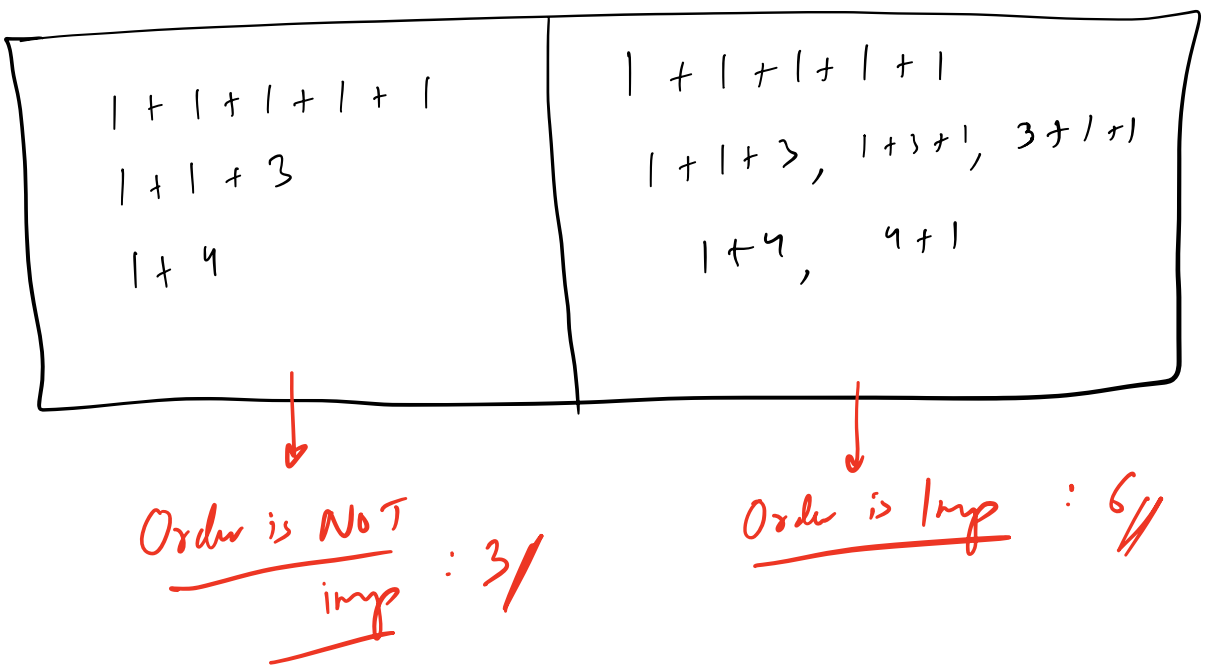
Q Given M different types of coins & their infinite copies!

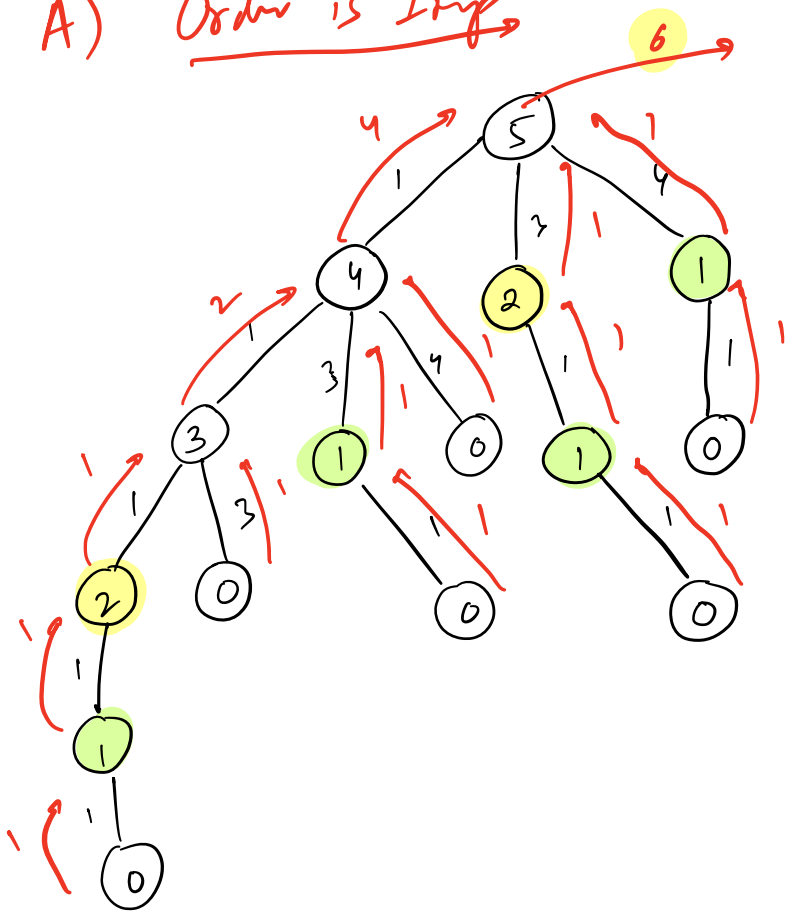
In how many ways can you make N sum using these coins!

A: [1, 3, 4] : M=3

N: 5



A) Order is Imp



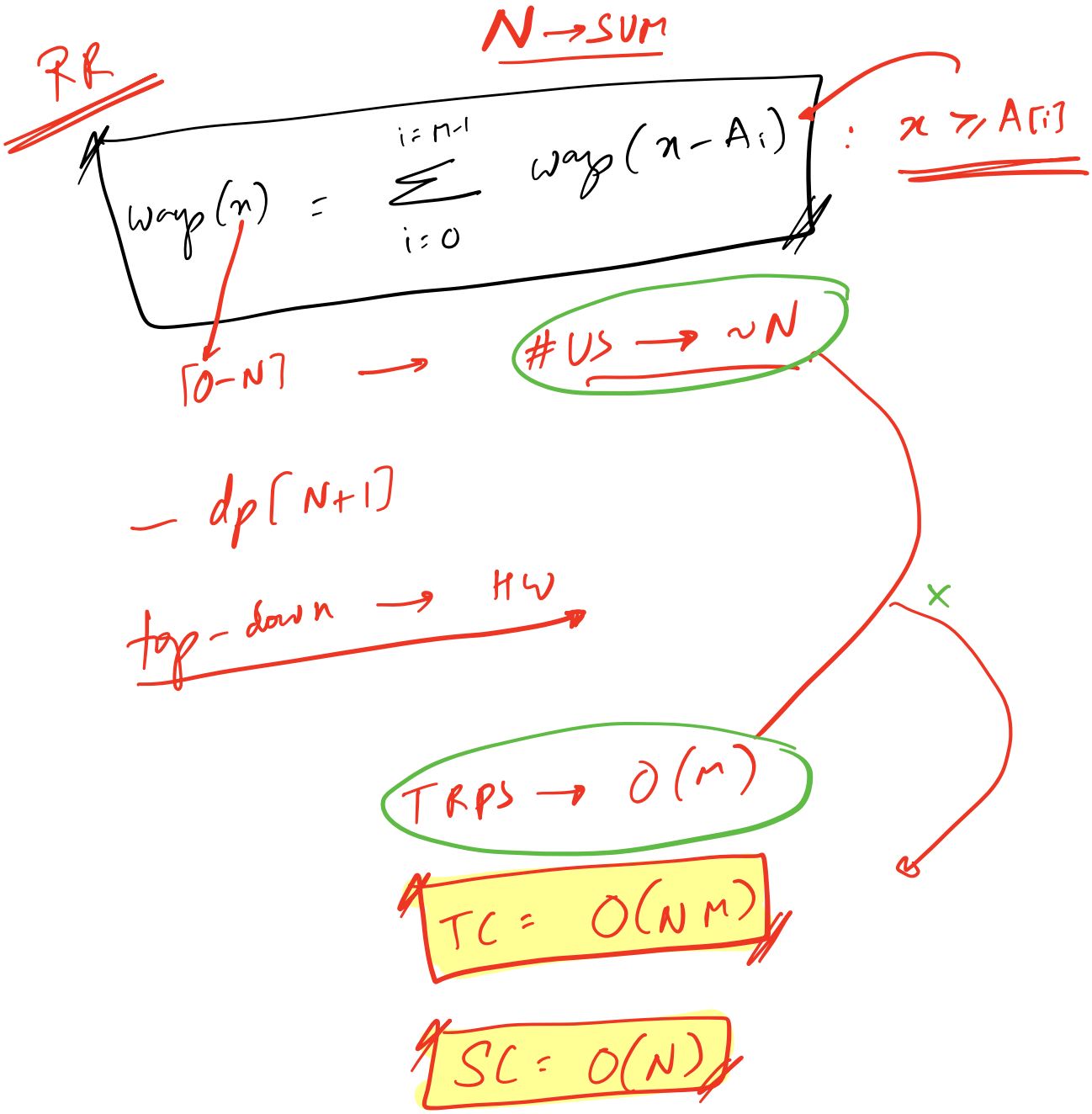
✓ Overlapping subproblems

✓ Optimal S.S.

⟹ DP !!!

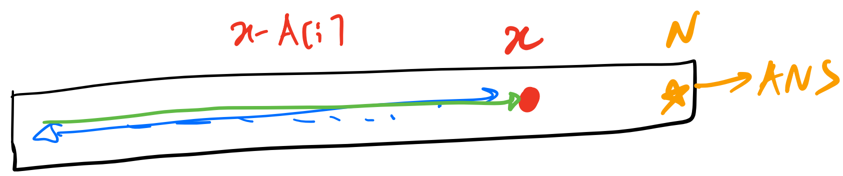
ways(x) → # of ways of making x a sum using any of the coins considering that the order is Imp

$$ways(x) = ways(x - A_0) + ways(x - A_1) + \dots + ways(x - A_{M-1})$$


RR
N → SUM
ways(n) = Σ_{i=0}^{i=n-1} ways(n - A_i)
: n >= A[i]
[0-N] →
#US → ~N
- dp[N+1]
top-down → HW
X
TRPS → O(n)
TC = O(N M)
SC = O(N)

Bottom Up

1, 2, 3, 5, 7 - -




```
for (i = 1 → N)        → N
{
    RR                 → M
}
ret dp[N];
```

TC = O(NM)

SC = O(N)

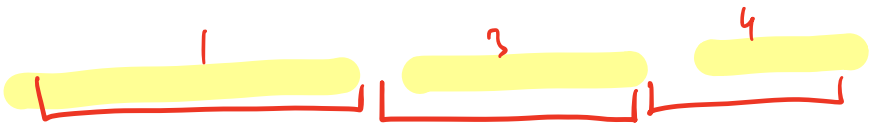
B) Order is NOT important

N = 5   A: [1, 3, 4]

1 + 1 + 1 + 1 + 1
1 + 1 + 3
1 + 4

: 3

Sequence pattern :



N = 5

1 + 1 + 1 + 1 + 1

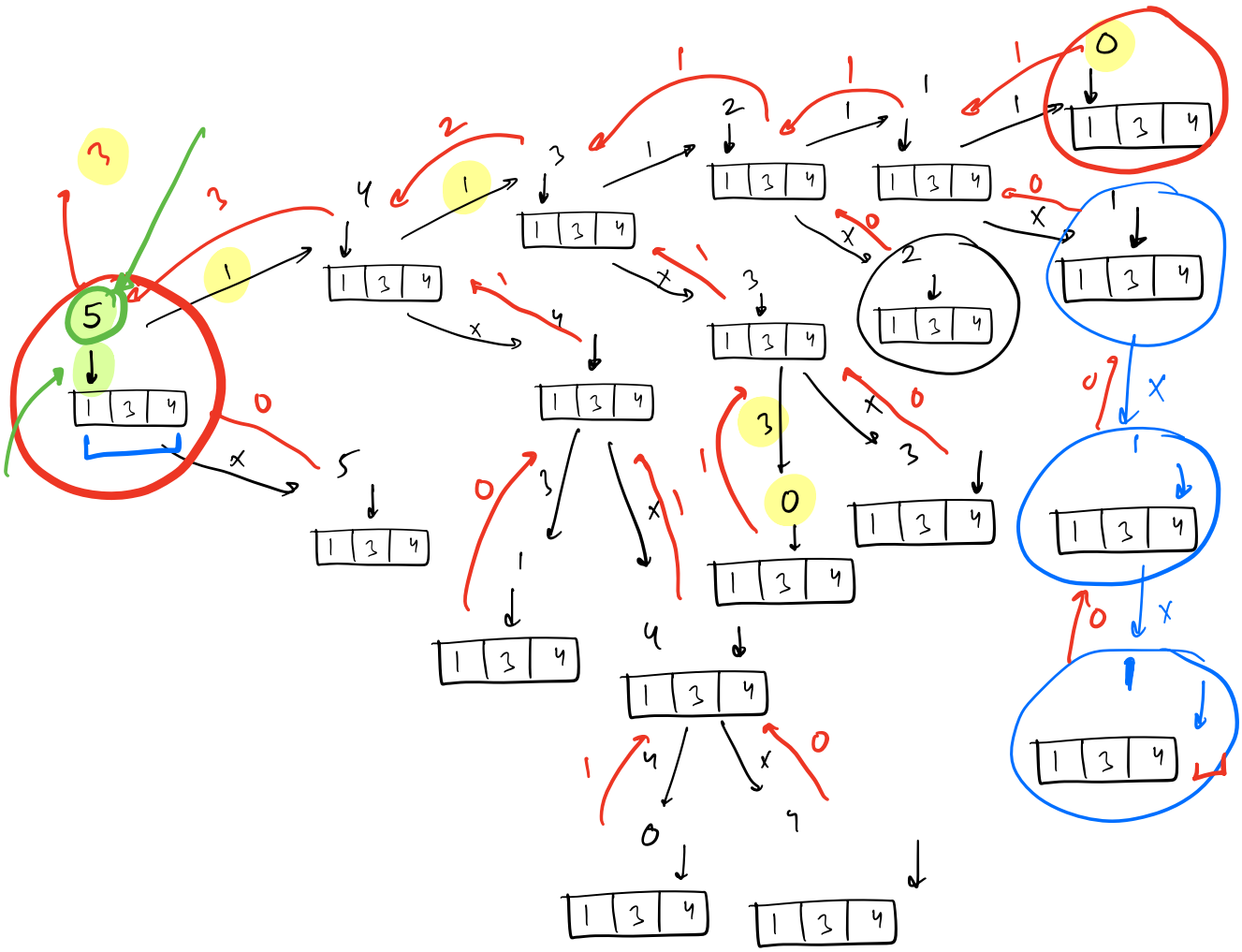
1 + 1 + 3

1 + 4

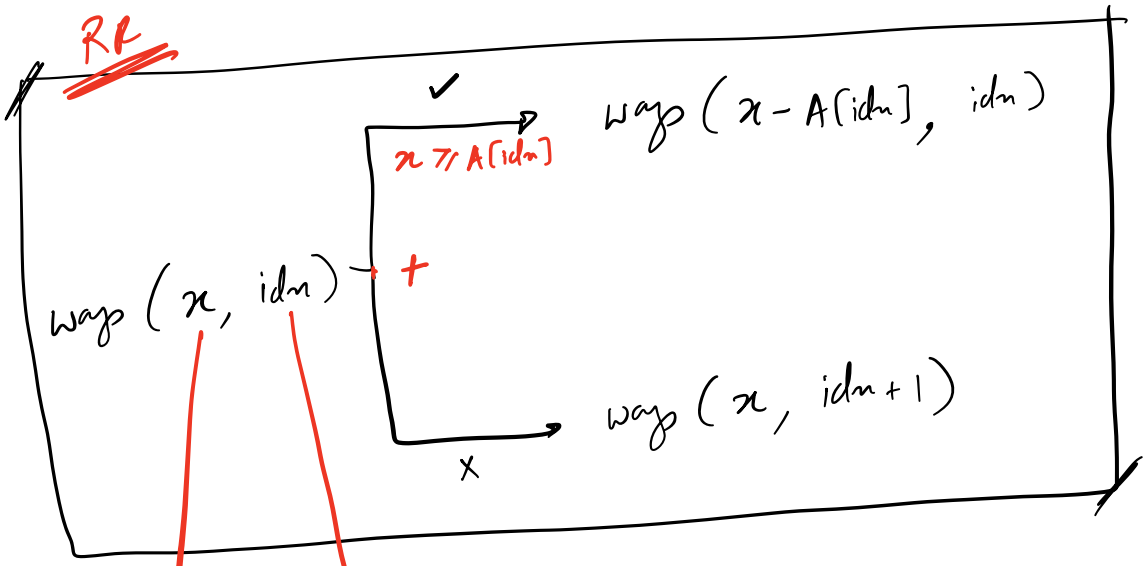
1 + 1 + 1 + 1 + 1

3 + 1 + 1

1 + 4



ways(n, idx) → No of ways of making n a sum
Considering the coins A[idx -- N-1]
: Order is NOT imp



[0-N] [0-M-1]

#US → O(NM)

1 <= N, M <= 10^3

TRPS → O(1)

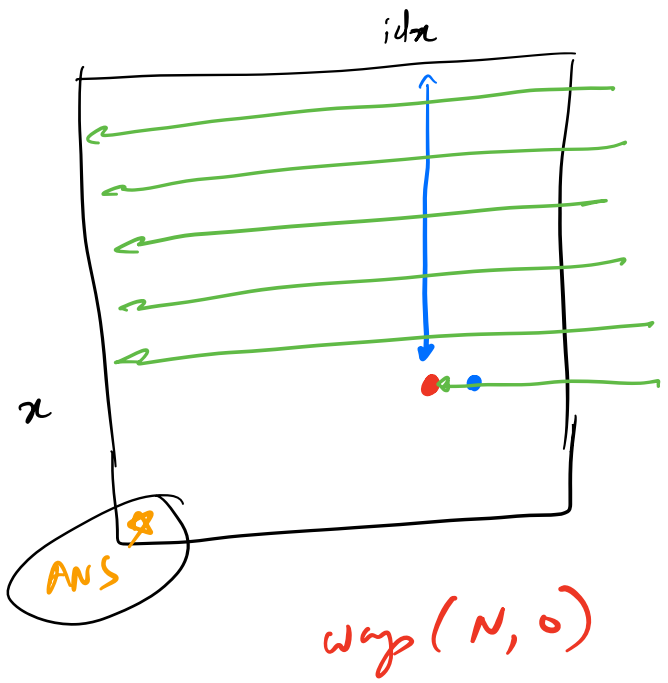
```
int dp[N+1][M] = {-1};
int ways ( x, idx ) {
    if ( x==0) ret 1;
      if (idx == M) ret 0;

    if (dp[x][idx] != -1) {
            ret dp [x][idx];

}
```

TC = O(NM)

SC = O(NM).

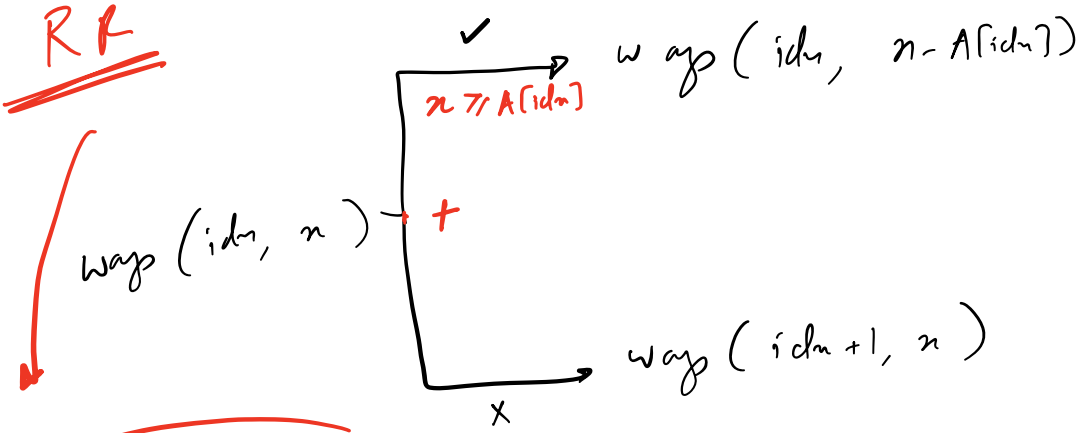
```
ANS = way(n, idx+1);
if( n >= A[idx] ){
    ANS += way( n-A[idx], idx );
}
dp[n][idx] = ANS;
ret ANS;
}
```

④ Bottom Up Analysis

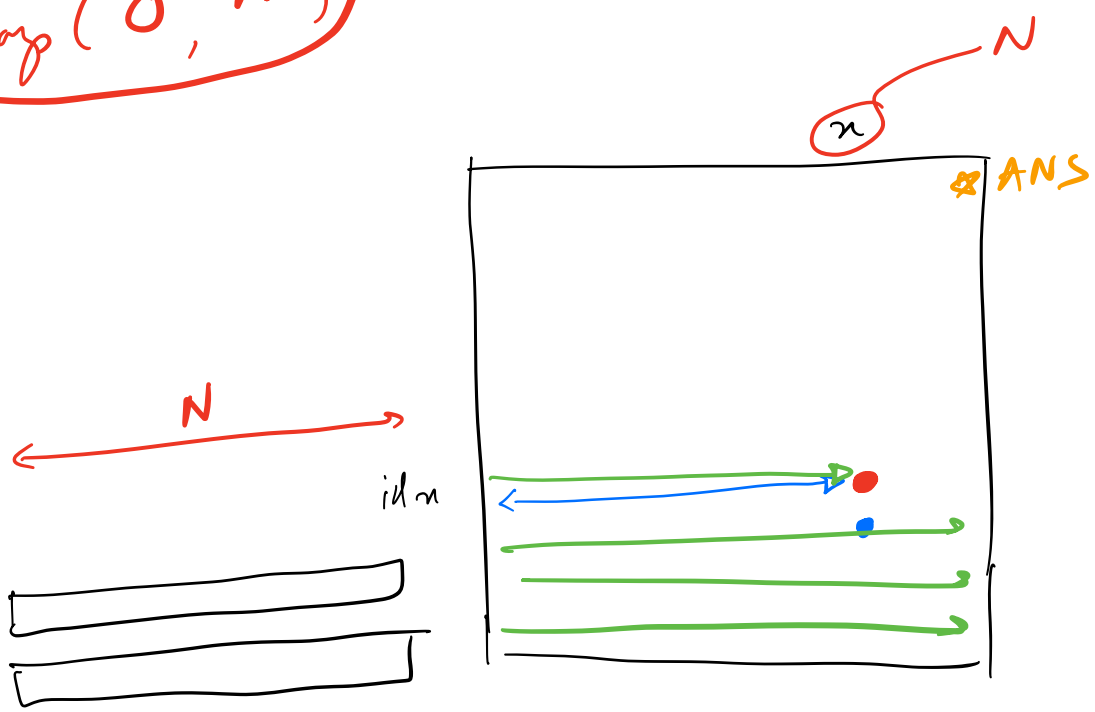


way(N, 0)

RF



way (0 , N)



2 rows

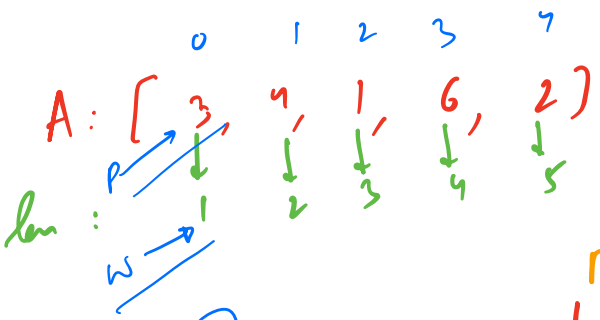
TC = O(NM)

SC = O(N)

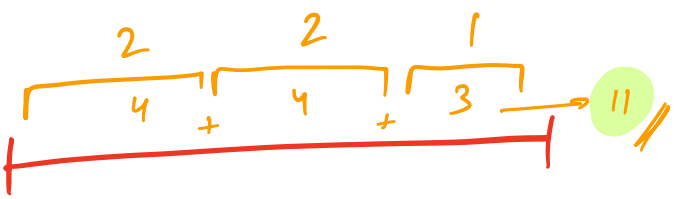
Given a rod of length N & an array of size N.

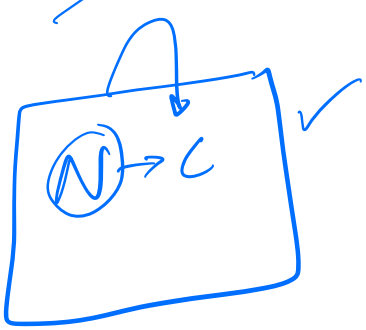
$A[i] \longrightarrow$ price of the rod of length $(i+1)$

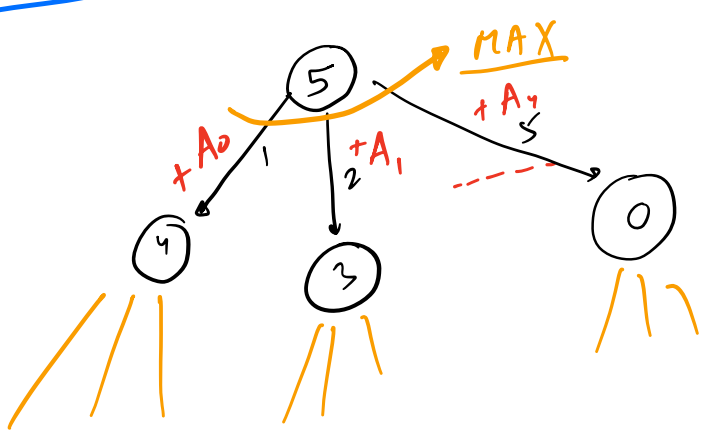
Find the MAX value that can be obtained by cutting up the rod & selling the pieces!



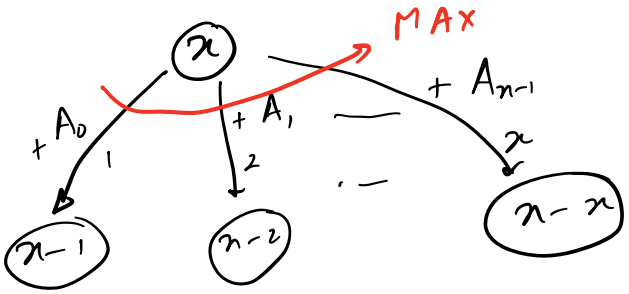
$N = 5$







$mp(x) \longrightarrow$ MAX profit we can make with rod of length x



$$mp(x) = \max_{L=1}^{L=x} \left(A_{L-1} + mp(x-L) \right)$$

[0-N]

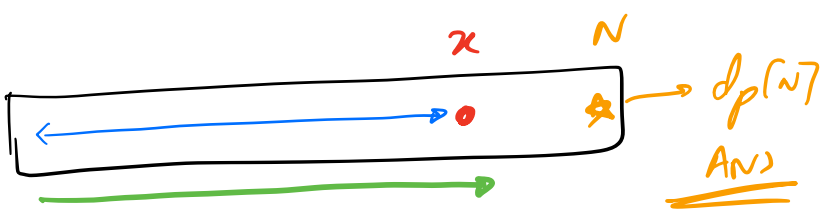
VS → ~N

TRPS → N

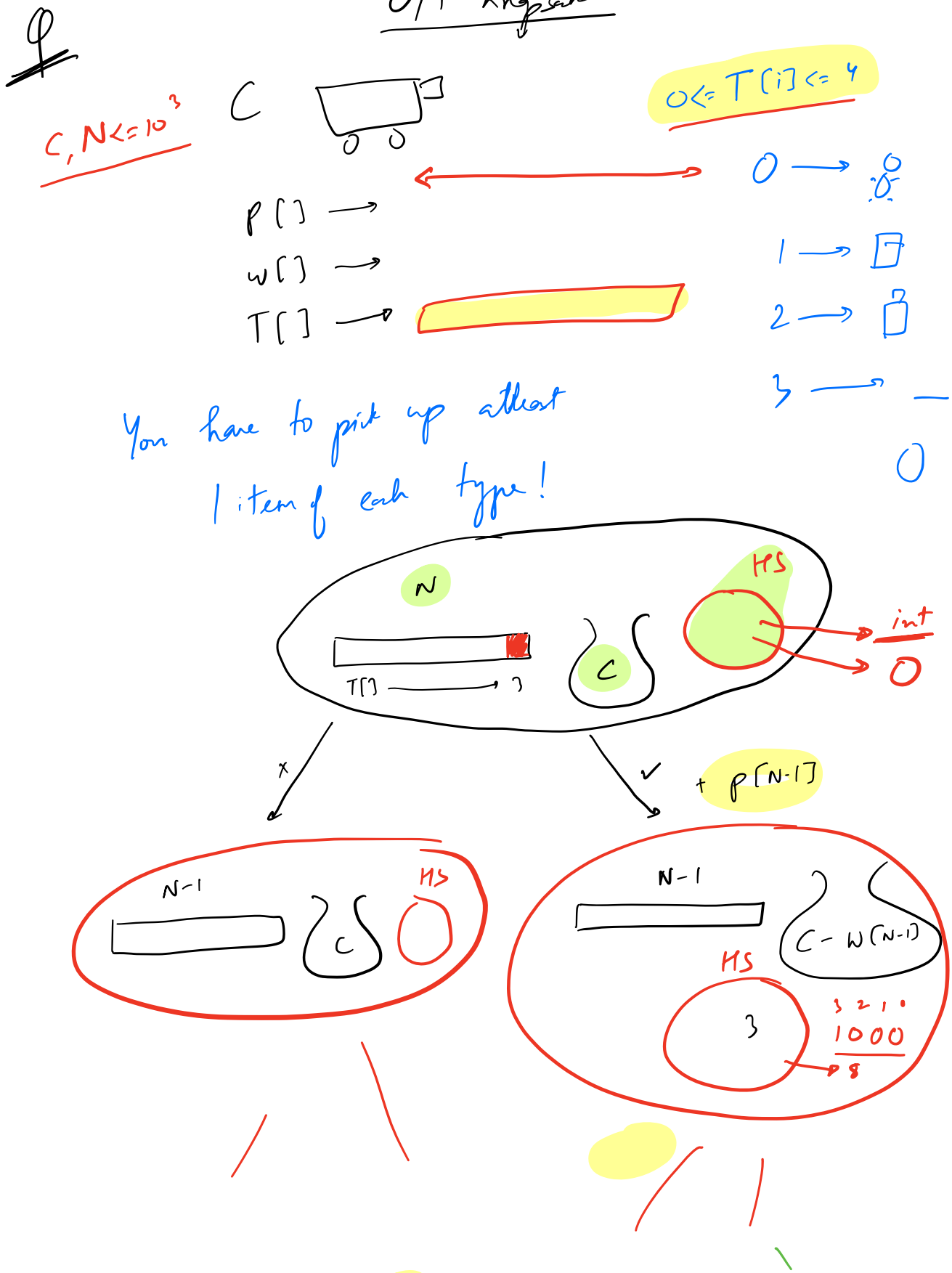
TC = O(N²)

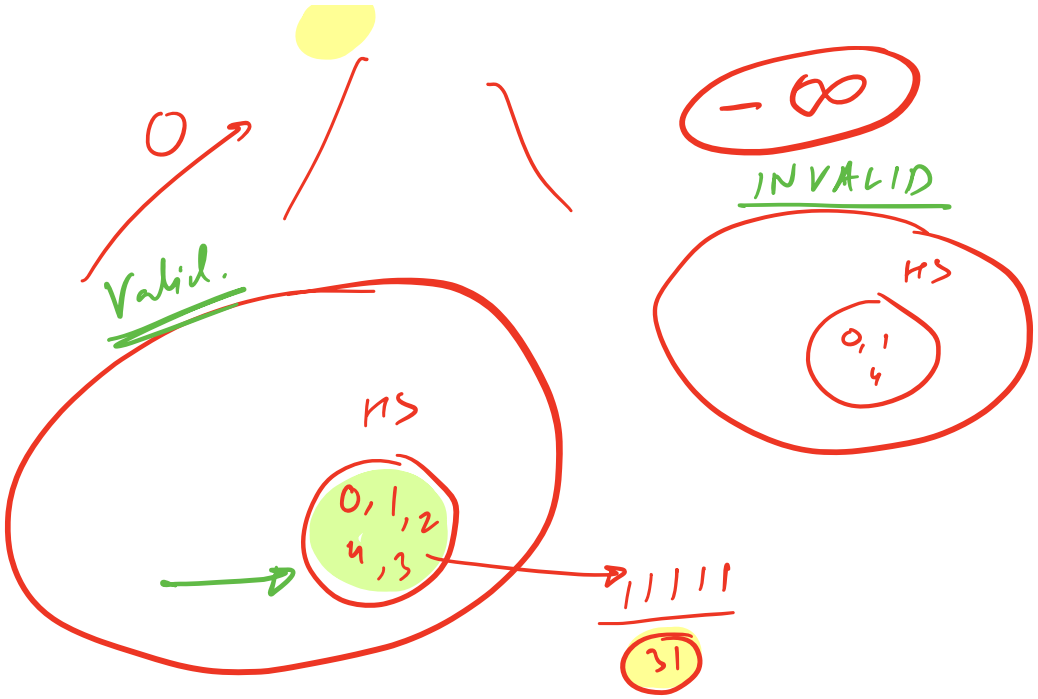
SC = O(N)

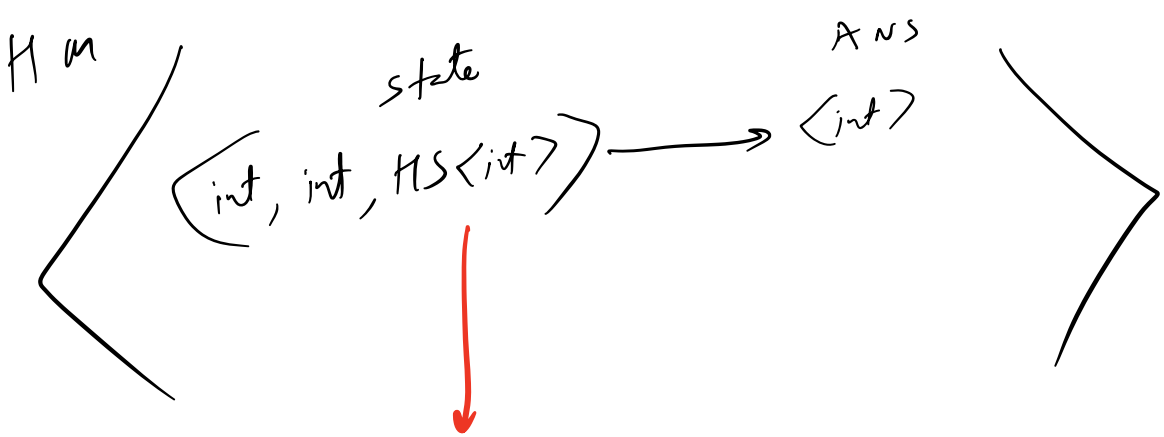
BU



0/1 Knapsack


C, N<=10^3
C
0<= T[i] <= 4
0 →
p[] →
w[] →
T[] →
1 →
2 →
3 →
You have to pick up atleast
1 item of each type!
N
HS
int
T[]
C
x
✓
+ p[N-1]
N-1
C
HS
N-1
C - W[N-1]
HS
3
3 2 1 0
1000
8



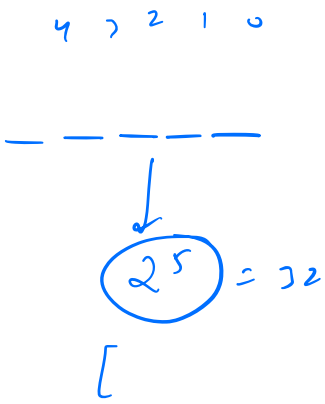


HM
3D ARRAY
<<int, int, int> → <int>>

[0-N] [0-C] [0-31]

#US → N x C x 32

TRPS → O(1)



TC = O(N.C .32) 0 <= T <= K

TC = O(N C 2^K)

SC