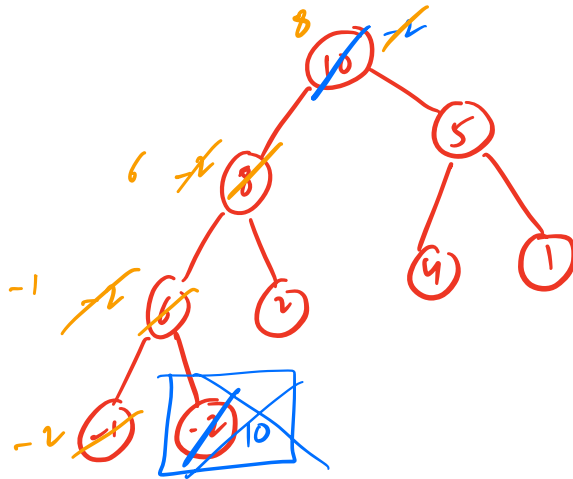


Q Given a max-heap. Can you delete the MAX element?



A: [10, 8, 5, 6, 2, 4, 1, -1, -2]

Steps:

- 1) Swap ( A[0], A[A.size()-1] )  $\rightarrow O(1)$
- 2) A.pop-back();  $\rightarrow O(1)$
- 3) heapify(0);  $\rightarrow O(\log N)$

**TC =  $O(\log N)$**

Heap Sort

$$TC = O(N \log N)$$

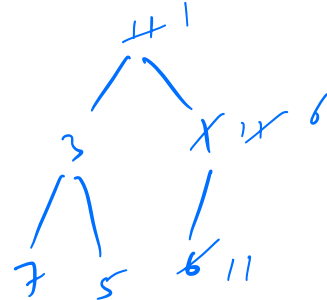
$$SC = O(1)$$

A: [11, 3, 1, 7, 5, 6]

MIN HEAP

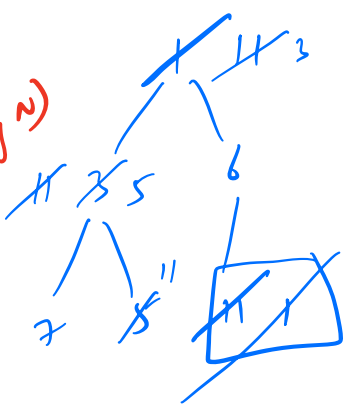
$$O(N)$$

A: [1, 3, 6, 7, 5, 11]



Ans: [1]

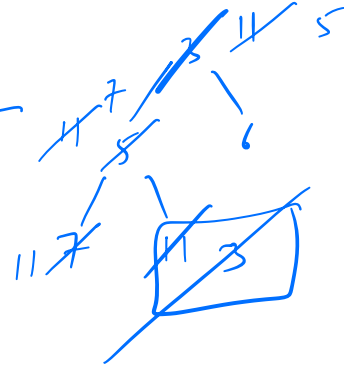
get min()  $\rightarrow O(1)$   
del min()  $\rightarrow O(\log N)$

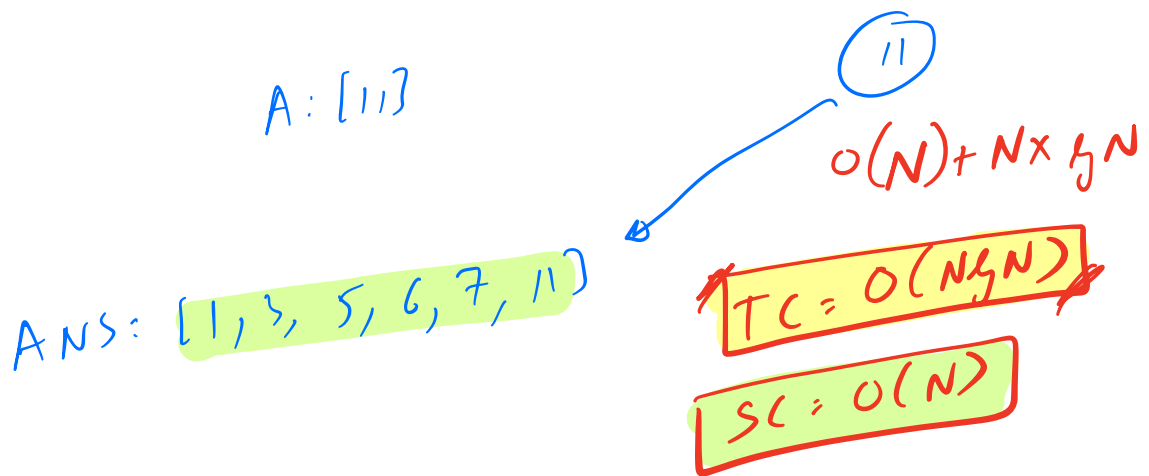
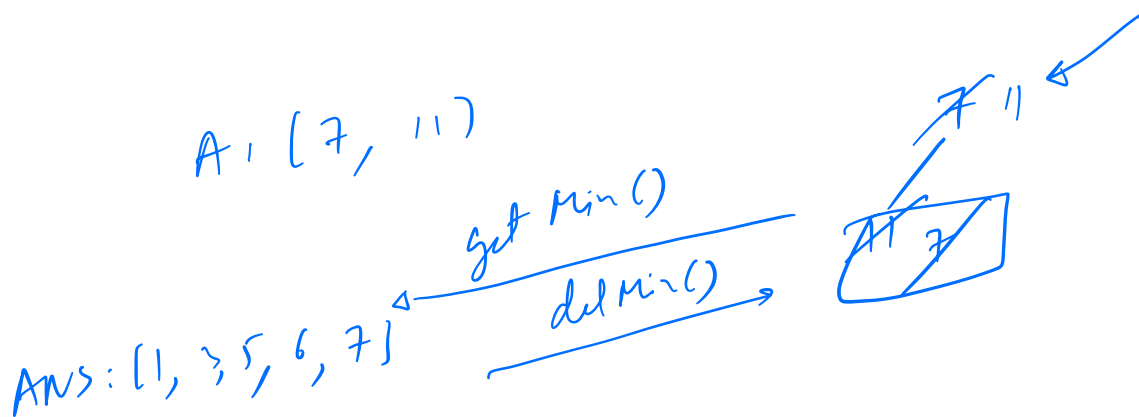
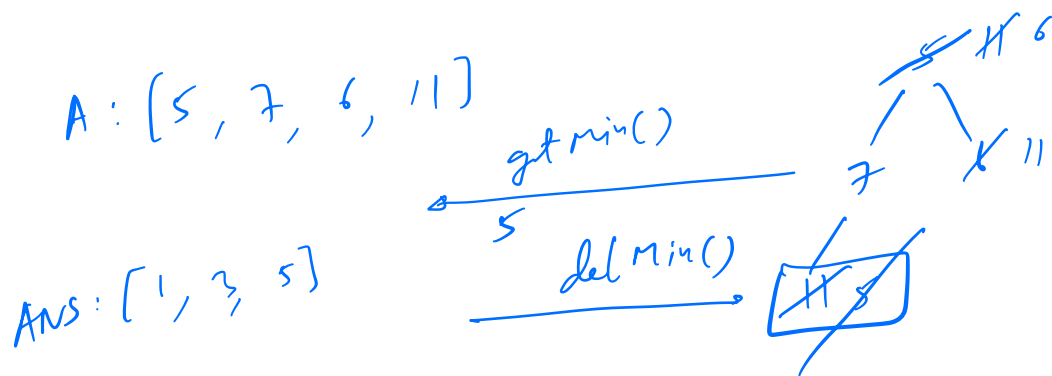


A: [3, 5, 6, 7, 11]

Ans: [1, 3]

get min()  $\rightarrow$   
del min()  $\rightarrow$



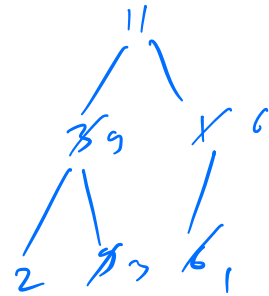


## II Create a MAX heap

$A: [11, 3, 1, 2, 9, 6]$

---

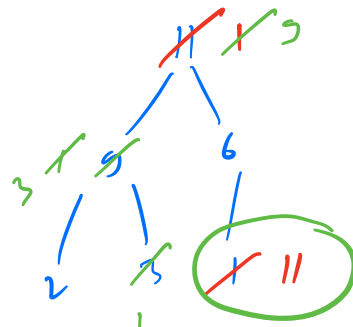
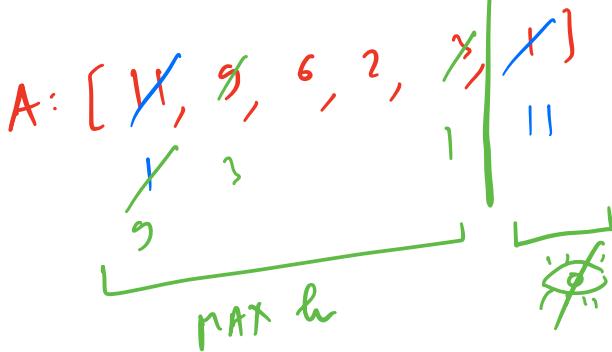
↓ MAX heap:



$A: [11, 9, 6, 2, 3, 1]$

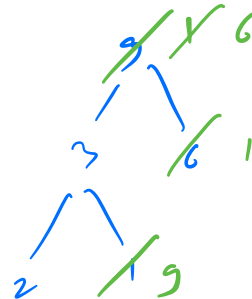
$N = A.size()$

del MAX() → N--



del MAX()

$A: [9, 3, 6, 2, 1, 11]$



$A: [6, 3, 1, 2, 9, 11]$





STEPS:

// A[]

1)  $N = A.size();$

2) Convert A[]  $\rightarrow$  MAX heap.

3)  $Swap(A[0], A[N-1]);$

4)  $N--;$

5)  $heapify(0)$

$del MAX()$   $\rightarrow$  N

$heapify()$   $\rightarrow$  N

$\times N$

$$TC = N + N \times \log N$$

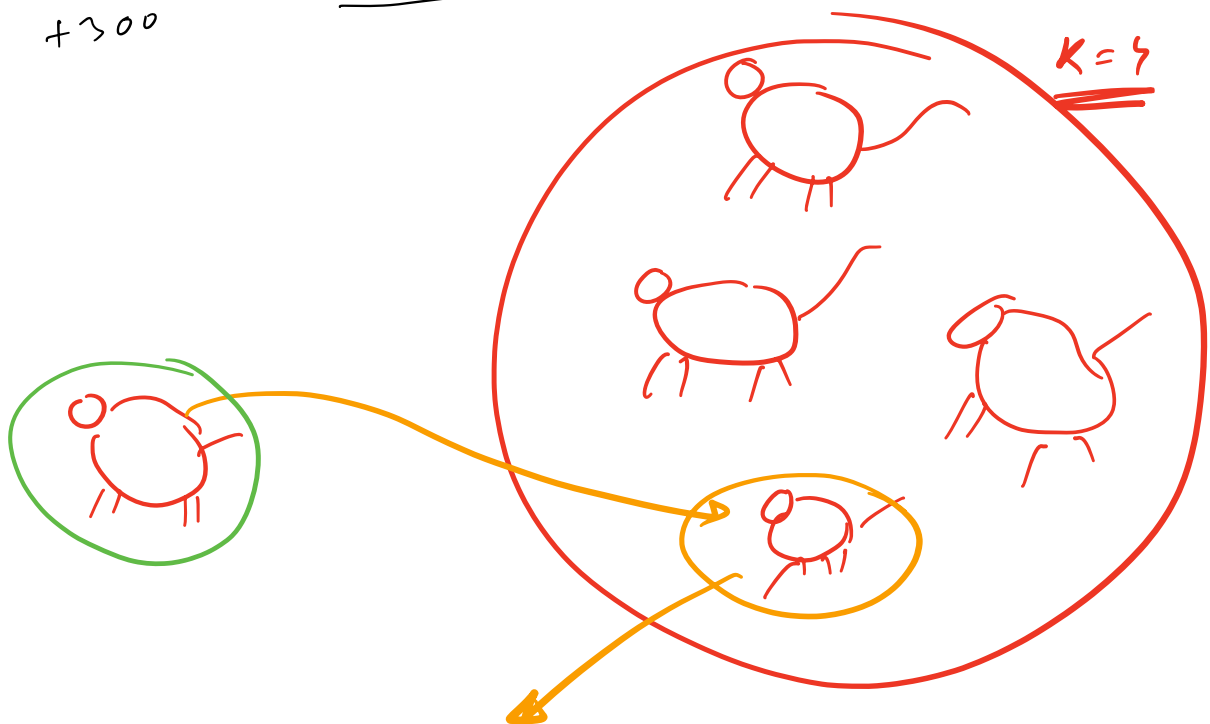
$$TC = O(N \log N)$$

$$SC = O(1)$$

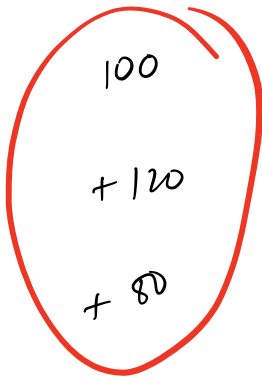
Q Given a stream of integers.  
At any point, give the largest K elements till that point!

K = 3

	<u>top K</u>
100	[ 100 ]
+ 120	[ 100, 120 ]
+ 80	[ 100, 120, 80 ]
+ 40	[ 100, 120, 80 ]
+ 200	[ 100, 120, 200 ]
+ 300	[ 300, 120, 200 ]

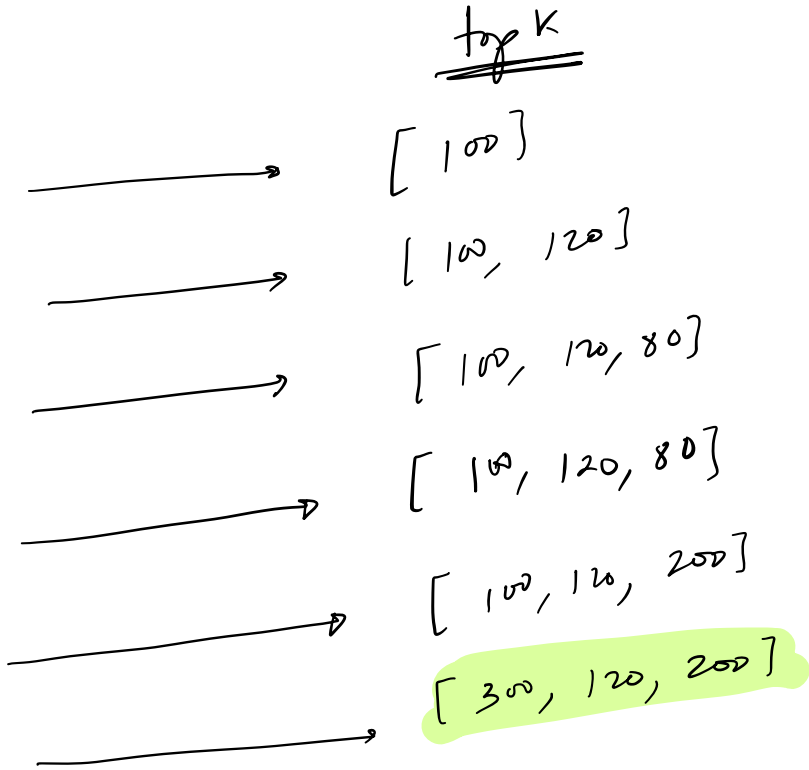
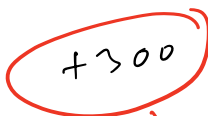


K = 3

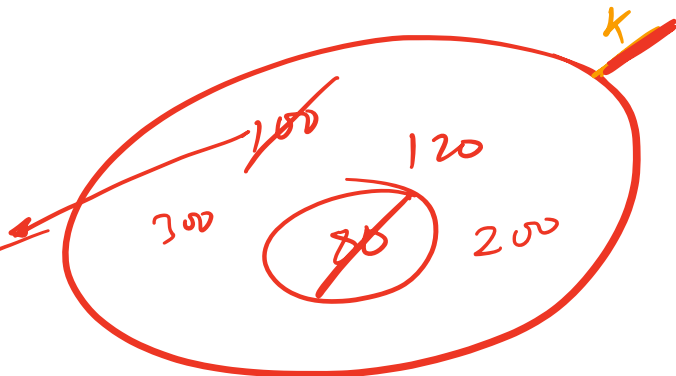


✗ + 40

✓ + 200



MIN Heap



A[] = [ - - - ]

## STEPS

+ x

```
if (MH.size() < K) {  
    MH.insert(x);
```

```
}  
else {
```

```
    if (MH.getMin() < x)  
        MH.delMin();  
    MH.insert(x);  
}
```

log(K)

getTopK()

return MH[];

NOTE : MH.size() would always  $\leq K$

SC :  $O(K)$   
 $O(1)$

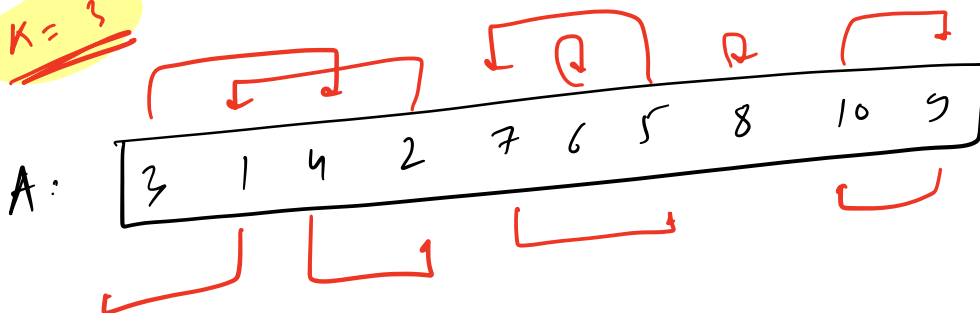


Q Given an almost sorted array.

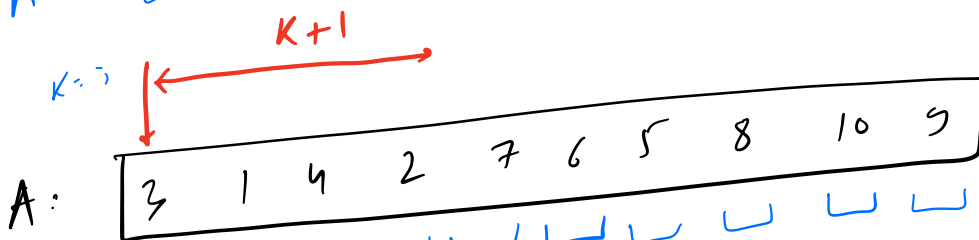
Sort it!

Every element is at MAX  
K distance from its sorted pos!

K = 3



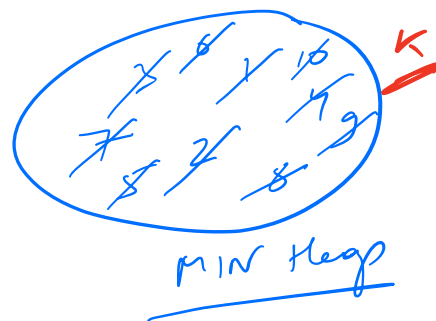
A: [ 1 2 3 4 5 6 7 8 9 10 ]



Sorted(A): [ 1 2 3 4 5 6 7 8 9 10 ]

get min()

(create a min heap of size K+1)  
→ O(K)

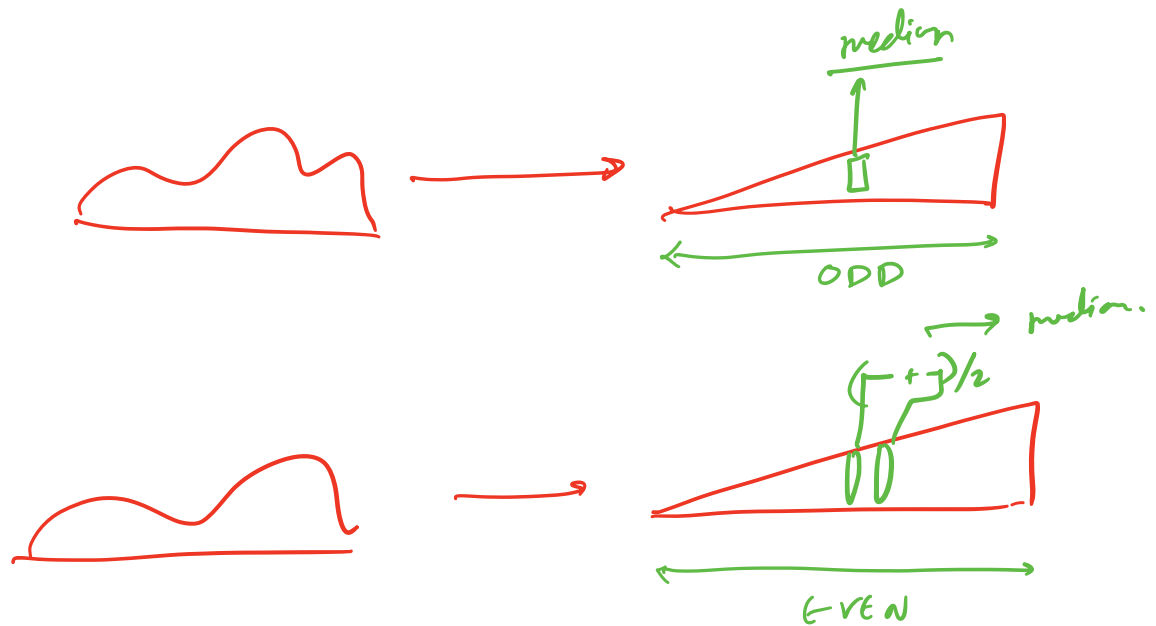


$N-1 \times \log(K)$

$T.C = O(N \log K)$

$S.C = O(K)$

Q Given a stream of integers.  
Find the median at every point!



+10 [10]  
+20 [10, 20]  
+1 [1, 10, 20]  
+7 [1, 7, 10, 20]  
+11 [1, 7, 10, 11, 20]

Median

10

15

10

8.5

10



(+2)

[ 1 2 ]

[ 10 20 ]

6

(+30)

[ 1 2 ]

[ 10 20 30 ]

10

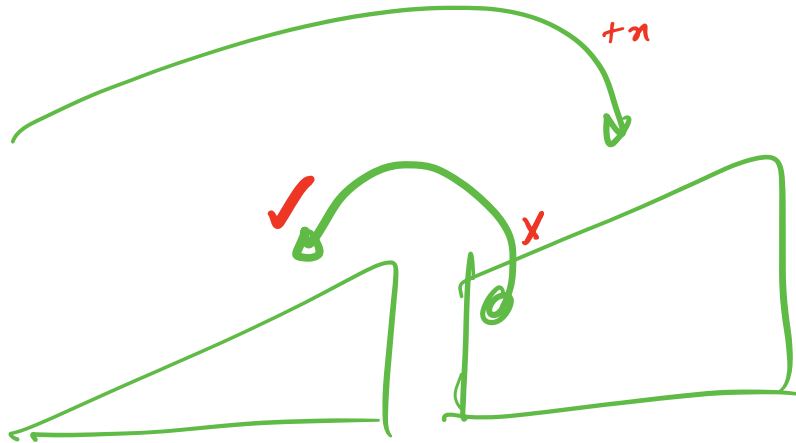
(+15)

[ 1 2 10 ]

[ 15 20 30 ]

12.5

(+ $\pi$ )



$T_C = O(\log N)$

$SL = O(N)$