Multi-classifiers

Department of Computer Languages and Systems

Terminologies

- Lots of terms are used to refer to multi-classifiers:
  - ensemble of classifiers
  - combining classifiers
  - decision committee
  - multiple classifier system
  - mixture of experts
  - committee-based learning
  - etc.

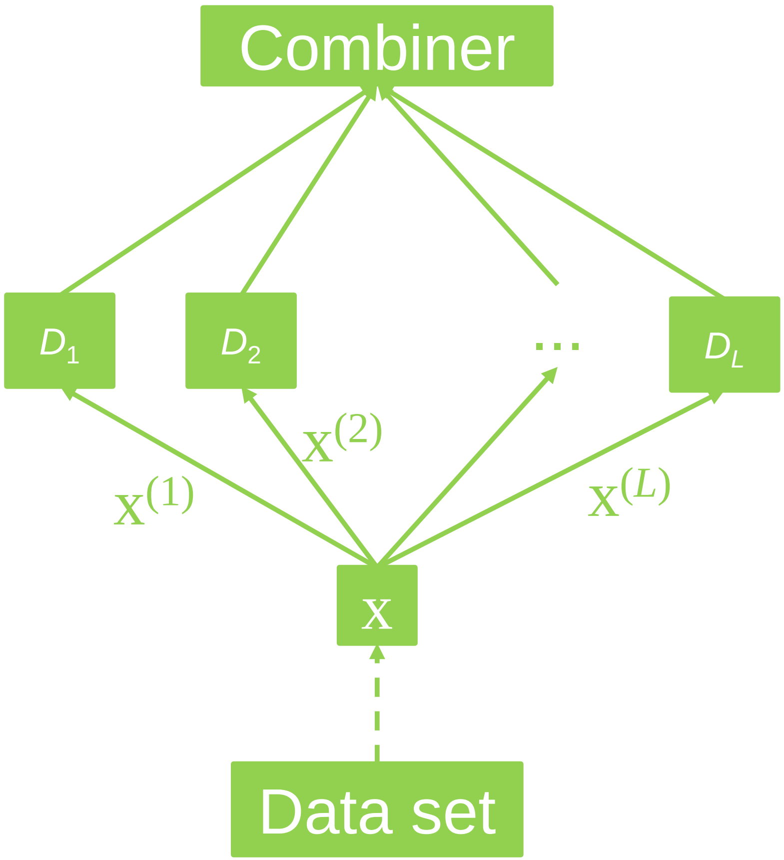
Introduction: motivation

- When you have to face a complex classification problem:
  - which learning algorithm to use?
  - which parameters to choose?
  - how to use the training data?
  - which vector space to map the data onto? What is the most discriminating representation?

Introduction: motivation

- Different models may appear while searching for a solution, but often none of them is better than the rest
  - In this case, a reasonable choice is to keep them all and create a final system integrating the pieces
  - The core idea behind this is to aggregate multiple models to obtain a combined model D that outperforms every single model D_i in it
  - Each single model D_i is called base learner (classifier) or individual learner (classifier)

Strategies to build a multi-classifier



- Combination level: design different combiners
- Classifier level: use different base classifiers
- Data level: use different data subsets
- Feature level: use different feature subsets

Source: *http://www.ccas.ru/voron/download/books/machlearn/kuncheva04combining.pdf*

Combination level: fusion vs. selection

Fusion

- each ensemble member is supposed to have knowledge of the whole feature space
- some combiner such as the average and majority vote is applied to label the input object x

Selection

- each ensemble member is supposed to know well a part of the feature space and to be responsible for objects in this part
- one member is chosen to label the input object x

Combination level (ii): fusion vs. selection

Fusion

- competitive classifiers
- ensemble approach
- multiple topology

Selection

- cooperative classifiers
- modular approach
- hybrid topology

Fusion: majority vote

Decision rule: to choose the class most voted by the base classifiers

Three consensus patterns:

- Unanimity (all agree)
- Simple majority (50%+1)
- Plurality (most votes)

Source: http://www.ccas.ru/voron/download/books/machlearn/kuncheva04combining.pdf

Fusion (ii): majority vote

Let it be

- $\left[d_{i,1}, \ldots, d_{i,C}\right]^{\mathrm{T}} \in \{0,1\}^C, i = 1, \ldots, L$, where $d_{i,j} = 1$ if D_i labels x in class ω_j, and 0 otherwise

Then, the plurality vote rule will result in an ensemble decision for class ω_k if

$$\sum_{i=1}^{L} d_{i,k} = \max_{j=1,\ldots,c} \sum_{i=1}^{L} d_{i,j}$$

This rule coincides with the simple majority rule if $C = 2$

Fusion (iii): majority vote

A thresholded plurality vote: we increase the set of classes with one more class ω_{c+1}, for objects for which the ensemble does not determine a class label with a sufficient confidence. Now, the decision is

$$\begin{cases} \omega_k, & \text{if } \sum_{i=1}^{L} d_{i,k} \geq \alpha \cdot L \\ \omega_{c+1}, & \text{otherwise} \end{cases}$$

where $0 < \alpha \leq 1$. If $\alpha = 1$, this becomes the unanimity vote rule

Fusion (iv): majority vote

Weighted majority vote:

- an adequate option when the base classifiers are not of very similar accuracy
- it attempts to give the more competent classifiers more power in making the final decision

Fusion (v): majority vote

Weighted majority vote:

- we can represent the outputs as

$$d_{i,j} = \begin{cases} 1 & \text{if } D_i \text{ labels x in } \omega_j \\ 0 & \text{otherwise} \end{cases}$$

- then, the decision is ω_k if

$$\sum_{i=1}^{L} w_i d_{i,k} = \max_{j=1,\ldots,c} \sum_{i=1}^{L} w_i d_{i,j}$$

where $w_i \geq 0$ ($\sum_{i=1}^{c} w_i = 1$) is a weight for classifier D_i

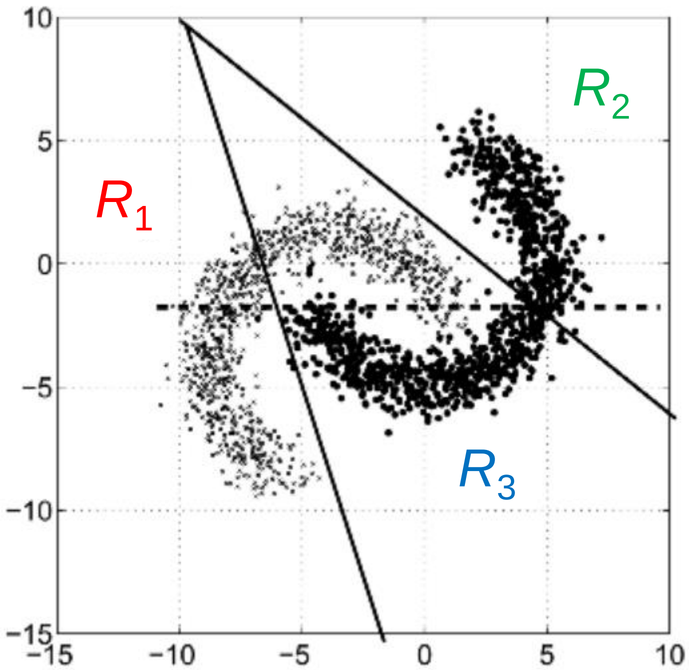
Selection

Suppose an ensemble $D = \{D_1, \ldots, D_L\}$ of classifiers already trained. Then, the feature space $\mathbb{R}^d$ is divided into $K > 1$ selection regions (or regions of competence), which are denoted by $R_1, \ldots, R_K$

- usually, $K = L$
- each region R_i is associated with a classifier, which will be responsible for deciding on the input objects in this part of the space
- these regions are not associated with specific classes, nor do they need to be of a certain shape or size

Selection (ii)

Example: suppose a data set with 2000 points and two classes ω_1 and ω_2, and we have an ensemble with three classifiers D_1, D_2, D_3, each one associated with regions R_1, R_2, R_3



- D_1 always predicts ω_1
- D_2 always predicts ω_2
- D_3 is a linear classifier whose discriminant function is shown as a dashed line

- Accuracy of the individual classifiers or that of a majority vote (fusion) is approximately 0.5
- Accuracy of the selection combiner will be close to 1

Source: http://www.ccas.ru/voron/download/books/machlearn/kuncheva04combining.pdf

Data level: bagging

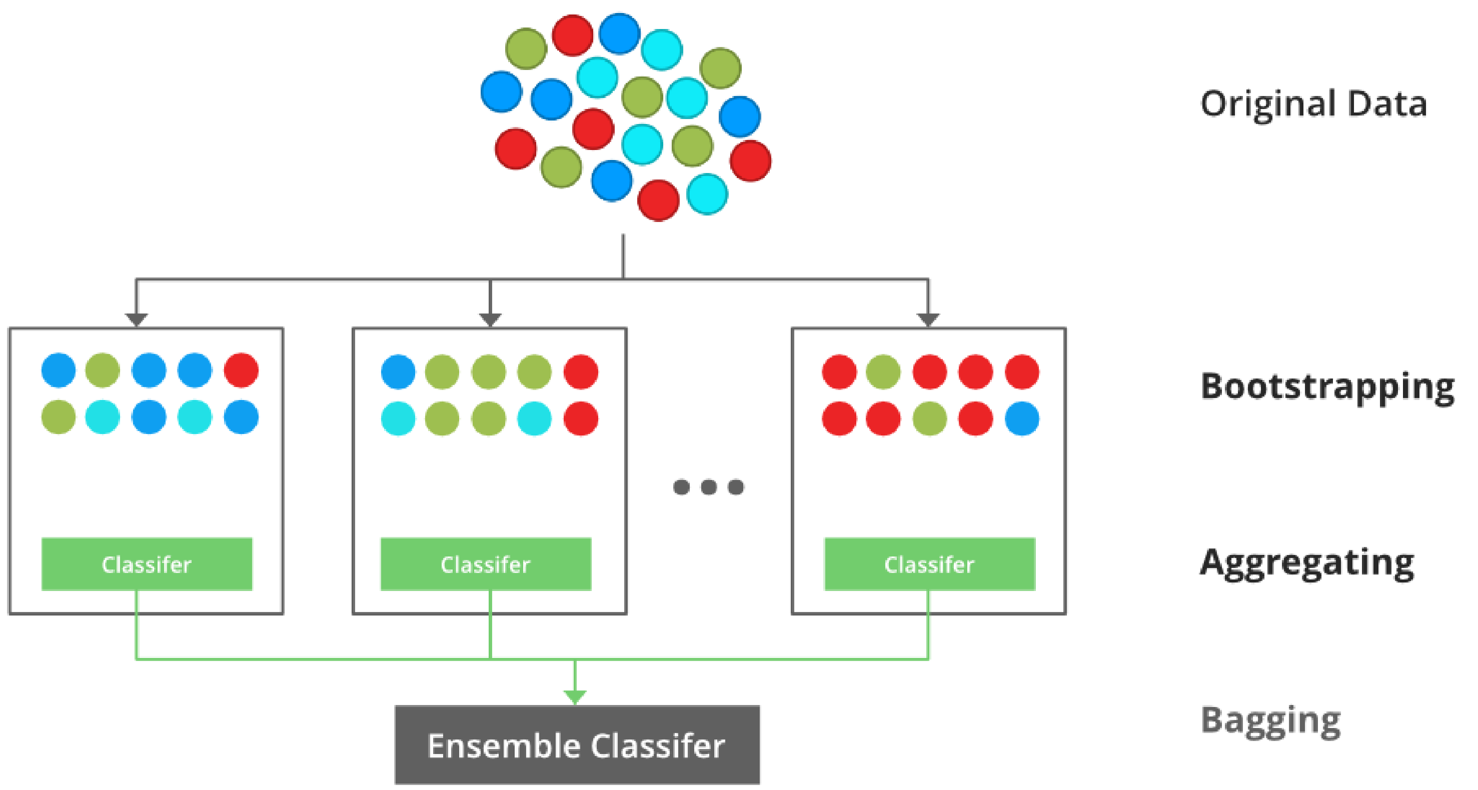
Idea:

- the ensemble is made of classifiers built on bootstrap replicates of the training set $T_{tra} = \{x_1, \ldots, x_n\}$
- the classifier outputs are combined by the plurality vote

Comments:

- we sample with replacement from the original T_{tra} to create L new training sets (often, also of size n)
- all L base classifiers are the same classification model
- the base classifier should be unstable (small changes in T_{tra} lead to large changes in the classifier output (neural networks and decision trees are unstable, k-NN is stable)
- this is a parallel algorithm in both its training and operational phases

Data level (ii): bagging



Data level (iii): bagging

Training phase

1. Initialize the parameters

   $D = \emptyset$, the ensemble

   L, the number of classifiers to train

2. For $k = 1, \ldots, L$

   Take a bootstrap sample S_k from the original training set T_{tra}

   Build a classifier D_k using S_k as the training set

   Add the classifier to the current ensemble, $D = D \cup D_k$

3. Return D

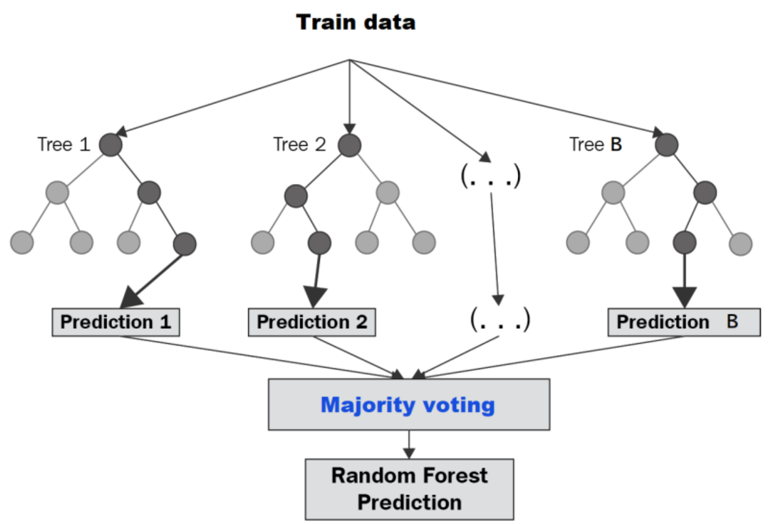
Classification phase

1. Run $D_1, \ldots, D_L$ on the input x
2. Assign x to the class with the maximum number of votes

Data level (iv): variants of bagging

Random forest

- a collection of decision trees, each built using a random bootstrap sample
- the trees can be built by sampling from the feature set, from the training set, or just varying randomly some of the parameters of the tree



Data level (vi): boosting

Idea:

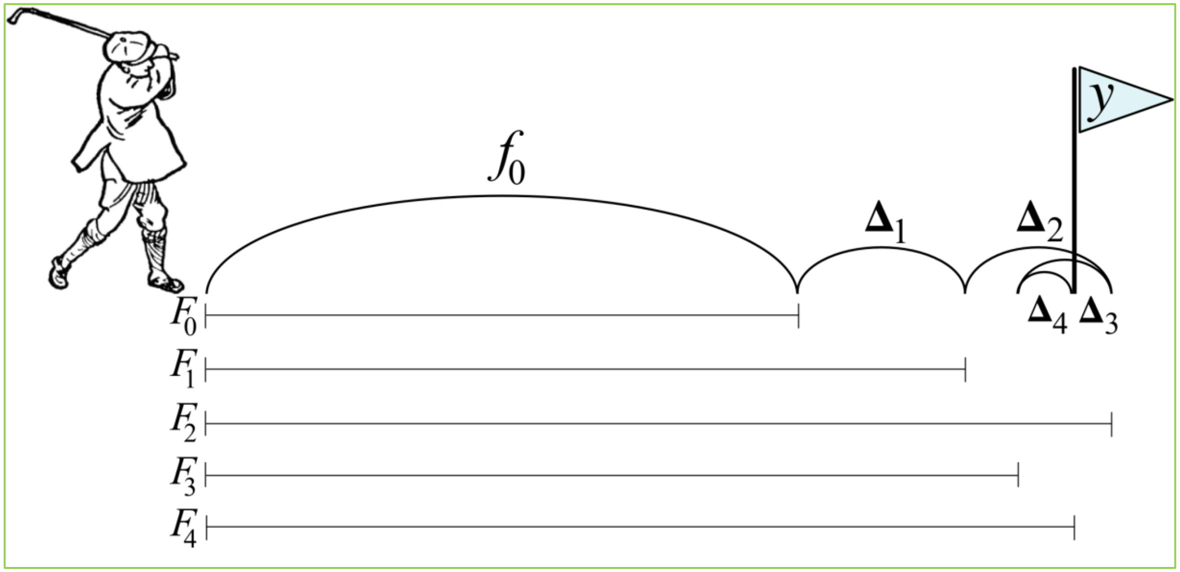
- to develop the ensemble D incrementally, adding one base classifier at a time
- some classifiers have more say in the classification than others
- the classifier D_i is made by taking the mistakes of the classifier D_{i-1} into account

Comments:

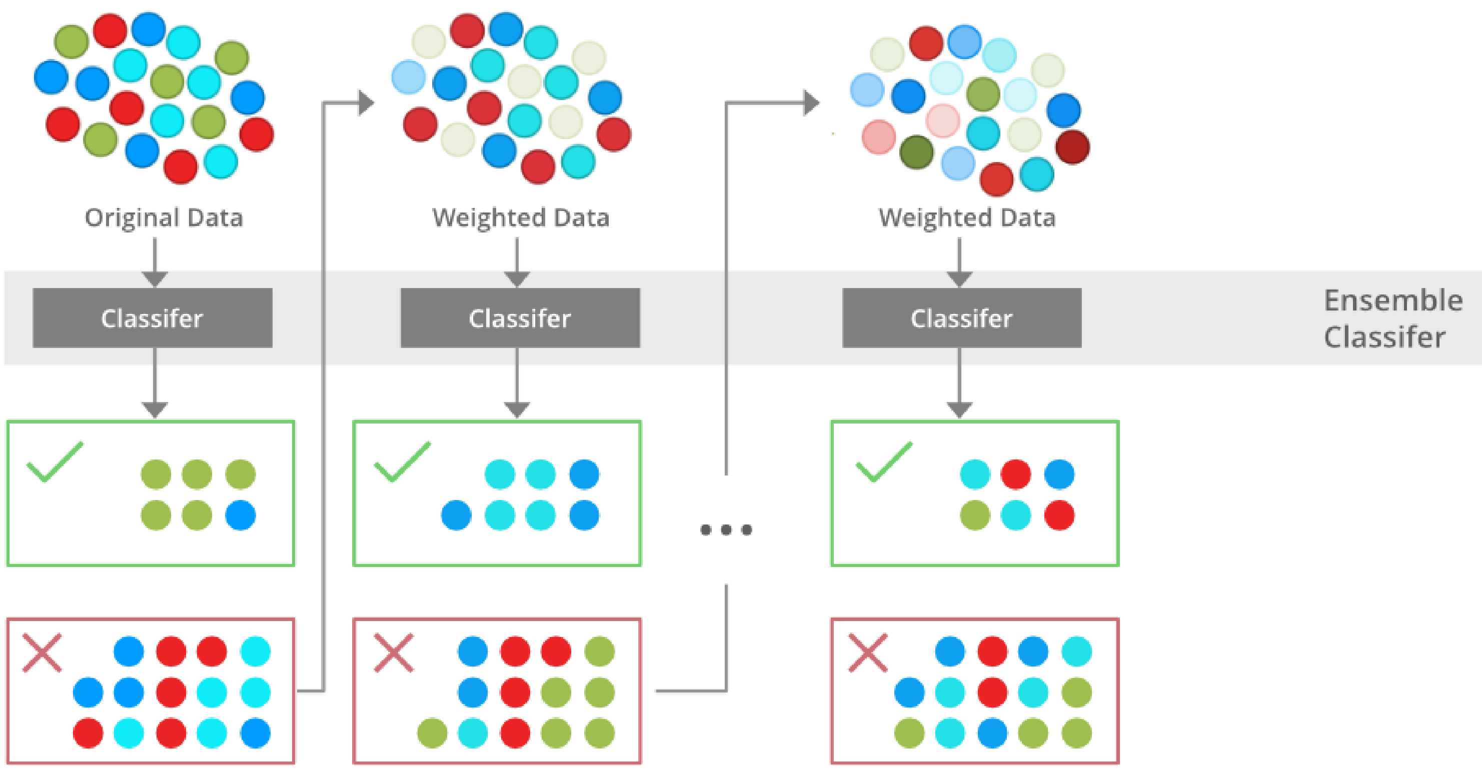
- this is a sequential algorithm
- the errors that the first classifier makes influence how the second classifier is made, and so on

Data level (vii): boosting

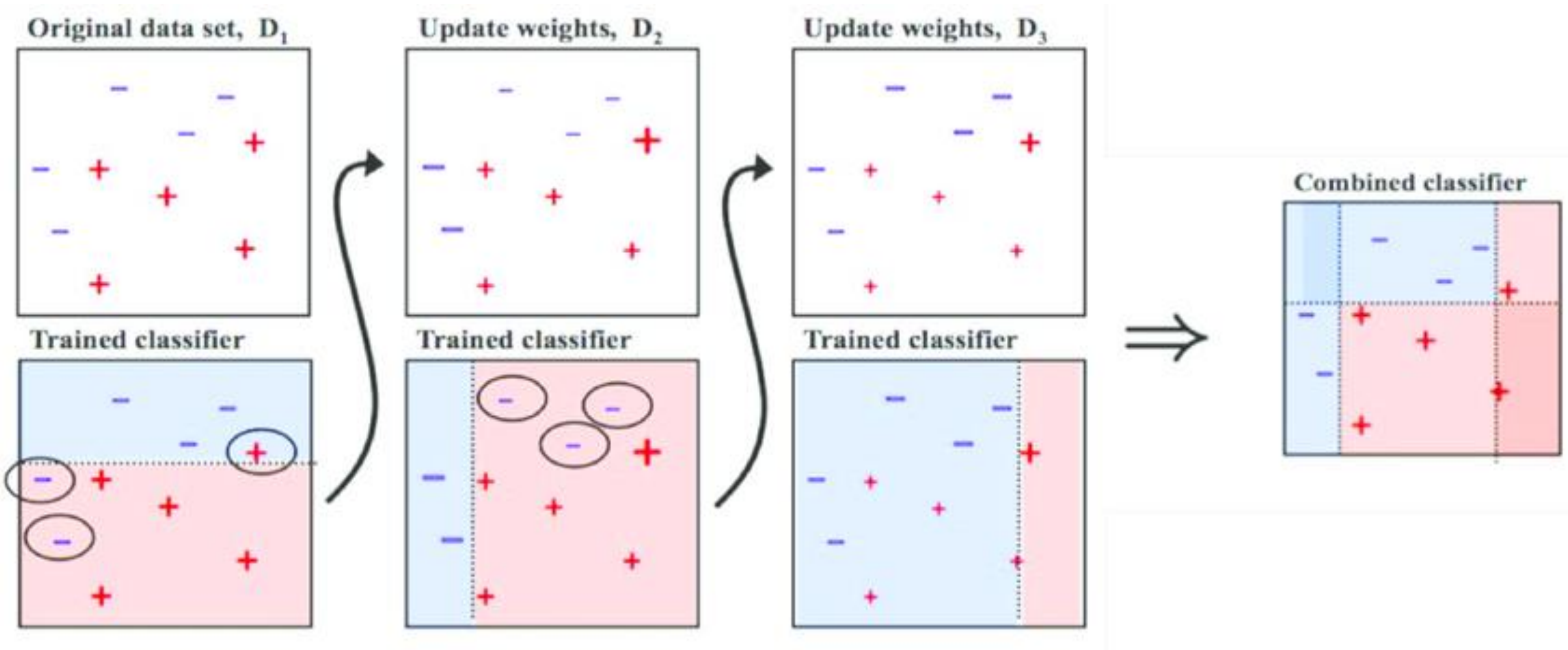
The idea of boosting could be seen as a golfer who initially hits a golf ball towards the hole at position y, but only goes as far as f_0. The golfer then repeatedly hits the ball more gently, moving it toward the hole a little at a time and after reassessing the direction and distance to the hole with each shot.



Data level (viii): boosting



Data level (ix): boosting



Data level (x): boosting (AdaBoost)

Training phase

1. Initialize the parameters

   Set the weights $w^i = 1/n$ (equal weights to each data point)

   $D = \emptyset$, the ensemble

   L, the number of classifiers

2. For $k = 1, \dots, L$

   Build a classifier D_k with the training data using w^i

   Compute the weighted error err_k

   $err_k = \frac{\sum_{i=1}^{n} w_i^k l_k^i}{\sum_{i=1}^{n} w_i^k}$, ($l_k^i$=1 if D_k misclassifies x_i and l_k^i= 0 otherwise)

   Compute $\alpha_k = \log[(1 - err_k)/err_k]$

Data level (xi): boosting (AdaBoost)

cont.

Update weights for $i = 1, \dots, n$

$w_i^k \leftarrow w_i^k \cdot e^{\alpha_k l_k^i}$ and renormalize w_i^k to sum to 1

Data level (xii): variants of boosting

Gradient boosting

- the process of additively generating base models is formalized as a gradient descent algorithm over an objective function
- it iteratively trains an ensemble, with each iteration using the residual errors of the previous model as labels to fit the next model
- the final prediction is a weighted sum of all model predictions
- there is a technique called Gradient Boosted Trees whose base learner is CART (Classification and Regression Trees)

Data level (xiii): variants of boosting

Extreme gradient boosting (XGBoost)

- an implementation of gradient boosted decision trees
- it allows parallel training
- it implements early stopping so we can stop model evaluation when additional trees offer no improvement
- it provides some parameters to help reduce model complexity and avoid overfitting

Data level (xiv): variants of boosting

Categorical boosting (CatBoost)

- it is designed to work on heterogeneous data (categorical, numerical, logical, …)
- it works well with less data
- improved accuracy by reducing overfitting

Data level (xv): variants of boosting

Light gradient boosting machine (LightGBM)

- it chooses the leaf with the largest loss to grow
- it is called “Light” because of its computation power and giving results faster
- it takes less memory to run and is able to deal with large amounts of data
- it is not for small data sets (it can easily overfit small data due to its sensitivity)

Feature level: random subspace

Idea:

- the ensemble is made of classifiers built on random subsets of features (with replacement) of predefined size d_{rs} ($d_{rs} < d$)
- the classifier outputs are combined by the plurality vote

Comments:

- an attractive choice for high-dimensional problems where the number of features (d) is much larger than the number of training points (n)
- it works best when the discriminative information is "dispersed" across all the features

Data level (ii): random subspace

Training phase

1. Initialize the parameters

   $D = \emptyset$, the ensemble

   L, the number of classifiers to train

2. For $k = 1, \ldots, L$

   Pick up d_{rs} features from d with replacement

   Build a classifier D_k using the subspace sample

   Add the classifier to the current ensemble, $D = D \cup D_k$

3. Return D

Classification phase

1. Run $D_1, \ldots, D_L$ on the input x
2. Assign x to the class with the maximum number of votes