

# MACHINE LEARNING University Master's Degree in Intelligent Systems

## artificial neural networks

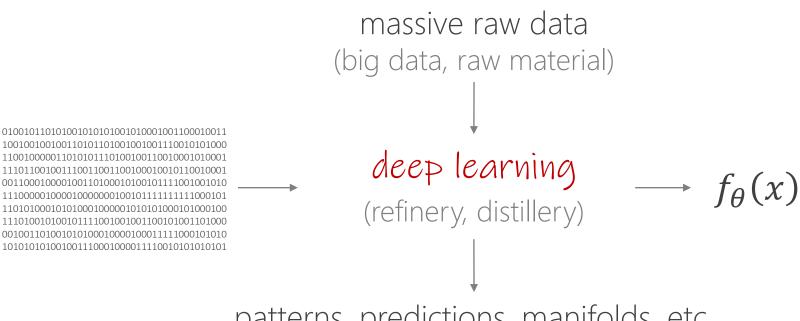
Ramón A. Mollíneda Cárdenas

#### fuel of the future

#### The Economist, May 6th 2017

https://www.economist.com/briefing/2017/05/06/data-is-giving-rise-to-a-new-economy.

"AN OIL refinery... Data centres... the two have much in common. For one thing, both are stuffed with pipes. In refineries these collect petrol, propane and other components of crude oil ... In big data centres ... tens of thousands of computers ... extract value —patterns, predictions and other insights— from raw digital information."



patterns, predictions, manifolds, etc. (product with practical value)

some context

#### **ARTIFICIAL INTELLIGENCE**

Programs with the ability to learn and reason like humans

#### **MACHINE LEARNING**

Algorithms with the ability to learn without being explicitly programmed

#### **DEEP LEARNING**

Subset of machine learning in which artificial neural networks adapt and learn from vast amounts of data

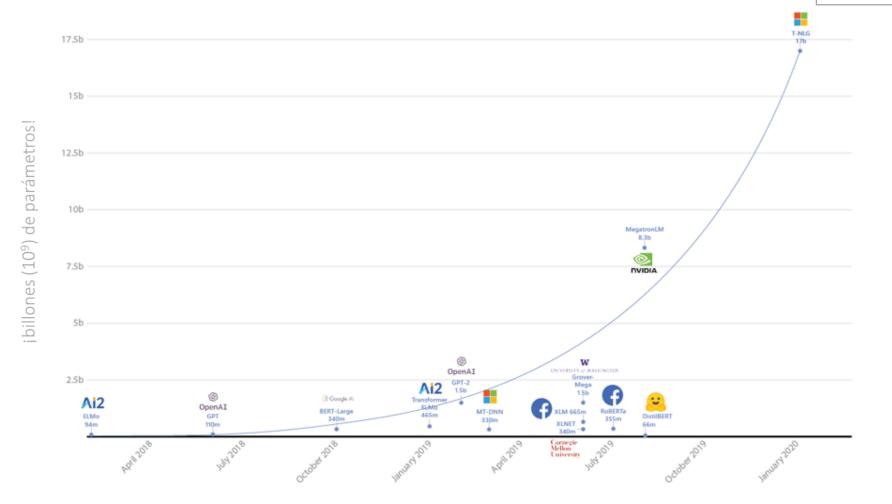
## artificial neural networks outperform humans

- o object and image recognition (ILSVRC, ¿aceptas el reto?)
- o imitation of art styles (<u>DeepArt.io</u>, <u>transferencia de estilo</u>)
- o medical diagnosis (<u>DL 95% senior doctors 87%</u> in the diagnosis of malignant melanomas -distinguishing them from benign moles-)
- o computer gaming (modelos que aprenden, AlphaGo vs. Lee Sedol)
- o lipreading (<u>LipNet 95% humanos 52%</u> according to grammar "command(4) + color(4) + preposition(4) + letter(25) + digit(10) + adverb(4)")
- o big data analytics (<u>demografía a partir de Google Street View</u>).

"Using deep learning ..., we determined the make, model, and year of all motor vehicles encountered .... Data from this census of motor vehicles, which enumerated 22M automobiles in total (8% of all automobiles in the US), was used to accurately estimate income, race, education, and voting patterns ..." (Gebru et al. 2017)

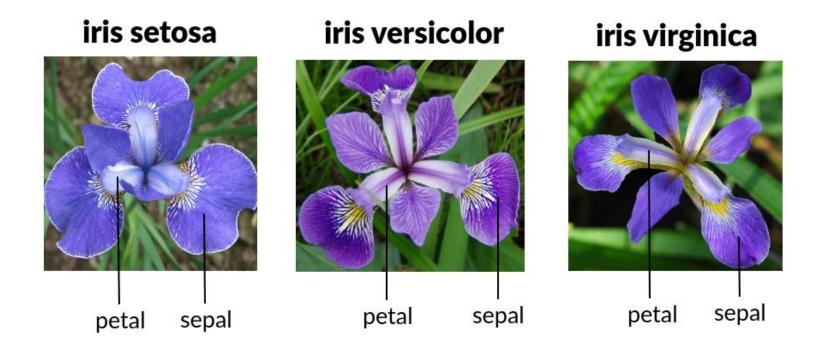
natural language generation models

OpenAl GPT-3 175 billion parameters



applications to summarize texts, generate responses (chatbots), write stories, etc.

# IRIS Data Set classes



#### IRIS Data Set

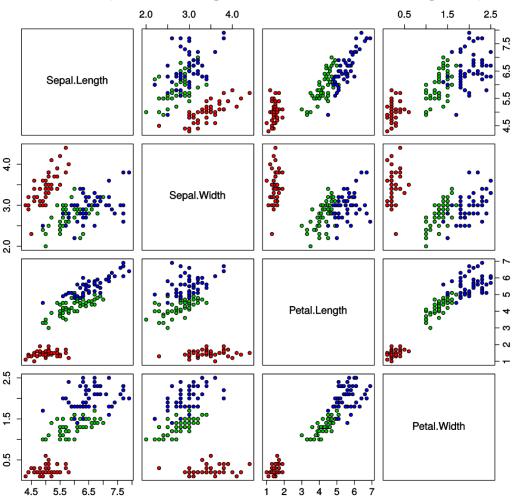
#### description

- most popular database in machine learning
- 150 examples distributed among 3 classes :
  - Iris Versicolor (50)
  - Iris Setosa (50)
  - Iris Virginica (50)
- examples described by 4 measurements (in cm):
  - sepal length
  - sepal width
  - petal length
  - petal width

## IRIS Data Set

## distributions by classes in 2D subspaces

#### Iris Data (red=setosa,green=versicolor,blue=virginica)



# IRIS Data Set original paper

Fisher, R.A. (1936). The use of multiple measurements in taxonomic problems. Annual Eugenics, 7, Part II, 179-188 (acceder).

a fully connected neural network f is a composition of nonlinear transformations of linear models (linear combinations of features)

$$f(x|w_k, \dots w_2, w_1) = g^* \left( w_k^t g_{k-1} \left( \dots g_2 \left( w_2^t g_1(w_1^t x) \right) \right) \right)$$

#### being...

- x, the input vector to the network
- k, the number of layers (or transformations)
- $w_i$ , weight matrix of the layer i,  $1 \le i \le k$
- $g_i$ , nonlinear activation function of the layer i (they are generally the same)
- $g^*$ , output layer activation function (can be the identity function)
- f, network function composed from chains of linear and nonlinear functions

a fully connected neural network f is a composition of nonlinear transformations of linear models (linear combinations of features)

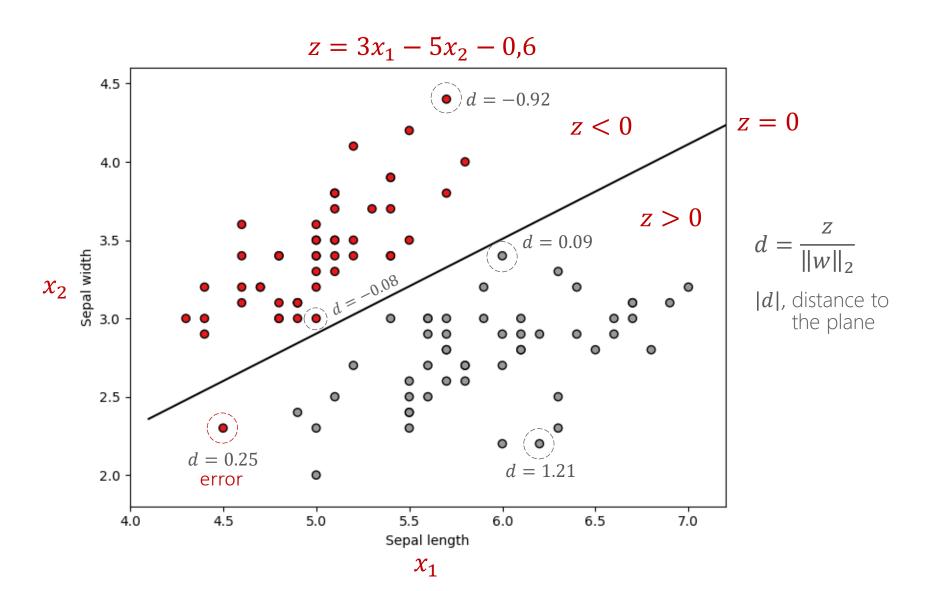
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$$\text{linear model}$$

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- f, network function composed from chains of linear and nonlinear functions

classes Setosa (red) and Versicolor (gray)



## affine/linear transformation

$$z = \mathbf{w}^T \mathbf{x} + b$$
$$z = \sum_{i=1}^d \mathbf{w}_i \mathbf{x}_i + b$$

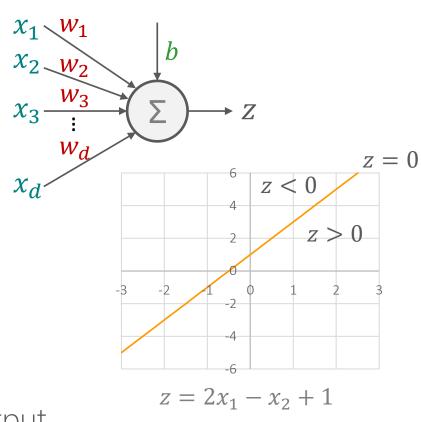
#### being:

w, weight vector

x, input data vector

b, scalar; bias, threshold

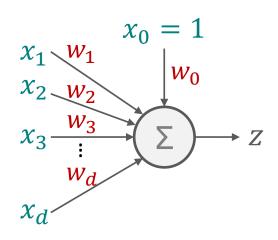
z, scalar; affine transformation output



linear division of space

affine/linear transformation: compact notation

$$z = \mathbf{w}^T \mathbf{x}$$
$$z = \sum_{i=0}^d \mathbf{w}_i \mathbf{x}_i$$



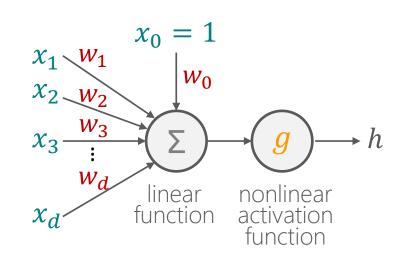
#### being:

w, weight vector ( $w_0$ , bias)

- x, input data vector
- z, scalar; affine transformation output

activation function

$$h = g(z) = g(w^T x)$$



#### being:

- z, scalar; affine transformation output
- g, nonlinear activation function
- h, scalar; nonlinear function output

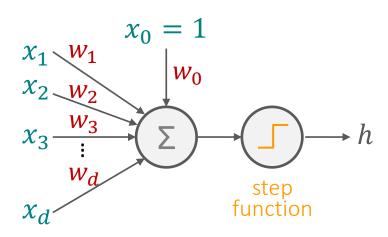
Perceptron (Frank Rosenblatt, 1958)

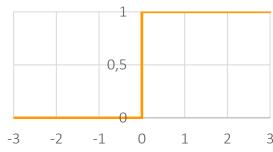
$$h = g(z) = g(w^T x)$$

#### being:

z, affine transformation output g, step function

$$h \in \{0,1\}$$





$$g(z) = \begin{cases} 1, & \text{si } z \ge 0 \\ 0, & \text{si } z < 0 \end{cases}$$

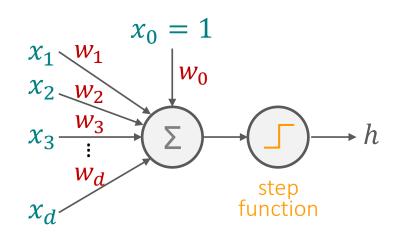
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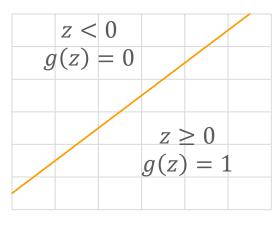
$$h = g(z) = g(w^T x)$$

#### being:

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$$h \in \{0,1\}$$





$$z = 2x_1 - x_2 + 1$$

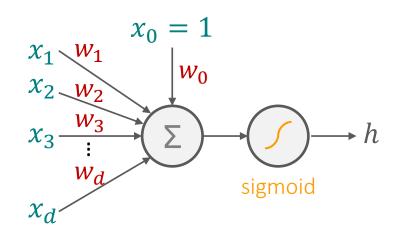
logistic regression: a linear decision boundary

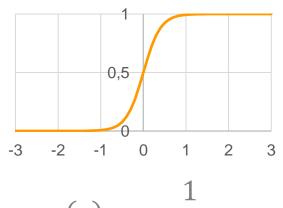
$$h = g(z) = g(w^T x)$$



z, affine transformation output g, sigmoid function

$$h \in (0,1)$$
$$h = Pr(y = 1|x, w)$$





logistic regression

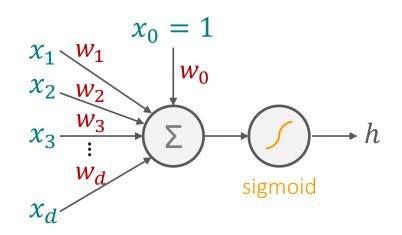
logistic regression: a linear decision boundary

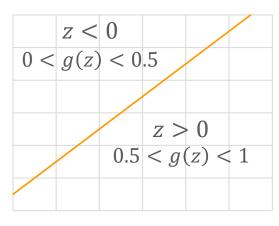
$$h = g(z) = g(w^T x)$$



z, affine transformation output g, sigmoid function

$$h \in (0,1)$$
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$$z = 2x_1 - x_2 + 1$$

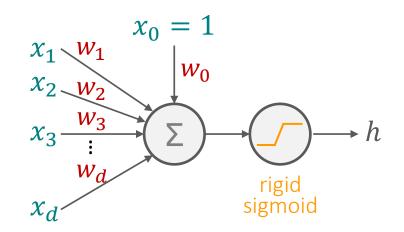
approximate (or rigid) logistic regression

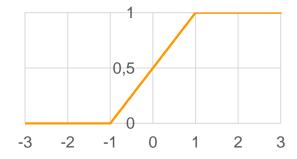
$$h = g(z) = g(w^T x)$$

#### being:

z, affine transformation output g, rigid sigmoid function

$$h \in (0,1)$$
$$h = Pr(y = 1|x, w)$$





$$g(z) = max\left(0, min\left(1, \frac{x+1}{2}\right)\right)$$

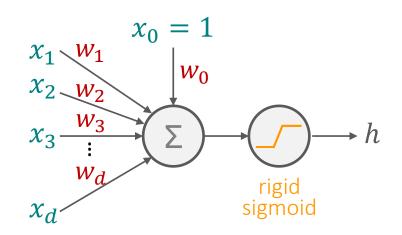
approximate (or rigid) logistic regression

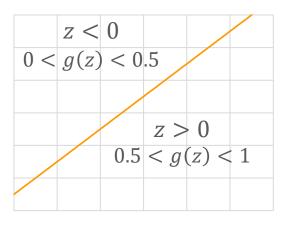
$$h = g(z) = g(w^T x)$$

#### being:

z, affine transformation output g, rigid sigmoid function

$$h \in (0,1)$$
$$h = Pr(y = 1|x, w)$$



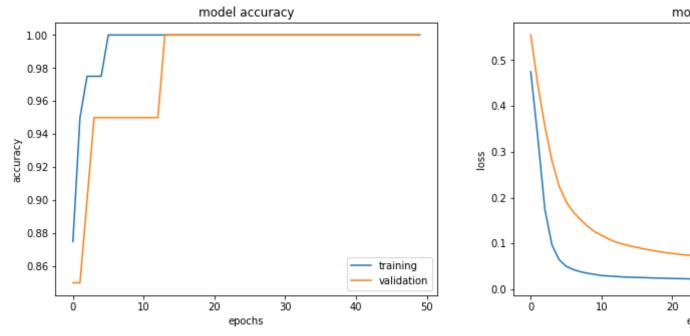


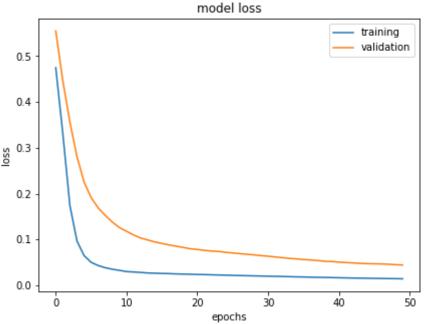
$$z = 2x_1 - x_2 + 1$$

approximate perceptron on Setosa and Versicolor classes

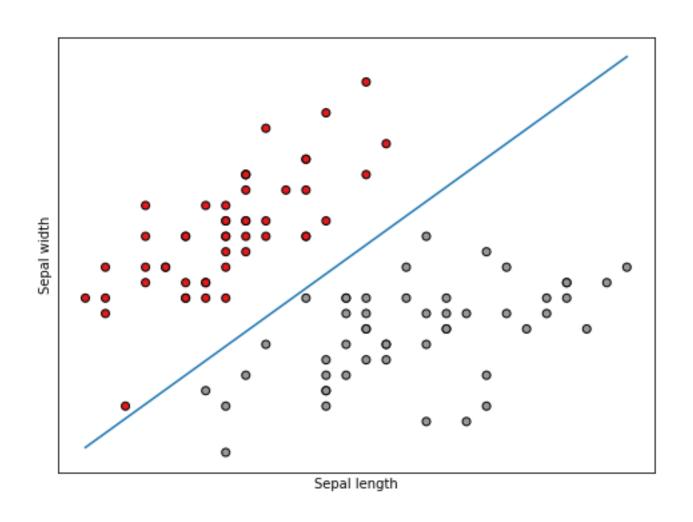
#### script simplificado usando numpy, sklearn y keras:

approximate perceptron on Setosa and Versicolor classes



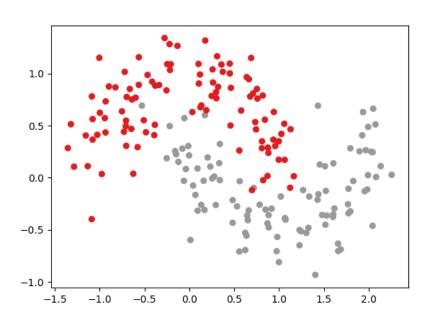


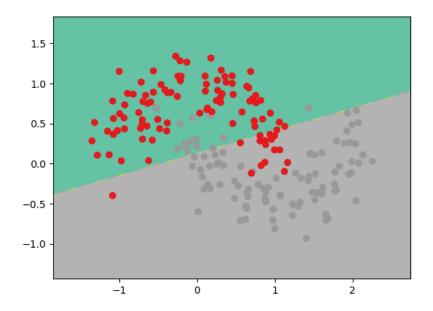
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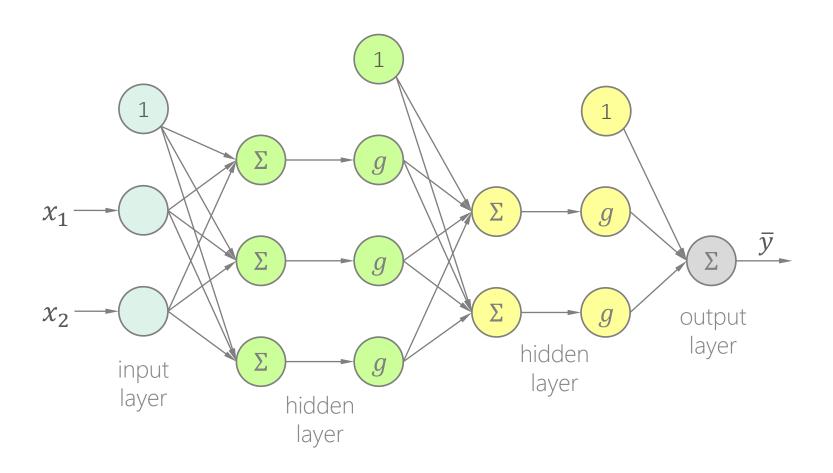


# linear model limitations

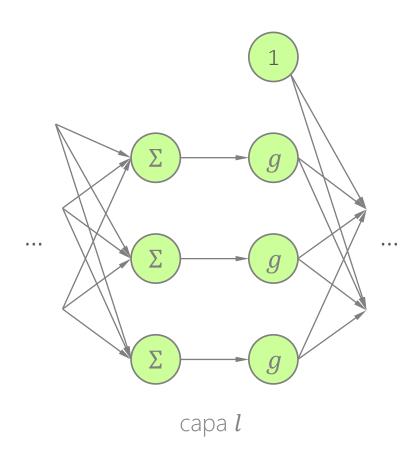
linear classifier
logistic regression => "sigmoid" activation function







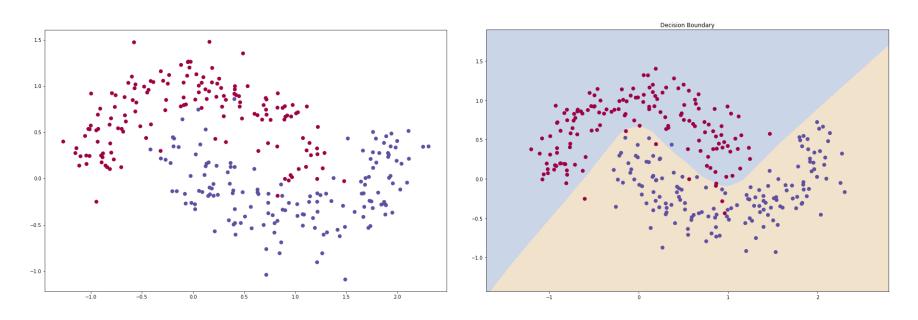
g: función de activación (e.g. sigmoid, tanh, softmax, relu, ...)



- cada capa consiste en: función lineal + activación no lineal
- función de transformación en capa l:  $z_l = W^l \cdot I_l$   $I_{l+1} = g(z_l)$
- sea  $H_l$  el número de unidades de la capa l; el tamaño de  $W^l$  sería ...  $H_l \times H_{l-1}$
- $I_l$  es de tamaño  $H_{l-1} \times 1$

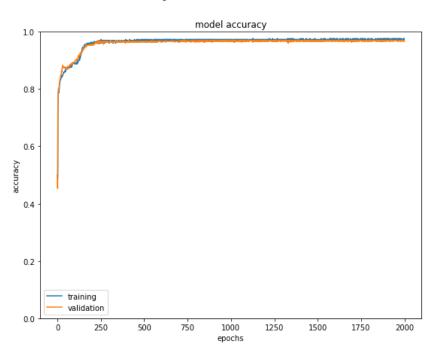
multilayer network

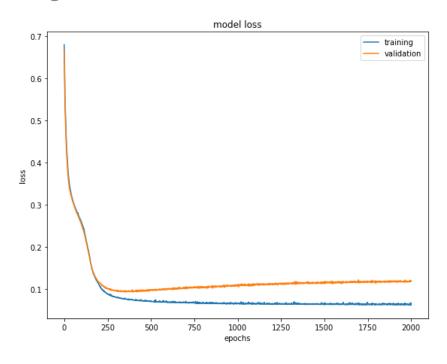
3 layers of 12+6+1 units, "relu" and "sigmoid" activation functions



multilayer network

3 layers of 12+6+1 units, "relu" and "sigmoid" activation functions





learning goal

progressive adjustment (iterative optimization) of weights that minimize the error/loss of the network on a representative set of training samples.

potentially, a network with sufficient learning capacity could achieve zero error on the training set; however, it is <u>not</u> usually a desirable goal (overfitting risk).

training a neural network (mini-batches)

```
initialize network weights w(0) with small arbitrary values
for epoch = 1...K, do
   for batch = 1...N/batch size, do
      batch <- randomly choose batch size instances</pre>
      X, y <- preprocess(batch)
      z <- network(X) (forward execution)</pre>
                                                      backpropagation
      \ell \leftarrow loss(z, y)
   g <- gradients(l, w) (backward execution)</pre>
      w(t+1) \leftarrow w(t) - \gamma \cdot g (weight optimization/fitting)
   end for
end for
```

fundamentals: chain rule

some examples of composite of two (differentiable) functions f(g(x)):

- $f(g(x)) = (2x+1)^3$
- $\circ f(g(x)) = \sin(x^2)$
- $\circ f(g(x)) = \sin(x)^2$

their derivative functions  $f' = f'(g(x)) \cdot g'(x)$ , or  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$ 

$$0 \frac{\partial f}{\partial x} = 3(2x+1)^2 \cdot 2$$

$$0 \frac{\partial f}{\partial x} = \cos(x^2) \cdot \frac{2x}{x}$$

$$\circ \frac{\partial f}{\partial x} = 2 \cdot \sin(x) \cdot \cos(x)$$

## fundamentals: gradient descent

gradient descent (steepest descent) is an iterative optimization algorithm for finding a local minimum of a differentiable function.



#### strategy

- starting from an initial value of the parameter
- move "downhill" along the surface of the function, in the direction of the negative gradient, looking for a parameter value that minimizes it
- stop when the minimum of the function is reached

fundamentals: gradient descent

#### goal

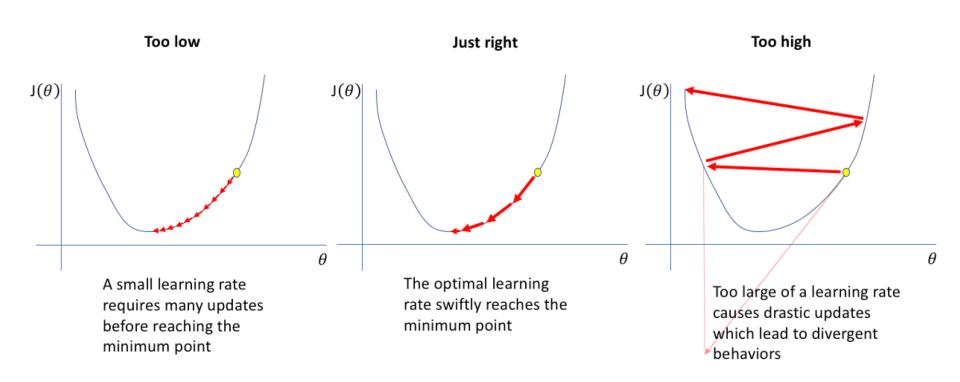
to find  $\theta = \theta^*$  such that  $L(\theta^*)$  is the minimum value of L

#### method

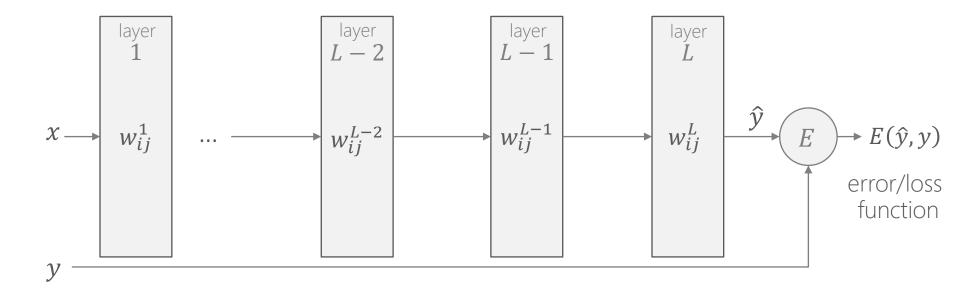
- let  $L(\theta)$  be a function defined by the parameter  $\theta$
- initialization :  $\theta_0 = \theta_{inicial}$
- updating:  $\theta_{i+1} = \theta_i \gamma \frac{\partial L}{\partial \theta}(\theta_i)$ ,
  - $-\gamma\ll 1$  is the learning rate; it determines the magnitude of change
- repeat as long as  $L(\theta_{i+1}) < L(\theta_i)$
- solution:  $\theta^* = \theta_i$

fundamentals: gradient descent

effect of the learning coefficient  $\gamma$ 

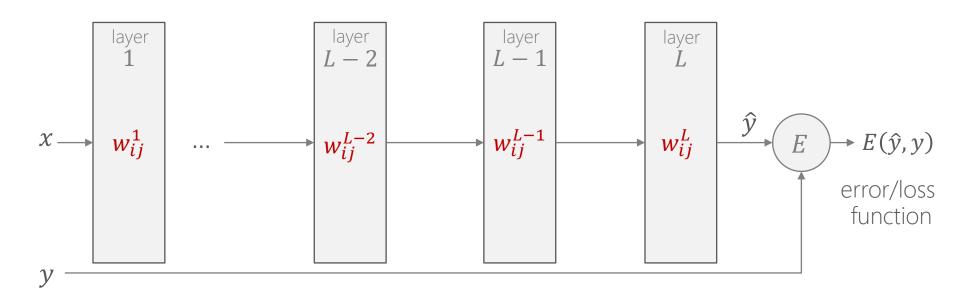


solution scheme



solution scheme



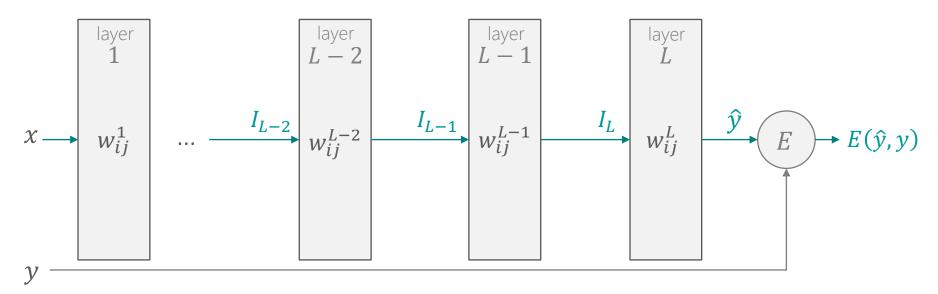


### solution scheme

- 1. initialize weights, choose learning coefficient, choose stop criterion
- 2. create an arbitrary batch  $(X, y) \longrightarrow (subset of instances, expected output)$ .
- 3. forward execution: walk X through the network and get output z.
- 4. compute loss(y, z)
- 5. backward execution (backpropagation)
  - o compute sensitivity coefficient  $\delta_L = f(\delta_{L+1})$  from input to layer L.
  - o compute gradients  $\frac{\partial E}{\partial w_{ij}^{L-1}} = f(\delta_L)$
- 6. weight optimization
- 7. check stopping criteria; if it is fulfilled, then finish; otherwise, go to step 2.

solution scheme

#### forward execution

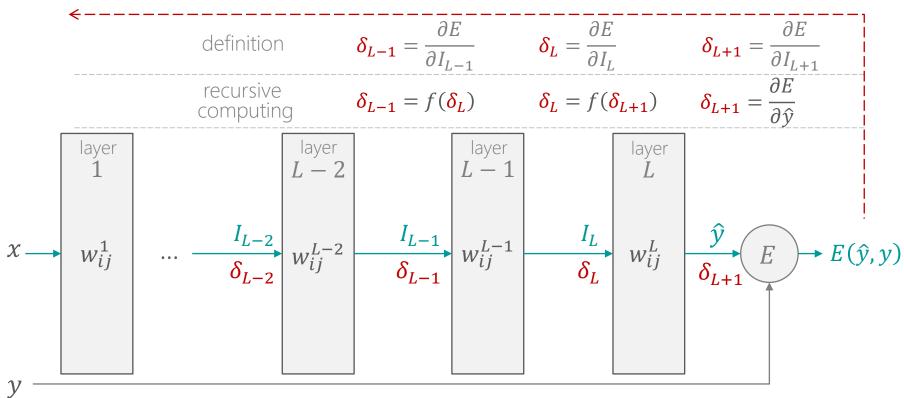


the following pieces of information are calculated:

- input  $I_l$  to each layer l (matches the output of the layer l-1),
- actual network output  $\hat{y}$
- value of the error or loss function  $E(\hat{y}, y)$

solution scheme

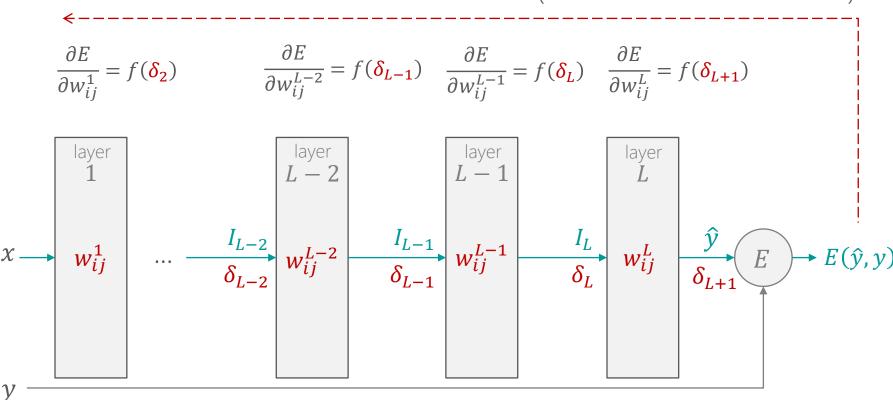
backward execution (calculation of sensitivities)



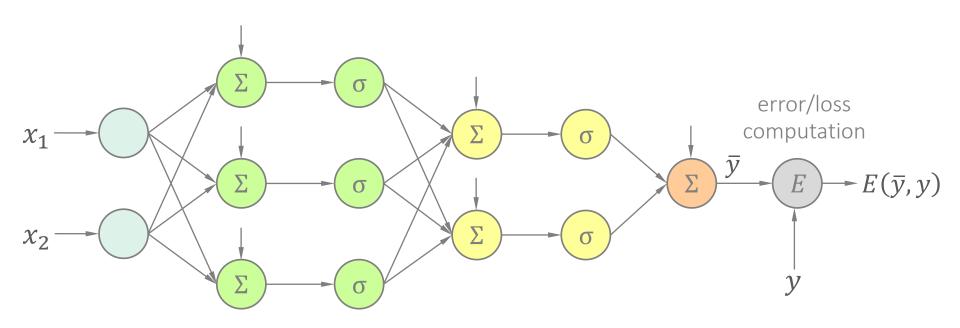
- $\delta_l$  measures the sensitivity of E to changes in  $I_l$
- $\delta_l$  is computed recursively from  $\delta_{l+1}$
- $\delta_l$  allows efficient computation of  $\frac{\partial E}{\partial w_{ij}^{l-1}}$

solution scheme

backward execution (calculation of sensitivities)

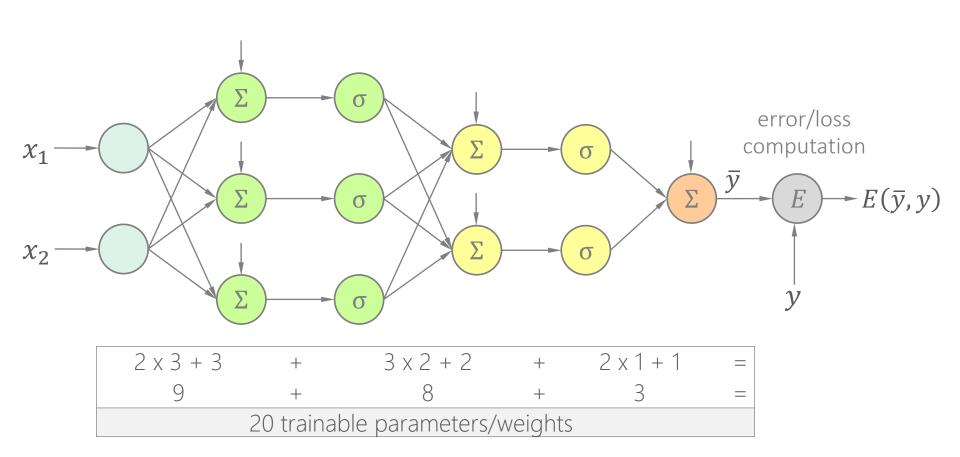


case study: fully connected network with 3 layers



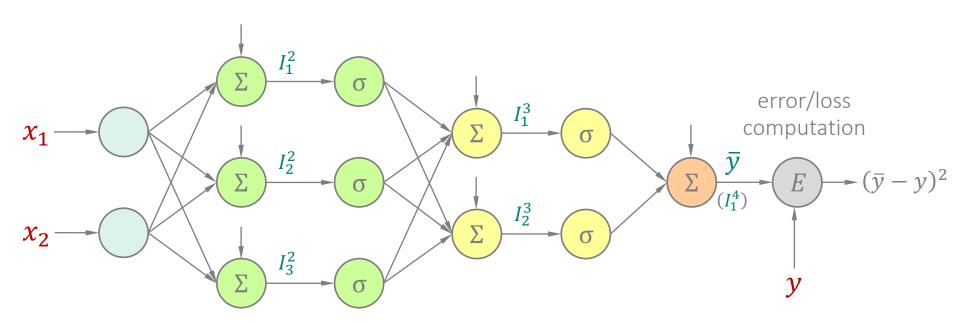
 $\sigma$ : activation function (e.g. sigmoid, tanh, relu, etc.)

case study: fully connected network with 3 layers



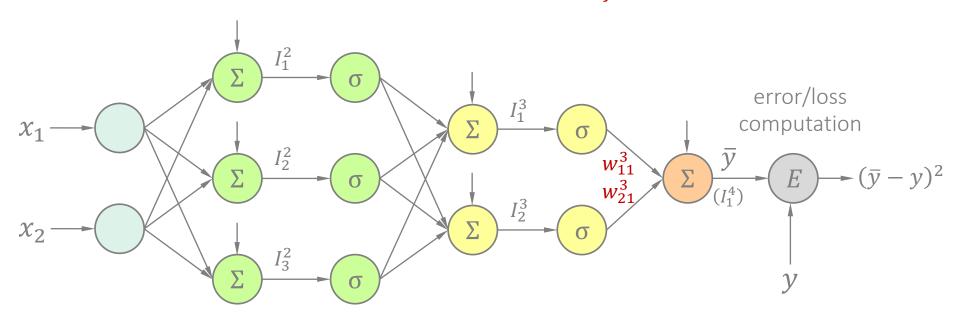
case study: fully connected network with 3 layers

forward execution: introduce (x,y) and compute  $I_i^l$  and  $ar{y}$ 



case study: fully connected network with 3 layers

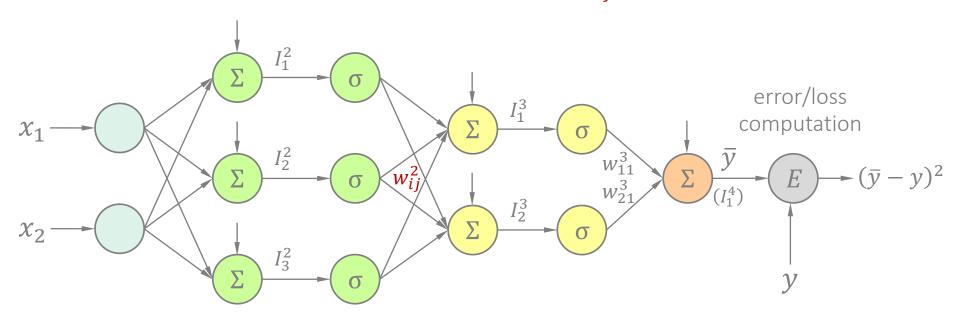
backward execution: compute gradients  $\frac{\partial E}{\partial w_{ij}^l}$ 



$$\frac{\partial E}{\partial w_{ij}^3} = \frac{\partial I_1^4}{\partial w_{ij}^3} \cdot \delta_1^4 = \sigma(I_i^3) \cdot 2(\bar{y} - y)$$

case study: fully connected network with 3 layers

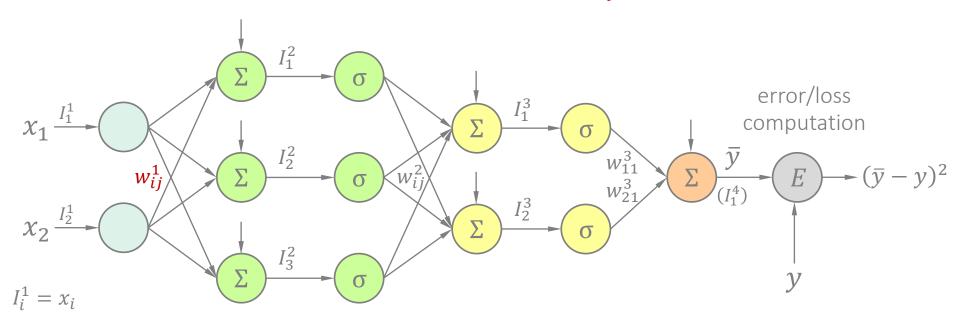
backward execution: compute gradients  $\frac{\partial E}{\partial w_{ij}^l}$ 



$$\frac{\partial E}{\partial w_{i,i}^2} = \frac{\partial I_j^3}{\partial w_{i,i}^2} \cdot \delta_j^3 = \sigma(I_i^2) \cdot \sigma'(I_j^3) \cdot w_{j,1}^3 \cdot \delta_1^4$$

case study: fully connected network with 3 layers

backward execution: compute gradients  $\frac{\partial E}{\partial w_{ij}^l}$ 



$$\frac{\partial E}{\partial w_{ij}^1} = \frac{\partial I_j^2}{\partial w_{ij}^1} \cdot \delta_j^2 = \sigma(I_i^1) \cdot \sigma'(I_j^2) \sum_{k=1}^2 w_{jk}^2 \cdot \delta_k^3$$

fully connected neural network for the MNIST digits task

## implementation in Keras

- deep learning framework: Python library for creating neural networks
- high-level interface based on Tensorflow, Theano, or the Microsoft Cognitive Toolkit
- it allows defining and assembling pieces in neural networks such as layers, objective functions, activation, optimizers, etc.

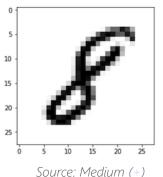
fully connected neural network for the MNIST digits task

#### MNIST (Modified National Institute of Standards and Technology database)

handwritten digits image collection (official website)



- 60,000 training images
- 10,000 test images
- 10 classes
- image size: 28x28
- grey images

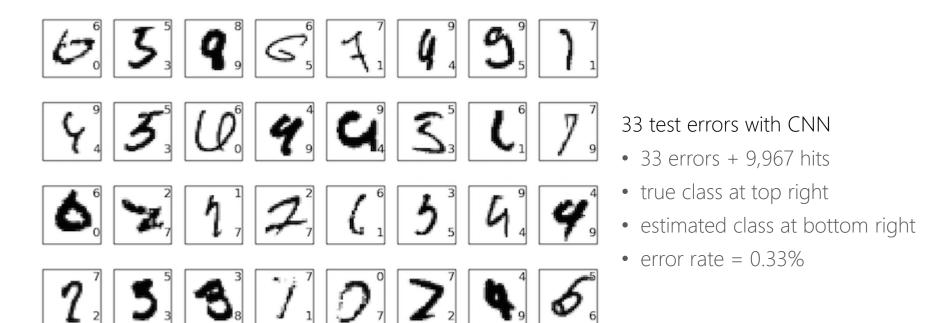


Source: Wikipedia (+)

fully connected neural network for the MNIST digits task

#### MNIST (Modified National Institute of Standards and Technology database)

- relatively simple task
- although it includes complex cases that are difficult to read (noise)



Source: Michael Nielsen (2019). Neural Networks and Deep Learning, online book ( $\pm$ ).

fully connected neural network for the MNIST digits task

## simplified script

```
(x_train, y_train), (x_test, y_test) = mnist.load_data()
x_{train} = x_{train.reshape}(60000, 784).astype('float32')/255
x_{test} = x_{test.reshape}(10000, 784).astype('float32')/255
y_train = keras.utils.to_categorical(y_train, num_classes)
y_test = keras.utils.to_categorical(y_test, num_classes)
model = Sequential()
model.add(Dense(1024, activation='relu', input_shape=(784,)))
model.add(Dense(1024, activation='relu'))
model.add(Dense(1024, activation='relu'))
model.add(Dense(10, activation='softmax'))
sgd=SGD(Ir=0.01, decay=1e-6, momentum=0.9)
model.compile(loss='categorical_crossentropy', optimizer=sgd, metrics=['accuracy'])
history = model.fit(x_train, y_train,batch_size=100, epochs=5,
                   validation_data=(x_test,y_test))
```

fully connected neural network for the MNIST task

How many parameters does this fully connected network have?

fully connected neural network for the MNIST task

layers	input	hidden 1	hidden 2	hidden 3	output	TOTAL
units	784	1.024	1.024	1.024	10	3.082
weights		802.816	1.048.576	1.048.576	10.240	2.910.208
bias		1.024	1.024	1.024	10	3.082
TOTAL		803.840	1.049.600	1.049.600	10.250	2.913.290

fully connected neural network for the MNIST task

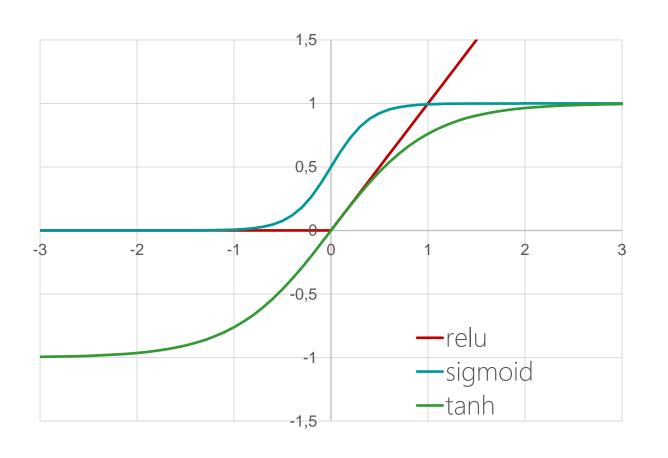
#### hyperparameters

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model = Sequential()
model.add(Dense(1024, activation='relu', input_shape=(784,)))
model.add(Dense(1024, activation='relu'))
model.add(Dense(1024, activation='relu'))
model.add(Dense(10, activation='softmax'))
sgd=SGD(Ir=0.01, momentum=0.9)
model.compile(loss='categorical_crossentropy', optimizer=sqd, metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=100, epochs=5,
                   validation data=(x test,y test))
```

## fully connected neural network for the MNIST task

activation	'relu'	•	Rectified Linear Unit (ReLU) a piecewise linear activation function (hidden u)
activation	'softmax'	•	output layer activation function probability distribution over K outputs
loss	'categorical_crossentropy'	•	one-hot vector + softmax + cross entropy loss function measures discrepancy between two distributions
optimizer	'sgd'	•	stochastic gradient descent basic optimization method
batch_size	100	•	mini-batch based training stochastic method (random subsets) weights update after each batch
epochs	5	•	learning loop over all training samples

relu, sigmoid, tanh



#### relu

$$g(z) = max\{0, z\}$$

sigmoid
$$g(z) = \frac{1}{1 + e^{-z}}$$

tanh

$$g(z) = 2\sigma(2z) - 1$$

softmax (normalized exponential function)

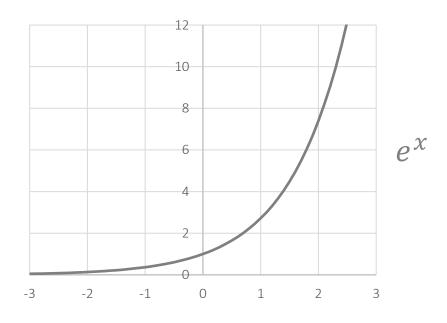
given a vector  $z \in \mathbb{R}^K$ 

softmax projects a vector of real data onto a "probability distribution"

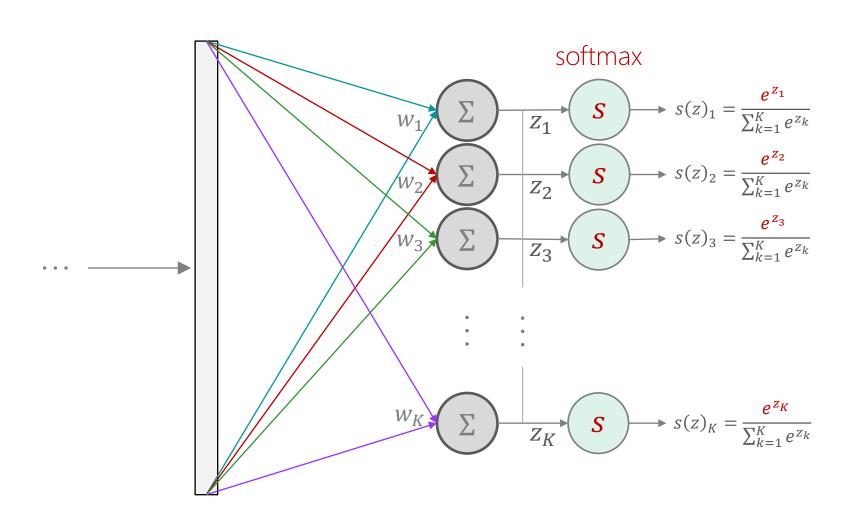
$$s: \mathbb{R}^K \to [0,1]^K, \quad \sum_i s(z)_i = 1$$

#### formulation:

$$S(z)_i = \frac{e^{z_i}}{\sum_{k=1}^K e^{z_k}}$$



softmax (normalized exponential function)



softmax (normalized exponential function)

softmax is increasing: if  $z_i < z_j$ , then  $s(z_i) < s(z_j)$ 

## example

$z_i$	$e^{z_i}$	$s(z)_i$	
-1	0,37	0,01	
0	1,00	0,02	penalizes non-maximun
1	2,72	0,04	activation values
2	7,39	0,11	
4	54,60	0,83	reinforces the highest
	66,07	1,00	activation value

softmax (normalized exponential function)

softmax saturates if  $\min_i z_i \ll \max_i z_i$  ejemplo

$z_i$	$e^{z_i}$	$s(z)_i$	
-1	0,37	0,00	
0	1,00	0,00	
1	2,72	0,00	
2	7,39	0,00	
8	2.980,96	1,00	the winner takes it all
	2.992,43	1,00	lakes il aii

## operating principle

#### given

- **y**, expectation (ground truth)
- $\hat{y}$ , prediction, estimation

#### a los/cost/error function $L(y, \hat{y})$

- ullet measures the distance, difference, or discrepancy between y e  $\widehat{y}$
- when y e  $\hat{y}$  are very different, then L is large (high loss)
- when y e  $\hat{y}$  are very similar, then L is small (low loss)
- when y e  $\hat{y}$  are equal, then L=0

#### learning objective

• find parameters of the model that minimize L over the validation set (part of the training data, not the test data!)

one-hot encoding (output encoding)

categorical/nominal variable: takes symbolic values, not numeric ones.

#### examples:

pet: "dog", "cat", "bird"

model: "sedan", "minivan", "bus", "truck"

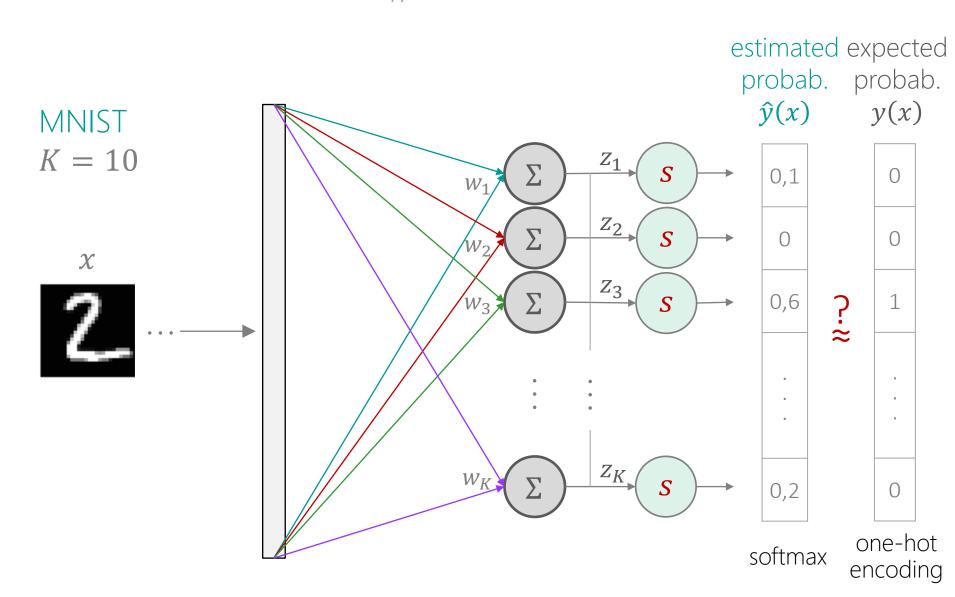
dígitos: '0', '1', '2', ...'9'

limitation: do not support numerical comparisons/operations

## one-hot encoding:

	sedan	minivan	bus	truck
$x_{sed}$	1	0	0	0
$x_{min}$	0	1	0	0
$x_{bus}$	0	0	1	0
$x_{tru}$	0	0	0	1

how to measure the difference between two distributions?



## cross entropy

#### given two probability distributions:

- y (e.g. expected, ground truth)
- $\hat{y}$  (e.g. predicted, observed)

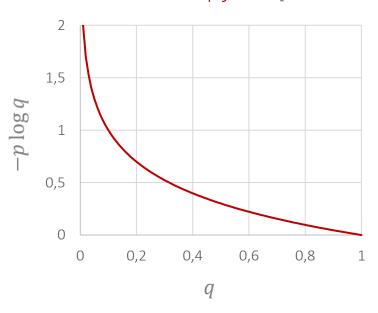
cross entropy (between two distributions):

$$E(y, \hat{y}) = -\sum_{i} y_{i} \log \hat{y}_{i}$$

#### effect:

- if y and  $\hat{y}$  are similar, then  $E(y, \hat{y})$  is small
- if y and  $\hat{y}$  are different, then  $E(y, \hat{y})$  is large

#### cross entropy for p = 1



cross entropy

#### two opposite scenarios

#### favorable scenario

у	ŷ	$-y_i \log \widehat{y_i}$
0	0,05	0,00
0	0,05	0,00
1	0,80	0,10
0	0,05	0,00
0	0,05	0,00
	loss	0,10

## unfavorable scenario

У	ŷ	$-y_i \log \widehat{y_i}$
0	0,20	0,00
0	0,20	0,00
1	0,20	0,70
0	0,20	0,00
0	0,20	0,00
	loss	0,70

only the  $\widehat{y}_i$  associated with  $y_i = 1$  contributes (log 1 is the smallest loss)

## optimization problem

MNIST dataset

given 60.000 training digit images, with their classes annotated as one-hot vectors...

$$T = \{(x_i, y_i)\}_{i=\overline{1.60.000}}$$
 such that  $x_i \in [0,1]^{784}$ ,  $y_i \in \{0,1\}^{10}$ 

goal: to find optimal values for the 2.913.920 model parameters

$$W^* = \arg\min_{W} L(W), \quad W \in \mathbb{R}^{2.913.920}$$

with L(W) being the loss function defined as follows:

$$L(W) = \frac{1}{60.000} \sum_{i=1}^{60.000} E(y_i, \hat{y}_i) = -\frac{1}{60.000} \sum_{i=1}^{60.000} \sum_{j=1}^{10} y_{ij} \log \hat{y}_{ij}$$

## optimizer

sgd + momentum + weight decay

#### stochastic gradient descent (sgd):

$$\theta_{t+1} = \theta_t - \gamma \frac{\partial L}{\partial \theta}(\theta_t)$$

sgd + momentum + weight decay ( $\gamma$  denotes the learning rate):

$$\theta_{t+1} = \theta_t + v_t$$

$$v_t = momentum \cdot v_{t-1} - \gamma \frac{\partial L}{\partial \theta}(\theta_t)$$

$$\gamma = \gamma \cdot \frac{1}{1 + decay \cdot iteración}$$

# hyperparameters overview

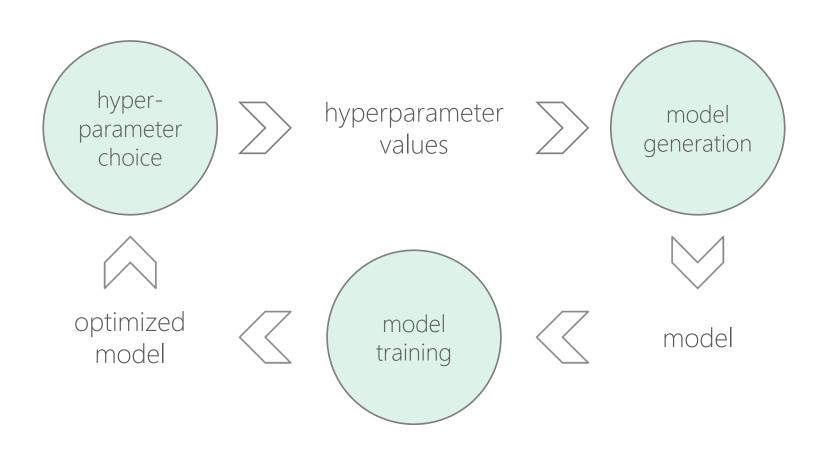
hyperparameters are parameters needed to generate the model or to define the training process

hyperparameters determine the structure or configuration of the model or characteristics of the learning process; their values are chosen before each training session.

their optimal values depend on the complexity of the task, the nature of the data (dimensionality, distribution, quantity, etc.), and the interdependence with other hyperparameters.

# hyperparameters

overview



## hyperparameters

fully connected neural network for the MNIST task

hyperparameters of the network architecture defined for MNIST:

- 4 trainable layers
- layer 1: 1024 units, ReLU activation
- layer 1: 1024 units, ReLU activation
- layer 1: 1024 units, ReLU activation
- layer 4: softmax activation

## hyperparameters

fully connected neural network for the MNIST task

hyperparameters of the learning process defined for MNIST:

- optimizer: SGD(lr=0.01, decay=1e-6, momentum=0.9)
- loss = 'categorical\_crossentropy'
- batch\_size = 100
- epochs = 5

## summary

- a machine learning algorithm optimizes models from data
- a linear model solves only tasks with linear boundaries
- Perceptron is a linear model of binary classification
- a fully connected multilayer network is composed of a sequence of fully connected layers.
- each unit consists of a linear function + nonlinear activation
- a multilayer network could learn any decision boundary
- backpropagation is an algorithm for training feedforward networks in supervised learning; includes calculation of gradients, not how to use them.