Vector problem

1 12^{th} Maths - Chapter 12

This is Problem-6 from Exercise 10.5

1. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $a = 2\hat{i} + 3\hat{j} - \hat{k}$ and $b = \hat{i} - 2\hat{j} + \hat{k}$
   Solution:

2. sum of the resultant vectors a and b is

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{2}$$

$$\mathbf{c} = \mathbf{a} + \mathbf{b} \tag{3}$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\tag{6}$$

3. The unit direction of the vector c is

$$\hat{c} = \frac{\mathbf{c}}{|\mathbf{c}|}$$

$$\hat{c} = \frac{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{3^2 + 1^2}}$$

$$\hat{c} = \frac{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{10}}$$

4. Hence, the vector of magnitude 5 units and parallel to the resultant of vectors a and b is $\pm 5\hat{c}$

$$\implies \pm \frac{5}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \tag{7}$$

$$\implies \pm \begin{pmatrix} \frac{3\sqrt{10}}{2} \\ \frac{\sqrt{10}}{2} \end{pmatrix} \tag{8}$$