⊛ Sub-Array → [S.A.]

? Sub Array → Contiguous part of an Array

→ Single element

→ Entire array

	0	1	2	3	4
A:	3	5	7	2	6

idx: [0, 1, 3] → X   2 is missing!

idx: [2, 3] → ✓

idx: [2] → ✓

→ Unique identifier of a S.A. ?

	0	1	2	3	4	5
A:	4	5	6	2	9	8

[L, R] → start & end index of the S.A → $\boxed{A[1, 4]}$

$0 =< L <= R < N$

0 … N-1

Q Given an Array. Given [L, R] of a S.A.
Print the S.A.

		L				R
	0	1	2	3	4	5
A:	5	6	2	5	2	9

print → 6, 2, 5, 2, 9 ✓

L = 1
R = 5

```
f(i = L; i <= R; i++){
    print(A[i]);
}
```

L

TC: O(R-L+1)

↓

$\boxed{TC: O(R-L)}$

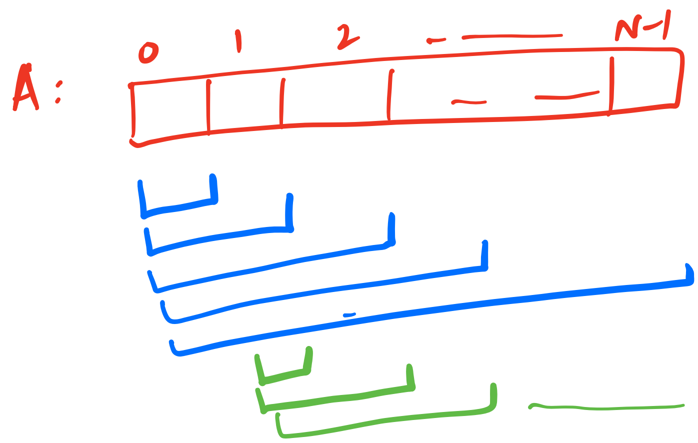
↓

$\boxed{O(N)}$

$\boxed{SC: O(1)}$

$\boxed{\text{\# elements in } [L, R] = R - L + 1}$

Q Given an array of size N.
Find the # of S.A s → depends on N and not on content!



0^{th} idx	1^{st} idx	2^{nd} i	----	$(N-1)^{th}$ idx
[0,0]				
[0,1]	[1,1]		----	
[0,2]	[1,2]	[2,2]		
⋮	[1,3]	[2,3]		
	⋮	⋮		
[0,N-1]	[1,N-1]	[2,N-1]		[N-1,N-1]
N +	N-1 +	N-2 +	---- +	1

Total # of SA = N + N-1 + N-2 + ---- 1

$$\boxed{\text{Total \# SA} = \frac{N(N+1)}{2}} \approx O(N^2)$$

Q. Given an Array. Print all the SA.

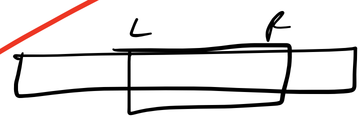
$$A : [\overset{0}{4}, \overset{1}{5}, \overset{2}{2}]$$

[L, R]	{
[0, 0] →	(4)
[0, 1] →	(4, 5)
[0, 2] →	(4, 5, 2)
[1, 1] →	(5)
[1, 2] →	(5, 2)
[2, 2] →	(2)

$$\boxed{0 <= L <= R < N}$$

```
f(L=0; L<N; L++){
    f(R=L; R<N; R++){
        // [L,R]
        f(i=L; i<=R; i++){
            print(A[i]);
        }
        printNewLine();
    }
}
```

SA: $N^2 \times N/2$



$$\boxed{TC : O(N^3)}$$

Q. Given an Array.

Print the sum of All Sub Arrays!

$$A : \begin{array}{ccc} 0 & 1 & 2 \\ \hline [\,4, & 5, & 2\,] \end{array}$$

?

[L,R]		
[0, 0] →	(4) →	4
[0, 1] →	(4, 5) →	9
[0, 2] →	(4, 5, 2) →	11
[1, 1] →	(5) →	5
[1, 2] →	(5, 2) →	7
[2, 2] →	(2) →	2

1. Create the PS Array!

2.
```
for(L = 0; L < N; L++) {          → N^2
    for(R = L; R < N; R++) {
        // [L,R]
        print( PS[R] - PS[L-1] );    → take care L==0
    }
}
```

$$\boxed{TC = O(N^2)}$$

$$\boxed{SC = O(N)}$$

Q Given an array!
Print the sum of all SAs which start at 2nd index!

	0	1	2 (L)	3	4	5
A :	5	2	3	6	5	2

→ 3 → 9 → 14 → 16

```
sum = 0;

for(R=2; R<N; R++){
    sum += A[R];
    print(sum);

}
```

$\boxed{TC = O(N)}$

$\boxed{SC = O(1)}$

BACK TO ORIGINAL QUES [PRINT ALL SA SUMS]

	0	1	2	3
A:	3	5	7	2

start

0 :	3	8	15	17
1 :		5	12	14
L 2 :			7	9
3 :				2

s=0

CARRY FORWARD →

```
f(L=0; L<N; L++){
    sum=0
    f(R=L; R<N; R++){
        sum += A[R];
        print(sum);
    }
}
```

$\boxed{TC: O(N^2)}$

$\boxed{SC = O(1)}$

	0	1	2	3
A:	3	5	7	2
	3	8	15	17
		5	12	14
			7	9
				2

Q Given an array. Find the MAX Sub-Array Sum!

A: 3 -7 4 2 -5 7

$\underbrace{4 \quad 2 \quad -5 \quad 7}_{8}$

```
ANS = -∞;
f(L=0; L<N; L++){
    sum=0
    f(R=L; R<N; R++){
        sum += A[R];
        ANS = MAX(ANS, sum);
    }
}
ret ANS;
```

$\boxed{TC = O(N^2)}$

$\boxed{SC = O(1)}$

KADANE's ALGO

└ MAX SA SUM → $\boxed{TC: O(N)}$ $\boxed{SC: O(1)}$

Q. Given an Array.
Find the sum of sum of all subArrays!

	0	1	2
A :	1	3	5

[1] → 1
+
[1, 3] → 4
+
[1, 3, 5] → 9
+
[3] → 3
+
[3, 5] → 8
+
[5] → 5

$\boxed{30}$

```
ANS = 0
f( L = 0; L < N; L++){
    sum = 0
    f(R = L; R < N; R++){
        sum += A[R];
        ANS += SUM;
    }
}
```

$\boxed{TC: O(N^2)}$

$\boxed{SC: O(1)}$

⊙ CONTRIBUTION TECHNIQUE

$$A : \begin{array}{ccc} 0 & 1 & 2 \\ \left[\,1, \right. & 3, & \left. 5\,\right] \end{array}$$

$(1) \longrightarrow 1$

$(1, 3) \longrightarrow + \; 4$

$(1, 3, 5) \longrightarrow + \; 9$

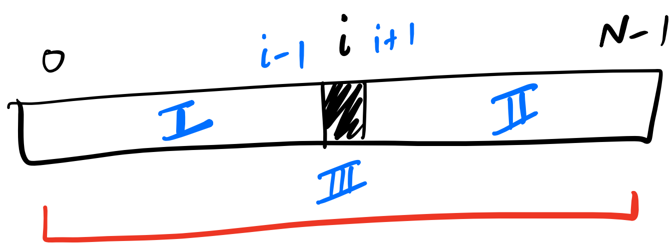
$(3) \longrightarrow + \; 3$

$(3, 5) \longrightarrow + \; 8$

$(5) \longrightarrow + \; 5$

30   30

element	#SA		
1	× 3	=	3
3	× 4	=	+ 12
5	× 3	=	+ 15
			$\boxed{30}$



1) Total #SA $\longrightarrow \dfrac{N(N+1)}{2}$

$\mathrm{I} \longrightarrow \dfrac{i(i+1)}{2}$ SAs

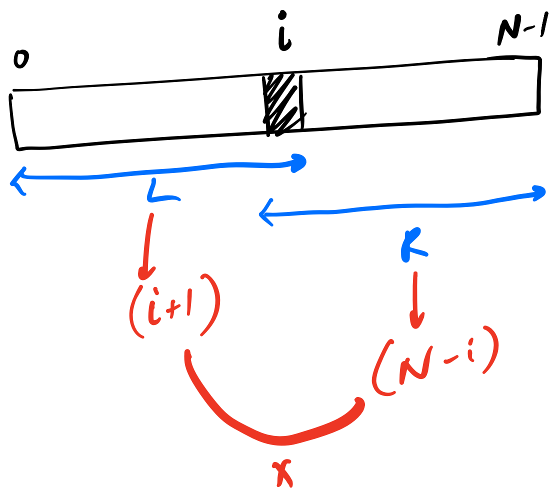
$\mathrm{II} \longrightarrow \dfrac{(N-i-1)(N-i)}{2}$ SAs

SA having i^{th} element

$$\text{III} = \boxed{\frac{N(N+1)}{2} - \left[\frac{i(i+1)}{2} + \frac{(N-i-1)(N-i)}{2}\right]}$$

L : [0, i]

R : [i, N-1]



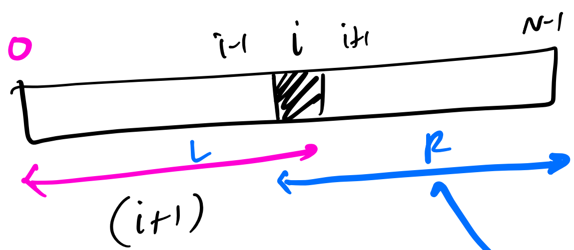
$$\boxed{\#\text{SA } i^{th} \text{ element is a part of} = (i+1)(N-i)}$$

$$\boxed{\text{ANS} = \sum_{i=0}^{N-1} A[i] \times (i+1)(N-i)}$$

```
ANS = 0
f(i=0; i<N; i++) {
    ANS += A[i] × (i+1) × (N-i);
}
ret ANS;
```

$\boxed{TC = O(N)}$

$\boxed{SC = O(1)}$



$[i, N-1]$

$N-\cancel{1}-i+\cancel{1}$

$N-i$