



NORTHWESTERN POLYTECHNIC
UNIVERSITY

Maze Project

Prepared For:

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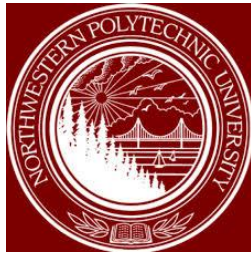
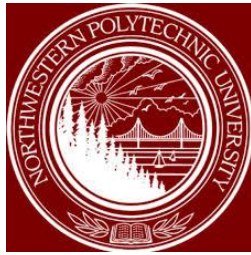


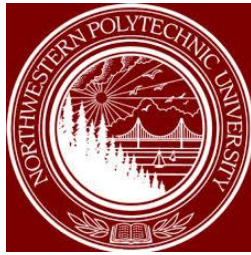
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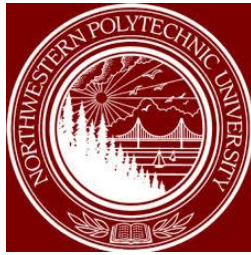
Introduction

- Maze solving is the act of finding a route through the maze from the start to finish. Some maze solving methods are designed to be used inside the maze by a traveler with no prior knowledge of the maze, whereas others are designed to be used by a person or computer program that can see the whole maze at once.



Maze-solving algorithms

- There are a number of different maze-solving algorithms, that is, automated methods for the solving of mazes.
 - Wall follower
 - Random mouse algorithm
 - Pledge algorithm
 - Trémaux's algorithm
 - Dead-end filling



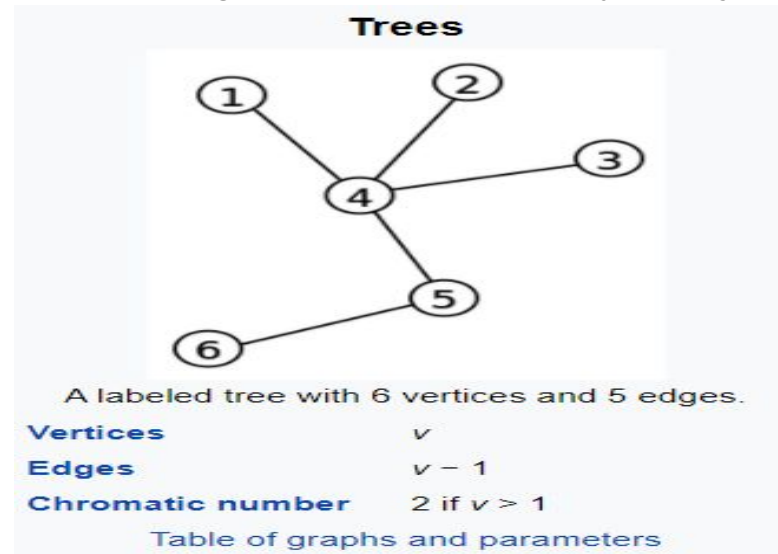
Maze-solving algorithms - Continues

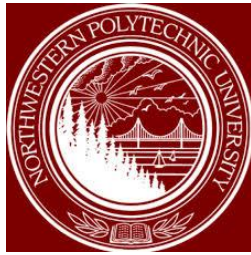
- Maze-routing algorithm
- Recursive algorithm
- Shortest path algorithm
- Mazes containing no loops are known as "simply connected", or "perfect" mazes, and are equivalent to a tree in graph theory. Thus many maze-solving algorithms are closely related to graph theory.



What is a Graph Theory?

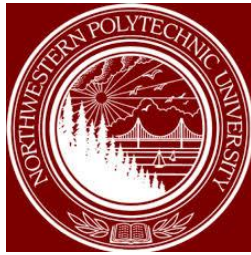
- In graph theory, a tree is an undirected graph in which any two vertices are connected by exactly one path, or equivalently a connected acyclic undirected graph. A forest is an undirected graph in which any two vertices are connected by at most one path, or equivalently an acyclic undirected graph, or equivalently a disjoint union of trees.





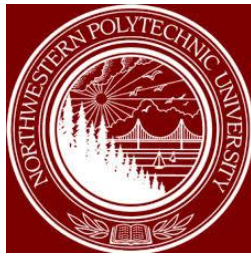
Shortest path algorithm

- When a maze has multiple solutions, the solver may want to find the shortest path from start to finish. One such algorithm finds the shortest path by implementing a **breadth-first search**, while another, the **A* algorithm**, uses a **heuristic** technique. The breadth-first search algorithm uses a **queue** to visit cells in increasing distance order from the start until the finish is reached.



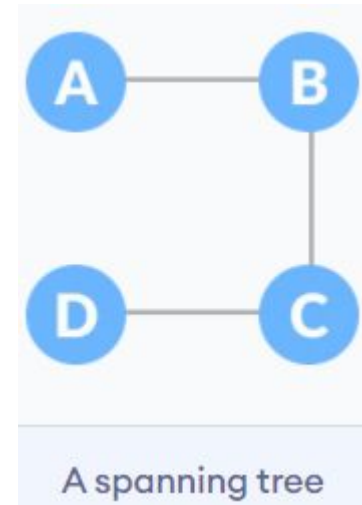
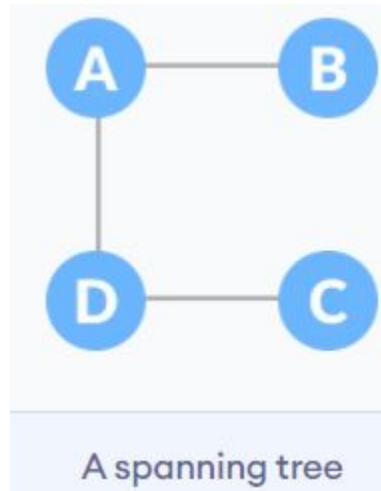
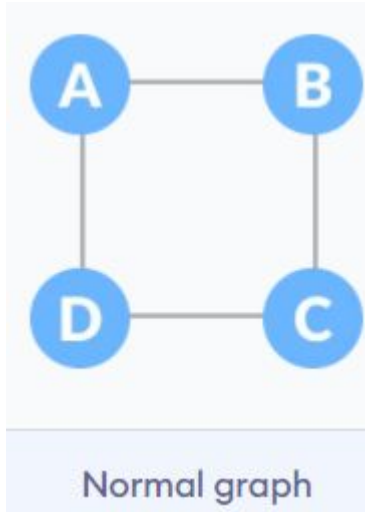
Spanning Tree

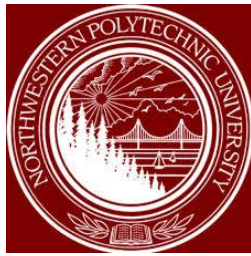
- A spanning tree is a sub-graph of an undirected connected graph, which includes all the vertices of the graph with a minimum possible number of edges. If a vertex is missed, then it is not a spanning tree.
- The edges may or may not have weights assigned to them.
- The total number of spanning trees with n vertices that can be created from a complete graph is equal to n^{n-2} .



Spanning Tree - Example

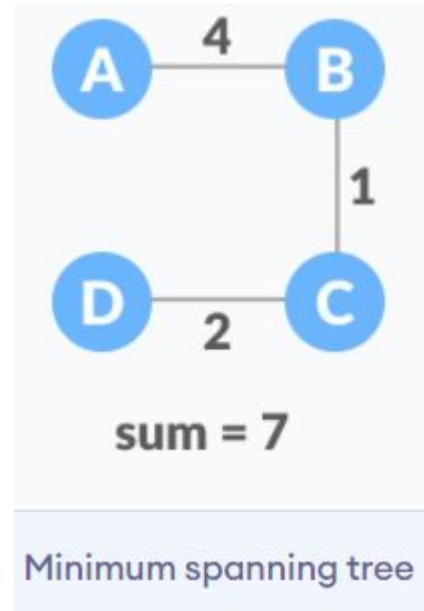
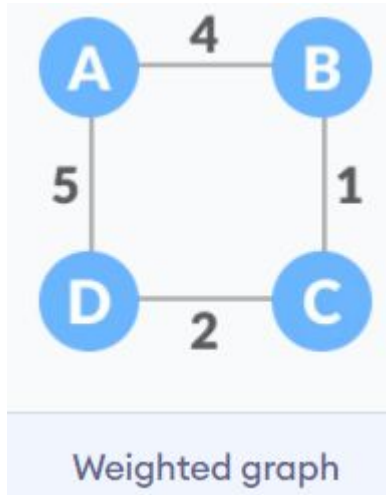
- If we have $n = 4$, the maximum number of possible spanning trees is equal to $4^{4-2} = 16$. Thus, 16 spanning trees can be formed from a complete graph with 4 vertices. Examples -

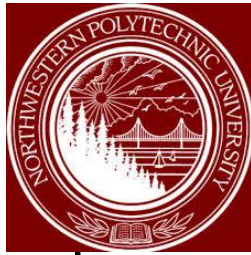




Minimum Spanning Tree

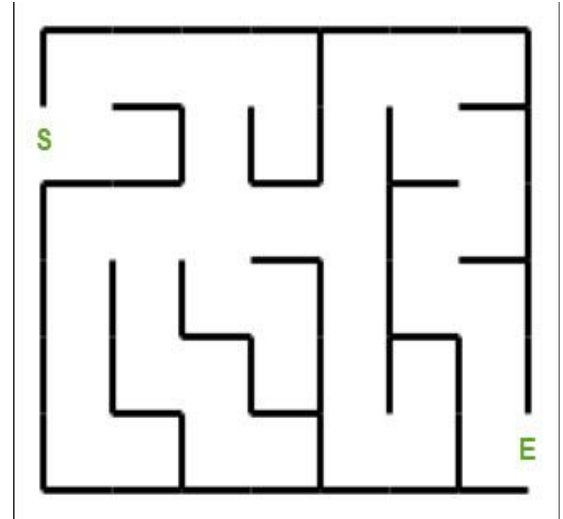
- A minimum spanning tree is a spanning tree in which the sum of the weight of the edges is as minimum as possible.

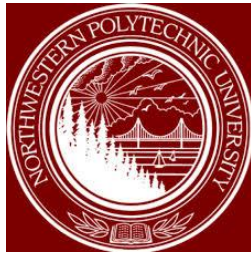




Find the minimum length spanning tree

- The weight of a tree is just the sum of weights of its edges. Obviously, different trees have different lengths. The problem: how to find the minimum length spanning tree? This problem can be solved by many different algorithms.
- In graph theory, there are below algorithms for calculating the minimum spanning tree (MST):
 - Kruskal's algorithm
 - Prim's algorithm
 - Boruvka's algorithm



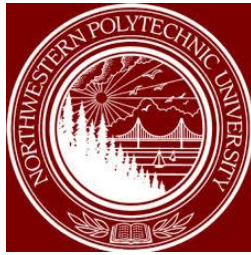


Prim's Algorithm

- Prim's start from one vertex and keep adding edges with the lowest weight until we reach our goal.

The steps for implementing Prim's algorithm are as follows:

- Initialize the minimum spanning tree with a vertex chosen at random.
- Find all the edges that connect the tree to new vertices, find the minimum and add it to the tree
- Keep repeating step 2 until we get a minimum spanning tree

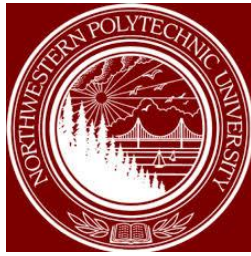


Kruskal's algorithm

- Kruskal start from the edges with the lowest weight and keep adding edges until we reach our goal.

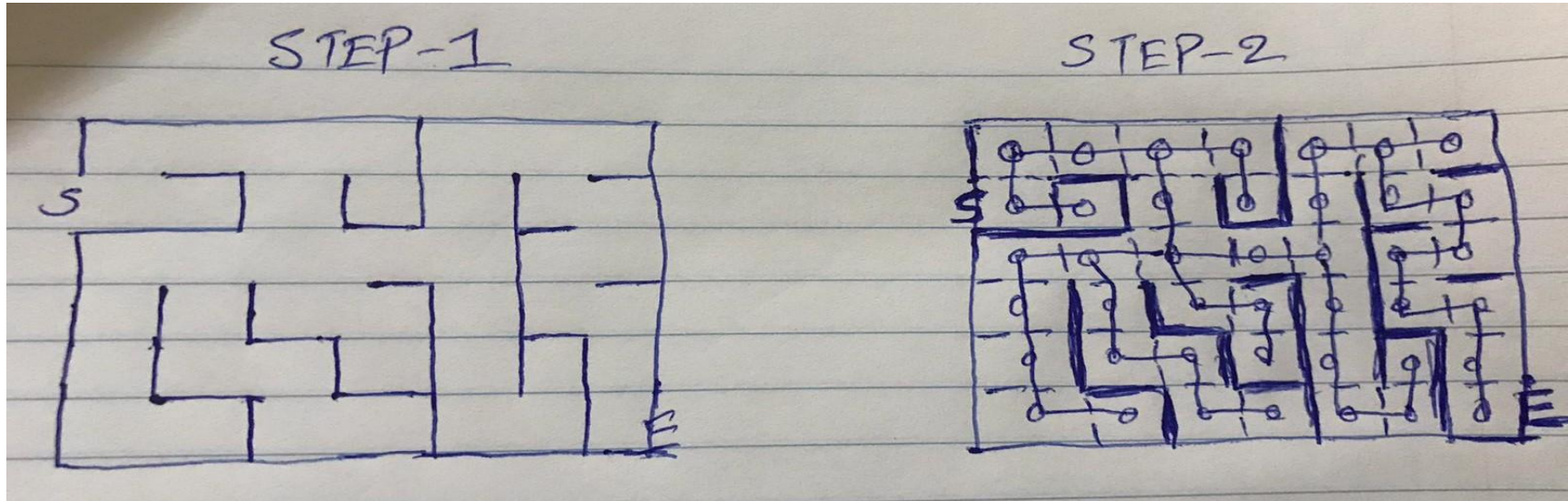
The steps for implementing Kruskal's algorithm are as follows:

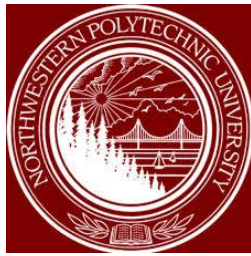
- Sort all the edges from low weight to high
- Take the edge with the lowest weight and add it to the spanning tree. If adding the edge created a cycle, then reject this edge.
- Keep adding edges until we reach all vertices.



Convert Maze to Graph

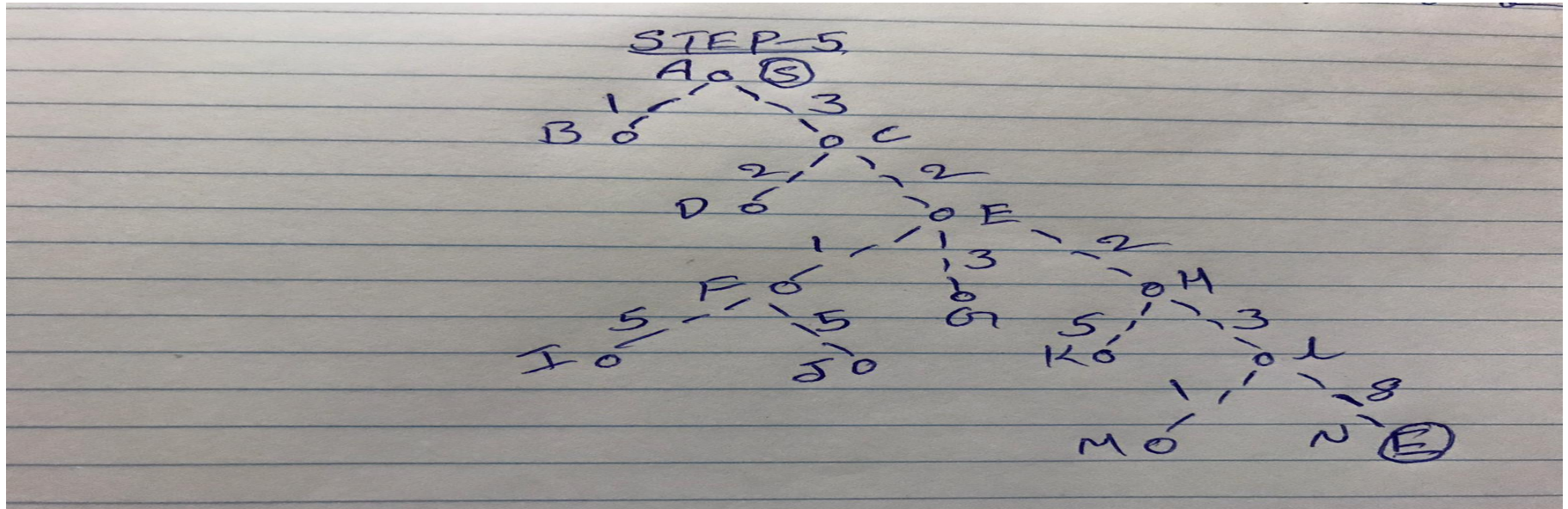
Step 1: Initialize distances according to the algorithm.





Tree or Graph

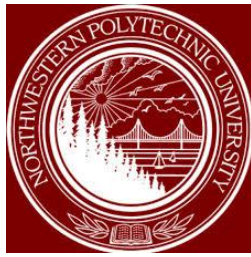
- A tree is an undirected graph in which any two vertices are connected by exactly one path, or equivalently a connected acyclic undirected graph





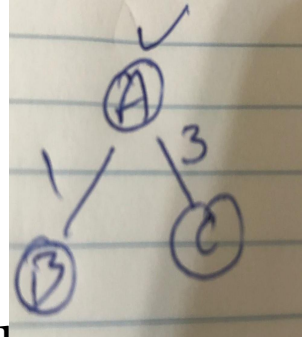
Prim's Minimum Spanning Tree (MST) - Steps

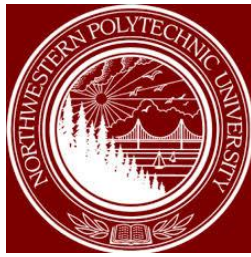
- A. Create a set **mstSet** that keeps track of vertices already included in **MST**.
- B. Assign a **key value** to **all vertices** in the **input graph**.
 - 1. Initialize all key values as **INFINITE**.
 - 2. Assign **key value** as **0** for the **first vertex** so that it is **picked first**.
- C. While **mstSet** doesn't include **all vertices**
 - 3. Pick a **vertex u** which is not there in **mstSet** and has **minimum key value**.
 - 4. Include **u** to **mstSet**.
 - 5. Update **key value** of all of **u's adjacent vertices** which are **not in mstSet**.
 - For every **adjacent vertex v** which are **not in mstSet**, if **weight of edge u-v** is less than the previous **key value** of **v**, update **v's key value** as **weight of u-v**.



Prim's algorithm to solve maze

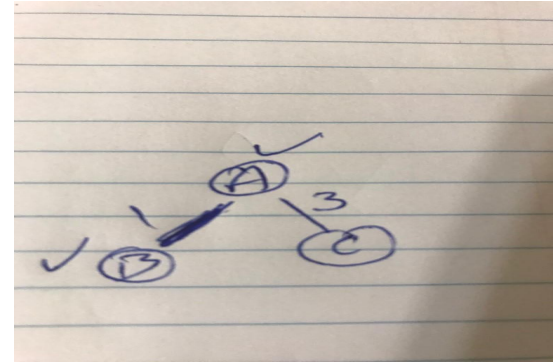
1. The set **mstSet** is initially **empty** and **keys** assigned to **vertices** are $\{0, \text{INF}, \text{INF}, \text{INF}, \text{INF}, \text{INF}, \text{INF}\}$ where **INF** indicates **infinite**.
 - Now pick the vertex with **minimum key value**. The **vertex A** include it in **mstSet**. So **mstSet** becomes **{A}**.
 - After including to **mstSet**, update **key values** of **adjacent vertices**. Adjacent vertices of **A** are **B and C**.
 - The **key values** of **A and B** are updated as **1 and 3**.
 - Following subgraph shows **vertices** and their **key values**, only the **vertices** with **finite key values** are shown. The **vertices** included in **MST** are shown in **Tick Mark**.

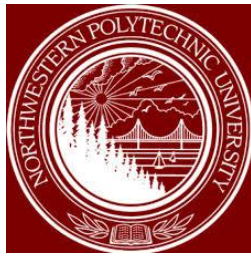




Steps - Continues

2. Pick the **vertex** with **minimum key value** and **not already** included in **MST** (i.e., not in **mstSET**).
- a. The **vertex B** is picked and added to **mstSet**.
 - b. So **mstSet** now becomes **{A,B}**.
 - c. Update the **key values** of adjacent **vertices** of **B**. There are no adjacent **vertices** of vertex B.

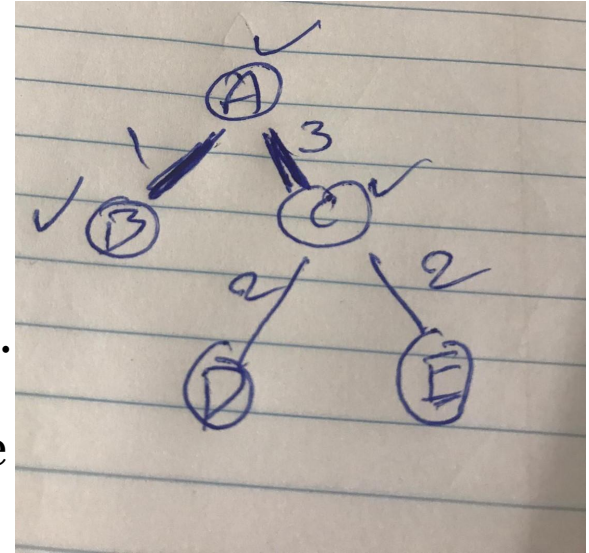


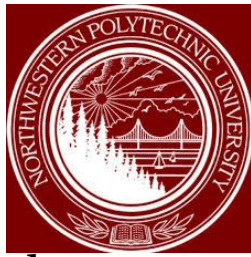


Steps - Continues

3. Pick the vertex with minimum key value and not already included in MST (i.e., not in mstSet).

- The vertex C is picked and added to mstSet.
- So mstSet now becomes **{A, B, C}**.
- Update the key values of adjacent vertices of C.
- The key value of vertex D and E becomes finite (2 and 2 respectively).



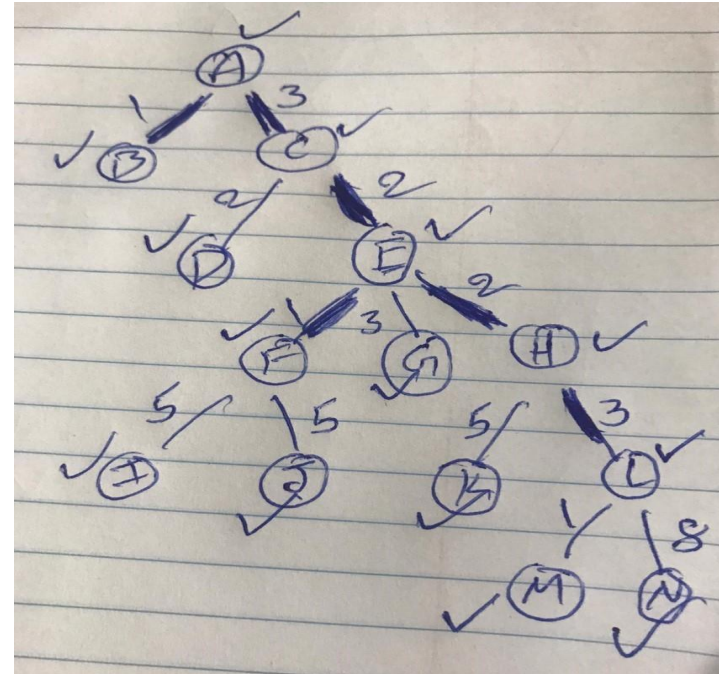


Final Steps - Continues

- We repeat the above steps until mstSet includes all vertices of given graph. Finally, we get the following graph.

14. Pick the **vertex** with **minimum key value** and **not already** included in **MST** (i.e., not in **mstSET**).

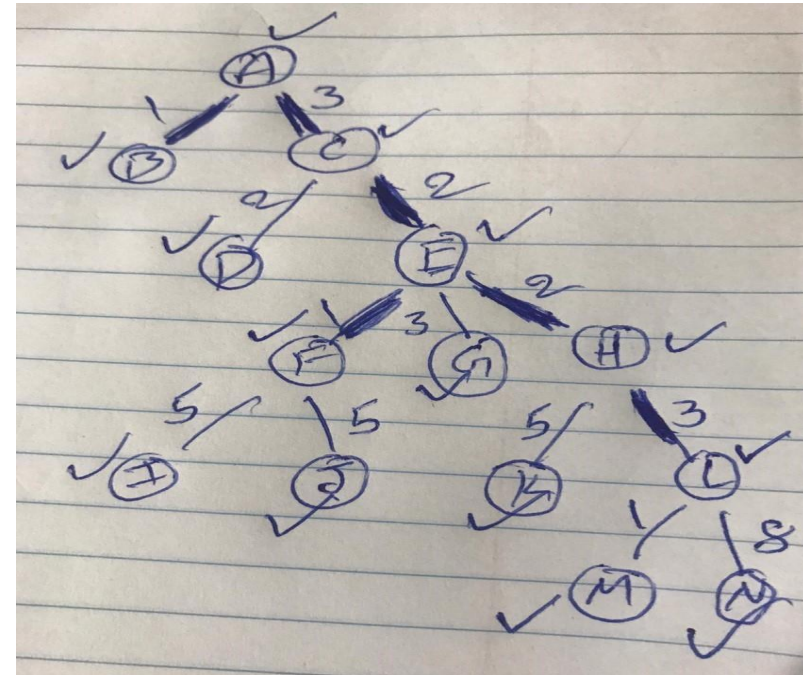
- The **vertex N** is picked and added to **mstSet**.
- So **mstSet** now becomes **{A, B, C, E, F, H, D, L, M, G, I, J, K, N}**.
- Update the **key values** of **adjacent vertices** Of **N**. There are no adjacent **vertices** of **vertex N**.

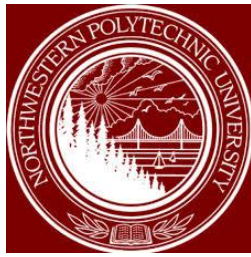




Cost of the spanning tree

- As all the vertices are visited, now the algorithm stops.
- The cost of the spanning tree is $(1 + 3 + 2 + 1 + 2 + 2 + 3 + 3 + 1 + 5 + 5 + 5 + 8) = 41$. There is no more spanning tree in this graph with cost less than 41.

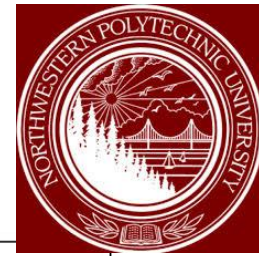




Kruskal's Algorithm to solve maze

- Sort all the edges in non-decreasing order of their weight.
- Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If cycle is not formed, include this edge. Else, discard it.
- Repeat step#2 until there are $(V-1)$ edges in the spanning tree.

The graph contains 14 vertices and 13 edges. So, the minimum spanning tree formed will be having $V-1$ ($14 - 1$) = 13 edges. Since the number of edges equals to $V-1$, the algorithm stops here.

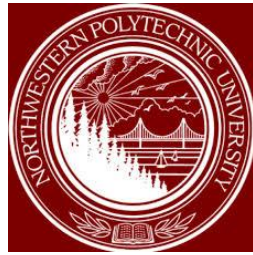


Kruskal's Algorithm to solve maze

After Sorting -

Weight	<u>Src</u>	Dest
1	A	B
1	E	F
1	L	M
2	C	D
2	C	E
2	E	H
3	A	C
3	E	G
3	H	L
5	F	I
5	F	J
5	H	K
8	L	N

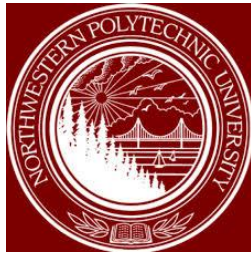
Sorted list of edges



Now pick all edges one by one from sorted list of edges

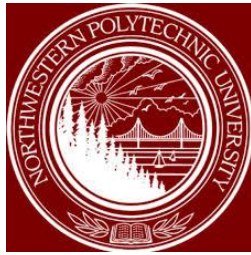
1. *Pick edge A-B: No cycle* is formed, include it.
2. *Pick edge E-F: No cycle* is formed, include it.
3. *Pick edge L-M: No cycle* is formed, include it.
4. *Pick edge C-D: No cycle* is formed, include it.
5. *Pick edge C-E: No cycle* is formed, include it.
6. *Pick edge E-H: No cycle* is formed, include it.
7. *Pick edge A-C: No cycle* is formed, include it.
8. *Pick edge E-G: No cycle* is formed, include it.
9. *Pick edge H-L: No cycle* is formed, include it.
10. *Pick edge F-I: No cycle* is formed, include it.
11. *Pick edge F-J: No cycle* is formed, include it.
12. *Pick edge H-K: No cycle* is formed, include it.
13. *Pick edge L-N: No cycle* is formed, include it.

Since the number of edges equals to $(V - 1) \Rightarrow (14 - 1) \Rightarrow 13$, the algorithm stops here.



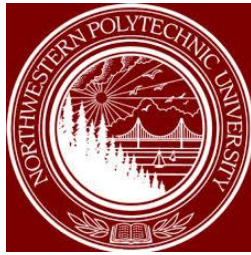
Kruskal Time Complexity

- Since the complexity is **$O(E * \log(V))$** , the Kruskal algorithm is better used with sparse graphs, where we don't have lots of edges.
- However, since we are examining all edges one by one sorted on ascending order based on their weight, this allows us great control over the resulting MST. Since different MSTs come from different edges with the same cost, in the Kruskal algorithm, all these edges are located one after another when sorted.



Prim's Time Complexity

- In the beginning, we add the source node to the queue with a zero weight and without an edge. Also, we initialize the total cost with zero and mark all nodes as not yet included inside the MST.
- After that, we perform multiple steps. In each step, we extract the node with the lowest weight from the queue. For each extracted node, we increase the cost of the MST by the weight of the extracted edge. Also, in case the edge of the extracted node exists, we add it to the resulting MST.
- When we finish handling the extracted node, we iterate over its neighbors. In case the neighbor is not yet included in the resulting MST, we use the addOrUpdate function to add this neighbor to the queue. Also, we add the weight of the edge and the edge itself.
- The complexity of Prim's algorithm is **$O(E + V \log(V))$** , where E is the number of edges and V is the number of vertices inside the graph.



Analysis

- The advantage of Prim's algorithm is its complexity, which is better than Kruskal's algorithm. Therefore, **Prim's algorithm is helpful when dealing with dense graphs that have lots of edges.**
- However, **Prim's algorithm doesn't allow us much control over the chosen edges when multiple edges with the same weight occur.** The reason is that only the edges discovered so far are stored inside the queue, rather than all the edges like in Kruskal's algorithm.
- Also, unlike Kruskal's algorithm, **Prim's algorithm is a little harder to implement.**



Comparison between Prim's & Kruskal

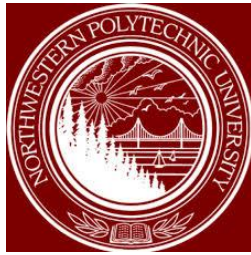
- Kruskal algorithm is better to use regarding the easier implementation and the best control over the resulting MST. However, Prim's algorithm offers better complexity.

	Kruskal	Prim
Multiple MSTs	Offers a good control over the resulting MST	Controlling the MST might be a little harder
Implementation	Easier to implement	Harder to implement
Requirements	Disjoint set	Priority queue
Time Complexity	$O(E \cdot \log(V))$	$O(E + V \cdot \log(V))$



CONCLUSION

- Here, we explained the main two algorithms for calculating the minimum spanning tree of a graph. Firstly, we explained the term MST. Secondly, we presented Kruskal's and Prim's algorithms and provided analysis for each one.
- Kruskal's has better running times if the number of edges is low, while Prim's has a better running time if both the number of edges and the number of nodes are low.
- So, of course, the best algorithm depends on the graph and if you want to bear the cost of complex data structures.



References:

- Maze

https://npu85.npu.edu/~henry/npu/classes/algorithm/graph_alg/slide/maze.html

- Spanning Tree

https://npu85.npu.edu/~henry/npu/classes/algorithm/tutorialpoints_daa/slide/spanning_tree.html

- Google Slides URL -

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