

$\text{gcd}(a, b) \rightarrow [1, \min(a, b)]$

$\text{TC} = \min(a, b)$

```

f(i = min(a, b); i >= 1; i--) {
    if (a % i == 0 && b % i == 0) {
        return i;
    }
}

```

④ LCM  
Least Common Multiple

$$\text{LCM}(16, 24) \rightarrow 48$$

16: 16 32 48 64

24: 24 48 - - -

$$\text{LCM}(1, 10) \rightarrow 10$$

1: 1 2 3 4 -

10: 10 20 30

$$\text{LCM}(8, 7) \rightarrow 56$$

$$\text{LCM}(a, b) \rightarrow [\max(a, b), a \times b]$$

$$\boxed{\text{LCM}(a, b) \times \text{GCD}(a, b) = a \times b}$$

$$a: 10$$

$$b: 8$$

$$\text{gcd}(10, 8) = 2 \rightarrow 10 \times 8$$

$$\text{LCM}(10, 8) = 40$$

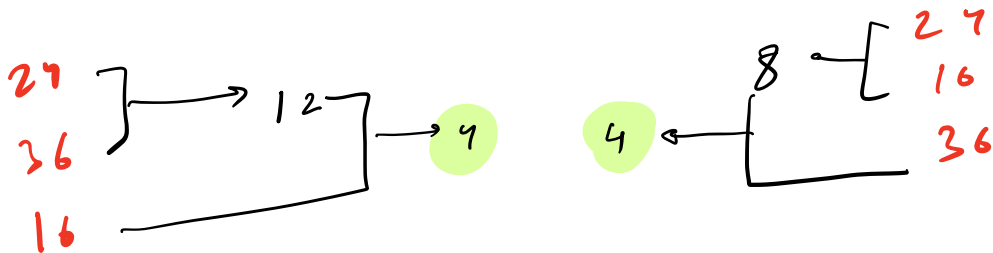
$$\boxed{\text{LCM}(a, b) = \frac{a \times b}{\text{GCD}(a, b)}}$$

$\rightarrow O(\min(a, b))$

④ Properties of GCD

①  $\text{GCD}(a, b) = \text{GCD}(b, a)$

②  $\text{GCD}(a, \text{GCD}(b, c)) = \text{GCD}(\text{GCD}(a, b), c)$



③  $\boxed{\text{GCD}(a, 0) = a}$

$2 \times 0 = 0$

0: 1, 2, 3, 4, ~~5~~  $\infty$   
 a: 1, —, —, ~~a~~

④  $\boxed{\text{GCD}(a, 1) = 1}$

a: 1, —, —, a  
 1: 1

# EUCLID →

$$\boxed{GCD(a, b) = GCD(a-b, b)} \quad \boxed{a \geq b}$$

$$a = x \cdot K_1$$

$$b = x \cdot K_2$$

$$a-b = (K_1 - K_2) \cdot x$$

$$b = x \cdot K_2$$

$$\begin{aligned} &gcd(75, 25) \\ &gcd(75-25, 25) \\ &gcd(50, 25) \\ &gcd(50-25, 25) \\ &gcd(25, 25) \\ &gcd(25-25, 25) \\ &gcd(0, 25) \end{aligned}$$

↓

25

$$\begin{aligned} &gcd(48, 32) \\ &gcd(48-32, 32) \\ &gcd(16, 32) \\ &gcd(32-16, 16) \\ &gcd(16, 16) \\ &gcd(16-16, 16) \\ &gcd(0, 16) \end{aligned}$$

↓

16 ✓

$$\begin{aligned} \textcircled{1} \quad &gcd(10, 7) \\ &gcd(10-7, 7) \\ &gcd(3, 7) \\ &gcd(7, 7-3) \\ &\quad (3, 4) \\ &\quad (7, 4-3) \end{aligned}$$

$$(3, 1) \quad (2, 1) \quad (1, 1) \quad (0, 1)$$

10. -

$$\begin{aligned} &\textcircled{1} \\ &(0, 1) \\ &(1, 1) \end{aligned}$$

①  $\gcd(100, 2)$

$(98, 2)$

$(96, 2)$

$(94, 2)$

$\vdots$

$(0, 2)$

$\downarrow$

$(2)$

50

$\gcd(a, b)$   $a > b$

$\gcd(a - b, b)$

$\gcd(a - 2b, b)$

$\gcd(a - 3b, b)$

$\vdots$

$(a \% b, b)$

$a \% b$

$\gcd(100, 7)$

$\gcd(100 \% 7, 7)$

$\gcd(2, 7)$

$\gcd(2, 7 \% 2)$

$(2, 1)$

$(2 \% 1, 1)$

$(0, 1)$

$\downarrow$

$(1)$

$\boxed{a > b}$

$\boxed{\gcd(a, b) = \gcd(a \% b, b)}$

$(45, 35)$

$(45 \% 35, 35)$

$(10, 35)$

$(10, 35 \% 10)$

$(10, 5)$

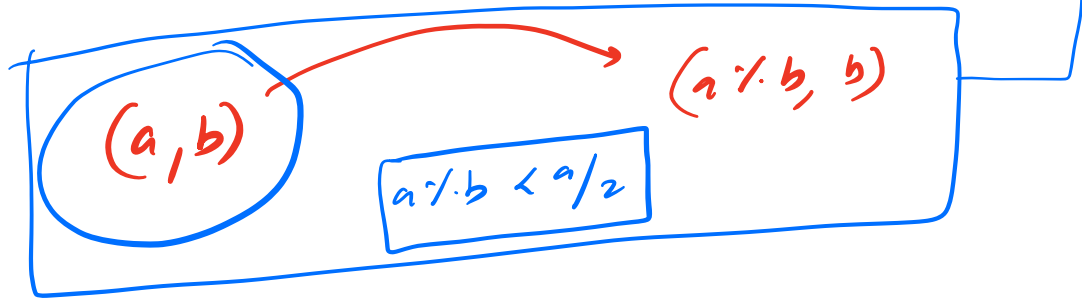
$(10 \% 5, 5)$

$(0, 5)$

$\rightarrow 5$

$\text{gcd}(a, b) \rightarrow \text{gcd}(a \% b, b) : a \geq b$

SL  $\rightarrow$   $\boxed{\text{TC} = O(\log(\min(a, b)))}$



CODE

```
int gcd(a, b) {
    if (a == 0) return b;
    if (b == 0) return a;
    if (a > b)
        return gcd(a % b, b);
    else
        return gcd(a, b % a);
}
```

```
int gcd(a, b) {
    if (b == 0) return a;
    return gcd(b, a % b);
}
```

4, 8

(8, 4)

8, 4  
8 % 4 = 0  
(8, 4)

Q Given an Array. Find the GCD of ALL no's.

$A: [32, 64, 12, 16] \rightarrow$

$g = 0;$

$f(i=0 \rightarrow N-1) \{$

$g = \text{gcd}(g, A[i]);$

$\}$

$\text{ret } g;$

$N \leq 10^5$

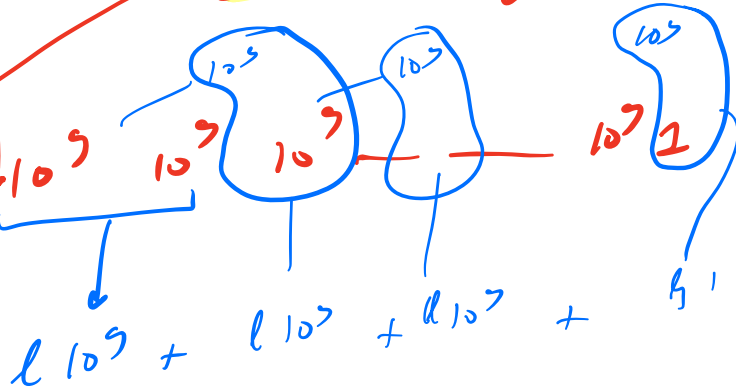
$A[i] \leq 10^3$

$(\times 10^6)$

$TC = O(N \cdot \log(\text{MAX}(A)))$

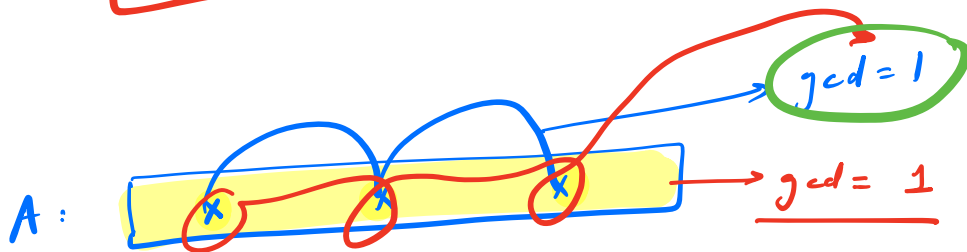
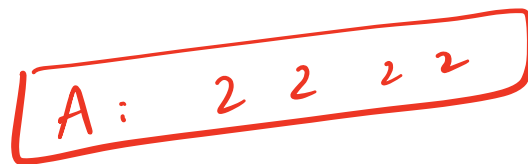
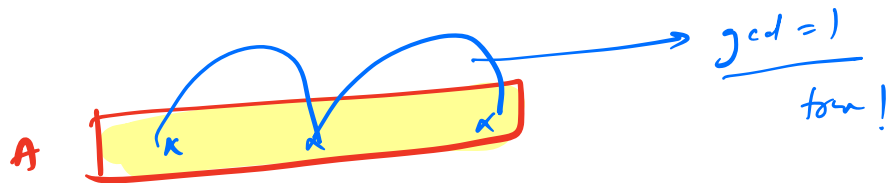
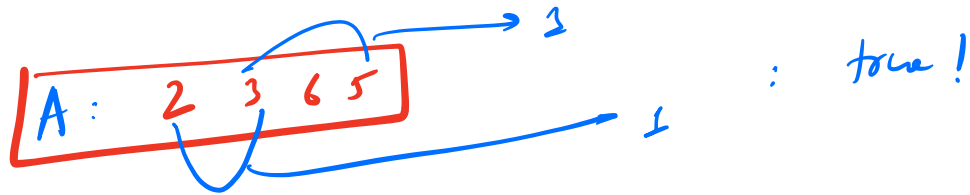
$SC = O(\log(\text{MAX}(A)))$

$O(1)$   
: iterative!



$N \cdot \log$

Q Given an array.  
check if any sub-sequence which has  $\gcd = 1$ .



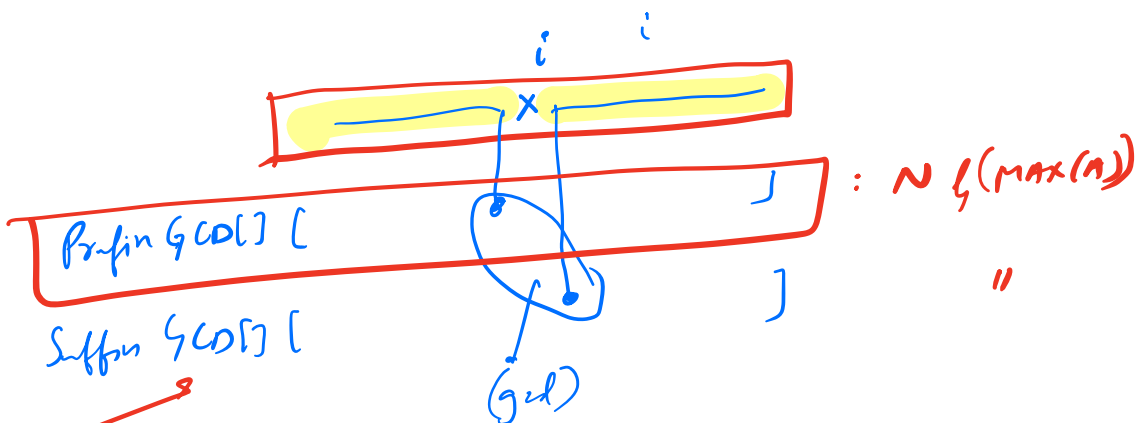
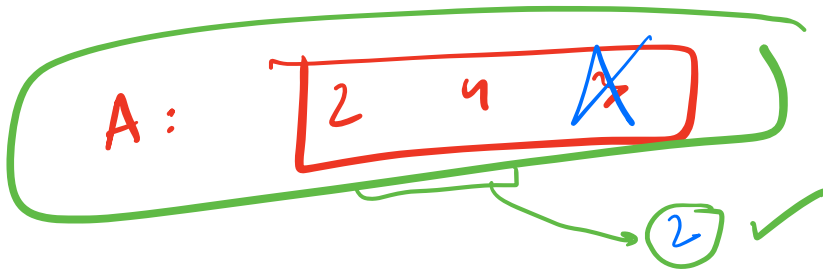
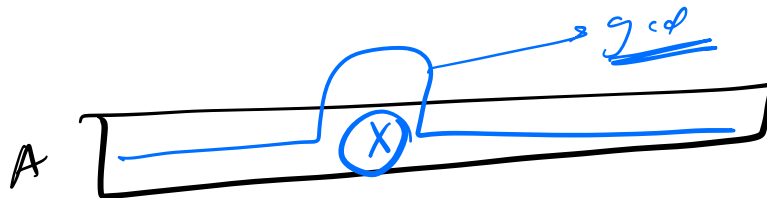
Idea: If  $\gcd(A[i]) \rightarrow 1$   
 $\Rightarrow$  there exists a sub-sequence with  $\gcd = 1$

$$TC = O(N \cdot \log(\max(A)))$$

$$SC = O(\log(\max(A)))$$



Q Given an Array. Delete EXACTLY 1 element,  
 S.t. the GCD of remaining elements is MAXIMIZED.  
 $\Rightarrow$  find the MAX such GCD of the rem. arr!



GCD after removing  $i^{\text{th}}$  element =  $\text{GCD}(\text{PrefixGCD}[i-1], \text{SuffixGCD}[i+1])$

TC =  $O(N \log \max(A))$

SC =  $O(N)$

$N \cdot \log(\max(A))$

PUB4

Q

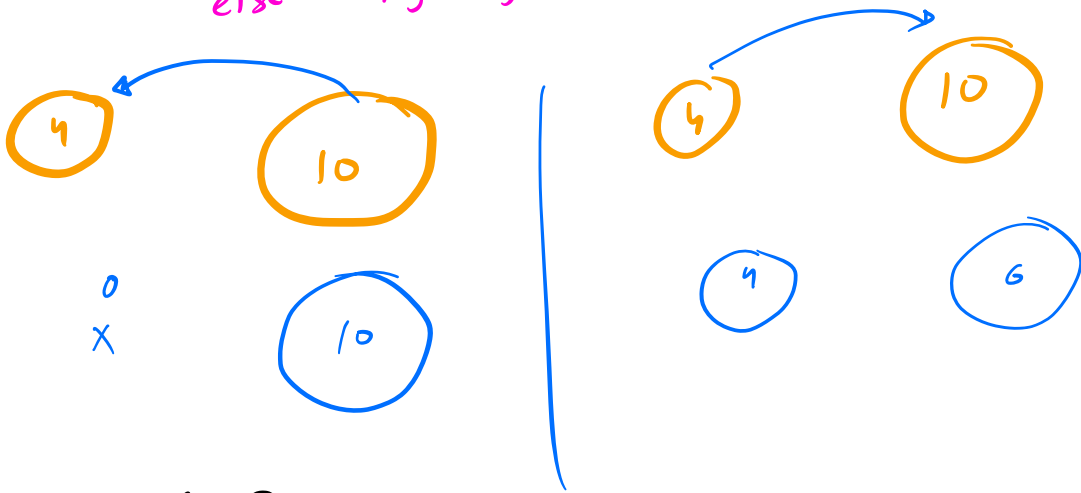
$N$  people & their healths are given.

$A[i] \rightarrow$  health of the  $i^{\text{th}}$  person!

$i^{\text{th}}$  person attacks the  $j^{\text{th}}$  person:

if  $(A_i \geq A_j) : A_j = 0 \rightarrow j \rightarrow \text{die!}$

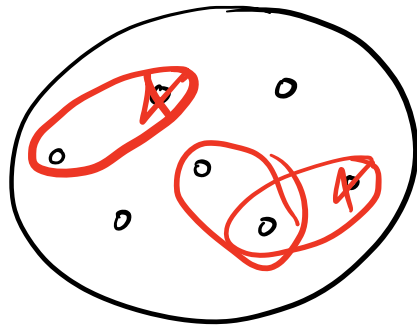
else  $A_j = A_j - A_i$

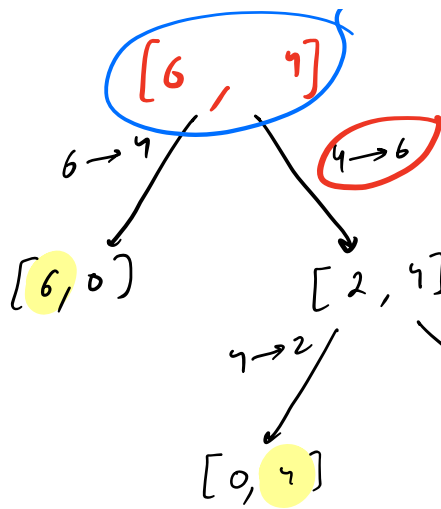


You're the GOD

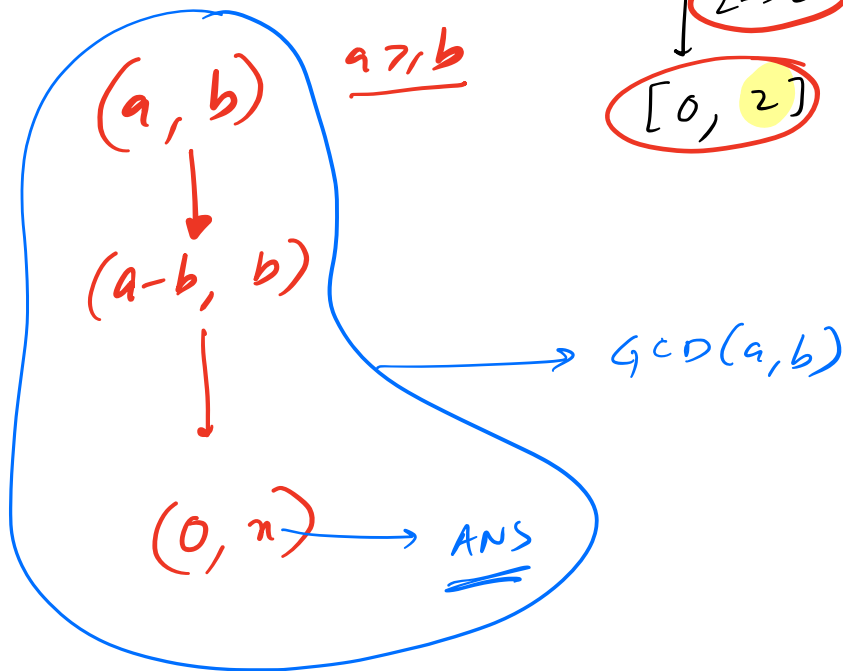
GOAL

find the MIN health  
of the last person standing!

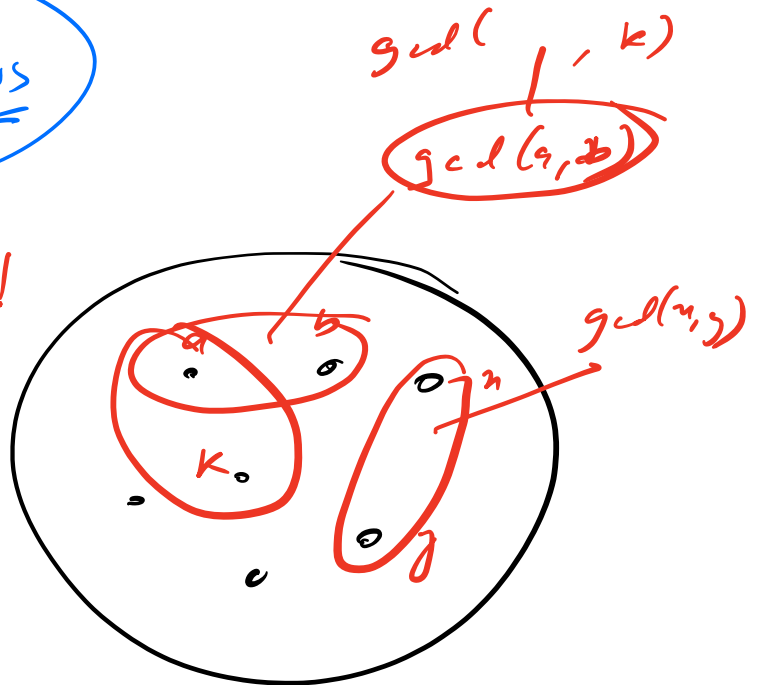




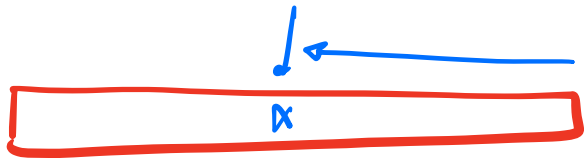
Idea: small guy  
hit large  
guy!



ANS = GCD of ALL!



A :



PG :

