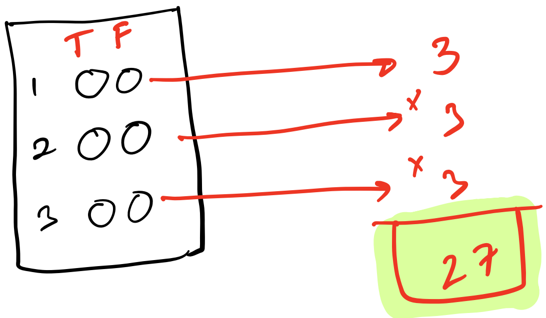
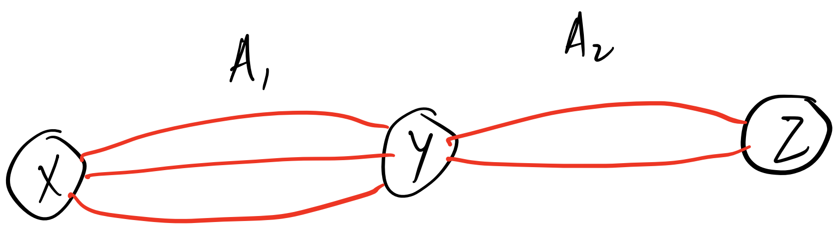


① Product Rule

If 2 events A_1 & A_2 can occur in N & M ways.
then, the no. of ways of doing A_1 & A_2 event

A_1 : N
A_2 : M



A_1
N : 3

A_2
M : 2

$3 \times 2 = 6$

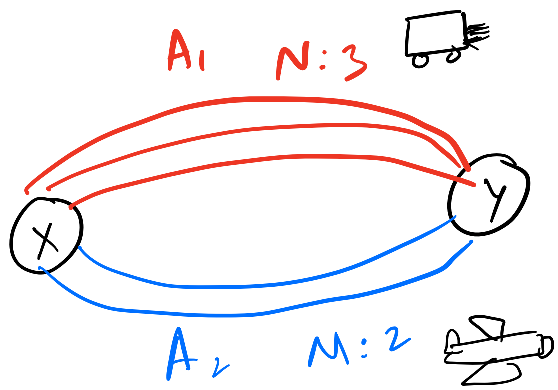
$\boxed{N \times M}$

Addition Rule

A_1 : N

A_2 : M

A_1 or A_2



$3 + 2 = 5$ ✓

$\boxed{N+M}$

Permutation

→ Act of arranging elements in a particular order!

$\boxed{(1,2) \neq (2,1)}$ → 2 diff Arrangemt

$(a, b, c) \longrightarrow$

$N = 3$

$$\left.\begin{matrix}(a, b, c)\\(a, c, b)\\(b, a, c)\\(b, c, a)\\(c, a, b)\\(c, b, a)\end{matrix}\right\} \rightarrow 6$$

(a, b, c)

$N = 3$

$$\underset{A_1}{\underline{3}} \times_{\text{AND}} \underset{A_2}{\underline{2}} \times_{\text{AND}} \underset{A_3}{\underline{1}} = 6$$

In general

$$\boxed{\text{\# of permutation of } N \text{ distinct items } = N \times (N-1) \times \cdots \times 1 = N!}$$

Q Given N distinct element. Select R elements from it & then permutate them. Find the # of ways!

(a, b, c, d, e)

N = 5

R = 2

ab ba
ac ca bd
db

$5 \times 4 = 20$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = \frac{5!}{3!}$$

$$= \frac{5!}{(5-2)!} = \frac{N!}{(N-R)!}$$

ab

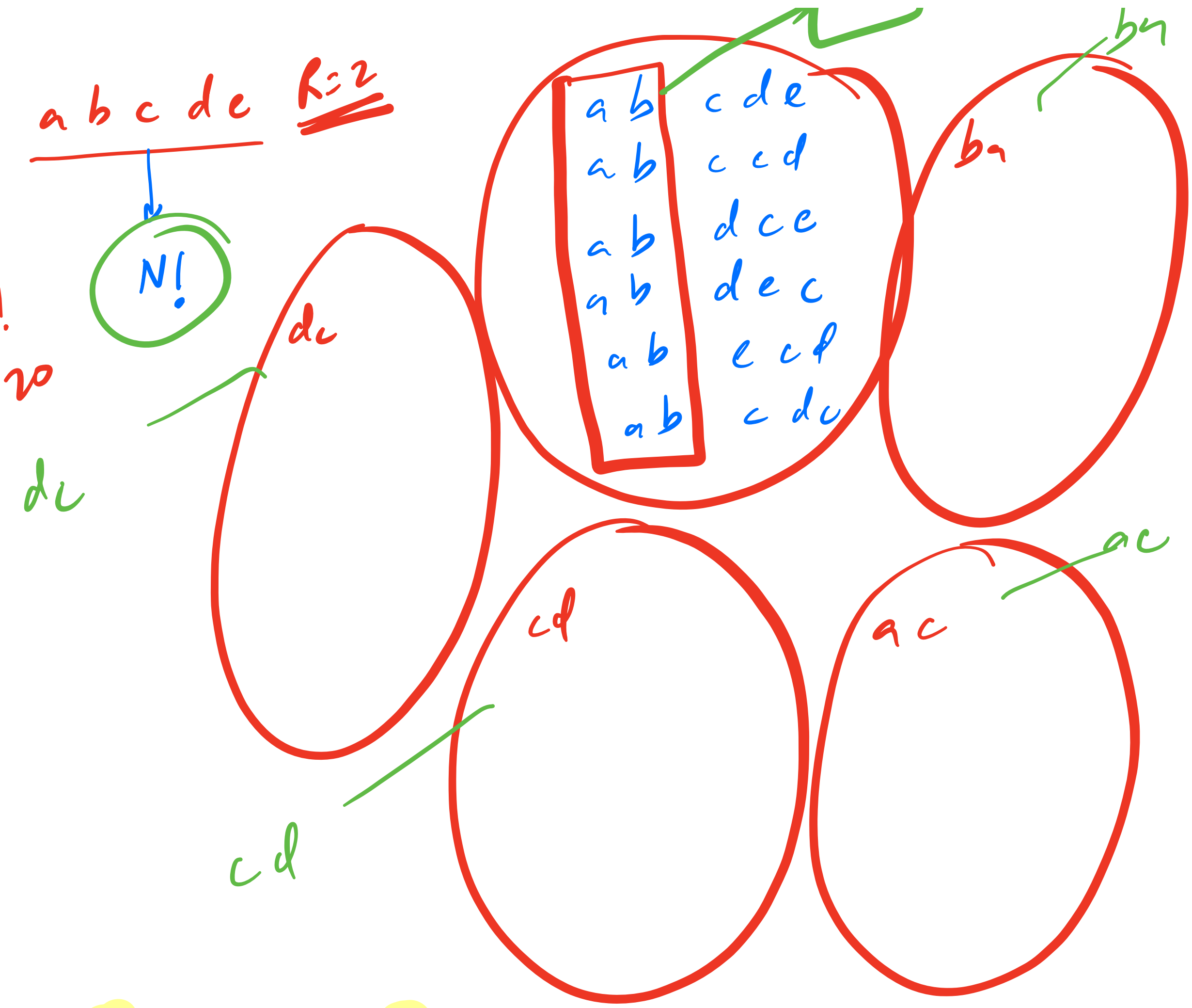
a b c d e $\underline{R=2}$

$N=5$

$5!$

$=120$

$N!$



$6 \rightarrow 1$

$120 \rightarrow 120/6$

$\rightarrow \dfrac{N!}{(N-R)!}$

$6 : (N-R)!$

$120 : N!$

$$\boxed{{}^{N}P_{R} = \frac{N!}{(N-R)!}}$$

① $S =$ "$a\ b_1\ b_2\ c$" # permutations?

$4! = N!$
$= 24$

$a\ b_1\ b_2\ c \rightarrow abbc$
$a\ b_2\ b_1\ c \rightarrow abbc$

$a\ c\ b_1\ b_2 \rightarrow acbb$
$a\ c\ b_2\ b \rightarrow acbb$

$b_1\ c\ b_2\ a \rightarrow bcba$
$b_2\ c\ b_1\ a \rightarrow bcba$

$2 \rightarrow 1$

$N! \rightarrow \frac{N!}{2}$

① $a\ b_1\ c\ b_2\ b_3$ $R = 3$

$3!$ ✓

$N = 5$

$5! = 120$

b_1	a	b_2	c	b_3
b_1	a	b_3	c	b_2
b_2	a	b_1	c	b_3
b_2	a	b_3	c	b_1
b_3	a	b_1	c	b_2
b_3	a	b_2	c	b_1

$b\ a\ b\ c\ b$

$6 \rightarrow 1$

$3! \rightarrow 1$

$5! \rightarrow \frac{5!}{3!} = \frac{N!}{R!}$

① $\boxed{\text{N items, R are repeating} \quad \text{\# permutations} = \frac{N!}{R!}}$

⊙ $S = a\ b_1\ c_1\ b_2\ c_2\ c_3\ d$

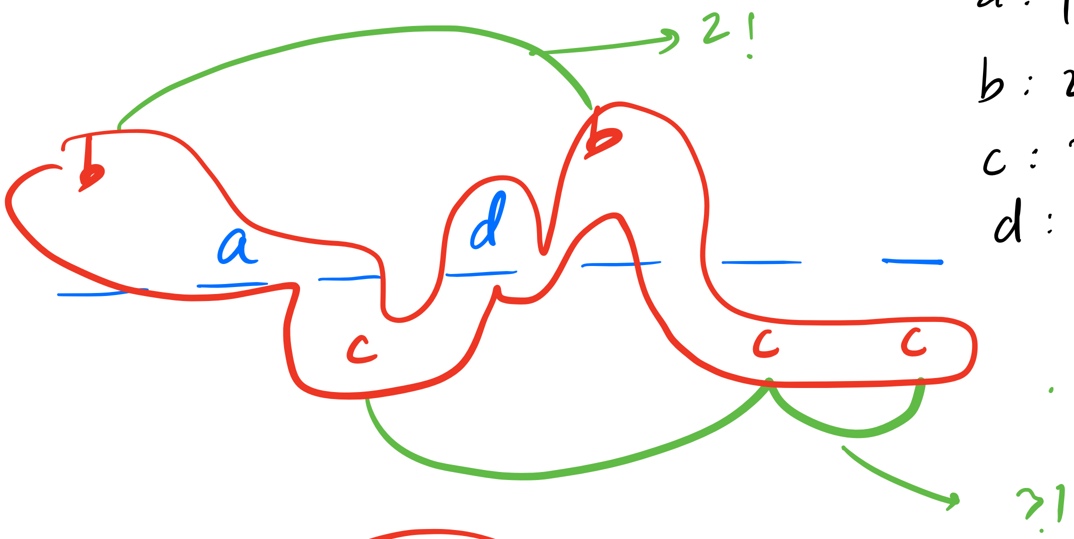
$N = 7$

$a : 1 : R_1$

$b : 2 : R_2$

$c : 3 : R_3$

$d : 1 : R_4$



$2! \times 3! \rightarrow 1$

$7! \rightarrow \dfrac{7!}{2! \times 3!}$

⊙ N items:

R_1 are repeating

R_2 ______

R_3 ______

⋮

R_k ______

$$\boxed{\#\ \text{permutations} = \frac{N!}{R_1! \times R_2! \times R_3! \cdots \times R_k!}}$$

Q Find the # of permutations of the word

MISSISSIPPI

N : 11

'M' : 1 : R_1

'I' : 4 : R_2

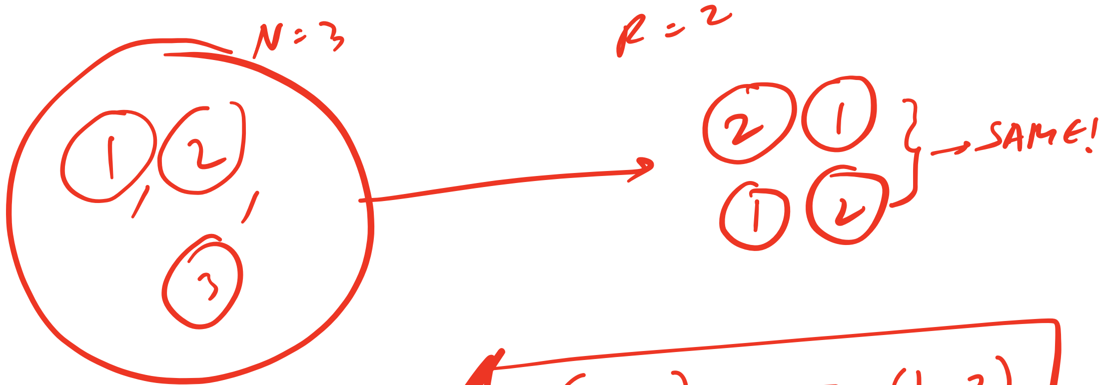
'S' : 4 : R_3

'P' : 2 : R_4

$$\boxed{\frac{11!}{1! \times 4! \times 4! \times 2!}}$$

Combinations

→ It deals with selecting items!



$$\boxed{(2,1) = (1,2)}$$

ARRANGEMENT does not matter!

① N items, choose R items

$$\# \text{ way} = \boxed{{}^{N}C_{R} = \frac{N!}{(N-R)!\,R!}}$$

a b c d e

$N=5$

choose 3 items $R=3$

$${}^{5}C_{3} = \frac{5!}{(5-3)!\,3!}$$

permutations

a b d
a d b
b a d
b d a
d a b
d b a

combinations

{a, b, d}

1

$3!$

$R!$

perm   # comb

$R! \longrightarrow 1$

$${}^{N}P_{R} \longrightarrow {}^{N}P_{R}/R!$$

$$\left(\frac{N!}{(N-R)!}\right)\Big/R! = \frac{N!}{(N-R)!\,R!}$$

combinations

$$\boxed{{}^{N}P_{R} = {}^{N}C_{R} \times R!}$$

${}^{n}C_{r} \longrightarrow$ Binomial Coefficient

$$\boxed{(x+y)^{N} = {}^{N}C_{0}\, x^{0} y^{N} + {}^{N}C_{1}\, x^{1} y^{N-1} + \dots + {}^{N}C_{N}\, x^{N} y^{0}}$$

⊙ Properties of Combinations

$$\boxed{{}^{n}C_{r} = {}^{n}C_{n-r}}$$

∴ Choosing R items is the same as discarding N-R items!

a, b, c

${}^{3}C_{2}$ ${}^{3}C_{1}$

ab ——— c

ac ——— b

bc ——— a

$^5C_3$ a b c d e $^5C_2$

$\frac{5!}{3!\,(5-3)!}$

$\frac{5 \times \cancel{4} \times \cancel{3!}}{\cancel{3!}\ \cancel{2!}}$

$= 10$

abc → de
abd → ce
abe → cd
acd → be
ace → bd
ade
bcd
bce
bde
cde

$\frac{5!}{(5-2)\,2!}$

$\frac{5!}{3!\,2!}$

10

① $\boxed{{}^NC_0 = 1}$

② $\boxed{{}^NC_1 = N}$

③ $\boxed{{}^NC_N = 1}$

① $^{N}C_{R}$: INVALID $\quad R > N$

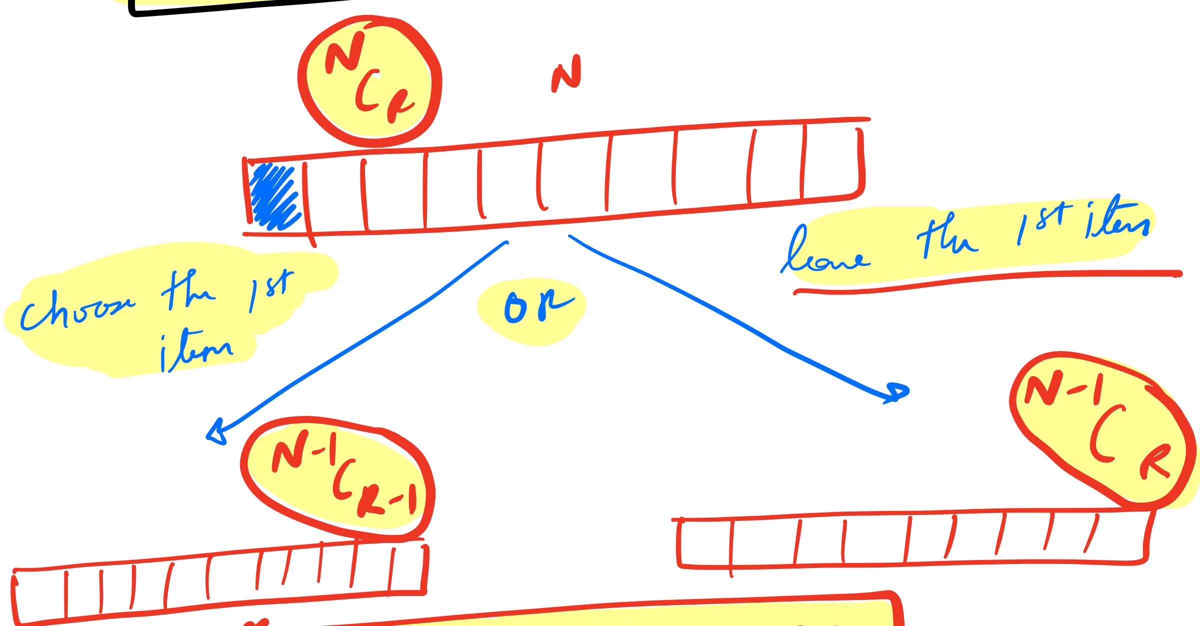
$^{N}C_{R} \to 0$

② $$\boxed{{}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}}$$

$n \to n-1$

$r \to r-1$

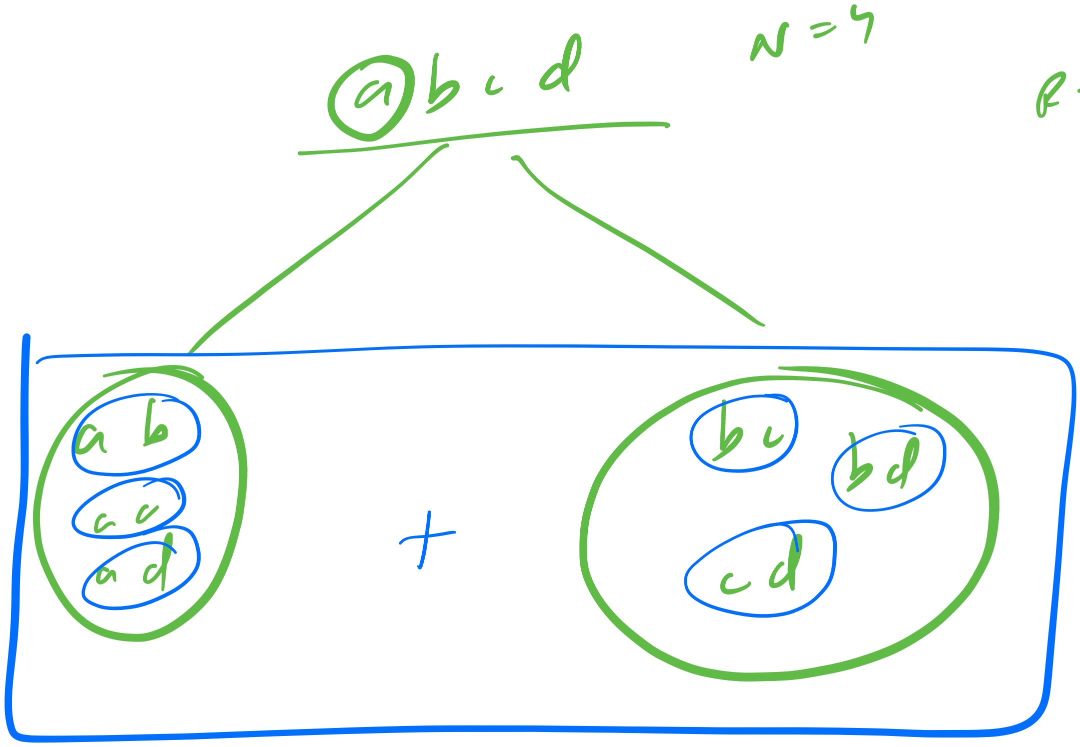
$$\boxed{{}^{n-1}C_{r-1} + {}^{n-1}C_{r} = {}^{n}C_{r}}$$



$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R-1} + {}^{N-1}C_{R}}$$

$N = 4$

$R = 2$



$$^NC_0 + {}^NC_1 + {}^NC_2 + {}^NC_3 + \dots + {}^NC_N = 2^N$$

Choose Nothing

1 item

2 item

3 ..

K

All

Choose ANY no. of items

a b c

$^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 8 = 2^3$

$^3C_0$	$^3C_1$	$^3C_2$	$^3C_3$
( )	(a)	(a b)	(a b c)
	(b)	(a c)	
	(c)	(b c)	

N=3

a	b	c	
✗	✗	✗	→ ( )
✗	✗	✓	→ (c)
✗	✓	✗	→ (b)
✗	✓	✓	→ (b, c)
✓	✗	✗	→ (a)
✓	✗	✓	→ (a, c)
✓	✓	✗	→ (a, b)
✓	✓	✓	→ (a, b, c)

8

a	b	c
2	2	2
✓	✓	✓
✗	✗	✗

$2 \times 2 \times 2 = 2^3 \rightarrow 8$

In general : 2^N subsets!

Q $\boxed{{}^{100}C_{45} \;\%\; P}$.

$$\left(\frac{100!}{(45)!\,(100-45)!}\right) \% \; P$$

Inv Mod ..

Q ${}^{100}C_{45} \;\%\; M$

$2 <= M <= 10^5$

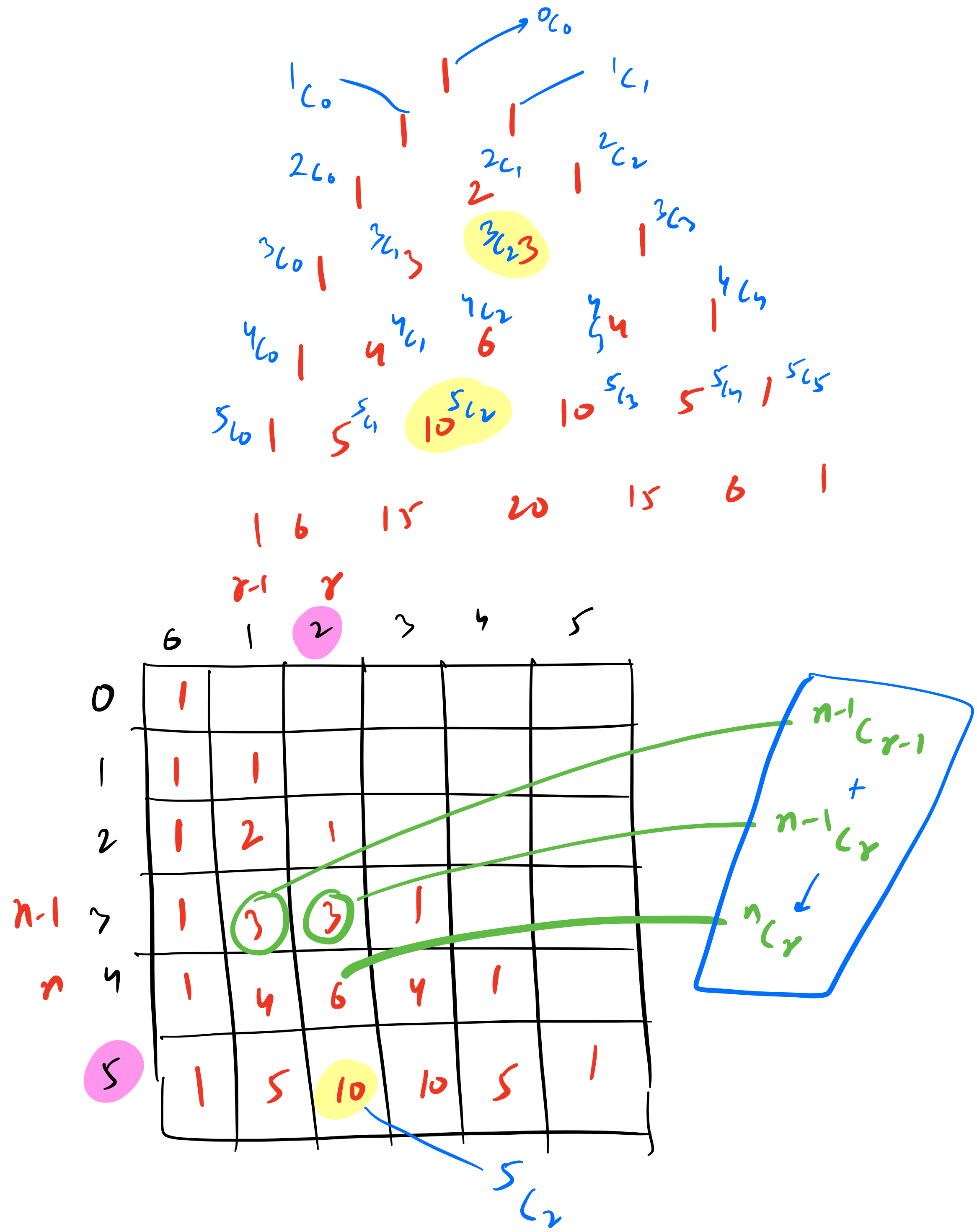
$$\frac{100!}{(45)!\,(100-45)!} = \frac{100!}{45!\,55!} \;\%\; M$$

Can't find Inv modulo easily

$\% \; M$

$\boxed{O(M)}$

PASCAL TRIANGLE

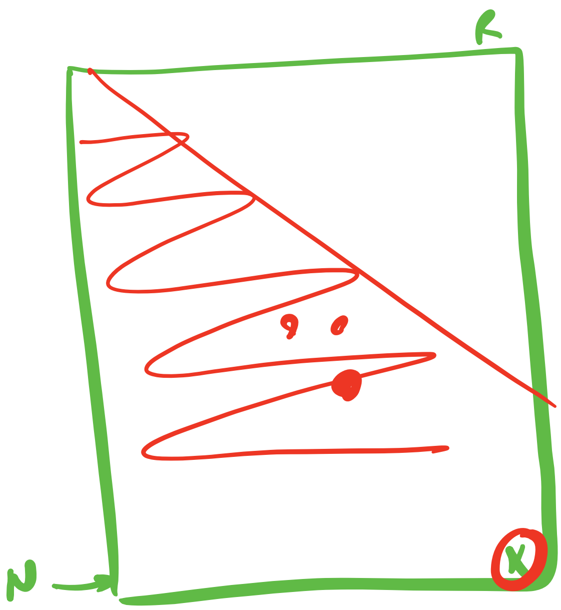


$^{100}C_{95} \% M$

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

$$\left({}^{n-1}C_{r-1} + {}^{n-1}C_{r}\right) \% M$$

$$^{N}C_{R} \% M$$



$O(1)$

$N \times R$

N^2

$$\boxed{TC = O(NR)}$$

$$N \times R \simeq 10^6$$

$$\boxed{R <= N <= 10^5}$$