E1+TE1

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

FALL SEMESTER 2022-23
CONTINUOUS ASSESSMENT TEST – II
Programme Name: M.Sc. Data Science
Course Code: MAT 5012
Course Name: Probability theory and distributions

Class Number(s): VL2022230105743 **Exam Duration: 90 minutes** **Maximum Marks: 50**

General instruction(s): Statistical tables are permitted. **Answer all the questions** ($5 \times 10 = 50$)

Q1 In a given city, 4% of all licensed drivers will be involved in at least 1 road accident in any given year. Determine the probability that among 150 licensed drivers randomly chosen in this city

(a) only 5 will be involved in at least 1 accident in any given year

(b) at most 3 will be involved in at least 1 accident in any given year.

Ans:- p = prob(licensed Drivers involve in accident)

$= 4\% = 0.04$; $q = 1-p = 1-0.04 = 0.96$

(i) $P(X=5) = 150C_5\, p^5 q^{145} = 0.1628$

(ii) $P(X \leq 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$= 0.1457$

Q2 Fit a Poisson distribution for the following data and hence find the expected frequencies.

X	0	1	2	3	4	5	6
f	314	335	204	86	29	9	3

Ans:- $X \sim Pois(\lambda)$; $E(X) = \lambda = \bar{X} = \Sigma f_i x_i / \Sigma f_i$

$N = 980$; $\Sigma f_i x_i = 1180$

$\bar{X} = 1.2041$

$\therefore \lambda = 1.2041$

$e^{-\lambda} = e^{-1.2041} - 0.29996$

$P(0) = e^{-\lambda} = 0.29996 = 980 * P(0) = 293.96 \simeq 294$

$P(1) = \left(\frac{\lambda}{x+1}\right) P(0) = 0.36118 = 980 * P(1) = 353.95 \simeq 354$

$P(2) = \frac{\lambda}{2} \cdot P(1) = 0.21744 = N * P(2) = 213.09 \simeq 213$

$P(3) = \lambda/3 \cdot P(2) = 0.08727 = N * P(3) = 85.526 \simeq 86$

	$P(4) = \lambda/_4 * P(3) = 0.02627 \quad = N * P(4) = 25.744 \simeq 26$ $P(5) = \lambda/_5 * P(4) = 0.006326 \quad = N * P(5) = 6.1997 \simeq 6$ $P(6) = \lambda/_6 * P(5) = 0.001269 = N * P(6) = 1.24411 \simeq 1$
Q3	The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer. (a) What is the probability that a line width is greater than 0.62 micrometer? (b) What is the probability that a line width is between 0.47 and 0.63 micrometer? (c) The line width of 90% of samples is below what value?
	$X \sim N(0.5, 0.05)$ (a) $P(X > 0.62) = P\left(\frac{X-\mu}{\sigma} > 0.62 - 0.5/0.05\right)$ $= P(Z > 2.4) = 1 - P(Z < 2.4)$ $1 - P(-\infty < Z < 2.4)$ $\Rightarrow 1 - 0.9918$ $= 0.0082$ $1 - 0.5 - P(0 < Z < 2.4)$ $\Rightarrow 0.5 - 0.4918$ $= 0.0082$ (b) $P(0.47 \le x \le 0.63) = P\left(\frac{0.47-0.5}{0.05} \le Z \le \frac{0.63-0.5}{0.05}\right)$ $= P(-0.6 \le Z \le 2.6)$ $\Rightarrow F(2.6) - F(-0.6)$ $P(-\infty < Z \le 2.6) - P(-\infty < Z \le -0.6)$ $\Rightarrow 0.9953 - 0.2743$ $= 0.721$ $P(-0.6 \le Z \le 0) + P(0 \le Z \le 2.6)$ $\Rightarrow P(0 \le Z \le 0.6) + P(0 \le Z \le 2.6)$ $\Rightarrow 0.2257 + 0.4953$ $= 0.721$ (c) $P(X < x) = 0.90 \Rightarrow P(Z \le z_1) = 0.9$ $\Rightarrow P(-\infty < Z \le z_1) = 0.9$ $z_1 = 1.28$ $\therefore \boxed{X = 0.564}$ $z_1 = \frac{X - 0.5}{0.05}$ $X = 0.5 + z_1(0.05)$ $\therefore X = 0.564$ $\left\{ 0.5 + P(0 < Z < z_1) = 0.9 \right.$ $\left. \therefore P(0 < Z < z_1) = 0.4 \Rightarrow z_1 = 1.28 \right\}$

Q4	The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours. (a) What is the probability that you do not receive a message during a two-hour period? (b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?
	Ans:- (a) $E(X) = 1/\lambda = 2\,hrs \Rightarrow \lambda = 1/2 = 0.5$ $P(X>2) = e^{-\lambda x} = e^{-2\times 0.5} = e^{-1} = 0.3678$ (b) $P(X>6 \mid X>4) = P(X>2) = e^{-1}$
Q5	Let $\boldsymbol{X}$ be a Poisson random variable with the parameter, find the moment generating function of $\boldsymbol{X}$ and use the M.G.F to find $\boldsymbol{E}(\boldsymbol{X})$ and $\boldsymbol{V}(\boldsymbol{X})$.
	Ans:- $M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \cdot \lambda^x}{x!}$ $= e^{-\lambda} \sum_{x=0}^{\infty} (\lambda e^t)^x / x!$ $M_x(t) = e^{-\lambda} \cdot e^{\lambda e^t} = e^{\lambda(e^t - 1)}$ $E(X) = V(X) = \lambda$