

가우시안 회귀 모델과 제어 장벽 함수를 활용한 불확실한 상황속 안전 경로 생성

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Safe Path Planning Under Uncertainties using Gaussian Process Model and Control Barrier function

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ABSTRACT (영문)

This paper focuses on the uncertainties of opponent vehicles in the context of autonomous driving control. Autonomous driving systems are defined as a safety critical system meaning that safety is the main concern of the system. In such systems, uncertainties are a major threat to the safety of the system. This problem is dealt by incorporating a Gaussian process model prediction of opponent vehicle states and using a control barrier function for collision avoidance in a model predictive controller. The effectiveness of the proposed controller is yet shown qualitatively. Additional research is required to show its effectiveness quantitatively. The simulation scripts can be found at https://github.com/sansoke/MPC_CBF_GP

1. Introduction

a. Motivation

Recent advancement in artificial intelligence is facilitating the development of Autonomous driving. Nevertheless, currently only level 2(SAE criteria) is released in the market even though the technology for level 3, 4 is available. This is due to risk of failure of the autonomous driving system which could lead to severe damage. One main reasons of failure is due to uncertainties. Uncertainties can lie in the system model equation, state measurement, state noise or uncertainty of opponent vehicles. This paper focuses on the uncertainty of opponent vehicles by using a Control Barrier Function(CBF) and a Gaussian process model(GP) that predicts the opponent vehicle's state. These two tools are incorporated in the model predictive controller(MPC) of the ego vehicle. The MPC takes into account of the opponent vehicle's uncertainties and creates a safe trajectory by using the CBF as a collision avoidance safety constraint.

The proposed controller is expected to enforce robust

safety to autonomous driving systems and enable commercialization of higher levels of autonomous driving.

b. Related work

This paper challenges to improve the work of [1] in terms of safety by using a CBF safety constraint. The former paper utilizes a trained GP model inside a model predictive contour controller of the ego vehicle(EV). The GP model gives the future state prediction of the opponent vehicle(target vehicle, TV) and its variance. The variance is used to create an uncertainty expanded safety boundary along side with a slack variable. The safety boundary is in the form of an ellipse where the standard deviation (square root of variance) is added to the major and minor axes. The safety boundary works as a state-wise constraint, meaning that the constraint consists of state variables. The slack variable relaxes the constraint for feasibility. Due to the nature of state wise constraints and slack variables the safety constraint formulated in [1] does not incorporate control input and also does not guarantee safety. That is, collisions can be tolerated in certain situations.

c. Contribution

This paper adds a CBF constraint to the model predictive controller with GP model (MPC–GP–CBF) to guarantee safety which is the main contribution of the paper. The MPCC–GP controller presented in [1] is the nominal controller that is compared with the one presented in this paper. The system model of EV and notions of CBF, GP, and MPC are stated in section 2–a. Unlike the work of [1] a MPC controller is used instead of a MPCC controller and the GP model in this paper uses less state variables for training. The problem formulation and construction of the controller and training of the GP model is explained in section 2. The results are shown in section 3, followed by the conclusion in section 4.

2. Research Method

a. Preliminaries

A bicycle model is used for modeling the EV dynamics. There are six state variables of the system which include the Frenet coordinates namely s , n , α , the linear velocity, and the throttle and steering of the vehicle. Frenet coordinates describe the position of the vehicle with reference to the track. The variable s is the distance along the track from the start point to the point on the track that is closest to the current position, simply put, s is the track progress. The variable n is the lateral deviation and α is the heading error between the heading of the vehicle and the tangent of the track at point s . The input of the system has two variables which are the derivatives of the throttle and steering. The state space model of the system is formulated as below.

$$\dot{x} = \begin{bmatrix} \dot{s} \\ \dot{n} \\ \dot{\alpha} \\ \dot{v} \\ \dot{D} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} v \cos(\alpha + C_1 \delta) \\ \frac{1 - \kappa(s)n}{v \sin(\alpha + C_1 \delta)} \\ v C_2 \delta - \kappa(s) \dot{s} \\ \frac{F_{x,d}}{m} \cos(C_1 \delta) \\ \dot{D} \\ \dot{\delta} \end{bmatrix}$$

$C_1 = 0.5$, $C_2 = 15.5$ are scaling parameters of the vehicle.

CBF is a popular method of enforcing forward invariance of a safe set for a control affine system. A control affine system is a system that is linear with respect to the control input $u(x)$.

$$\dot{x} = f(x) + g(x)u(x) \dots (1)$$

Here $f(x)$ is that system matrix and $g(x)$ is the control matrix. The state space model of EV stated above shows that the control inputs \dot{D} and $\dot{\delta}$ appear linearly. A safe set is defined as a set of states that satisfy a safety condition. In the context of autonomous driving a safe set is defined as the states that are outside of a safety boundary. The safety boundary in this paper is defined as an ellipse as below.

$$h(X) = \frac{(s - s_o)^2}{a^2} + \frac{(n - n_o)^2}{b^2} - 1$$

The s , n coordinates denote the position of EV and the ones with subscripts o denote the position of the opponent, TV. Also variables a and b are the major and minor axes of the ellipse. Then the safe set is defined as below.

$$C = \{X | h(X) \geq 0\}$$

The term forward invariance means that if the state at a certain time step is an element of the safe set C , then all the following states are also an element of the set. It can be stated as below.

$$X(t_0) \in C \rightarrow \text{for all } t \geq t_0, X(t) \in C$$

The CBF constraint is a sufficient and necessary condition for the forward invariance of the safe set C and is stated as below.

$$\dot{h} + \alpha(h) \geq 0$$

Here, $\alpha(\cdot)$ is a class k function that starts at the origin and increases monotonically. In this paper we will use $\alpha(x) = \gamma x$, where γ is a positive real number. The first term in the left side of the inequality is the time derivative of the CBF, $h(X)$. Using the state space model defined in (1) the CBF constraint is expressed as follows.

$$\frac{dh}{dx} (f(x) + g(x)u(x)) + \gamma h = L_f h + L_g h u + \gamma h \geq 0$$

The terms $L_f h$ and $L_g h$ are Lie derivatives of h which is simply the direction derivative of h in the direction of f , and g respectively. Notice that the control input is included in the CBF constraint. This acts as a constraint against the control input whereas the work of [1] uses a state-wise safety constraint. The constraint in paper [1] is formulated as below.

$$h(X) \geq 0$$

Notice that the control input isn't included in this inequality constraint. This is the main difference between the CBF constraint used in this paper and the state-wise safety constraint used in [1]. The CBF constraint is used

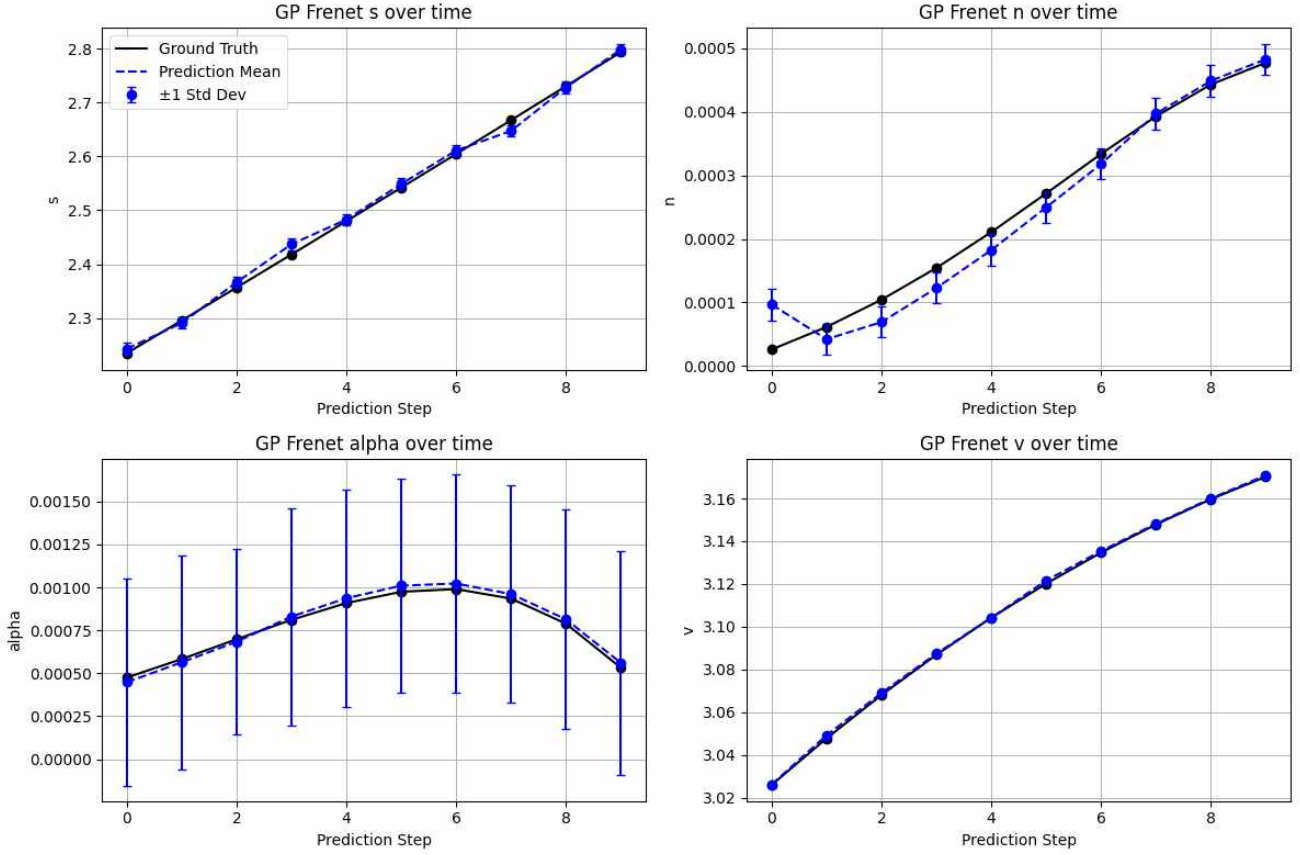


Fig. 1 : 10 step GP prediction of TV state at timestep 50 of the validation dataset.

inside the MPC for EV.

MPC is a controller that uses predictions of the future states for a predefined time horizon. The prediction are obtained from the state space model equation used as an equality constraint inside the MPC. A performance index also called an objective function or cost function is formulated as the quadratic form of error between the reference state and predicted states. The cost function also includes a quadratic form of the control input. The optimal input vector is a set of control inputs at each timestep in the time horizon that minimizes the cost function. At the current step the first control input in the optimal input vector is used as the optimal control input for the current state. This process of calculating the optimal control input vector and using its first value is repeated at every step. The MPC for EV is formulated as below.

$$u^* = \underset{u}{\operatorname{argmax}} \sum_{i=1}^N (x - x_{ref})^T Q (x - x_{ref})^T + u^T R u$$

$$\dot{x} = f(x) + g(x)u(x) \quad \dots(1)$$

$$x_{\min} \leq x \leq x_{\max} \quad \dots(2)$$

$$u_{\min} \leq u \leq u_{\max} \quad \dots(3)$$

$$\dot{h} + \gamma h \geq 0 \quad \dots(4)$$

The reference state is the Frenet coordinates s and α of a predefined centerline of a lane which the vehicle is tracking. Equation (1) is the state space model used as an equality constraint. (2) is an inequality constraint for the state which confines the vehicle inside the lane and limits the amount of steering and throttle. (3) is an inequality constraint for control input which limits the rate of change of steering and throttle. (4) is the CBF inequality constraint for collision avoidance. It acts as a constraint to the control input because the time derivative of CBF contains the control input variable. Also the CBF incorporates GP model predictions of the opponent vehicle.

This paper uses GP as a regression model that finds a function that best represents a dataset. This function then gives a prediction for an input value as a multivariate gaussian distribution where the mean vector gives the predictions of each variable and the diagonal terms in the covariance matrix indicate their uncertainty. The GP model is trained to give predictions of the next 10 time step state variables of TV given the current time step state variables of EV and TV as input. The prediction follows a multivariate gaussian distribution as stated above. The CBF uses the predicted s and n values among

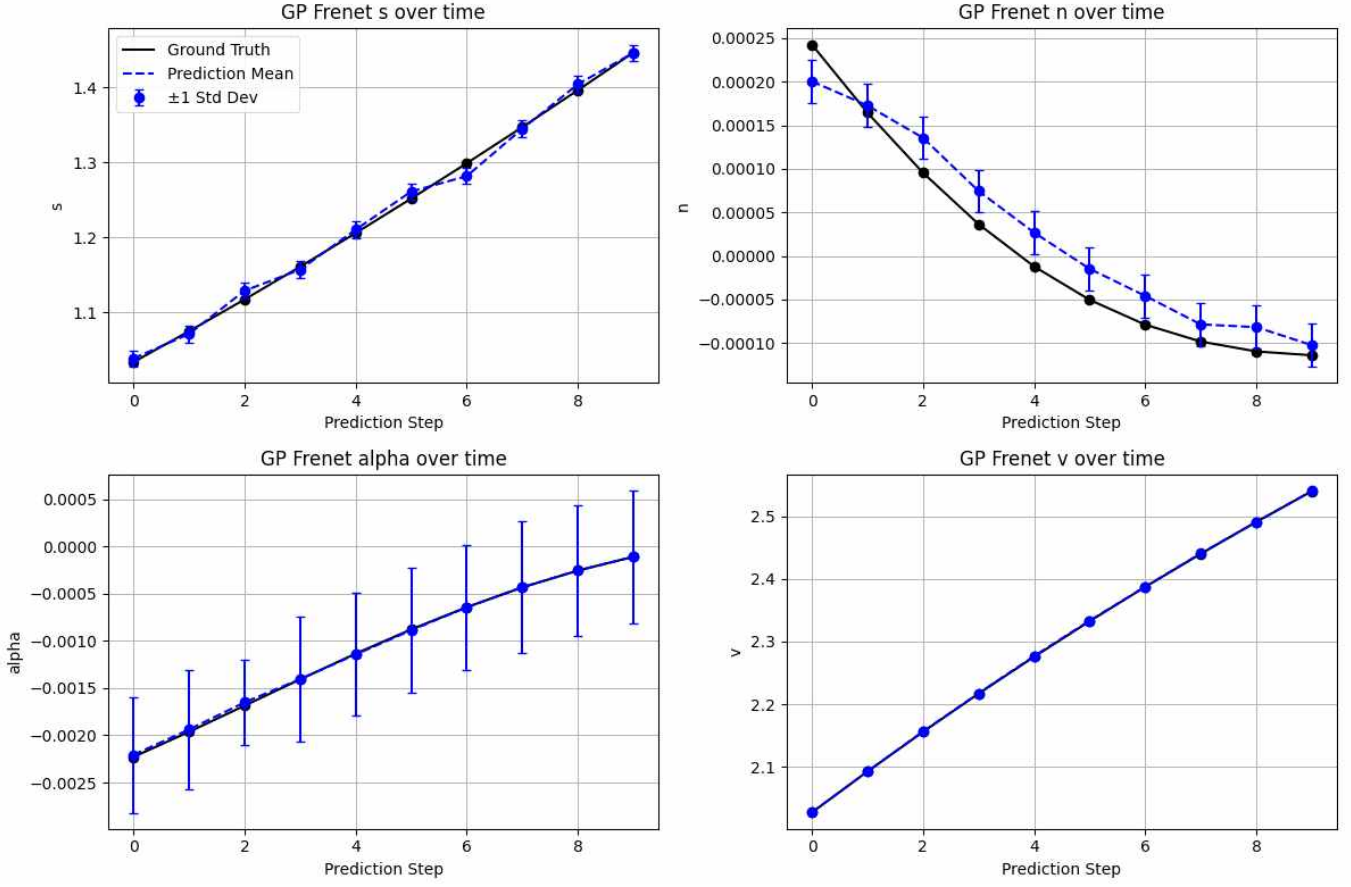


Fig. 2 : 10 step GP prediction of TV state at timestep 15 of the simulation dataset.

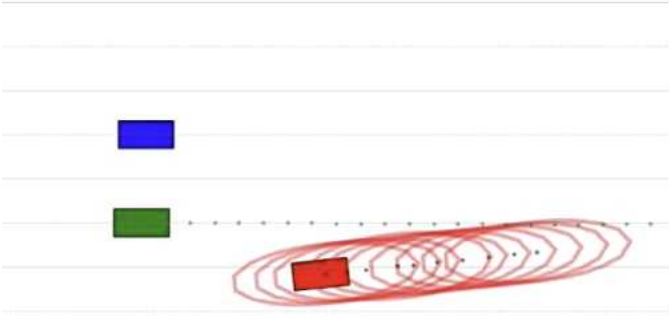


Fig 3: Simulation of MPC-GP-CBF at timestep 2 of the simulation.

the opponent state variables as the center coordinates of the ellipse. Also the standard deviation(square root of variance) of s and n are each added to the major and minor axes of the ellipse. Doing so, we can define an uncertainty expanded ellipse as below.

$$\begin{aligned}
 a_i &= a + \sqrt{\text{var}(s_i)} \\
 b_i &= b + \sqrt{\text{var}(n_i)} \\
 (i &= 1 \sim 10) \\
 h(X_i) &= \frac{(s - s_i)^2}{a_i^2} + \frac{(n - n_i)^2}{b_i^2} - 1
 \end{aligned}$$

The dataset for training and validating the GP model is obtained from a single simulation rollout of two vehicles

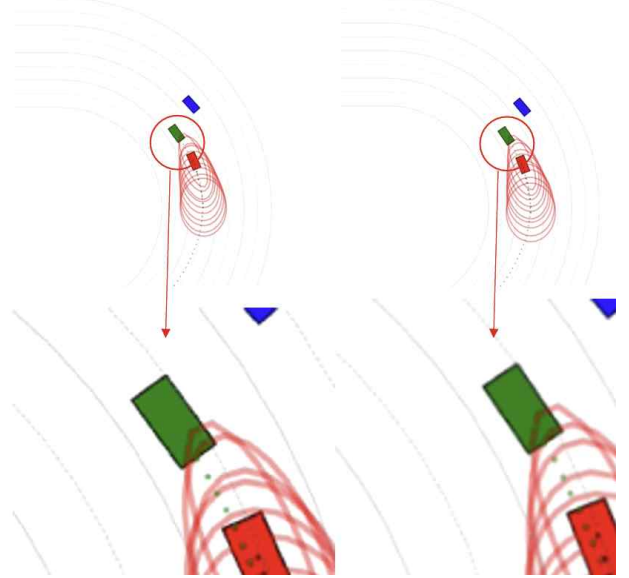


Fig 4: Simulation comparison between MPC-GP-CBF(left) and MPC-GP(right) at timestep 15 of the simulation.

each controlled by MPCs. The dataset consists of 740 sets of (X, Y) where X consists of the current time step state variables s , n , α and v for EV and TV and Y consists of the next 10 time step prediction of TV state variables obtained from the MPC of TV. The dataset is

split so that 20 percent(148 sets) is used for validation. The validation result of the GP model and simulation comparison between MPC-GP-CBF and MPC-GP result are stated in the following section.

3. Results and Discussion

a. GP validation result

The accuracy of the trained GP model is measured in terms of root mean square error(rsme), negative log likelihood(nll), and percent of predictions where the true value lies within 1 standard deviation. The rsme is 0.0067, nll is -4.5155 , and the prediction accuracy within 1 standard deviation(one sigma) is 62.11 percent. The prediction result of a random sample at timestep 50 is shown in figure 1. The predictions of s , n , v , α are shown in clockwise order. The y axis of s and n are in units of [m], v in units of [m/s] and α in units of [rad/sec]. The x axis indicates the prediction timesteps. At each timestep, predictions of 10 steps are obtained from the GP model so the x axis shows 10 timesteps. The error bar indicates the standard deviation where the length of the bar is its magnitude. Longer bars indicate higher uncertainties of the prediction. The black and blue dots indicate the ground truth and prediction values respectively. The error bar shows the range of 1 standard deviation of the prediction. At the sampled time step the prediction accuracy of s and v is low compared to n and α . Also the large error bars of α show that the prediction uncertainty of α is higher compared to the other three state variables.

b. Simulation Study

Simulations results of our controller(MPC-GP-CBF), and the nominal controller(MPC-GP) are shown in figure 3 and figure 4. Looking at figure 3, the green and red rectangles represent the EV and TV respectively. The green dots scattered in front of the EV show the predicted states of the EV obtained from the its MPC state prediction. The black dots scattered in front of the TV show the predicted states of TV obtained from the GP model. The red ellipses are the uncertainty expanded safety boundaries. Figure 4 shows a comparison of the simulation result at timestep 15 of our controller and the nominal one. At timestep 15 it is observable that the vehicles are cornering and the safety boundaries are expanded

larger due to higher uncertainties. Figure 2 which is the GP prediction at timestep 15 during the simulation shows that the standard deviation values for n is large which is accountable for the increased minor axis of the ellipses. In figure 4 the left image is the result of our controller and the right image is the result of the nominal controller. The difference is very subtle but the result of our controller is shows to maintain the safety boundary distance slightly more. The image shows that the nominal controller intrudes the ellipse narrowly more than ours.

4. Conclusion

The GP model in the work of [1] uses 5000 datasets for training shows lower error in s and n values. The accuracy of GP model is critical for formulating a feasible CBF, hence using a larger training dataset to improve the current GP model accuracy is necessary. Also the GP model of [1] is single step prediction model that repeats the prediction for ten steps to obtain a 10 step prediction whereas our GP model is trained to predict 10 steps in a single shot. This difference could also have an affect on the accuracy of the prediction result. A single step prediction model could be more applicable to MPCs. Further research is required in finding a better implementation of the GP model.

Along with improving the GP model, the comparison between ours and the nominal controller should be conducted with a quantitative metric such as the number of collisions or average distance between the EV and TV in order to clearly show the improvement in safety with the use of CBF. In theory the CBF constraint enforces the EV to maintain a safety distance, but the simulation results do not show a significant difference between a controller that uses a CBF constraint and one that doesn't. Improvement in the formulation of MPC-GP-CBF for each component of the controller to work seamlessly together remains a task.

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