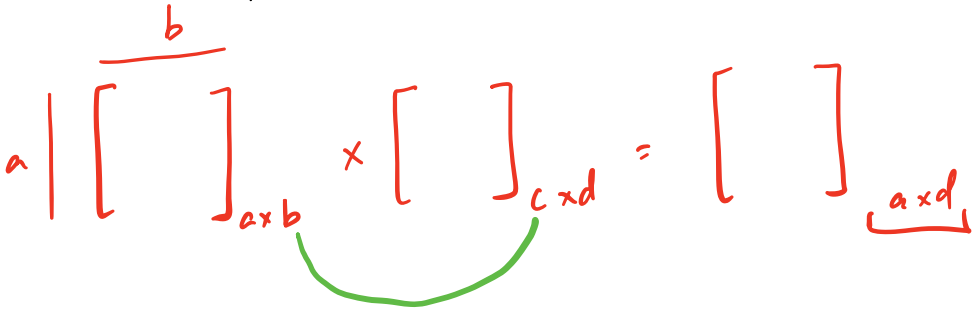
Matrix Chain Multiplication



multiplication is possible only if $b == c$

if $M_1 \times M_2$ is possible

is $M_2 \times M_1$ ——— ? → NO

$M_1 \times M_2 = \checkmark$

$2 \times 3 \quad 3 \times 5$

$M_2 \times M_1 = X$

$3 \times 5 \quad 2 \times 3$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3} \times \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}_{3\times3} = \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}_{2\times3}$$

$ax_1+by_1+cz_1$ $\quad ax_2+by_2+cz_2$

1 element → 3 *

All ——— → 2x3x3 *

$$\underset{a\times b}{[\ ]} \times \underset{b\times c}{[\ ]} = \underset{a\times c}{[\ ]}$$

No. of multiplication req'd = $a \times b \times c$

Cost of multiplying matrix = $a \times b \times c$

$$\underset{5\times 6}{[\ ]}\ \underset{6\times 12}{[\ ]} = \underset{5\times 12}{[\ ]}$$

Cost of multiplying = $5 \times 6 \times 12$

Q Given N matrices. Find the minimum cost of multiplying them!

M_1 (5×4) × M_2 (4×6) × M_3 (6×2) × M_4 (2×7)

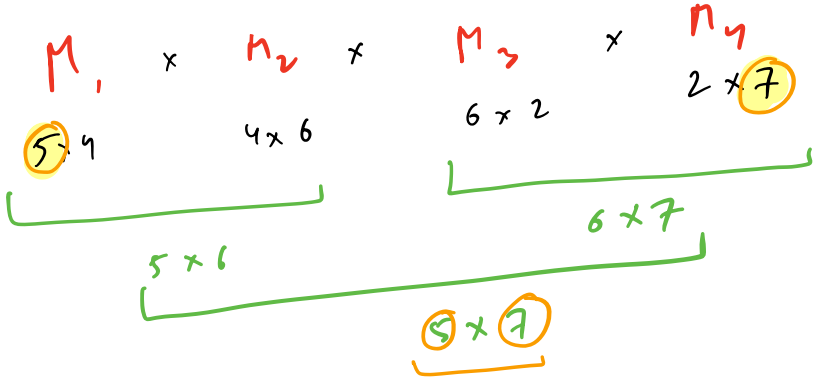
5×6

5×2

5×7

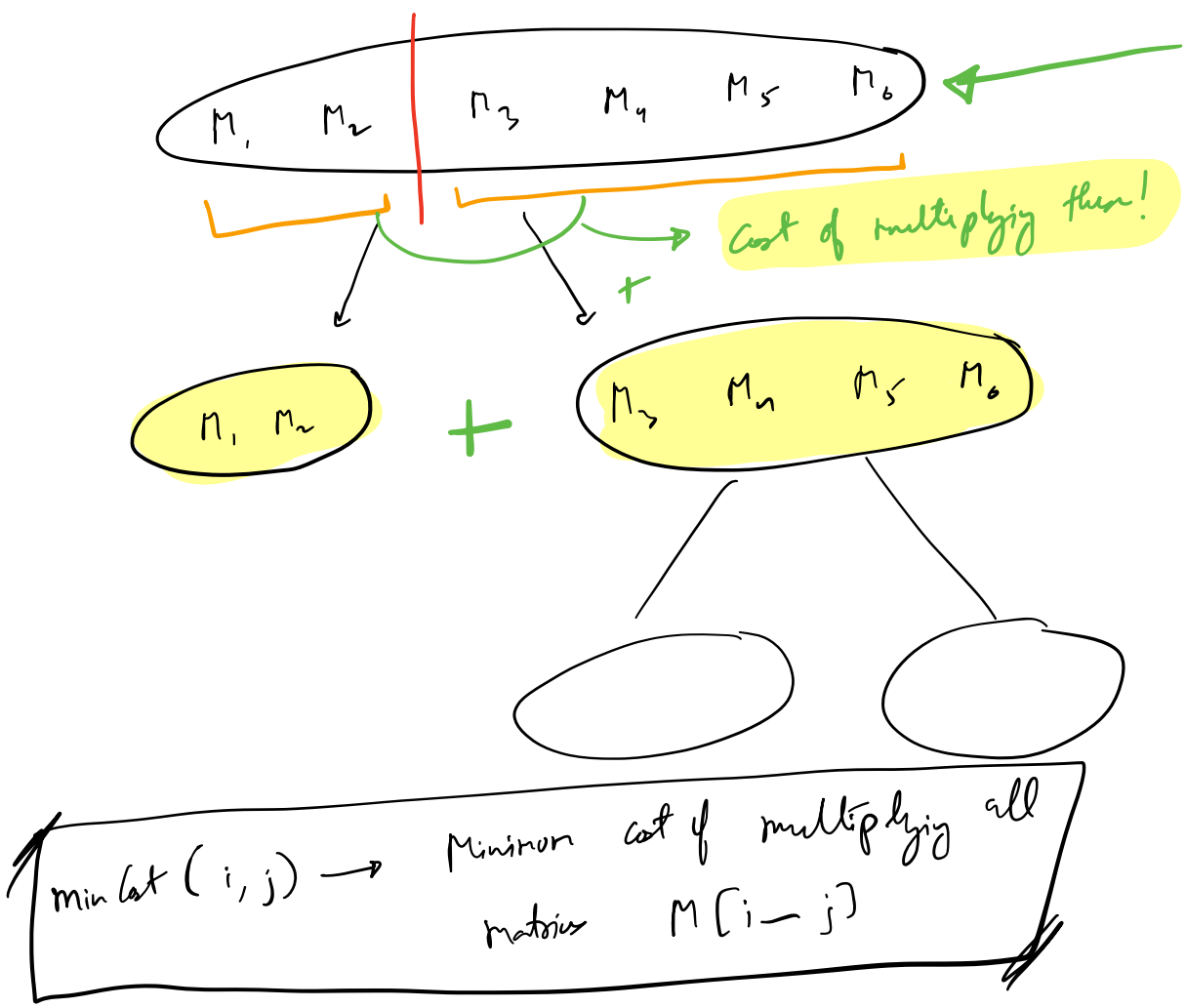
Cost = $5\times4\times6 + 5\times6\times2 + 5\times2\times7 = 120 + 60 + 70$

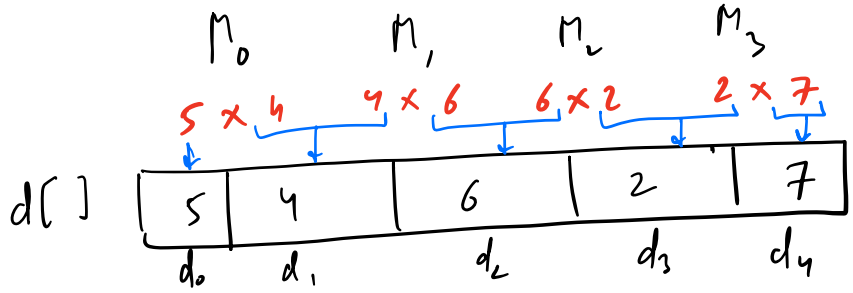
$= 250$ //



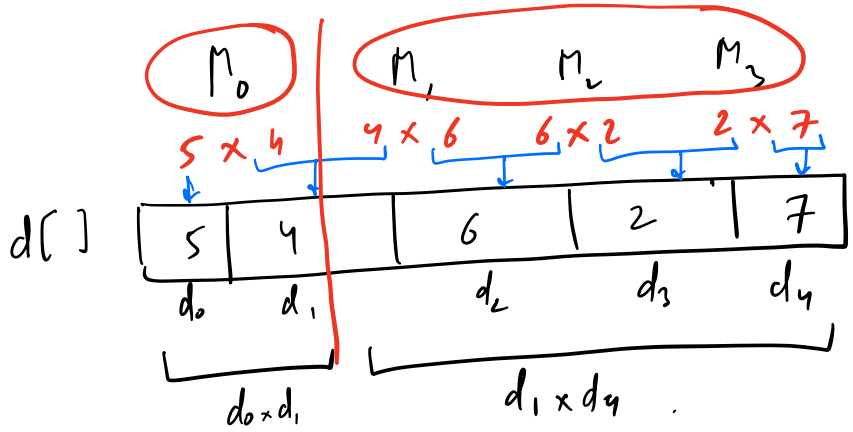
Cost: $5 \times 4 \times 6 + 6 \times 2 \times 7 + 5 \times 6 \times 7$

$= 120 + 84 + 210 = 414$ //

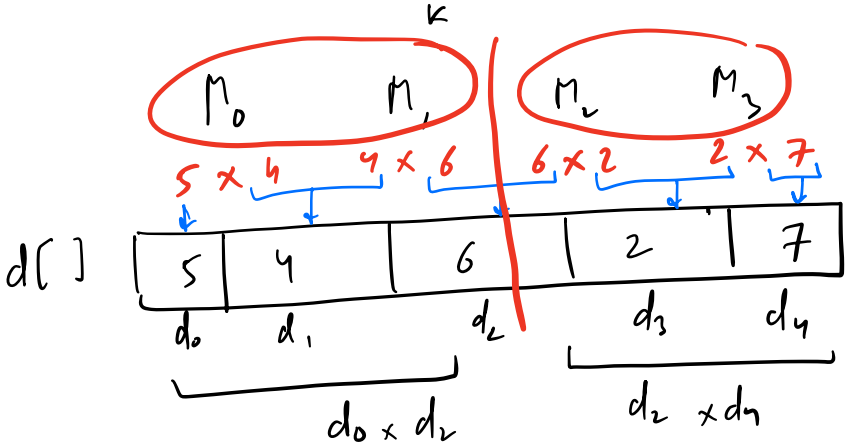




1)



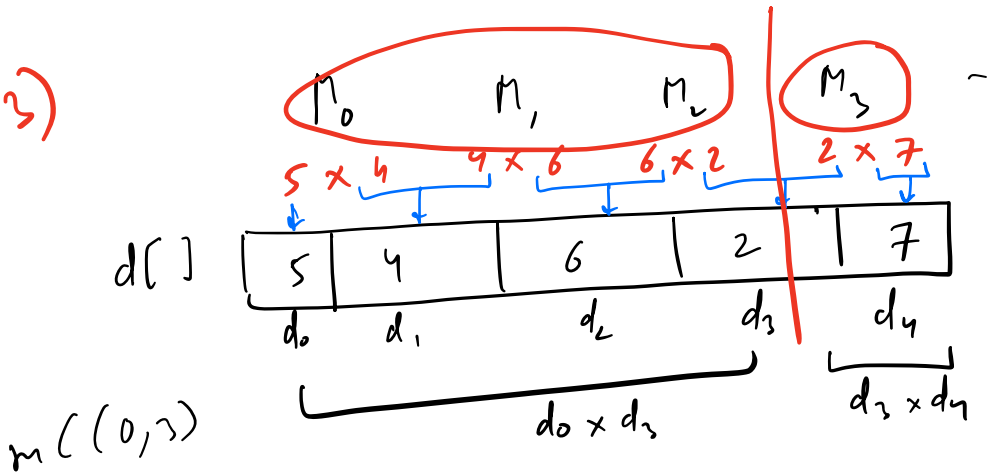
$= m((0,0) + m(1,3) + d_0 \times d_1 \times d_4$



2)

$= m((0,1) + m(2,3) + d_0 \times d_2 \times d_4$

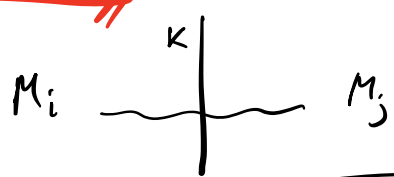
3)



$m(0,3)$

$= m(0,2) + m(3,3) + d_0 \times d_3 \times d_4$

$$m(0,3) = \min \begin{cases} 1) \\ 2) \\ 3) \end{cases}$$



$$m(i,j) = \min_{K=i}^{j-1} m((i,k) + m(k+1,j) + d_i \times d_{k+1} \times d_{j+1}$$

$[0-N-1]$ $[0-N-1]$

#VS $\rightarrow N^2$

TRPS $\rightarrow N$

ANS → mc(0, N-1)

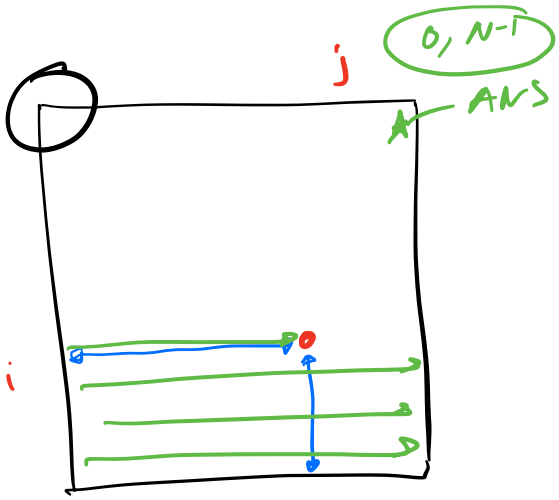
TC = O(N³)

SC = O(N²)

```
// d[] ;
int dp[N][N] = {-1};
int mc(i, j) {
    if(i >= j) ret 0;

    if(dp[i][j] != -1) ret dp[i][j];

    ANS = ∞;
    f(K = i ⟶ j-1) {
        ANS = min(ANS, mc(i, K) + mc(K+1, j)
                                + d_i d_{K+1} d_{j+1});

    }
    dp[i][j] = ANS;
    ret ANS;

}
```

Mx

Bottom Up



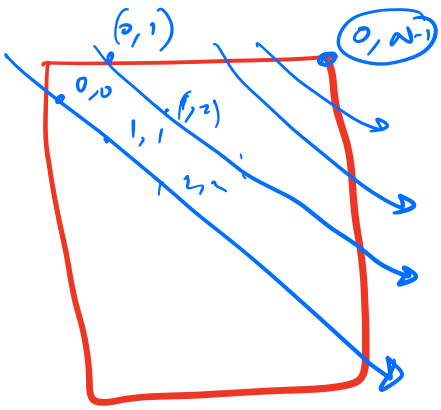
K

i → j-1

(i, K) (K+1, j)

TC = O(N³)

SC = O(N²)

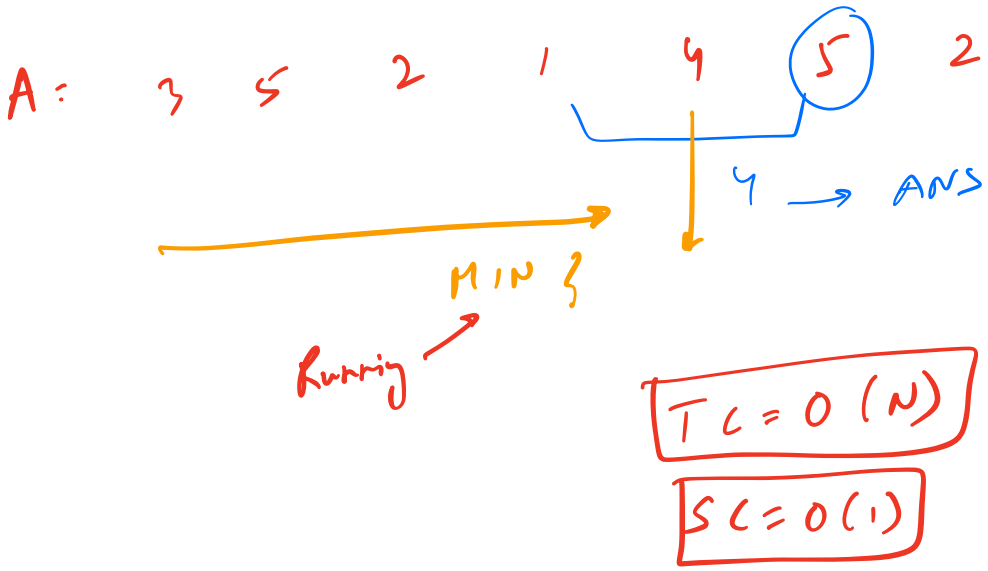


Q Given stock price on N days.

You can buy on any day & sell on any day after (>=) it.

Once !

MAX profit ?



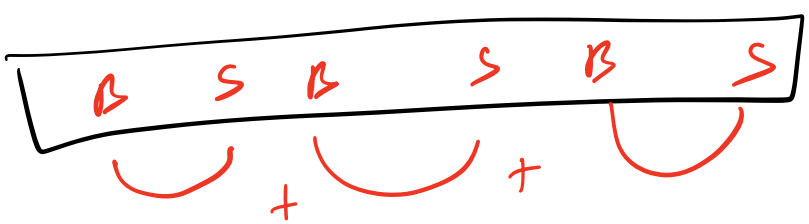
$TC = O(N)$

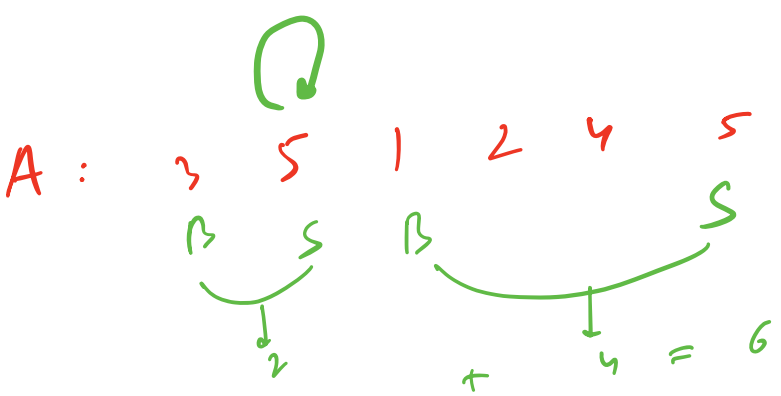
$SC = O(1)$

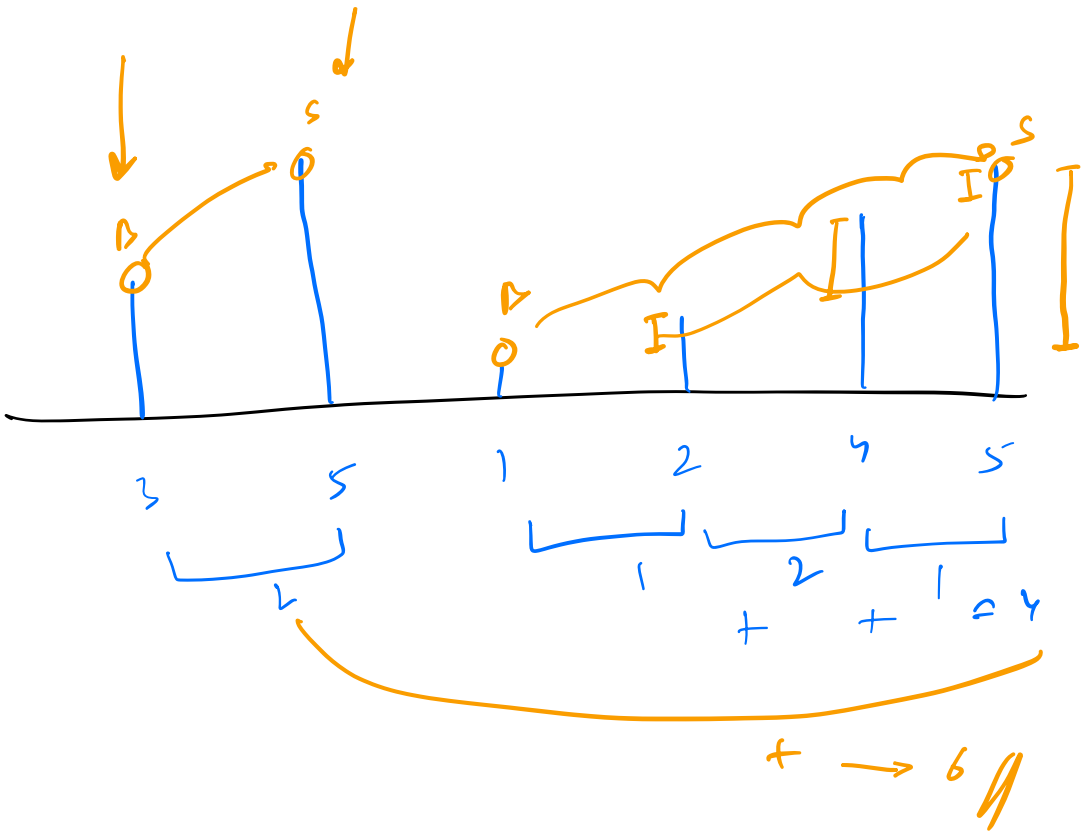
Q SAME QUES.

ALLOWED to do ANY NO. of Buy & sell!

Once you buy, you have to sell before another day!





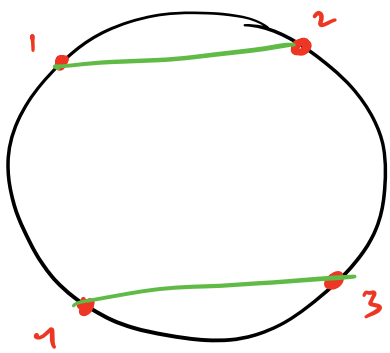


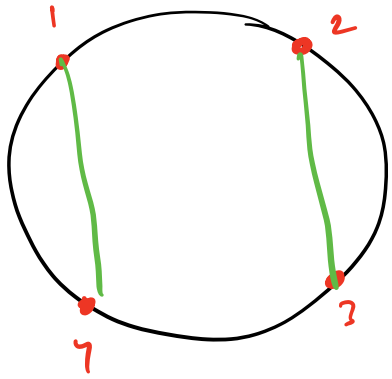
TC = O(N)

SC = O(1)

Q Find the no. of ways of drawing N chords in a circle with $2N$ points on it's circumference. s.t. No 2 chords intersect!

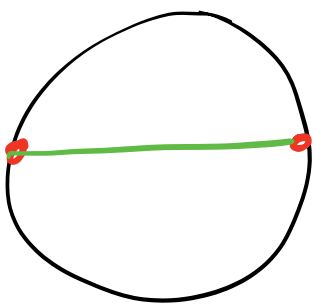
$N = 2$





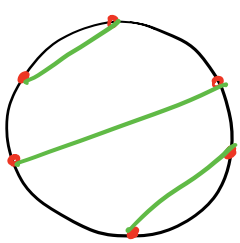
→ 2

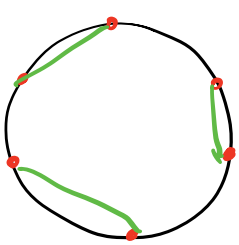
$N = 1$

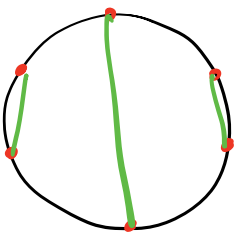


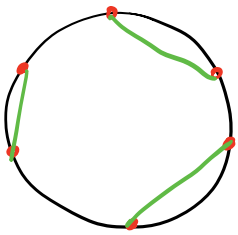
→ 1

$N = 3$

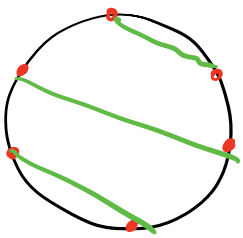




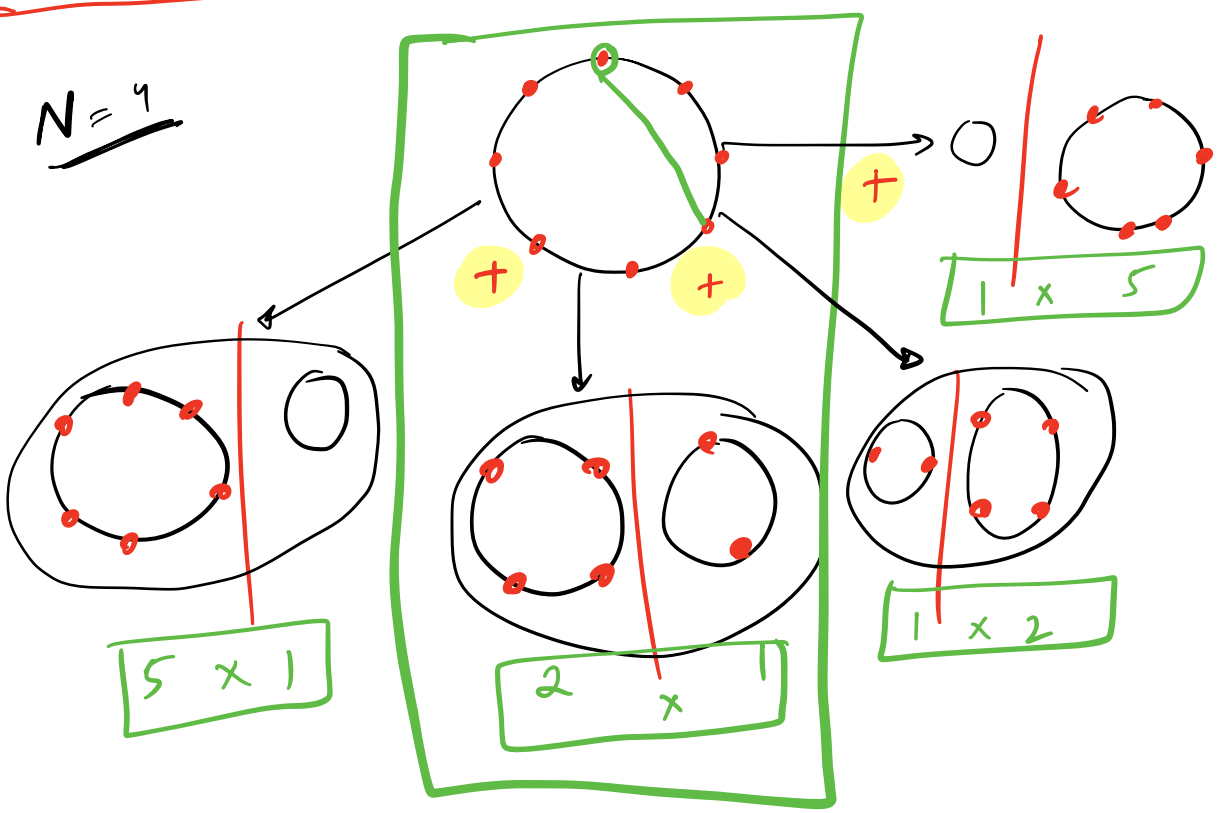


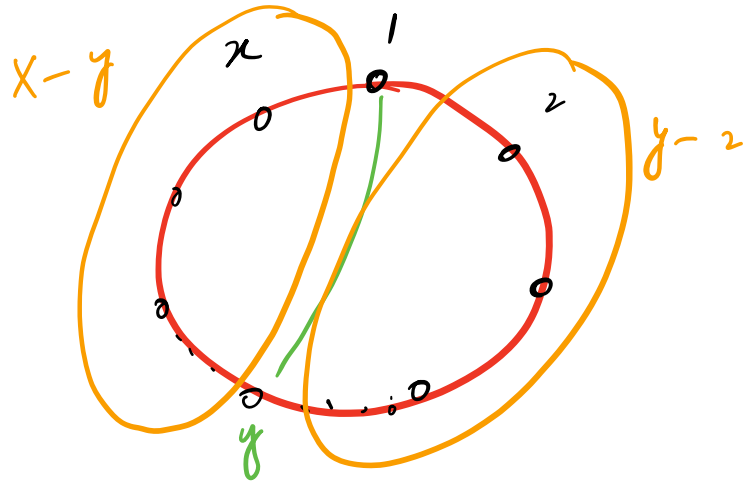


7 5



N = 4





$ways(n) \longrightarrow \sum_{y=2}^{y=n} ways(y-2) \times ways(n-y)$

n: even

[y: even]

N^2

BC

$N = 0 \rightarrow ANS = 1$

HW

CATALAN NO's

$\frac{2N C_N}{N+1}$

$O(N)$!