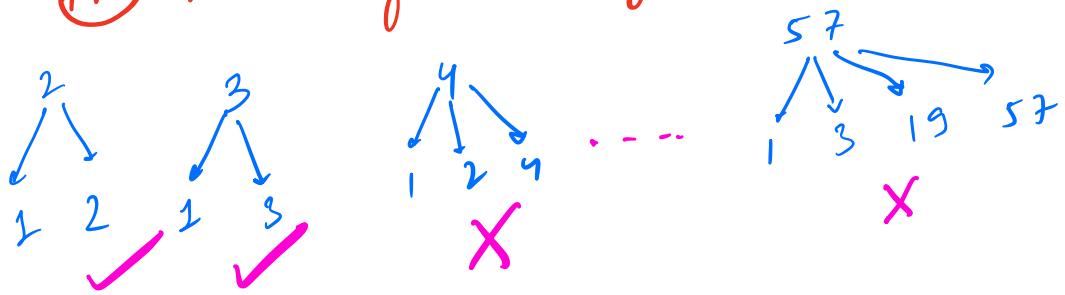
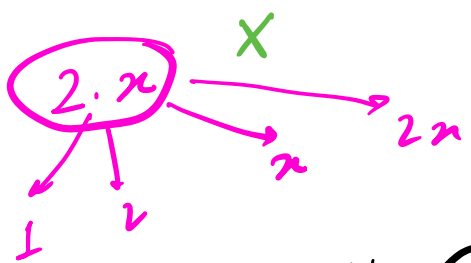
Prime No

+ve no.'s having Exactly 2 factor



Prime NO: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 - -

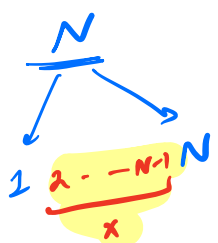
ONLY EVEN PRIME NO!



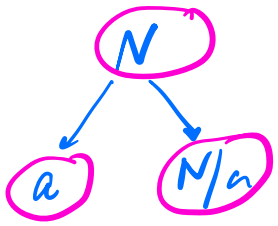
Given N. Check if it is a prime no. or not!

N != 1

```
if(N==1) ret false;
for(i=2; i<N; i++) {
    if(N%i==0) {
        ret false;
    }
}
ret true;
```



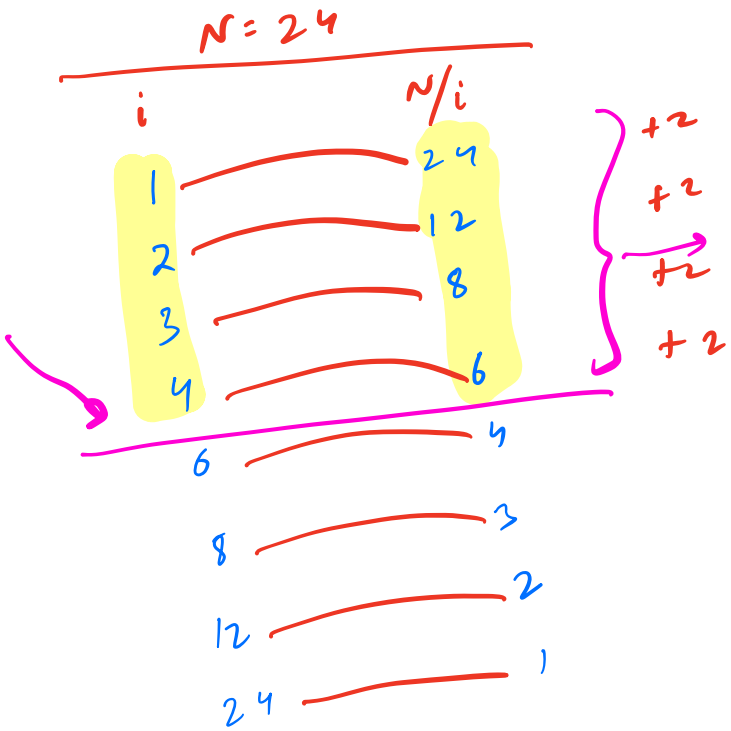
TC = O(N)



$N = a \times b$

$b = N/a$

$12 = 3 \times 4$



$i < N/i$

$i \times i < N$

$i^2 < N$

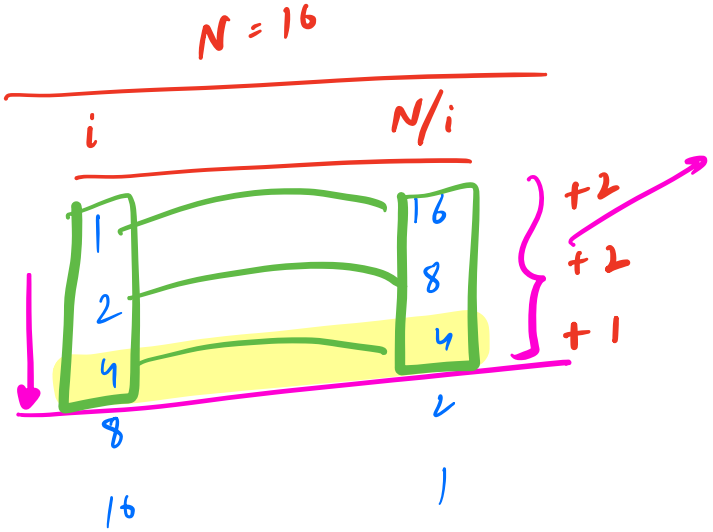
$i < \sqrt{N}$ X

6 4

8 3

12 2

24 1



$i <= N/i$

$i^2 <= N$

$i <= \sqrt{N}$

8 2

16 1

```
int countfactors(N) {
    cnt=0
    for(i = 1; i*i <= N; i++) {
        if(N%i==0) {
            if( i == N/i){
                cnt++;
            }
            else {
                cnt+=2;
            }
        }
    }
    ret cnt;
}
```

$i <= x$

$x = sqrt(N)$

$TC = O(\sqrt{N})$

```
bool isPrime(N) {
    if ( countfactors(N) != 2) ret false;
    ret true;
}
```

$TC = O(\sqrt{N})$

Q Given N. Find all the prime nos in [1, N].

N = 10 . [1, 10] : {2, 3, 5, 7}

```
for( i = 1 ; i <= N ; i++) {
    if( isPrime(i) == true) {
        print(i);
    }
}
```

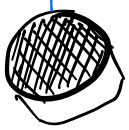
$TC = O(N\sqrt{N})$

$\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{N}$

$N = 10^6$ $\quad 10^6 \cdot \sqrt{10^6}$

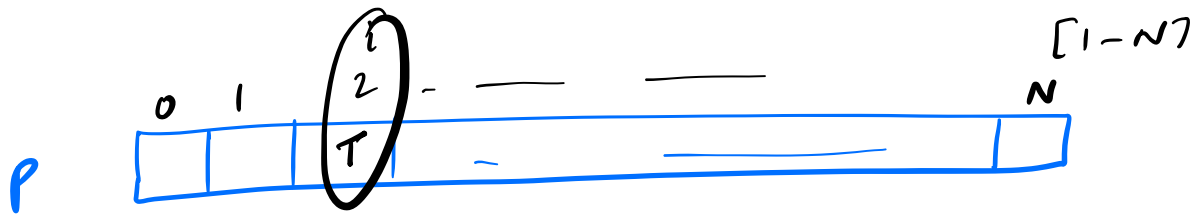
$= 10^9$ ops

SIEVE OF ERATOSTHENES



$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
$7 \times 5 = 35$
$7 \times 6 = 42$
$7 \times 7 = 49$

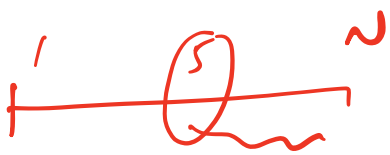
~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	~~47~~	~~48~~	~~49~~	~~50~~




```
bool P[N+1] = {true};
    P[0] = false,   P[1] = false;

for(i=2; i*i <= N; i++) {
    if( P[i] == true) {
        for(j= i*i ; j<=N ; j = j+i)
            P[j] = false;
    }
}
cnt = 0;
for(i=2; i<=N ; i++) {
    if( P[i] == true) cnt++;
}
return cnt;
```

N



$[N/2 + N/3 + N/5 + N/7 + \cdots + N/P]$

$P <= \sqrt{N}$

$N\left[\underbrace{1/2 + 1/3 + 1/5 + \cdots + 1/P}\right]$

$N \times \log(\log(N))$

$TC = O(N \cdot \log(\log(N)))$

$N <= 10^6$

$N \times \log(\log(10^6))$

$N \times \log(20)$

$N \times 5 \approx O(N)$

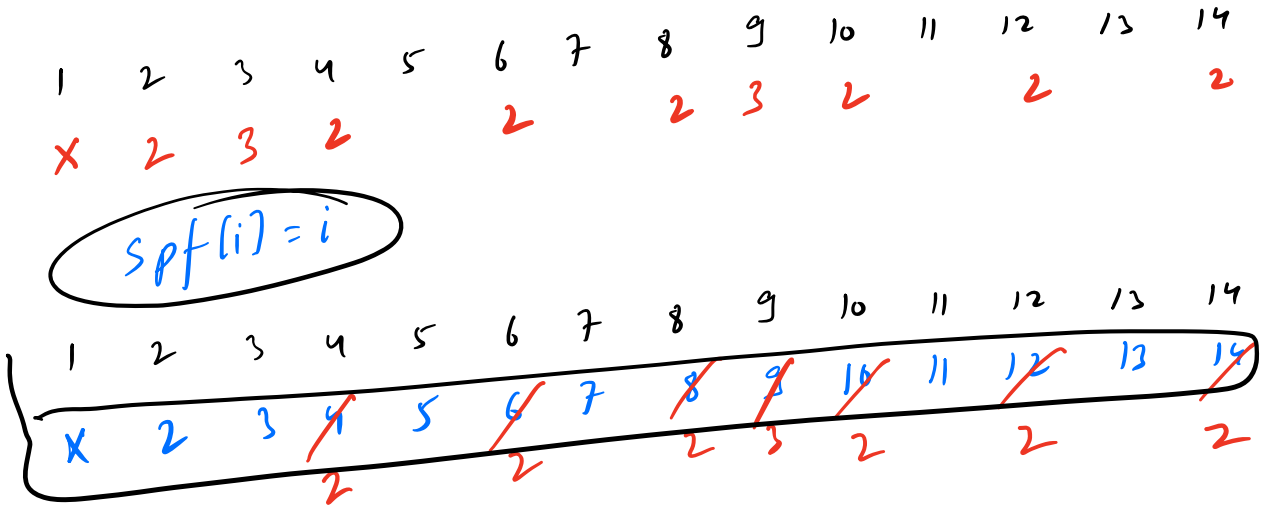
$SC = O(N)$

Q Given N. Find the smallest prime factor for all nos from [2, N].

5 6

2 7

1	2	3	4	5	6	7	8	9	10	11	12	13
spf	X	2	3	2	5	2	7	2	3	2	11	2	13




```
int spf[N+1];
for(i=2; i<=N; i++) {
    spf[i] = i;
}
spf[0] = -1, spf[1] = -1;

for(i=2; i*i<=N; i++) {
    if(spf[i]==i) {
        for(j=i*i; j<=N; j=j+i) {
            if(spf[j]==j) {
                spf[j]=i;
            }
        }
    }
}
ret spf[]
```

TC = O(N log(log(N)))

Q Given N. Count the no. of divisors of it!

$N = 12 \longrightarrow 1, 2, 3, 4, 6, 12$: 6

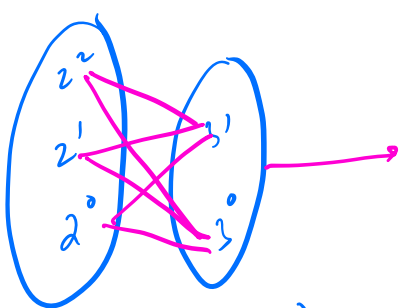
: 2

$N = 13 \longrightarrow 1, 13$

I) countfactors(N) $\longrightarrow$ $O(\sqrt{N})$

II) $N = 12$

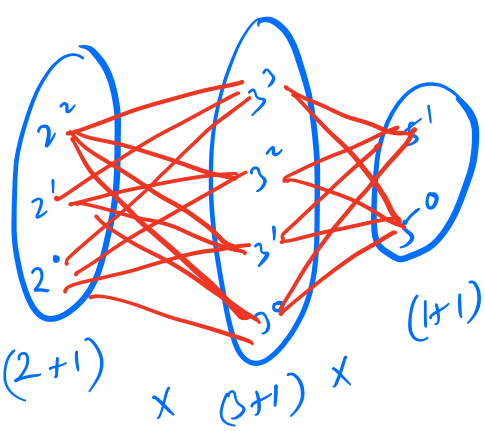
$12 = 2^2 \cdot 3^1$



$3 \times 2 = 6$

$(2+1) \quad (1+1)$

540 : $2^2 \cdot 3^3 \cdot 5^1$



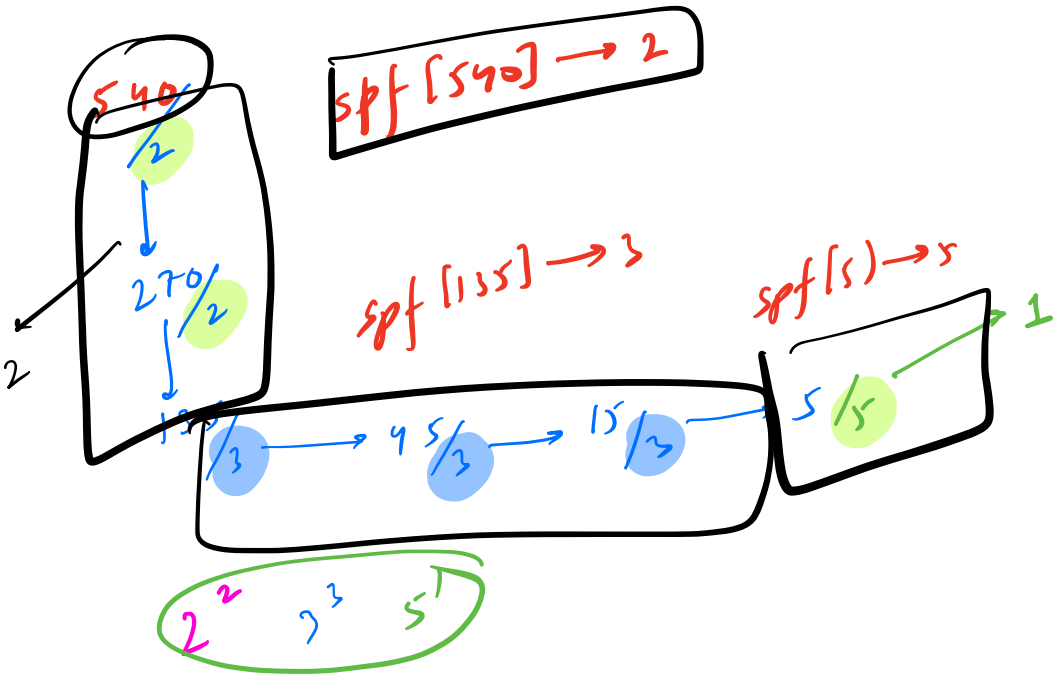
= 24

In general

$$N = p_1^{a_1} \times p_2^{a_2} \times p_3^{a_3} \times \cdots \times p_k^{a_k}$$

$p \rightarrow$ prime

$$\#\text{ of factors}(N) = (a_1+1) \times (a_2+1) \times \cdots \times (a_k+1)$$

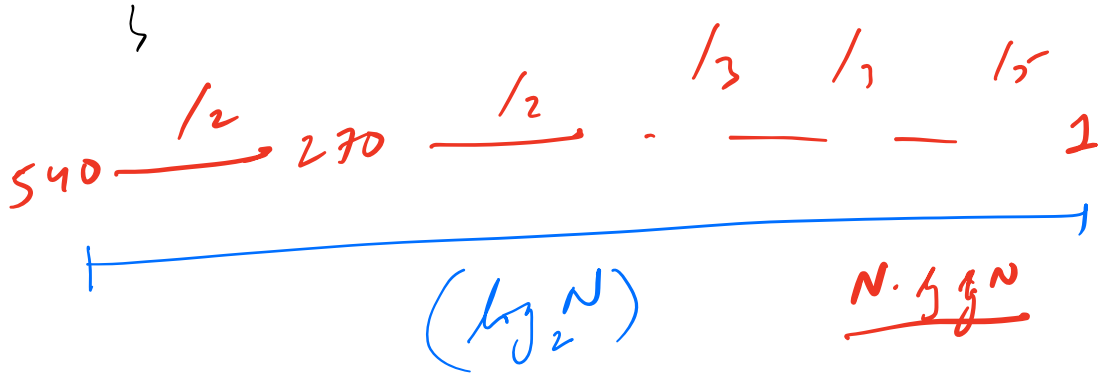


$$(2+1) \times (3+1) \times (1+1)$$

```
// N, spf[]   ANS=1
while(N>1){
    p = spf[N];
    cnt = 0;
    while(N%p==0){
        N = N/p;
        cnt++;
    }
    ANS = ANS * (cnt+1);
}
```

spf[] → N log(log N)

TC = O(log N)



Q Given an Array of size N.
∀ A[i] → find the # of factors!

spf[max(A)+1]

1. spf[] → BUILD : max(A) log(log(Max(A)))
2. Solve the prev Q for every A[i] + : N x log(max(A))