Poincare Sphere and Antenna Models

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1 Introduction

In this report,the Poincare sphere is discussed. Also the models for rectangular patch antenna and some of the design considerations are discussed. [1].

2 Poincare Sphere

The state of polarization can be given by the shape of the ellipse which the wave draws and the shape of the ellipse is characterized by two parameters axial ratio(AR) and tilt angle(τ). Tilt angle is the angle that X-axis makes with the major axis of the ellipse.

A state of polarization can be defined either in terms of electrical parameters or in terms of wave parameters.

(i) Electrical parameters: $\frac{E_2}{E_1}, \delta$

(ii) Wave parameters: $\pm AR, \tau$

The quantity $\frac{E_2}{E_1}$ and $\pm AR$ are numbers and δ and τ are angles. Let us represent the quantities which are numbers also in terms of angles.

$$\gamma = tan^{-1}(\frac{E_2}{E_1}) \tag{1}$$

$$\epsilon = cot^{-1}(\pm AR) \tag{2}$$

So, now essentially we are defining the state of polarization of an electromagnetic wave by the pairs of angles (γ, δ) and (ϵ, τ).

The corresponding ranges of the angles that they can vary are:

$0 \le \gamma \le \pi$

$-\frac{\pi}{2} \le \delta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \epsilon \le \frac{\pi}{4}$

$0 \le \tau \le \pi$

Once these things are defined, then mathematicians have found a very nice way of compacting these angles. By using these angles, it is possible to construct a sphere and every point on that sphere can be uniquely represented by either (γ, δ) or (ϵ, τ). This compact representation gives some kind of visual representation of states of polarization. This sphere is called as the Poincare sphere.

In Fig(1), Poincare sphere is shown. From the reference point h, all the longitudes are measured and the angle which is in vertical direction, that is measured in terms of latitude. If I take the pair (ϵ, τ) for representation, where 2ϵ is the longitude and 2τ is the latitude of the observation point p, then all possible combinations of ϵ and τ are covered by the surface of the sphere. And every point on the sphere is having a unique combination of ϵ and τ. It means that every point on the sphere is representation a unique state of polarization.

Relationship between electrical and wave parameters

For the observation point p, we have-

$\angle moh = 2\tau$

$\angle mop = 2\epsilon$

Let us consider a horizontal plane which is passing through the equator, a circle which is passing through the centre of the sphere and a plane which is passing the observation point p. The angle of the arc which is measured from the reference point h to the observation point p is 2γ. The angle between the two planes, one containging the arc passing through the observation point and the other plane is passing through the equator, that angle is given by δ.

Now by using the spherical trigonometry, the relationship between (ϵ, τ) and (γ, δ) can be found.

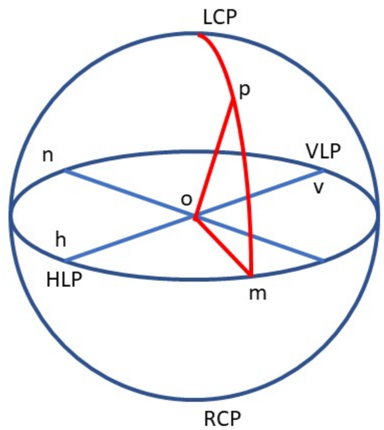

Figure 1: Poincare Sphere

Conversion from electrical to ellipse parameters

$$tan(2\tau) = tan(2\gamma)cos\delta \tag{3}$$

$$sin(2\epsilon) = sin(2\gamma)sin\delta \tag{4}$$

Conversion from ellipse to electrical parameters

$$cos(2\gamma) = cos(2\epsilon)cos(2\tau) \tag{5}$$

$$tan\delta = \frac{tang(2\epsilon)}{sin(2\tau)} \tag{6}$$

Let us see some special state of polarization in Poincare sphere.

At reference point, $\tau = 0, \epsilon = 0$ that means the tilt angle of the line is zero and the axial ratio is infinity. This reference point p corresponds to horizontal linearly polarized wave. If I move on the equator, the tilt angle τ changes, but the angle ϵ remains zero and so the axial ratio remains infinite. That means the polarization remains linear.

So, on the equator, the reference point represents a horizontally polarized wave whereas on the equator, the diagonally opposite point of the reference point represents a vertically polarized wave. Again, as we move further in the equator, the angle will go on tilting and when it comes back the line become again horizontally polarized. Thus, the points which lie on the equator of the sphere represents all linear states of polarization.

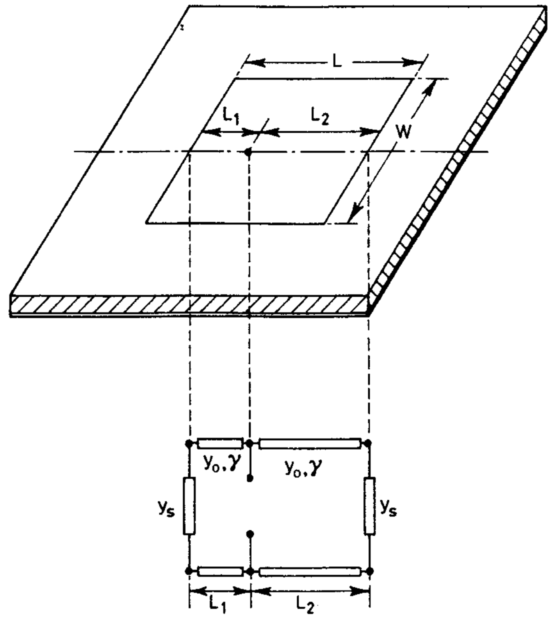

Figure 2: Probe fed rectangular micro-strip antenna and transmission line model(Equivalent circuit based on simple transmission line model)

For North pole(at the top), $2\epsilon = +90°$, so $\epsilon = +45°$, which implies axial ratio, $AR = +1$. So, it becomes a circle and tilt angle lost it meaning. And as the axial ratio with positive value of 1 corresponds to the left handed circular polarization, so the north pole represents the left handed circular polarization.

For south pole, $2\epsilon = -90°$, so $\epsilon = -45°$, which implies that the axial ratio, $AR = -1$. As negative angle is a sign of right handed sense of rotation. So, the south pole represents the right handed circular polarization.

For any point which is lying on the northern hemisphere, δ and axial ratio is positive. So, its a sign of left handed sense of rotation. But the angle ϵ is neither 45° nor 0°, so the northern hemisphere represents the left handed elliptical polarization.

Similarly, if I take any point on the southern hemisphere, it will correspond to a right handed elliptical polarization.

3 Models for Rectangular Patch Antenna

Transmission line model analysis

The antenna has a physical structure derived from a micro-strip transmission line. Therefore, the transmission line model is the first and obvious choice for the analysis and design of a rectangular patch. In this model, the micro-strip antenna is modeled as a length of the transmission line of characteristic impedance Z_0 and propagation constant $\gamma = \alpha + j\beta$. The fields vary along the length of the patch which is usually a half-wavelength, and remain constant across the width. Radiation occurs mainly from the fringing field at the open ends. The effect of radiation is accounted for by the radiation admittance called self-admittance Y_s attached at the open ends of the transmission line.

The probe-fed patch radiator along with the transmission line equivalent circuit is shown in Fig(2). The input impedance of the patch based on this equivalent circuit is easily obtained as-

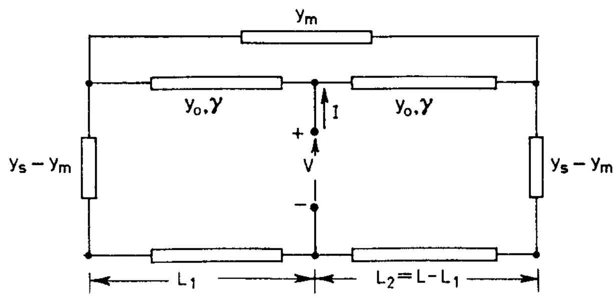

Figure 3: Equivalent circuit including mutual coupling

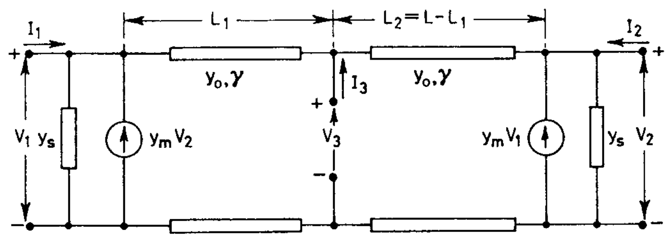

Figure 4: Three port alternative of fig.3

$$Z_{in} = jX_L + Z_1 \tag{7}$$

where X_L is the probe reactance and $Z_1(=\frac{1}{Y_1})$ is obtained from transmission line transformation of Y_s, that is,

$$Y_1 = Y_0(\frac{Y_0 + jY_s tan(\beta L_1)}{Y_s + jY_0 tan(\beta L_1)}) + (\frac{Y_0 + jY_s tan(\beta L_2)}{Y_s + jY_0 tan(\beta L_2)}) \tag{8}$$

$$L_1 + L_2 = L$$

Here β is the phase constant in a micro-strip line of width W.

The simple transmission line transformation line predicts the radiation pattern of the antenna correctly, but the input impedance only approximately.

Improved Transmission Line Model

It takes into account the mutual coupling between the radiating apertures. It is shown in Fig(3).The mutual coupling effect, denoted by Y_m is connected between the two ends of the transmission line. To solve for the voltage at the feed point, the transmission line section on either side of the feed source is converted into $\pi-$network equivalents. The resulting equivalent circuit is then simplified using the star-delta and delta-star transmission. Pues and Van de Capelle included the mutual admittances shown in Fig()4 This approach requires the equivalent circuit to be a three-port network. For a micro-strip antenna fed at one of the radiating edges, the input admittance is obtained as-

$$Y_{in} = \frac{Y_0^2 + Y_s^2 - Y_m^2 + 2Y_0 Y_s coth(\gamma L) - 2Y_0 Y_m csch(\gamma L)}{Y_s + Y_0 coth(\gamma L)} \tag{9}$$

4 Design Consideration for Rectangular Patch Antenna

Substrate selection

The first design step is to choose a suitable dielectric substrate of appropriate thickness h and loss tangent. A thicker substrate, besides being mechanically strong, will increase the radiated power, reduce conductor loss, and improve impedance bandwidth. However, it will also increase weight, dielectric loss, surface wave loss, and extraneous radiations from the probe feed. A rectangular patch antenna stops resonating for substrate thickness greater than $0.11\lambda_0$ ($\epsilon_r = 2.55$) due to inductive reactance of the probe feed. The substrate dielectric constant ϵ_r plays a role similar to that of substrate thickness. A low value of ϵ_r for the substrate will increase the fringing field at the patch periphery, and thus the radiated power. Therefore, substrates with $\epsilon_r \leq 2.5$ are preferred unless a smaller patch size is desired. An increase in the substrate thickness has a similar effect on antenna characteristics as a decrease in the value of ϵ_r. A high loss tangent increases dielectric loss and therefore reduces antenna efficiency. The four most commonly used substrate materials are honeycomb($\epsilon_r = 1.07$), Duroid($\epsilon_r = 2.32$), quartz($\epsilon_r = 3.8$), and alumina($\epsilon_r = 10$).

Table 1: Electromagnetic Spectrum

Region	**Wavelength Range**	**Applications**
Radio Waves	1 mm to 100 km	Radio broadcasting, radar
Microwaves	1 mm to 1 m	Microwave ovens, wireless communication
Infrared	700 nm to 1 mm	Remote sensing, thermal imaging
Visible Light	400 nm to 700 nm	Human vision, optical communication
Ultraviolet	10 nm to 400 nm	UV lamps, sterilization
X-rays	0.01 nm to 10 nm	Medical imaging, security screening
Gamma Rays	Less than 0.01 nm	Nuclear medicine, astronomy

5 Conclsuion

In this report, poincare sphere, a represenation of different types of polarization is being discussed. Also, the different types of models for Rectangular Patch antenna is discused.

References

[1] R. Garg, P. Bhartia, I. Bahl, and A. Ittipiboon, *Microstrip Antenna Design Handbook.* Antennas and Propagation Library, Artech House, 2001.