1 Place your Permanent / Temporary Student ID card on the desk during the examination for verification by the Invigilator.
2 Reading material such as books (unless open book exam) are not allowed inside the examination hall.
3 Borrowing writing material or calculators from other students in the examination hall is prohibited.
4 If any student is found indulging in malpractice or copying in the examination hall, the student will be given 'F' grade for the course and may be debarred from writing other examinations.

$Q^T A Q = D$

Best of Luck

$Q^T A^T Q =$

SECTION A

1. Answer the following questions: **(True /False)**

~~(a)~~ For a limear transform **T:** $V \to V$, dim (**Ker(T)**)-dim(**Im (T)**) = dim (**V**) **--True /False**
~~(b)~~ All Hermitian matrices are unitary **--True /False**
~~(c)~~ If a square matrix **A** has a zero row, then **detA=1--True /False**
~~(d)~~ If **A** is a Hermitian matrix, then the eigen values of **A** are complex**--True /False**
~~(e)~~ The identity matrix of any order is positive definete**--True /False**
~~(f)~~ Singular values of orthogonal matrix are always equal to **1--True /False**
~~(g)~~ The right singular vector represents perpendicular distance from the data point to the best fit line **--True /False**
~~(h)~~ If A is orthogonally diagonalizable, then A is skew symmetric **--True /False**
~~(i)~~ Any set of **m** vectors in $\mathbf{R}^n$ is linearly dependent if **m>n --True /False**
~~(j)~~ If the matrix A is invertible, then the system of equation **Ax=0** has only trivial solution**--True /False** **[1x10=10]**

SECTION B

2. Answer the following questions:

~~(a)~~ If **A** is similar to **B**, then show that $\mathbf{A}^T$ is similar to $\mathbf{B}^T$.

~~(b)~~ Find the conjugate transpose of the matrix **A**= $\begin{pmatrix} 1+i & -i & 1+5i \\ 1 & 4-i & 11 \\ 3+7i & -9i & 4-3i \end{pmatrix}$

~~(c)~~ Find the inverse of the elementary matrix $\begin{pmatrix} 1 & 0 & 0 \\ -3/5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

~~(d)~~ Prove that the distance **d(u,v)**= $\sqrt{\|u\|^2+\|v\|^2}$ if and only if **u** and **v** are orthogonal.

~~(e)~~ Is the matrix **A**= $\begin{pmatrix} 1 & 1-i & 0 \\ 1+i & 1 & i \\ 0 & -i & 1 \end{pmatrix}$ Hermatian? Justify

[5x2=10]

SECTION C

3. a) Find the orthogonal diagonalization of the following matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ b) Determine wether the following is linear transformationor not **T:** $\mathbf{R}^2 \to \mathbf{R}^2$ defined by **T** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-y \\ x+2y \end{pmatrix}$

[7+3=10]

4. a) Find the pseudo inverse of the matrix **A**= $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ b) Find the symmetric matrix **A** associated with given quadratic form $5a^2 - b^2 + 2c^2 + 2ab - 4ac + 4bc$. c) Let **B** be an invertible matrix, show that $\mathbf{B}^T\mathbf{B}$ is posotive definite. **[4+2+4=10]**

5. a) Compute **A**= $\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}^k$ b) Diagonalize the matrix **M**= $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ **[4+6=10]**

6. Find the eigen value, eigen vector, the charecterastic plynomial, geometric multiplity and algebric multiplicity of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ -2 & 1 & 2 & -1 \end{pmatrix}$. b) Find out whether the following set of vectors spans $\mathbf{R}^3$ or not: S={ $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ } **[(2 .5+ 2 .5+1+1+1)+2=10]**

7. a) Find the **QR** factorization of the matrix $\begin{pmatrix} 2 & 8 & 2 \\ 1 & 7 & -1 \\ -2 & -2 & 1 \end{pmatrix}$ b) Show that W= $\begin{pmatrix} a \\ -a \\ 2a \end{pmatrix}$ is a subspace of the vector space $\mathbf{R}^3$ with respect to standard vector addition and scalar multiplication in $\mathbf{R}^3$. c) Prove that every vector space has an unique zero vector. **[5+3+2=10]**

8. a) Find the outer product form of the **SVD** of the matrix **A**= $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}_{2\times 3}$ b) If **A** is a **nxn** matrix, show that **adj A** is also invertible and $(\mathbf{adj\ A})^{-1} = \mathbf{A/det\ A = adj\ (A^{-1})}$ **[7+3=10]**

$A^{-1} = \dfrac{\text{adj}\,A\,A}{\det A}$

$(\text{adj}\,A)^{-1} = A/\det A$

$3\ 0\ 1$ $\begin{pmatrix} -1 & 2 & -3 \\ -1 & -2 & 3 \\ -1 & -4 & 3 \end{pmatrix}$

$-3-1$

$3(-1)+16(-1-2)$

$-3-3$ -6

$2(-1)+1(-3-1)$

$-2-4=-6$

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Best of Luck

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9. a) Apply Gram Schmidt process to obtain the othonormal basis from the set of vectors{ $\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$,

$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$ }in $\mathbf{R}^4$. ~~b)~~ Consider the set $\mathbf{M}_{nxn}$ **(R)** is the set of all real square matrices. Find out whether **<A,B>** = **Tr(AB^T)** for all $A,B\in M_{nxn}(R)$ is an inner product or not. **[5+5=10]**

10. ~~a)~~ Find a unitary matrix **U** and a diagonal matrix **D** such that **U*AU=D** for **A=**$\begin{pmatrix}-1 & 1+i\\1-i & 0\end{pmatrix}$, ~~b)~~ Solve the following system of equations by Cramer's rule **2x+y+3z=1, y+z=1,z=1. [7+3=10]**

Note: R is the set of Real numbers and C is the set of complex numbers

---END---

$+2$

$\begin{matrix}1 & 1+i\\ 1-i & 2\end{matrix}$

$x+(1+i)y=0$

$i)x+2y=0$

$x=-(1+i)y$

$(i-1)(i+1)y$

i^2

$2y$

$2-(1-i)(i+i)$

$2-(1+2)$

$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ $\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$

$(\bar{U})^T U = I$

$\begin{pmatrix}\frac{1-i}{2} & \phi\\ 0 & 0\end{pmatrix}\begin{pmatrix}\frac{1+i}{2} & 0\\ 1 & 0\end{pmatrix}$

$\frac{(1-i)(1+i)}{2}+1 \quad 0$

0

$\begin{matrix}1 & 1+i\\ 1-i & 2\end{matrix}$

$\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$