Colombian Warfare Model Project

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1 Introduction

Throughout the history of Colombia, different conflicts or wars have been noticed in the country. Nowadays, the war between different guerrillas and the Colombian State is remarkable since these conflicts were and continue to be an event that has impacted the entire Colombian population in one way or another throughout the last two centuries.

Moreover, it is important to be aware of the violence to which the country has been subjected, in this way, in this project a modeling of the conflict with the guerrillas, paramilitaries, drug traffickers and the National Army is proposed in order to represent a multilateral conflict. In such manner, it will be taken into account 2 alliances, army with paramilitaries and narcos with guerrilla. thereby, it will be possible to have a different point of view than the one that most of the population are used to, allowing the arrival of new ideas to improve the present situation and the future of the country.

2 Objectives

- Model a discrete time problem using differential equations in order to describe the warfare context.
- Understand the history of the war inside Colombia posing a realistic conflict situation.
- Find certain values that achieve the victory of each side, being the victory defined as the last standing side. Also find values that achieve an equilibrium state.
- Apply the knowledge acquired in the course Modeling of Dynamical Systems to model and analyze linear and non-linear models and compare their effectiveness on the representation of the problem.

3 Variables and tools used

For the development of this project, MATLAB and Simulink were used. Additionally, different variables were taken into account for both the linear and non-linear models:

Non-linear model variables:

- g_a: Portion of army killed by guerrilla.
- n_a: Portion of army killed by narcos.
- p_g: Portion of guerrilla killed by paramilitaries.
- a_g: Portion of guerrilla killed by army.
- p_n: Portion of narcos killed by paramilitaries.
- a_n: Portion of narcos killed by army.
- g_p: Portion of paramilitares killed by guerrilla.
- n_p: Portion of paramilitares killed by narcos.

Let $k_e \in \{g_A, g_P, n_A, n_P, a_G, a_N, p_N, p_G\}$

Such that $k_e = (T_k + e_{ek}), T_k, e_{ek} \in [0, 0.4]$, where T_k is the effectiveness of the army k over the terrain T and e_{ek} is the effectiveness of the army k fighting army e. Logically it follows that $e_{ke} = 0.4 - e_{ke}$, where 0.4 corresponds to the 100% of effectiveness.

To reflect if an army is alive, $b(K(t))$ will be used, as a piece-wise function for the army $K(t)$:

$$b(K(t)) = \begin{cases} 0 & K(t) = 0 \\ 1 & K(t) > 0 \end{cases}$$

For the recruitment of each army let $J \in \{a, g, p, n\}$, then $J_r \in [0.03, 0.09]$ is the constant of recruitment for the army J.
In regards to the possible advantages for the sides,

- When there is an improvement in the combat variables, it can mean that the troop or alliance is better trained that the other one or that it has better techniques.
- Concerning the recruitment variables, they show the appeal or capacity to recruit that each troop has.
- Respecting to terrain variables, it shows the experience that each infantry has.

4 Models

Let
A(k) be the population of army at a moment (day) k.
G(k) be the population of guerrilla at a moment (day) k
P(k) be the population of paramilitares at a moment (day) k
N(k) be the population of narcos at a moment (day) k

$$\mathbb{X}(k) = \begin{pmatrix} A(k) \\ G(k) \\ P(k) \\ N(k) \end{pmatrix}, \mathbb{X}(k+1) = \begin{pmatrix} A(k+1) \\ G(k+1) \\ P(k+1) \\ N(k+1) \end{pmatrix}$$

4.1 Linear model

$$\mathbb{X}(k+1) = \begin{pmatrix} 1+a_r & -g_a & 0 & -n_a \\ -a_g & 1+g_r & -p_g & 0 \\ 0 & -g_p & 1+p_r & -n_p \\ -a_n & 0 & -p_n & 1+n_r \end{pmatrix} \mathbb{X}(k)$$

To better understand this linear model, we can take:

$$A(k+1) = A(k)(1+a_r) + (-G(k)g_a - N(k)n_a)$$

where:

- $A(k)(1+a_r)$ is the army recruitment constant $a_r \in$ plus the existing soldiers.
- $(-G(k)g_a - N(k)n_a)$ represents the number of killed soldiers of the troop A, where in $G(k)g_a$ it is necessary to consider the number of aggressors in the proportion of killed soldiers.

4.2 Linear model analyis

We will use simulink to run the model of this multilateral conflict, find the state transition matrix to analyze specific intervals of the conflict and find its limit to analyze the behavior of the conflict if it goes on forever. Also try different initial sizes of each troop to see which one yield equilibrium states and which ones lead to a specific side winning the conflict.

4.2.1 Equilibrium points:

For this analysis the trace-determinant parable formula is gonna be taken into account.

$$\delta = Tr(A)^2 - 4det(A)$$

- All 4 troops have the same values on each constant:

$$\begin{pmatrix} 1.03 & -0.4 & 0 & -0.4 \\ -0.4 & 1.03 & -0.4 & 0 \\ 0 & -0.4 & 1.03 & -0.4 \\ -0.4 & 0 & -0.4 & 1.03 \end{pmatrix} \Rightarrow eigenvalues: \begin{pmatrix} 0.23 \\ 1.03 \\ 1.03 \\ 1.83 \end{pmatrix} \Rightarrow \begin{array}{r} Tr(A) = 4.12 \\ Det(A) = 0.4465 \\ \delta = 15.18 > 0 \end{array}$$

Thus, 0 is the only equilibrium point and by trace-determinant plane it is unstable.

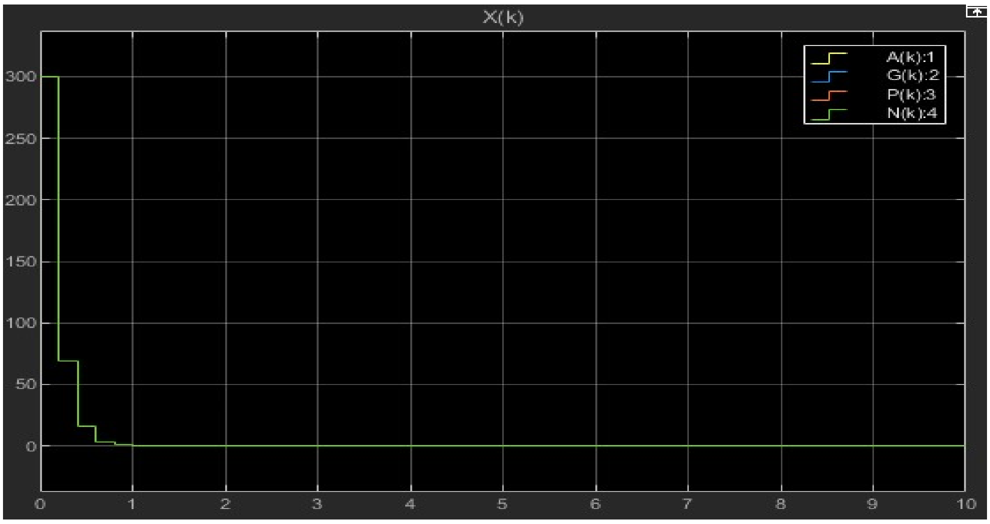

Figure 1: Every troop has the same values.

Hence, with this graph it is clear that 0 is an equilibrium point. Thereby, since all the troops have the same values, it is evidently that the war ends up in draw.

- Alliance advantage:

$$\begin{pmatrix} 1.03 & -0.3 & 0 & -0.3 \\ -0.5 & 1.03 & -0.5 & 0 \\ 0 & -0.3 & 1.03 & -0.3 \\ -0.5 & 0 & -0.5 & 1.03 \end{pmatrix} \Rightarrow eigenvalues : \begin{pmatrix} 0.2554 \\ 1.03 \\ 1.8 \\ 1.03 \end{pmatrix} \Rightarrow \begin{array}{r} Tr(A) = 4.12 \\ Det(A) = 0.489 \\ \delta = 15.01 > 0 \end{array}$$

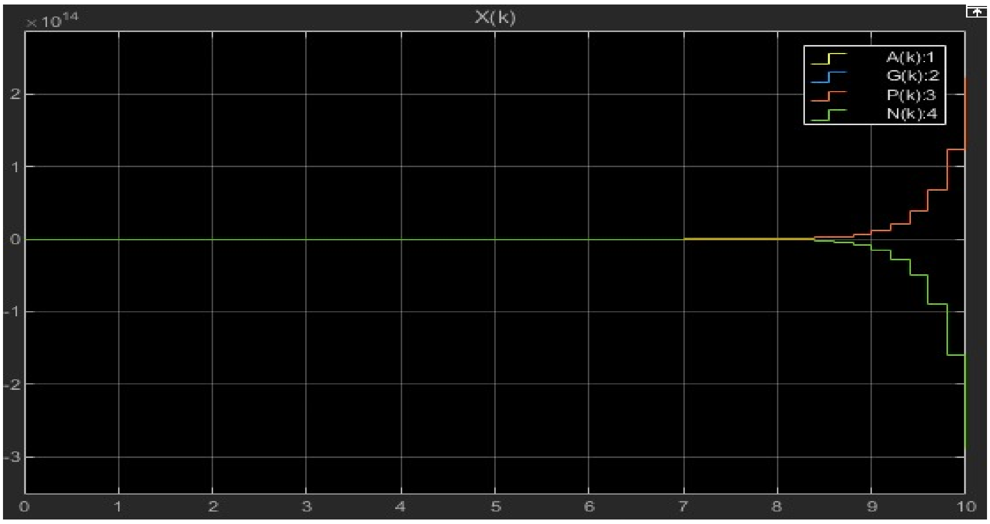

Figure 2: Bigger combat effectiveness for alliance 1: 0.3 vs 01

Thus, 0 is the only equilibrium point and by trace-determinant plane it is unstable. As the combat effectiveness is greater in one of the alliances, then since the combat effectiveness is complementary for any 2 enemy armies, this effectiveness will be lower for the Alliance 2, thus resulting in less soldiers of the other side getting killed.

Resulting in Alliance 1 killing more soldiers of the Alliance 2 and losing less soldiers in the process, eventually since this model does not consider if a Troop is still alive(number of soldiers greater than 0), eventually it will lead to negative values for alliance 2.

It follows that, when that new X(k), $X \in \{N, G\}$ is evaluated in the winner side(Alliance 1), it is going to be 1 - (negative value), thus, the winner troop will have more soldiers or how we called this phenomena, "zombies". In doing so, the winner alliance will grow rapidly while the other, Alliance will decrease in an inversely proportional way.

- Alliances balance advantages:

$$\begin{pmatrix} 1.03 & -0.45 & 0 & -0.45 \\ -0.4 & 1.03 & -0.4 & 0 \\ 0 & -0.45 & 1.03 & -0.45 \\ -0.4 & 0 & -0.4 & 1.03 \end{pmatrix} \Rightarrow eigenvalues: \begin{pmatrix} 0.1815 \\ 1.03 \\ 1.87 \\ 1.03 \end{pmatrix} \Rightarrow \begin{array}{r} Tr(A) = 4.12 \\ Det(A) = 0.36 \\ \delta = 15.52 > 0 \end{array}$$

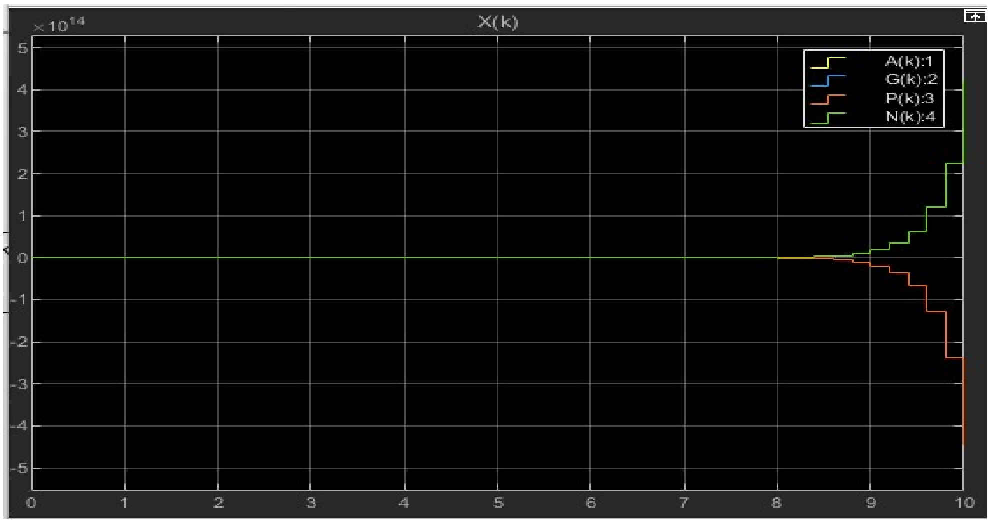

Figure 3: Combat: 0.3 vs 01, Terrain: 0.1 vs 0.35

Thus, 0 is the only equilibrium point and by trace-determinant plane it is unstable. As the combat effectiveness is greater in one of the alliances, then since the combat effectiveness is complementary for any 2 enemy armies, this effectiveness will be lower for the Alliance 1, thus resulting in less soldiers of the other side getting killed.

Resulting in Alliance 2 killing more soldiers of the Alliance 1 and losing less soldiers in the process, eventually since this model does not consider if a Troop is still alive(number of soldiers greater than 0), eventually it will lead to negative values for alliance 1.

It follows that, when that new X(k), $X \in \{A, P\}$ is evaluated in the winner side(Alliance 2), it is going to be 1 - (negative value), thus, the winner troop will have more soldiers or how we called this phenomena, "zombies". In doing so, the winner alliance will grow rapidly while the other, Alliance will decrease in an inversely proportional way.

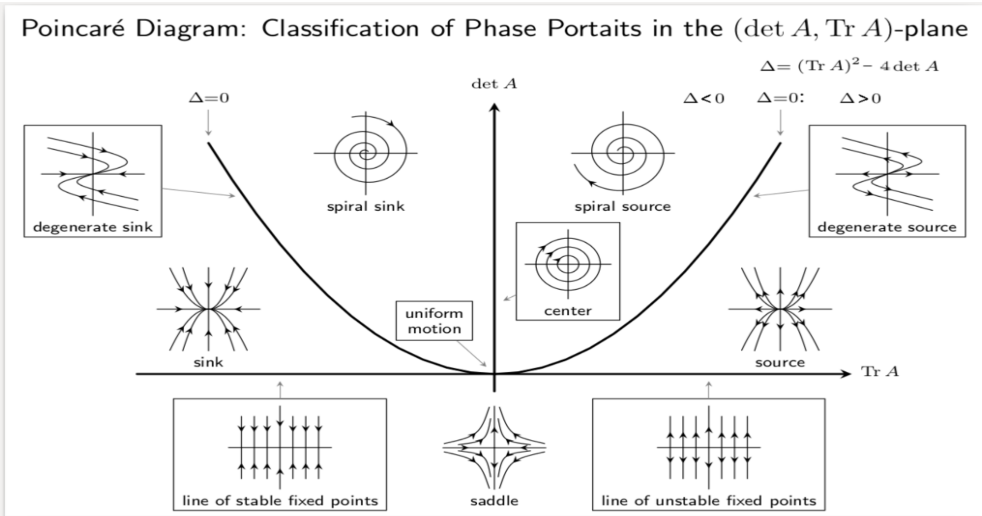

Figure 4: Pointcaré diagram - Trace determinant plane

It is also possible to notice that in all the three cases, $\Delta > 0, Tr > 0, det > 0$, thus, all of them will be located in the first quadrant and they can be considered as source. Hence, the equilibrum point 0 will be unstable.

Linear model problems

- It is not possible to recruit or kill 0.j soldiers, thus, a floor or ceiling functions are needed. Where $j \in \mathbb{Z}_{10} - \{0\}$
- Zombie Phenomena, explained later after some analysis.

4.3 Non-linear model

In order to fix the issues that the linear model has, a non-linear model is proposed:

$$\mathbb{X}(k+1) = \begin{pmatrix} \lceil 1+a_r \rceil b(A(k)) & \lfloor -g_a b(A(k)) \rfloor & 0 & \lfloor -n_a b(A(k)) \rfloor \\ \lfloor -a_g b(G(k)) \rfloor & \lceil 1+g_r \rceil b(G(k)) & \lfloor -p_g b(G(k)) \rfloor & 0 \\ 0 & \lfloor -g_p b(P(k)) \rfloor & \lceil 1+p_r \rceil b(G(k)) & \lfloor -n_p b(P(k)) \rfloor \\ \lfloor -a_n b(N(k)) \rfloor & 0 & \lfloor -p_n b(N(k)) \rfloor & \lceil 1+n_r \rceil b(N(k)) \end{pmatrix} \mathbb{X}(k)$$

To better understand it, let's take:

$$A(k+1) = (\lceil A(k)(1+a_r) \rceil + \lfloor (-G(k)g_a - N(k)n_a) \rfloor) b(A(k))$$

Where $b(H(k))$, where $H \in \{A, G, P, N\}$ is the piecewise function that acts as a boolean variable previously defined. It is necessary since when a troop dies, negative values can show up and grow in a negative way, resulting in the **zombie phenomena** previously explained.

- All 4 troops have the same values on each constant:

  Thus, 0 is the only equilibrium point and it is stable.

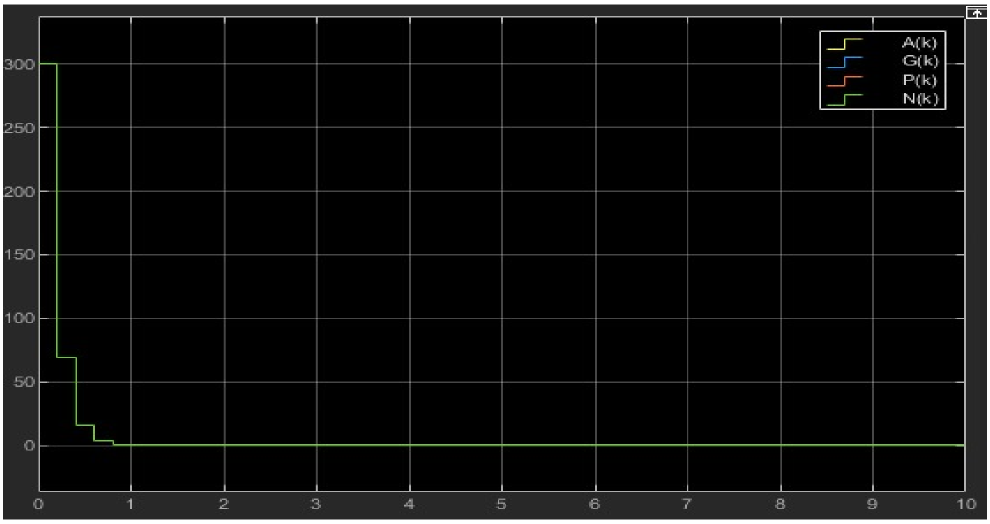

Figure 5: Every troop has the same values.

Hence, with this graph it is clear that 0 is an equilibrium point, it confirms the one we found in the linear system. Thereby, since all the troops have the same values, it is evidently that the war ends up in draw.

- Alliance advantage::

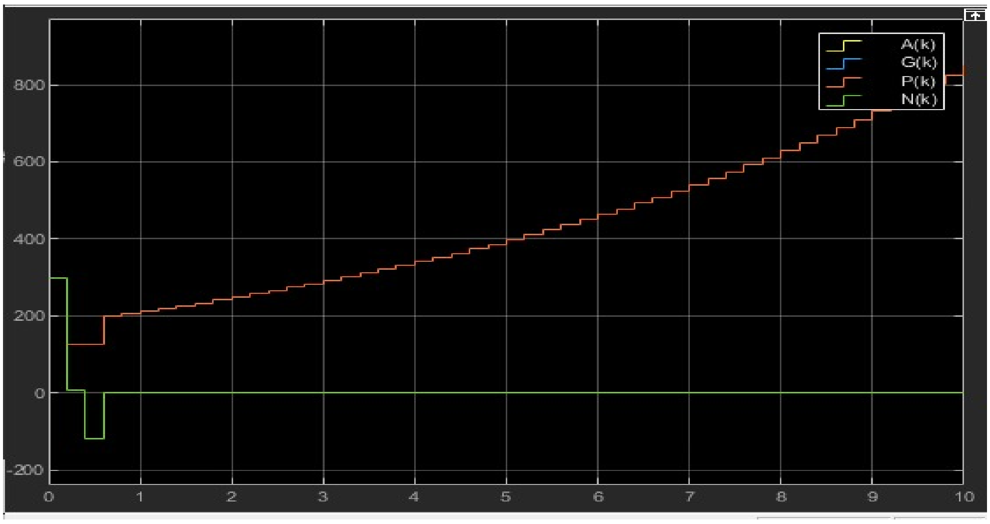

Figure 6: Alliance 1 has the combat effectiveness advantage.

Hence, with this graph it is clear that 0 is an equilibrium point. Moreover, it is possible to see the alliance 1 defeated the other side. Additionally, it can be seen that the alliance 2 at some point is below zero, however, it rapidly becomes zero, this proves that the piece-wise function b is working perfectly and that it also fix the **zombie phenomena** of the linear model.

- Alliances balance advantages:

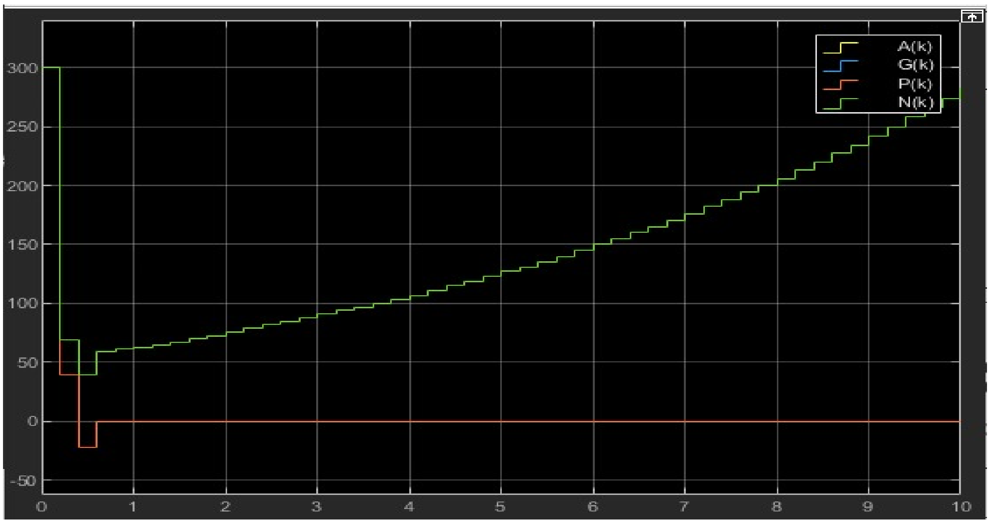

Figure 7: Combat alliance 1 vs Terrain alliance 2

Hence, with this graph it is clear that 0 is an equilibrium point. Moreover, it is possible to see the alliance 2 defeated the other side. Additionally, it can be seen that the alliance 1 at some point is below zero, however, it rapidly becomes zero, this proof that the piece-wise function b is working perfectly and that it also fix the model.

5 Conclusions

5.1 Linear model

- This model was discarded since it did not considered that if a troop has a total population of 0 or below, thus it shouldn't intervene in the warfare.
- In regards to equilibrium point this one is uniquely the 0 vector, it is remarkable that it is always defined as a source by the Trace-Determinant plane, thus, it is always unstable.
- If one of the alliances has the combat advantage, at some point, it will be the winner since it starts to grow as all the dead soldiers or "zombies" from the other side are added to it. Analogously, the losing alliance will decrease in an inversely proportional way.
- As it was seen in the previous graphs, combat effectiveness is the most influential variable since whatever alliance that has it will always win.

5.2 Non-Linear model

- All the issues in the linear model were fixed with the implementation of the piece-wise function b and the floor and ceiling functions, reaching a better representation of the problematic that we were trying to model.

- As in the linear model, it is seen that the combat effectiveness variable influence the most to determine the victory of one alliance, thus the training of each troop is essential to win any war.