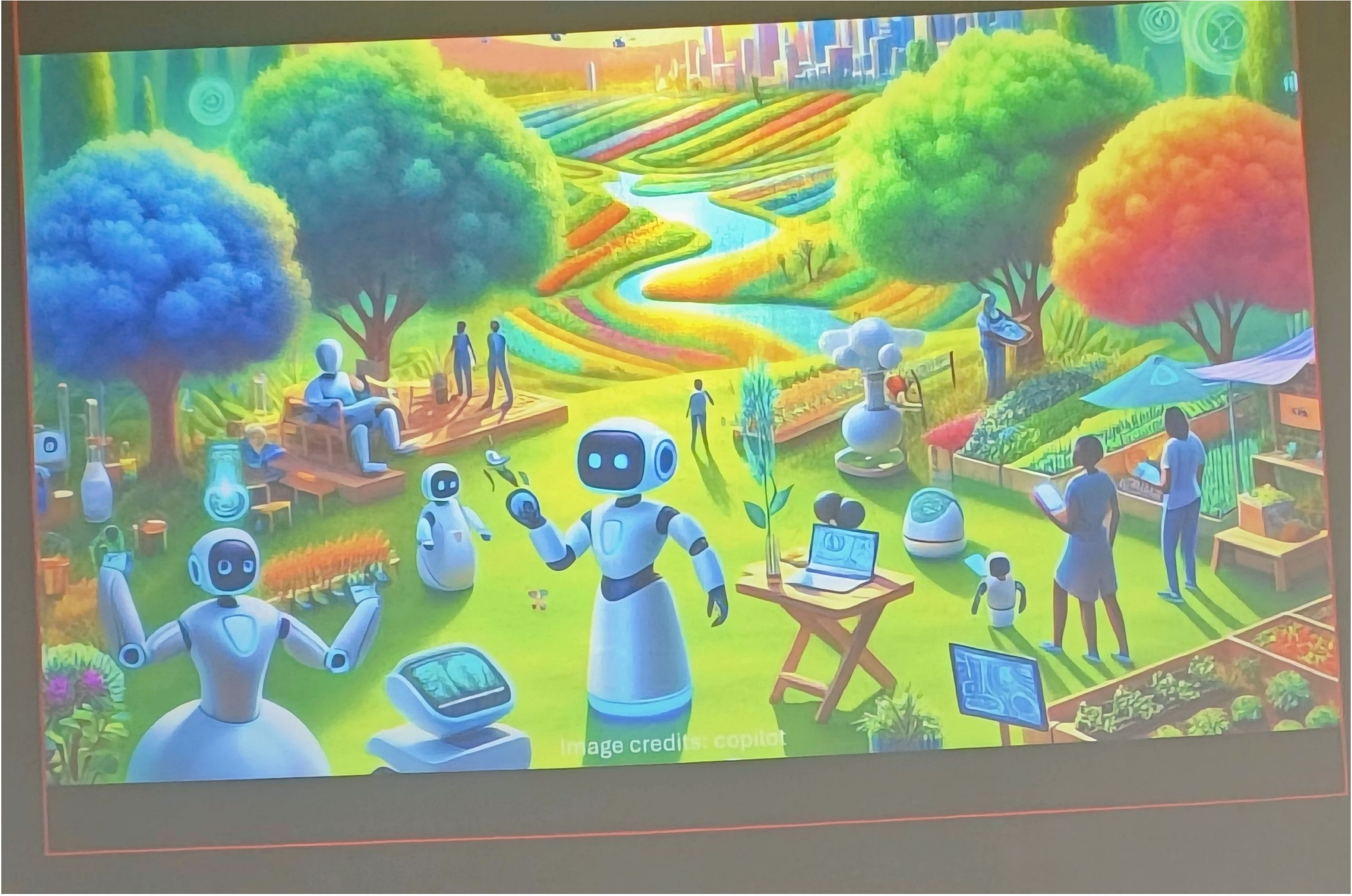
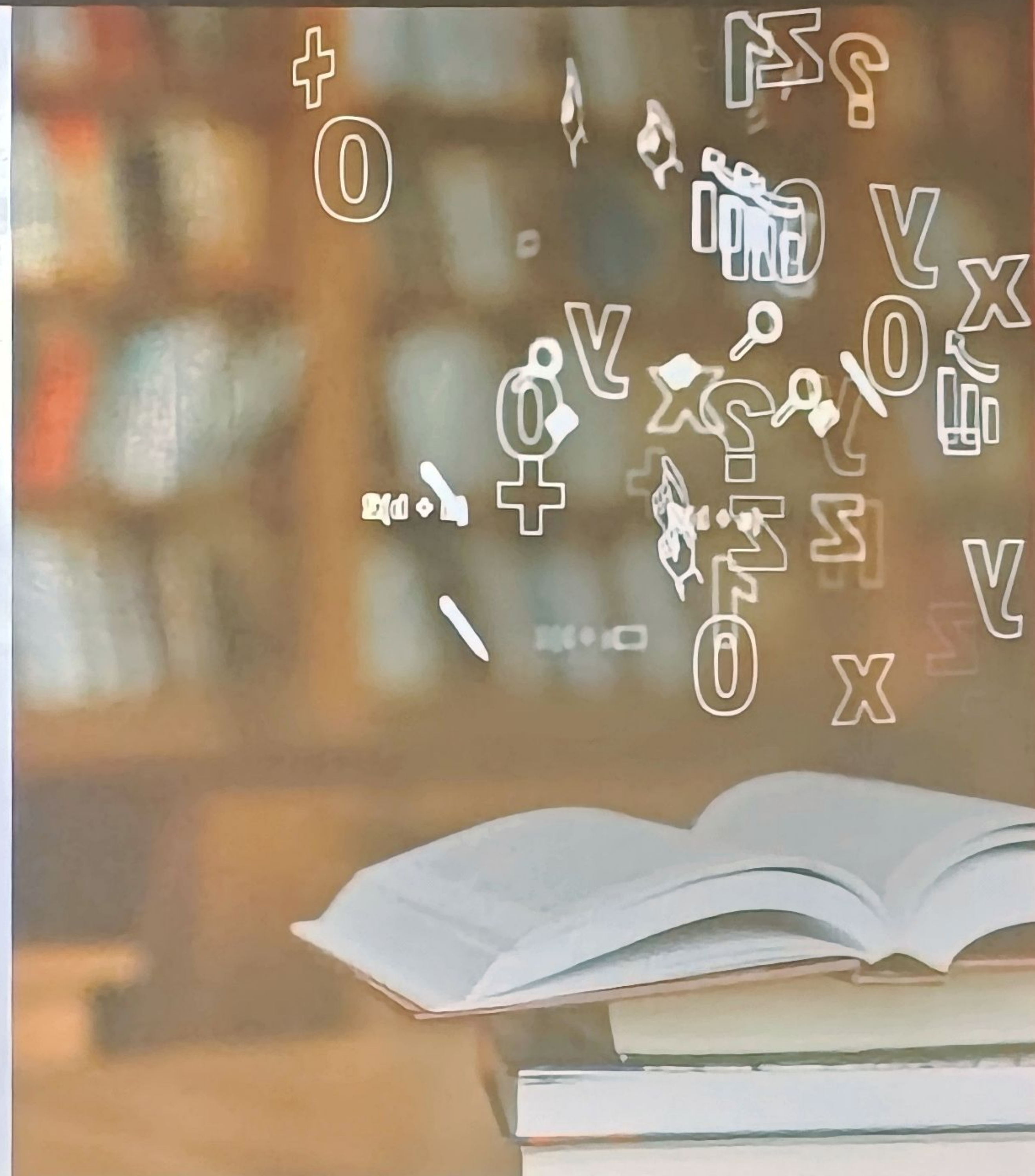


Image credits: copilot

Knowledge Base

- Knowledge Base – Set of sentences represented in knowledge representation language

- Knowledge representation language– Expressing knowledge explicitly in a computer-tractable way
  - (logic)



Formal vs Informal

Formal	Informal
Uses formal language	Natural language
Study of logical truths	Studies informal fallacies, Critical Thinking,…
Symbolic logic, mathematical logic,	

Why formal languages

Natural languages exhibit ambiguity

The boy saw a girl with a telescope

Our shoes are guaranteed to give you a fit

Ambiguity makes reasoning difficult / incomplete

Formal languages

promote rigour and thereby reduce possibility of human error

help reduce implicit / unstated assumptions by removing familiarity with subject matter

help achieve generality due to possibility of finding alternative interpretations for sentences and arguments.

Formal logic

- **Formal logic**: Uses syllogisms to make inferences, and examines how conclusions follow from premises based on the structure of arguments
  - **Mathematical logic**: Uses mathematical symbols to prove theoretical arguments
  - **Symbolic logic**: Uses symbols to accurately map out valid and invalid arguments.
  - **Propositional Logic**

Why is it important?

- Core of AI
  - Possibility of *automating* reasoning
    Reasoning: draw inferences from knowledge
  - answer queries
  - discover facts that follow from the knowledge base decide what to do etc.
- In AI, propositional logic is essential for knowledge representation, reasoning, and decision-making processes

Impact of Logic in AI

Technology | CYBERTIMES

December 10, 1996

Computer Math Proof Shows Reasoning Power

By GINA KOLATA

Computers are whizzes when it comes to the grunt work of mathematics. But for creative and elegant solutions to hard mathematical problems, nothing has been able to beat the human mind. That is, perhaps, until now.

Credit: [illegible] / The New York Times

Dr. William McCune at Argonne Labs, Illinois in his office with computer. The "Proof of Robbins Conjecture" problem is on the screen.

A computer program written by researchers at Argonne National Laboratory in Illinois has come up with a major mathematical proof that would have been called creative if a human had thought of it. In doing so, the computer has, for the first time, got a toehold into pure mathematics, a field described by its practitioners as more of an art form than a science. And the implications, some say, are profound, showing just how powerful computers can be at reasoning itself, at mimicking the flashes of logical insight or even genius that have characterized the best human minds.

Computers have found proofs of mathematical conjectures before, of course, but those conjectures were easy to prove. The difference this time is that the computer has solved a conjecture that stumped some of the best mathematicians for 60 years. And it did so with a program that was designed to reason, not to solve a specific problem. In that sense, the program is very different from chess-playing computer programs, for example, which are intended to solve just one problem: the moves of a chess game.

"It's a sign of power, of reasoning power," said Dr. Larry Wos, the supervisor of the computer reasoning project at Argonne. And with this result, obtained by a colleague, Dr. William McCune, he said, "We've taken a quantum leap forward."

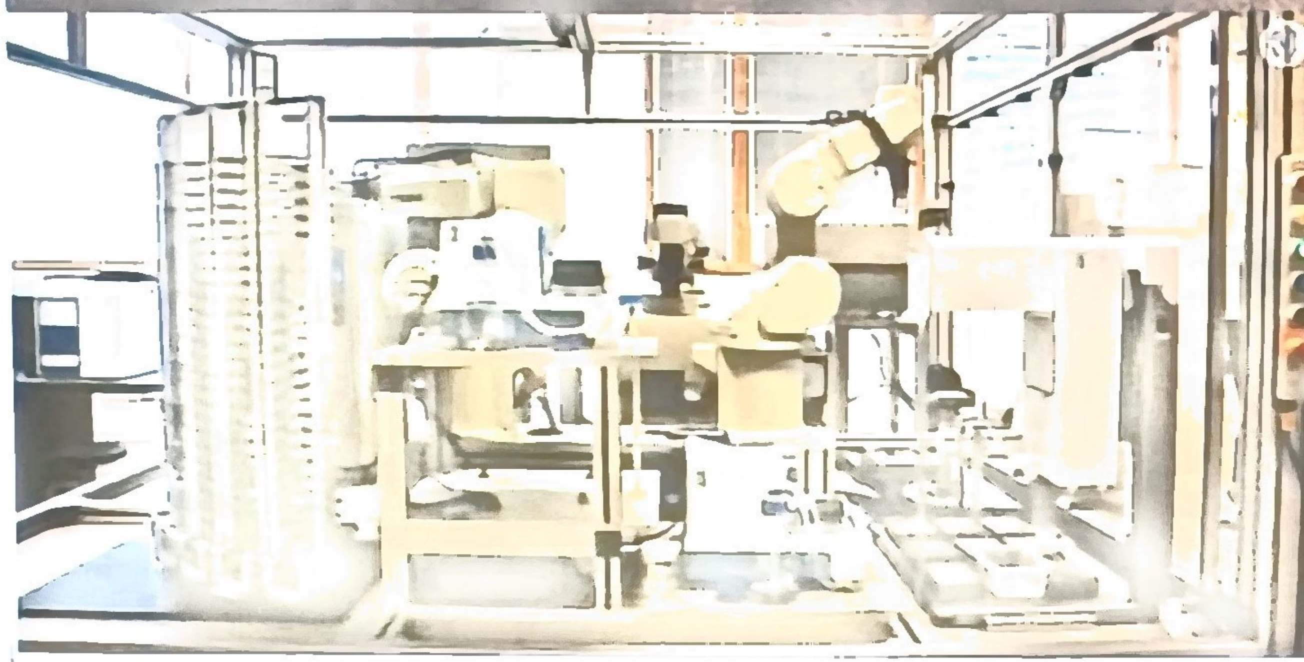
Wos predicts that the result may mark the beginning of the end for mathematics research as it is now practiced, eventually freeing mathematicians to focus on discovering new conjectures, and leaving the proof to computers.

But the result also may challenge the very notion of creative thinking, raising the possibility that computers could take a parallel path to reach the same

- https://archive.nytimes.com/www.nytimes.com/library/cyber/week/1210math.html?pagewanted=all

Impact of Logic in AI

Artificially-intelligent Robot Scientist 'Eve' could boost search for new drugs



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Logical Arguments

- (A) All humans have 2 eyes.
- (B) Sujit is a human.
  - Therefore (P) Sujit has 2 eyes.

- (C) All humans have 4 eyes.
- (B) Sujit is a human.
  - Therefore (Q) Sujit has 4 eyes.

- Which of P and Q are true / false ?
- Is deducing P from A and B correct? Q from B and C?

- All humans have 2 eyes.
- Kishore has 2 eyes.
  - Therefore (P) Kishore is a human.

- No human has 4 eyes.
- Kishore has 2 eyes.
  - Therefore (Q) Kishore is not human.

- Which of P and Q are true / false ?

- All humans have 2 eyes.
- Kishore has 2 eyes.
  - Therefore (P) Kishore is a human.

- No human has 4 eyes.
- Kishore has 2 eyes.
  - Therefore (Q) Kishore is not human.

- Which of P and Q are true / false ?
- Is deducing P correct? Q?

- ...fallacy conclusion may be correct. The reasoning is incorrect

Propositional Logic

Deals with propositions which are true or false

Also known as propositional calculus

Zero-order logic

Foundations for first order and higher order logics

Order of 'Logic'

- The order of a logic refers to the degree of quantification that can be performed over sets:
  - **First-order** logic: Quantifies only over individuals. It is also known as predicate logic, predicate calculus, or quantificational logic.
  - **Second-order** logic: Quantifies over sets.
  - **Third-order** logic: Quantifies over sets of sets.
  - **Higher-order** logic: The union of first-, second-, third-, and higher-order logic. It allows quantification over sets that are nested arbitrarily deeply.

Propositional Logic

- Propositions are Declarative Statements
- Atomic Propositions:
  - Simple, indivisible statements, cannot be broken down further
  - Each atomic proposition represents a basic fact or condition
  - Example: "The door is closed."
- Compound Propositions:
  - Multiple atomic propositions can be combined using logical connectives (like AND, OR, NOT) to create compound propositions.
  - Example: "The door is closed AND the heater is on."

Excercise

- Which of the following are propositions
  - (P) Today is Wednesday
  - (Q) It is raining today
  - (R) It will be raining tomorrow
  - (S) Close the door
  - (S) $2 + 7 = 9$
  - (T) $3+9 = 10$
  - (U) $X+2 = 1$

Logical connections

AND ($\wedge$) conjunction	S: P AND Q S: P ^ Q	S is true if both P and Q are true
OR (V) disjunction	S: P OR Q S: P V Q	S is true if any of P , Q is true
NOT ($\neg$): Negation.	S: $\neg$P	S is true only if P is false
IMPLIES ($\rightarrow$)	S: P $\rightarrow$ Q S: $\neg$P V Q	S is true if P implies Q
IFF ($\leftrightarrow$)	S: P $\leftrightarrow$ Q S: $\neg$P XOR Q	S is true if P and Q are true or false together

$P \rightarrow Q$

The only time $P \rightarrow Q$ evaluates to False is when

- P is True and Q is False

If $P \rightarrow Q$ is True, then:

- P is a sufficient condition for Q
- Q is a necessary condition for P

$P \leftrightarrow Q$

$P \rightarrow Q$

$Q \rightarrow P$

If $P \Leftrightarrow Q$ is true

- P and Q are equivalent
- P is necessary and Sufficient for Q
- Q is necessary and Sufficient for P

Truth Table

- A truth table is a breakdown of all the possible truth values returned by a logical expression

- Write down truth tables for, P, Q, $\neg P$, , $P \vee Q$, $P \rightarrow Q$, $P \leftrightarrow Q$, $\neg P \vee Q$, $\neg P$ XOR Q

  - One row for each possible assignment of True/False to propositional variables
  - Important: Above P and Q can be any sentence, including complex sentences

Exercise

Given: A and B are true; X and Y are false, determine truth values of:

$\neg(A \vee X)$

$A \vee (X \wedge Y)$

$A \wedge (X \vee (B \wedge Y))$

$[(A \wedge X) \vee \neg B] \wedge \neg[(A \vee X) \vee \neg B]$

$(X \wedge Y) \wedge (\neg A \vee X)$

$[(X \wedge Y) \to A] \to [X \to (Y \to A)]$

Some Terminology

- A sentence is valid if it is **True** under all possible assignments of True/False to its propositional variables (e.g. $P \vee \neg P$)
- Valid sentences are also referred to as **tautologies**
- SAT: A sentence is satisfiable if and only if there is some assignment of True/False to its propositional variables for which the sentence is True
- A sentence is unsatisfiable if and only if it is not satisfiable (e.g. $P \wedge \neg P$)

Towards Formal Proof

- $\alpha \Rightarrow \beta$ — whenever all the formulae in the set α are True, β is True
- This is a *semantic* notion; it concerns the notion of *Truth*
- To determine if $\alpha \Rightarrow \beta$ construct a truth table for α, β
  $\alpha \Rightarrow \beta$ if, in any row of the truth table where all formulae of α are true, β is also true

Modus Ponens

- Write Truth Table for P, P → Q
- (and what can you conclude about Q)
- Therefore, P, P→Q ⇒ Q
- α : P ∧ (P→Q), β: Q