

Decimal Number System  $\rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 Base  $\rightarrow 10$

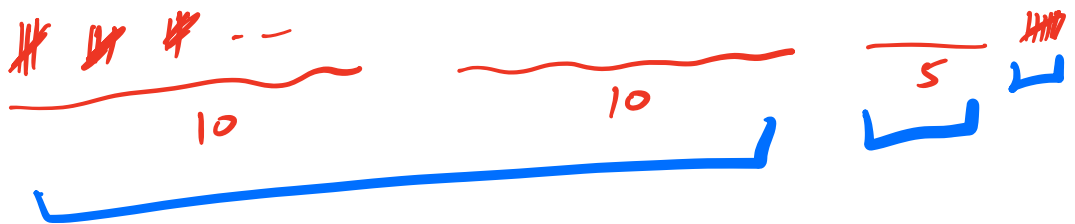
$$(345)_{10} \rightarrow 345$$

$$(345)_{10} \rightarrow 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$$

$$3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$$

$$(15276)_{10} \rightarrow 1 \times 10^4 + 5 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 6 \times 10^0$$

$$259 \rightarrow 2 \times 10^2 + 5 \times 10^1 + 9 \times 10^0$$



Hexadecimal Base  $\rightarrow 16$   
 $\{0, 1, \dots, 9, A, B, C, D, E, F\}$

$$(9523)_{16} = 9 \times 16^3 + 5 \times 16^2 + 2 \times 16^1 + 3 \times 16^0$$

$$\rightarrow ( )_{10}$$

$$\begin{array}{cccccc}
 4 & 3 & 2 & 1 & 0 & \\
 (A & 0 & F & 9 & 6)_{16} & \rightarrow
 \end{array}
 \begin{array}{c}
 A \times 16^4 + 0 \times 16^3 + F \times 16^2 + 9 \times 16^1 + 6 \times 16^0 \\
 \hline
 (\quad)_{10}
 \end{array}$$

④ Binary Number System  $\rightarrow \{0, 1\}$  base  $\rightarrow 2$

$(01(2)1)_2$  X INVALID

④ BIN  $\rightarrow$  DEC

$$\begin{array}{cccc}
 3 & 2 & 1 & 0 \\
 (1 & 1 & 0 & 1)_2 \rightarrow 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
 & & & 8 + 4 + 0 + 1 \\
 & & & = (13)_{10}
 \end{array}$$

$$\begin{array}{cccccc}
 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
 (1 & 0 & 1 & 0 & 1 & 0 & 1)
 \end{array}$$

$$2^6 + 2^4 + 2^2 + 2^0$$

$$64 + 16 + 4 + 1 = 85$$

④ DEC  $\rightarrow$  BIN

$(25)_{10} \rightarrow ( \quad )_2$

	<u>rem</u>
2   25	1
2   12	0
2   6	0
2   3	1
2   1	1
0	

$(11001)_2$

$$2^4 + 2^3 + 2^0$$

$$16 + 8 + 1 = 25 //$$

$(45)_{10} \rightarrow ( \quad )_2$

2   45	1
2   22	0
2   11	1
2   5	1
2   2	0
2   1	1
0	

$(101101)_2$

# ADDITION →

$$\begin{array}{r}
 \begin{array}{ccc}
 2 & 1 & 0 \\
 1 & 1 & \\
 3 & 6 & 8 \\
 + & 4 & 5 & 6 \\
 \hline
 8 & 2 & 4
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{digit} \rightarrow d = S \% 10 \rightarrow 4 \\
 \text{carry} \rightarrow c = S / 10 \rightarrow 1
 \end{array}$$

$$\begin{array}{r}
 8 \\
 + 6 \\
 \hline
 S = 14
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{ccc}
 1 & 1 & \\
 3 & 6 & 8 \\
 + & 4 & 5 & 6 \\
 \hline
 8 & 2 & 4
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 S = 14 \\
 d = 14 \% 10 \rightarrow 4 \\
 c = 14 / 10 \rightarrow 1
 \end{array}$$

$$\begin{array}{l}
 S = 12 \\
 d = 12 \% 10 \rightarrow 2 \\
 c = 12 / 10 \rightarrow 1
 \end{array}$$

$$\begin{array}{l}
 S = 8 \\
 d = 8 \% 10 \rightarrow 8 \\
 c = 8 / 10 \rightarrow 0
 \end{array}$$

## BINARY ADDITION

$$\begin{array}{r}
 \begin{array}{ccccccc}
 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
 + & 1 & 0 & 1 & 1 & 1 & 1 \\
 \hline
 1 & 0 & 0 & 0 & 1 & 0 & 0
 \end{array}
 \end{array}$$

$1111111 \rightarrow 23$   
 $1111111 \rightarrow 11$   
 $1000100 \rightarrow 37$

$$\begin{array}{l}
 d \rightarrow s \cdot 2 \\
 c \rightarrow s / 2
 \end{array}$$

$$\begin{array}{l}
 d \rightarrow 1 \cdot 2 \rightarrow 1 \\
 c \rightarrow 1 / 2 \rightarrow 0
 \end{array}$$

cd

$$\begin{array}{l}
 0 + 0 \rightarrow 00 \\
 0 + 1 \rightarrow 01 \\
 1 + 0 \rightarrow 01 \\
 1 + 1 \rightarrow 10
 \end{array}$$

## BITWISE OPERATORS

(operates on bits).

&  
AND

|  
OR

^  
XOR

~  
NEG

<<  
left shift

>>  
Right Shift

A	B	A & B	A   B	A ^ B	~ A
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

SAME  $\rightarrow 0$   
DIFF  $\rightarrow 1$

## ④ Bitwise Operators on Numbers

5 & 6  
↓  
4

$$\begin{array}{r}
 \begin{array}{c} 2 \quad 1 \quad 0 \\ 5 \longrightarrow 1 \quad 0 \quad 1 \\ 6 \longrightarrow 1 \quad 1 \quad 0 \\ \hline \text{\&} \\ \hline 1 \quad 0 \quad 0 \longrightarrow 4 \end{array}
 \end{array}$$

int a = (5 & 6);  
print(n); → 4 //

20 | 45 →

1	0	1	0	0
1	0	1	1	0
1	1	1	1	0

→ 61 //

20 ^ 45 →

$$\begin{array}{r}
 \begin{array}{c} 1 \quad 0 \quad 1 \quad 0 \quad 0 \\ \wedge \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \\ \hline 1 \quad 1 \quad 1 \quad 0 \quad 0 \end{array} \longrightarrow 57 //
 \end{array}$$

$$\begin{matrix} 4 & 3 & 2 & 1 & 0 \\ (1 & 1 & 1 & 1 & 1) \end{matrix}_2 \rightarrow 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

GP

$$\frac{a(r^n - 1)}{r - 1}$$

$$a = 1$$

$$r = 2$$

$$n = 5$$

$$\frac{1(2^5 - 1)}{2 - 1}$$

$$= 2^5 - 1$$

$$\begin{matrix} k & \dots & 3 & 2 & 1 & 0 \\ (1 & 1 & 1 & 1 & 1 & \dots & 1) \end{matrix}_2$$

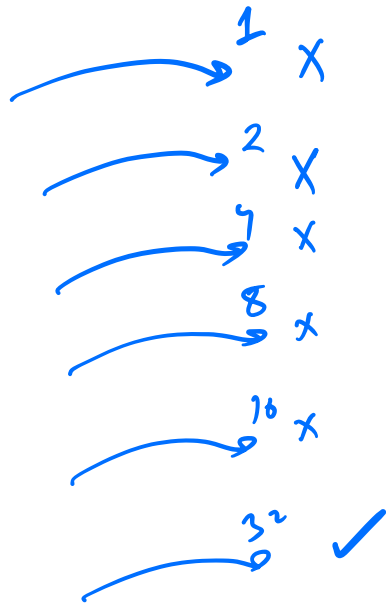
$$\rightarrow 2^0 + 2^1 + \dots + 2^k$$

$$2^{k+1} - 1$$

$$2^0 + 2^1 + 2^2 + \dots + 2^{k-1} = 2^k - 1$$

$$\Rightarrow 2^0 + 2^1 + \dots + 2^{k-1} < 2^k$$

$$2^0 + 2^1 + 2^2 + \dots < 2^{99} < 2^{100}$$



$$-1 -2 -4 -8 -16 + 32 = +1$$

Properties

$$(101010)_2$$

$$\rightarrow 2^5 + 2^3 + 2^1$$

$$\downarrow \text{even} + \downarrow \text{even} + \downarrow \text{even} = \text{even!}$$

$$\text{Even} + \text{Even} = \text{Even}$$

exception  $2^0 \rightarrow 1 : \text{ODD}$

$$\text{EVEN} + \text{ODD} \rightarrow \text{ODD}$$

1  $\rightarrow$  Set  
0  $\rightarrow$  Unset

$$110101$$

$$2^5 + 2^4 + 2^2 + 2^0$$

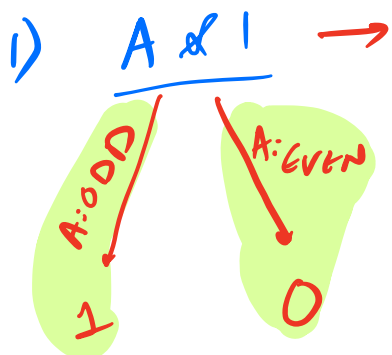
$$\downarrow \text{EVEN} + \downarrow \text{ODD} \rightarrow \text{ODD}$$



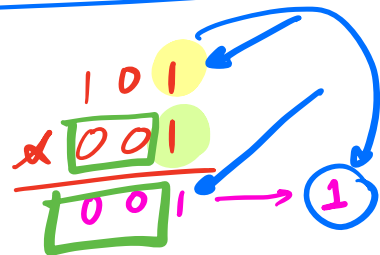
If a no. is ODD, its 0th bit  $\rightarrow 1$   
 \_\_\_\_\_ EVEN \_\_\_\_\_  $\rightarrow 0$

$n = 5 : 101$   
 $n = 15 : 1111$   
 $n = 7 : 111$   
 $n = 3 : 11$   
 $n = 11 : 1011$

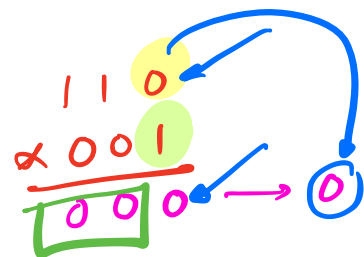
$n = 16 : 10000$   
 $n = 6 : 110$   
 $n = 9 : 1001$   
 $n = 12 : 1100$   
 $n = 17 : 11111$



ODD  
 $A: 5$   
 $5 \& 1 \rightarrow 1$



EVEN  
 $A: 6$   
 $6 \& 1 \rightarrow 0$



```

if (A % 2 == 1) {
    → ODD
} else {
    → EVEN
}

```

```

if ((A & 1) == 1) {
    → ODD
} else {
    → EVEN
}

```



NOTE: PRECEDENCE of bitwis operators  
is LESS!

$$\text{if } (A \times 1 == 0) \quad \times$$

$$\text{if } ((A \times 1) == 0) \quad \checkmark$$

$$2) \quad A \times 0 \rightarrow 0$$

$$3) \quad A \times A \rightarrow A$$

$$4) \quad A | 0 \rightarrow A$$

$$5) \quad A \wedge 0 \rightarrow A$$

$$6) \quad A \wedge A \rightarrow 0$$

$$\begin{array}{r} 10110 \\ \wedge 10110 \\ \hline 00000 \end{array}$$

④ Commutative property →

$$a \times b = b \times a$$

$$a | b = b | a$$

$$a \wedge b = b \wedge a$$

$$\frac{a \times b \times c}{n} = \frac{c \times a \times b}{n}$$

XOR,  
OR, AND

$$= a \times c \times b$$

$$b \times a \times c$$

$$b \times c \times a$$

$$c \times b \times a$$

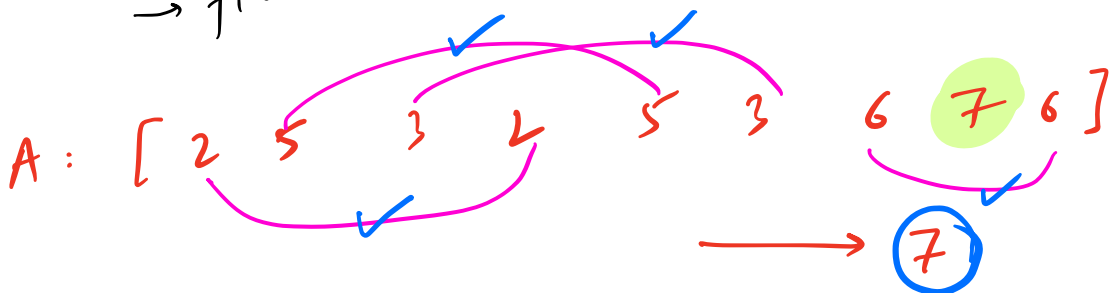
⑧ Associative property

$$\begin{aligned}(a \times b) \times c &= a \times (b \times c) \\ (a \div b) \div c &= a \div (b \div c) \\ (a \wedge b) \wedge c &= a \wedge (b \wedge c)\end{aligned}$$

Q  $a \wedge b \wedge a \wedge d \wedge b$

$$\begin{aligned}&= (a \wedge a) \wedge (b \wedge b) \wedge d \\&\quad \downarrow \quad \quad \downarrow \\&(0 \wedge 0) \wedge d \\&\quad \downarrow \\&(0 \wedge d) \\&\quad \downarrow \\&d\end{aligned}$$

Q Given an array.  
 → every element repeats twice  
 except one element which appears  
 EXACTLY ONCE!  
 → find that no!



$$x \wedge x = 0 \quad | \quad 0 \wedge x = x$$

$$2 \wedge 5 \wedge 3 \wedge 2 \wedge 5 \wedge 3 \wedge 6 \wedge 7 \wedge 6$$

$(2 \wedge 2) \wedge (5 \wedge 5) \wedge (3 \wedge 3) \wedge (6 \wedge 6) \wedge 7$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $0 \quad \wedge \quad 0 \quad \wedge \quad 0 \quad \wedge \quad 0 \quad \wedge \quad 7$   
 $\downarrow \quad \downarrow$   
 $0 \quad \wedge \quad 0 \quad \wedge \quad 7$   
 $\downarrow$   
 $0 \quad \wedge \quad 7$   
 $\downarrow$   
 $7$

ANS = 0 ;

$$\{ (i=0; i < n; i++) \}$$

$$ANS = ANS \wedge A[i];$$

ret ANS/

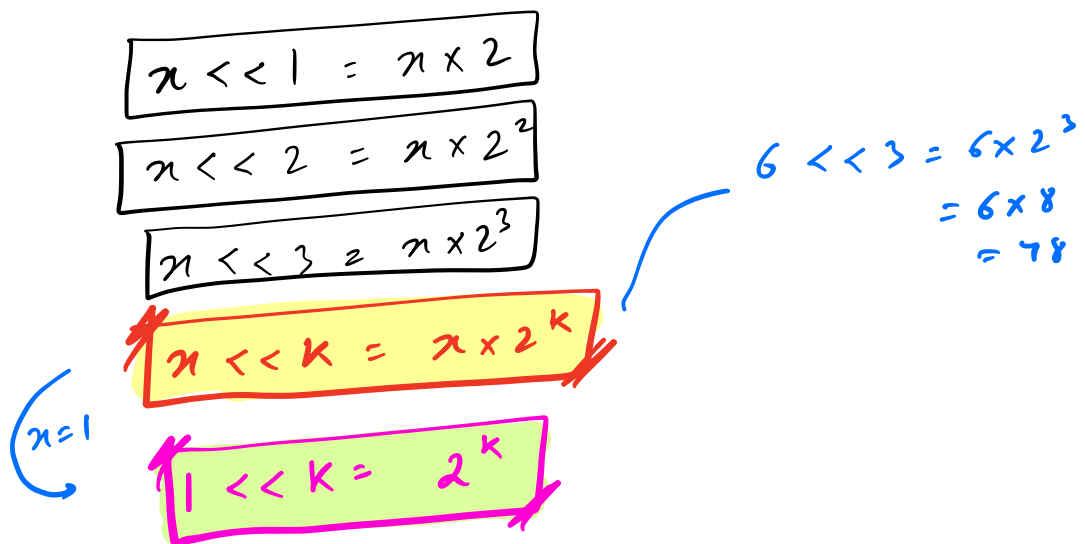
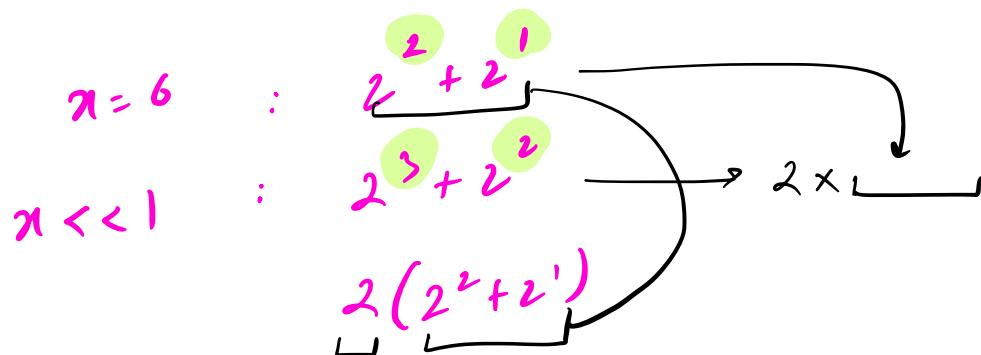
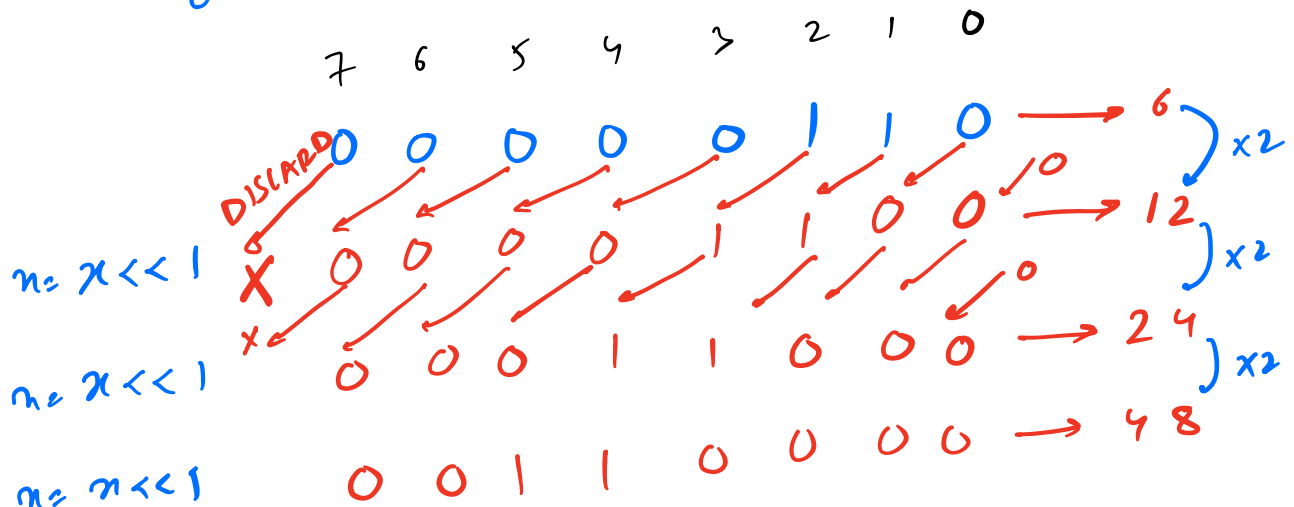
TC :  $O(N)$

SC :  $O(1)$

# ② Left Shift ( $\ll$ )

1 byte = 8 bits

byte  $x = 6$ ;



$$2^4 ? \rightarrow n = (1 \ll 4)$$

└─ 16

1: 0 0 0 0 0 0 0 1

$1 \ll 4$ : 0 0 0 1 0 0 0 0

$2^4 = 16$  ✓

$2^{20} \rightarrow (1 \ll 20)$

int n = 5;

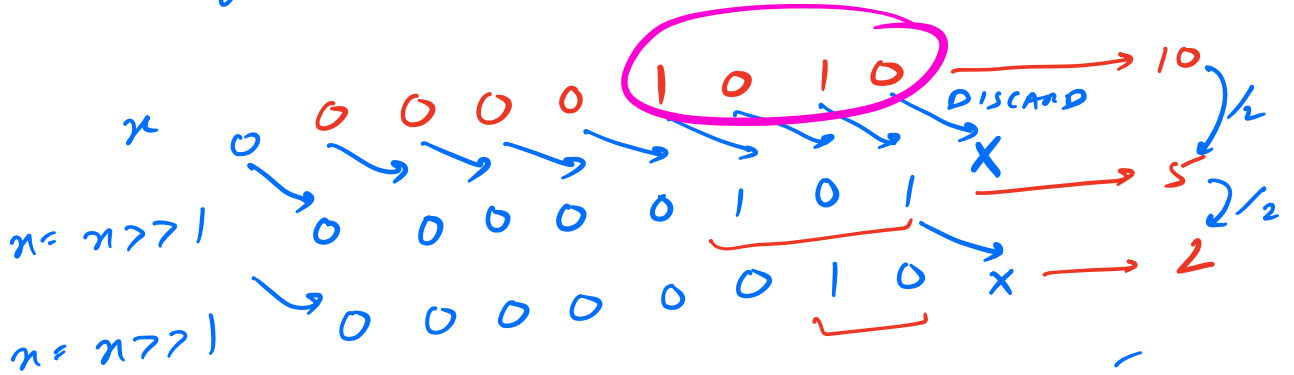
$n = n \ll 1;$

print(n) →

ASSIGN!

④ Right Shift ( $\gg$ )

byte  $n = 10$



10 :  $2^3 + 2^1$   
 10771 :  $2^2 + 2^0 = \underline{\underline{2^3 + 2^1}}$   
 2

$$n > 71 = n/2$$

$$n_{772} = n/2^2$$

$$n \gg 3 = n/2^3$$

$$n \gg K = \left\lfloor \frac{n}{2^K} \right\rfloor$$