1. Consider the modified logistic model

$$\frac{dN}{dt} = rN\left(1 - \frac{N^{\alpha}}{K}\right), \qquad r > 0,\ K > 0,\ \alpha > 0.$$

Find all fixed points of the system. [2]

2. For a scalar ordinary differential equation $\dot{x} = f(x)$, linearise the system about a fixed point x^* and derive the condition for stability in terms of $f'(x^*)$. [2]

3. Apply the result from part (2) to classify the fixed points of the modified logistic model obtained in part (1). [2]

4. Explain briefly why the population does not remain constant for an arbitrary nonzero initial condition $N(0)$, unless $N(0)$ is an equilibrium point. [1]

5. Write pseudocode to simulate the continuous modified logistic equation using the forward Euler method. (No derivation of the Euler method is required.) [2]

6. Write an algorithm or pseudocode for solving the linear system

$$Ax = b,$$

when A is an upper-triangular matrix, assuming that a unique solution exists. [2]

7. Consider the discrete map

$$x_{n+1} = 2.5\,x_n(1 - x_n), \qquad n = 0, 1, 2, \ldots,$$

with initial condition $x_0 = 0.3$. Explain what happens to x_n as n becomes very large. [2]

8. Write pseudocode to approximate the derivative of $f(x) = \sin x$ at the point $x_0 = \frac{\pi}{4}$, using a finite-difference method. [2]

9. What type of linear transformation preserves the Euclidean norm in linear least-squares problem
Prove your claim.

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