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Homework - 3

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Q1. find the derivative of the function

$$f(x) = 5(x+47)^2$$

→ using chain rule

$$h(x) = f(g(x))$$

then, $h'(x) = f'(g(x)) \cdot g'(x)$

Hence,

$$f'(x) = 5 * 2(x+47) * \frac{d(x+47)}{dx}$$

$$= 5 * 2(x+47) * (1+0)$$

$$= 5 * 2(x+47) * 1$$

$$= 5 * 2(x+47)$$

$$= 10(x+47)$$

$$f'(x) = 10x + 470$$

hence the derivative of the function $f(x) = 5(x+47)^2$ is $10x + 470$

Q2. determine the minimum and maximum of the function $f(x) = 3x^3 + 15x^2$ then sketch it.

→ To calculate the maximum & minimum of the function we need to find the derivative & double derivative of the given function

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using power rule $f(x) = x^n$

$f'(x) = nx^{n-1}$

solving for our given equation

$f(x) = 3x^3 + 15x^2$

first order derivative,

$f'(x) = 3 \times 3(x)^2 + 15 \times 2(x)^1$

$f'(x) = 9x^2 + 30x$

second order derivative,

$f''(x) = 9 \times 2(x) + 30 \times (1)$

$f''(x) = 18x + 30$

Now, lets set $f'(x) = 0$ in order to get values of x

$f'(x) = 9x^2 + 30x = 0$

this can be simplified as,

$3x(3x + 10) = 0$

so x can have 2 values

$3x = 0$ or $3x + 10 = 0$

$\therefore x = 0$

$3x = -10$

$x = -10/3$

$\therefore$ the values for x can be

$x = 0$ OR $x = -10/3$

To plot the graph of the given function we also need the y-coordinates.

Hence, we substitute these values back to find y-coordinates

$$f(x) = 3x^3 + 15x^2$$

for $x = 0$,

$$f(0) = 3(0)^3 + 15(0)^2$$

$$f(0) = 0$$

$f(x) = 0$ at $x = 0$ $\therefore$ Stationary point is $(0,0)$

for $x = -10/3 \approx x = -3.3$

$$f(-10/3) = 3 \times (-10/3)^3 + 15 \times (-10/3)^2$$

$$= \cancel{3} \times \frac{-1000}{\cancel{3} \times 9} + \frac{\overset{5}{\cancel{15}} \times 100}{\cancel{3} \times 3}$$

$$= \frac{-1000}{9} + \frac{500}{3}$$

$$= \frac{-1000}{9} + \frac{1500}{9}$$

$$= \frac{+500}{9}$$

$$f(-10/3) = 55.55$$

$\therefore$ the stationary point is $(-10/3, 500/9)$

$f(x) = 55.55$ at $x = -10/3$

finally to calculate Maximum & minimum of a function we substitute the calculated values of 'x' in the second order derivative $f''(x)$.

hence for $x = 0$

$$f''(0) = 18(0) + 30$$
$$= 30$$

And for $x = -10/3$

$$f''(-10/3) = 18 \times \left(\frac{-10}{3}\right) + 30$$
$$= 6 \times -10 + 30$$
$$= -60 + 30$$
$$= -30$$

Therefore

for $x = 0$, $f''(0) = 30$

$\because$ $f''(x) > 0$ evaluated at $x = 0$
the point $(0,0)$ is a local minimum for the function $f(x)$.

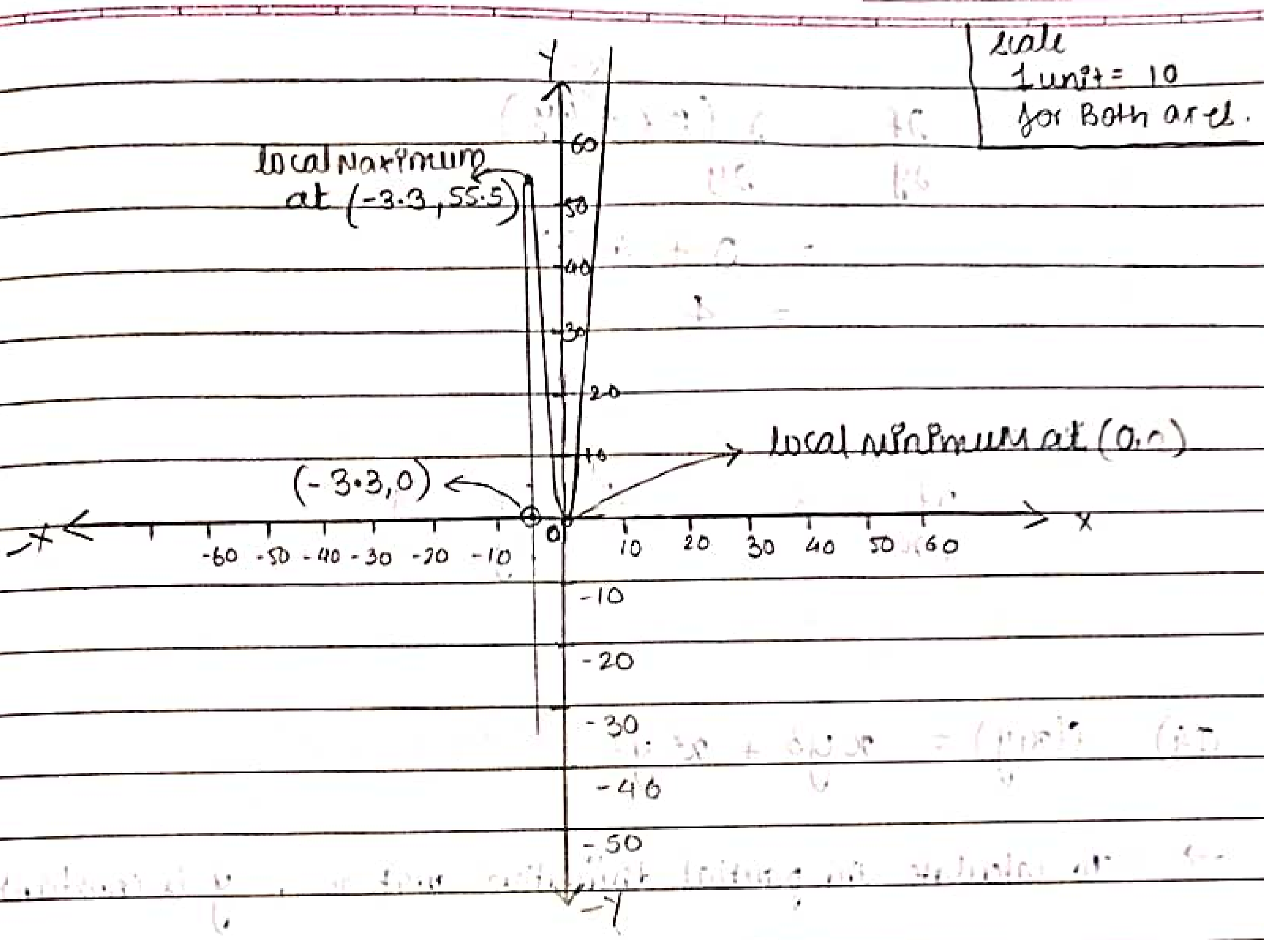
for $x = -10/3$, $f''(x) = -30$

$\because$ $f''(x) < 0$ evaluated at $x = -10/3$

the point $(-10/3, 500/9)$ is the local maximum for the function $f(x)$.

hence $\therefore$ $(0,0)$ is the minima
$\left(-10/3, \frac{500}{9}\right)$ is the maxima

Now lets sketch the graph



Q find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions:

Q3. $f(x,y) = 3x + 4y$

→ to calculate the partial derivative with respect to x we consider y as constant

$$\therefore \frac{\partial f}{\partial x} = \frac{\partial (3x+4y)}{\partial x}$$

$$= 3 \times (1) + 0$$

$$= 3$$

And to calculate partial derivative with respect to y we consider x as constant.

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$$\frac{\partial f}{\partial y} = \frac{\partial (3x + 4y)}{\partial y}$$
$$= 0 + 4(1)$$
$$= 4.$$

Hence

$$\frac{\partial f}{\partial x} = 3 \quad \& \quad \frac{\partial f}{\partial y} = 4$$

Q4) $f(x,y) = xy^3 + x^2y^2$

→ To calculate the partial derivative wrt x, y is constant

$$\frac{\partial f}{\partial x} = \frac{\partial (xy^3 + x^2y^2)}{\partial x}$$
$$= (1) \cdot y^3 + 2x \cdot y^2$$
$$= y^3 + 2xy^2$$

To calculate partial derivative wrt y, x is constant

$$\frac{\partial f}{\partial y} = \frac{\partial (xy^3 + x^2y^2)}{\partial y}$$
$$= x \cdot 3y^2 + x^2 \cdot 2y$$
$$= 3xy^2 + 2x^2y.$$

hence $\frac{\partial f}{\partial x} = y^3 + 2xy^2$ & $\frac{\partial f}{\partial y} = 3xy^2 + 2x^2y$

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Q5. $f(x,y) = x^3y + e^x$

→ to calculate the partial derivative wrt x, y is constant

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^3y + e^x)$$

$$= 3x^2 \cdot y + e^x$$

$$= 3x^2y + e^x$$

to calculate the partial derivative w.r.t y, x is constant

$$\frac{\partial f}{\partial y} = \frac{\partial (x^3y + e^x)}{\partial y}$$

$$= x^3 \cdot (1) + 0$$

$$= x^3$$

Hence, $\frac{\partial f}{\partial x} = 3x^2y + e^x$ & $\frac{\partial f}{\partial x} = x^3$

Q6. $f(x,y) = x \cdot e^{2x+3y}$

→ to calculate partial derivative wrt x, y is constant

$$\frac{\partial f}{\partial x} = \frac{\partial (x \cdot e^{2x+3y})}{\partial x}$$

using product Rule for derivation;

$$\frac{\partial f}{\partial x} = \frac{\partial (x)}{\partial x} \cdot e^{2x+3y} + x \cdot \frac{\partial (e^{2x+3y})}{\partial x}$$

$$= (1) \cdot e^{2x+3y} + x \cdot e^{2x+3y} \times (2 \cdot (1) + 0)$$

$$= e^{2x+3y} + x \cdot e^{2x+3y} \cdot 2$$

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$$= e^{2x+3y} + 2x\, e^{2x+3y}$$

$$\frac{\partial f}{\partial x} = e^{2x+3y}(2x+1)$$

to calculate partial derivative wrt y, x is constant

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x \cdot e^{2x+3y}\right)$$

~~Using product~~ $= x \cdot e^{2x+3y} \cdot (0 + 3 \cdot (1))$

$$= x \cdot e^{2x+3y} \cdot 3$$

$$\frac{\partial f}{\partial y} = 3x\, e^{2x+3y}$$

hence, $\frac{\partial f}{\partial x} = e^{2x+3y}(2x+1)$, $\frac{\partial f}{\partial y} = 3x\, e^{2x+3y}$

Q7. Given the function $J(w)$:

$$J(w_0, w_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(w_0 + w_1 x^{(i)} - y_i\right)^2$$

Determine, $\frac{\partial J(w)}{\partial w_0}$ and $\frac{\partial J(w)}{\partial w_1}$

→ The given function $J(w)$ can be simplified as;

$$J(w_0, w_1) = \frac{1}{2m}\left[(w_0 + w_1 x^1 - y_1)^2 + (w_0 + w_1 x^2 - y_2)^2 + \cdots \right.$$
$$\left. \cdots + (w_0 + w_1 x^m - y_m)^2 \right]$$

i.e it will have m terms

Now lets calculate the partial derivatives for each term. with respect to w_0.

for the 1st term. $\frac{\partial (w_0 + w_1 x^1 - y_1)^2}{\partial w_0}$ ⇒ using chain rule

$$= 2(w_0 + w_1 x^1 - y_1) \times [1 + 0 - 0]$$
$$= 2(w_0 + w_1 x^1 - y_1)$$

similarly for all other terms

$$\frac{\partial (w_0 + w_1 x^2 - y_2)^2}{\partial w_0} = 2(w_0 + w_1 x^2 - y_2)$$

$$\frac{\partial (w_0 + w_1 x^m - y_m)^2}{\partial w_0} = 2(w_0 + w_1 x^m - y_m)$$

hence,

$$\frac{\partial J(w)}{\partial w_0} = \frac{1}{2m}\left[2(w_0 + w_1 x^1 - y_1) + 2\cdot(w_0 + w_1 x^2 - y_2) + \right.$$
$$\left. \cdots + 2\cdot(w_0 + w_1 x^m - y_m)\right]$$

$$= \frac{2}{2m}\left[(w_0 + w_1 x^1 - y_1) + (w_0 + w_1 x^2 - y_2) + \cdots \right.$$
$$\left. \cdots + (w_0 + w_1 x^m - y_m)\right]$$

this can be simplified as,

$$\frac{\partial J(W)}{\partial W_0} = \frac{1}{m} \sum_{i=1}^{m} (W_0 + W_1 x^i - y_i)$$

Now partial derivative of J(W) w.r.t. W_1

for the 1st term,

$$\frac{\partial (W_0 + W_1 x^1 - y_1)^2}{\partial W_1}$$

using chain rule

$$= 2 (W_0 + W_1 x^1 - y_1) * (0 + x^1 - 0)$$

$$= 2 (W_0 + W_1 x^1 - y_1) * x^1$$

$$= 2 x^1 (W_0 + W_1 x^1 - y_1)$$

Similarly for all other terms,

$$\frac{\partial (W_0 + W_1 x^2 - y_2)}{\partial W_1} = 2 x^2 (W_0 + W_1 x^2 - y_2)$$

$$\frac{\partial (W_0 + W_1 x^m - y_m)}{\partial W_1} = 2 x^m (W_0 + W_1 x^m - y_m)$$

this can be simplified as,

$$\frac{\partial J(W)}{\partial W_1} \rightarrow$$

P.T.O

$$\frac{\partial J(w)}{\partial w_1} = \frac{1}{2m}\Big[2x^1(w_0+w_1x^1-y_1) + 2x^2(w_0+w_1x^2-y_2) + \cdots + 2x^m(w_0+w_1x^m-y_m)\Big]$$

this can be simplified as

$$= \frac{1}{2m}\sum_{i=1}^{m}(2x^i)(w_0+w_1x^i-y_i)$$

$$\frac{\partial J(w)}{\partial w_1} = \frac{1}{m}\sum_{i=1}^{m}(x^i)(w_0+w_1x^i-y_i)$$

Q8. Find the derivative of the function

$$f(x) = \frac{1}{1+e^{-x}}$$

→ this equation can be simplified as,

$$f(x) = \frac{1}{1+1/e^x}$$

$$= \frac{1}{(e^x+1)/e^x}$$

$$f(x) = \frac{e^x}{(e^x+1)}$$

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Now using Quotient rule,

$$h(x) = \frac{f(x)}{g(x)} \cdot \quad h'(x) = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{g(x)^2}$$

$$\therefore \quad f'(x) = \frac{e^x (e^x+1) - (e^x+0) \cdot e^x}{(e^x+1)^2}$$

$$= \frac{e^x (e^x+1) - e^{2x}}{(e^x+1)^2}$$

$$= \frac{e^x (e^x + 1 - e^x)}{(e^x+1)^2}$$

$$= \frac{e^x \cdot (1)}{(e^x+1)^2}$$

$$= \frac{e^x}{(e^x+1)^2}$$