Unit 2. Introduction

2.1 Control architectures

2.2 Actuator and sensor models

2.3 Vision-based control

SJK015 – Intelligent Control

Gabriel Recatalá

Bibliography

- Corke, P. (2023) Robotics, vision and control : fundamental algorithms in Python. Cham, Switzerland: Springer Nature. Chapters 2, 3, 15.

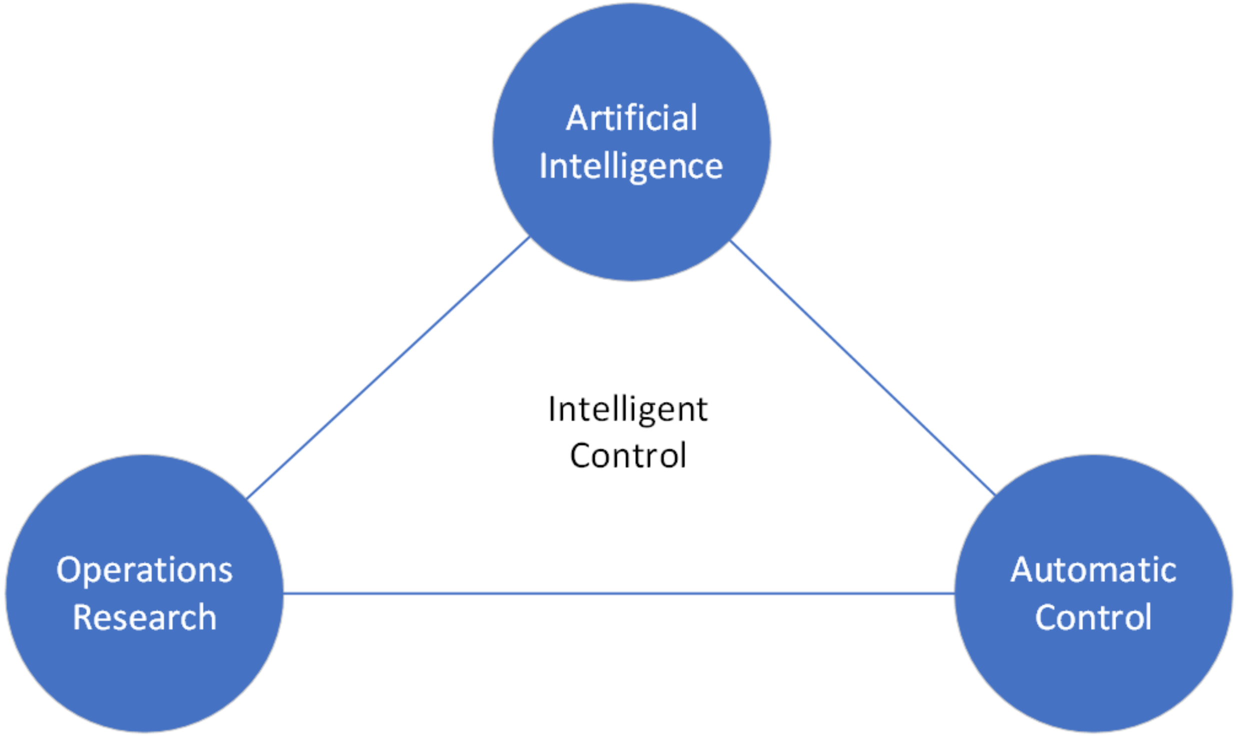
Bibliography

Articles

- Corke, P. et al. (2007) An Introduction to Inertial and Visual Sensing. The International journal of robotics research. [Online] 26 (6), 519–535.
- Four-legged friends and other robots: ESA astronaut tests avatar control between ISS and Earth. From DLR (last read: February 2024).

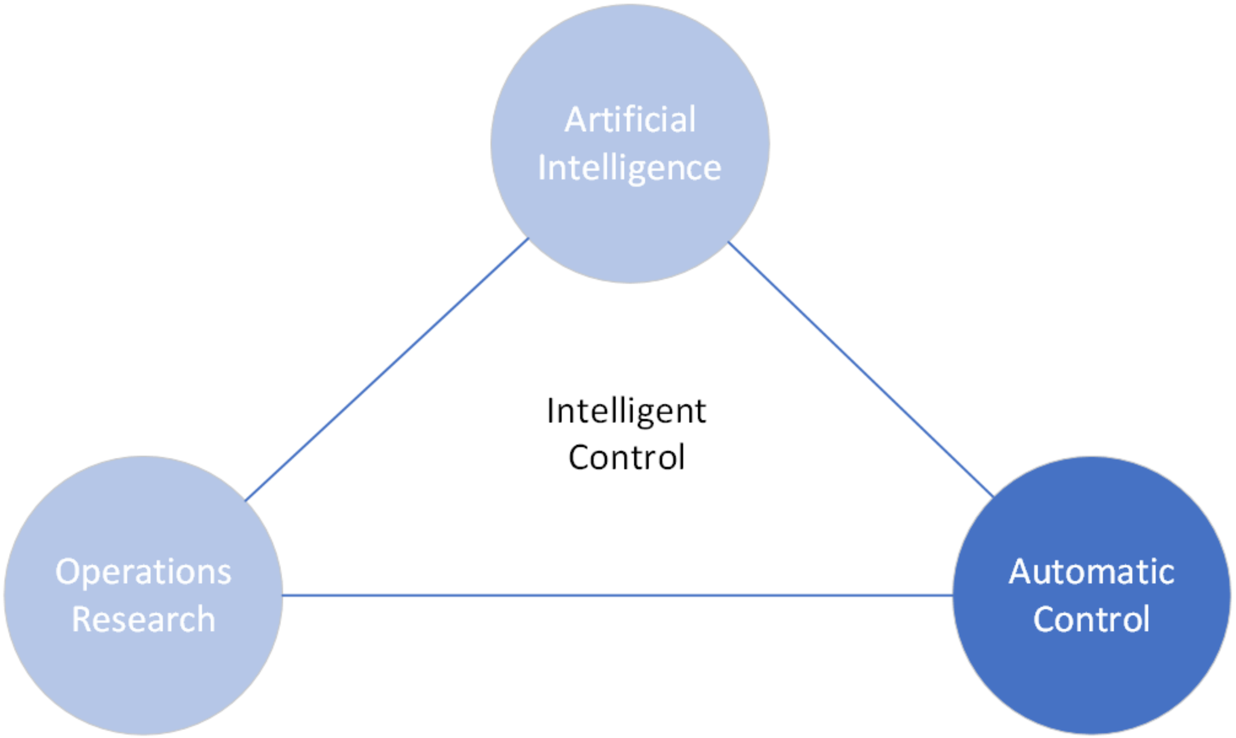
2.1 Control architectures

Intelligent Control



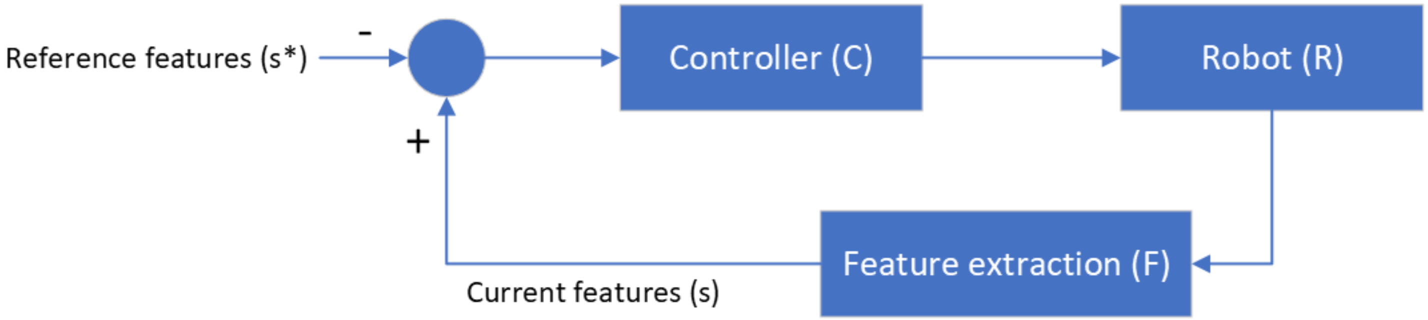
2.1 Control architectures

Automatic
Control



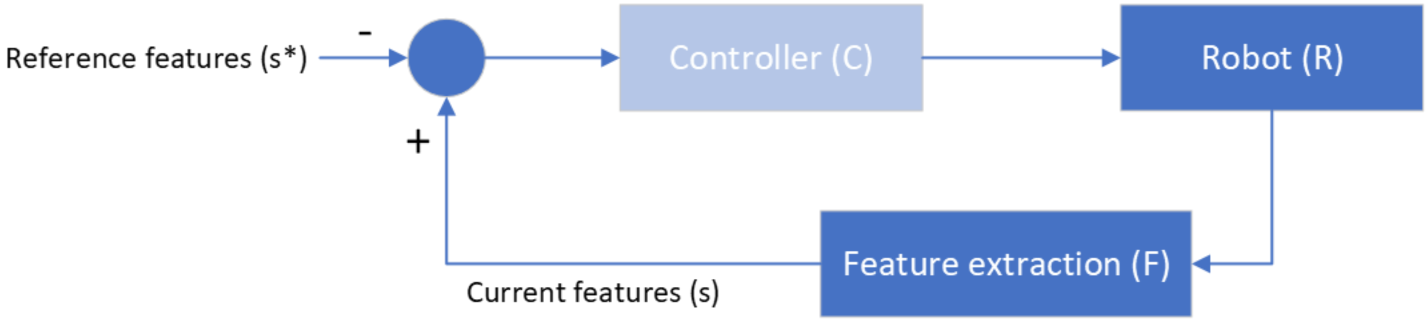
2.1 Control architectures

Basic control loop



2.2 Actuator and sensor models

Modelling Sensors (e.g. Vision), Actuators (e.g. Robot), Environment (e.g. Objects)

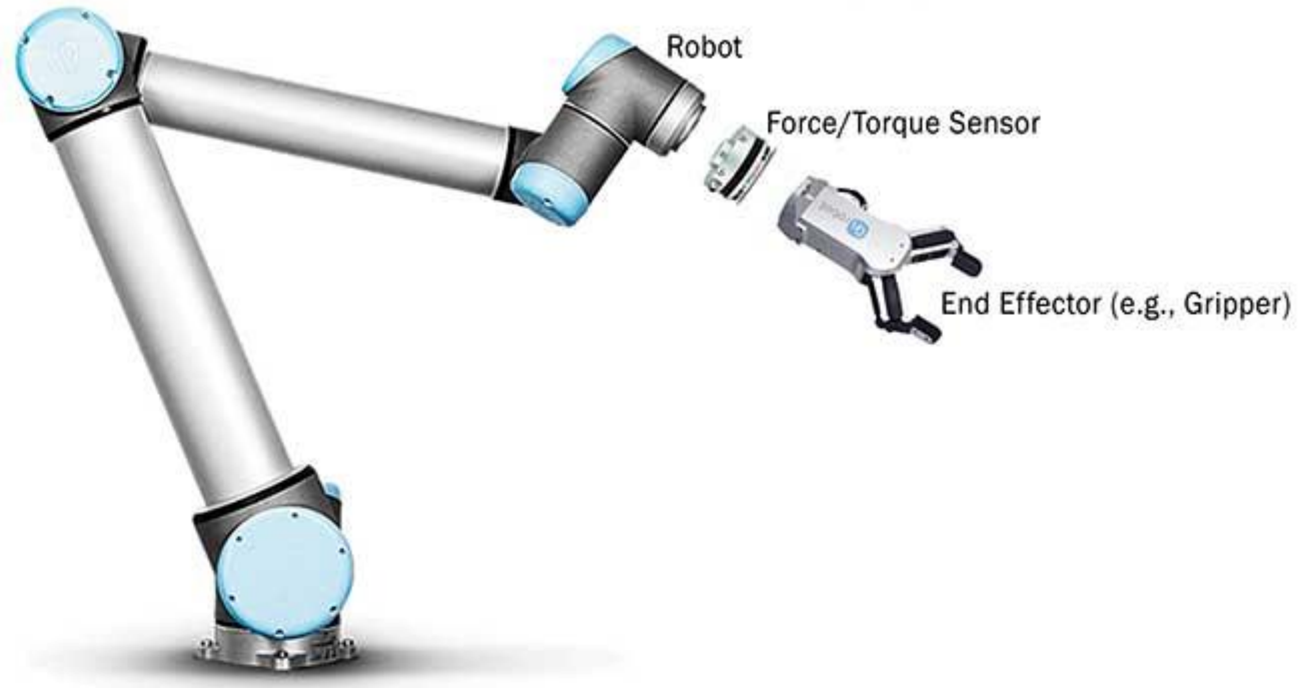


2.2 Actuator and sensor models

Sensor data:
vision

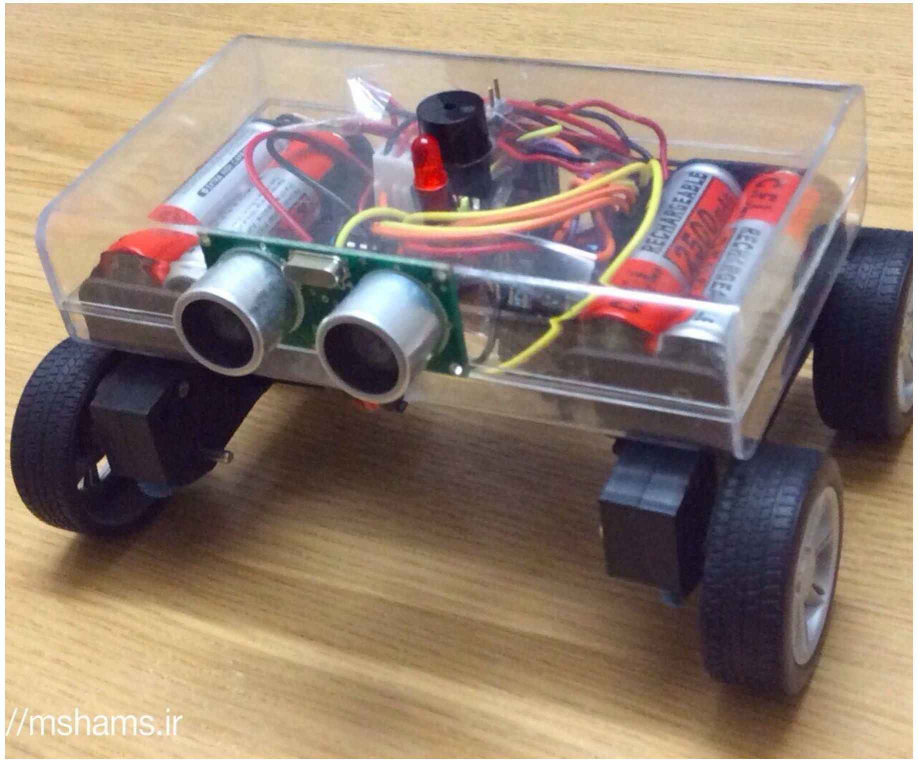
2.2 Actuator and sensor models

Sensor data:
vision
force / torque



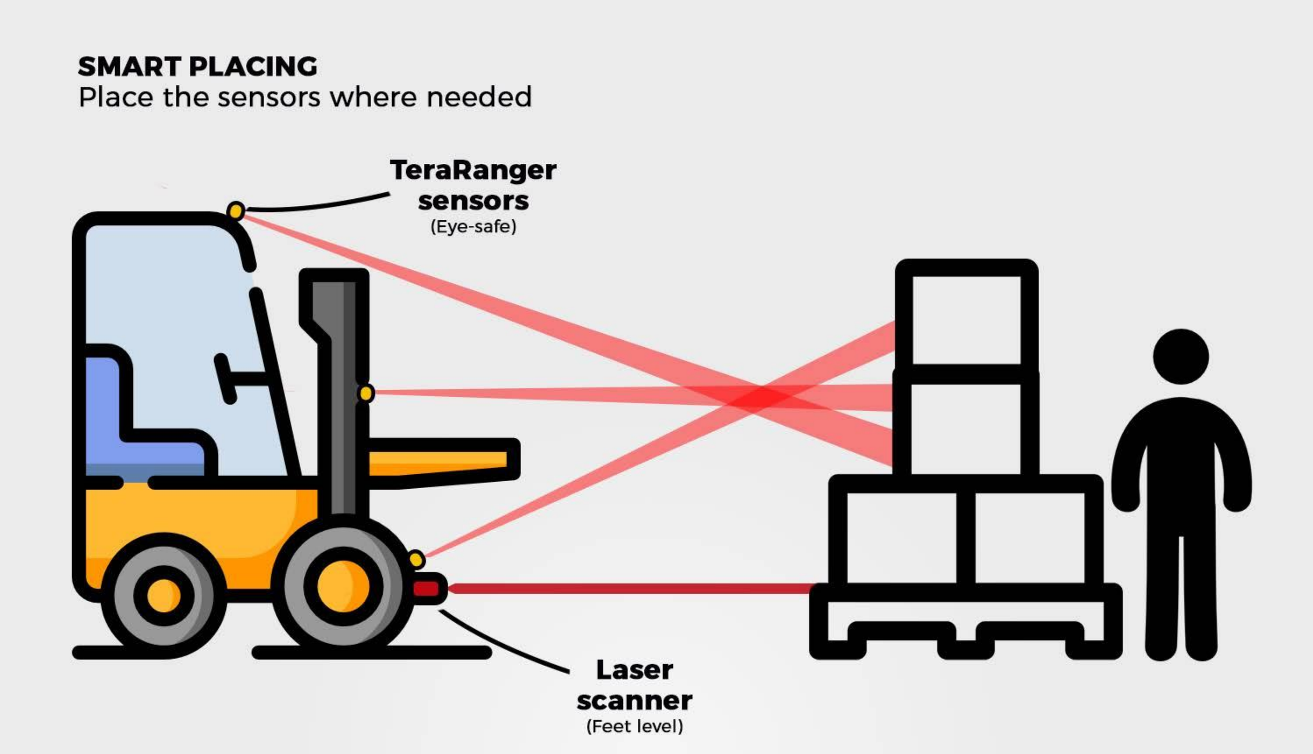
2.1 Control architectures

Sensor data:
vision
force / torque
sonar



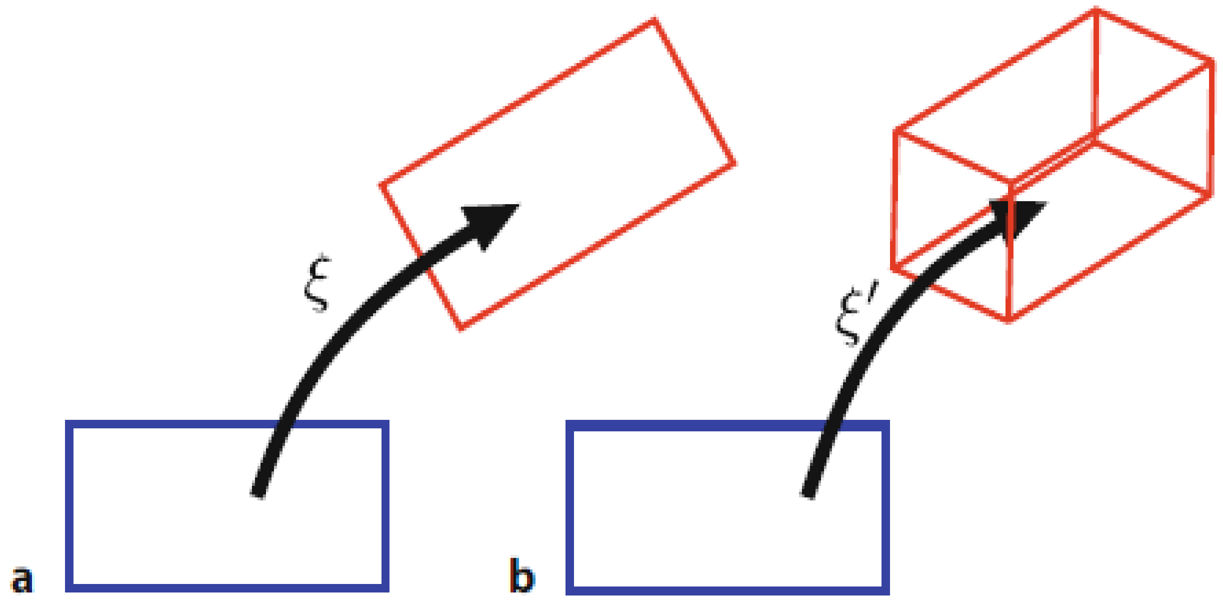
2.2 Actuator and sensor models

Sensor data:
vision
force / torque
sonar
laser
etc



2.2.1 Position and orientation

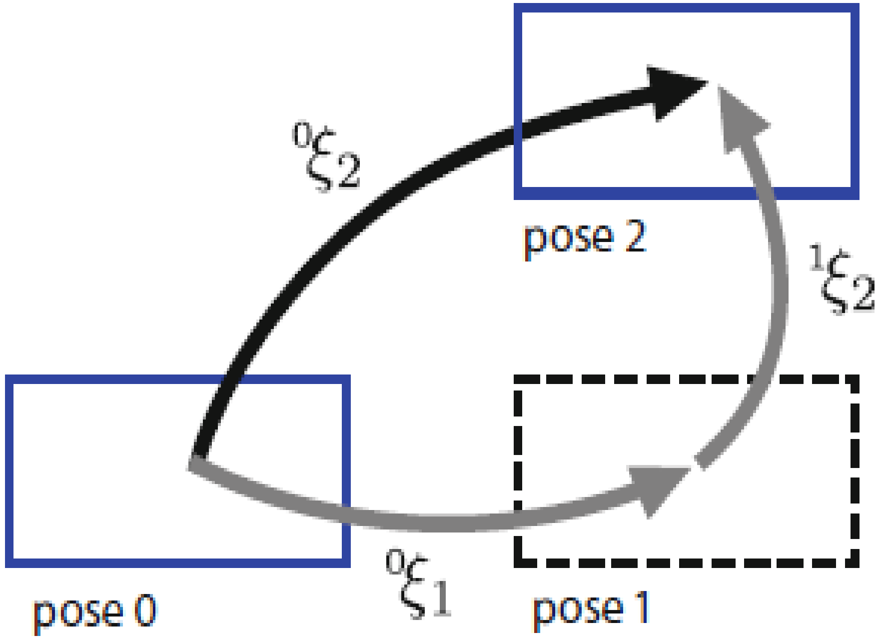
Rigid motion



2.2.1 Position and orientation

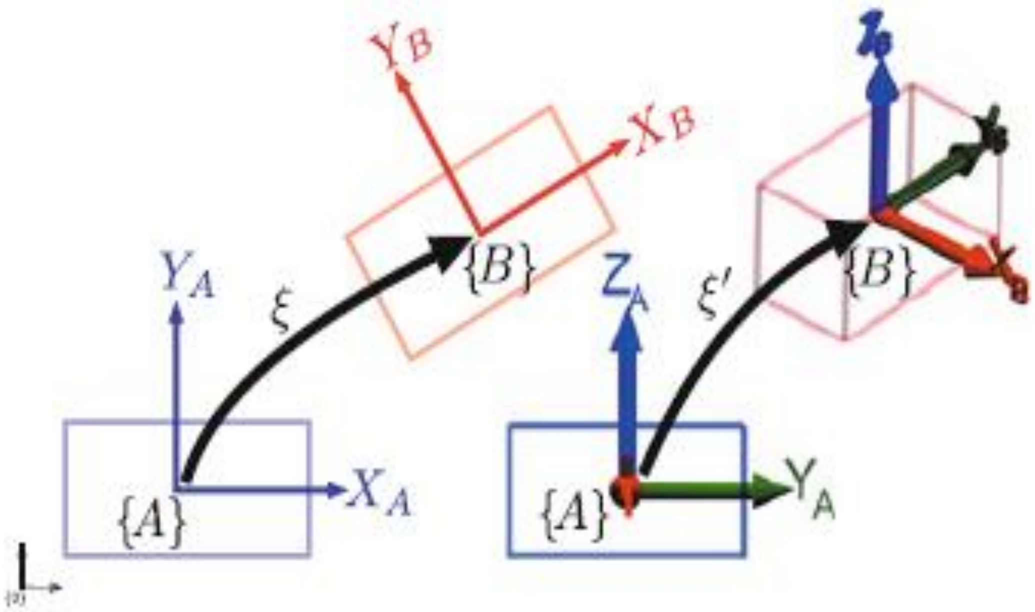
Rigid motion

$^{0}\xi_{2} = {^{0}\xi_{1}} \oplus {^{1}\xi_{2}}$



2.2.1 Position and orientation

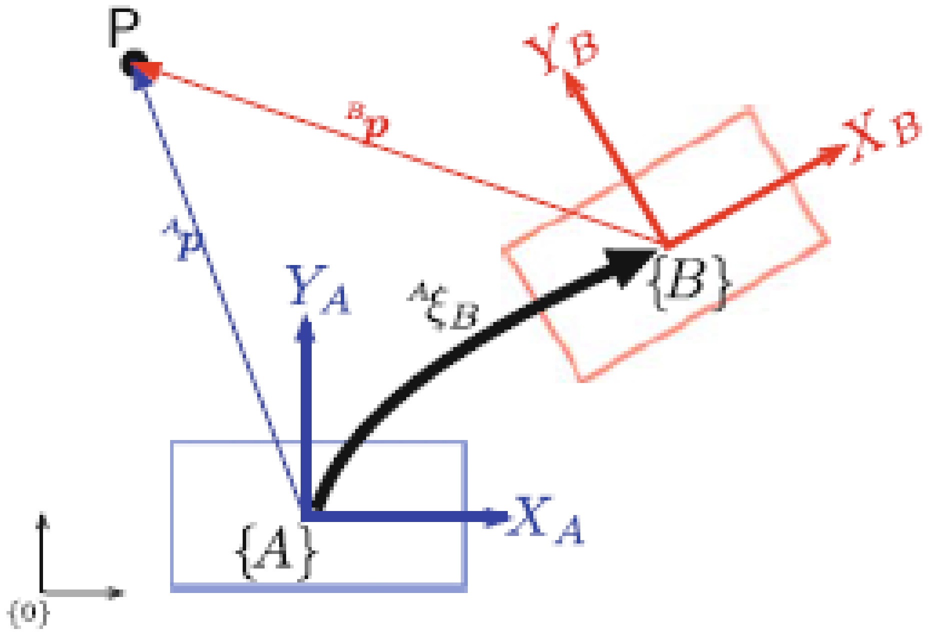
Coordinate frames



2.2.1 Position and orientation

Coordinate frames

$$^{A}p = {}^{A}\xi_{B} \cdot {}^{B}p$$

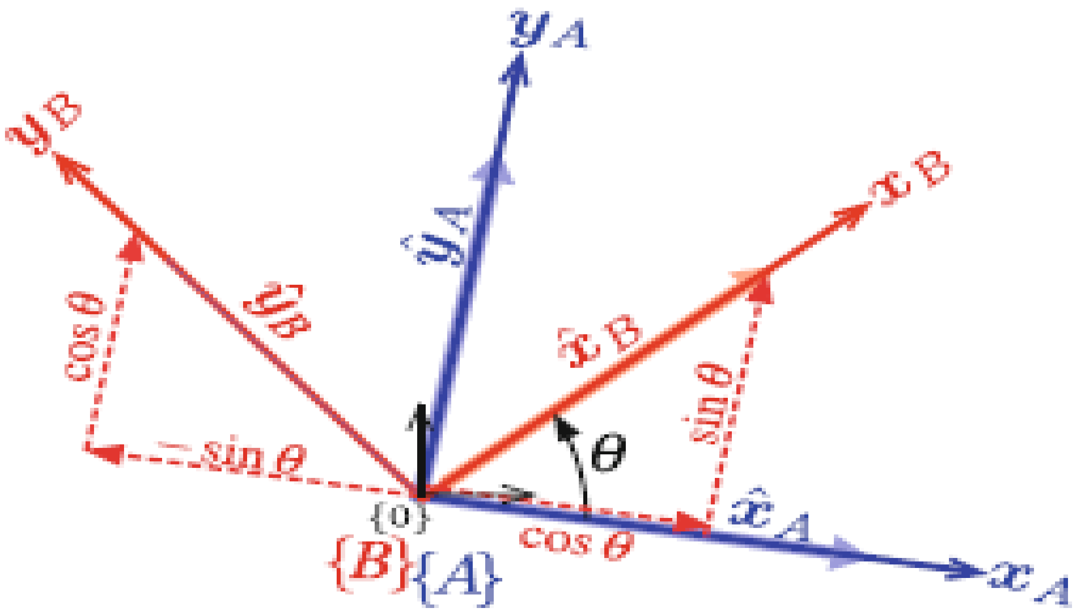


2.2.1 Transformations in 2D

Rotation

$$(\hat{x}_B \quad \hat{y}_B) = (\hat{x}_A \quad \hat{y}_A) \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{{}^A\mathbf{R}_B(\theta)}$$

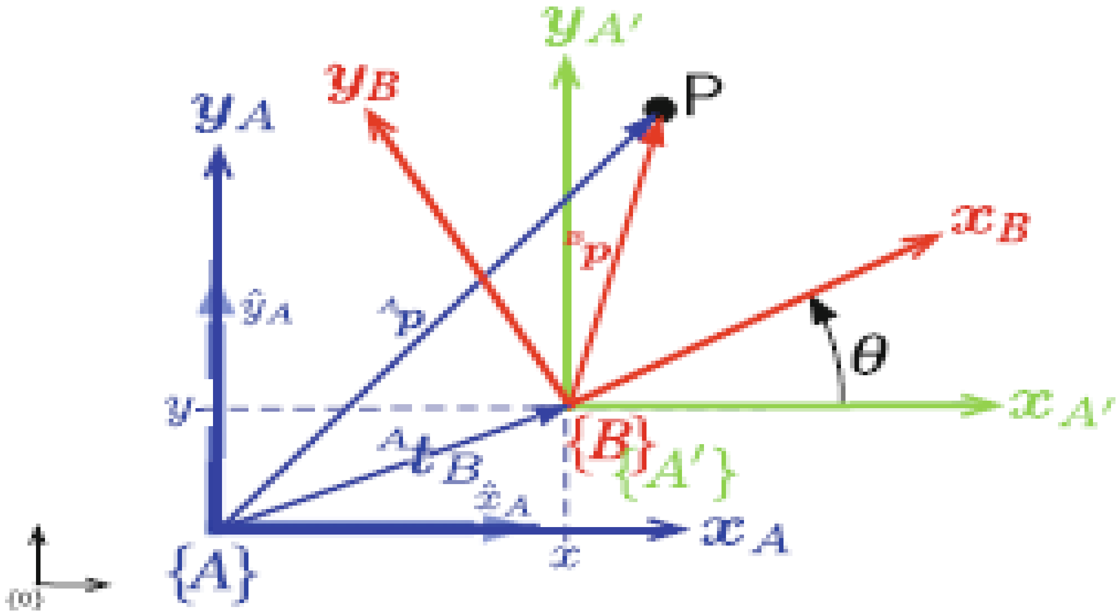
$${}^A\mathbf{R}_B(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



2.2.1 Transformations in 2D

Translation

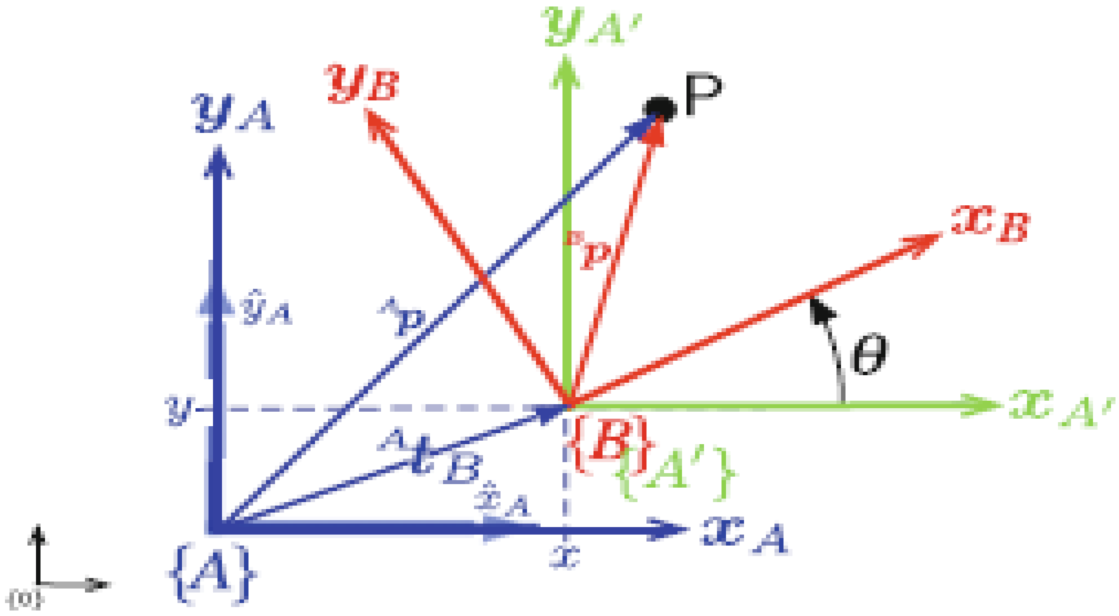
$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \end{pmatrix} = \begin{pmatrix} {}^{A'}x \\ {}^{A'}y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$



2.2.1 Transformations in 2D

Pose

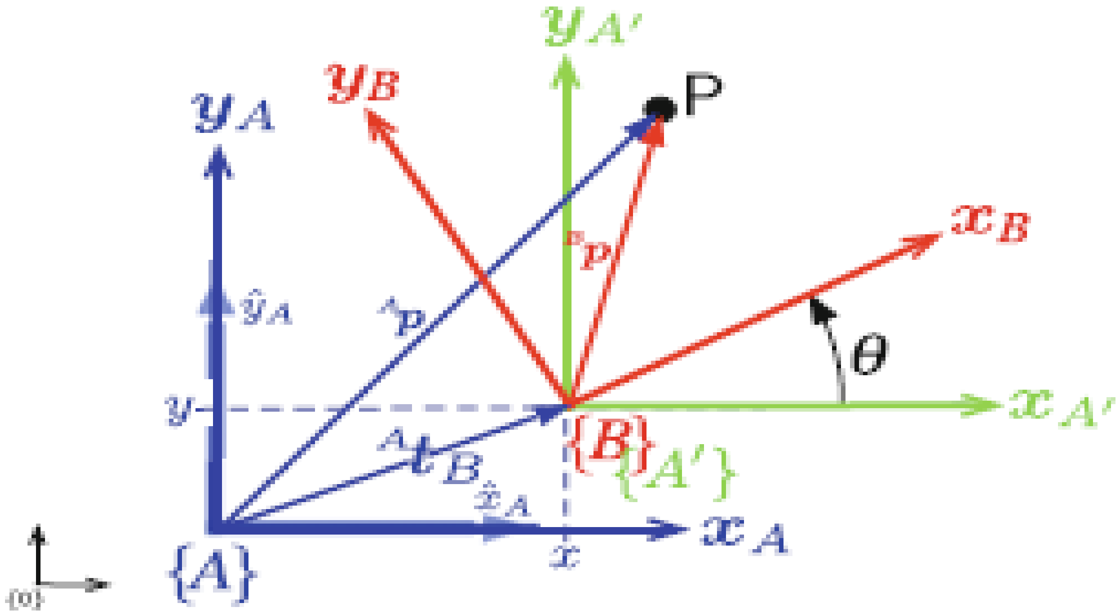
$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{A}\mathbf{R}_{B}(\theta) & {}^{A}t_{B} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ 1 \end{pmatrix}$$



2.2.1 Transformations in 2D

Pose

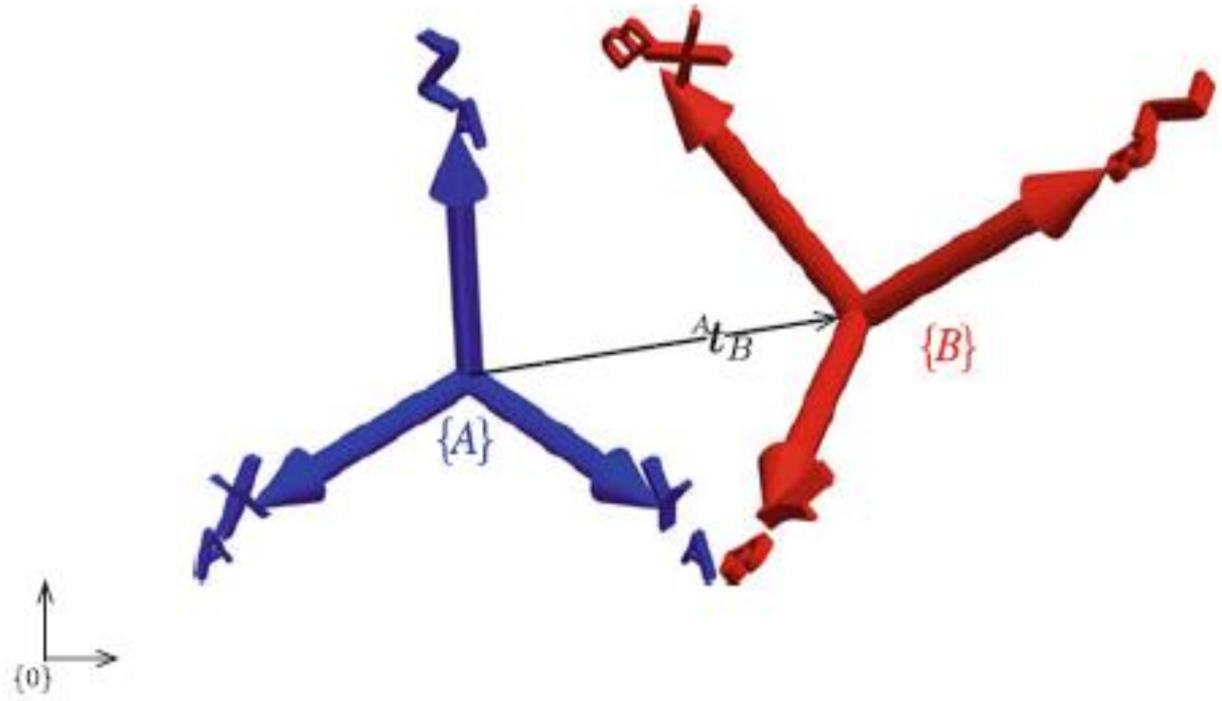
$$
\begin{aligned}
{}^{A}\tilde{p} &= \begin{pmatrix} {}^{A}\mathbf{R}_{B}(\theta) & {}^{A}t_{B} \\ \mathbf{0}_{1\times 2} & 1 \end{pmatrix} {}^{B}\tilde{p} \\
&= {}^{A}\mathbf{T}_{B}\, {}^{B}\tilde{p}
\end{aligned}
$$



$^{A}T_{B}$ →Homogeneous transformation

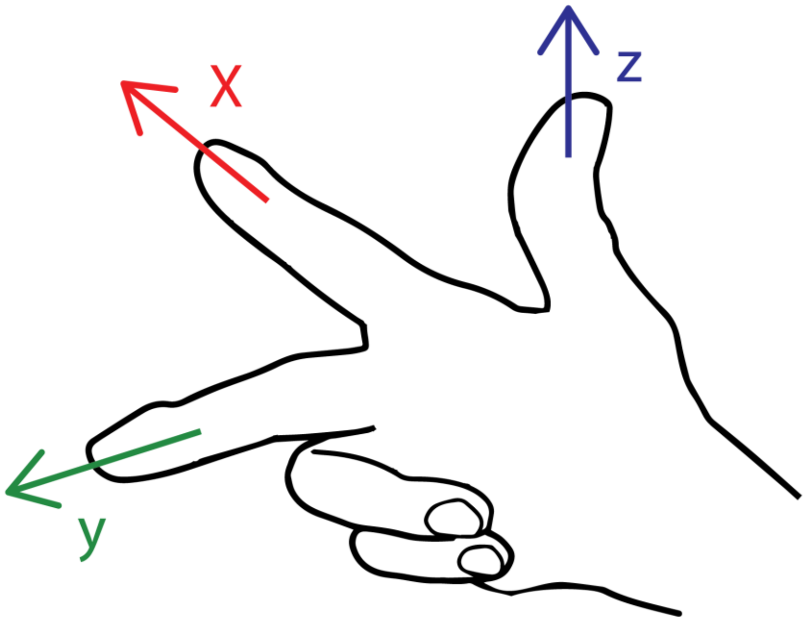
2.2.1 Transformations in 3D

Rotations



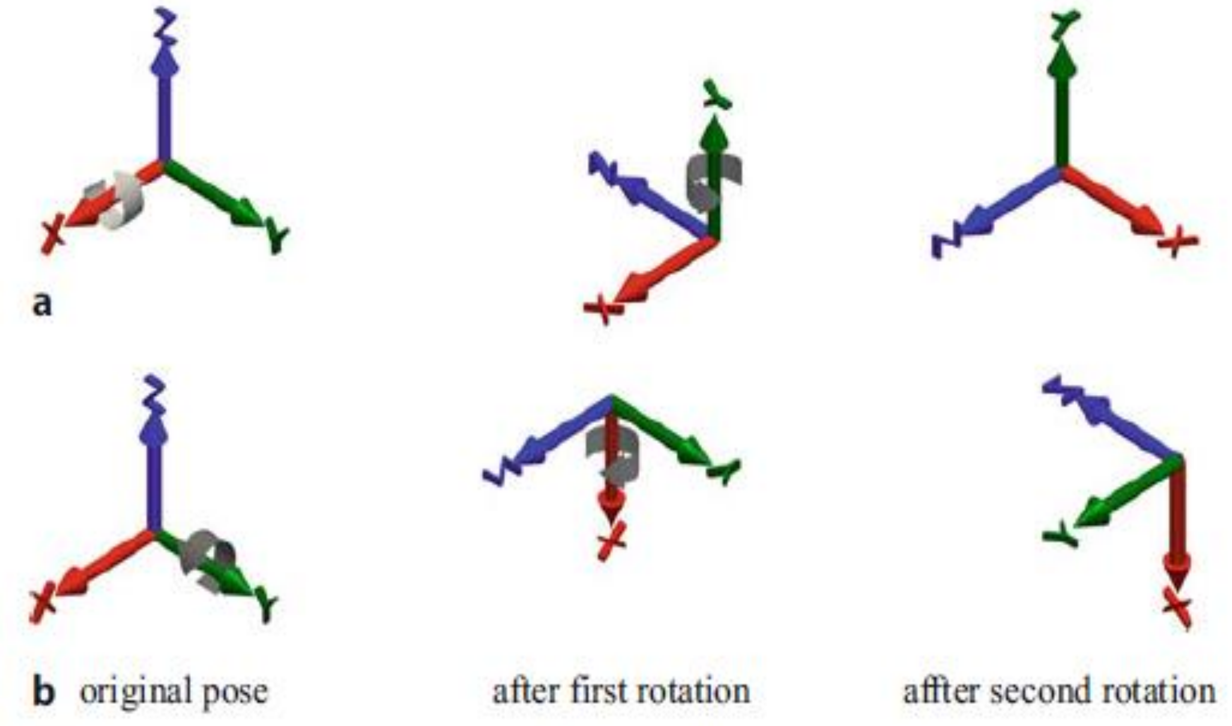
2.2.1 Transformations in 3D

Right-hand rule



2.2.1 Transformations in 3D

Rotations



2.2.1 Transformations in 3D

3D rotation matrix

$$\begin{pmatrix} {}^{A}p_x \\ {}^{A}p_y \\ {}^{A}p_z \end{pmatrix} = {}^{A}\mathbf{R}_B \begin{pmatrix} {}^{B}p_x \\ {}^{B}p_y \\ {}^{B}p_z \end{pmatrix}$$

2.2.1 Transformations in 3D

3D rotation matrix

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2.1 Transformations in 3D

3D rotation matrix:
ZYZ sequence

$$\mathbf{R}(\phi, \theta, \psi) = \mathbf{R}_z(\phi)\,\mathbf{R}_y(\theta)\,\mathbf{R}_z(\psi)$$

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2.1 Transformations in 3D

3D rotation matrix:
roll (x), pitch (y), yaw (z) angles

$$\mathbf{R}(\alpha, \beta, \gamma) = \mathbf{R}_z(\gamma)\,\mathbf{R}_y(\beta)\,\mathbf{R}_x(\alpha)$$

(ZYX sequence)

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2.1 Transformations in 3D

3D rotation matrix:
roll (x), pitch (y), yaw (z) angles

$$\mathbf{R}(\alpha, \beta, \gamma) = \mathbf{R}_x(\gamma)\, \mathbf{R}_y(\beta)\, \mathbf{R}_z(\alpha)$$

(XYZ sequence)

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2.1 Transformations in 3D

Pose:
3D homogeneous transformation matrix

$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{A}\mathbf{R}_B & {}^{A}\mathbf{t}_B \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{pmatrix}$$

2.2.1 Transformations in 3D

Pose:
3D homogeneous transformation matrix (${}^{A}T_{B}$)

$$
\begin{aligned}
{}^{A}\tilde{p} &= \begin{pmatrix} {}^{A}\mathbf{R}_{B} & {}^{A}t_{B} \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix} {}^{B}\tilde{p} \\
&= {}^{A}\mathbf{T}_{B}\, {}^{B}\tilde{p}
\end{aligned}
$$

2.2.2 Motion

Orientation defined with respect to time

$[\cdot]_\times$ → skew-symmetric matrix

$$ {}^{A}\mathbf{R}_B(t) = \mathrm{e}^{\theta(t)\left[{}^{A}\hat{\omega}_B(t)\right]_\times} \in \mathbf{SO}(3) $$

2.2.2 Motion

Rate of change of orientation

$$ {}^{A}\dot{\mathbf{R}}_{B} = \left[{}^{A}\omega_{B} \right]_{\times} {}^{A}\mathbf{R}_{B} \in \mathbb{R}^{3\times 3} $$

2.2.2 Motion

Rate of change of pose

$$
{}^{A}\mathbf{T}_{B} = \begin{pmatrix} {}^{A}\mathbf{R}_{B} & {}^{A}t_{B} \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix} \in \mathbf{SE}(3)
$$

$$
{}^{A}\dot{\mathbf{T}}_{B} = \begin{pmatrix} {}^{A}\dot{\mathbf{R}}_{B} & {}^{A}\dot{t}_{B} \\ \mathbf{0}_{1\times 3} & 0 \end{pmatrix} = \begin{pmatrix} \left[{}^{A}\omega_{B}\right]_{\times} {}^{A}\mathbf{R}_{B} & {}^{A}\dot{t}_{B} \\ \mathbf{0}_{1\times 3} & 0 \end{pmatrix} \in \mathbb{R}^{4\times 4}
$$

2.2.2 Motion

Spatial velocity vector
(linear velocity, angular velocity)

$$^{A}\mathrm{v}_{B} = \left({}^{A}v_{B},\ {}^{A}\omega_{B} \right) \in \mathbb{R}^{6}$$

2.2.2 Motion

Expressing the spatial velocity vector from frame {B} to frame {A}

${}^{A}J_{B}(\cdot) \rightarrow$
Jacobian or interaction matrix between two frames {A} and {B}

$$ {}^{A}\boldsymbol{\nu} = \begin{pmatrix} {}^{A}\mathbf{R}_B & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & {}^{A}\mathbf{R}_B \end{pmatrix} {}^{B}\boldsymbol{\nu} = {}^{A}\mathbf{J}_B({}^{A}\mathbf{T}_B)\ {}^{B}\boldsymbol{\nu} $$

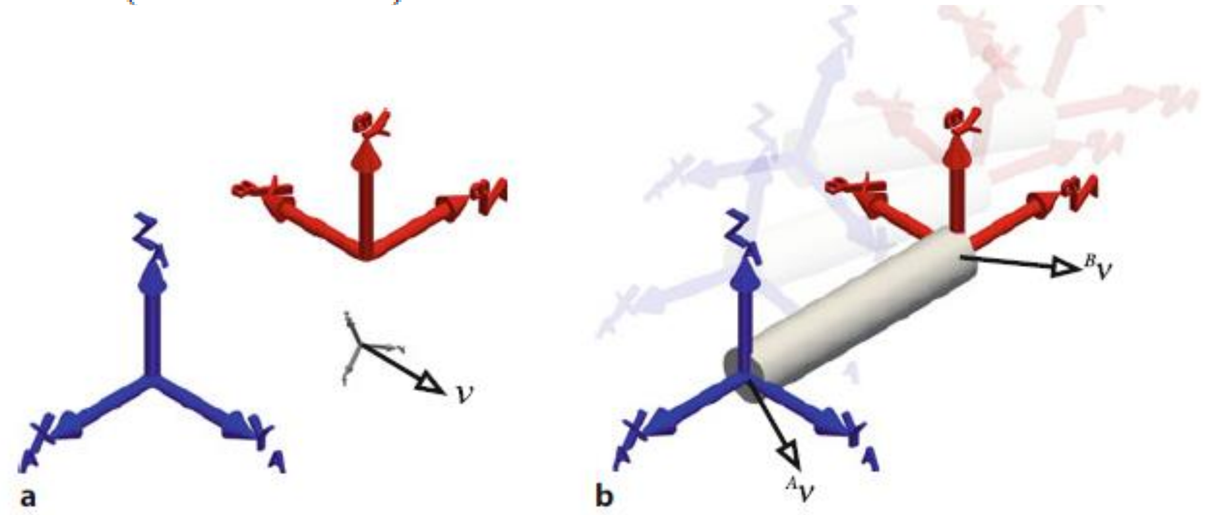
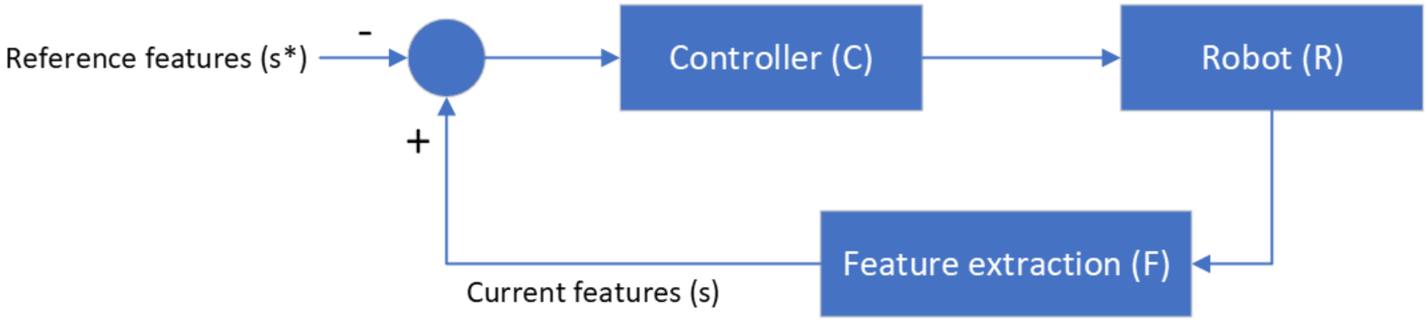


Fig. 3.1 Spatial velocity can be expressed with respect to frame {A} or frame {B}. **a** Both frames are stationary and the object frame is moving with velocity $\boldsymbol{\nu}$, **b** both frames are moving together

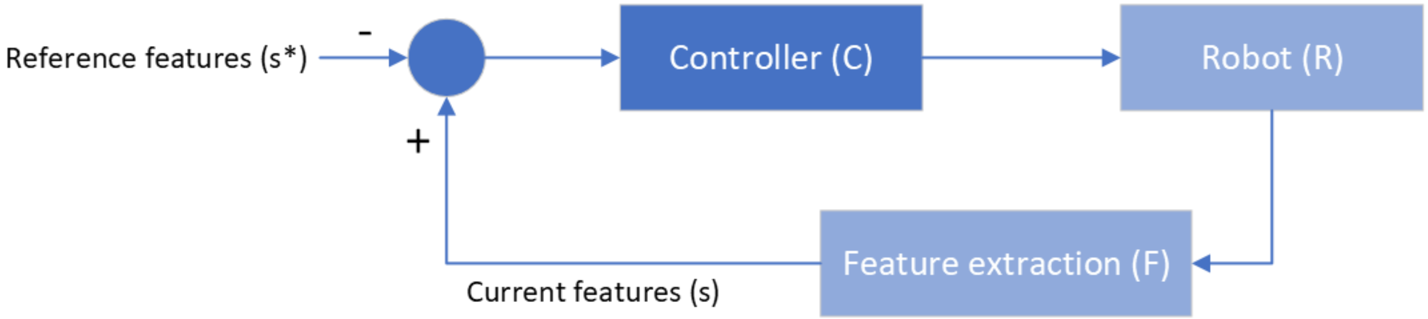
2.3 Vision-based control

Basic control loop



2.3 Vision-based control

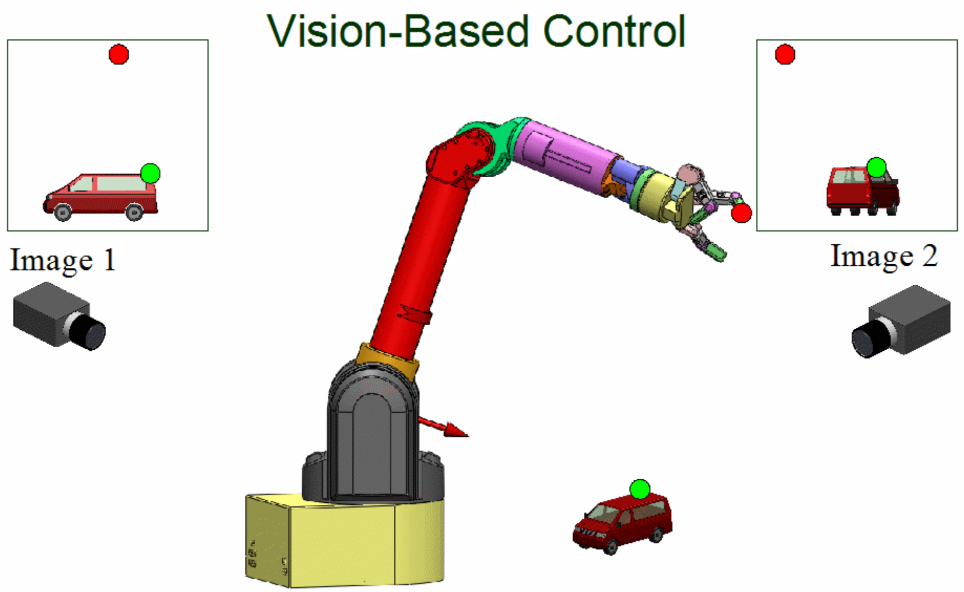
Modelling the Controller



2.3 Vision-based control

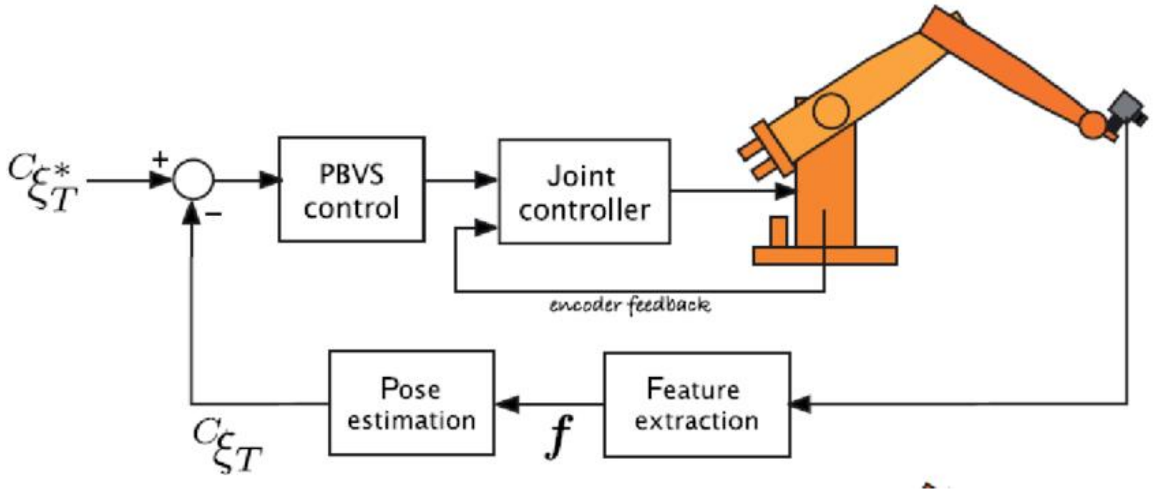
Visual servoing

Sensor data from vision



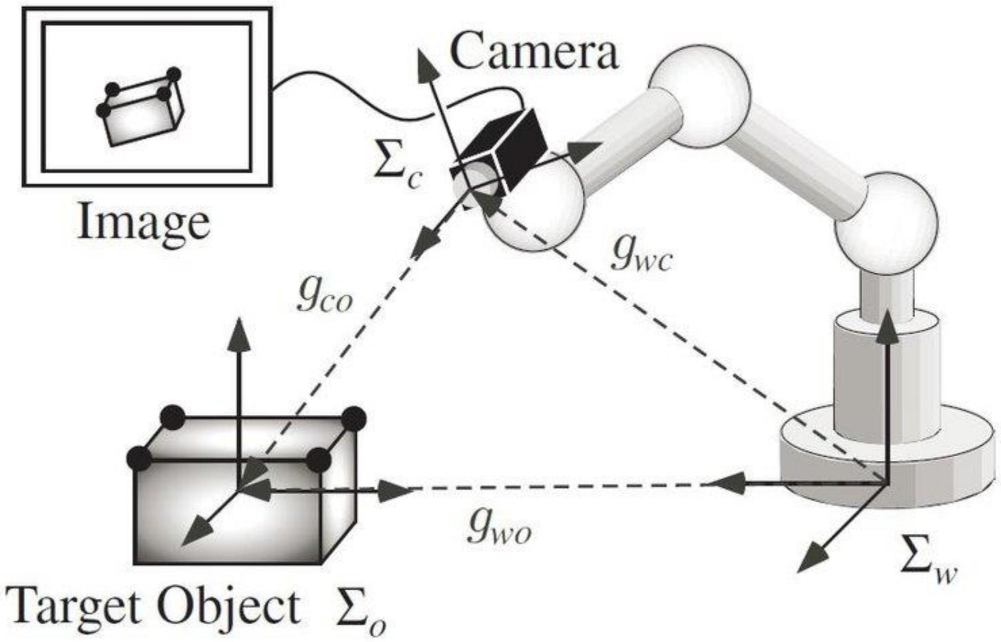
2.3 Vision-based control

Position-based visual servoing



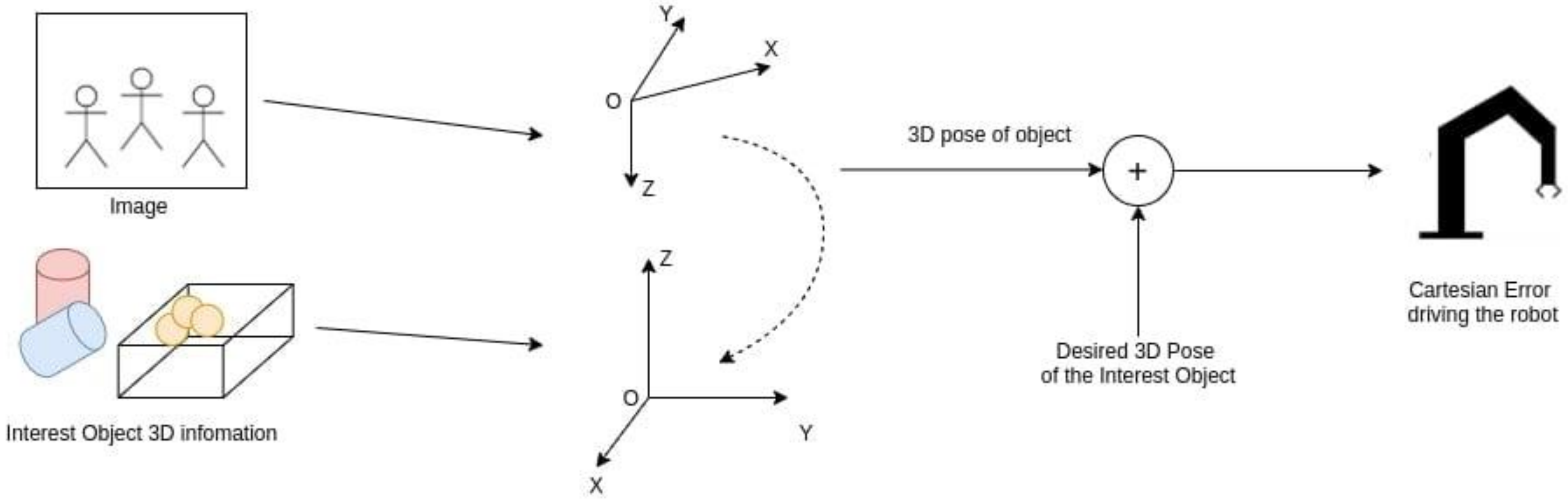
2.3 Vision-based control

Position-based visual servoing



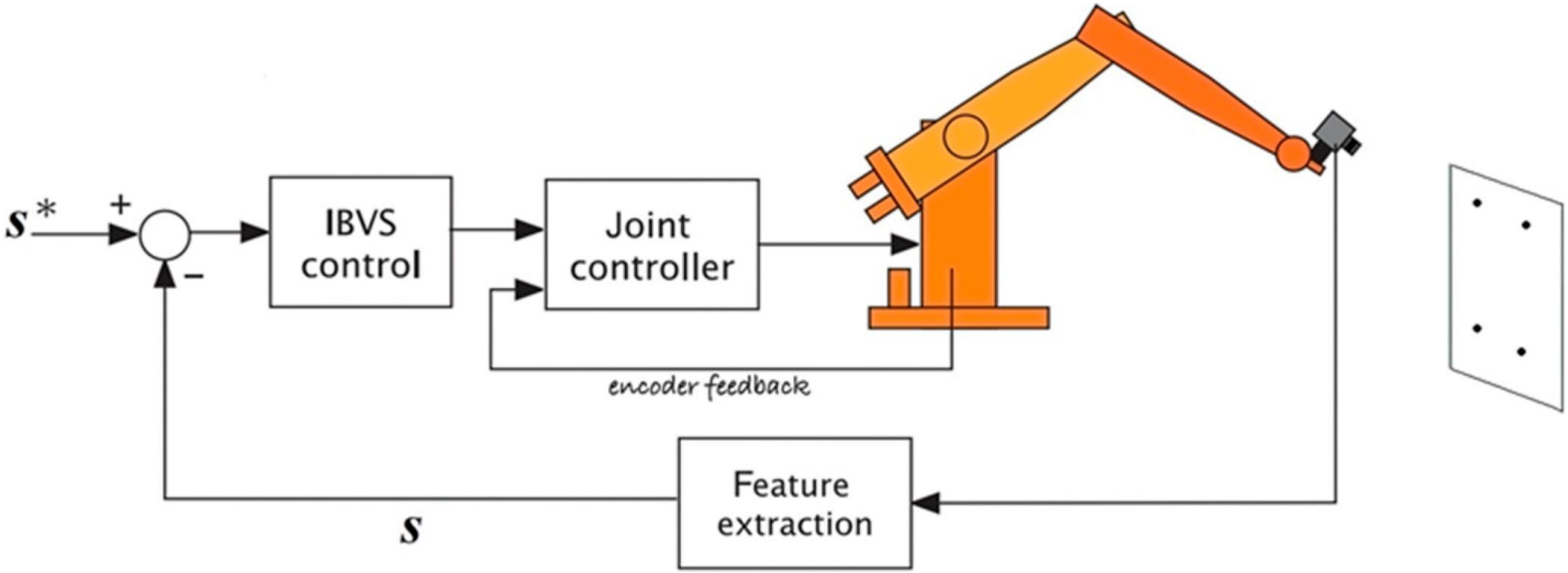
2.3 Vision-based control

Position-based visual servoing



2.3 Vision-based control

Image-based visual servoing



2.3 Vision-based control

Image-based visual servoing