Not so surprize Quiz 2: Probability and Statistics (25 mks)

Each question: 5 marks

1. (Memory test:)
   - Define convergence in probability and convergence in r^{th} mean. (3 marks)
   - When do we say that an estimator Θ is strongly consistent. Give an example. (2 mks)
2. Find the stationary distribution π for the Markov chain with transition matrix

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{bmatrix}.$$

$\left(\frac{b}{2} \quad b \quad \frac{b}{2} \right)$

(5 marks)

3. Give a method to simulate n successive states of a Markovian coin with initial distribution $\mu = [0.5, 0.5]$ and

$$P = \begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}.$$

You are given access to only samples from a Uniform $[0, 1]$ random variable.

Each question 10mks

Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $E[X_i] = \theta$ and $Var(X_i) = \sigma^2$. Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and c denote a known constant. Derive the Mean square error (MSE) for the following estimators for θ:

- $\hat{\Theta}_1 = 2\bar{X}_n - X_1$ (5mks). (Hint: $\bar{X}_n$ and X_1 are not independent, so careful with covariance) $\frac{8\sigma^2}{n} + c$
- $\hat{\Theta}_2 = \frac{1}{3}(X_1 + X_3)$, (2.5mks) $\frac{2\sigma^2 + \theta^2}{9}$
- $\hat{\Theta}_3 = c\bar{X}_n$, (2.5mks)

$\frac{c^2\sigma^2}{n} + \theta^2(c-1)^2$

$MSE = Var(\hat{\Theta}) + ($

$B(\hat{\Theta}) =$

hY