

Q Given N. Count the no. of factors.

$x$  is a factor of  $y$

$$y \% x \rightarrow 0$$

3 is a fac of 12 ✓

$$3 \times 4 = 12 !$$

7 ——— 12 ✗

$$7 \times \underline{\quad} \neq 12$$

$N=10$  : { 1, 2, 5, 10 }  $\longrightarrow$  4

$$1 \times 10 = 10$$

$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

$$10 \times 1 = 10$$

$N=12$  : { 1, 2, 3, 4, 6, 12 }

Given  $N$ , check if  $x$  is a factor?  $\downarrow$   
 $[1, N]$  factor range

↳ if ( $N \% x == 0$ )

↳  $x$  is a factor of  $N$

else

↳ NOT

cnt = 0;

```
{ ( i=1; i<=N; i++) {  
    if (N%i==0) {  
        cnt++;  
    }  
}
```

```
}  
ret cnt;  
}
```

N iterations

Ass:  $10^8 \text{ it} \rightarrow 1 \text{ sec}$

1 it  $\rightarrow \frac{1}{10^8} \text{ sec}$

$10^9 \text{ it} \rightarrow 10^9 \times \frac{1}{10^8} \text{ sec}$

$10^{9-8} \text{ sec}$

10 sec

$N = 10^9$

$N = 10^{18}$

$10^{18} \text{ it} \rightarrow 10^{18} \times \frac{1}{10^8} \text{ sec}$

$\rightarrow 10^{10} \text{ sec}$

$\sim 317 \text{ yrs}$

You  $\rightarrow$  child  $\rightarrow$  gc  $\rightarrow$  ggc  $\rightarrow$  — 6/7.

Optimize →

$i$  is a factor of  $N$

$$i \times j = N$$

$\Rightarrow j \times i = N \Rightarrow j$  is also a factor of  $N!$

$$i \times j = N \rightarrow j = N/i$$

$$2 \times 5 = 10$$

If  $i$  is a factor of  $N$ .

$\Rightarrow N/i$  is also a factor of  $N$ .

$N = 24$

$i$	$N/i$
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24. The first four factors (1, 2, 3, 4) are circled in blue. The last four factors (6, 8, 12, 24) are listed below the table. The value 8 is boxed in blue.

$N = 35$

$i$	$N/i$
1	35
5	7
7	5
35	1

Factors of 35: 1, 5, 7, 35. The first two factors (1, 5) are circled in orange. The value 4 is boxed in blue.

$$i < N/i$$

$N = 16$

$i$	$N/i$	
1	16	+2
2	8	+2
4	4	+1
8	2	
16	1	

5

$$i \leq N/i$$

$$i \times i \leq N$$

$$i^2 \leq N$$

$$i \leq \sqrt{N}$$

⇒ All no's have even no. of factors except perfect square no's.

cnt = 0

```

for (i = 1; i <= sqrt(N); i++) {
    if (N % i == 0) {
        if (i != N/i) {
            cnt += 2;
        }
        else {
            cnt += 1;
        }
    }
}
return cnt;

```

$i: [1, \sqrt{N}]$

+2, +1

Sqrt

1.  $i \leq \text{sqrt}(N)$  ✓

2.  $sq = \text{sqrt}(N)$  ✓

$i \leq sq$

3.  $i \leq N/i$  ✓

4.  $i \times i \leq N$  ✓

#it  $\rightarrow \sqrt{N}$

$$N = 10^9 \quad \sqrt{N} \text{ it} \rightarrow \sqrt{10^9} \text{ it} \\ \downarrow \\ 3 \times 10^4 \text{ it}$$

$$1 \text{ it} \rightarrow \frac{1}{10^8} \text{ sec}$$

$$3 \times 10^4 \text{ it} \rightarrow 3 \times 10^4 \times \frac{1}{10^8} \text{ sec}$$

$$10 \mu\text{s} \text{ vs } \sim 3 \times 10^{-4} \text{ sec}$$

$$N = 10^{18}$$

$$\sqrt{10^{18}} \text{ it} \rightarrow \sqrt{(10^9)^2} \\ \rightarrow 10^9 \text{ it}$$

$$1 \text{ it} \rightarrow \frac{1}{10^8} \text{ sec}$$

$$10^9 \text{ it} \rightarrow 10^9 \times \frac{1}{10^8} \text{ sec}$$

$$317 \text{ yrs} \text{ vs } \rightarrow 10 \text{ sec}$$

⑧ Prime Number  
 → A <sup>no</sup> ~~no~~ having exactly **2 factors!**

[1 & itself]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Neither prime nor composite

● → prime nos

● → composite no's

↓  
 > 2 factors

Q Given N.  
 Check if N is a prime no. or not!

N > 0

// N

if ( cnt Factors (N) == 2 ) {  
     ret true;

}  
 ret false;

Modify the  
cnt Factors ()

to stop early →  
 cnt > 2

Break till : 10<sup>36</sup>

Q

# GAUSS SUM

Find out the sum of all no's from 1-100!

$$\begin{array}{rcl}
 S & = & 1 + 2 + 3 + \dots + 100 \\
 + \quad S & = & 100 + 99 + 98 + \dots + 1
 \end{array}$$

---


$$2S = 101 + 101 + 101 + \dots + 101$$

$$2S = 100 \times 101$$

$$S = \frac{100 \times 101}{2} \Rightarrow 5050$$

Q → Sum [1, N]

$$\begin{array}{rcl}
 S_N & = & 1 + 2 + \dots + N \\
 + \quad S_N & = & N + N-1 + \dots + 1
 \end{array}$$

---


$$2S_N = \underbrace{N+1 + N+1 + \dots + N+1}$$

$$2S_N = N(N+1)$$

$$S_N = \frac{N(N+1)}{2}$$

Sum of 1<sup>st</sup> N Natural no's

① Log basis

"log a base b"

$$\log_b a = c \Rightarrow \boxed{b^c = a}$$

↳ to what power should I raise b to get a?

a ———  
 $\log_2 64 \rightarrow$   
b ———  
6

$$2^6 = 64$$

$\log_2 32 \rightarrow$   
5

$$2^5 = 32$$

$\log_{10} 1000 \rightarrow$   
3

$$10^3 = 1000$$

$\log_2 10$   
3. —

$$\begin{aligned} 2^3 &= 8 \\ 2^{3.} &= 10 \\ 2^4 &= 16 \end{aligned}$$

$\log_2 40$

$$\begin{aligned} 2^5 &= 32 \\ 2^{5.} &= 40 \\ 2^6 &= 64 \end{aligned}$$



$$\log_2 2^{20} \rightarrow 2^{\overline{20}} = 2^{20}$$

20

$$\log_b b^k = k$$

$$\log_2 N = k \iff 2^k = N$$

$$\log_c (a \times b) = \log_c a + \log_c b$$

Q

HW

Given  $N (N > 0)$

Find how many times you need to divide it by 2 so that you reach 1

$$N = 45$$

$$22 \rightarrow /2$$

$$11 \rightarrow /2$$

$$5 \rightarrow /2$$

$$2 \rightarrow /2$$

$$1 \rightarrow /2$$

$$1 \rightarrow /2$$

$$\rightarrow 5$$

① Perfect square

↳ product of 2 same integers

1, 4, 9, 16, 25, 36 —  $x^2$  —

Q

Given  $N$  ( $N > 0$ )

Given  $N$  is a perfect square!

Find out its square root!

$N = 100$

→ 10

$N = 25$

→ 5

$N = 36$

→ 6

$N = 25$

$i: [1, 2, 3, 4, 5]$   
1x1, 2x2, 3x3, 4x4, 5x5

$N/2$   $N$   $\infty$

```
{ i=1; i<=N; i++ }  
  { if (i*i == N) {  
    return i;  
  }  
}
```

$N = 10000$   
 $i: [1, \dots, 100]$   
 $\sqrt{N}$

$\sqrt{N}$  times

Q Given  $N$  ( $N > 0$ )  
Find its  $\text{floor}(\sqrt{N})$

$\text{floor}(n)$   
↳ greatest integer  $\leq n$

$\lfloor 44.5 \rfloor \rightarrow 44$

$\lfloor 44 \rfloor \rightarrow 44$

$N = 35 \rightarrow 5$

$N = 36 \rightarrow 6$

$N = 37 \rightarrow 6$

$N = 48 \rightarrow 6$

$N = 49 \rightarrow 7$

$N = 27$

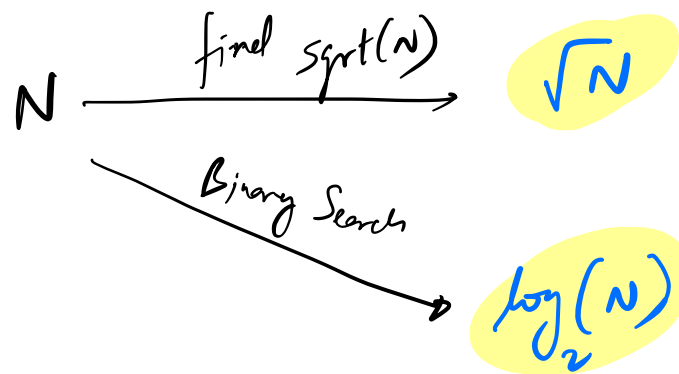
1 2 3 4 5 6 7

$6 \times 6 > 27$

ans

```
ans = 0;
for (i = 1; i <= N; i++) {
    if (i * i <= N) {
        ans = i;
    } else {
        break;
    }
}
return ans;
```

$\sqrt{N}$  times



$$N = 10^{12}$$

$$\sqrt{N}$$

$$\sqrt{10^{12}}$$

$$10^6$$

vs

$$\log_2(N)$$

$$\log_2(10^{12})$$

$$\sim 60$$