

Vector problem

1 12th Maths - Chapter 12

This is Problem-6 from Exercise 10.5

1. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Solution:

2. sum of the resultant vectors \mathbf{a} and \mathbf{b} is

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (2)$$

$$\mathbf{c} = \mathbf{a} + \mathbf{b} \quad (3)$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (4)$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \quad (5)$$

$$(6)$$

3. The unit direction of the vector \mathbf{c} is

$$\hat{\mathbf{c}} = \frac{\mathbf{c}}{\|\mathbf{c}\|}$$

$$\hat{\mathbf{c}} = \frac{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{3^2 + 1^2}}$$

$$\hat{\mathbf{c}} = \frac{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{10}}$$

4. Hence, the vector of magnitude 5 units and parallel to the resultant of vectors \mathbf{a} and \mathbf{b} is $\pm 5\hat{\mathbf{c}}$

$$\Rightarrow \pm \frac{5}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \tag{7}$$

$$\Rightarrow \pm \begin{pmatrix} \frac{3\sqrt{10}}{2} \\ \frac{\sqrt{10}}{2} \\ 0 \end{pmatrix} \tag{8}$$