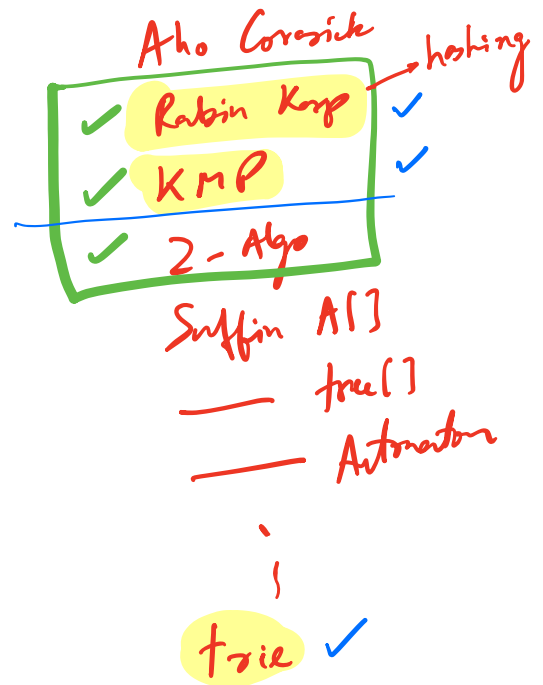


String Pattern Matching



Given a string S of size N .

Prefix string : substring of S , which starts at $S[0]$
: $S[0 - i]$ $0 \leq i < N$

Suffix string : ————— ends at $S[N-1]$
: $S[i - N-1]$ $0 \leq i < N$

$S = \underline{abab}$

prefix string

a
ab
aba
~~abab~~

suffix strings

b x
ab ✓
bab x
~~abab~~

LPS of a string: Length of the longest prefix which is also a suffix of the string!
except the full string.

$$LPS(abab) = 2$$

$S = abcba \rightarrow LPS: 1$

prefix

a
ab
abc
abcb
~~abcba~~

suffix

a ✓
ba x
cba x
bcba x
~~abcba~~

① LPS ("a") \rightarrow 0

Prefix

a

Suffix

a

② LPS ("aaaaa") \rightarrow 4

P

a

aa

aaa

aaaa

aaaaa

S

a

aa

aaa

aaaa

aaaaa

aaaaa

✓

✓

✓

✓

③ S = abcab

P

a

ab

abc

abca

abcab

abcab

S

b

ab

cab

bca

abcab

abcab

time

\rightarrow 1

\rightarrow 2

\rightarrow 3

\rightarrow 4

\vdots

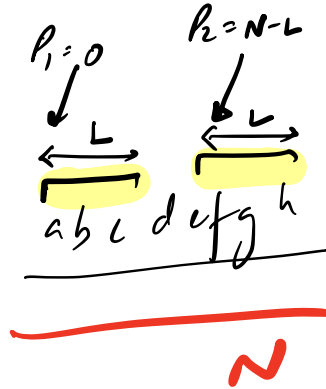
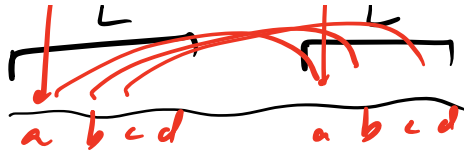
\rightarrow N-1

$\frac{(N-1)(N)}{2}$

TC: $O(N^2)$

P_1

P_2



ANS = 0;

f (L = 1; L < N; L++) {

ok = true;

p1 = 0, p2 = N - L;

f (i = 0; i < L; i++) {

if (s[p1, i] != s[p2, i]) {

ok = false;

break;

}

p1++, p2++;

}

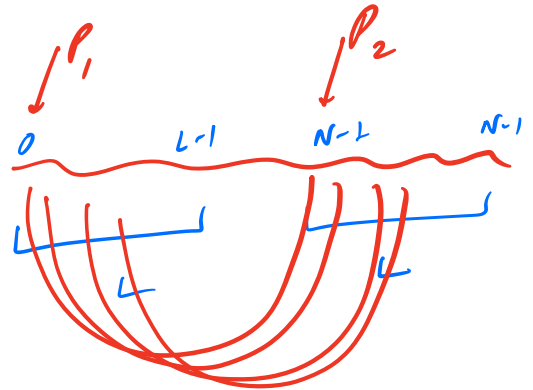
if (ok == true)

ANS = L;

}

return ANS;

TC = O(N²)



Q Given a string S of length N .
Return the LPS Array!

$LPS[i] =$ LPS value of the substring $s[0 \dots i]$!

	0	1	2	3	4	5	6
$S =$	a	a	b	a	a	b	a
$LPS[]$	0	1	0	1	2	3	4

KMP $O(N)$ $N \times O(N^2)$
 $TL = O(N^3)$

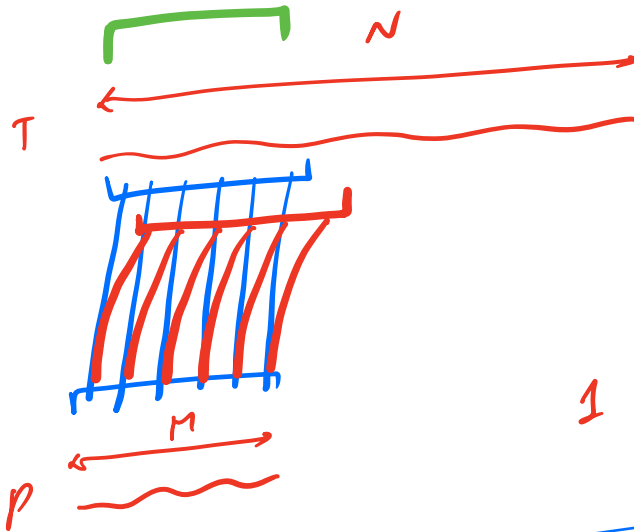
S : a a b a c a a b a

LPS : 0 1 0 1 0 1 2 3 4

Q Given a text (T) & a pattern (P).
 Check if the pattern is present in the text
 as a substring!

length
 $N \leftarrow T = \text{a a b a c d} \rightarrow \text{text}$

$m \leftarrow P = \text{a b a c}$



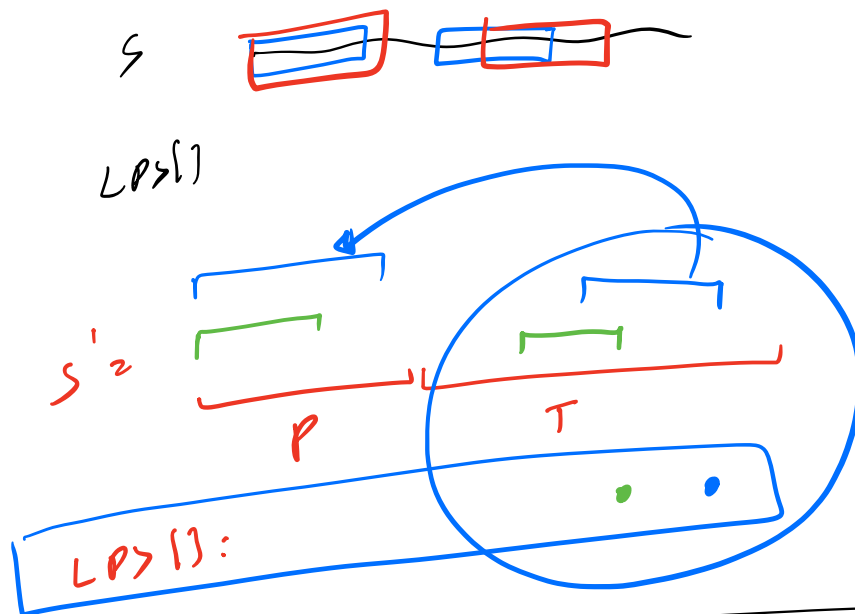
SS $\rightarrow N - m + 1$
 of m

1 SS comparison
 $\rightarrow O(m)$

$$T.C = O((N - m) m) \approx NM$$

$$1 \leq m \leq N \leq 10^6$$

II LPS



$T = aabacd$

$P = abac$

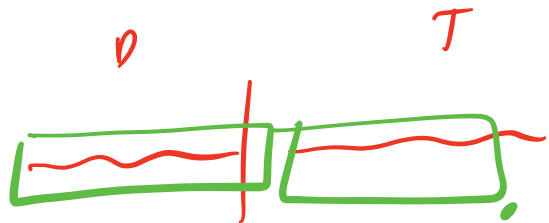
$S' = P + T$

$s' = abac | aabacd$

$LPS[]: 0010 | 112340$

Diagram illustrating the recursive step of the Longest Palindromic Substring (LPS) algorithm. It shows a string S' partitioned into two parts, P and T . A recursive call $LPS[1]$ is shown for the substring T , which is circled. A label $LPS[1]$ is also shown in a box, pointing to the recursive call. The diagram uses various colors (red, blue, green) to highlight different parts of the string and the recursive call.

$T = a$ $N = 1$
 $P = aa$ $M = 2$



$S' = P + T$



$LPS[]: 0 \ 1 \ 2$

idea: $S' = P + @ + T$

delimiter
 [Not be in
 P or T]

$S' = \overset{\text{2}}{aa} @ a$
 $LPS[]: 0 \ 1 \ 0 \ 1$ X

$T = a, \quad P = aaa$

$S' = P + @ + T$



$LPS: 0 \ 1 \ 2 \ 0 \ 1$ X

$S' = \overbrace{a a}^M @ \overbrace{a a}$
 $LPS[] = 0 \ 1 \ 0 \ 1 \ 2 \rightarrow m$ ✓

STEPS:

1. $S' = P + @ + T$ $\rightarrow O(N+M)$
2. Construct $LPS[]$ for S' $\xrightarrow{\text{KMP}} O(N+M)$
3. find for any i
 if $(LPS[i] == m)$
 \downarrow \rightarrow is a match
 $O(N+M)$

$TC = O(N+M)$

$SC = O(N+M)$

$S', LPS[]$

Q Given T & P . Count the no. of occurrences of P in T .

$T = \text{abababab} \longrightarrow 3$
 $P = \text{aba}$

$M = 3$
 $S' = P + @ + T$
 $\text{aba} @ \text{abababab}$
 LPS[]: 0 0 1 0 1 2 3 2 3 2 3 1 2
 (The LPS array is shown with the values 3, 2, 3, 2, 3 circled, and a bracket underneath pointing to the text "Count the # of occ. of M")

Q Given a string S . Find out the no. of cyclic rotations of S which are equal to S .

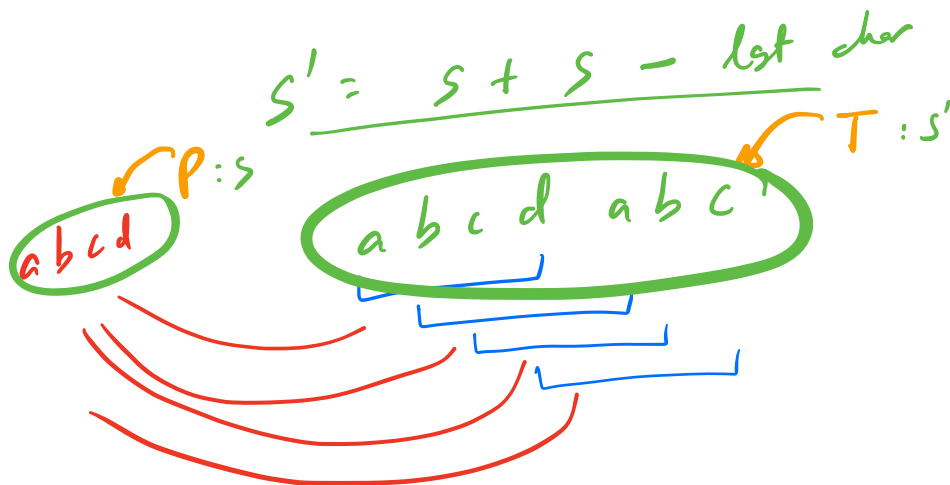
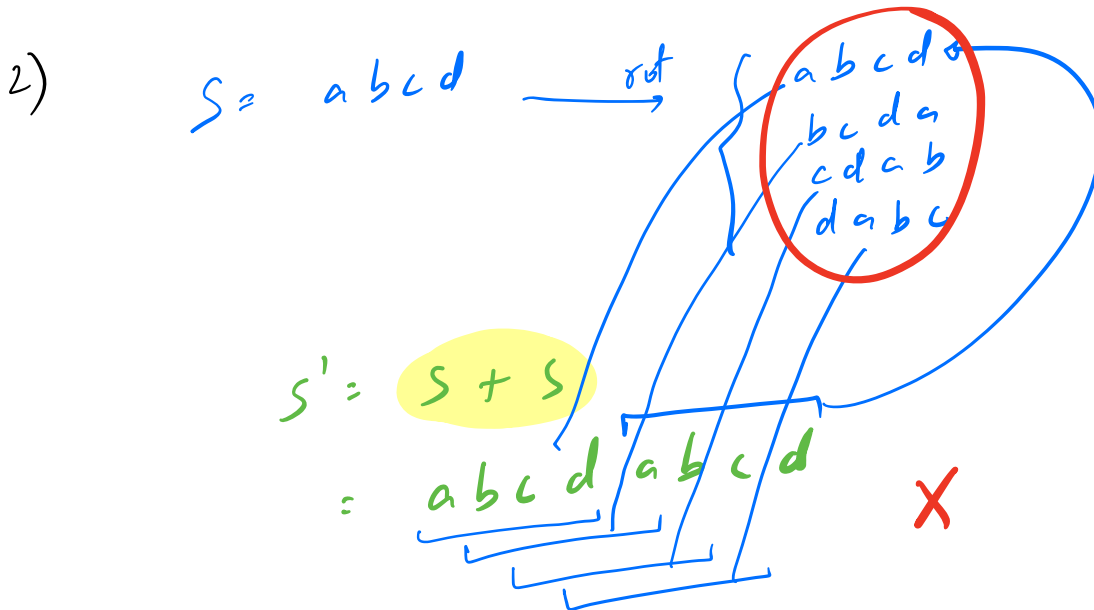
$S = \text{abab}$
 (Diagram showing a cyclic rotation of S with a green arrow and the text "1 rot")

Cyclic rotations

	3	0	1	2
1:	b	a	b	a
2:	a	b	a	b
3:	b	a	b	a
4:	a	b	a	b

} 2

1) BF $\rightarrow N^2$



$S'' = P + @ + T$
 $S + @ + S'$
 S'' $abcd @ abcdabc$
 LPS: 0 0 0 0 0 1 2 3 4 1 2 3

$$S = abab$$

$$S' = S + S - 1$$

$$: 2N - 1$$

$$: abababab$$

$$S'' = P + @ + T$$

$$: 3N$$

$$S + @ + S'$$

$\overbrace{abab} \quad @ \quad \overbrace{abababab}$

LPS[] 0 0 1 2 0 1 2 3 4 3 4 3

→ 2

$N = 4$

$$T = O(N)$$

$$SC = O(N)$$