

Recursion

STEPS:

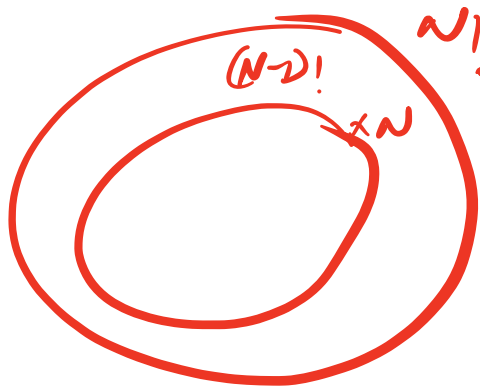
1. Assumption: Decide what your fⁿ should do!
& then assume that it does it!
2. Main Logic: Solve the problem using the solutions of the subproblem!

3. Base Case:

Q Given N. Calculate N! [N ≥ 0]
N = 5 : 5! = 5 × 4 × 3 × 2 × 1 = 120

$$N! = 1 \times 2 \times 3 \times \dots \times (N-1) \times N$$

$$\cancel{N!} = \cancel{(N-1)!} \times N$$

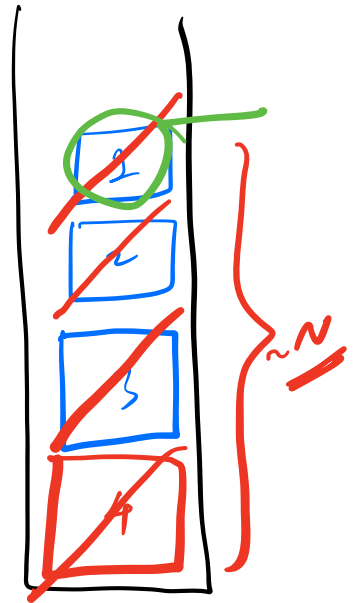
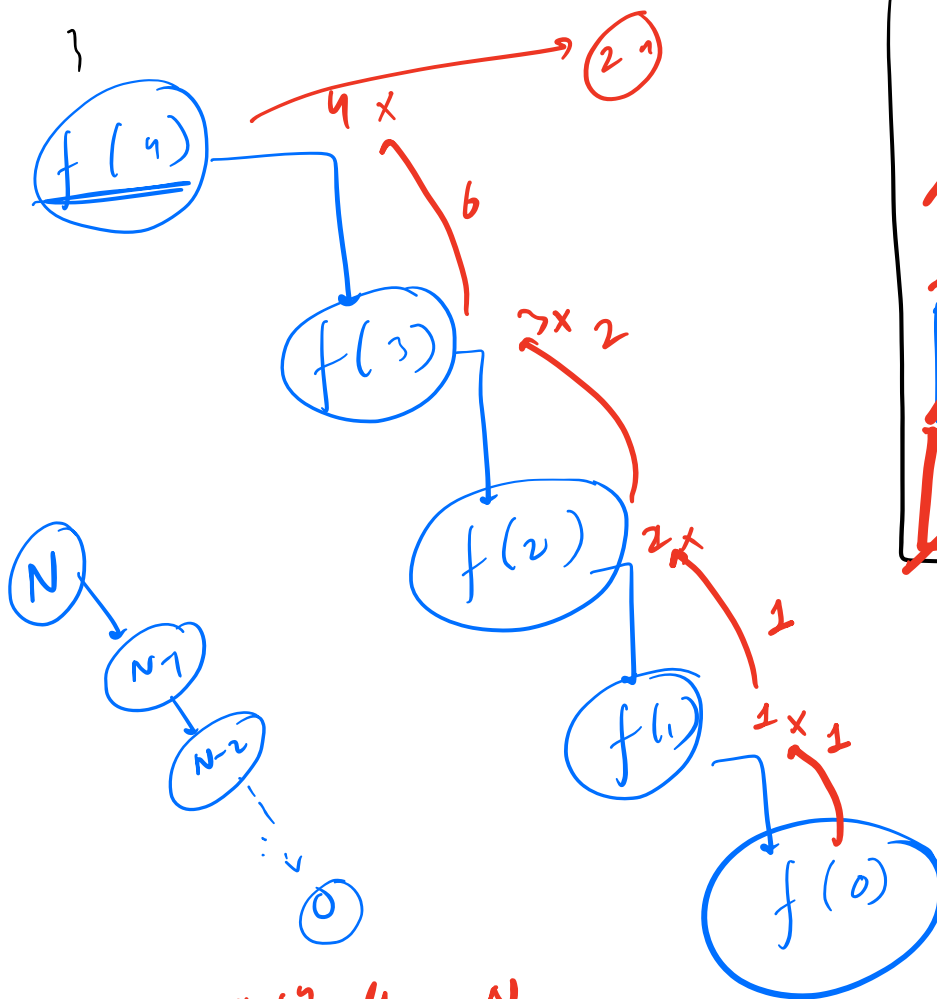


```

int fact(N) {
  // Ass: return N!
  if (N == 0) return 1;
  return fact(N-1) * N;
}

```

$$N! = (N-1)! \times N$$

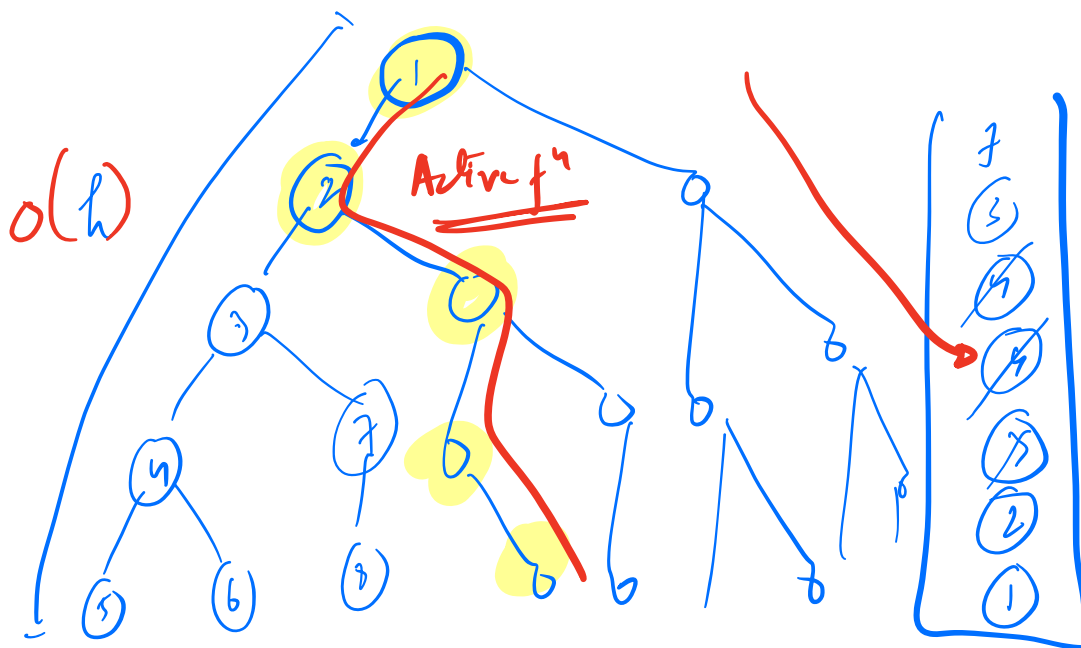
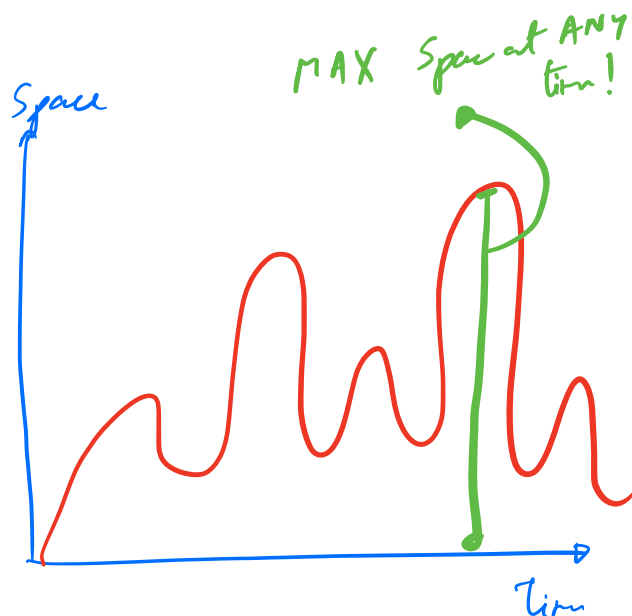


f^{th} calls = N
 time taken / f^{th} call $\rightarrow O(1)$

$$TC = O(N)$$

$$TC = \# f^n \text{ calls} \times \text{time taken} / f^n \text{ calls}$$

$$SC = O(N)$$



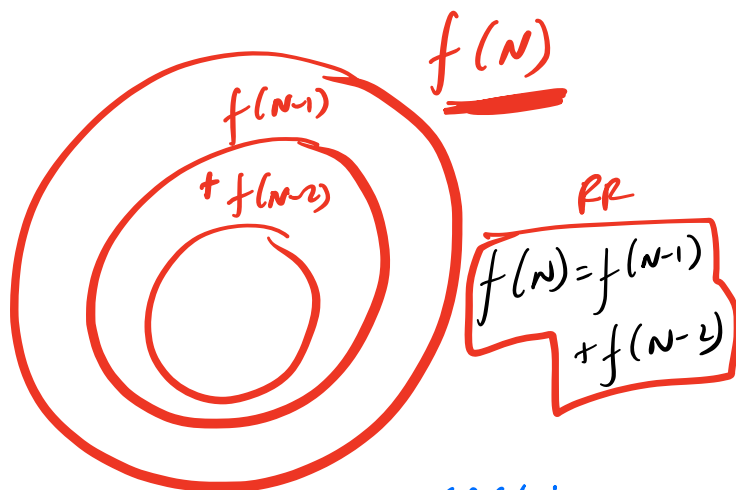
Fibonacci No

1	2	3	4	5	6	7	8	9	10	...
0	1	1	2	3	5	8	13	21	34	...

Arrows indicate: $N-2$ from 5 to 3, $N-1$ from 6 to 5. The value 8 at index 7 is highlighted in yellow.

Given N find N^{th} fib No!

```
int f(N) {
  // Ans: ret  $N^{\text{th}}$  fib. No.
  if ( $N \leq 2$ ) ret  $N-1$ ;
  ret  $f(N-1) + f(N-2)$ ;
}
```



BASE CASE:

1) Cases Not defined by R.R.

✓ $f(2) = f(1) + f(0)$

✓ $f(1) = f(0) + f(-1)$



$TC = O(2^N)$

$N = 20$

$\sim 2^{20}$

$\sim 10^6$

SC

$$SC = O(N)$$

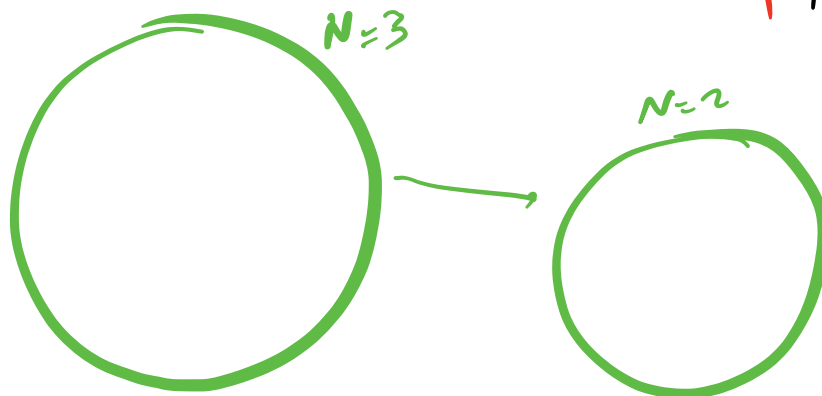
Q Given N . Generate all binary strings of len N .
 2^N

$N=3$

- "000"
- "001"
- "010"
- "011"
- "100"
- "101"
- "110"
- "111"

$N=2$

- 00
- 01
- 10
- 11
- 00
- 01
- 10
- 11



`List<string> gen(N) {`
// Ass: Ret the list of the N first binary strings!
// N > 0
`if (N == 1) { ret {"0", "1"}; } N = 3`

`List<string> p = gen(N-1);`

`List<string> ans;`

`f(s: p) {`
`ans.add("0"+s);`

`}`

`f(s: p) {`
`ans.add("1"+s);`

`}`

`ret ans;`

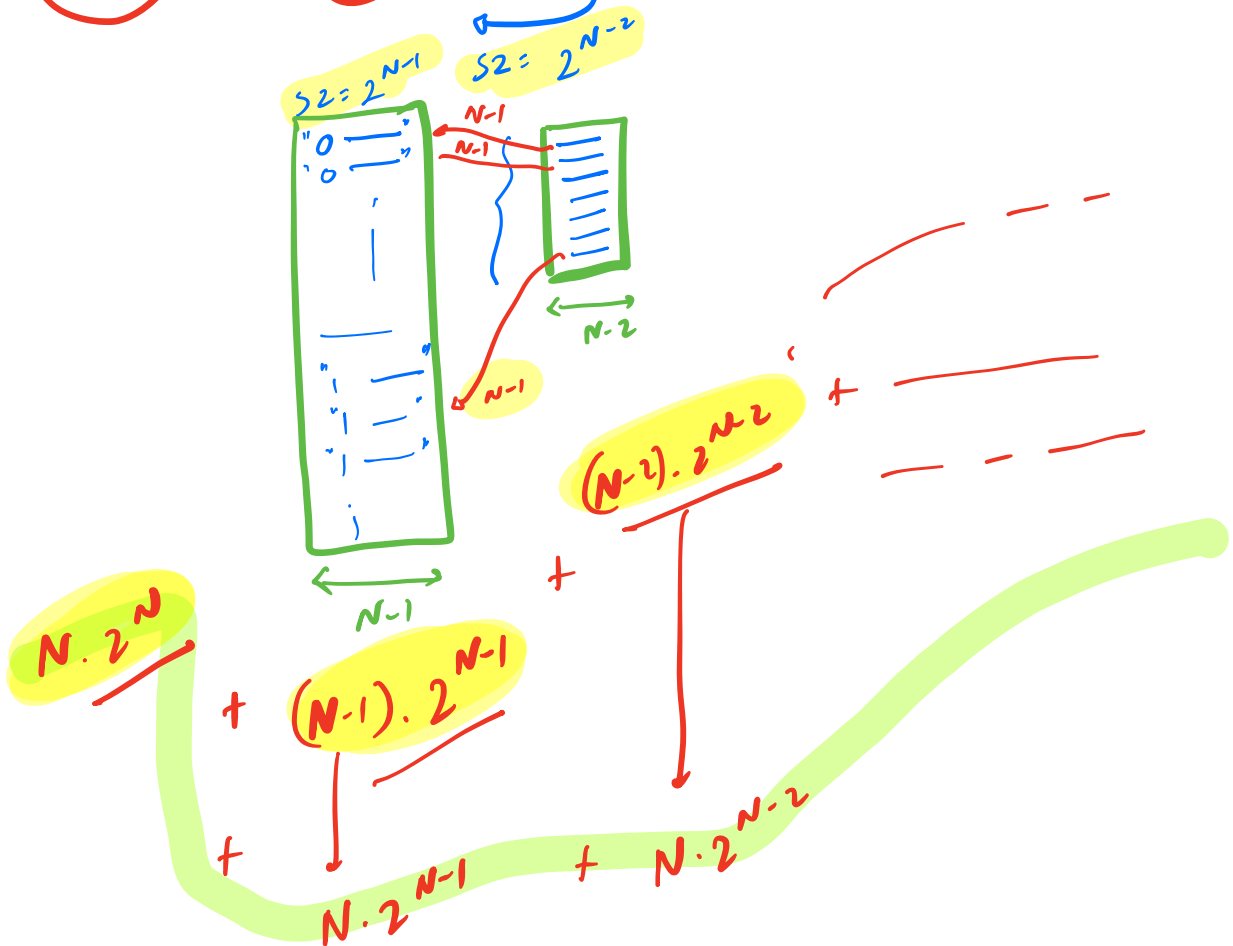
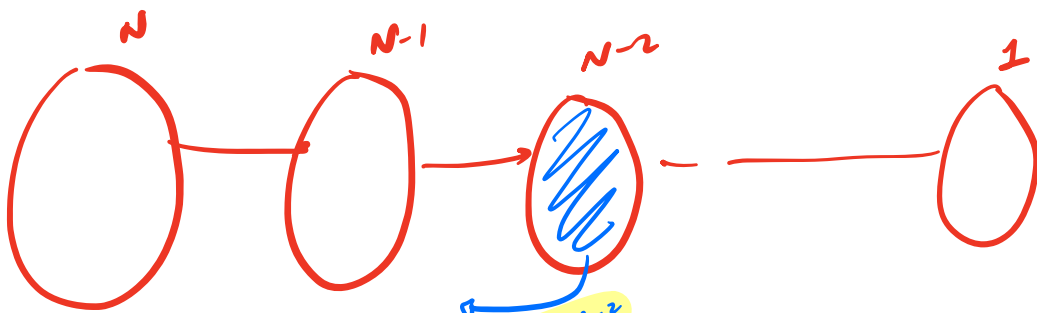
`}`

\uparrow
`<00`
`01`
`10`
`11>`

N = 1

`<"0"`
`"1">`

`if (N == 0)`
`ret {""};`



$$N [2^N + 2^{N-1} + \dots + 2^0]$$

$$N \cdot [2^{N+1} - 1]$$

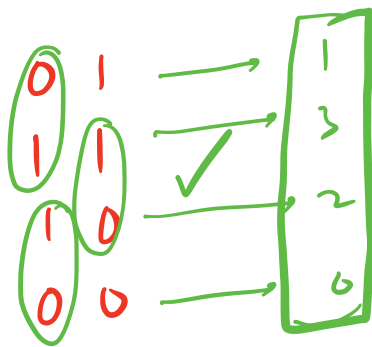
[TC]

$O(N \cdot 2^N)$

✓

1 0
1 1
0 1
0 0

|



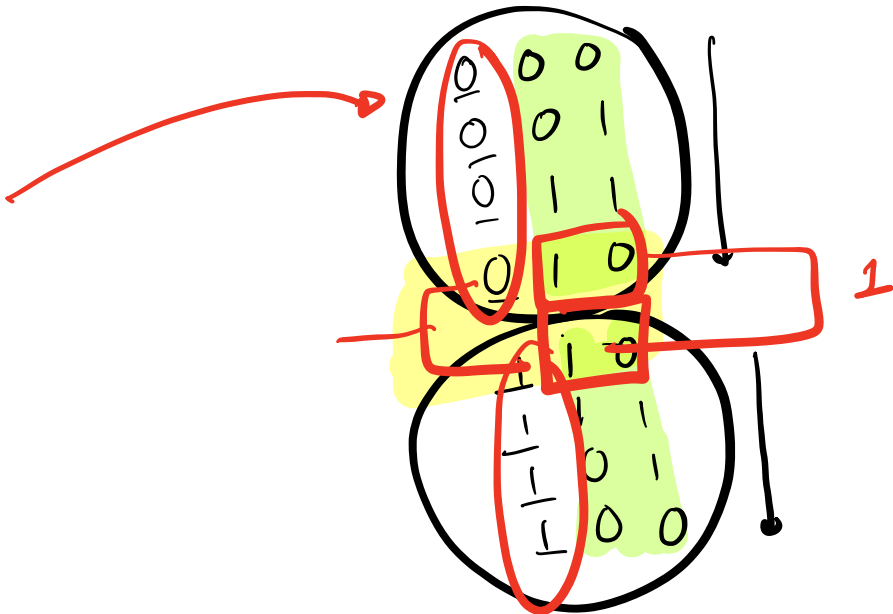
N=3

0 0 0
0 0 1
0 1 1
0 1 0
1 1 0
1 0 1
1 0 0

N=2

0 — 0 0
1 — 0 1
3 — 1 1
2 — 1 0

N=3



0
1
3
2
6
7
5
4

```
List <int> gen(N) {
    if (N == 1) { ret (0, 1); }
```

$list<int> p = giv(N-1);$

List $\langle \text{int} \rangle$ as i

```
f(s: p) {  
  arr.add(s);  
}
```

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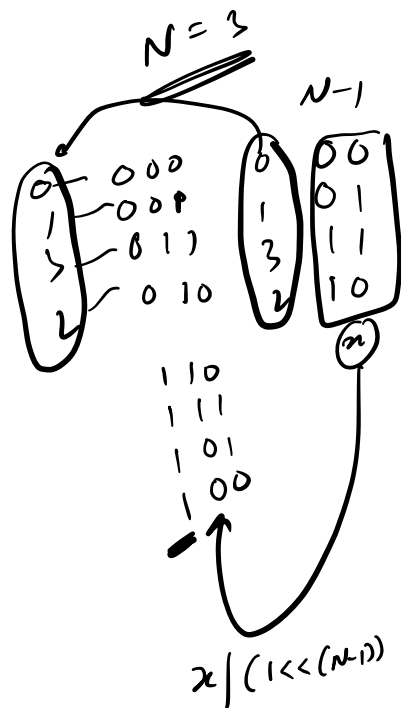
$$f(s: \text{row}(p)) \{$$

$$\text{any.add}(s \mid (1 \leq (N-1))) ;$$
 γ

yet any;

)

N70



TL / SL ?