$$2^0 + 2^1 + 2^2 + \cdots + 2^K = 2^{K+1} - 1$$

Sum of GP $\longrightarrow$ $a\left[\frac{r^t - 1}{r-1}\right]$

$a = 2^0 = 1$

$r = 2$

$t = K+1$

$$1\left[\frac{2^{K+1} - 1}{2-1}\right] \rightarrow 2^{K+1} - 1$$

$$1 + 2 + 4 + 8 + 16 + 32 = 64 - 1$$

④ Number Systems

1) Decimal No. System

Symbols: $\{0, 1, 2 \ldots 9\}$

Base = 10

$$\overset{3}{3}\ \overset{2}{4}\ \overset{1}{6}\ \overset{0}{2}$$

$$3 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 2 \times 10^0$$

2) Binary No Systems → Symbols $\{0, 1\}$

Base : 2

BIN → DEC

$$(\overset{4}{1}\ \overset{3}{0}\ \overset{2}{1}\ \overset{1}{0}\ \overset{0}{1})_2$$

$$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 2^4 + 2^2 + 2^0$$

$$16 + 4 + 1 = (21)_{10}$$

DEC → BIN

2	56	
2	28	0
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

$$(111000)_2 = (56)_{10}$$

$(54)_{10} = (\_\_\_\_\_)_2$

2	54	
2	27	0
2	13	1
2	6	1
2	3	0
2	1	1
	0	1

$(110110)_2$

BITWISE OP

&, |, ^, ~, <<, >>

AND, OR, XOR, TILDE <NEG>, Left SHIFT, Right SHIFT

A	B	A & B	A | B	A ^ B	~A	
0	0	0	0	0	1	
0	1	0	1	1	1	
1	0	0	1	1	0	
1	1	1	1	0	0	

a : 29 : 11101

b : 18 : 10010

a | b : .... 1 1 1 1 1 → $(31)_{10}$

a & b : ... 1 0 0 0 0 → $(16)_{10}$

a ^ b : ... 0 1 1 1 1 → $(15)_{10}$

```
int a = 29, int b = 18
int z = (a & b);
print(z) → 16
```

Left Shift (<<)

int a : 5

31 ... 3 2 1 0

0 0 0 ... 0 1 0 1 → $2^2 + 2^0$

a = (a << 1) x ... 0 1 0 1 0 → $2^3 + 2^1 = 2(2^2 + 2^0)$ ×2

a = (a << 1) x 0 0 ... 1 0 1 0 0 → $2^4 + 2^2 = 2(2^3 + 2^1)$ ×2

INT → 4B → 32 bits

LONG → 8B → 64 bits

$$\boxed{a << 1 = (a \times 2)}$$

$$a << 2 = a \times 2^2$$

$$\boxed{a << k = a \times 2^k}$$

$$\boxed{1 << k = 2^k}$$

2^{20}

int x = (1 << 20);

$a = 5$

a = (a << 1);

print(a) $\longrightarrow$ 10

Right Shift (>>)

	31					2	1	0		
$a = 5$	0	—	0	0	0	1	0	1	$\longrightarrow$	$2^2 + 2^0 = 5$
a = (a >> 1)	0	0	0	—	—	0	1	0	$\longrightarrow$	/2 $\quad 2^1 = 2$
a = (a >> 1)	0	0	0	0	—	—	—	1	$\longrightarrow$	/2 $\quad 2^0 = 1$

$$a >> 1 = \left\lfloor \frac{a}{2} \right\rfloor$$

$$\boxed{a >> k = \left\lfloor \frac{a}{2^k} \right\rfloor}$$

$$1 >> 10 = \left\lfloor \frac{1}{2^{10}} \right\rfloor \longrightarrow 0$$

find AVG:

a, b : $(a+b)/2$

$(a+b) >> 1$

Q Given N.
Check if ith bit is set or not!

set → 1

	…	4	3	2	1	0
N = 25	… 0	1	1	0	0	1

$\boxed{i : 2} \rightarrow$ F

$\boxed{i : 3} \rightarrow$ T

i = 3:

	5	4	3	2	1	0
N :	0	1	1	0	0	1
(1<<i) &	0	0	1	0	0	0
	0	0	1	0	0	0

> 0

i = 2:

	5	4	3	2	1	0
N :	0	1	1	0	0	1
& (1<<i)	0	0	0	1	0	0
	0	0	0	0	0	0

0

$\boxed{(N \,\&\, (1<<i))}$

- 0 : ith bit is UNSET
- > 0 : ith bit is SET

Q Given N. Set it's i^{th} bit !

$$N = 25 \qquad \begin{array}{cccccc} & 4 & 3 & \overset{i}{2} & 1 & 0 \\ & 1 & 1 & 0 & 0 & 1 \\ \mid & 0 & 0 & 1 & 0 & 0 \\ \hline & 1 & 1 & 1 & 0 & 1 \end{array}$$

$$N = 25 \qquad \begin{array}{cccccc} & 4 & \overset{i}{3} & 2 & 1 & 0 \\ & 1 & 1 & 0 & 0 & 1 \\ \mid & 0 & 1 & 0 & 0 & 0 \\ \hline & 1 & 1 & 0 & 0 & 1 \end{array}$$

$$\boxed{N = (N \mid (1 << i))}$$

Q Given N. toggle the i^{th} bit.

$1 \rightarrow 0$

$0 \rightarrow 1$

XOR

$0 \wedge 1 = 1$

$1 \wedge 1 = 0$

$$\begin{array}{cccccc} & & & i & & \\ & 1 & 1 & 0 & 0 & 1 \\ \wedge & 0 & 0 & 1 & 0 & 0 \\ \hline & 1 & 1 & 1 & 0 & 1 \end{array}$$

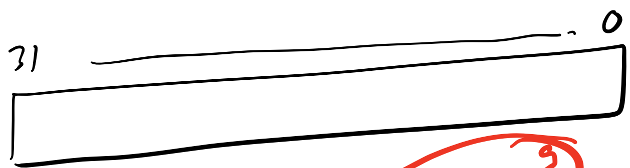
$$\boxed{N = (N \wedge (1 << i))}$$

Q Given N (int). Find the no. of SET bits!

N = 25 : 11001 → 3

N = 64 : 1000000 → 1

N:



$N = 10^9$

```
cnt = 0;
for (i = 0; i < 32; i++) {
    if ((N & (1<<i)) > 0) {
        cnt++;
    }
}
ret cnt
```

→ 32 → O(1)

N : 25 :	1	1	0	0	1	→	cnt++
N=(N>>1)	0	1	1	0	0	→	X
N=(N>>1)	0	0	1	1	0	→	X
N=(N>>1)	0	0	0	1	1	→	cnt++
N=(N>>1)	0	0	0	0	1	→	cnt++
N=(N>>1)	0	0	0	0	0	→	N==0 STOP

Check if 0^{th} bit is set?

(N & 1)

0^{th} bit is set → >0

unset → 0

```
cnt = 0
while( N > 0) {
    if( (N&1) > 0) cnt++;
    N = (N>>1);  // N = N/2
}
ret cnt;
```

$O(\log N)$

$2^5 + (2^3 - 1) \to 2^5 + \underbrace{2^2 + 2^1 + 2^0}$

N = 1 0 1 0 0 0 → 40

N-1 = & 1 0 0 1 1 1 → 39

1 0 0 0 0 0

N-1 will have all the bits including and after the last set bit of N flipped!

N = 1 1 1 0 1 0 → 58

N-1 = & 1 1 1 0 0 1 → 57

1 1 1 0 0 0

N = 1 1 1 1 1

N-1 = & 1 1 1 1 0

1 1 1 1 0

N = 1 0 0 0 0

N-1 = & 0 1 1 1 1

0 0 0 0 0

$2^4 - 1$

$2^0 + 2^1 + 2^2 + 2^3$

2^0 — — $2^k = 2^{k+1} - 1$

$2^0 + \text{—} + 2^3 = 2^4 - 1$

NOTE: $N = N \& (N-1) \rightarrow$ UNSET the last set bit (LSB)

$N = 11001$

$$
\begin{array}{rl}
N-1 \quad \& & 11000 \\
\hline
N \rightarrow & 11000
\end{array}
\rightarrow cnt{+}{+}
$$

$$
\begin{array}{rl}
N-1 \quad \& & 10111 \\
\hline
 & 10000
\end{array}
\rightarrow cnt{+}{+}
$$

$$
\begin{array}{rl}
N-1 \quad \& & 01111 \\
\hline
 & 00000
\end{array}
\rightarrow cnt{+}{+}
$$

$00000 \rightarrow N == 0$ STOP!

```
cnt = 0;
while (N > 0) {
    cnt++;
    N = (N & (N-1));
}
ret cnt;
```

$\boxed{O(\#\ 1\ \text{set bits})}$

Q. Given N. Check if it is a power of 2!

$N = 32$? $\longrightarrow$

$$\begin{array}{rr} & 100000 \\ \& & 011111 \\ \hline & 000000 \end{array}$$

$2^k \longrightarrow$ 1 set bit

```
if(N!=0 && (N & (N-1)) == 0)
   └──→ N is power of 2
else ──→ NOT a pow of 2
```

Q. Given N numbers. Every element repeats twice except 1 element, find that!

A = [3 2 3 4 6 4 2]

```
X = 0
f(i:0 ──→ N-1) {
    X = X ^ A[i].
}
print(X)
```

$\boxed{TC = O(N)}$

$\boxed{SC = O(1)}$

3	0	1	1
2	0	1	0
3	0	1	1
4	1	0	0
6	1	1	0
4	1	0	0
2	0	1	0

$\boxed{110}$

Q Given an Array. Every element repeats thrice except 1 element → occurs once. find this!

A : [5, 7, 5, 4, 7, 11, 11, 9, 11, 7, 5, 4, 4] → 9

1) BF : O(N²) O(1)

2) HM : O(N) O(N)

3) SORT O(N log N) O(1)

4)

	3	2	1	0
5	0	1	0	1
7	0	1	1	1
5	0	1	0	1
4	0	1	0	0
7	0	1	1	1
11	1	0	1	1
11	1	0	1	1
9	1	0	0	1
11	1	0	1	1
7	0	1	1	1
5	0	1	0	1
4	0	1	0	0
4	0	1	0	0

i

3 9 6 9 → sum of set bits NOT including ANOMALY

4 9 6 10 → Multiple of 3

%3

1 0 0 1 → 9

MAX(A) :

#bits

$\boxed{\log(\text{Max}(A))}$

$\boxed{TC = O(N \cdot \log(\text{Max}(A)))}$

$\boxed{SC = O(1)}$

N

A[]