Range

int : $[-2 \times 10^9, 2 \times 10^9]$

long : $[-9 \times 10^{18}, 9 \times 10^{18}]$

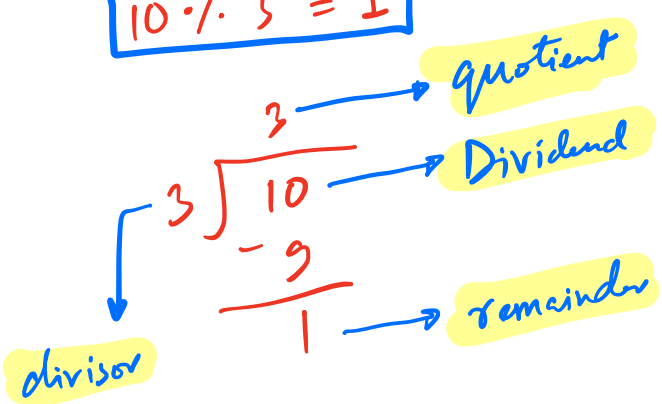
Modular Arithmetic → +, /, -, *

Modulo → %

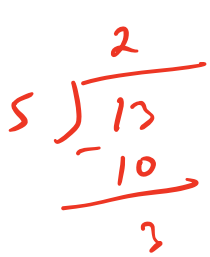
"n modulo a"

n % a → Remainder when n is divided by a!

10 % 3 = 1



13 % 5 = 3



$10 = 3 \times 3 + 1$

$13 = 5 \times 2 + 3$

Dividend = divisor × quotient + remainder

remainder = dividend − divisor × quotient

$32 \% 5 = 32 - 5 \times 6$

$= 32 - 30 = 2$

$17 \% 4 = 17 - 4 \times 4 = 1$

Remainder : Dividend − greatest multiple of divisor <= Dividend

$x \% 3$

0 <= remainder < divisor

0 1 2

$a \% b \longrightarrow [0, b-1]$

x8

$32 \% 4 \longrightarrow 0$

$12 \% 6 \longrightarrow 0$

x2

if $((a \% b) == 0)$

→ a is a multiple of b

① 150 %. 11 → 150 − greatest mult. of 11 <= 150

= 150 − 143

= 7 //

② 100 %. 7 → 100 − greatest mult. of 7 <= 100

100 − 7 x 14

= 2 //

③ 5 %. 12 → 5 − greatest mult. of 12 <= 5

5 − 12 x 0

5 − 0

= 5 //

④ 7 %. 15 = 7 //

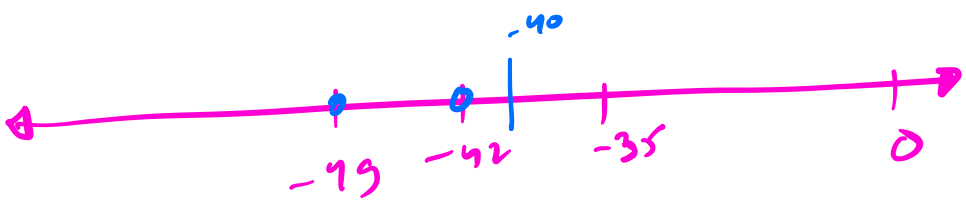
a %. b → a : 0 <= a < b

① $-40 \% 7$ = -40 − greatest mult. of 7 <= -40

$[0, 6]$

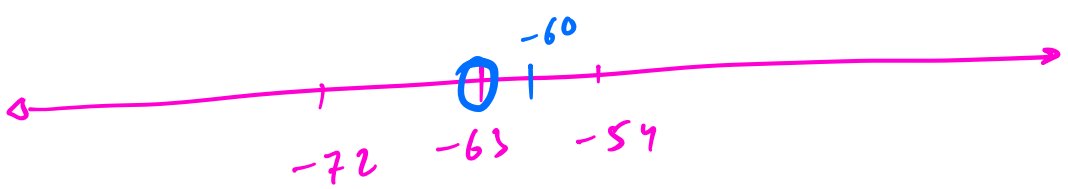
$= -40 - (-42)$

$-40 + 42 = 2$ //



② $-60 \% 9$ → -60 − greatest mult. of 9 <= -60

$-60 - (-63) = 3$ //



C++ / Java

$-40 \% 7$ → -5 (+7) → 2

$-60 \% 9$ → -6 (+9) → 3

Python

$-40 \% 7$ → 2

$-60 \% 9$ → 3

C++

36 % 7 ⟶ 36 − greatest mult. of 7 <= 36

$7 \times x <= 36$

$x <= 36/7 \rightarrow 5$

$36 - (36/7) \times 7$

$36 - 5 \times 7$

$36 - 35 = 1$

$a \% b = a - (a/b) \times b$

$-40 \% 7 = -40 - (-40/7) \times 7$

$-40 - (-5) \times 7$

$-40 + 35 = -5$

python →

$36 \% 7 \longrightarrow 36 - (36/7) \times 7$

↓ INSTEAD

$36 - \text{floor}(36/7) \times 7$

$36 - \text{floor}(5.1..) \times 7$

$36 - 5 \times 7$

$36 - 35 = 1$

floor(x) = ⌊x⌋ = greatest integer <= x

$a \% b = a - \text{floor}(a/b) \times b$

$-40 \% 7 = -40 - \text{floor}(-40/7) \times 7$

$-40 - \text{floor}(-5.) \times 7$

↓

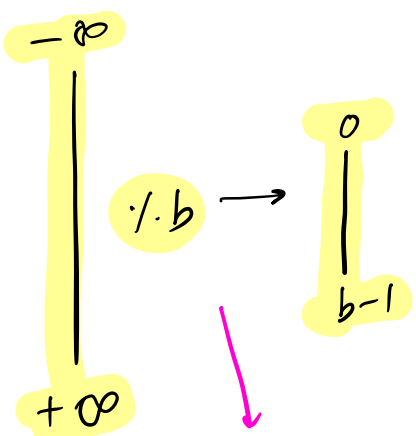
$-40 - (-6 \times 7)$

$-40 + 42$

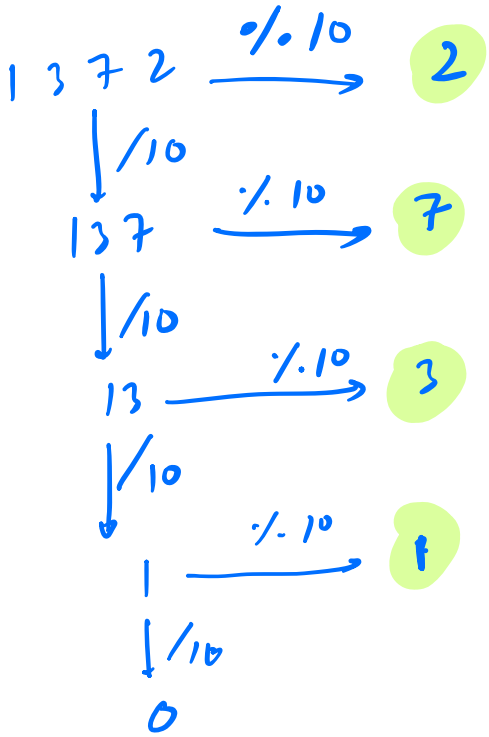
$= 2$

⊙ Why % ?

- Used in HashMap / HashSet
- Circular Arrays, Queues
- extracting digits from a no
- Cryptography
- Consistent Hashing !

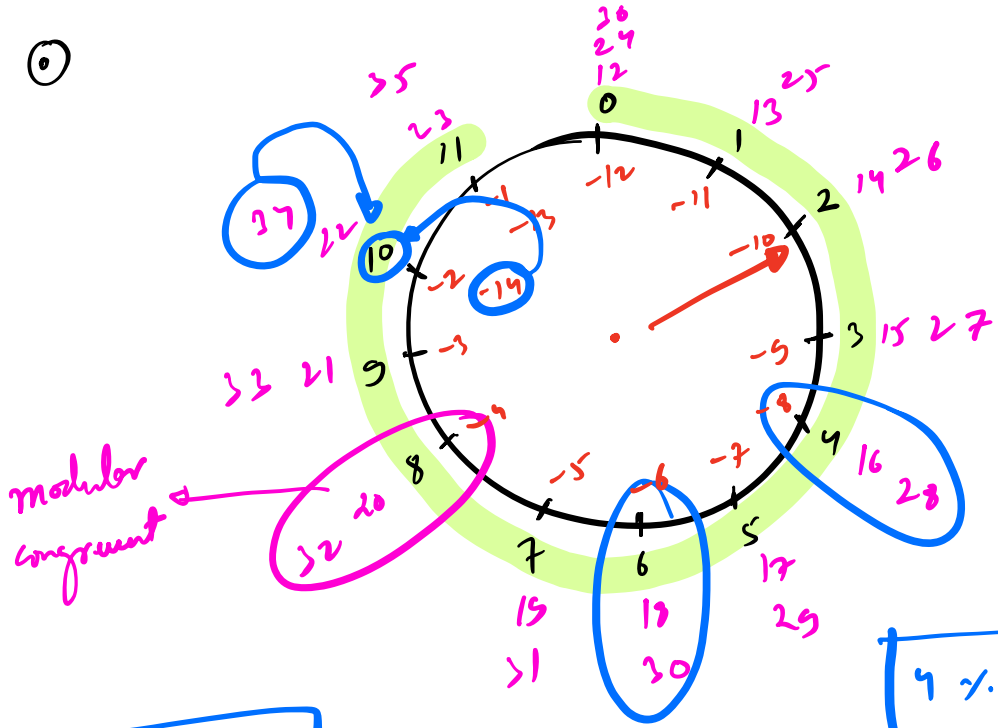


limits our data to required range!



①

$-14 \% 12$



$6 \% 12 \rightarrow 6$
$18 \% 12 \rightarrow 6$
$30 \% 12 \rightarrow 6$

$4 \% 12 \rightarrow 4$
$16 \% 12 \rightarrow 4$
$28 \% 12 \rightarrow 4$

$a \% n = b \% n$

a is modular congruent to b w.r.t. n

④ Modular Arithmetic →

$(40 + 37) \% 3 \mid 77 \% 3 \to 2$

$((40 \% 3) + (37 \% 3)) \% 3$

$(1 + 1) \% 3 = 2$

1) ADDITION →

$$(a + b) \% M = ((a \% M) + (b \% M)) \% M$$

$[0, M-1]$ $[0, M-1]$ $[0, M-1]$ x

$0 \text{ — } 2M-2$

2) MULTIPLICATION →

$$(a \times b) \% M = ((a \% M) \times (b \% M)) \% M$$

$a = 10$

$b = 14$

$m = 7$

$(10 \times 14) \% 7$

$= 140 \% 7$

$= 0$

$((10 \% 7) \times (14 \% 7)) \% 7$

$(3 \times 0) \% 7$

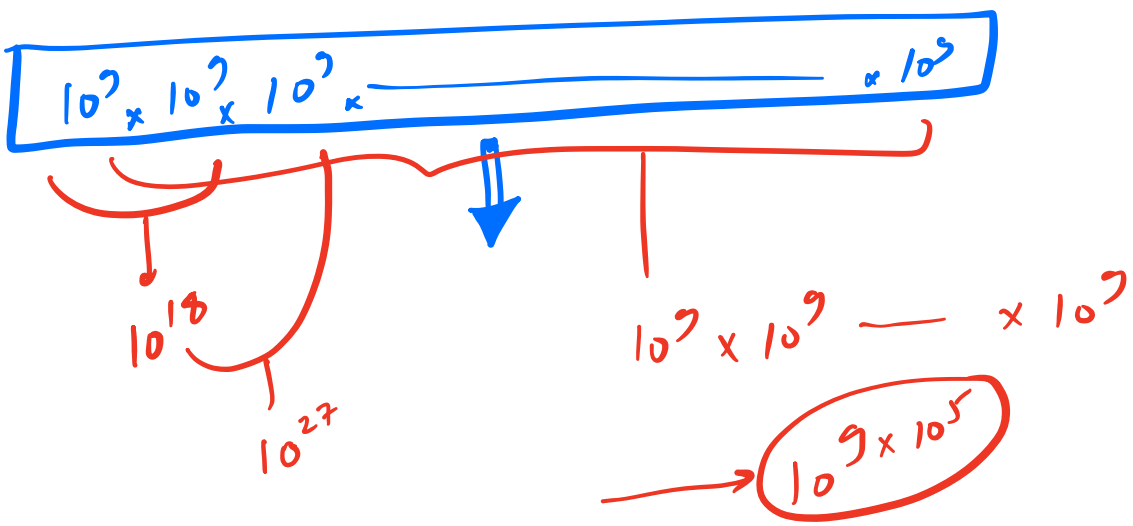
$= 0$

$a \% b = (a \% b) \% b = ((a \% b) \% b) \% b \ldots$

Q Given an array. find the product of all no.'s % 10^9+7

$1 <= N <= 10^5$

$1 <= A[i] <= 10^9$



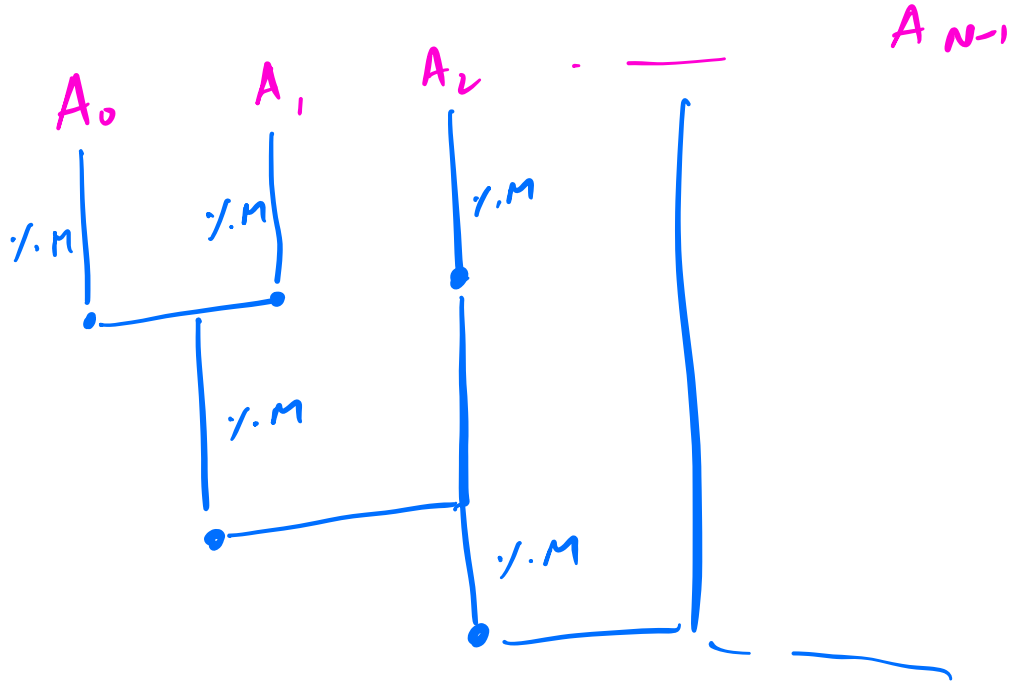
$M \rightarrow 10^9+7$

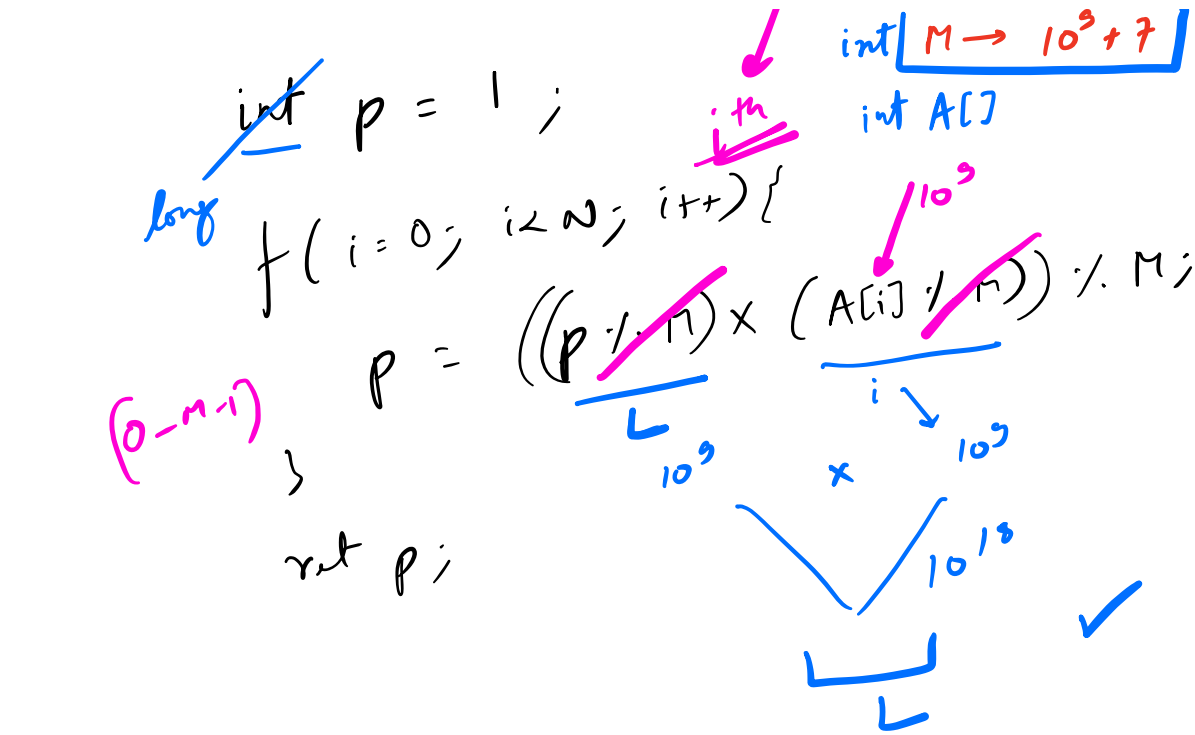
$$\left(\frac{A_0}{x} \times \frac{A_1 \times A_2 \times \cdots \times A_{N-1}}{y}\right) \% M$$

$$\left(\left((A_0 \% M) \times (A_1 \cdots A_{N-1}) \% M\right) \% M\right)$$

$$\left(\left((A_0 \% M) \times (A_1 \% M) \times (A_2 \% M) \cdots \times (A_{N-1} \% M)\right) \% M\right)$$

$$\left(\left(\left(\left((A_0 \% M) \times (A_1 \% M)\right) \% M \times (A_2 \% M)\right) \% M \times (A_3 \times M)\right) \% M \cdots\right)$$





$$C = (b \times a + c) \times d$$

$$C = (((((b \% M) \times (a \% M)) \% M + (c \% M)) \% M) \times (d \% M)) \% M$$

HW

$a^b \% M$

$1 <= a <= 10^9$

$1 <= b <= 10^6$

$M = 10^9 + 7$

Q Given a no. in an Array!

Calculate the no % M! → 10^9+7

$0 <= A[i] < 10$

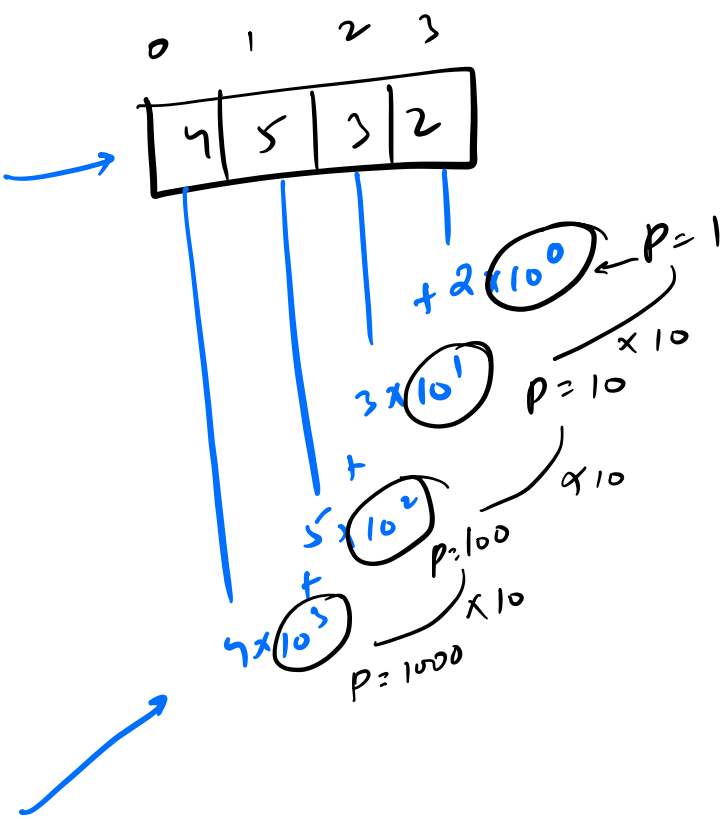
$1 <= N <= 10^5$

A :

4	5	2	6	7	8	3	2	5	3	2	5	8	4	3

$(45267832532983) \% 10^9+7$

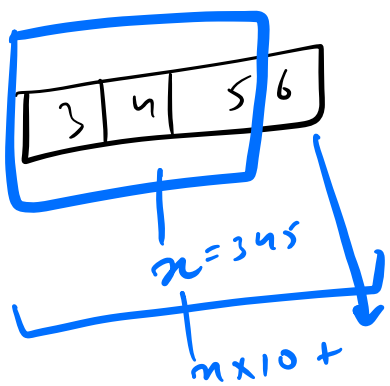
?

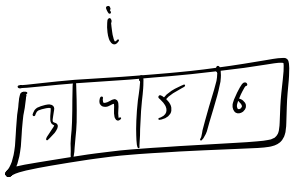


```
int num = 0, p = 1;
for (i = N-1; i >= 0; i--) {
    num = ((num % M) + (A[i] % M * p % M) % M) % M;
    p = ((p % M) * (10 % M)) % M;
}
ret num;
```

Long

II





num = 0x10+3
→ 3

= 3x10+4
→ 34

= 34x10+5
345

=345x10+6
3456

HW %.? data type?

```
num = 0;
for (i = 0; i < N; i++) {
    num = num x 10 + A[i];
}
ret num;
```

Divisibility Rule →

⊙ Div. Rule by 3 ?

→ Sum of digits is div. by 3.

987 → (9 + 8 + 7) → 24

⊛ (1234) % 3

$(1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0) \% 3$

$(1 \times 10^3\%3 + 2 \times 10^2\%3 + 3 \times 10^1\%3 + 4 \times 10^0\%3) \% 3$

1 1 1 1

(1 + 2 + 3 + 4) % 3

⊙ Div. Rule for 4

→ No. formed by last 2 digits is div by 4.

Eg: 4856 ? → 4 × 14

(1234) % 4

$(1 \times 10^3 + 2 \times 10^2 + 34) \% 4$

$(1 \times 10^3 \% 4 + 2 \times 10^2 \% 4 + 34) \% 4$

$0 + (0 + 0 + 34) \% 4$

(34) % 4