COURSE: LINEAR ALGEBRA
Course Code: MA2.101

Spring-2025

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Assignment 1: [Released date: 27.03.2025] [Submission Date: 06.04.2025]

Full Marks- 25

1. Obtain an orthogonal basis for the subspace R^3 spanned by the vectors

$x=\begin{pmatrix}1\\1\\0\end{pmatrix}, y=\begin{pmatrix}3\\4\\2\end{pmatrix}$, by Gram- Schmidt orthogonalisation process. Here the inner product is the standard inner product defined for R^n **[5]**

2. a) Prove that if A and B are square matrices and AB is invertible , then both A and B are invertible. b) Prove that if a symmetric matrix is invertible, then its inverse is symmetric also. **[2.5+2.5=5]**

3) Find the value of k for which A is invertible

a) A= $\begin{pmatrix}k & -k & 3\\0 & k+1 & 1\\k & -8 & k-1\end{pmatrix}$ b) A= $\begin{pmatrix}k & k & 0\\k^2 & 2 & k\\0 & k & k\end{pmatrix}$

4) Show that following vectors **{$\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$}** forms an orthogonal basis for $\mathbf{R}^3$ (where R is the set of all real numbers) and then find the coordinate of the vector **w** with respect to the basis **{$\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$}**

$\mathbf{u}_1$= $\begin{bmatrix}1\\1\\1\end{bmatrix}$, $\mathbf{u}_2$= $\begin{bmatrix}1\\-1\\0\end{bmatrix}$, $\mathbf{u}_3$= $\begin{bmatrix}1\\1\\-2\end{bmatrix}$; **w**= $\begin{bmatrix}7\\9\\10\end{bmatrix}$

[a= $\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}$ and b = $\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$,the inner product $\langle a,b\rangle = a_1b_1+ a_2b_2+ a_3b_3$. **[5]**

5. (a) Determine whether the given matrix is orthogonal. If it is, then find its inverse

$$\begin{pmatrix} 1/3 & 1/2 & 1/5 \\ 1/3 & -1/2 & 1/5 \\ -1/3 & 0 & 2/5 \end{pmatrix}$$

(b) If **Q** be a 2x2 orthogonal matrix, then **Q** must have the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ or $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$