★ Build LPS using KMP

S = c a c y c a c a

LPS[] →

LPS[i] → LPS(s[0..i])

LPS(string) → the length of longest prefix which is also a suffix.

NOTE: do not consider complete string!

LPS(ab abab) → 4

P	S	
a	b	✗
ab	ab	✓
aba	bab	✗
abab	abab	✓
ababa	babab	✗

S = c a c y c a c a

LPS[] → 0 0 1 0 1 2 3 2

BF → $\boxed{TC = O(N^3)}$

S : $s_0\ s_1\ s_2\ s_3\ s_4$ —— $s_{i-5}\ s_{i-4}\ s_{i-3}\ s_{i-2}\ s_{i-1}\ s_i$ — s_{N-1}

≥ 4 $\quad 5$

LPS[]

Assume: LPS[i] = 5

$$s_0\, s_1\, s_2\, s_3\, \cancel{s_4} = s_{i-4}\, s_{i-3}\, s_{i-2}\, s_{i-1}\, \cancel{s_i}$$

$$LPS[i-1] \geq 4$$
$$\geq 5-1$$

$$LPS[i-1] \geq LPS[i]-1$$

$$\boxed{LPS[i] \leq LPS[i-1]+1}$$

i-1 $\quad$ i

LPS[] —— 6 $\leq$ 7

0 1 2 3 i-1 i

S = a b a y a b a ch

LPS → 0 0 1 0 1 2 3

? = 4

<= 4

LPS[i-1]

if (s[3] == ch)

LPS[i] → 4

0 1 2 3 4 5 6 7 i-1 i

S = b c a d c b c a d ch

LPS[] → 0 0 0 0 0 1 2 3 4

≤ 5

LPS[i-1]

if (s[4] == ch)

LPS[i] → 5

x x-1 x

i-1 i

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

S = c a c y c a c a b c a c y c a c y

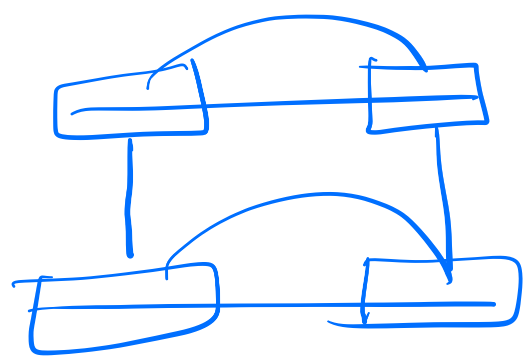
LPS[]: 0 0 1 0 1 2 3 2 0 1 2 3 4 5 6 7 → 4

≤ 8

x = LPS[i-1]

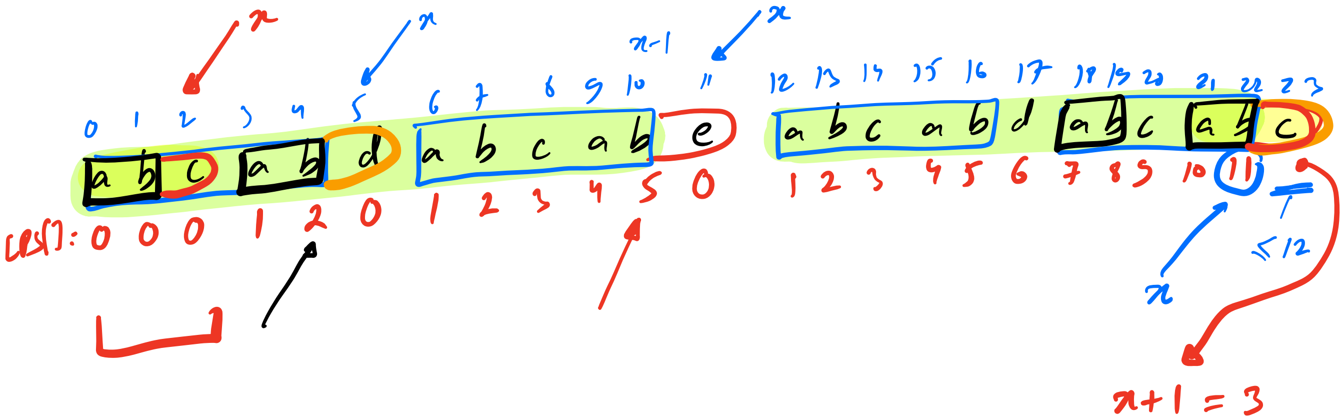
if (s[i] == s[x])

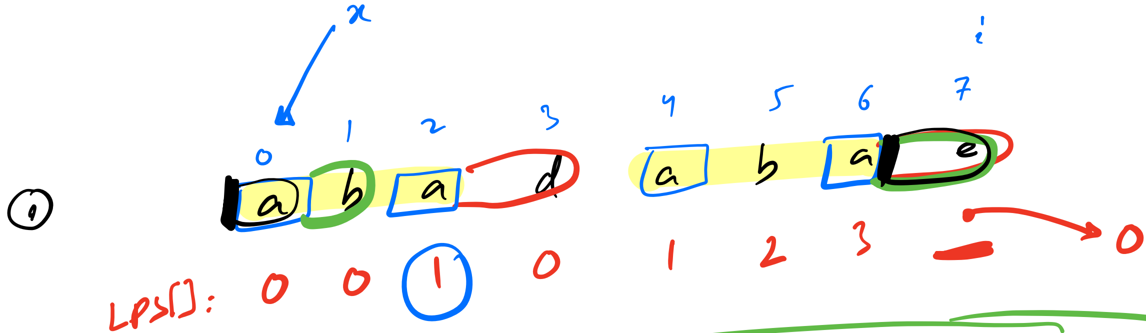
LPS → x+1 X



10 11

x k

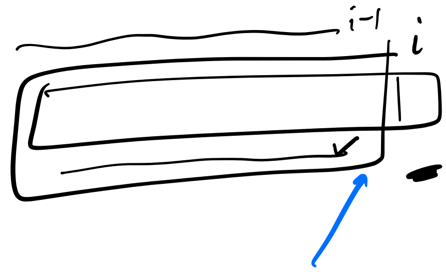




NOTE: If at x == 0, we do not match → STOP!

CODE :

```
LPS[N];
LPS[0] = 0;
f( i=1; i<N; i++) {
    x = LPS[i-1];
    while ( s[x] != s[i]) {
        if(x==0) { x=-1; break; }
        x = LPS[x-1];
    }
    LPS[i] = x+1 ;
}
rt LPS[];
```



TC :

N

S : 0

LPS : 0 1 2 3 4 0 1 2 3 4 - — 10 0

ops : 0 1 1 1 1 5 1 1 1 1 1 - - - 1 11 - —

4

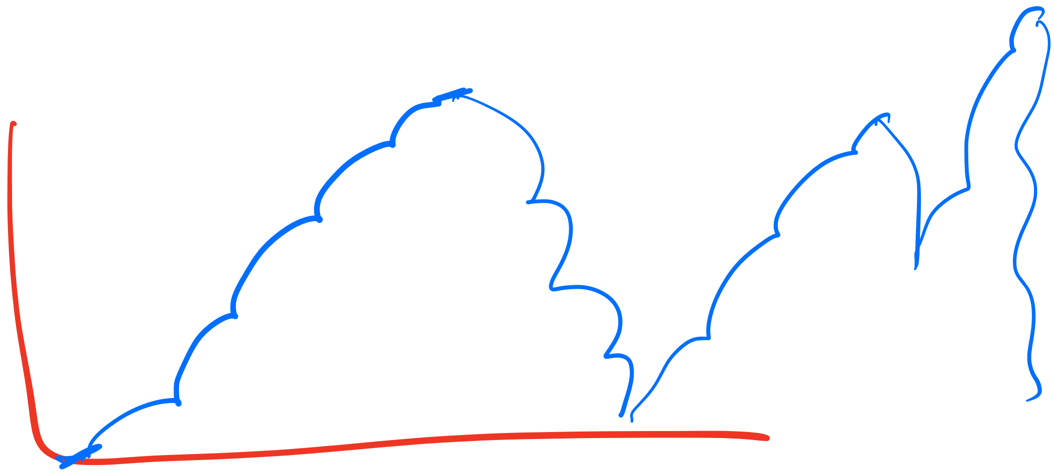
\# dec <= # inc

$\#inc \rightarrow N$

$\# dn \rightarrow N$ $\Big\} \rightarrow 2N$

$\boxed{TC = O(N)}$

LPS



n i

a b c d e f g

0 0 0 0 0 0 0

a a a a a a n

0 1 2 3 4 5 0

1 1 1 1 5

$\boxed{SC = \rightarrow O(N)}$

⊙ Polynomial Rolling Hash

→ find the hash of a string

a	b	c	—	z
0	1	2		25

abc : $0+1+2 \rightarrow 3$

aaa : $0+0+0 \rightarrow 0$

aa : $0+0 \rightarrow 0$

a : $0+0 \rightarrow 0$

a	b	—	z
1	2		26

aaa : $1+1+1 \rightarrow 3$

aa : $1+1 \rightarrow 2$

a : $1 \rightarrow 1$

aa $\rightarrow$ 2

b $\rightarrow$ 2

abc : $1+2+3 \rightarrow 6$

acb

bac

bca

cab

cba

$\rightarrow 6$

5 4 1 6 :

DEC : $\rightarrow \underbrace{5 \times 10^3 + 4 \times 10^2 + 1 \times 10^1 + 6 \times 10^0}$

(10) 11

ALP:

(a b — z)

(27)

1 — 26

[redacted]

0 — 9

3 2 1 0

a c d a

$a \times 27^3 + c \times 27^2 + d \times 27^1 + a \times 27^0$

$\underbrace{1 \times 27^3 + 3 \times 27^2 + 4 \times 27^1 + 1 \times 27^0}$

integer

$S : (S_0 \; S_1 \; S_2 \; — \; S_{N-1})$

$h(S) = S_0 + S_1 \times 27^1 + S_2 \times 27^2 + — + S_{N-1} \, 27^{N-1}$

$h(S) = (S_0 + S_1 \times 27^1 + S_2 \times 27^2 + — + S_{N-1} \, 27^{N-1}) \% \; 10^9 + 7$

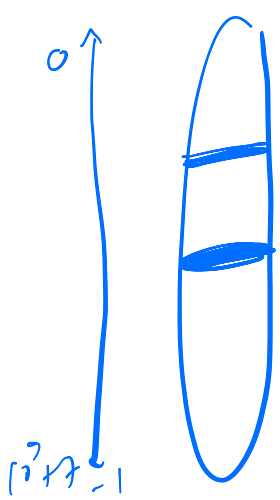
$$h(s) = \left(s_0 + s_1 \times M^1 + s_2 \times M^2 + \cdots + s_{N-1} M^{N-1}\right) \% P$$

M, P : prime No

M: 29, 31

P: 10^9+7, 10^9+9

$N <= 10^5$



		P
$s_1 \rightarrow$	$h_1 \rightarrow$	0
$s_2 \rightarrow$	$h_2 \rightarrow$	$1/(10^9+7)$
$s_3 \rightarrow$	$h_3 \rightarrow$	$2/(10^9+7)$
$\vdots$		
$s_N \rightarrow$	$h_N \rightarrow$	$(N-1)/(10^9+7)$

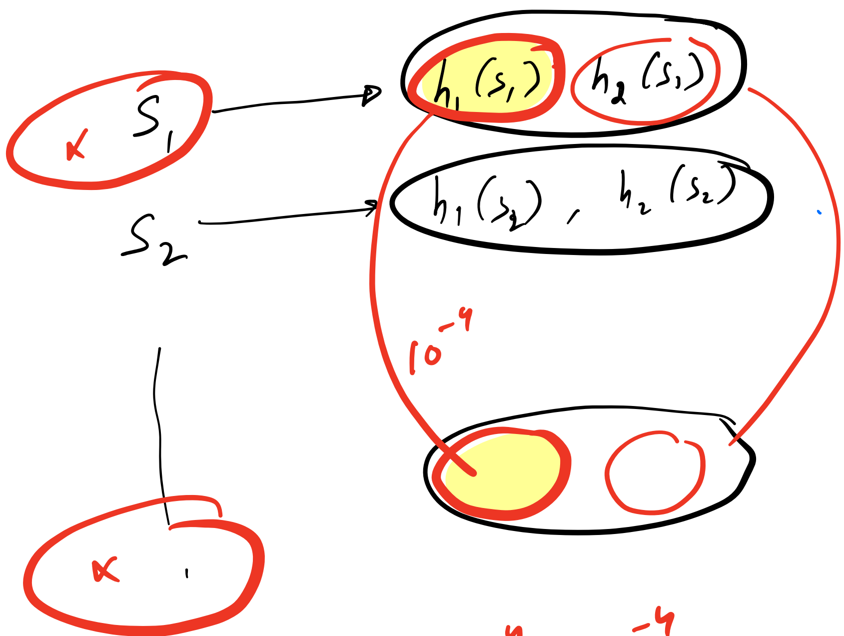
$$\sim \frac{10^5}{10^9} \sim 10^{-4}$$

$$h(s) = \left(S_0 + S_1 \times M^1 + S_2 \times M^2 + \dots + S_{N-1} M^{N-1}\right) \% \, P$$

$h_1(s) \longrightarrow M: 29, \quad P = 10^9 + 7$

$h_2(s) \longrightarrow M: 31, \quad P = 10^9 + 9$

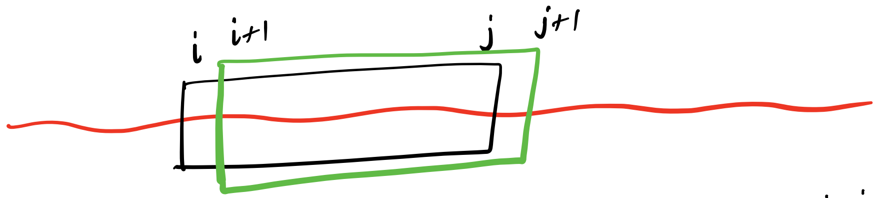
$N <= 10^5$



$10^{-4} \times 10^{-4}$

$= 10^{-8}$

Polynomial Rolling Hash



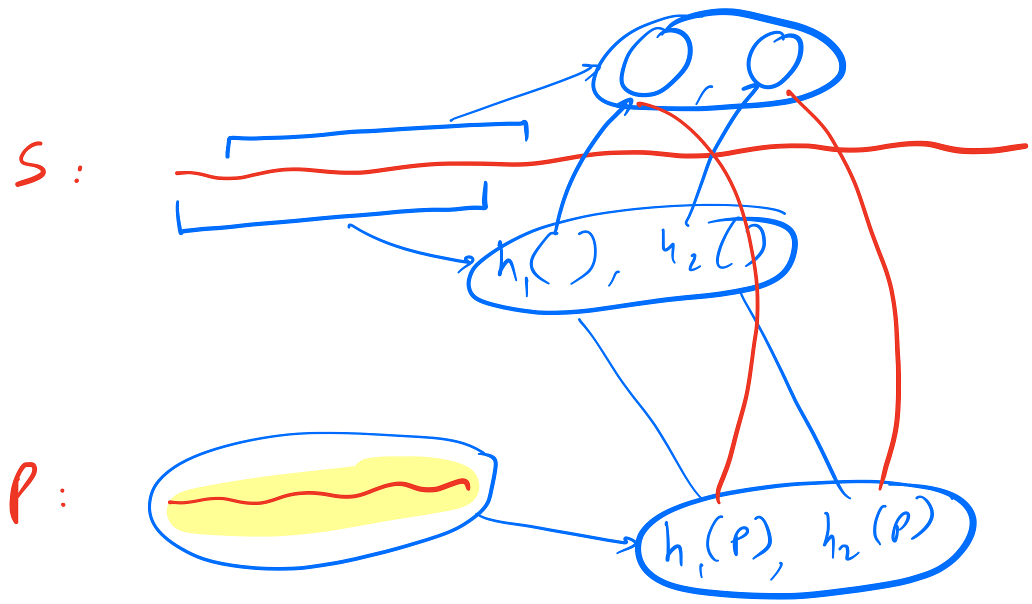
$$h(s[i..j]) = \left(s[i] + s[i+1]\,M + \dots + s[j]\cdot M^{j-i} \right) \% \, p$$

$$h(s[i+1..j+1]) = \left(s[i+1] + s[i+2]\,M + \dots + s[j+1]\,M^{j-i} \right) \% \, p$$

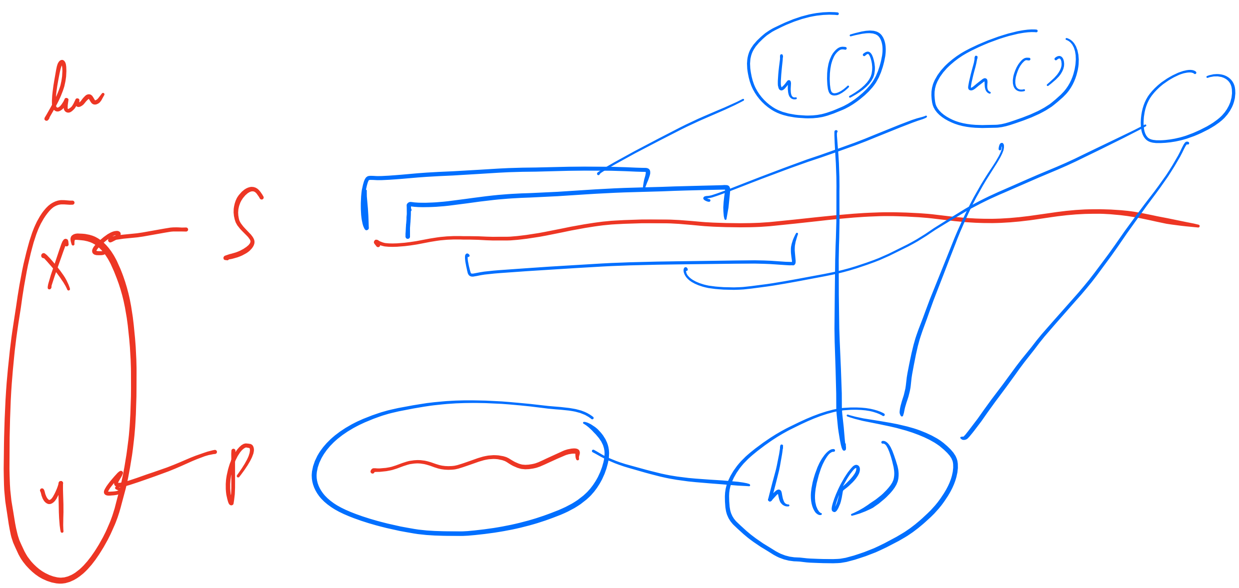
$$\boxed{h(s[i+1..j+1]) = \left[\frac{\left(h(s[i..j]) - s[i]\right)}{M} + s[j+1]\,M^{j-i} \right] \% \, p}$$

Inv. Modulo → $\log_2(p) / O(1)$

Q. Given a text (T) & pattern (P). Find if P is in T as a substring!



RABIN KARP



$$TC = y + y + (x - y + 1) \times O(1) + h(p)$$

$$\boxed{TC = O(x + y + h(p))}$$