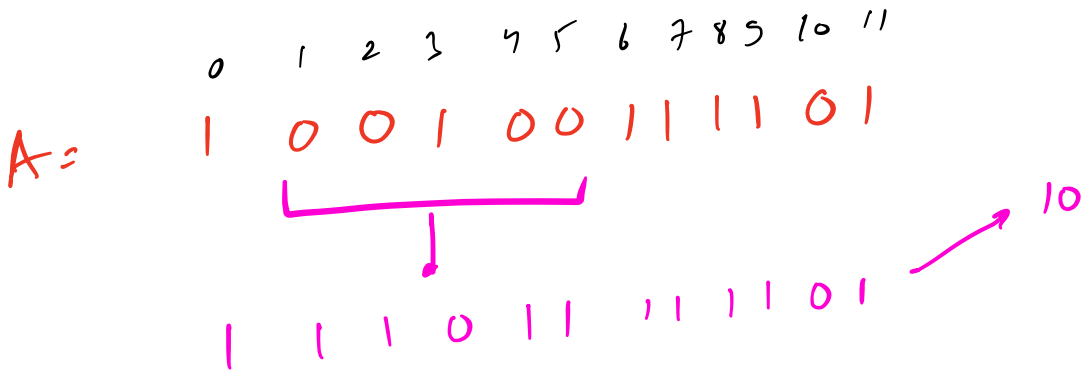
Q Given a binary Array. $A_i \in \{0, 1\}$

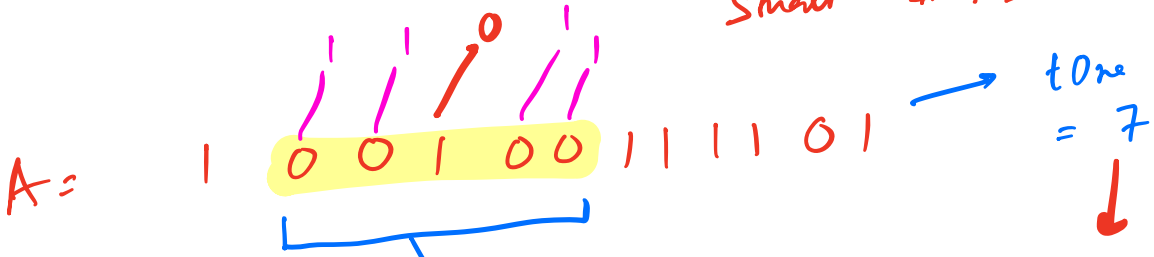
MAXIMIZE the total # of 1s by flipping any ONE subarray!

flip: $0 \to 1$, $1 \to 0$



Idea: Choose a SA. with large # 0's

Small # 1's



cnt0 = 1

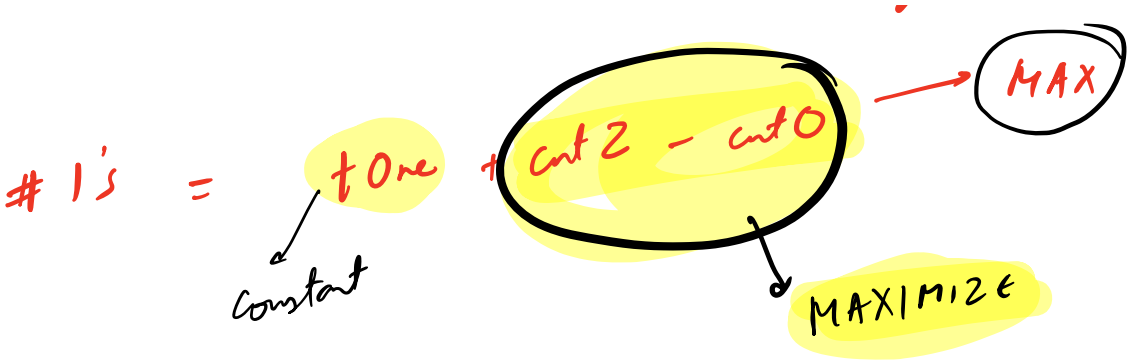
+1

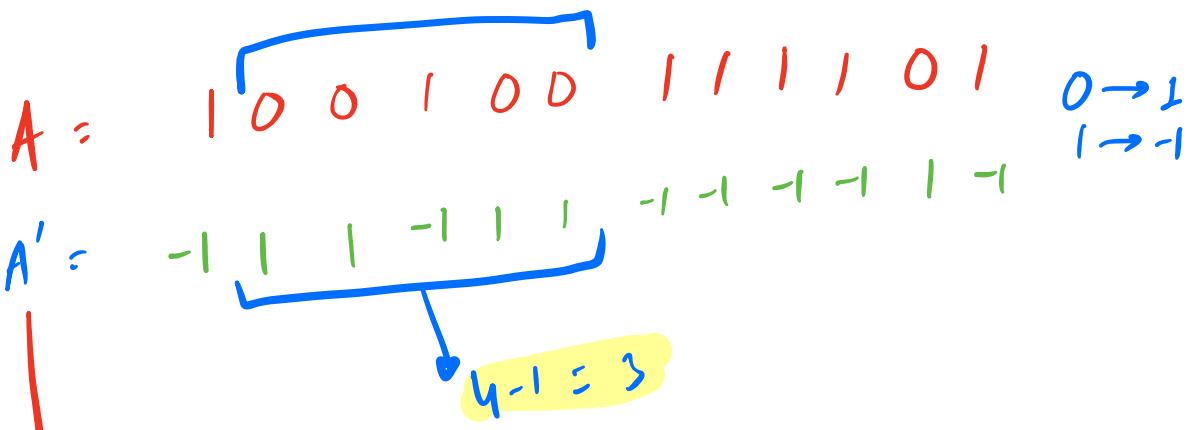
-1

#1's : tOne + cnt2 - cnt0

= 7 + 4 - 1 = 10 //

x



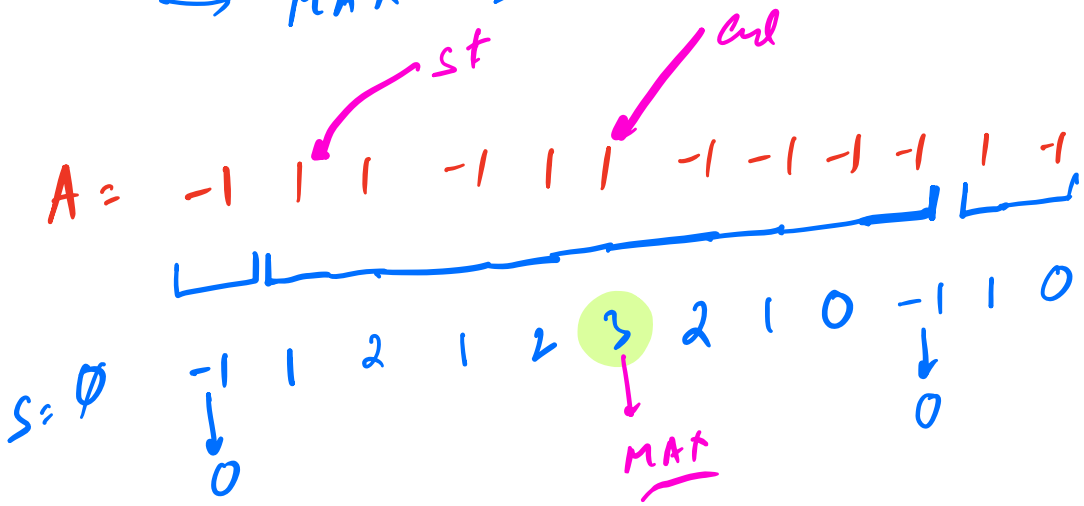


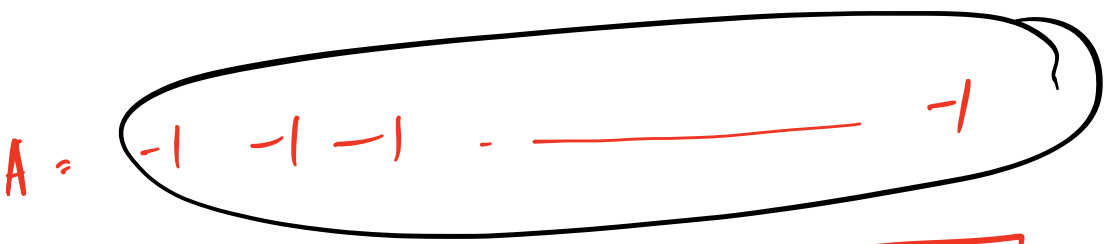
Idea: find the SA with MAX sum → (x)

ANS = tOne + x

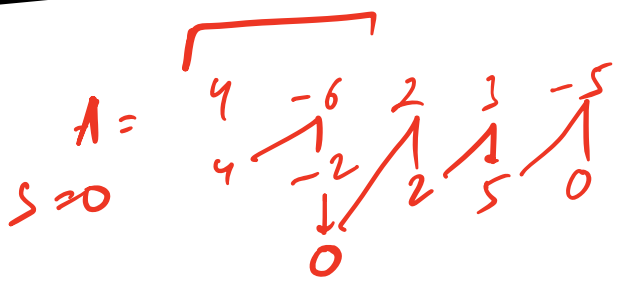
KADANE'S ALGO

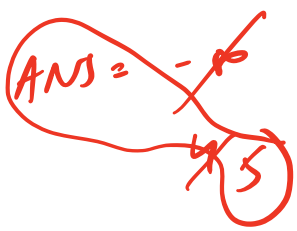
→ MAX SUBARRAY SUM !






```
ANS = -∞;
SUM = 0;
f( i=0; i<N; i++) {
    Sum += A[i];
    ANS = MAX(ANS, SUM);
    if (SUM < 0) {
        SUM = 0;
    }
}
ret ANS;
```





TC = O(N)

SC = O(1)

ORG 9 :

STEPS:

1. $A \xrightarrow[0 \to 1]{1 \to -1} A'$ → O(N)
2. find tOne in A. → O(N)
3. Use KADANE's ALGO to find MAX S.A. sum in A' → (x) → O(N) → O(1)
4. ANS = tOne + x !

TC = O(N)

SC = O(N) → Same Array : O(1)

O(1) :

Assume 0 → 1
1 → -1
while doing KADANE.

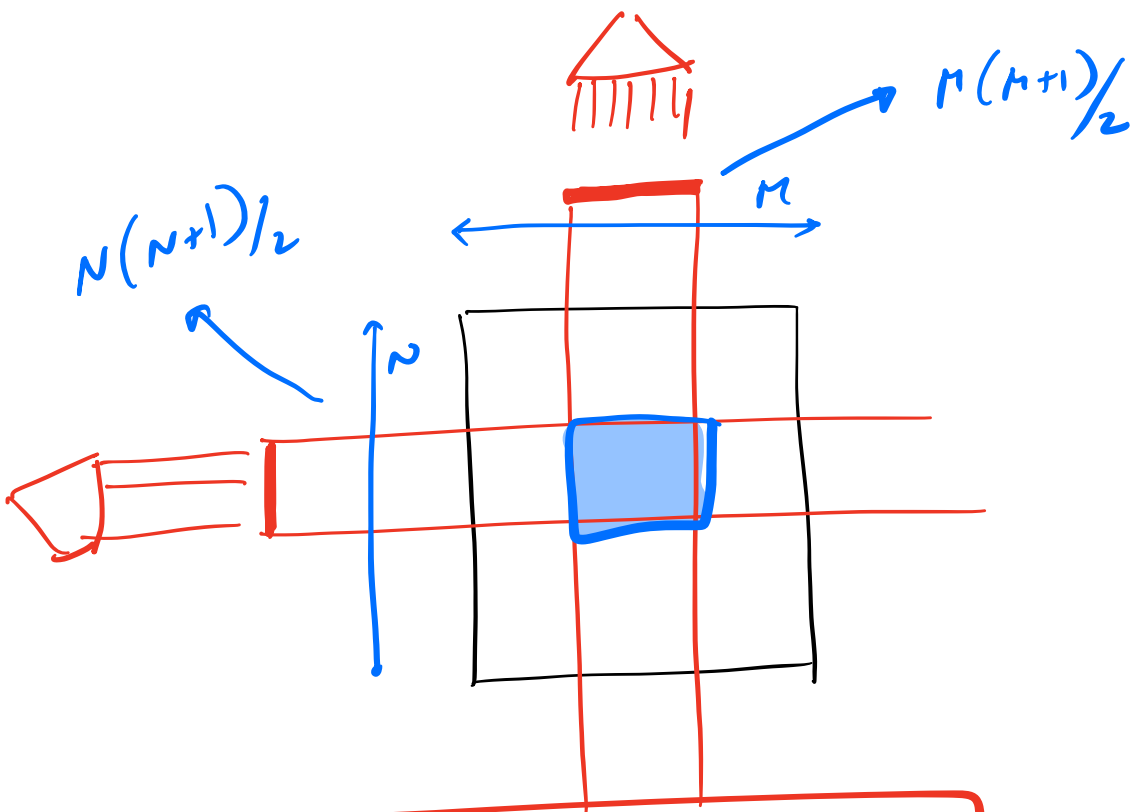
Q → Given a 2D Array (N x M)
Find the sum of sum of all sub-matrices!

$$\begin{bmatrix} 9 & 6 \\ 5 & 4 \end{bmatrix}$$

[9] [6] [9 6] [7] [6] [3 6]
[5] [4] [5 4] [5] [4] [5 7]

9, 6, 15, 14, 10, 24
5, 4, 9

= 96



Sub Mat = N(N+1)/2 x M(M+1)/2

NM

O(N²M²)

1) BF

∀ Sub matrix → $N^2 M^2$

find the sum → NM

$TC = O(N^3 M^3)$

$1 <= N, M <= 10^3$

Idea: find the contribution of every element

$A_{i,j}$ → find the # S.M. it is a part of!

$\begin{bmatrix} 9 & 6 \\ 5 & 4 \end{bmatrix}$ →

[9] [6] [9 6] $\begin{bmatrix} 7 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$

[5] [4] [5 4]

9 → 4 +

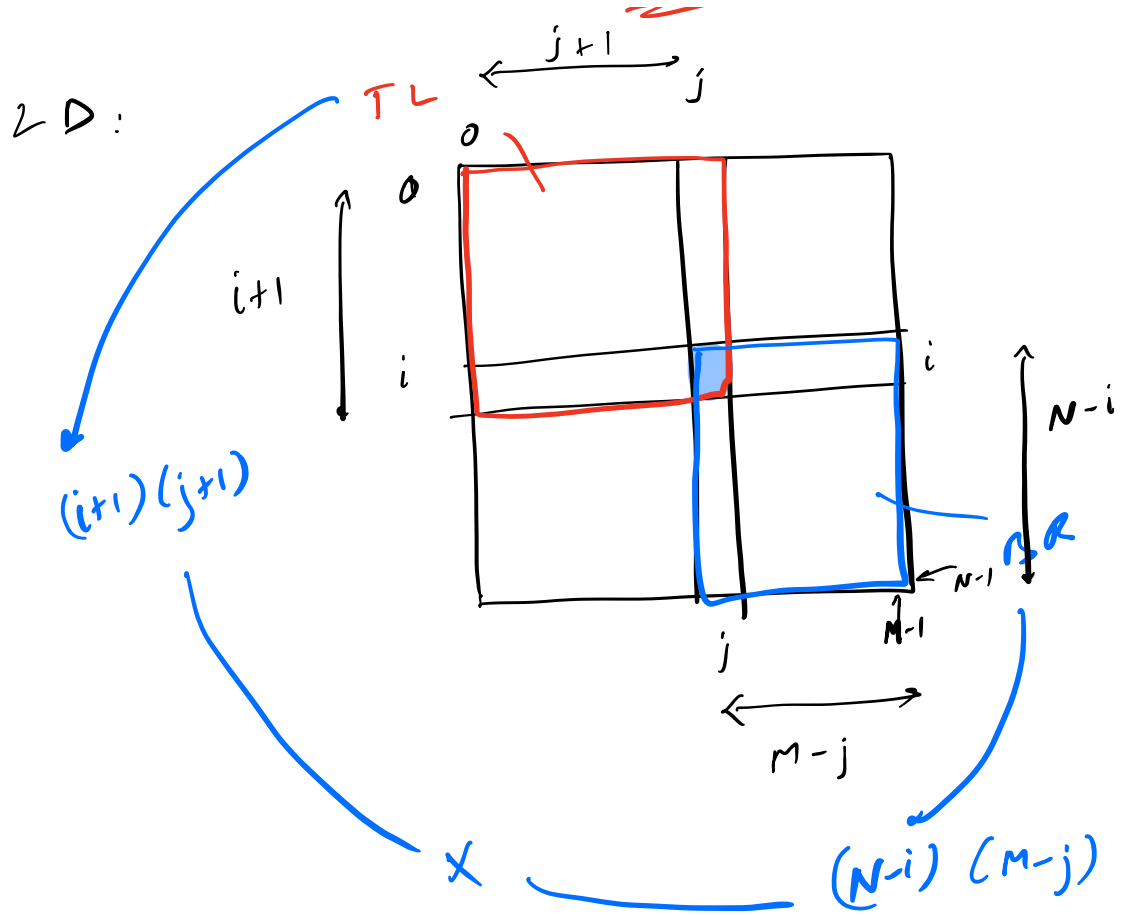
6 → 4 +

5 → 1 +

y : xy

→ ANS

2 D :



# of sub Matrix containing $(i,j)^{th}$ element

$= (i+1)(j+1)(N-i)(M-j)$

Contribution of A_{ij} : $A_{ij} \times (i+1)(j+1)(N-i)(M-j)$

$$ANS = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} A_{ij} \times (i+1)(j+1)(N-i)(M-j)$$

TC = O(NM)

SC = O(1)

Q Given a 2D array & Q queries.
∀ query, find the sum of the submatrix defined in the query!

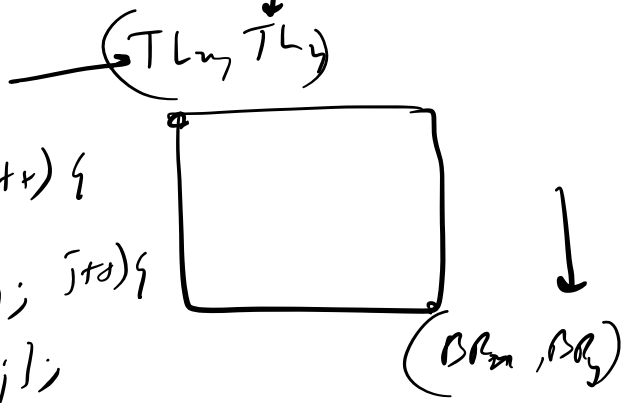
[TL_n, TL_j, BR_n, BR_j]

	0	1	2	3
0	1	5	2	3
1	2	1	2	2
2	4	1	3	2

[0,0,1,3] → 18

[1,1,1,1] → 1

[0,1,2,3] → 21



I) BF

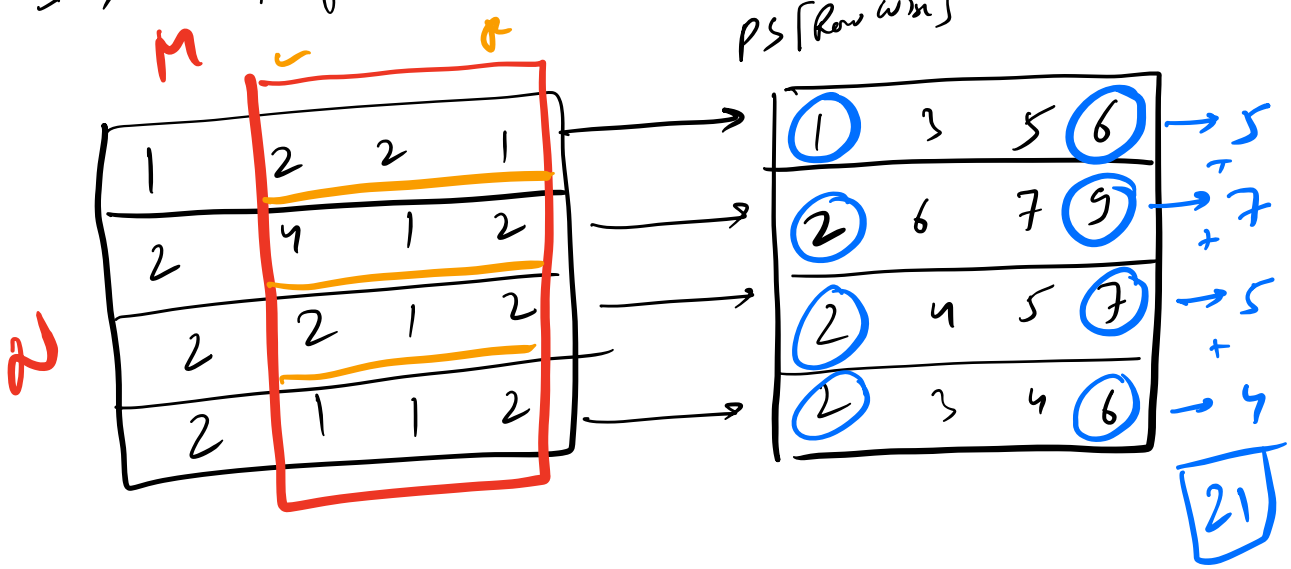
1 Q

```
for (i = TLn; i <= BRn; i++) {
    for (j = TLj; j <= BRj; j++) {
        sum += A[i][j];
    }
}
```

1 → O(NM)

TC = O(Q.NM)

II) Prefix Sum (Rows)



2) 1 Qu → O(N)

Q Qu → O(Q.N)

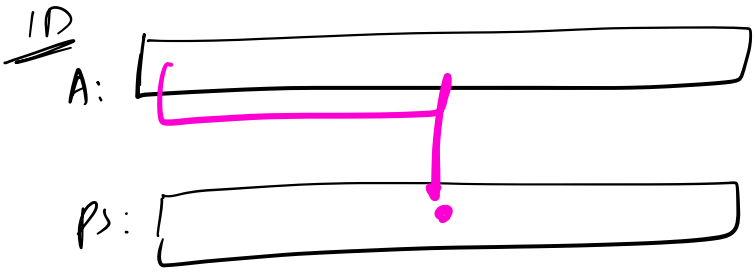
1) Build Row wise PS matrix → O(NM)

TC = O(NM + Q.N)

SC = O(~~NM~~) → O(1) : SAME

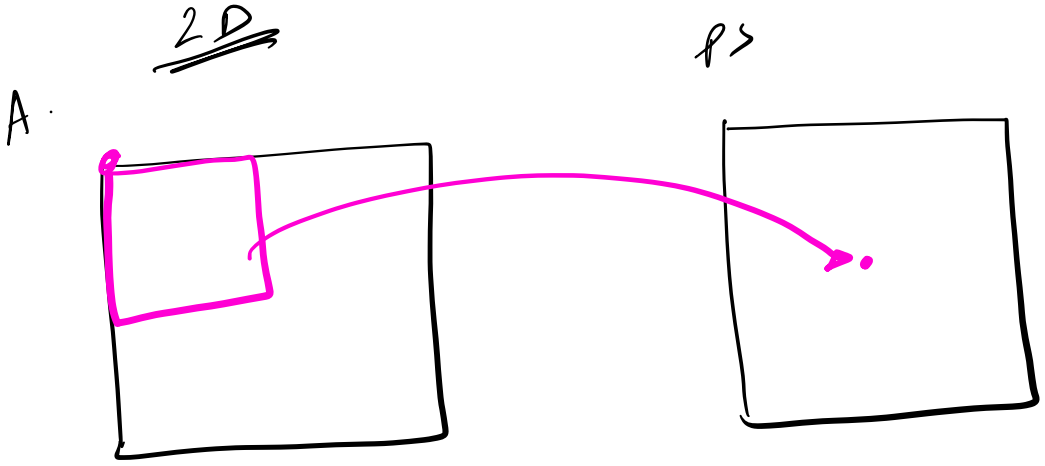
III

1D



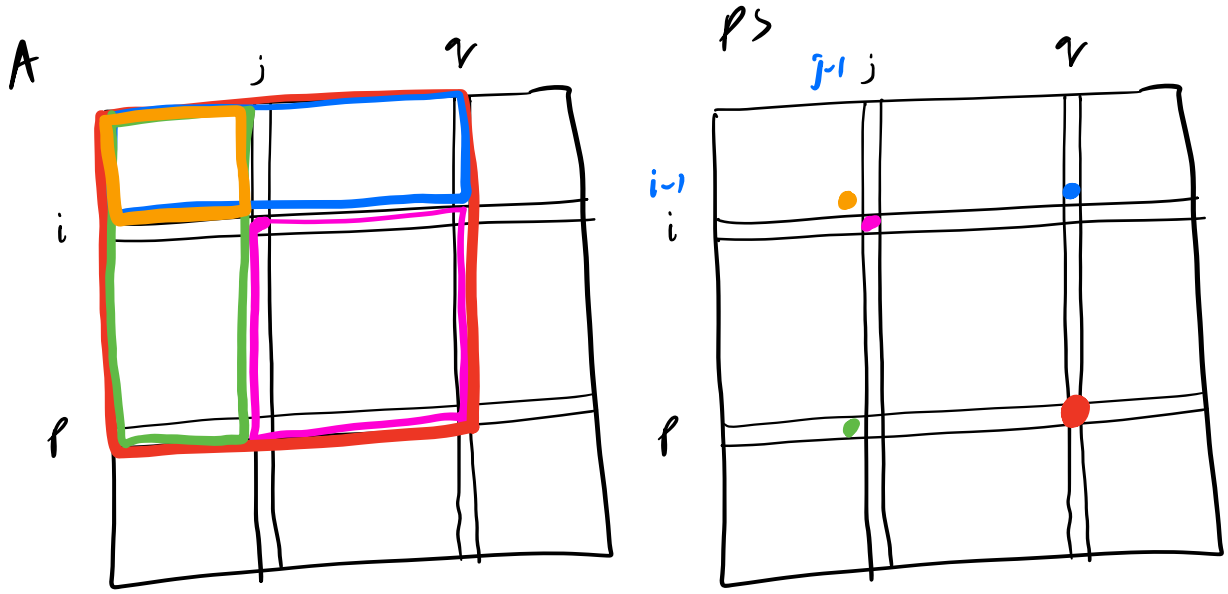
PS[i]
=
sum(0_i)

2D



PS(i,j) = sum(0,0, i,j)

PTO



$$P = R - B - Q + O$$

$$sum\left(\begin{matrix}(i,j)\\(p,q)\end{matrix}\right) = ps[p][q] - ps[i-1][q] - ps[p][j-1] + ps[i-1][j-1]$$

1 $q_n \longrightarrow O(1)$

q $q_n \longrightarrow O(q)$

Build the 2D PS

A

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

O(NM)

Row wise PS

$$\begin{bmatrix} 1 & 3 & 6 & 10 \\ 1 & 3 & 4 & 5 \\ 2 & 3 & 5 & 6 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

Column wise PS O(NM)

$$\begin{bmatrix} 1 & 3 & 6 & 10 \\ 2 & 6 & 10 & 15 \\ 4 & 9 & 13 & 21 \\ 7 & 13 & 20 & 27 \end{bmatrix}$$

PS

BUILD

TC = O(NM)

SC = O(NM)

Q

O(1) : SAME

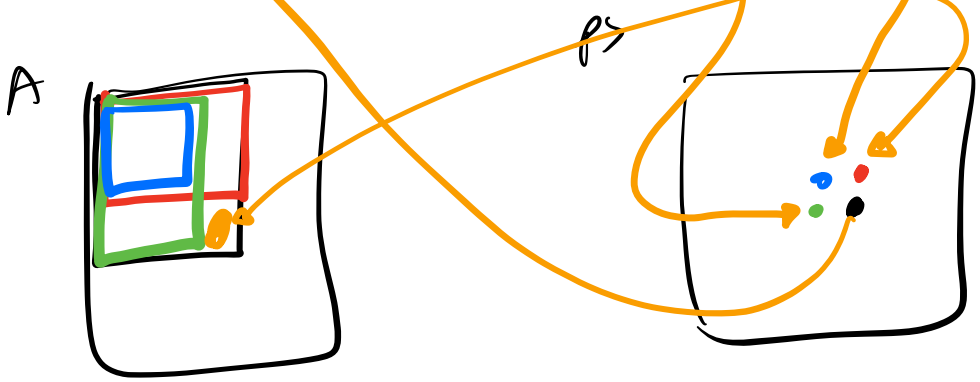
TC = O(NM + Q)

SC = O(NM)

O(1) : SAME

II BUILD

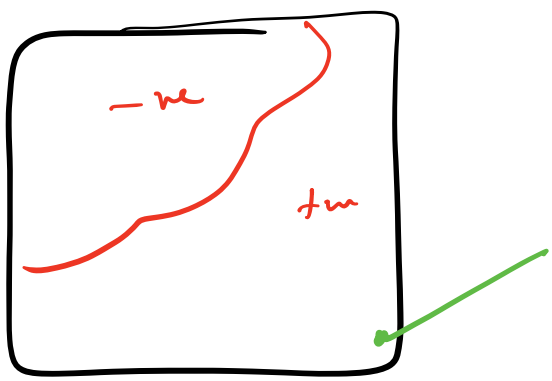
$$PS_{ij} = PS_{i-1\,j} + PS_{i\,j-1} - PS_{i-1\,j-1} + A_{ij}$$



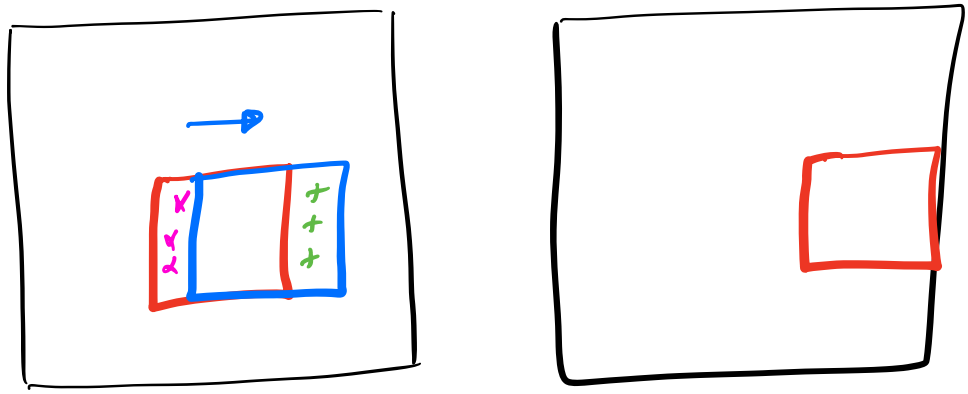
I Given a row-wise & col-wise sorted Matrix!
Find the MAX Sub-matrix sum!

$$A: \begin{bmatrix} -5 & -4 & -2 & -1 \\ -3 & -2 & -1 & 0 \\ -2 & -1 & 0 & 10 \\ -1 & 0 & 1 & 11 \end{bmatrix}$$

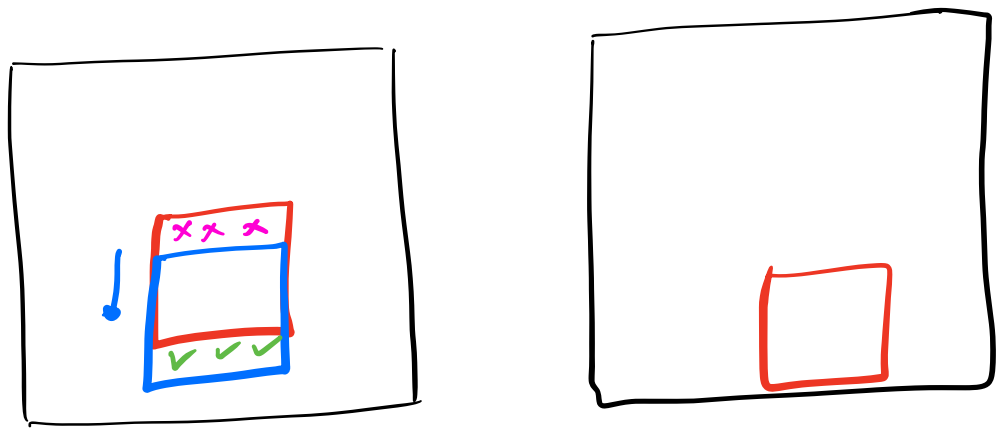
22



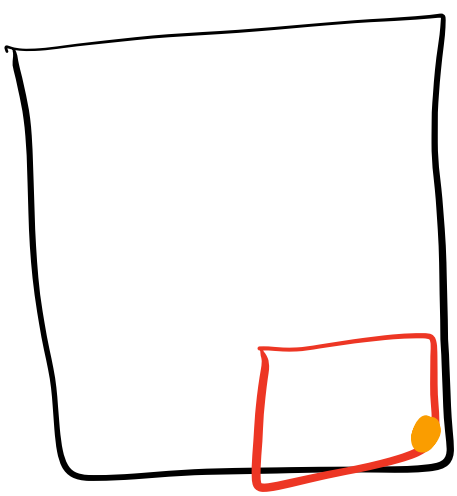
I



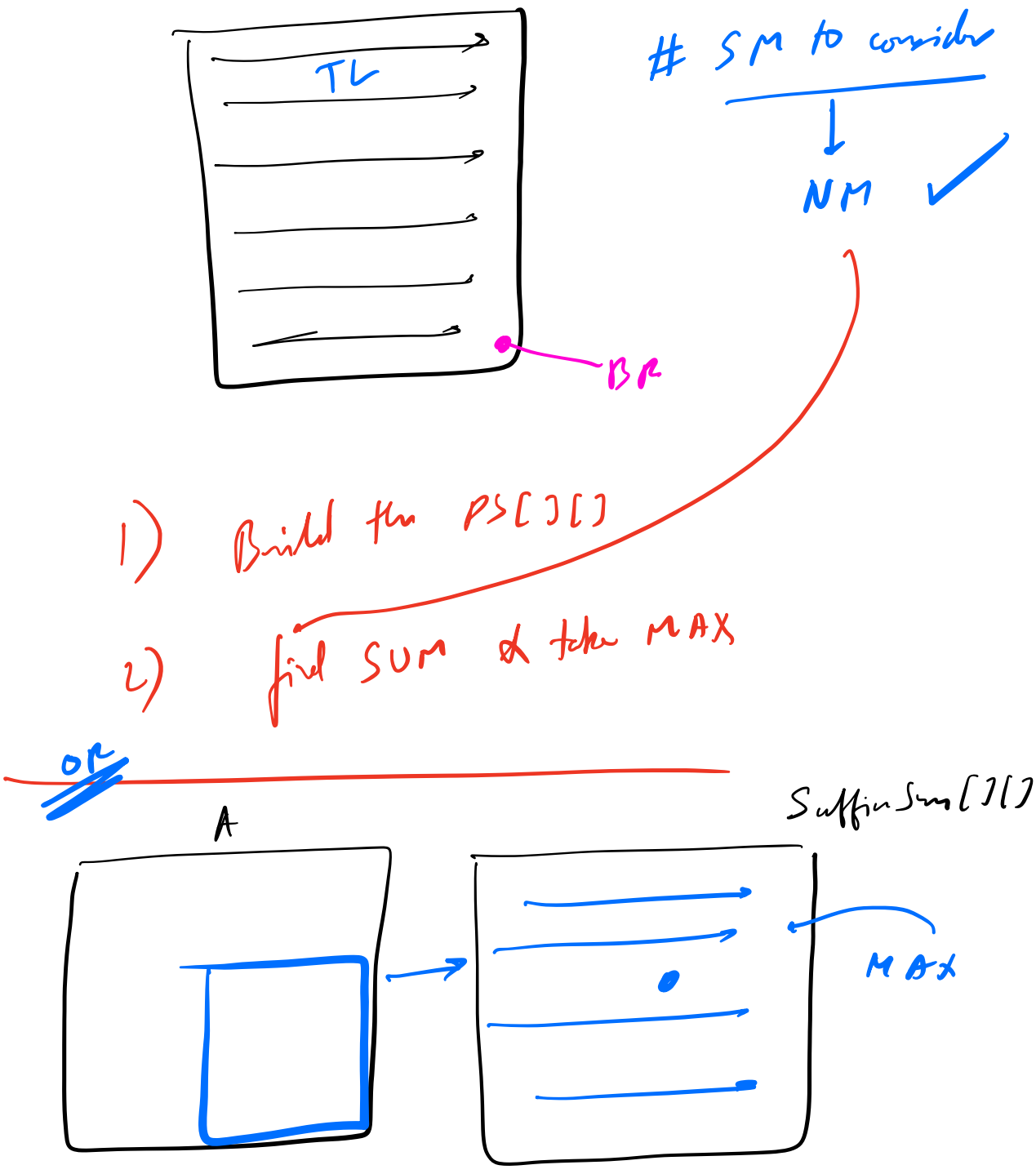
II



I & II : Conclusion:



NOTE:
ANS. S.M would have BR at (N-1, M-1).


TL
BR
SM to consider
NM
1) Build the PS[][]
2) find SUM & take MAX
OR
A
SuffixSum[][]
MAX