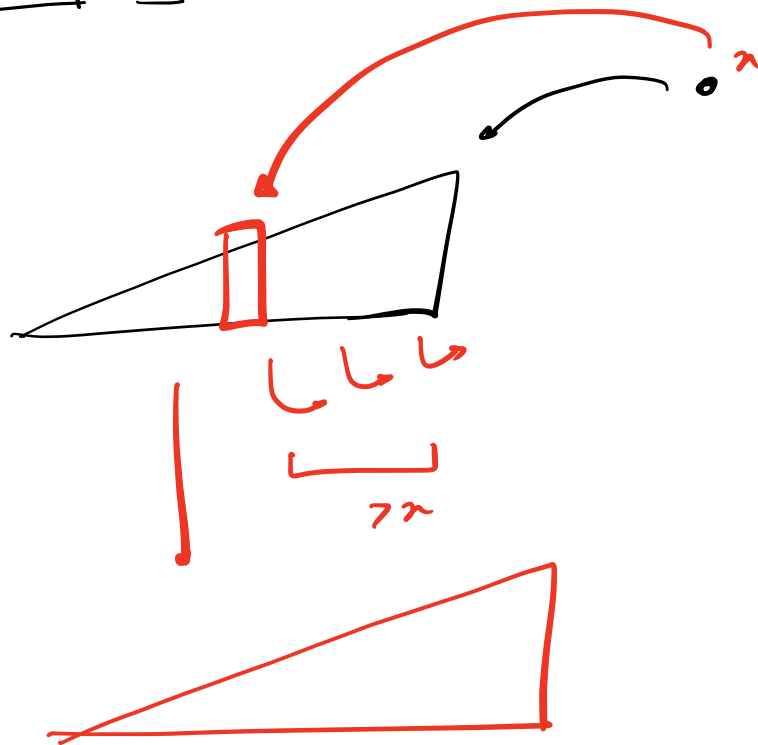
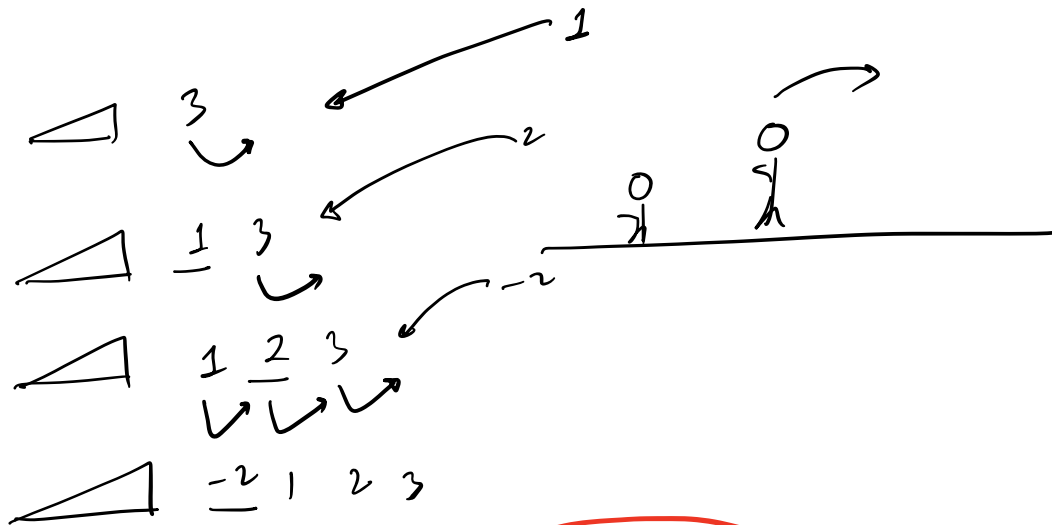
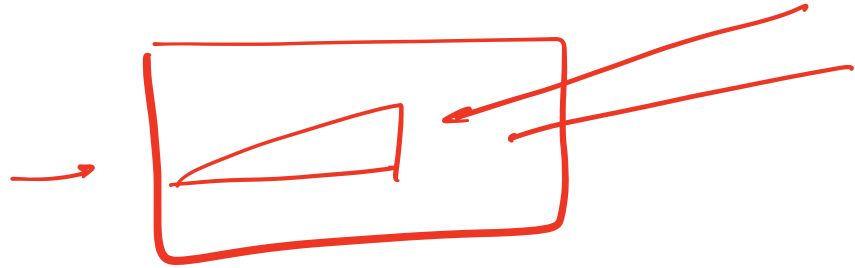


# ④ Insertion Sort



A: 

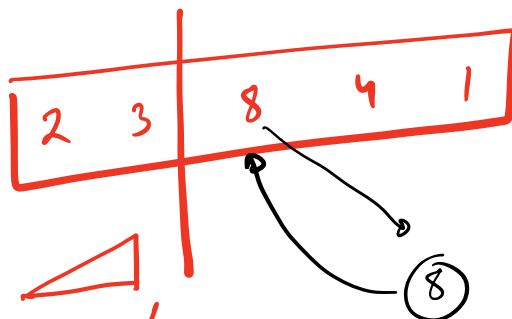
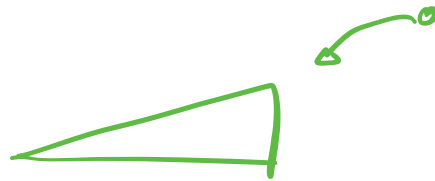
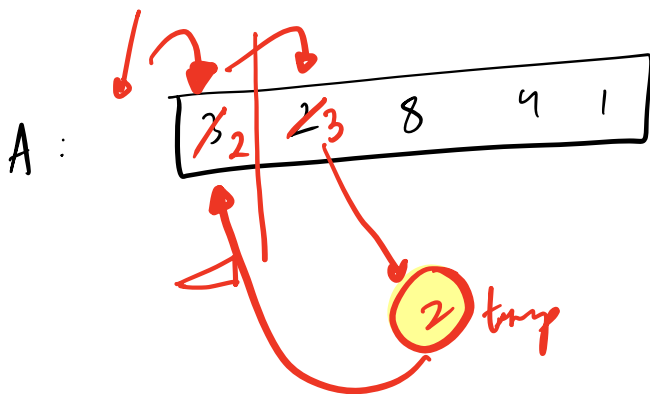
3	2	8	4	1
---	---	---	---	---

TC:  $O(N^2)$

SC:  $O(N)$

B: 

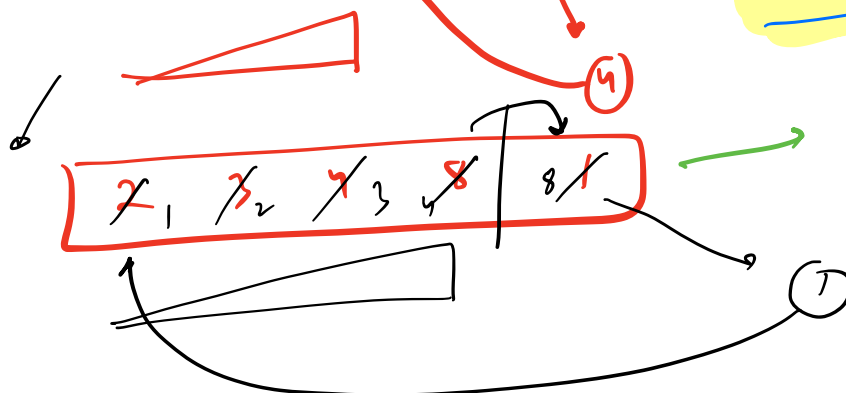
1	2	3	4	8
---	---	---	---	---



SC =  $O(1)$

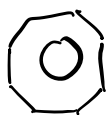
INPLACE!

ONLINE



Q

$N$  nuts &  $N$  bolts



○ Every nut is distinct in size.

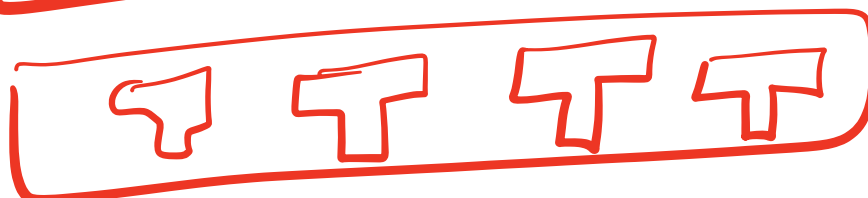
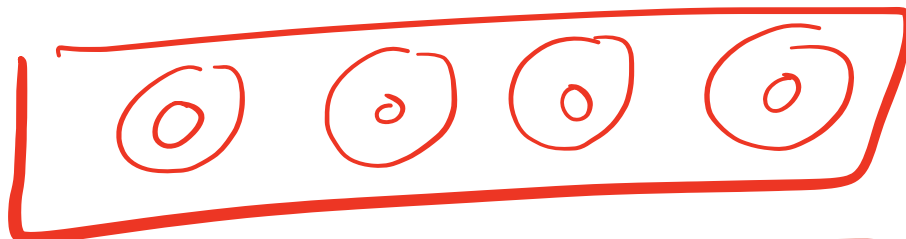
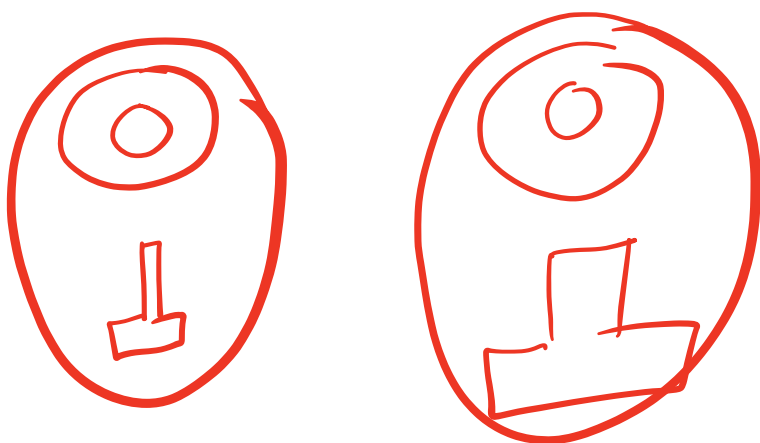
○ Every bolt is distinct in size.

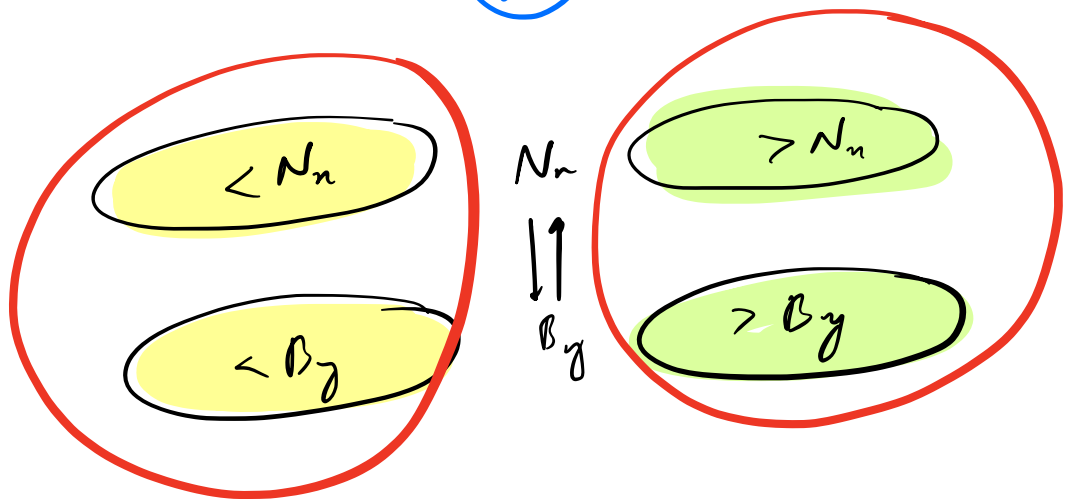
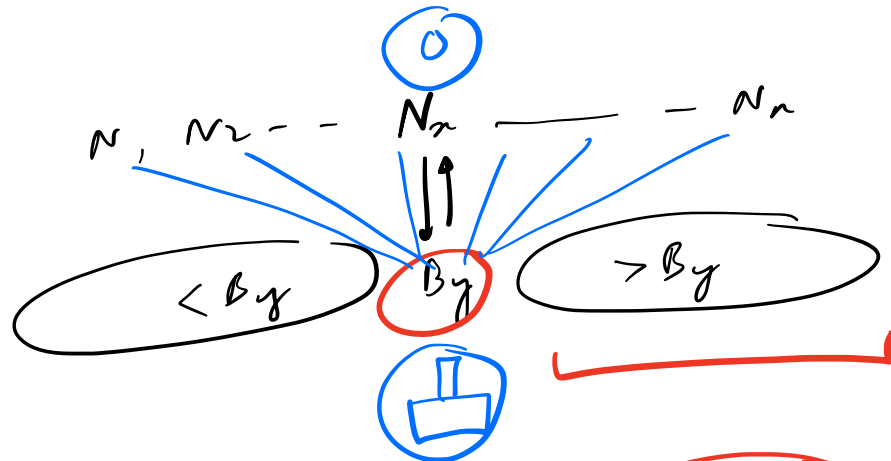
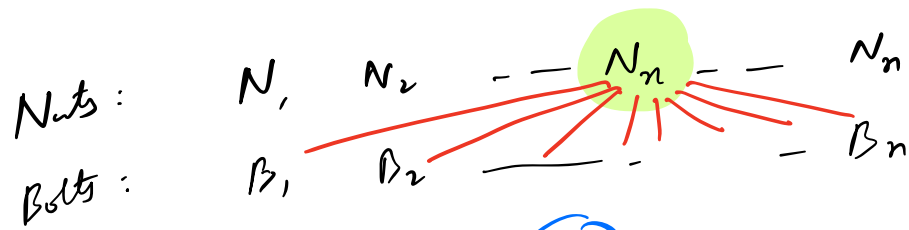
○ Every nut has a corresponding bolt!

Sort the nuts & bolts in  $INC$  order of size!

→ You cannot compare b/w the size of 2 nuts or 2 bolts

→ You can compare a nut & a bolt





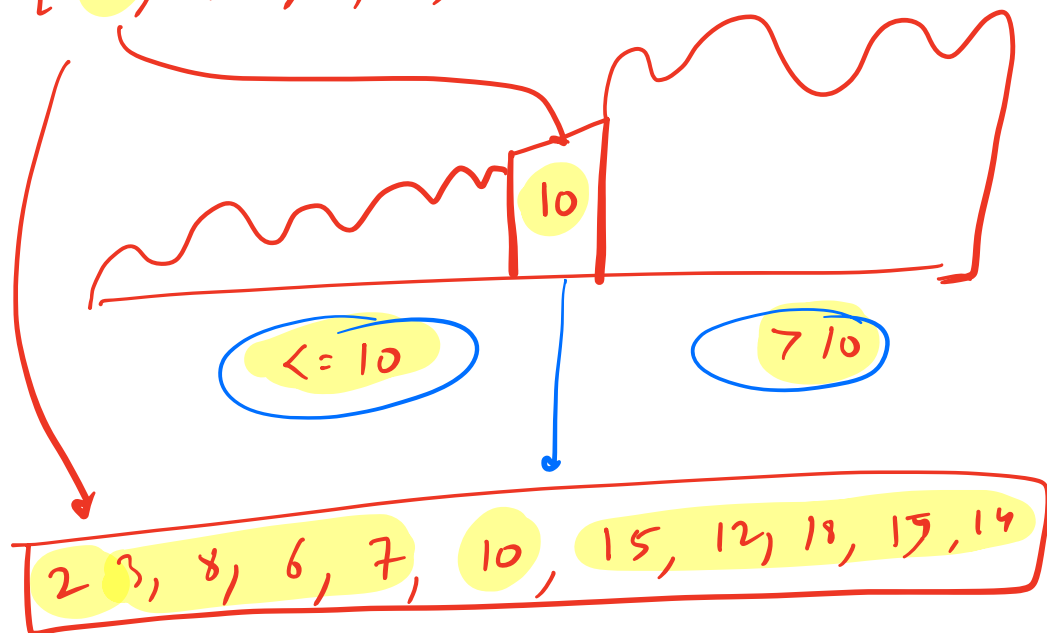
QUICK SORT : idea

Q: Given an Array.  
Rearrange the array s.t.

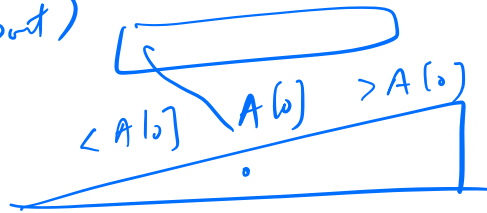
- $A[0]$  should go to its right sorted position!
- All the elements  $\leq A[0]$  should go to the left of the right sorted pos of  $A[0]$

→ \_\_\_\_\_  $> A[0]$  \_\_\_\_\_  
right

A: [10, 3, 8, 15, 6, 12, 2, 18, 7, 15, 14]



1) Sort (Merge Sort)



$TC: O(N \log N)$

2)

A: [10, 3, 8, 15, 6, 12, 2, 18, 7, 15, 17]

# Count  $< 10$   
= 5

$\leq 10$

$> 10$

$TC = O(N)$

A:

[2, 10, 4, 1, 9]

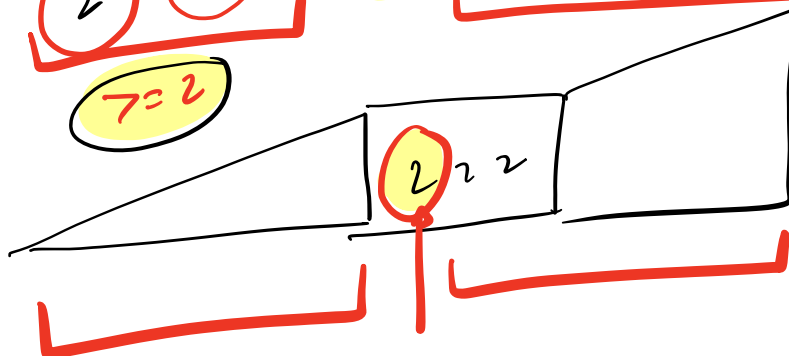
# Count  $< 2$   
= 1

7 2

A:

2 10 2 4 2 1 0

7 = 2



```
// A[N]; cut = 0, x = A[0];
for (i = 1; i < N; i++) {
    if (A[i] < x) cut++;
}
```

PARTITIONING  
ALGO

```
swap(A[0], A[cut]);
```



```
P1 = 0, P2 = N-1;
while (P1 < cut && P2 > cut) {
```

```
    while (A[P1] < x) P1++;
```

```
    while (A[P2] >= x) P2--;
```

```
    swap(A[P1], A[P2]);
```

```
    P1++, P2--;
```

```
    return cut;
```

TC = O(N)

SC = O(1)

Q. Given A[], // [s, e]  
Move A[s] in its right sorted place  
in the SA: A[s, e]

move left  $\leq A[s]$   
— right  $> A[s]$

```
// s, e
// A[N]; cnt = 0, x = A[s];
for (i = s+1; i <= e; i++) {
    if (A[i] < x) cnt++;
}
```

int rearrange(  
A[], s, e)

```
swap(A[s], A[s+cnt]);
```

```
P1 = s, P2 = e;
while (P1 < start && P2 > start) {
```

```
    while (A[P1] < x) P1++;
```

```
    while (A[P2] >= x) P2--;
```

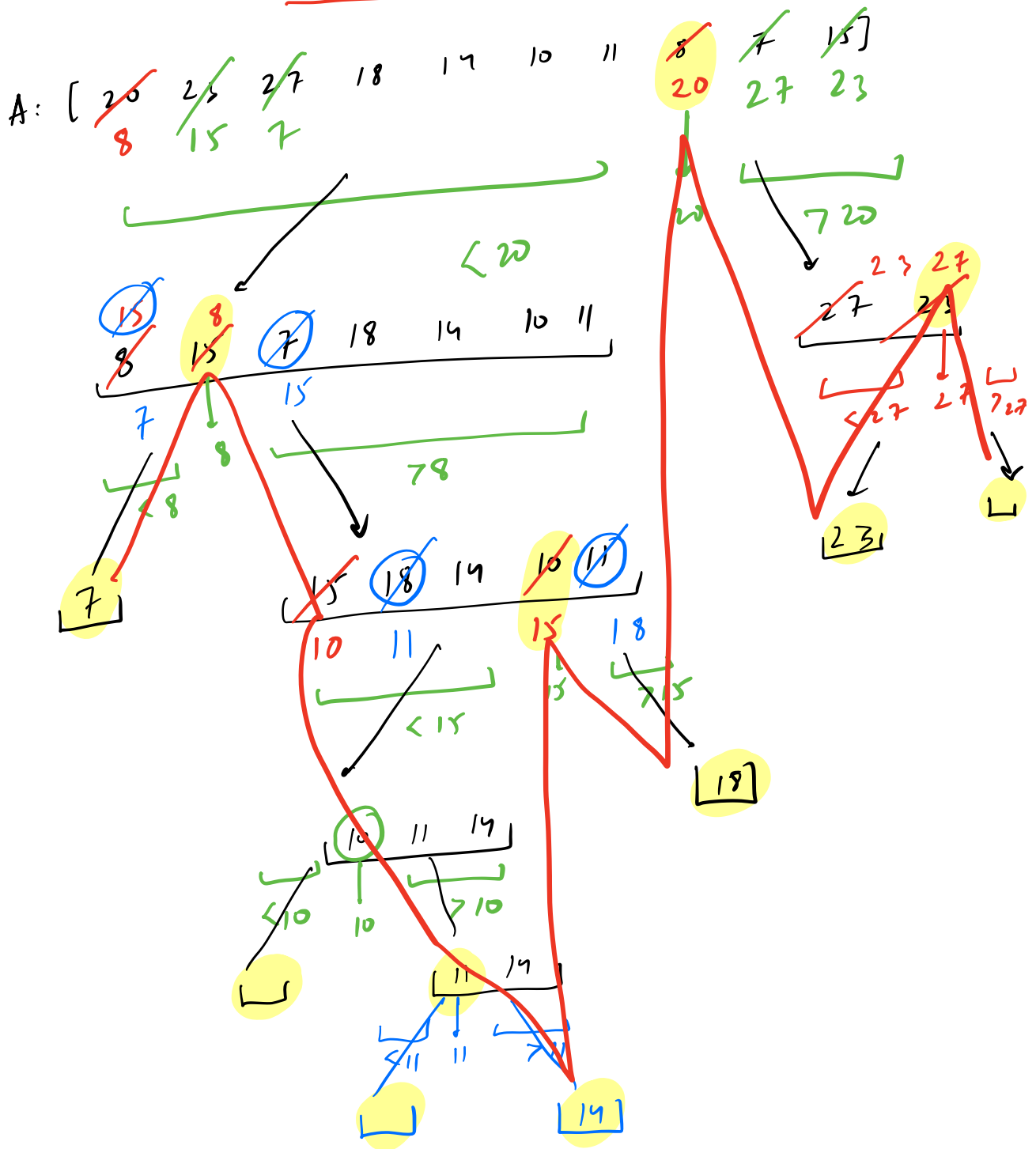
```
    swap(A[P1], A[P2]);
```

```
    P1++, P2--;
```

```
}
```

```
return s+cnt;
```





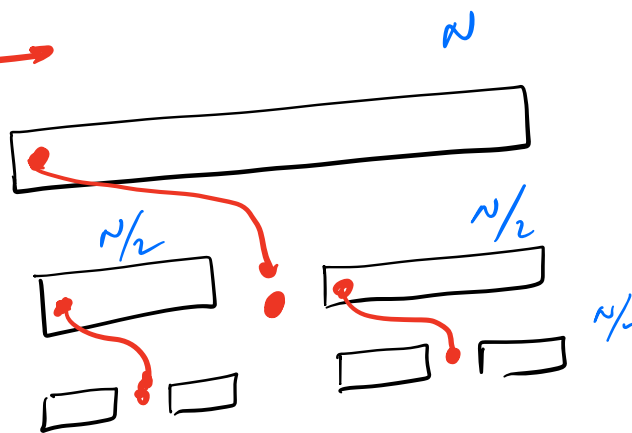
```

void qSort ( A[], s, e ) { 0 A[], 0, n-1)
    if ( s >= e ) return;
    p = partition ( A[], s, e );
    qSort ( A[], s, p-1 );
    qSort ( A[], p+1, e );
}

```

## ④ Complexity

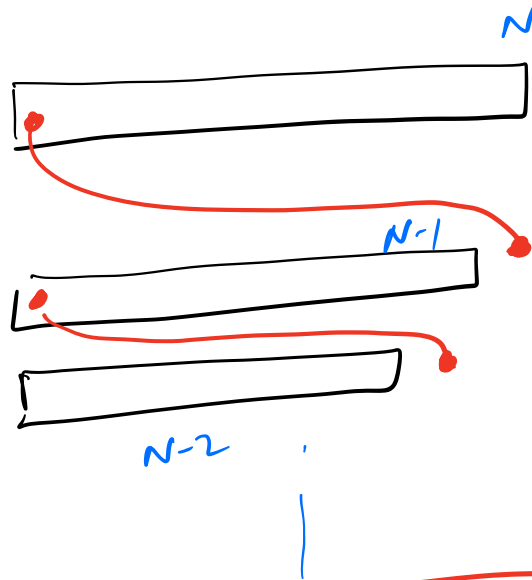
1)  
CASE



$$T(N) = N + 2T(N/2)$$

TC  $\rightarrow$   $N \log N$  ✓

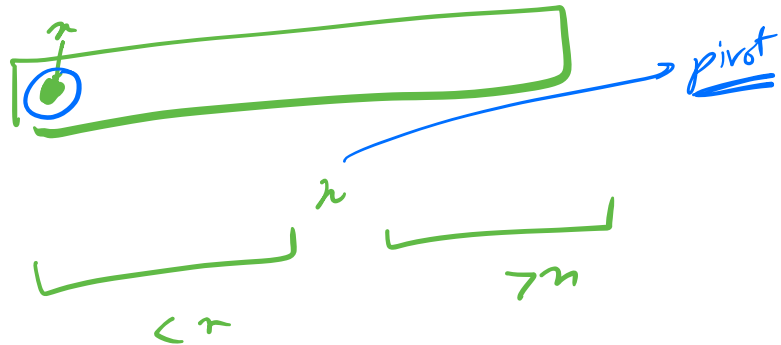
2  
CASE



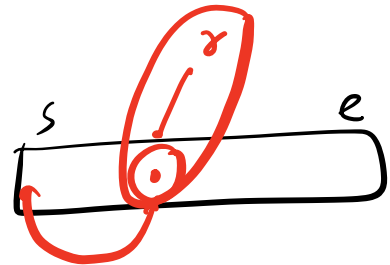
$$T(N) = N + T(N-1)$$

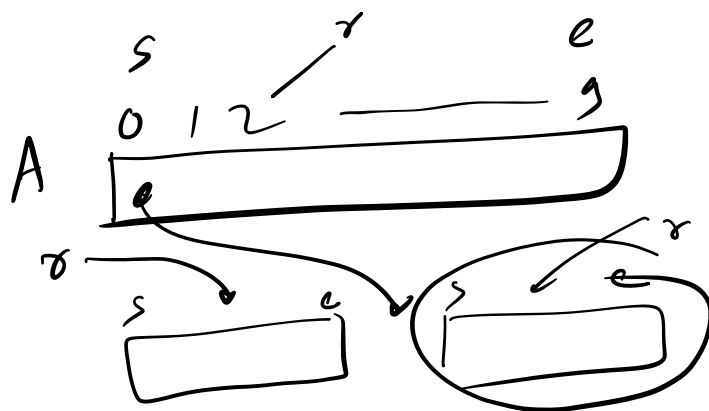
TC  $\rightarrow N^2$

① How to improve it?



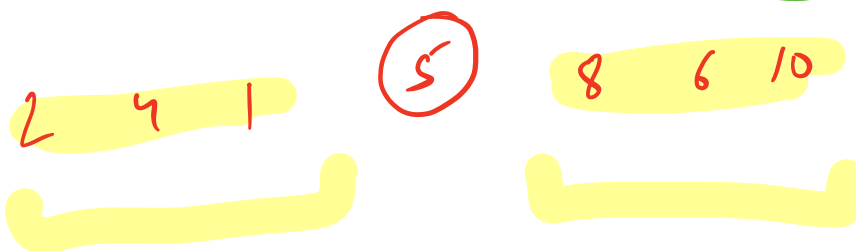
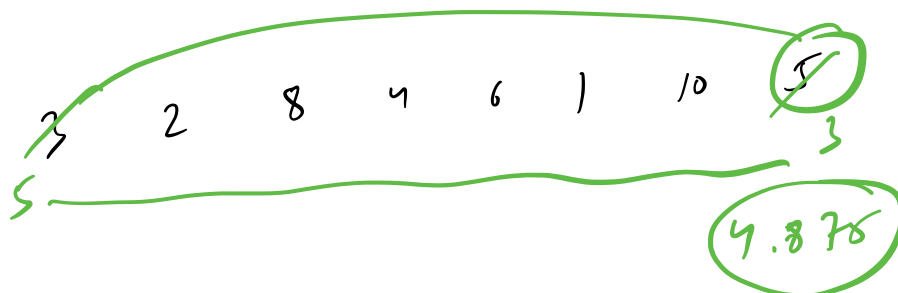
1) Randomly choose the pivot!  
 $r = \text{random}(s, e)$   
 $\text{swap}(A[s], A[r]);$   
 $\text{return } \left( \frac{\quad}{1} \right)$





MEAN

A:

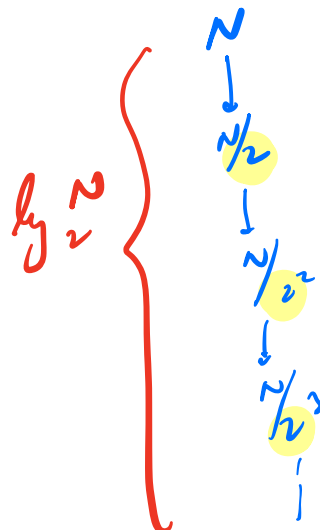
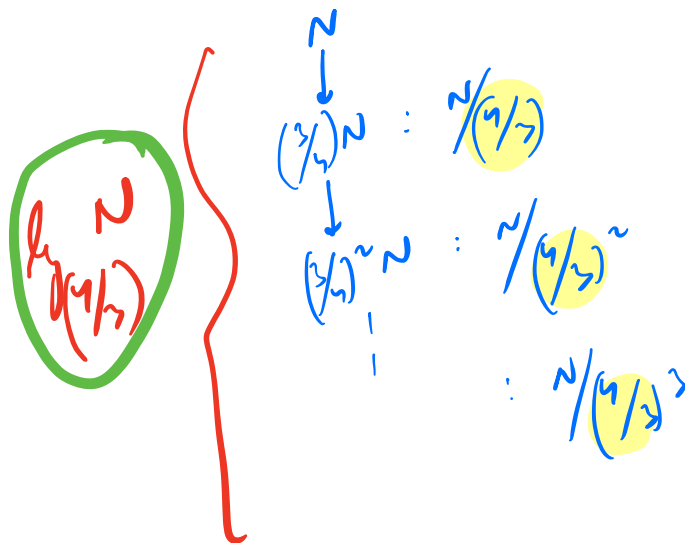
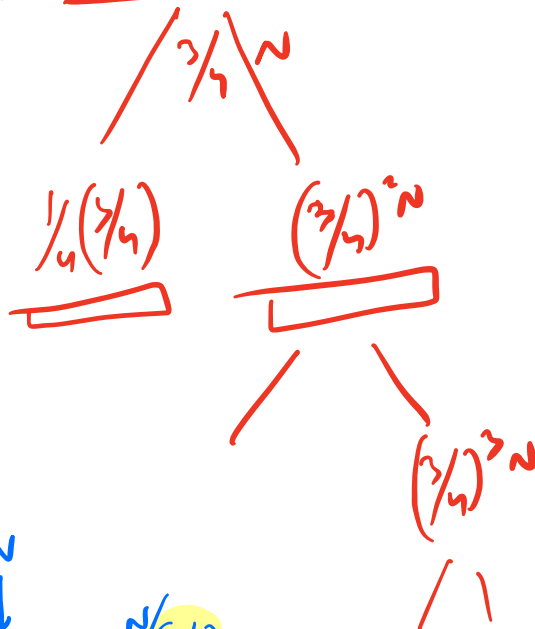
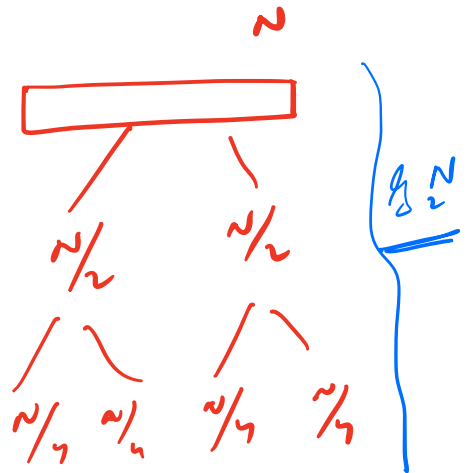
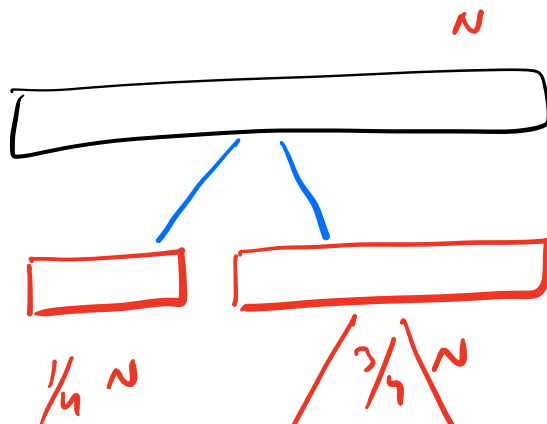


MODE

MEDIAN

# AVERAGE

$r = \text{random}(s, e)$   
 $\text{swap}(A[s], A[r]);$   
 $\text{return } \left( \frac{\quad}{1} \right)$



Av. Case: # levels =  $\log_{(4/3)} N$

Amount of Work done on every level  
=  $O(N)$

$$TC = N \cdot \log_{(4/3)} N$$

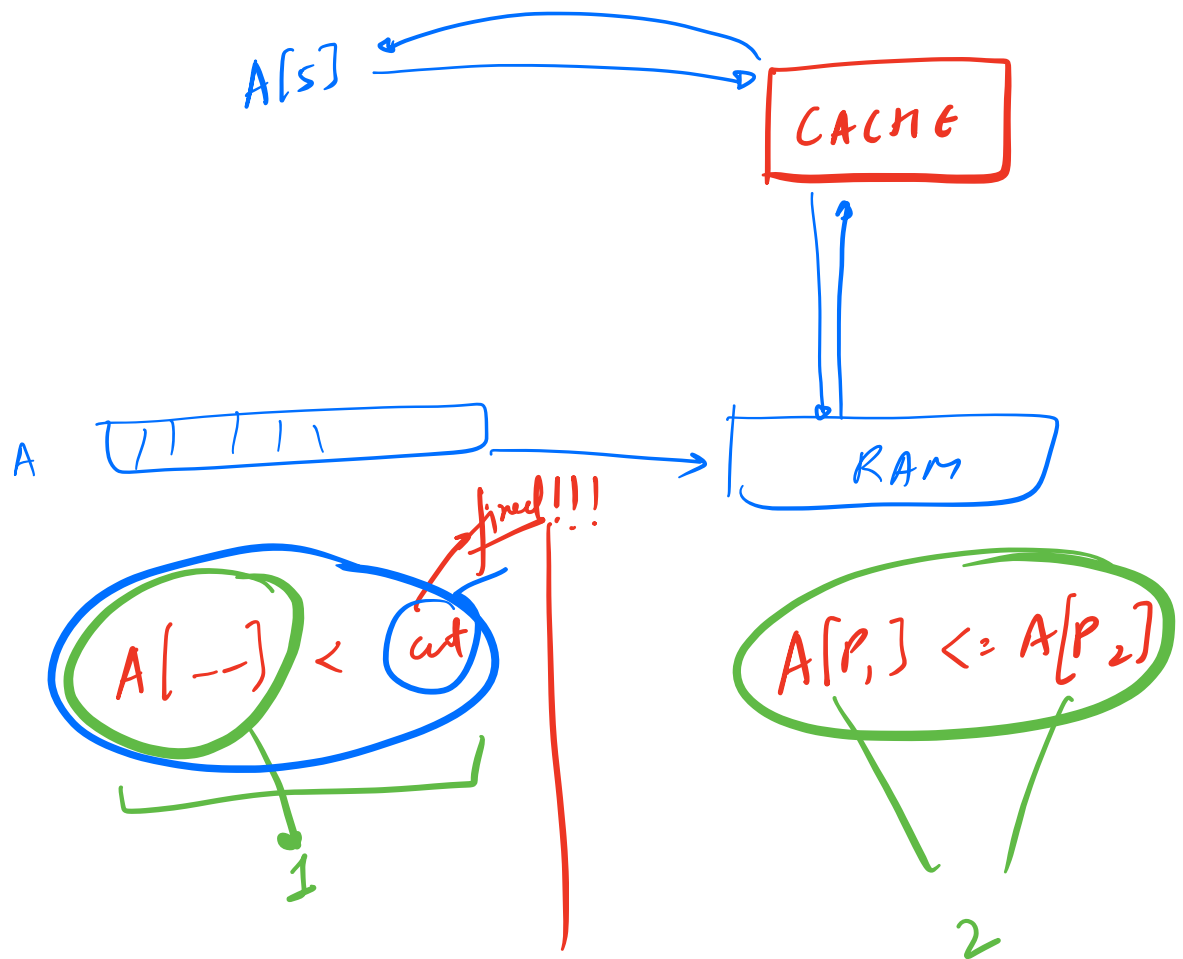
$$TC = O(N \log N) \quad \checkmark$$

$$SC = O(\log N) \quad \checkmark$$

Why **Quick Sort** over Merge Sort?

1) Space:  $O(\log n)$   $O(n)$

2) Quick's Runtime is less than Merge's —



⑧ Quick Sort is CACHE friendly!