Today's class? → Find # of iterations

- → TC & SC X
- → Asymptotic Analysis X
- → Big O X
- → TLE X

Quiz 1:

Sum of first N natural no's.

$$1 + 2 + 3 + \dots + N = \frac{N(N+1)}{2}$$

Quiz 2:

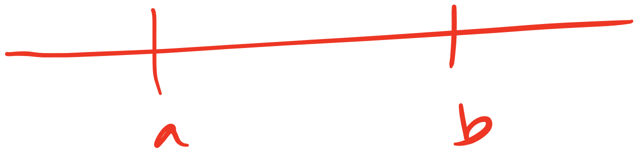
$[3, 10] \rightarrow 3, 4, 5, 6, 7, 8, 9, 10 \rightarrow 8$

[ ] → Inclusive

( ) → Exclusive

[a, b]	[a, b)	(a, b)
b-a+1	b-a	b-a-1

a+1 - - b-1



A.P : Arithmetic Progression

Series : $4 \quad 7 \quad 10 \quad 13 \quad 16 \quad 19 \quad 22$

$\underbrace{4 \quad 7}_{3} \quad \underbrace{7 \quad 10}_{3} \quad \underbrace{10 \quad 13}_{3} \quad \underbrace{13 \quad 16}_{3} \quad \underbrace{16 \quad 19}_{3} \quad \underbrace{19 \quad 22}_{3}$

In general : $\overset{1}{a} \quad \overset{2}{a+d} \quad \overset{3}{a+2d} \quad \overset{4}{a+3d} \quad - - \quad \overset{N}{a+(N-1)d}$

$$\boxed{\text{Sum of } AP_N = \frac{N}{2}\left[2a+(N-1)d\right]}$$

a : first term

d : common diff

N : no. of terms

$$\boxed{\log_a a^x = x}$$

GP : Geometric Progression

eg : $3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \quad 24 \quad 48 \quad 96$

General : $\overset{1}{a} \quad \overset{2}{a.r} \quad \overset{3}{ar^2} \quad \overset{4}{ar^3} \quad \_\_\_ \quad \overset{N}{ar^{N-1}}$

$$\boxed{\text{Sum of first } N \text{ terms of a GP} = a\left[\frac{r^N-1}{r-1}\right]}$$

→ $r \neq 1$

→ $r > 1$

$$a\left[\frac{1-r^N}{1-r}\right]$$

↓ $r < 1$

a : first term

r : common ratio

N : no. of terms

Q

```
void fun(int N){
    S=0
    f(i=1; i<=N; i++){
        S=S+i;
    }
    return S;
}
```

iterations

i : 1, 2, 3 _ N

i : [1, N]

$\boxed{\text{\# it} \rightarrow N}$

Q

```
void func(int N, int M){
    f(i=1; i<=N; i++){
        if(i%2==0){
            print(i);
        }
    }
    f(i=1; i<=M; i++){
        if(i%2==0){
            print(i);
        }
    }
}
```

i : 1,2 -- N

i : [1, N]

it → N

i : 1, 2, -- M

i : [1, M]

it → M

$\boxed{\text{total \# it} \rightarrow N + M}$

Q

```
int func(int N) {
    s=0
    f(i=1; i<=N; i=i+2) {
        s=s+i;

    }

}
```

$\underbrace{i=1, 3, 5, 7, -- N}$

$N=10 \rightarrow 1, 3, 5, 7, 9 \rightarrow 5$

$N=7 \rightarrow 1, 3, 5, 7 \rightarrow 4$

$(N+1)/2$

$\boxed{\#it \rightarrow (N+1)/2}$

odd no's [1, N]

Q

```
int func ( int N) {
    S = 0
    f(i= 0; i<=100; i++) {
        S= S + i + i^2
    }
    return S;
}
```

$\underbrace{i = 0, 1, 2, 3 \ldots, 100}$

$i : [0, 100]$

$\boxed{\# it \rightarrow 101}$

Q

```
void func (N) {
    f(i=1; i*i <=N; i++) {
        S= S+i^2
    }
    return S;
}
```

$i \times i <= N$

$i^2 <= N$

$i <= \sqrt{N}$

$f(i : 1 \rightarrow \sqrt{N})$

$i : 1, 2, -- \sqrt{N}$

$[1, \sqrt{N}]$

$\boxed{\# it \rightarrow \sqrt{N}}$

Q

```
void func(N) {
    i = N;
    while (i > 1) {
        i = i/2;
    }
}
```

iteration	$i \rightarrow N$
1	$N/2 \rightarrow N/2^1$
2	$N/4 \rightarrow N/2^2$
3	$N/8 \rightarrow N/2^3$
|	|
K	$1 = N/2^K$

$$1 = N/2^K$$

$$2^K = N$$

Take $\log_2$ on both sides

$$\log_2 2^K = \log_2 N$$

$$\boxed{K = \log_2 N}$$

$\#it \rightarrow \log_2 N$

$$\begin{tikzcd} N \arrow[r, bend left, "/2"] & N/2 \arrow[l, bend left, "\times 2"] \arrow[r, bend left, "/2"] & N/4 \arrow[l, bend left, "\times 2"] \arrow[r, dash, "/2"] & 1 \arrow[l, bend left, "\times 2"] \end{tikzcd}$$

Q

```
void func (N)
    s=0
    f(i=0; i<N; i=i*2){
        s=s+i;
    }
}
```

it	i:0
1	0
2	0
3	0
∞	0

$\boxed{\# it \rightarrow \infty}$

Q

```
void func (N) {
    s=0
    f(i=1; i<=N; i=i*2){
        s=s+i;
    }
}
```

it	i
1	2 → 2^1
2	4 → 2^2
3	8 → 2^3
4	16 → 2^4
K	$\boxed{N \rightarrow 2^K}$

$\boxed{K = \log_2 N}$

$\# it \rightarrow \log_2 N$

Q

```
void  func (N) {
    f (i=1; i<=10; i++) {
        f (j=1; j<=N; j++) {
            print(..);
        }
    }
}
```

i	j : [1,N]	#it
1	[1,N]	N
2	[1,N)	+ N
3	[1,N]	+ N
⋮	⋮	+ ⋮
10	[1,N]	+ N

$$\overbrace{N + N + \cdots + N}^{10}$$

$$\boxed{\#it = 10N}$$

Q

```
void  func (N) {
    f (i=1; i<=N; i++) {
        f (j=1; j<=N; j++){
            print(i*j);
        }
    }
}
```

i	j : [1,N]	#it
1	[1,N]	N
2	[1,N)	+ N
3	[1,N]	+ N
⋮	⋮	⋮
N	[1,N)	N

$$\boxed{\text{total } \#it \rightarrow N^2}$$

Q

```
void func(N) {
    f(i=0; i<N; i++) {
        f(j=0; j<=i; j++) {
            print(i+j);
        }
    }
}
```

i	j:[0,i]	# it
0	[0,0]	1
1	[0,1]	+ 2
2	[0,2]	+ 3
		+
N-1	[0,N-1]	N

$\boxed{\# it \rightarrow N(N+1)/2}$

Q

```
void func(N) {
    f(i=1; i<=N; i++) {
        f(j=1; j<=N; j=j*2) {
            print(i+j);
        }
    }
}
```

i	j:[1,N]	#it
1	1 2 4 . . N	$\log_2 N$
2	1 2 4 8 . . N	+ $\log_2 N$
3		+
N	1 2 4 8 - - N	+ $\log_2 N$

$\boxed{\# it = N \log_2 N}$

Q

```
void func(N) {
    f(i=1; i<=2^N; i++){
        print(i);
    }
}
```

$i: [1, 2, 3, 4 \ldots 2^N]$

$i: [1, 2^N]$

$$\boxed{\# it \rightarrow 2^N}$$

Q

```
void func(N){
  f(i=1; i<=N; i++){
     f(j=1; j<=2^i; j++){
        print(i*j);
     }
  }
}
```

i	$j: [1, 2^i]$	$\# it$
1	$[1, 2^1]$	2^1
2	$[1, 2^2]$	$+ 2^2$
3	$[1, 2^3)$	$+ 2^3$
|	|	$+$ |
N	$[1, 2^N]$	$+ 2^N$

$$2^1 + 2^2 + 2^3 + \text{——} + 2^N$$

$$\text{Sum} = a\left[\frac{r^N - 1}{r - 1}\right]$$

$$2\left[\frac{2^N - 1}{2 - 1}\right] = 2\left[\frac{2^N - 1}{1}\right]$$

$$\boxed{\# it \rightarrow 2^{N+1} - 2}$$

Q

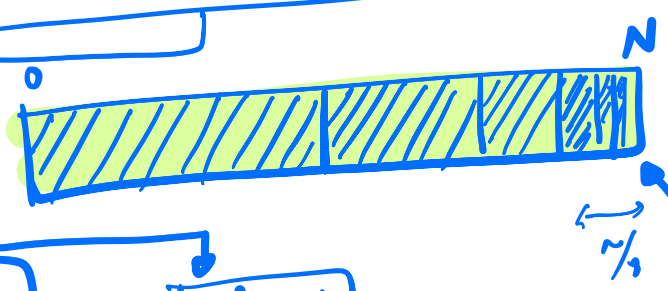
```
f(i=N; i>0; i=i/2){
    f(j=1; j<=i; j++){
        print(i*j);
    }
}
```

	i	j:[1, i]	#it
1	N	[1, N]	N
2	N/2	[1, N/2]	+ N/2
3	N/4	(1, N/4]	+ N/4
			+
log	1	[1, 1)	+ 1

$N + N/2 + N/4 + N/8 + ——— 1$

Sum of GP: a: N
r: 1/2
N: $\log_2 N$ → HW

$[N] + [N/2 + N/4 + N/8 + - - - + 1]$



$N + \boxed{N-1}$ → $2N-1$

Compares

$$\boxed{\log N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < N^3 < 2^N}$$

$N!$ HW !