le e^{\max}_s \quad \forall\, s \in S, \qquad i_s \in I_s$$

$$e_{i_s+1} = \min(e^{\max}_s, e_{i_s} - \Delta e^O_{i_s} \cdot x^O_{j,s,k} - \Delta e^D_{i_s} + \Delta e^{\text{sun}}_{i_s}) \quad \forall\, s \in S, \qquad i_s \in I_s$$

Initialising the energy stored, limiting the energy in the bounds at all times. And energy stored after a task is energy during the preceding task minus energy consumed for observation and downloading plus gained energy

Thank You

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Appendix : Big M method

In operations research, the Big M method is a method of solving linear programming problems using the simplex algorithm. The Big M method extends the simplex algorithm to problems that contain "greater-than" constraints. It does so by associating the constraints with large negative constants which would not be part of any optimal solution, if it exists.

The steps in the algorithm are as follows:

1. Multiply the inequality constraints to ensure that the right hand side is positive.
2. If the problem is of minimization, transform to maximization by multiplying the objective by -1.
3. For any greater-than constraints, introduce surplus s_i and artificial variables a_i (as shown below).
4. Choose a large positive Value M and introduce a term in the objective of the form $-M$ multiplying the artificial variables.
5. For less-than or equal constraints, introduce slack variables s_i so that all constraints are equalities.
6. Solve the problem using the usual simplex method.

Assumption: The time to change attitude for various tasks is assumed to be part of time window

Appendix: Nomenclature

$\mathbb{C}_{j,s}$	=	satellite-task capability indicator
d_{i_s}, e_{i_s}	=	amount of data and energy at end of i_s
$d_s^{\min}$, $e_s^{\min}$, $d_s^{\max}$, $e_s^{\max}$	=	minimum and maximum of data and energy in satellite s
d_s^{start}, e_s^{start}	=	initial amount of data and energy in satellite s
I_s, i_s	=	set of time intervals and its element that belong to satellite s, $i_s \in I_s$
J, S, G	=	set of observation tasks, satellites, and ground stations, respectively
j, s, g	=	indices of J, S, and G, respectively
$\mathbb{P}_{j_1,j_2}$	=	task precedence indicator
$p_{j,s}^O$, $p_{s,g,k}^G$	=	processing time for each satellite s to observe j and download in $\text{TW}_{s,g,k}^G$, respectively
$r_{j,s,k}^O$, $r_{s,g,k}^G$, $r_{s,k}^D$, $r_{s,k}^E$	=	release times of $\text{TW}_{j,s,k}^O$, $\text{TW}_{s,g,k}^G$, $\text{TW}_{s,k}^D$, and $\text{TW}_{s,k}^E$, respectively
$\text{TW}_{j,s}^O$, $\text{TW}_{j,s,k}^O$	=	set of time windows of satellite s related to observation j and its kth element
TW_s^D, $\text{TW}_{s,k}^D$	=	set of time windows of satellite s for downloading data and its kth element
TW_s^E, $\text{TW}_{s,k}^E$	=	set of time windows of satellite s when sunlight is available and its kth element
TW_s^G, $\text{TW}_{s,g,k}^G$	=	set of time windows of satellite s related to ground station (GS) and its kth element
t_{i_s}, t_{i_s+1}	=	start time and end time of interval i_s
t_j^O, $t_{s,g,k}^G$	=	start time of observation j and download in $\text{TW}_{s,g,k}^G$, respectively
$w_{j,s}$	=	weight (or importance) of observation task j conducted by satellite s
w_1, w_2	=	coefficients of download interval and constellation mission scheduling problem, respectively
$x_{j,s,k}^O$, $x_{s,g,k}^G$	=	indicators of assignment on $\text{TW}_{j,s,k}^O$ and $\text{TW}_{s,g,k}^G$, respectively
$\Delta d_{i_s}^O$, $\Delta e_{i_s}^O$	=	amount of data and energy to be changed if observation is completed in i_s
$\Delta e_{i_s}^{\text{sun}}$	=	amount of energy to be increased if sunlight is available in i_s
$\delta_{j,s,k}^O$, $\delta_{s,g,k}^G$, $\delta_{s,k}^D$, $\delta_{s,k}^E$	=	due times of $\text{TW}_{j,s,k}^O$, $\text{TW}_{s,g,k}^G$, $\text{TW}_{s,k}^D$, and $\text{TW}_{s,k}^E$, respectively
$\theta_{g,(s_1,k_1),(s_2,k_2)}^{G2S}$	=	support variable to force ground station g to communicate with only single satellite
$\theta_{j,s,k}^{wj}$, $\theta_{j,s,k}^{jw}$	=	support variable to determine whether observation is assigned in $\text{TW}_{j,s,k}^O$
$\theta_{j_1,j_2,s}^O$	=	support variable to prevent overlap between observations
$\theta_{s,(g_1,k_1),(g_2,k_2)}^{S2G}$	=	support variable to force satellite s to communicate with only single ground station
$\tau_{g,(s_1,k_1),(s_2,k_2)}^{G2S}$	=	setup time of antenna of ground station g from TW_{s_1,g,k_1}^G to TW_{s_2,g,k_2}^G
$\tau_{s,(g_1,k_1),(g_2,k_2)}^{S2G}$	=	setup time of satellite s from TW_{s,g_1,k_1}^G to TW_{s,g_2,k_2}^G
$\tau_{s,(j_1,k_1),(j_2,k_2)}^{S2O}$	=	setup time of satellite s from TW_{j_1,s,k_1}^O to TW_{j_2,s,k_2}^O