



IIT KHARAGPUR



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CERTIFICATION COURSES

Data Mining

Week 1: Introduction, Association Rules

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Course Outline:

- Introduction: KDD Process
- Data Preprocessing
- Association Rule Mining
- Classification
- Clustering and Anomaly Detection
- Regression
- Case Studies

Data Mining

Introduction

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Why Data Mining?

- The Explosive Growth of Data: from terabytes to petabytes
 - Data collection and data availability
 - Automated data collection tools, database systems, Web, computerized society
 - Major sources of abundant data
 - Business: Web, e-commerce, transactions, stocks, ...
 - Science: Remote sensing, bioinformatics, scientific simulation, ...
 - Society and everyone: news, digital cameras, YouTube
- We are drowning in data, but starving for knowledge!
- “Necessity is the mother of invention”—Data mining—Automated analysis of massive data

What Is Data Mining?

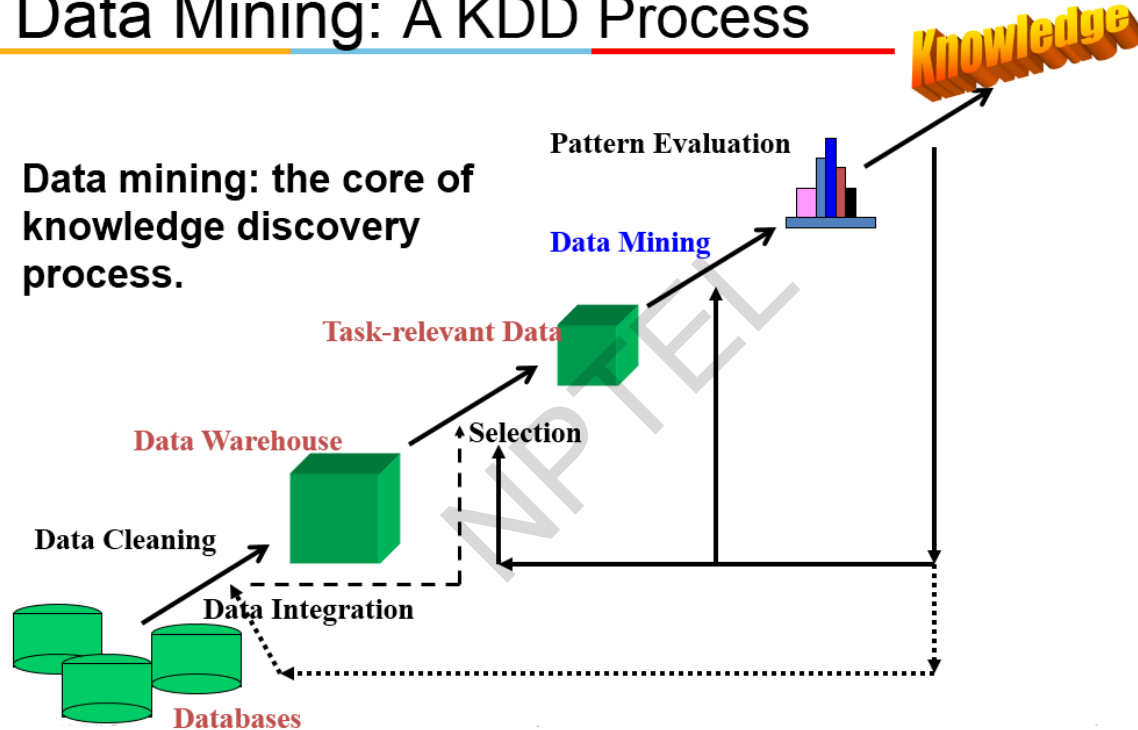


- Data mining (knowledge discovery from data)
 - Extraction of interesting (non-trivial, implicit, previously unknown and potentially useful) patterns or knowledge from huge amount of data
- Alternative names
 - Knowledge discovery (mining) in databases (KDD), knowledge extraction, data/pattern analysis, data archeology, data dredging, information harvesting, business intelligence, etc.
- Watch out: Is everything “data mining”?
 - Simple search and query processing
 - (Deductive) expert systems

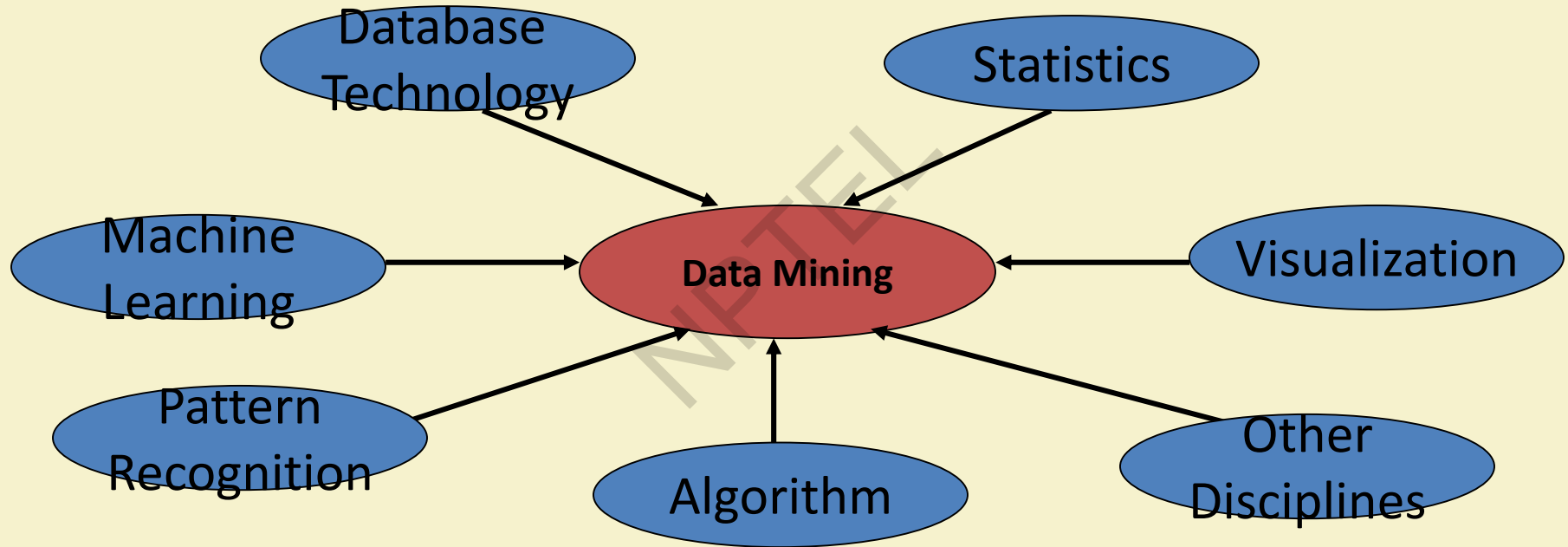


Data Mining: A KDD Process

Data mining: the core of knowledge discovery process.



Data Mining: Confluence of Multiple Disciplines



Why Not Traditional Data Analysis?

- Tremendous amount of data
 - Algorithms must be highly scalable to handle such as tera-bytes of data
- High-dimensionality of data
 - Micro-array may have tens of thousands of dimensions
- High complexity of data
 - Data streams and sensor data
 - Time-series data, temporal data, sequence data
 - Structure data, graphs, social networks and multi-linked data
 - Heterogeneous databases and legacy databases
 - Spatial, spatiotemporal, multimedia, text and Web data

Data Mining: On What Kinds of Data?

- Database-oriented data sets and applications
 - Relational database, data warehouse, transactional database
- Advanced data sets and advanced applications
 - Data streams and sensor data
 - Time-series data, temporal data, sequence data (incl. bio-sequences)
 - Structure data, graphs, social networks and multi-linked data
 - Object-relational databases
 - Heterogeneous databases and legacy databases
 - Spatial data and spatiotemporal data
 - Multimedia database
 - Text databases
 - The World-Wide Web

Data Mining Functionalities

- Multidimensional concept description: Characterization and discrimination
 - Generalize, summarize, and contrast data characteristics, e.g., dry vs. wet regions
- Frequent patterns, association, correlation vs. causality
 - Tea → Sugar [0.5%, 75%] (Correlation or causality?)
- Classification and prediction
 - Construct models (functions) that describe and distinguish classes or concepts for future prediction
 - E.g., classify countries based on (climate), or classify cars based on (gas mileage)
 - Predict some unknown or missing numerical values

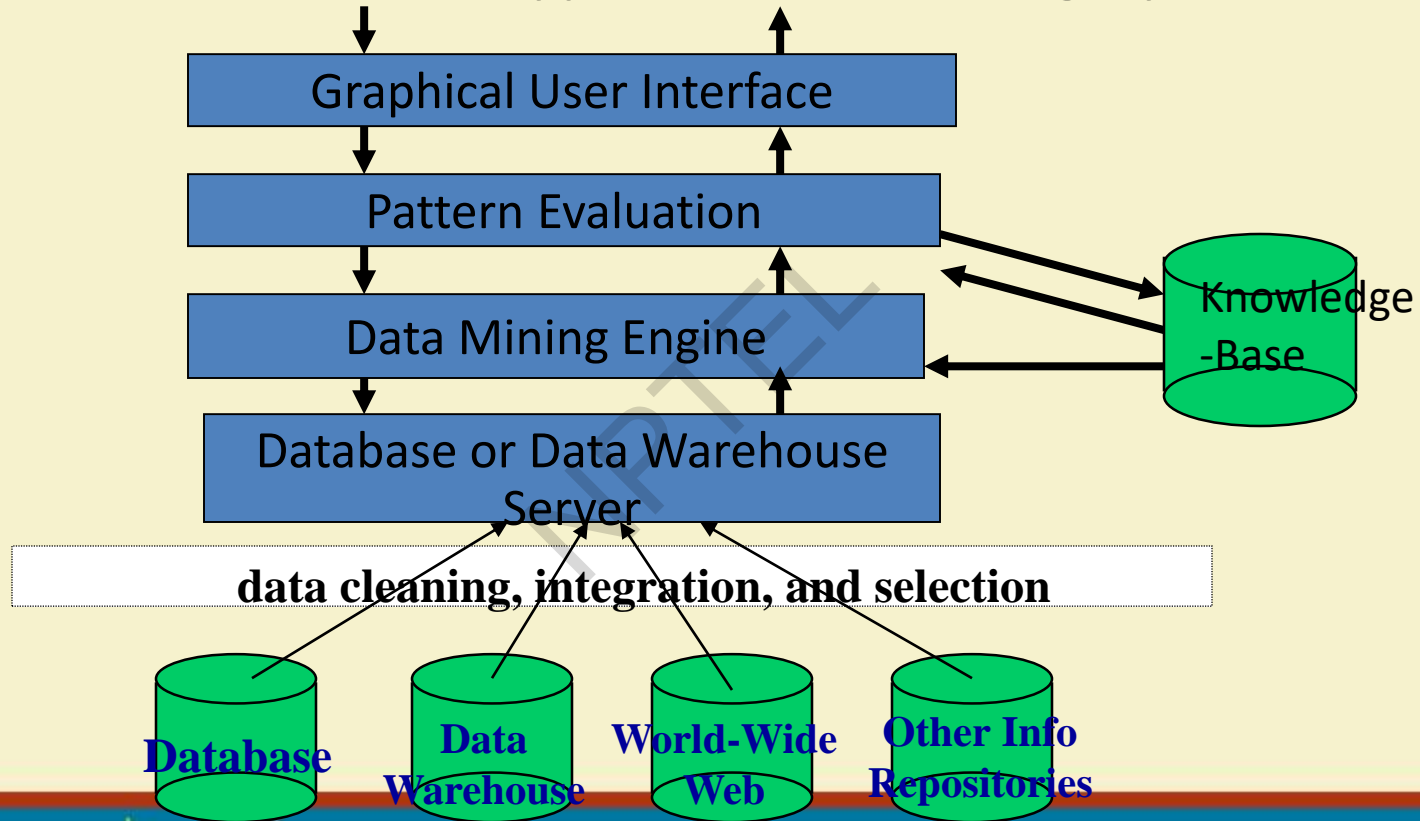
Data Mining Functionalities

- Cluster analysis
 - Class label is unknown: Group data to form new classes, e.g., cluster houses to find distribution patterns
 - Maximizing intra-class similarity & minimizing interclass similarity
- Outlier analysis
 - Outlier: Data object that does not comply with the general behavior of the data
 - Noise or exception? Useful in fraud detection, rare events analysis
- Trend and evolution analysis
 - Trend and deviation: e.g., regression analysis
 - Sequential pattern mining: e.g., digital camera → large SD memory
 - Periodicity analysis
 - Similarity-based analysis
- Other pattern-directed or statistical analyses

Major Issues in Data Mining

- Mining methodology
 - Mining different kinds of knowledge from diverse data types, e.g., bio, stream, Web
 - Performance: efficiency, effectiveness, and scalability
 - Pattern evaluation: the interestingness problem
 - Incorporation of background knowledge
 - Handling noise and incomplete data
 - Parallel, distributed and incremental mining methods
 - Integration of the discovered knowledge with existing one: knowledge fusion
- User interaction
 - Data mining query languages and ad-hoc mining
 - Expression and visualization of data mining results
 - Interactive mining of knowledge at multiple levels of abstraction
- Applications and social impacts
 - Domain-specific data mining & invisible data mining
 - Protection of data security, integrity, and privacy

Architecture: Typical Data Mining System



KDD Process: Summary

- Learning the application domain
 - relevant prior knowledge and goals of application
- Creating a target data set: data selection
- **Data cleaning** and preprocessing: (may take 60% of effort!)
- **Data reduction and transformation**
 - Find useful features, dimensionality/variable reduction, invariant representation
- Choosing functions of data mining
 - summarization, classification, regression, association, clustering
- Choosing the mining algorithm(s)
- **Data mining**: search for patterns of interest
- **Pattern evaluation and knowledge presentation**
 - visualization, transformation, removing redundant patterns, etc.
- Use of discovered knowledge

End of Introduction

Data Mining

Data Preprocessing

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What is Data?

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
 - Examples: eye color of a person, temperature, etc.
 - Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
 - Object is also known as record, point, case, sample, entity, or instance

Attributes

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Objects

Types of Attributes

- There are different types of attributes
 - **Nominal**
 - Examples: ID numbers, eye color, zip codes
 - **Ordinal**
 - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
 - **Interval**
 - Examples: calendar dates, temperatures in Celsius or Fahrenheit.
 - **Ratio**
 - Examples: temperature in Kelvin, length, time, counts

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
 - Distinctness: $= \neq$
 - Order: $< >$
 - Addition: $+ -$
 - Multiplication: $* /$
 - Nominal attribute: distinctness
 - Ordinal attribute: distinctness & order
 - Interval attribute: distinctness, order & addition
 - Ratio attribute: all 4 properties

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. (=, ≠)	zip codes, employee ID numbers, eye color, sex: { <i>male</i> , <i>female</i> }	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects (< >).	hardness of minerals, { <i>good</i> , <i>better</i> , <i>best</i> }, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. (+, -)	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, <i>t</i> and <i>F</i> tests
Ratio	For ratio variables, both differences and ratios are meaningful. (*, /)	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation



Discrete and Continuous Attributes

- Discrete Attribute
 - Has only a finite or countably infinite set of values
 - Examples: zip codes, counts, or the set of words in a collection of documents
 - Often represented as integer variables.
 - Note: binary attributes are a special case of discrete attributes
- Continuous Attribute
 - Has real numbers as attribute values
 - Examples: temperature, height, or weight.
 - Practically, real values can only be measured and represented using a finite number of digits.
 - Continuous attributes are typically represented as floating-point variables.

Types of data sets

- **Record**

- Data Matrix
- Document Data
- Transaction Data

- **Graph**

- World Wide Web
- Molecular Structures

- **Ordered**

- Spatial Data
- Temporal Data
- Sequential Data
- Genetic Sequence Data

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

<i>Tid</i>	<i>Refund</i>	<i>Marital Status</i>	<i>Taxable Income</i>	<i>Cheat</i>
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute
- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Text Data

- Each document becomes a 'term' vector,
 - each term is a component (attribute) of the vector,
 - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

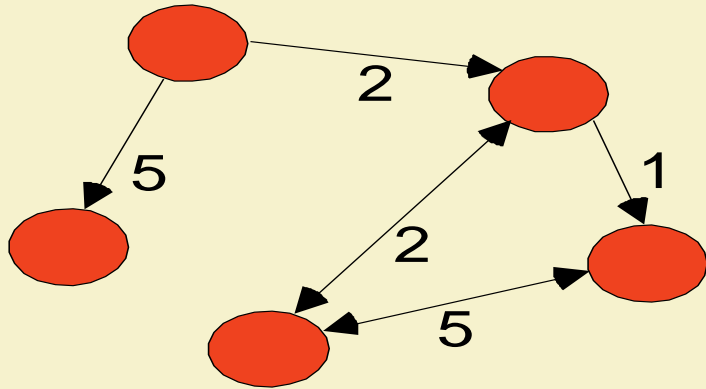
Transaction Data

- A special type of record data, where
 - each record (transaction) involves a set of items.
 - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

<i>TID</i>	<i>Items</i>
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Graph Data

- Examples: Facebook graph and HTML Links



Ordered Data

- Genomic sequence data

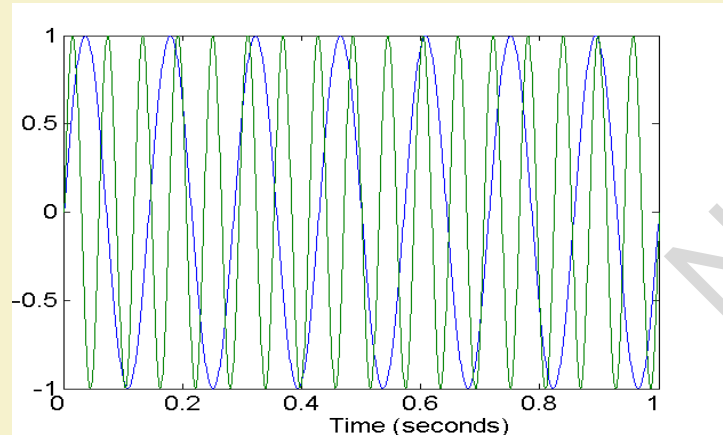
GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG

Data Quality

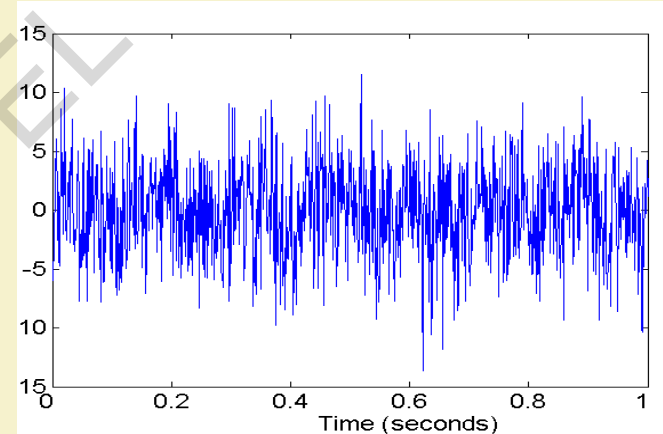
- What kinds of data quality problems?
- How can we detect problems with the data?
- What can we do about these problems?
- Examples of data quality problems:
 - Noise and outliers
 - missing values
 - duplicate data

Noise

- Noise refers to modification of original values
 - Examples: distortion of a person's voice when talking on a poor phone and “snow” on television screen



Two Sine Waves



Two Sine Waves + Noise

Outliers

- Outliers are data objects with characteristics that are considerably different than most of the other data objects in the data set



Missing Values

- Reasons for missing values
 - Information is not collected (e.g., people decline to give their age and weight)
 - Attributes may not be applicable to all cases (e.g., annual income is not applicable to children)
- Handling missing values
 - Eliminate Data Objects
 - Estimate Missing Values
 - Ignore the Missing Value During Analysis
 - Replace with all possible values (weighted by their probabilities)

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
 - Major issue when merging data from heterogenous sources
- Examples:
 - Same person with multiple email addresses
- Data cleaning
 - Process of dealing with duplicate data issues

Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

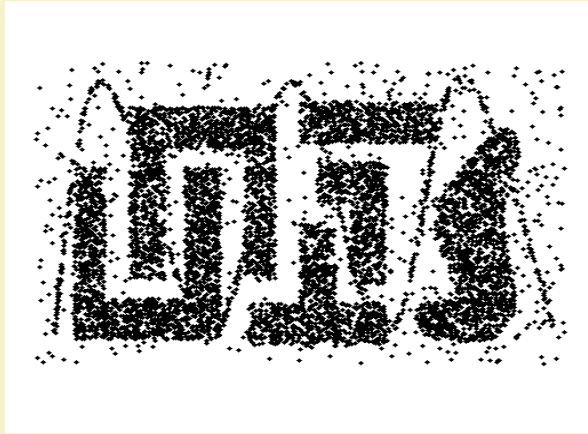
Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)
- Purpose
 - Data reduction
 - Reduce the number of attributes or objects
 - Change of scale
 - Cities aggregated into regions, states, countries, etc
 - More “stable” data
 - Aggregated data tends to have less variability

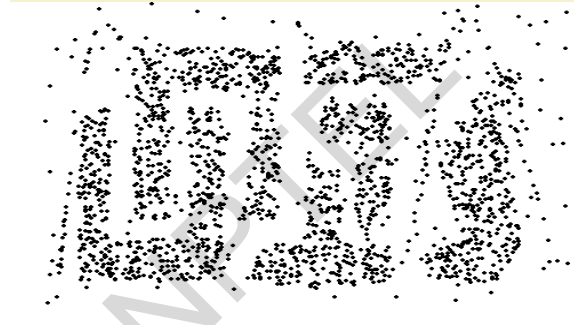
Sampling

- Sampling is the main technique employed for data selection.
 - It is often used for both the preliminary investigation of the data and the final data analysis.
- Statisticians sample because **obtaining** the entire set of data of interest is too expensive or time consuming.
- Sampling is used in data mining because **processing** the entire set of data of interest is too expensive or time consuming.

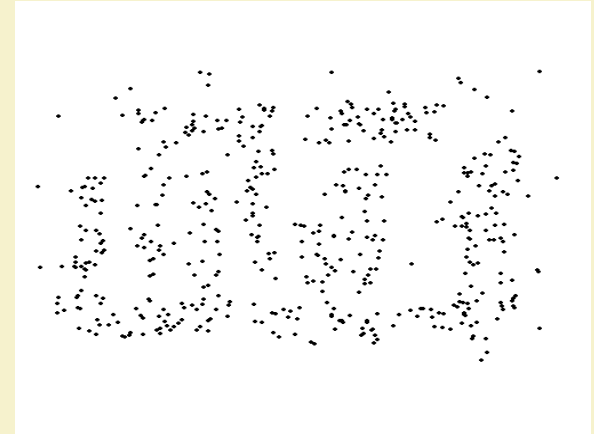
Sample Size



8000 points



2000 Points



500 Points

Sampling ...

- The key principle for effective sampling is the following:
 - using a sample will work almost as well as using the entire data sets, if the sample is representative
 - A sample is representative if it has approximately the same property (of interest) as the original set of data

Types of Sampling

- Simple Random Sampling
 - There is an equal probability of selecting any particular item
- Sampling without replacement
 - As each item is selected, it is removed from the population
- Sampling with replacement
 - Objects are not removed from the population as they are selected for the sample.
 - In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
 - Split the data into several partitions; then draw random samples from each partition

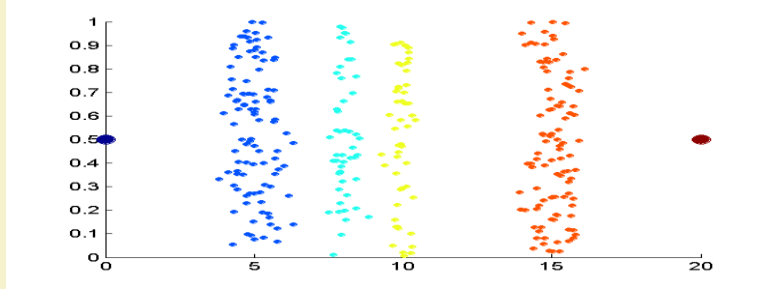
Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies
- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful

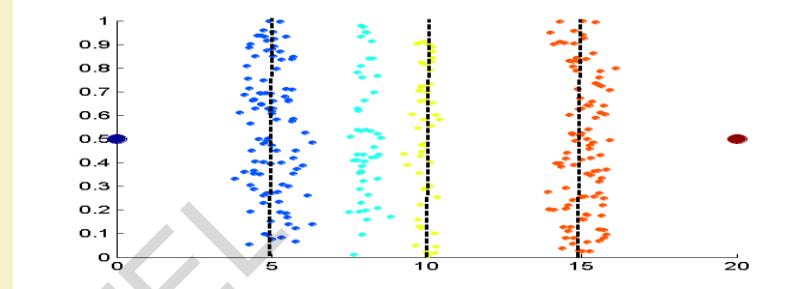
Dimensionality Reduction

- Purpose:
 - Avoid curse of dimensionality
 - Reduce amount of time and memory required by data mining algorithms
 - Allow data to be more easily visualized
 - May help to eliminate irrelevant features or reduce noise
- Techniques
 - Principle Component Analysis
 - Singular Value Decomposition
 - Others: supervised and non-linear techniques

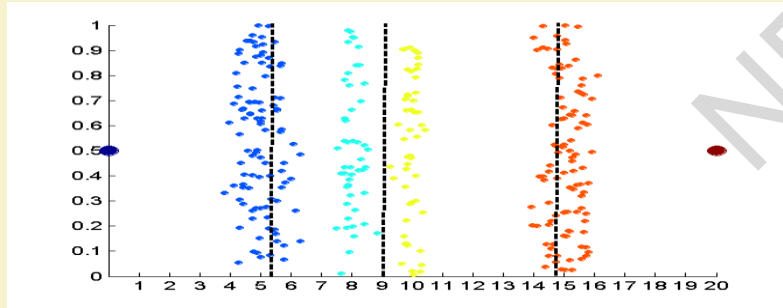
Discretization



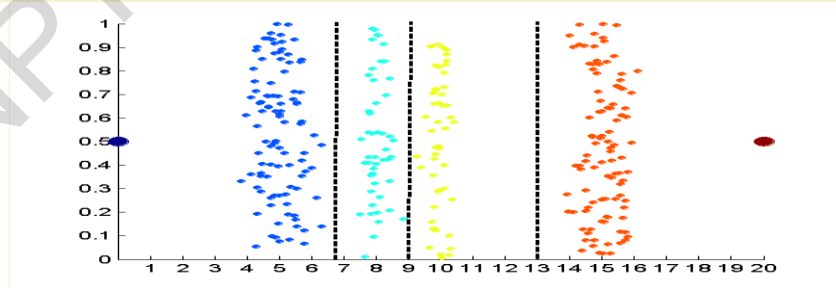
Data



Equal interval width



Equal frequency



K-means

Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
 - Simple functions: x^k , $\log(x)$, e^x , $|x|$
 - Standardization and Normalization

Similarity and Dissimilarity

- Similarity
 - Numerical measure of how alike two data objects are.
 - Is higher when objects are more alike.
 - Often falls in the range $[0,1]$
- Dissimilarity
 - Numerical measure of how different are two data objects
 - Lower when objects are more alike
 - Minimum dissimilarity is often 0
 - Upper limit varies
- Proximity refers to a similarity or dissimilarity

Similarity/Dissimilarity for Simple Attributes

p and q are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{ p-q }{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{ p-q }{n-1}$
Interval or Ratio	$d = p - q $	$s = -d, s = \frac{1}{1+d} \text{ or } s = 1 - \frac{d - \min_d}{\max_d - \min_d}$

Table 5.1. Similarity and dissimilarity for simple attributes

Euclidean Distance

- Euclidean Distance

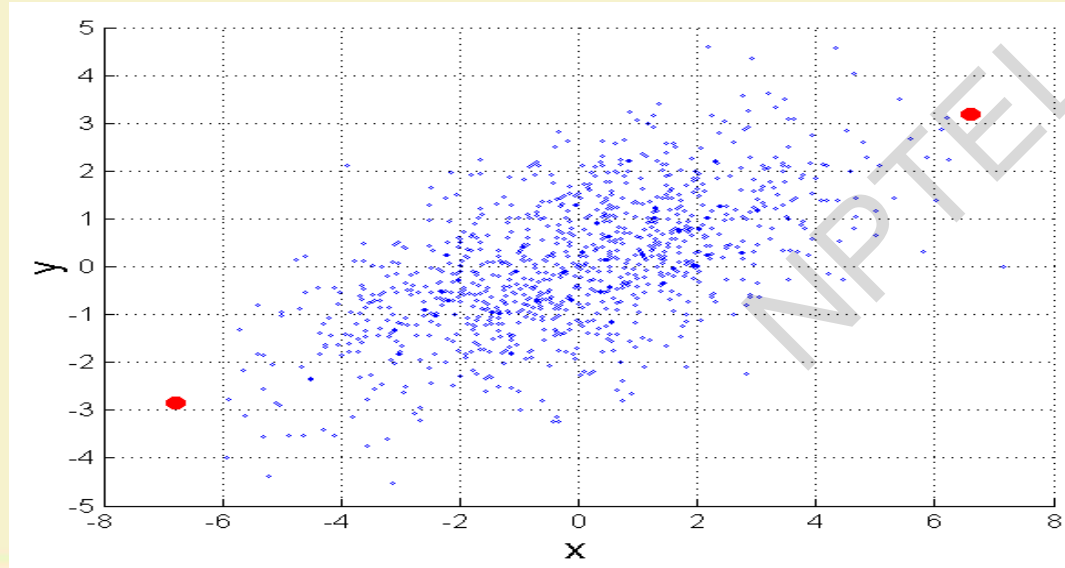
$$\mathbf{dist} = \sqrt{\sum_{k=1}^n (\mathbf{p}_k - \mathbf{q}_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k^{th} attributes (components) or data objects p and q .

- Standardization is necessary, if scales differ.

Mahalanobis Distance

$$\text{mahalanobis}(p, q) = (p - q)^T \Sigma^{-1} (p - q)$$



Σ is the covariance matrix of the input data X

$$\Sigma_{j,k} = \frac{1}{n-1} \sum_{i=1}^n (X_{ij} - \bar{X}_j)(X_{ik} - \bar{X}_k)$$



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For red points, the Euclidean distance is 14.7, Mahalanobis distance is 6.

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| ||d_2|| ,$$

where \bullet indicates vector dot product and $||d||$ is the length of vector d .

- Example:

$$d_1 = 3 \ 2 \ 0 \ 5 \ 0 \ 0 \ 0 \ 2 \ 0 \ 0$$

$$d_2 = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 2$$

$$d_1 \bullet d_2 = 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5$$

$$||d_1|| = (3*3 + 2*2 + 0*0 + 5*5 + 0*0 + 0*0 + 0*0 + 2*2 + 0*0 + 0*0)^{0.5} = (42)^{0.5} = 6.481$$

$$||d_2|| = (1*1 + 0*0 + 0*0 + 0*0 + 0*0 + 0*0 + 0*0 + 1*1 + 0*0 + 2*2)^{0.5} = (6)^{0.5} = 2.245$$

$$\cos(d_1, d_2) = .3150$$

Similarity Between Binary Vectors

- Common situation is that objects, p and q , have only binary attributes

- Compute similarities using the following quantities

M_{01} = the number of attributes where p was 0 and q was 1

M_{10} = the number of attributes where p was 1 and q was 0

M_{00} = the number of attributes where p was 0 and q was 0

M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

SMC = number of matches / number of attributes

$$= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$$

J = number of 11 matches / number of not-both-zero attributes values

$$= (M_{11}) / (M_{01} + M_{10} + M_{11})$$

Correlation

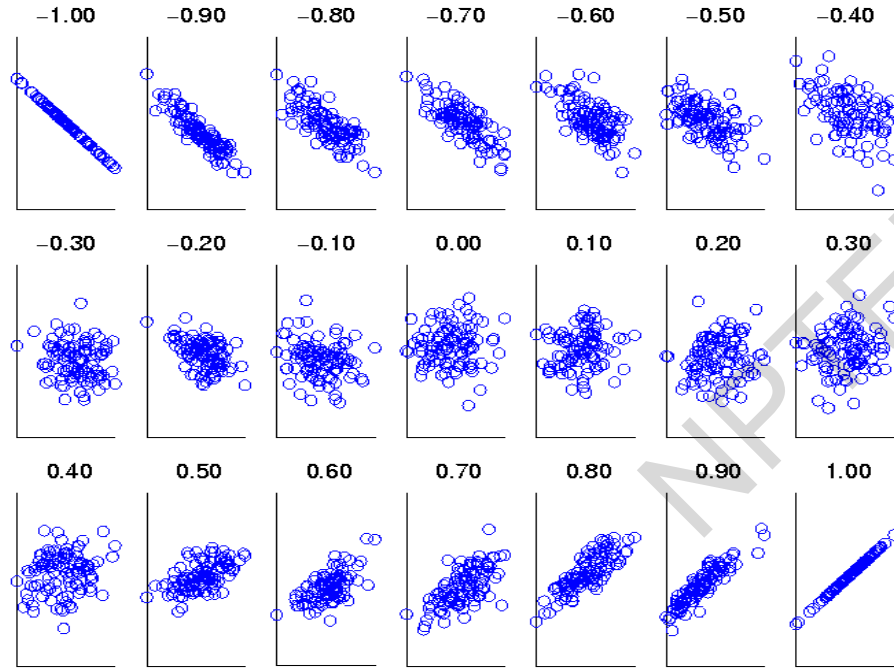
- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, p and q , and then take their dot product

$$p'_k = (p_k - \text{mean}(p)) / \text{std}(p)$$

$$q'_k = (q_k - \text{mean}(q)) / \text{std}(q)$$

$$\text{correlation}(p, q) = p' \bullet q'$$

Visually Evaluating Correlation



Scatter plots showing the similarity from -1 to 1 .

End of Data Preprocessing

Data Mining

Association Rules

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Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

$\{\text{Diaper}\} \rightarrow \{\text{Beer}\},$
 $\{\text{Milk, Bread}\} \rightarrow \{\text{Eggs, Coke}\},$
 $\{\text{Beer, Bread}\} \rightarrow \{\text{Milk}\},$

Implication means co-occurrence,
not causality!

Definition: Frequent Itemset

- **Itemset**
 - A collection of one or more items
 - Example: {Milk, Bread, Diaper}
 - k-itemset
 - An itemset that contains k items
- **Support count (σ)**
 - Frequency of occurrence of an itemset
 - E.g. $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$
- **Support**
 - Fraction of transactions that contain an itemset
 - E.g. $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$
- **Frequent Itemset**
 - An itemset whose support is greater than or equal to a *minsup* threshold

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Association Rule
 - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
 - Example:
 $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

- Rule Evaluation Metrics

- Support (s)
 - ◆ Fraction of transactions that contain both X and Y
- Confidence (c)
 - ◆ Measures how often items in Y appear in transactions that contain X

<i>TID</i>	<i>Items</i>
1	Bread, Milk
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3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

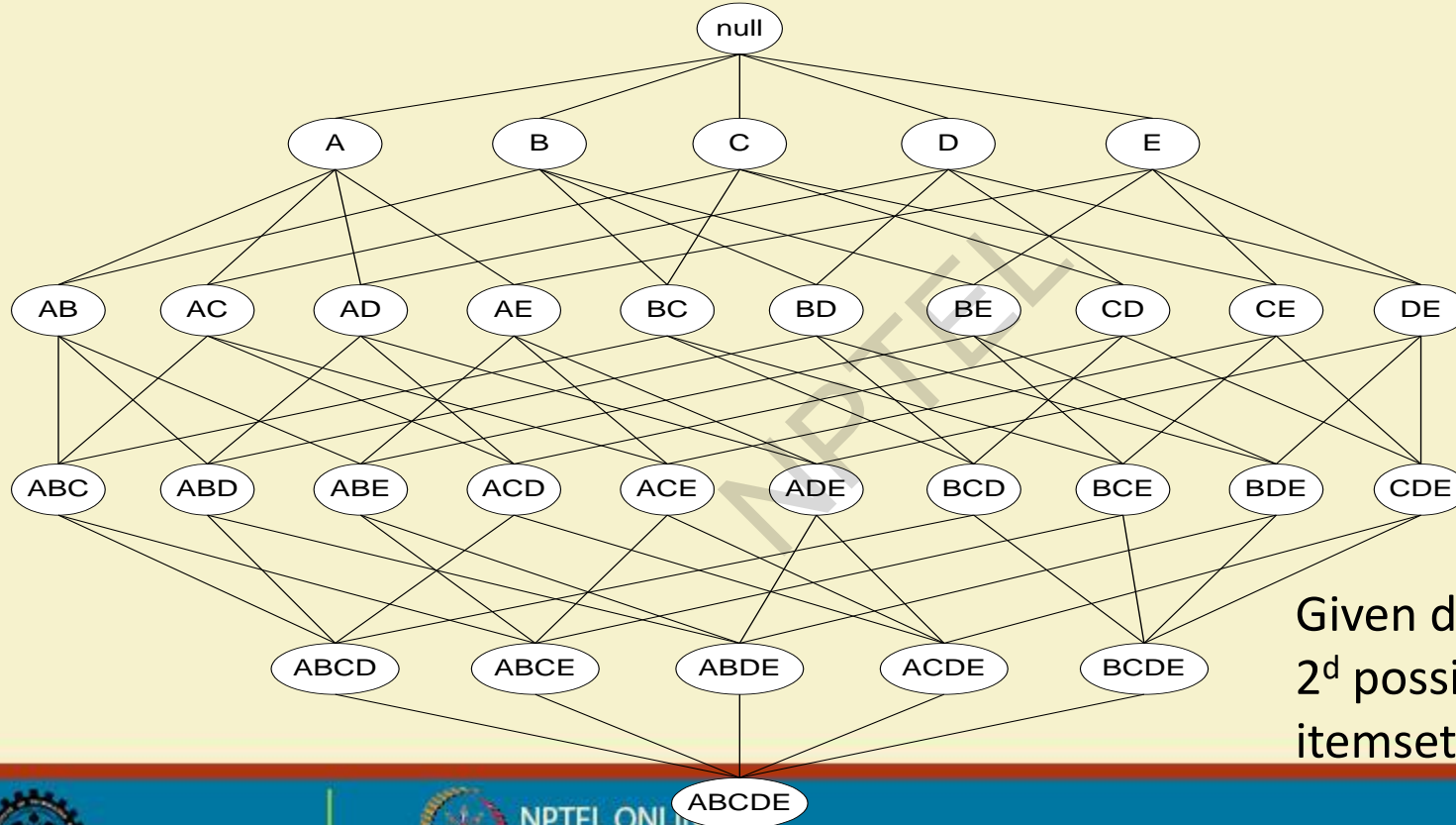
- Given a set of transactions T , the goal of association rule mining is to find all rules having
 - support \geq *minsup* threshold
 - confidence \geq *minconf* threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the *minsup* and *minconf* thresholds

⇒ **Computationally prohibitive!**

Mining Association Rules

- Two-step approach:
 1. **Frequent Itemset Generation**
 - Generate all itemsets whose support \geq minsup
 2. **Rule Generation**
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

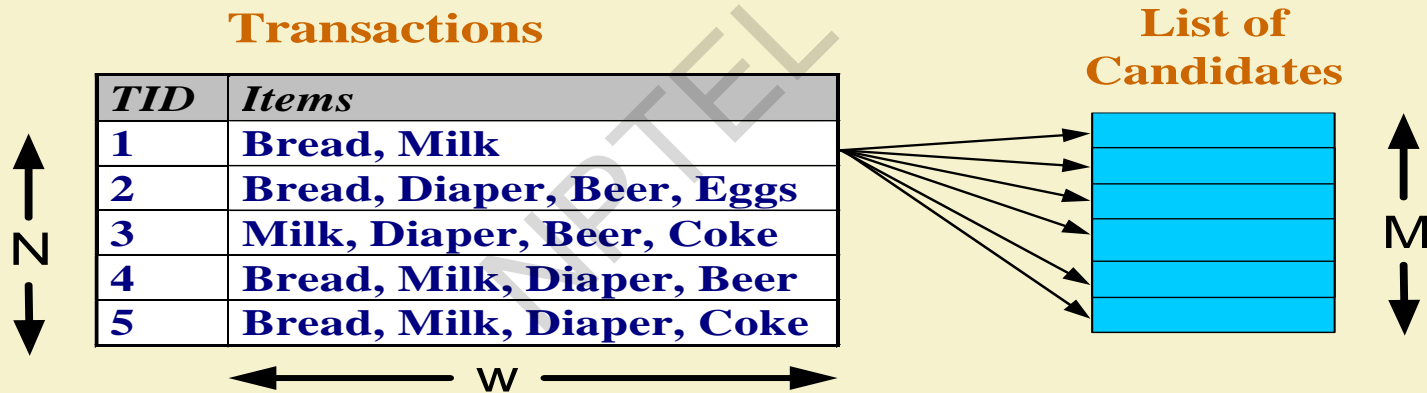
Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
 - Each itemset in the lattice is a **candidate** frequent itemset
 - Count the support of each candidate by scanning the database



- Match each transaction against every candidate
- Complexity $\sim O(NMw) \Rightarrow$ **Expensive since $M = 2^d$!!!**

Frequent Itemset Generation Strategies

- Reduce the **number of candidates** (M)
 - Complete search: $M=2^d$
 - Use pruning techniques to reduce M
- Reduce the **number of transactions** (N)
 - Reduce size of N as the size of itemset increases
 - Used by DHP and vertical-based mining algorithms
- Reduce the **number of comparisons** (NM)
 - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction

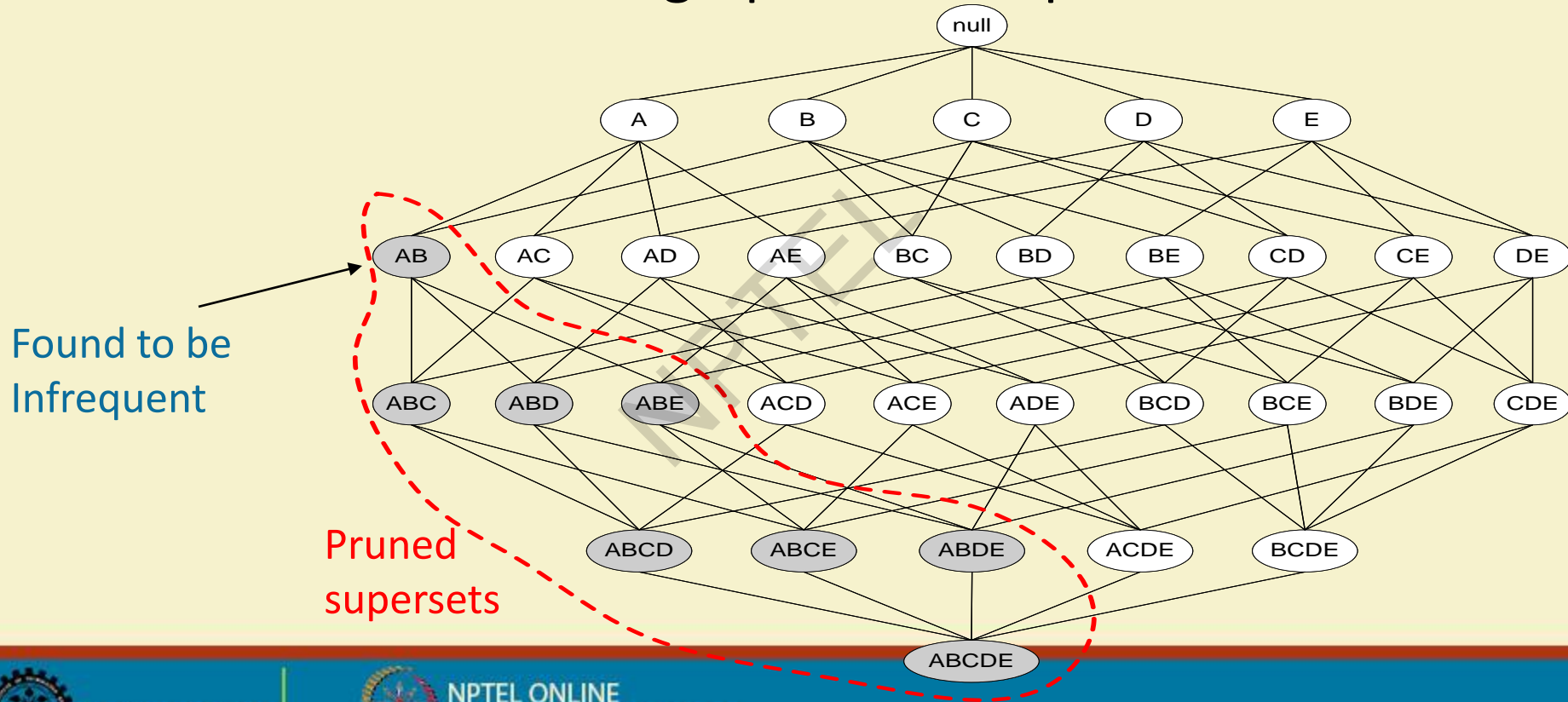
Reducing Number of Candidates

- **Apriori principle:**
 - If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the **anti-monotone** property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)



Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3



Minimum Support = 3

If every subset is considered,
 ${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
 $6 + 6 + 1 = 13$

Apriori Algorithm

- Method:
 - Let $k=1$
 - Generate frequent itemsets of length 1
 - Repeat until no new frequent itemsets are identified
 - Generate length $(k+1)$ candidate itemsets from length k frequent itemsets
 - Prune candidate itemsets containing subsets of length k that are infrequent
 - Count the support of each candidate by scanning the DB
 - Eliminate candidates that are infrequent, leaving only those that are frequent

Factors Affecting Complexity

- Choice of minimum support threshold
 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
 - Apriori makes multiple passes, run time of algorithm increase with number of transactions
- Average transaction width
 - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
 - In general, confidence does not have an anti-monotone property
 $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$
 - But confidence of rules generated from the same itemset has an anti-monotone property
 - e.g., $L = \{A, B, C, D\}$:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

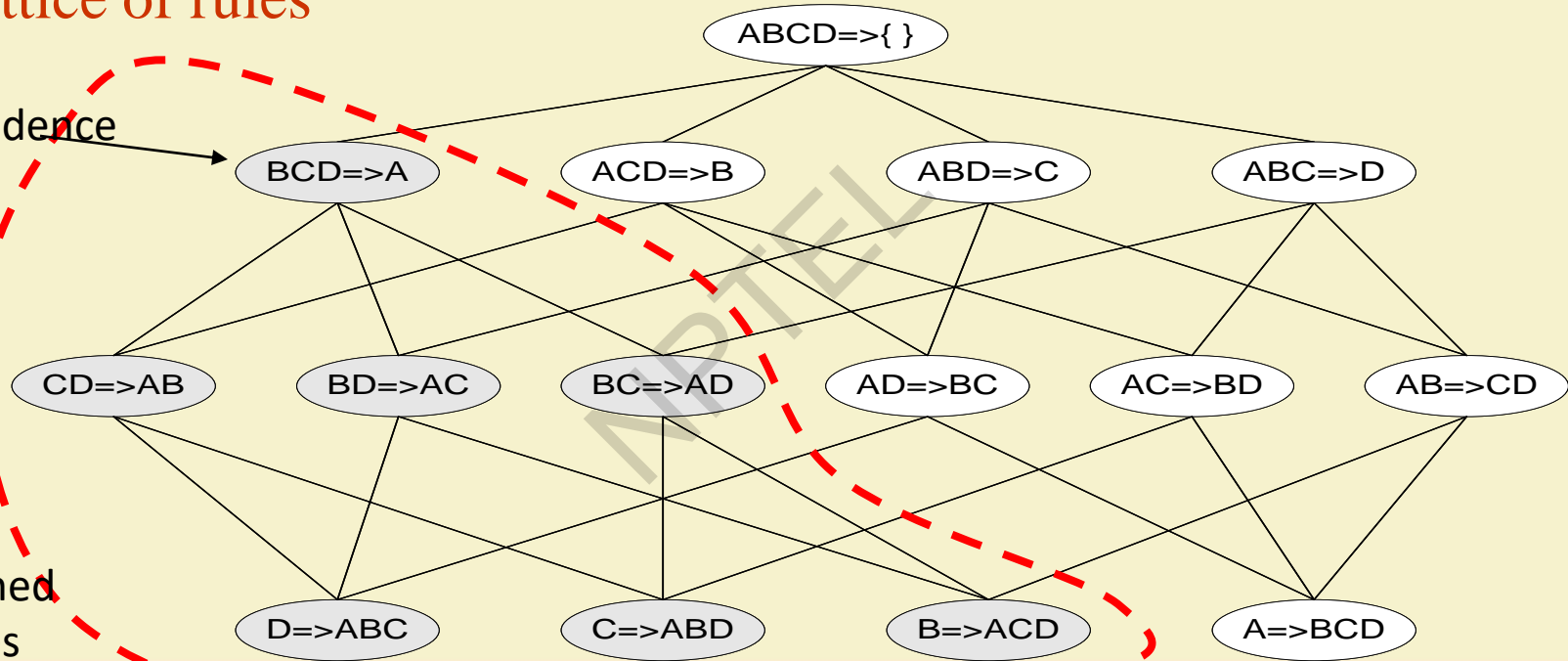
- Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules

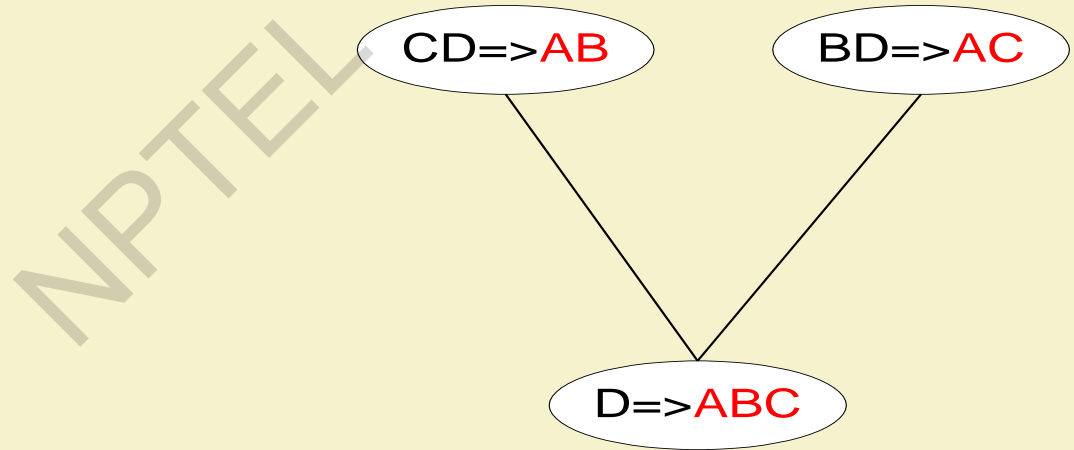
Low
Confidence
Rule

Pruned
Rules



Rule Generation for Apriori Algorithm

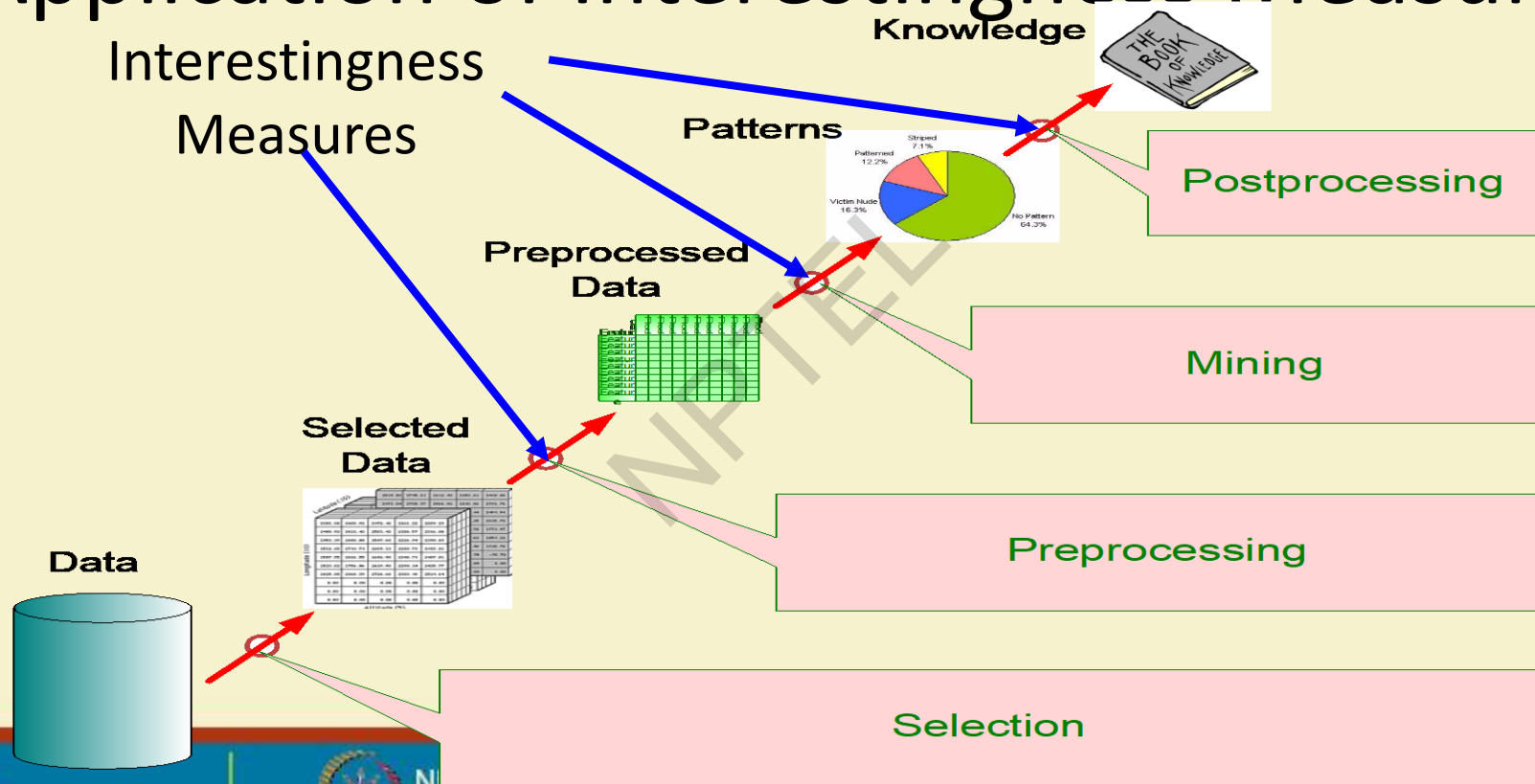
- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent
- $\text{join}(\text{CD} \Rightarrow \text{AB}, \text{BD} \Rightarrow \text{AC})$ would produce the candidate rule $\text{D} \Rightarrow \text{ABC}$
- Prune rule $\text{D} \Rightarrow \text{ABC}$ if its subset $\text{AD} \Rightarrow \text{BC}$ does not have high confidence



Pattern Evaluation

- Association rule algorithms tend to produce too many rules
 - many of them are uninteresting or redundant
 - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence
- Interestingness measures can be used to prune/rank the derived patterns
- In the original formulation of association rules, support & confidence are the only measures used

Application of Interestingness Measure



Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for supports $X \rightarrow Y$

	Y	\overline{Y}	
X	f_{11}	f_{10}	f_{1+}
\overline{X}	f_{01}	f_{00}	f_{0+}
	f_{+1}	f_{+0}	$ T $

Used to define various measures

- ◆ support, confidence, lift, Gini, J-measure, etc.

Statistical Independence

- Population of 1000 students
 - 600 students know how to swim (S)
 - 700 students know how to bike (B)
 - 420 students know how to swim and bike (S,B)
 - $P(S \cap B) = 420/1000 = 0.42$
 - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$
 - $P(S \cap B) = P(S) \times P(B) \Rightarrow$ Statistical independence
 - $P(S \cap B) > P(S) \times P(B) \Rightarrow$ Positively correlated
 - $P(S \cap B) < P(S) \times P(B) \Rightarrow$ Negatively correlated

Statistical-based Measures

- take into account statistical dependence

$$\textit{Lift} = \frac{P(Y | X)}{P(Y)}$$

$$\textit{Interest} = \frac{P(X, Y)}{P(X)P(Y)}$$

$$PS = P(X, Y) - P(X)P(Y)$$

$$\phi - \textit{coefficient} = \frac{P(X, Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

Example: Lift/Interest

	Coffee	<u>Coffee</u>	
<u>Tea</u>	15	5	20
Tea	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Confidence = $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

$\Rightarrow \text{Lift} = 0.75/0.9 = 0.8333 (< 1, \text{ therefore is negatively associated})$

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it

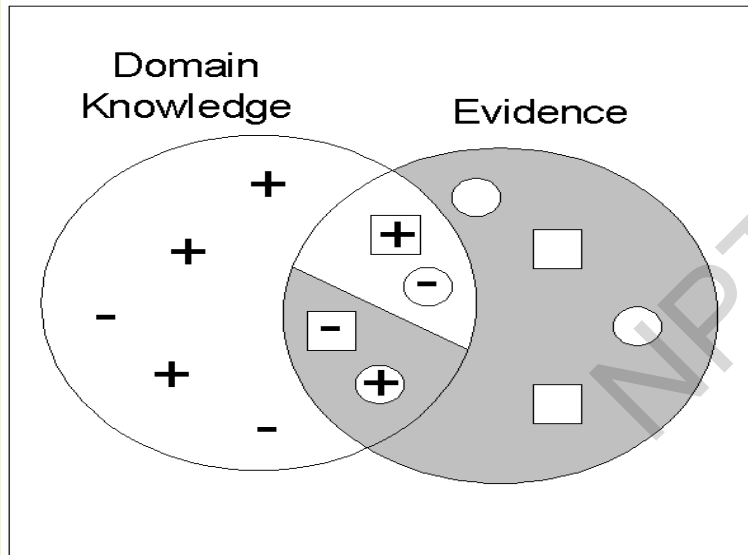
#	Measure	Formula
1	ϕ -coefficient	$\frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j, B_k) + \sum_k \max_j P(A_j, B_k) - \max_j P(A_j) - \max_k P(B_k)}{2 - \max_j P(A_j) - \max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\bar{A},\bar{B})}{P(A,\bar{B})P(\bar{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\bar{A}\bar{B}) - P(A,\bar{B})P(\bar{A},B)}{P(A,B)P(\bar{A}\bar{B}) + P(A,\bar{B})P(\bar{A},B)} = \frac{\alpha - 1}{\alpha + 1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\bar{A}\bar{B})} - \sqrt{P(A,\bar{B})P(\bar{A},B)}}{\sqrt{P(A,B)P(\bar{A}\bar{B})} + \sqrt{P(A,\bar{B})P(\bar{A},B)}} = \frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}$
6	Kappa (κ)	$\frac{P(A,B) + P(\bar{A},\bar{B}) - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure (J)	$\max \left(P(A, B) \log \left(\frac{P(B A)}{P(B)} \right) + P(\bar{A}\bar{B}) \log \left(\frac{P(\bar{B} \bar{A})}{P(\bar{B})} \right), \right. \\ \left. P(A, B) \log \left(\frac{P(A B)}{P(A)} \right) + P(\bar{A}\bar{B}) \log \left(\frac{P(\bar{A} \bar{B})}{P(\bar{A})} \right) \right)$
9	Gini index (G)	$\max \left(P(A)[P(B A)^2 + P(\bar{B} A)^2] + P(\bar{A})[P(B \bar{A})^2 + P(\bar{B} \bar{A})^2] \right. \\ \left. - P(B)^2 - P(\bar{B})^2, \right. \\ \left. P(B)[P(A B)^2 + P(\bar{A} B)^2] + P(\bar{B})[P(A \bar{B})^2 + P(\bar{A} \bar{B})^2] \right. \\ \left. - P(A)^2 - P(\bar{A})^2 \right)$
10	Support (s)	$P(A, B)$
11	Confidence (c)	$\max(P(B A), P(A B))$
12	Laplace (L)	$\max \left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2} \right)$
13	Conviction (V)	$\max \left(\frac{P(A)P(\bar{B})}{P(\bar{A}B)}, \frac{P(B)P(\bar{A})}{P(\bar{B}A)} \right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A, B) - P(A)P(B)$
17	Certainty factor (F)	$\max \left(\frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)} \right)$
18	Added Value (AV)	$\max(P(B A) - P(B), P(A B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B) + P(\bar{A}\bar{B})}{P(A)P(B) + P(\bar{A})P(\bar{B})} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A,B) - P(\bar{A}\bar{B})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A) + P(B) - P(A,B)}$
21	Kloggen (K)	$\sqrt{P(A, B)} \max(P(B A) - P(B), P(A B) - P(A))$

Subjective Interestingness Measure

- Objective measure:
 - Rank patterns based on statistics computed from data
 - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).
- Subjective measure:
 - Rank patterns according to user's interpretation
 - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
 - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

- Need to model expectation of users (domain knowledge)



- + Pattern expected to be frequent
- Pattern expected to be infrequent
- Pattern found to be frequent
- Pattern found to be infrequent
- + ○- Expected Patterns
- ○+ Unexpected Patterns

- Need to combine expectation of users with evidence from data (i.e., extracted patterns)

End of Association Rule