Dynamic Event-Triggered Asynchronous MPC of Markovian Jump Systems With Disturbances

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***Abstract*—This article investigates the model predictive control (MPC) for discrete-time Markov jump systems (MJSs). First, the asynchronization between the modes of the controller and those of the plant is studied. An asynchronous MPC controller is designed to tackle this issue. Next, to reduce the computational cost and communication burden, a version of the dynamic event-triggered mechanism (ETM) is presented. Finally, the exogenous disturbances are considered and the notion of mean-square input-to-state stability (ISS) is taken into account in the controller design. The highlight of this article is the introduction of both dynamic ETM and asynchronous control into the MPC design. The control algorithm is developed and formulated as a convex optimization problem. Moreover, the recursive feasibility and the closed-loop mean-square ISS are both studied. Finally, some simulations are given to show the effectiveness of the derived MPC method.**

***Index Terms*—Asynchronization, event-triggered control (ETC), Markov jump systems (MJSs), model predictive control (MPC).**

I. Introduction

THE PRACTICAL control systems have hybrid dynamic features [1]–[3]. Markov jump systems (MJSs) have received a great deal of attention since they are quite effective to model such processes. A wealth of valuable research results has been published, see, for example, filtering [3], sliding control [4], fuzzy control [5], reliable control [6], fault detection [7], nonfragile control [8], and so forth.

In modern control systems, the network is commonly introduced. In networked MJSs, it is difficult for the controller to precisely obtain the mode information of the plant due to the communication delays, dropouts, etc. In such cases, the asynchronization appears between the modes of the controller and the plant, which leads to the asynchronous control issue for MJSs [9]–[12]. In [9], the asynchronous passive control has been addressed for MJSs, in which the asynchronization is described by a hidden Markov model. Using the same approach, the results in [9] have been extended to the H_∞ asynchronous control [10], nonlinear asynchronous control [11], and stochastic asynchronous control [12]. Although many achievements have been made, the asynchronous control for MJSs has not been fully studied.

In control systems, the information transmission over a network may cause heavy communication burden. To alleviate the communication load, a strategy called event-triggered control (ETC) has been developed in the literature. The basic idea of ETC is that the computations and communications are performed only when the event is triggered. Therefore, network utilization can be greatly reduced compared to the conventional time-triggered control. ETC has been studied since the late 1990s [13]. It was first systematically studied in [14] for linear systems based on the Lyapunov theory. Since then, it was extended for nonlinear systems [15], multiagent systems [16], networked control systems [17], and MJSs [18]. In the literature, there are mainly two types of ETC strategies, that is: 1) the static ETC [14]–[18] and 2) the dynamic ETC [19], [20]. As pointed out in [19], the dynamic ETC introduces an additional dynamic variable and can further reduce the triggering instants in comparison with the static ETC.

The model predictive control (MPC) has gained considerable attention in past decades, due to its ability to handle hard constraints and model uncertainties [21]–[23]. It has been investigated for a variety of fields, including uncertain systems [21], [24]; nonlinear systems [25]; and MJSs [26], [27]. In the MPC, the optimization problem is solved at every instant, which may take up a lot of computing resources. Therefore, the introduction of ETC into MPC is of great significance because it can reduce the computational burden and communication load at the same time. In [28], the event-triggered MPC (ETMPC) has been investigated for uncertain networked systems. In [29], the networked systems with packet loss were considered, and an H_2/H_∞ ETMPC strategy has been studied. In [30], the ETMPC has been presented for networked nonlinear systems. Tang and Deng [31] further considered a case in which the states are unmeasurable. Then, output feedback ETMPC has been presented. It is worth mentioning that all these results

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mentioned above are based on the static ETC mechanism. It seems to be natural to introduce the dynamic ETC in the MPC design.

As discussed above, it is imperative to investigate the MPC for MJSs with an asynchronous phenomenon based on the dynamic ETC scheme. However, no related results are published in the literature. To fill the gap, this work is focused on this topic.

The remainder of this article is organized as follows. In Section II, the MJSs with asynchronous phenomenon and additive disturbance are introduced. Some preliminaries are presented and the ETMPC problem is then formalized. The design algorithm in terms of a set of linear matrix inequalities (LMIs) is developed in Section III. Some simulation results in Section IV are given to show the effectiveness of the obtained ETMPC. Finally, we summarize this article in Section V and end this work.

Notation: Throughout this work, $\text{Prob}\{X\}$, $\mathbb{E}\{X\}$, and $\mathbb{E}_k\{X\}$ refer to the occurrence probability, the mathematical expectation of random variable X, and the expected value of X conditional on the information available at time k. The symbols $\mathbb{N}$, $\mathbb{N}_+$, and $\mathbb{R}$ denote the set of non-negative integers, the set of positive numbers, and the set of real numbers. The notations $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix. For $A \in \mathbb{R}^{n\times m}$, its elements and ith row are denoted by $A_{(i,j)}$ and $A_{(i)}$, $i = 1, \ldots, n$, $j = 1, \ldots, m$. For $x \in \mathbb{R}^n$, its ith entry is represented by $x_{(i)}$ or x_i if no confusion is caused. For $x \in \mathbb{R}^n$ and a positive matrix $W \in \mathbb{R}^{n\times n}$, the notation $\|x\|_W = \sqrt{x^T W x}$ refers to its weighted vector 2-norm.

II. System Description and Preliminaries

A. System Model

Consider the following discrete-time MJS:

$$x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) + E(\theta(k))w(k) \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ are system state and control input, respectively. $w(k) \in \mathbb{R}^{n_w}$ is the persistent disturbance satisfying

$$w(k) \in \mathbb{W} \triangleq \left\{w : w^T w \leq d^2\right\} \tag{2}$$

where $d > 0$ is a known constant. $\theta(k)$ is a Markov chain valued on a finite set $\mathcal{N} = \{1, 2, \ldots, N\}$. Its transition probability matrix is given as $\Pi_\theta = [\pi_{i\eta}]$ and

$$\text{Prob}\{\theta(k+1) = \eta | \theta(k) = i\} = \pi_{i\eta},\ \ i, \eta \in \mathcal{N} \tag{3}$$

where $\sum_{\eta=1}^{N} \pi_{i\eta} = 1$ and $\pi_{i\eta} \geq 0$. For $\theta(k) = i \in \mathcal{N}$, $A_i \triangleq A(\theta(k))$, $B_i \triangleq B(\theta(k))$, and $E_i \triangleq E(\theta(k))$ are constant matrices with appropriate dimensions.

The control signals are subject to hard constraints described by

$$|u_c(\cdot)| \leq \bar{u}_c,\ \ c = 1, \ldots, n_u \tag{4}$$

where the upper bound $\bar{u} > 0$ is a known vector.

Note that the system mode $\theta(k)$ is not always detectable in the real control systems. Inspired by some existing works [9]–[11], another Markov chain $\sigma(k)$ is adopted to obtain an estimation of $\theta(k)$ with certain probability. It is assumed that $\sigma(k)$ takes values on a finite set $\mathcal{M} = \{1, 2, \ldots, M\}$ and satisfies the conditional probability matrix $\Pi_\sigma = [\tau_{ij}]$

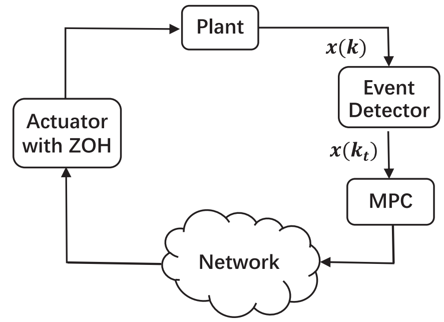

Fig. 1. Framework of the system with event-triggered scheme.

$$\text{Prob}\{\sigma(k) = j | \theta(k) = i\} = \tau_{ij},\ \ i \in \mathcal{N}, j \in \mathcal{M} \tag{5}$$

with $\tau_{ij} \geq 0$ and $\sum_{j=1}^{M} \tau_{ij} = 1$.

B. Event-Triggered Mechanism

In the networked MJS, the plant and the controller are connected through shared communication networks. The structure of the controlled system is shown in Fig. 1, involving a physical plant, an event detector, an MPC controller, and an actuator with zero-order hold (ZOH). The event detector samples the state $x(k)$ at every instant and judges when the events are triggered. Only when an event is triggered does the MPC controller compute a control signal to send over the network to the actuator. Otherwise, the computation and communication tasks are both skipped.

Denote $\{k_t | t \in \mathbb{N}\}$ by the instants when events are triggered. An asynchronous event-triggered state-feedback control law is applied

$$u(k) = K(\sigma(k))x(k_t),\ \ k \in \{k_t, \ldots, k_{t+1} - 1\}. \tag{6}$$

For $\sigma(k) = j$, $K_j \triangleq K(\sigma(k))$, $j \in \mathcal{M}$, are feedback gains to be determined.

Remark 1: In controller (6), the variable $\sigma(k)$ represents the received mode information from the original plant. In the practical applications, $\sigma(k)$ is not identical to $\theta(k)$ as the mode information is not always detectable. Then, the asynchronization between system (1) and controller (6) occurs. Particularly, when $M = N$ and $\Pi_\sigma = I$, the asynchronization disappears and the controller (6) achieves perfect synchrony with MJS (1). Moreover, when $\mathcal{M} = \{1\}$, controller (6) reduces to the mode-independent case.

Define an error signal $e(k)$ based on the latest triggered state $x(k_t)$ and the current measured state $x(k)$ as follows:

$$e(k) \triangleq x(k_t) - x(k),\ \ k \in \{k_t, \ldots, k_{t+1} - 1\},\ \ t \in \mathbb{N}. \tag{7}$$

Note that at triggering instants $\{k_t | t \in \mathbb{N}\}$, $e(k)$ is reset as $e(k_t) = 0\ \forall t \in \mathbb{N}$. In this article, a dynamic event-triggered mechanism (ETM) is employed to determine the time sequence $\{k_t | t \in \mathbb{N}\}$, which is given as follows:

$$k_0 = 0,\ \ k_{t+1} = \inf_{k\in\mathbb{N}}\{k > k_t : \varepsilon\ell(k) > \lambda(k)\} \tag{8a}$$

where $\ell(k)$ is given as follows:

$$\ell(k) = \|e(k)\|_\Phi^2 - \delta\|x(k)\|_\Phi^2, \ \ \Phi > 0. \tag{8b}$$

The matrix $\Phi \in \mathbb{R}^{n_x \times n_x}$ is an event-triggering parameter determined later. $\lambda(k)$ is the auxiliary internal variable described as

$$\lambda(k+1) = \rho\lambda(k) - \ell(k), \ \ \lambda(0) \geq 0. \tag{8c}$$

The scalars ρ, δ, and ε are given in advance and satisfy

$$0 < \rho < 1, \ \ 0 < \delta < 1, \ \ \varepsilon \geq 1/\rho. \tag{8d}$$

Below, we will first investigate the important features of the given dynamic ETM. From (8a), it is clear that the following inequality:

$$\varepsilon\ell(k) \leq \lambda(k) \tag{9}$$

holds for all $k \in \mathbb{N}$. Substituting (9) into (8c), one has $\lambda(k+1) \geq \rho\lambda(k) - (1/\varepsilon)\lambda(k)$, which further results in

$$\lambda(k+1) \geq \left(\rho - \frac{1}{\varepsilon}\right)^{k+1}\lambda(0).$$

In light of $\lambda(0) \geq 0$ and $\rho \geq 1/\varepsilon$, it yields

$$\lambda(k) \geq 0 \ \forall k \in \mathbb{N}. \tag{10}$$

It is worth mentioning that (9) and (10) are two important relationships in this work to derive the MPC design.

Remark 2: Note that the triggering condition in (8a) depends not only on the state $x(k)$ and $e(k)$ but also on the internal dynamic state $\lambda(k)$. It is called dynamic ETM in [19] and [20]. In particular, when $\varepsilon \to \infty$, the triggering condition (8a) reduces to

$$k_{t+1} = \inf_{k\in\mathbb{N}}\{k > k_t : \ell(k) > 0\} \tag{11}$$

which is the so-called static ETM [14]–[18]. In addition, when $\varepsilon \to \infty$ and $\delta = 0$, the triggering condition (8a) will hold at every instant. It reduces to the time-triggered scheme. In the dynamic ETM, an event is triggered when $\ell(k) > \lambda(k)/\varepsilon$, while in the static ETM, it is determined as long as $\ell(k) > 0$. For a previously triggered instant k_t, it is clear that the next instant k_{t+1} determined by dynamic ETM (8) will be no less than the one decided by static ETM (11). In the ETMPC literature, most of the existing works adopt the static ETM (see, for example, [28]–[31]). In this work, a dynamic ETM is employed and a much more general ETMPC design will be derived.

Based on (6) and (7), the controller can be rewritten as

$$u(k) = K(\sigma(k))(x(k) + e(k)). \tag{12}$$

The closed-loop system is then derived as (8c) and

$$\begin{aligned} x(k+1) = {} & (A(\theta(k)) + B(\theta(k))K(\sigma(k)))x(k) \\ & + B(\theta(k))K(\sigma(k))e(k) + E(\theta(k))w(k). \end{aligned} \tag{13}$$

The aim of this work is to determine the feedback gains K_j, $j \in \mathcal{M}$, and the triggering parameter Φ simultaneously for the closed-loop system (8c) and (13) at every instants k_t, $t \in \mathbb{N}$, such that the mean-square input-to-state stability (ISS) is reached with a certain level of performance. For later analysis, the definition of mean-square ISS is given as follows.

Definition 1 [24]: For the closed-loop system (8c) and (13) with initial state $x(0) = x_0$, $\lambda(0) = \lambda_0$ and initial mode $\theta(0) = \theta_0$, it is said to be mean-square ISS if there exist functions $\chi_1 \in \mathcal{KL}$ and $\chi_2 \in \mathcal{K}$ such that

$$\mathbb{E}_0\left\{\|x(k)\|_2^2\right\} \leq \chi_1\left(\|x_0\|_2^2 + \lambda_0, k\right) + \chi_2\left(\|w(k)\|_\infty^2\right).$$

C. MPC Formulation

From (7) and (8b), it follows that (9) is equivalent to:

$$\|e(k)\|_\Phi^2 \leq \delta\|x(k)\|_\Phi^2 + \frac{1}{\varepsilon}\lambda(k). \tag{14}$$

The error variable $e(k)$ satisfying (14) together with the disturbance $w(k)$ and the stochastic variables $\theta(k)$ and $\sigma(k)$ introduces the uncertainties into closed-loop system. In order to ensure robustness against all these uncertainties, a robust ETMPC algorithm will be derived in this work.

For $\theta(k+n|k) = i$, $\sigma(k+n|k) = j$, it follows from (8c), (12), and (13) that the prediction model is given by:

$$u(k+n|k) = K_j(x(k+n|k) + e(k+n|k)) \tag{15a}$$

and

$$\begin{aligned} x(k+n+1|k) = {} & \big(A_i + B_iK_j\big)x(k+n|k) \\ & + B_iK_je(k+n|k) + E_iw(k+n|k) \\ \lambda(k+n+1|k) = {} & \rho\lambda(k+n|k) - \ell(k+n|k) \end{aligned} \tag{15b}$$

where $\ell(k+n|k) = \|e(k+n|k)\|_\Phi^2 - \delta\|x(k+n|k)\|_\Phi^2$. The robust ETMPC technique is to minimize the worst case expected cost function in an infinite horizon

$$\min_{u(k+n|k)} \ \max_{e(k+n|k),\theta\in\mathcal{N},\sigma\in\mathcal{M}} J_\infty(k) \tag{16}$$

subject to (15) and

$$|u_c(k+n|k)| \leq \bar{u}_c, \ \ n \geq 0, c = 1, \ldots, n_u \tag{17}$$

where

$$\begin{aligned} J_\infty(k) &= \mathbb{E}_k\left\{\sum_{n=0}^{\infty} h(k+n|k)\right\} \\ h(k+n|k) &= \|x(k+n|k)\|_Q^2 + \|u(k+n|k)\|_R^2 \\ &\quad - \varphi\|w(k+n|k)\|_2^2 \end{aligned} \tag{18}$$

with $Q \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{n_u \times n_u}$ are given positive-definite weighting matrices, and $\varphi > 0$ is a given constant. In this article, $x(k+n|k)$, $e(k+n|k)$, and $u(k+n|k)$, $n \geq 1$, are the prediction based on the model (15) and its information at time k. Note that $x(k|k) = x(k)$. In order to handle the disturbance, the term $\varphi\|w\|_2^2$ is contained in $J_\infty(k)$, which is inspired by H_∞ control [25].

In this work, the optimization problem (16) is only solved at triggered instants $\{k_t | t \in \mathbb{N}\}$. At instant k_t, a sequence of control input $u(k_t+n|k_t)$, $n \geq 0$ in the form of (15) is obtained by solving the problem (16). The ETMPC only carries out the first component $u(k_t) = K(\sigma(k_t))(x(k_t) + e(k_t))$ to MJS (1). Note that at triggered instant k_t, the signal $e(k_t)$ is reset as $e(k_t) = 0$. Then, the actual control input is $u(k_t) = K(\sigma(k_t))x(k_t)$. At the next time k_t+1, the computational task will be executed again or skipped according to whether a new event is triggered or not.

III. Main Results

In order to design the MPC feedback gains and event-triggering parameter, the optimization problem (16) should be solved in an efficient way. In this section, the main objective is first to propose a tractable approach to solve it. Then, the closed-loop stability is investigated later.

A. Upper Bound for $J_\infty(k)$

The cost function $J_\infty(k)$ in (18) cannot be optimized directly as it contains the uncertain information of $w(k)$. In the following, by imposing a condition, it can be minimized indirectly. Define a quadratic function as follows:

$$\begin{aligned} V(k+n|k) &\triangleq V_1(k+n|k) + \lambda(k+n|k) \ \forall n \geq 0 \\ V_1(k+n|k) &\triangleq x^T(k+n|k)P(\theta(k+n|k))x(k+n|k). \end{aligned} \tag{19}$$

For $\theta(k+n|k) = i$, $P_i \triangleq P(\theta(k+n|k)) \in \mathbb{R}^{n_x\times n_x}$, $i \in \mathcal{N}$, are positive matrices to be determined. In order to optimize the cost function (18), consider the following constraint for $V(k+n|k)$:

$$\mathbb{E}_{k+n|k}\{V(k+n+1|k)\} - V(k+n|k) + h(k+n|k) < 0. \tag{20}$$

Taking the expected values on both sides of (20) conditional on the information available at time k, one has

$$\begin{aligned} &\mathbb{E}_k\{V(k+n+1|k)\} - \mathbb{E}_k\{V(k+n|k)\} \\ &\quad + \mathbb{E}_k\{h(k+n|k)\} < 0. \end{aligned}$$

Summing it from $n = 0$ to $n = \infty$, it yields

$$\begin{aligned} J_\infty(k) &\leq \mathbb{E}_k\{V(k|k)\} - \mathbb{E}_k\{V(\infty|k)\} \\ &\leq \mathbb{E}_k\{V(k|k)\}. \end{aligned}$$

Impose a new constraint as follows:

$$\mathbb{E}_k\{V(k|k)\} = V(k) \leq \gamma. \tag{21}$$

Then, the cost function $J_\infty(k)$ is upper bounded by γ. By minimizing the parameter γ, the performance index (18) is optimized in an indirect way.

From the discussion above, condition (20), together with (21), gives an upper bound for $J_\infty(k)$. In what follows, we will first derive sufficient conditions for (20) and (21).

Lemma 1: Consider the MJS (1) with the state measurement $x(k)$. For given weighting matrices Q and R and a scalar φ in (18), if there exist positive-definite matrices $\Phi \in \mathbb{R}^{n_x\times n_x}$, $P_i \in \mathbb{R}^{n_x\times n_x}$, and $L_{ij} \in \mathbb{R}^{n_x\times n_x}$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ and scalar $\gamma > 0$ satisfying the following conditions:

$$\begin{bmatrix} \Xi_1 & * & * & * \\ \Xi_2 & -I & * & * \\ \alpha\otimes\Xi_3 & 0 & -\Xi_4^{-1} & * \\ \delta^{1/2}I & 0 & 0 & -\Phi^{-1} \end{bmatrix} < 0 \tag{22}$$

$$\sum_{j=1}^{M} \tau_{ij}L_{ij} < P_i \tag{23}$$

$$x^T(k)P_ix(k) + \lambda(k) < \gamma \tag{24}$$

where $\alpha = \begin{bmatrix}\sqrt{\pi_{i1}} & \dots & \sqrt{\pi_{iN}}\end{bmatrix}^T$ and

$$\begin{aligned} \Xi_1 &= \mathrm{diag}\{-L_{ij}, -\Phi, -\varphi I\} \\ \Xi_2 &= \begin{bmatrix} Q^{1/2} & 0 & 0 \\ R^{1/2}K_j & R^{1/2}K_j & 0 \end{bmatrix} \\ \Xi_3 &= \begin{bmatrix} A_i + B_iK_j & B_iK_j & E_i \end{bmatrix} \\ \Xi_4 &= \mathrm{diag}\{P_1, P_2, \dots, P_N\} \end{aligned} \tag{25}$$

for all $i = 1, \dots, N$, $j = 1, \dots, M$, then the expected cost function $J_\infty(k)$ is upper bounded by γ.

Proof: In what follows, we will first verify that (20) is guaranteed by (22) and (23). Substituting the dynamics of $\lambda(k+n|k)$, it results in

$$\begin{aligned} &\lambda(k+n+1|k) - \lambda(k+n|k) \\ &\quad = (\rho-1)\lambda(k+n|k) - \ell(k+n|k) \\ &\quad \leq -\ell(k+n|k). \end{aligned}$$

The last inequality results from the fact that $\rho < 1$ and $\lambda(k+n|k) \geq 0$ $\forall n \geq 0$. From (19) and the predictive model (15), one has

$$\begin{aligned} &\mathbb{E}_{k+n|k}\{V(k+n+1|k)\} - V(k+n|k) \\ &\quad = \mathbb{E}_{k+n|k}\{V_1(k+n+1|k)\} - V_1(k+n|k) \\ &\qquad + \lambda(k+n+1|k) - \lambda(k+n|k) \\ &\quad \leq \mathbb{E}_{k+n|k}\{V_1(k+n+1|k)\} - V_1(k+n|k) - \ell(k+n|k). \end{aligned} \tag{26}$$

Then, it is clear that (20) is ensured if

$$\begin{aligned} &\mathbb{E}_{k+n|k}\{V_1(k+n+1|k)\} - V_1(k+n|k) - \ell(k+n|k) \\ &\quad + \|x(k+n|k)\|_Q^2 + \|u(k+n|k)\|_R^2 - \varphi\|w(k+n|k)\|_2^2 \\ &< 0. \end{aligned} \tag{27}$$

Define $\xi(\cdot) = \begin{bmatrix}x^T(\cdot) & e^T(\cdot) & w^T(\cdot)\end{bmatrix}^T$. For $\theta(k+n|k) = i$, $\sigma(k+n|k) = j$ and $\theta(k+n+1|k) = \eta$, the following three relations can be derived on the basis of (15):

$$\begin{aligned} &\mathbb{E}_{k+n|k}\{V_1(k+n+1|k)\} \\ &\quad = \sum_{j=1}^{M}\tau_{ij}\sum_{\eta=1}^{N}\pi_{i\eta}\big((A_i+B_iK_j)x + B_iK_je + E_iw\big)^T \\ &\qquad \times P_\eta\big((A_i+B_iK_j)x + B_iK_je + E_iw\big) \\ &\quad = \sum_{j=1}^{M}\sum_{\eta=1}^{N}\tau_{ij}\pi_{i\eta}\xi^T(k+n|k)\,\Xi_3^TP_\eta\Xi_3\xi(k+n|k) \end{aligned} \tag{28}$$

$$\begin{aligned} &- V_1(k+n|k) - \ell(k+n|k) - \varphi\|w(k+n|k)\|_2^2 \\ &\quad = -x^T(k+n|k)P(\theta(k+n|k))x(k+n|k) \\ &\qquad - \left(\|e\|_\Phi^2 - \delta\|x\|_\Phi^2\right) - \varphi\|w\|_2^2 \\ &\quad = \xi^T(k+n|k)\Omega_1\xi(k+n|k) \end{aligned} \tag{29}$$

$$\begin{aligned} &\|x(k+n|k)\|_Q^2 + \|u(k+n|k)\|_R^2 \\ &\quad = x^TQx + (x+e)^TK_j^TRK_j(x+e) \\ &\quad = \xi^T(k+n|k)\,\Xi_2^T\Xi_2\xi(k+n|k) \end{aligned} \tag{30}$$

where $\Omega_1 = \{-P_i + \delta\Phi, -\Phi, -\varphi I\}$, the matrices Ξ_2 and Ξ_3 are given in (25), and $x(k+n|k)$, $e(k+n|k)$, and $w(k+n|k)$ are denoted by x, e, and w, respectively, for notation simplicity.

Then, (27) is guaranteed by

$$\sum_{j=1}^{M}\sum_{\eta=1}^{N}\tau_{ij}\pi_{i\eta}\xi^T(k+n|k)\Xi_3^T P_\eta \Xi_3 \xi(k+n|k) + \xi(k+n|k)^T\big(\Omega_1 + \Xi_2^T\Xi_2\big)\xi(k+n|k) < 0$$

which is equivalent to

$$\sum_{j=1}^{M}\sum_{\eta=1}^{N}\tau_{ij}\pi_{i\eta}\Xi_3^T P_\eta \Xi_3 + \Omega_1 + \Xi_2^T\Xi_2 < 0. \quad (31)$$

Note that

$$\sum_{\eta=1}^{N}\pi_{i\eta}P_\eta = \big(\alpha\otimes I_{n_x}\big)^T\Xi_4\big(\alpha\otimes I_{n_x}\big) \quad (32)$$

where α is given in (25). Substituting (32) into (31), it results in

$$\sum_{j=1}^{M}\tau_{ij}\Xi_3^T\big(\alpha\otimes I_{n_x}\big)^T\Xi_4\big(\alpha\otimes I_{n_x}\big)\Xi_3 + \Omega_1 + \Xi_2^T\Xi_2 < 0. \quad (33)$$

From (23), it follows $\Omega_1 < \sum_{j=1}^{M}\tau_{ij}\Omega_2$, where:

$$\Omega_2 = \mathrm{diag}\big\{-L_{ij} + \delta\Phi, -\Phi, -\varphi I\big\}.$$

It implies that (33) is guaranteed by

$$(\alpha\otimes\Xi_3)^T\Xi_4(\alpha\otimes\Xi_3) + \Omega_2 + \Xi_2^T\Xi_2 < 0. \quad (34)$$

Applying the Schur complement, it results in (22).

Furthermore, from the definition of $V(\cdot)$ in (19), it is clear that (21) is equivalent to inequality (24). This completes the proof. ■

B. Invariant Set

The invariant set plays an important role in investigating input constraints and recursive feasibility property. In this section, a set of conditions ensuring an invariant for the closed-loop system will be first obtained. To begin with, define a set as follows:

$$\Theta_i = \big\{L\{x,\lambda\} : x^T P_i x + \lambda \le \gamma\big\} \quad (35)$$

where $P_i > 0$ and $\gamma > 0$ are a matrix and a scalar, respectively.

For the quadratic function $V(\cdot)$ defined in (19), condition (24) together with the following relation:

$$\mathbb{E}_{k+n|k}\{V(k+n+1|k)\} - (1-\mu)V(k+n|k) - \frac{\mu\gamma}{d^2}\|w(k+n|k)\|_2^2 < 0,\ \ 0<\mu<1-\rho \quad (36)$$

ensures an invariant set $\bigcap_{i=1}^{i=N}\Theta_i$ for (15). To see this, taking the expected values on both sides of (36) conditional on the information available at time k, one has

$$\mathbb{E}_k\{V(k+n+1|k)\} - (1-\mu)\mathbb{E}_k\{V(k+n|k)\} - \frac{\mu\gamma}{d^2}\mathbb{E}_k\Big\{\|w(k+n|k)\|_2^2\Big\} < 0. \quad (37)$$

Considering the case $n = 0$ in (37), one has

$$\mathbb{E}_k\{V(k+1|k)\} - (1-\mu)V(k) - \frac{\mu\gamma}{d^2}\|w(k)\|_2^2 < 0. \quad (38)$$

From (24), it infers that $V(k) < \gamma$. This together with (2) further leads to

$$\mathbb{E}_k\{V(k+1|k)\} < (1-\mu)\gamma + \frac{\mu\gamma}{d^2}d^2 < \gamma.$$

From the definition of $V(\cdot)$, one has

$$\mathbb{E}_k\{V(k+1|k)\} = \sum_{\eta=1}^{N}\pi_{i\eta}x^T(k+1|k)P_\eta x(k+1|k) + \lambda(k+1|k). \quad (39)$$

According to $P_\eta > 0$, $\pi_{i\eta} \ge 0\ \forall i,\eta \in \mathcal{N}$, and applying the mathematical induction, it infers that

$$x^T(k+n|k)P_i x(k+n|k) + \lambda(k+n|k) < \gamma \quad (40)$$

for all $n \ge 0$ and $i \in \mathcal{N}$. That is to say, $\bigcap_{i=1}^{N}\Theta_i$ is an invariant set for the prediction model (15).

As discussed above, the condition (36) together with (24) ensures an invariant set for the closed-loop system (15). Next, a set of conditions will be obtained to ensure (36), which is summarized in the following lemma.

Lemma 2: Consider the MJS (1) with the state measurement $x(k)$. For given weighting matrices Q and R and a scalar μ satisfying $0 < \mu < 1-\rho$, if there exist positive-definite matrices $\Phi \in \mathbb{R}^{n_x\times n_x}$, $P_i \in \mathbb{R}^{n_x\times n_x}$, and $L_{ij} \in \mathbb{R}^{n_x\times n_x}$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ and scalar $\gamma > 0$ satisfying (23), (24), and the following conditions:

$$\begin{bmatrix} \Xi_5 & * & * \\ \alpha\otimes\Xi_3 & -\Xi_4^{-1} & * \\ \delta^{1/2}I & 0 & -\Phi^{-1}\end{bmatrix} < 0 \quad (41)$$

where

$$\Xi_5 = \mathrm{diag}\Big\{-(1-\mu)L_{ij}, -\Phi, -\mu\gamma d^{-2}I\Big\} \quad (42)$$

for all $i = 1,\dots,N$, $j = 1,\dots,M$, then $\bigcap_{i=1}^{N}\Theta_i$ is an invariant set for the closed-loop system (15).

Proof: First, we will prove that (36) is guaranteed by (23) and (41). From (26), it follows that (36) is ensured by the following condition:

$$\mathbb{E}_{k+n|k}\{V_1(k+n+1|k)\} - (1-\mu)V_1(k+n|k) - \ell(k+n|k) - \frac{\mu\gamma}{d^2}\|w(k+n|k)\|_2^2 < 0. \quad (43)$$

For $\theta(k+n|k) = i$, we can obtain the relation as follows:

$$\begin{aligned} &-(1-\mu)V_1(k+n|k) - \ell(k+n|k) - \frac{\mu\gamma}{d^2}\|w(k+n|k)\|_2^2 \\ &= -(1-\mu)x^T(k+n|k)P_i x(k+n|k) \\ &\quad - \Big(\|e(k+n|k)\|_\Phi^2 - \delta\|x(k+n|k)\|_\Phi^2\Big) \\ &\quad - \frac{\mu\gamma}{d^2}\|w(k+n|k)\|_2^2 \\ &= \xi^T(k+n|k)\Omega_3\xi(k+n|k) \end{aligned} \quad (44)$$

where $\Omega_3 = \mathrm{diag}\{-(1-\mu)P_i + \delta\Phi, -\Phi, -\mu\gamma d^{-2}I\}$. From (28) and (44), it follows that (43) is ensured by:

$$\sum_{j=1}^{M}\sum_{\eta=1}^{N} \tau_{ij}\pi_{i\eta}\,\Xi_3^T P_\eta \Xi_3 + \Omega_3 < 0. \tag{45}$$

Substituting (32) into (45), it leads to

$$\sum_{j=1}^{M} \tau_{ij}\,\Xi_3^T\big(\alpha\otimes I_{n_x}\big)^T \Xi_4\big(\alpha\otimes I_{n_x}\big)\Xi_3 + \Omega_3 < 0. \tag{46}$$

From (23), it follows $\Omega_3 < \sum_{j=1}^{M}\tau_{ij}\Omega_4$, where:

$$\Omega_4 = \mathrm{diag}\Big\{-(1-\mu)L_{ij} + \delta\Phi, -\Phi, -\mu\gamma d^{-2}I\Big\}.$$

It implies that (46) is guaranteed by

$$(\alpha\otimes\Xi_3)^T\Xi_4(\alpha\otimes\Xi_3) + \Omega_4 < 0. \tag{47}$$

Using the Schur complement, it leads to (41).

From the discussion above, conditions (23), (24), and (41) ensure an invariant set $\bigcap_{i=1}^{N}\Theta_i$ for (15). ∎

C. ETMPC Algorithm

In Lemmas 1 and 2, a set of conditions is obtained to ensure an upper bound for $J_\infty(k)$ and to guarantee an invariant set for the prediction model (15). They provide elementary results for the MPC design. Based on the results obtained above, the entire ETMPC optimization problem is summarized as follows:

$$\begin{aligned}&\min_{P(k),\gamma}\ \gamma \\ &\text{s.t.}\quad (17),\ (22), (23),\ (24) \text{ and } (41).\end{aligned} \tag{48}$$

Note that inequalities (22)–(24) and (41) are bilinear. They cannot be solved efficiently using the exiting solvers. In the below, by taking into account the solvability of these conditions together with the input constraints (17), the criterion for the ETMPC design will be obtained.

Theorem 1: Consider the MJS (1). For given upper bound $\bar{u}$ in (5), weighting matrices Q and R and a scalar φ in (18), ETM constants ρ, δ, and ε satisfying (8d), a scalar μ satisfying $0 < \mu < 1-\rho$ and state measurement $x(k)$, if there exist positive-definite matrices $X_i \in \mathbb{R}^{n_x\times n_x}$, $H_{ij}\in\mathbb{R}^{n_x\times n_x}$, $Z\in\mathbb{R}^{n_x\times n_x}$, and $\Upsilon\in\mathbb{R}^{n_u\times n_u}$, matrices $Y_j\in\mathbb{R}^{n_u\times n_x}$ and $W_j\in\mathbb{R}^{n_x\times n_x}$, and positive scalar γ such that the following problem is feasible:

$$\min_{X_i,Y_j,Z,H_{ij},W_j,\Upsilon,\gamma}\ \gamma \tag{49}$$

$$\text{s.t.}\ \begin{bmatrix}\Gamma_1 & * & * & * \\ \Gamma_2 & -\gamma & * & * \\ \alpha\otimes\Gamma_3 & 0 & -\Gamma_4 & * \\ \delta^{1/2}W_j & 0 & 0 & -Z\end{bmatrix} < 0 \tag{50}$$

$$\begin{bmatrix}-X_i & * \\ \beta\otimes X_i & -\Gamma_5\end{bmatrix} < 0 \tag{51}$$

$$\begin{bmatrix}-1 & * & * \\ x(k|k) & -X_i & * \\ \lambda^{1/2}(k) & 0 & -\gamma\end{bmatrix} < 0 \tag{52}$$

$$\begin{bmatrix}\Gamma_6 & * & * \\ \alpha\otimes\Gamma_7 & -\Gamma_4 & * \\ \delta^{1/2}W_j & 0 & -Z\end{bmatrix} < 0 \tag{53}$$

$$\begin{bmatrix}-\Upsilon_{(c,c)} & \big(Y_j\big)_{(c)} \\ * & -W_j - W_j^T + X_i\end{bmatrix} < 0,\ \ \Upsilon_{(c,c)} < \bar{u}_c \tag{54}$$

where $\beta = \big[\sqrt{\tau_{i1}}\ \ \dots\ \ \sqrt{\tau_{iM}}\big]^T$ and

$$\begin{aligned}
\Gamma_1 &= \mathrm{diag}\{\Pi_1, \Pi_2, -\gamma\varphi I\}\\
\Pi_1 &= -W_j - W_j^T + H_{ij}\\
\Pi_2 &= -W_j - W_j^T + Z\\
\Gamma_2 &= \begin{bmatrix}Q^{1/2}W_j & 0 & 0\\ R^{1/2}Y_j & R^{1/2}Y_j & 0\end{bmatrix}\\
\Gamma_3 &= \big[A_iW_j + B_iY_j \quad B_iY_j \quad \gamma E_i\big]\\
\Gamma_4 &= \mathrm{diag}\{X_1, X_2, \dots, X_N\}\\
\Gamma_5 &= \mathrm{diag}\{H_{i1}, \dots, H_{iM}\}\\
\Gamma_6 &= \mathrm{diag}\Big\{\Pi_3, \Pi_2, -\mu d^{-2}I\Big\}\\
\Pi_3 &= -(1-\mu)\Big(W_j + W_j^T - H_{ij}\Big)\\
\Gamma_7 &= \big[A_iW_j + B_iY_j \quad B_iY_j \quad E_i\big]
\end{aligned}$$

for all $i\in\mathcal{N}$, $j\in\mathcal{M}$, then the MPC feedback gains in (6) and the ETM parameter Φ can be computed from $K_j = Y_jW_j^{-1}$ $\forall j\in\mathcal{M}$, and $\Phi = \gamma Z^{-1}$, respectively.

Proof: Multiplying diag{ $\gamma^{-1/2}W_j^T$, $\gamma^{-1/2}W_j^T$, $\gamma^{1/2}I, \dots, \gamma^{1/2}I$} and its transpose on both sides of (22), one has

$$\begin{bmatrix}\Lambda_1 & * & * & * \\ \Lambda_2 & -\gamma & * & * \\ \alpha\otimes\Lambda_3 & 0 & -\gamma\,\Xi_4^{-1} & * \\ \delta^{1/2}W_j & 0 & 0 & -\gamma\Phi^{-1}\end{bmatrix} < 0$$

where

$$\begin{aligned}
\Lambda_1 &= \mathrm{diag}\bigg\{-W_j^T\frac{L_{ij}}{\gamma}W_j, -W_j^T\frac{\Phi}{\gamma}W_j, -\gamma\varphi I\bigg\}\\
\Lambda_2 &= \begin{bmatrix}Q^{1/2}W_j & 0 & 0\\ R^{1/2}K_jW_j & R^{1/2}K_jW_j & 0\end{bmatrix}\\
\Lambda_3 &= \big[A_iW_j + B_iK_jW_j \quad B_iK_jW_j \quad \gamma E_i\big]
\end{aligned}$$

From $\gamma > 0$ and $L_{ij} > 0$, it implies $(W_j - \gamma L_{ij}^{-1})^T\gamma^{-1}L_{ij}(W_j - \gamma L_{ij}^{-1}) \geq 0$, which can be equivalently rewritten as

$$-\gamma^{-1}W_j^TL_{ij}W_j \leq -W_j - W_j^T + \gamma L_{ij}^{-1}. \tag{55}$$

Similarly, one has

$$-\gamma^{-1}W_j^T\Phi W_j \leq -W_j - W_j^T + \gamma\Phi^{-1}. \tag{56}$$

Resorting the relationship in (55) and (56), and introducing the variable definition

$$\begin{aligned}&\gamma P_i^{-1} = X_i,\ \ K_jW_j = Y_j\\ &\gamma\Phi^{-1} = Z,\ \ \gamma L_{ij}^{-1} = H_{ij}\end{aligned} \tag{57}$$

produce the LMIs in (50).

Note that

$$\sum_{j=1}^{M} \tau_{ij} L_{ij} = \big(\beta \otimes I_{n_x}\big)^T \Omega_5 \big(\beta \otimes I_{n_x}\big) \tag{58}$$

where $\Omega_5 = \mathrm{diag}\{L_{i1}, L_{i2}, \ldots, L_{iM}\}$. Substituting (58) into (23), and applying the Schur complement, it results in

$$\begin{bmatrix} -P_i & * \\ \beta \otimes I_{n_x} & -\Omega_5^{-1} \end{bmatrix} < 0. \tag{59}$$

Premultiplying and postmultiplying (59) with $\mathrm{diag}\{\gamma^{1/2} P_i^{-1}, \gamma^{1/2} I, \ldots, \gamma^{1/2} I\}$ and its transpose, and resorting to (57) lead to the LMIs in (51).

Dividing both sides of (24) by γ, it results in $x^T(k|k)\gamma^{-1} P_i x(k|k) + \gamma^{-1}\lambda(k|k) < 1$. Resorting to the Schur complement and (57), it leads to the LMIs in (52).

For the inequalities in (41), multiplying $\mathrm{diag}\{\gamma^{-1/2} W_j^T, \gamma^{-1/2} W_j^T, \gamma^{-1/2} I, \gamma^{1/2} I, \ldots, \gamma^{1/2} I\}$ on the left and its transpose on the right, one has

$$\begin{bmatrix} \Lambda_4 & * & * \\ \alpha \otimes \Lambda_5 & -\gamma\, \Xi_4^{-1} & * \\ \delta^{1/2} W_j & 0 & -\gamma \Phi^{-1} \end{bmatrix} < 0$$

where

$$\begin{aligned} \Lambda_4 &= \mathrm{diag}\Big\{\Lambda_6, -\gamma^{-1} W_j^T \Phi W_j, -\mu d^{-2} I\Big\} \\ \Lambda_5 &= \big[(A_i + B_i K_j) W_j \quad B_i K_j W_j \quad E_i\big] \\ \Lambda_6 &= -(1-\mu)\gamma^{-1} W_j^T L_{ij} W_j. \end{aligned}$$

Resorting the relationship in (55) and (56) and the variable definition in (57) leads to the LMIs in (53).

The input constraint (17) can also be incorporated into the ETMPC design. From (40), one has

$$\begin{aligned} &\max_{n\geq 0} \big|u_c(k+n|k)\big|^2 \\ &= \max_{n\geq 0} \Big|\big(K_j\big)_{(c)} x(k+n|k)\Big|^2 \\ &\leq \max_{n\geq 0} \Big\|\big(K_j\big)_{(c)} P_i^{-1/2}\Big\|^2 \Big\|P_i^{1/2} x(k+n|k)\Big\|^2 \\ &= \Big\|\big(K_j\big)_{(c)} P_i^{-1/2}\Big\|^2 x^T(k+n|k) P_i x(k+n|k) \\ &\leq \gamma \big(K_j\big)_{(c)} P_i^{-1} \big(K_j\big)_{(c)}^T. \end{aligned}$$

The last inequality results from the fact that $\bigcap_{i=1}^N \Theta_i$ is an invariant set for (15). Then, a sufficient condition for (17) is obtained as $(K_j)_{(c)}(P_i/\gamma)^{-1}(K_j)_{(c)}^T < \bar{u}_c$, which is equivalent to

$$\begin{bmatrix} -\Upsilon_{(c,c)} & \big(K_j\big)_{(c)} \\ * & -\gamma^{-1} P_i \end{bmatrix} < 0,\ \ \Upsilon_{(c,c)} < \bar{u}_c. \tag{60}$$

Performing the congruence transformation by $\mathrm{diag}\{I, W_j^T\}$, it further leads to

$$\begin{bmatrix} -\Upsilon_{(c,c)} & \big(K_j W_j\big)_{(c)} \\ * & -W_j^T \gamma^{-1} P_i W_j \end{bmatrix} < 0,\ \ \Upsilon_{(c,c)} < \bar{u}_c.$$

From $-W_j^T \gamma^{-1} P_i W_j < -W_j - W_j^T + \gamma P_i^{-1}$ and the variable definition in (57), it results in the LMIs in (54).

Algorithm 1 : Dynamic Event-Triggered Asynchronous MPC

1) Initialization: Select the upper bound $\bar{u}$ in (4). Set the ETM constants ε, δ, ρ satisfying (8d) and a scalar μ satisfying $0 < \mu < 1-\rho$. Choose the weighting matrices Q, R and a scalar φ in (18). Give the probability matrix Π_θ and Π_σ.
2) At time $k \geq 0$, obtain the state $x(k)$ and determine the triggering instant k_t. If $k = 0$ or an event is triggered, go to step 3; otherwise go to step 6.
3) Solve the ETMPC optimization problem (49)-(54) to compute the MPC feedback gains K_j $\forall j \in \mathcal{M}$, and triggering matrix Φ.
4) Detect the mode information $\sigma(k)$.
5) Calculate $u(k)$ by (6).
6) Implement the control input obtained at the latest triggered instant k_t.
7) Go to step 2 at time $k+1$.

Finally, according to the variable definition in (57), the MPC feedback gains and the event-triggering parameter can be computed. The proof is completed. ∎

Remark 3: In Theorem 1, the ETMPC design is formulated as a convex optimization problem (49)–(54). Its computational complexity depends on the number of modes in $\theta(k)$ and $\theta(k)$. The computation burden increases when the modes of $\theta(k)$ and $\sigma(k)$ increase. Fortunately, with the aid of dynamic ETC, the computation cost can be saved to some extend.

The ETMPC design can be specified in Algorithm 1.

Theorem 2: For the MJS (1), if the problems (49)–(54) are feasible at time k, then the closed-loop MJS (13) is mean-square ISS with respect to the additive disturbance $w(k)$.

Proof: The proof is composed of two step.

Step 1 (Recursive Feasibility): Assume that the problems (49)–(54) are feasible at triggering instant k_t. The corresponding optimal solution is denoted as $\{X_i^*(k_t), Y_j^*(k_t), Z^*(k_t), H_{ij}^*(k_t), W_j^*(k_t), \Upsilon^*(k_t), \gamma^*(k_t)\}$, $\forall i \in \mathcal{N}$ $\forall j \in \mathcal{M}$. At the next triggering instant k_{t+1}, only the LIMs in (52) depend explicitly on the updated measurement $x(k_{t+1})$. Thus, to show the recursive feasibility, we only need to prove that (52) is still satisfied at k_{t+1}. At instant k_{t+1}, (52) can be rewritten as $x^T(k_{t+1}) X_i^{-1}(k_{t+1}) x(k_{t+1}) + \gamma^{-1}(k_{t+1})\lambda(k_{t+1}) < 1$. Construct a set of feasible solution at time k_{t+1} as follows:

$$\{X_i(k_{t+1}), \ldots, \gamma(k_{t+1})\} = \big\{X_i^*(k_t), \ldots, \gamma^*(k_t)\big\}. \tag{61}$$

From (61), it follows that (52) is further equivalent to:

$$x^T(k_{t+1}) X_i^{*-1}(k_t) x(k_{t+1}) + \frac{\lambda(k_{t+1})}{\gamma^*(k_t)} < 1. \tag{62}$$

On the other hand, the feasibility of (51)–(53) ensures an invariant set $\bigcap_{i=1}^N \Theta_i$ for the prediction model (15). It follows $x^T(k_{t+1}|k_t) P_i^*(k_t) x(k_{t+1}|k_t) + \lambda(k_{t+1}|k_t) < \gamma^*(k_t)$. From (57), one has $\gamma^{*-1}(k_t) P_i^*(k_t) = X_i^{*-1}(k_t)$. This further leads to

$$x^T(k_{t+1}|k_t) X_i^{*-1}(k_t) x(k_{t+1}|k_t) + \frac{\lambda(k_{t+1}|k_t)}{\gamma^*(k_t)} < 1. \tag{63}$$

Note that the state $x(k_{t+1}|k_t)$ is predicted by model (15), which is affected by the additive disturbance $w(k)$, stochastic parameters $\theta(k)$ and $\sigma(k)$, and uncertain parameter $e(k)$. The relationship in (63) is satisfied for all parameters $w(k)$, $\theta(k)$, $\sigma(k)$, and $e(k)$. This means that the real state measurement $x(k_{t+1})$ also satisfies the relationship (63). It leads to the inequality in (62). That is to say, (52) still holds at instant k_{t+1}. Similarly, it is also satisfied at instants k_{t+2}, $k_{t+3}, \ldots$. The recursive feasibility of problem (49)–(54) is proved.

Step 2 (Mean-Square ISS): The feasibility of (49)-(54) leads to the conditions (22)–(24) in Lemma 1, which further results in inequality (27). In what follows, we will prove the mean-square ISS based on (27). For constant ρ in (8d), there exists a parameter $\nu \in (0,\ 1-\rho)$ such that $\rho - 1 + \nu < 0$. This together (27) implies

$$\begin{aligned}&\mathbb{E}_k\{V_1(k+1|k)\} - V_1(k|k) - \ell(k|k)\\&\quad + \|x(k|k)\|_Q^2 + \|u(k|k)\|_R^2 - \varphi\|w(k|k)\|_2^2\\&\quad + (\rho - 1 + \nu < 0)\lambda(k|k) < 0.\end{aligned}$$

Note that $\rho\lambda(k) - \ell(k) - \lambda(k) = \lambda(k+1) - \lambda(k)$. It further infers to

$$\begin{aligned}&\mathbb{E}_k\{V(k+1|k)\} - V(k) + \|x(k)\|_Q^2\\&\quad + \nu\lambda(k) + \|u(k)\|_R^2 - \varphi\|w(k)\|_2^2 < 0.\end{aligned} \tag{64}$$

Similar to the procedure from (63) to (62), it further implies

$$\begin{aligned}&\mathbb{E}_k\{V(k+1)\} - V(k) + \|x(k)\|_Q^2\\&\quad + \nu\lambda(k) + \|u(k)\|_R^2 - \varphi\|w(k)\|_2^2 < 0.\end{aligned} \tag{65}$$

Based on the definition of $V(\cdot)$ in (19), we have

$$\begin{aligned}&\lambda_{\min}(P)\|x(k)\|_2^2 + \lambda(k)\\&\quad \le V(k) \le \lambda_{\max}(P)\|x(k)\|_2^2 + \lambda(k)\end{aligned} \tag{66}$$

where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximal and minimal eigenvalue of a matrix, respectively. From (66), it further leads to

$$\varrho_1\|\zeta(k)\|_2^2 \le V(k) \le \varrho_2\|\zeta(k)\|_2^2 \tag{67}$$

where $\varrho_1 = \min\{\lambda_{\min}(P), 1\}$, $\varrho_2 = \max\{\lambda_{\max}(P), 1\}$, and $\zeta(k) = \begin{bmatrix}x^T(k) & \lambda^{1/2}(k)\end{bmatrix}^T$. From (65), it follows:

$$\mathbb{E}_k\{V(k+1)\} - V(k) < -\varrho_3\|\zeta(k)\|_2^2 + \varphi\|w(k)\|_2^2 \tag{68}$$

where $\varrho_3 = \min\{\lambda_{\min}(Q), \nu\}$. Based on [24, Lemma 6], conditions in (67) and (68) lead to

$$\mathbb{E}_0\Big\{\|\zeta(k)\|_2^2\Big\} \le \chi_1\Big(\|\zeta(0)\|_2^2, k\Big) + \chi_2\Big(\|w(k)\|_\infty^2\Big)$$

where $\chi_1 \in \mathcal{KL}$ and $\chi_2 \in \mathcal{K}$. Note that $\|x(k)\|_2^2 \le \|\zeta(k)\|_2^2$ and $\|\zeta(0)\|_2^2 = \|x(0)\|_2^2 + \lambda(0)$. It immediately produces the condition in Definition 1. ■

IV. Numerical Example

This section gives some simulations to show the effectiveness of the ETMPC algorithm proposed in this work. Consider an MJS with 2-mode of the form (1). Its system matrices are given as follows:

$$A_1 = \begin{bmatrix}-0.05 & -0.55\\ 0.05 & 0.85\end{bmatrix}, B_1 = \begin{bmatrix}1.0 & 0.1\\ 0.1 & 0.1\end{bmatrix}, E_1 = \begin{bmatrix}-0.1\\ 0.1\end{bmatrix}$$
$$A_2 = \begin{bmatrix}-0.65 & -1.0\\ -0.45 & -0.15\end{bmatrix}, B_2 = \begin{bmatrix}0.5 & 0.8\\ 0.2 & 0.1\end{bmatrix}, E_2 = -E_1.$$

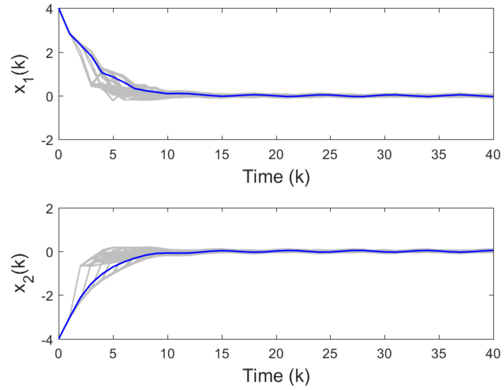

Fig. 2. System states with 1000 repetitions.

The weighting parameters in (18) are selected as $Q = I$, $R = 0.01I$, and $\varphi = 5$. The constraint for inputs is given by $|u(\cdot)| \le \begin{bmatrix}10 & 10\end{bmatrix}^T$. The transition probability matrix Π_θ and the conditional probability matrix Π_σ are chosen as

$$\Pi_\theta = \begin{bmatrix}0.9 & 0.1\\ 0.5 & 0.5\end{bmatrix},\ \Pi_\sigma = \begin{bmatrix}0.8 & 0.2\\ 0.6 & 0.4\end{bmatrix}.$$

The scalars in (8d) are set as $\rho = 0.5$, $\delta = 0.5$, and $\varepsilon = 2$. The constant μ in (35) is chosen as $\mu = 0.1$. The additive disturbance is assumed to be $w(k) = 0.5\sin(k)$, which satisfies $w^T(k)w(k) \le d^2$ with $d = 0.5$. The initial conditions for system (1) and (8c) are given as $x(0) = \begin{bmatrix}4 & -4\end{bmatrix}^T$ and $\lambda(0) = 5$, respectively.

It can be seen that the system with these parameters is unstable as the eigenvalues of A_2 are -1.1159 and 0.3159. The aim here is to design an EMMPC controller and triggering matrix Φ simultaneously at each triggering instant such that the MJS is stabilized with a certain level of performance.

By solving the optimization problem in (48)–(53), the feedback gains K_j, $j = 1, 2$, and the ETM parameter Φ can be computed. Through 1000 repeated simulations, the state trajectories (in gray lines) and their mean values (in blue lines) are plotted in Fig. 2. In one of these simulations, the control inputs and trigger instants are depicted in Fig. 3, in which 1 denotes an event triggered and 0, otherwise. From these results, it can be seen that the state trajectories converge to the neighborhood of the orign in the presence of additive disturbance.

Next, we will compare the dynamic ETM (8) with the static ETM (11). Attention is paid to the performance $J_\infty(k)$ and the reduction of computation that is measured by the triggering rate χ defined by

$$\chi = \frac{\text{Total number of events}}{\text{System runtime}}.$$

Based on 1000 simulation repeats, the mean values of their performance upper bounds γ are shown in Fig. 4 and the mean

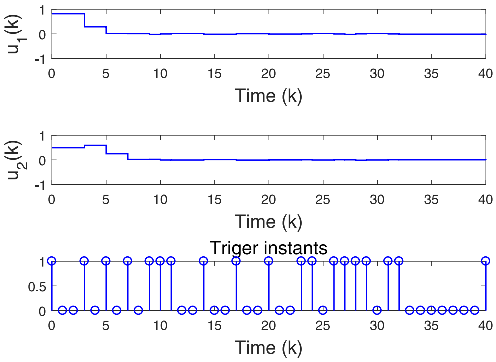

Fig. 3. System inputs and trigger instants.

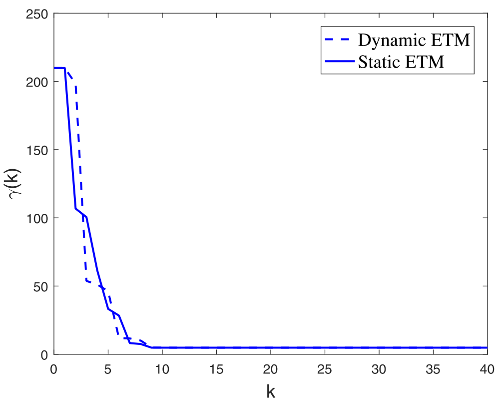

Fig. 4. Upper bound of $J_{\infty}(k)$.

TABLE I
OBTAINED χ

	χ
Dynamic ETM (8)	0.4210
Static ETM (11)	0.5361

values of their triggering rate χ are given in Table I. From these results, it follows that the dynamic ETM can reduce the number of triggering instants while keeping almost the same performance as the static ETM.

V. CONCLUSION

This article has investigated the MPC for a class of MJSs. Both the additive disturbance and asynchronous phenomenon have been considered. A dynamic ETM has been adopted to alleviate the computation and communication burden. A set of conditions for the ETMPC design has been derived. It has been shown that by virtue of the ETMPC technique, the closed-loop systems can be stochastically stabilized with a certain level of performance. Simulation results have been proposed to show the effectiveness of the ETMPC algorithm obtained in this work.

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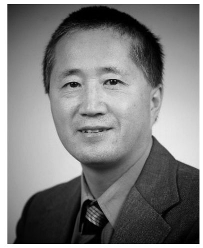

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