Q Given N. Count the no. of factors.

x is a factor of y

y % x → 0

3 is a fac of 12 ✓ $3 \times 4 = 12$!

7 ——— 12 X $7 \times ? \neq 12$

N = 10 : { 1, 2, 5, 10 } → 4

$1 \times 10 = 10$
$2 \times 5 = 10$
$5 \times 2 = 10$
$10 \times 1 = 10$

N = 12 : { 1, 2, 3, 4, 6, 12 }

Given N, check if x is a factor ? [1, N] factor range

N

↳ if (N % x == 0)
↳ x is a factor of N

else ↳ ——NOT——

```
cnt = 0;
for ( i=1; i<=N; i++) {
    if ( N % i == 0) {
        cnt++;
    }
}
ret cnt;
```

N iterations

Ass: 10^8 it → 1 sec

1 it → $\frac{1}{10^8}$ sec

$N = 10^9$

10^9 it → $10^9 \times \frac{1}{10^8}$ sec

10^{9-8} sec

10 sec

$N = 10^{18}$

10^{18} it → $10^{18} \times \frac{1}{10^8}$ sec

→ 10^{10} sec ~ 317 yrs

Yrs → child → gc → ggc → — 6/7 -

Optimize

i is a factor of N

$i \times j = N$

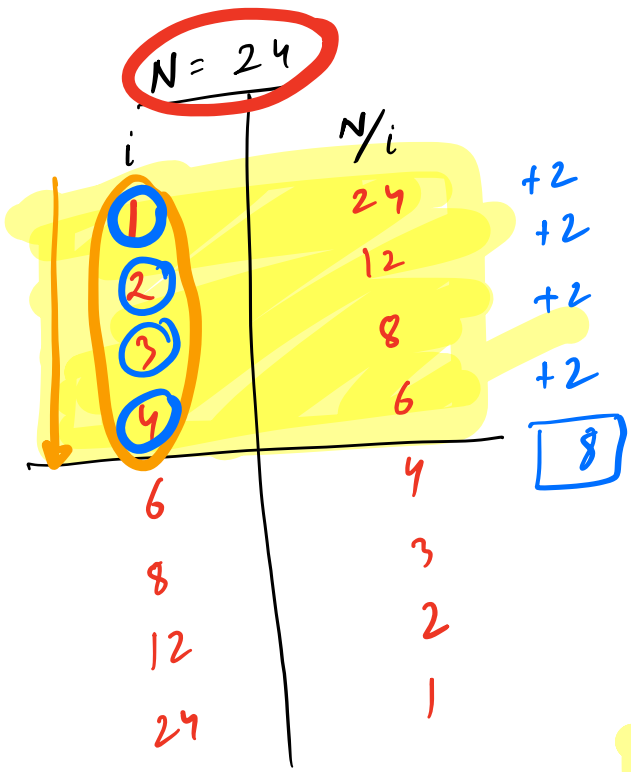
$\Rightarrow j \times i = N \Rightarrow$ j is also a factor of N!

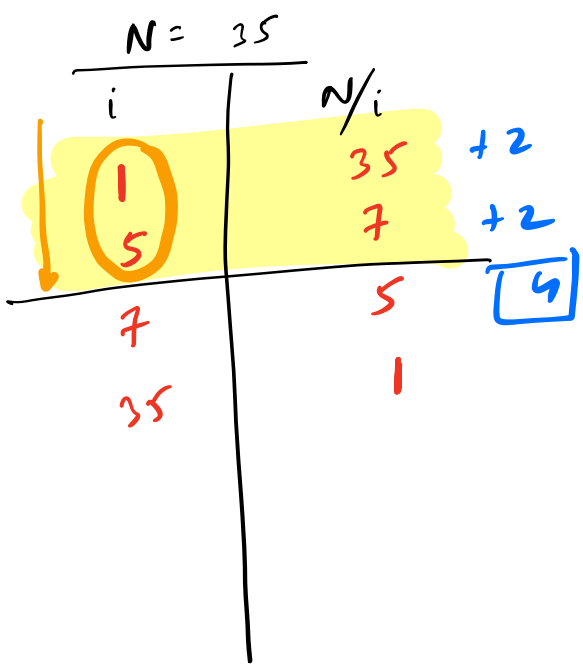
$2 \times 5 = 10$

$i \times j = N \rightarrow j = N/i$

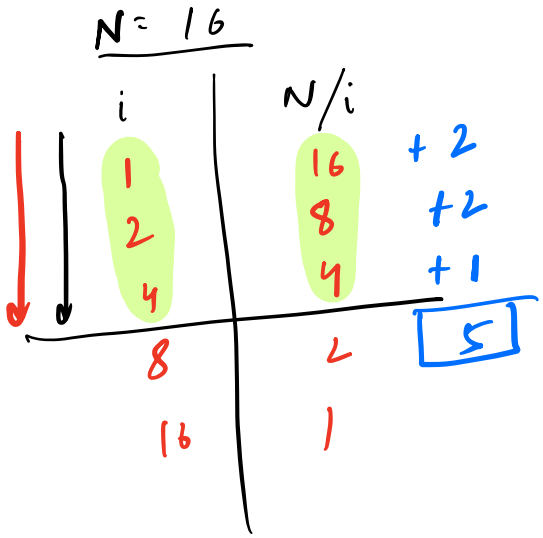
If i is a factor of N.

$\Rightarrow N/i$ is also a factor of N.





$i < N/i$



$i <= N/i$

$i \times i <= N$

$i^2 <= N$

$i <= \sqrt{N}$

⇒ All no's have even no. of factors except perfect square no's.

$i : [1, \sqrt{N}]$

+2, +1

```
cnt = 0
for(i=1; i<=√N; i++) {
    if(N % i == 0){
        if(i != N/i) {
            cnt += 2;
        }
        else {
            cnt += 1;
        }
    }
}
ret cnt;
```

Sqrt

1. $i <= sqrt(N)$ ✓
2. $sq = sqrt(N)$ ✓

   $i <= sq$
3. $i <= N/i$ ✓
4. $i \times i <= N$ ✓

it $\longrightarrow \sqrt{N}$

$N = 10^9$ $\quad \sqrt{N}$ it $\longrightarrow \sqrt{10^9}$ it

$\downarrow$

3×10^4 it

1 it $\longrightarrow 1/10^8$ sec

3×10^4 it $\longrightarrow 3 \times 10^4 \times 1/10^8$ sec

10 sec vs $\sim 3 \times 10^{-4}$ sec

$N = 10^{18}$ $\quad \sqrt{10^{18}}$ it $\longrightarrow \sqrt{(10^9)^2}$

$\longrightarrow 10^9$ it

1 it $\longrightarrow 1/10^8$ sec

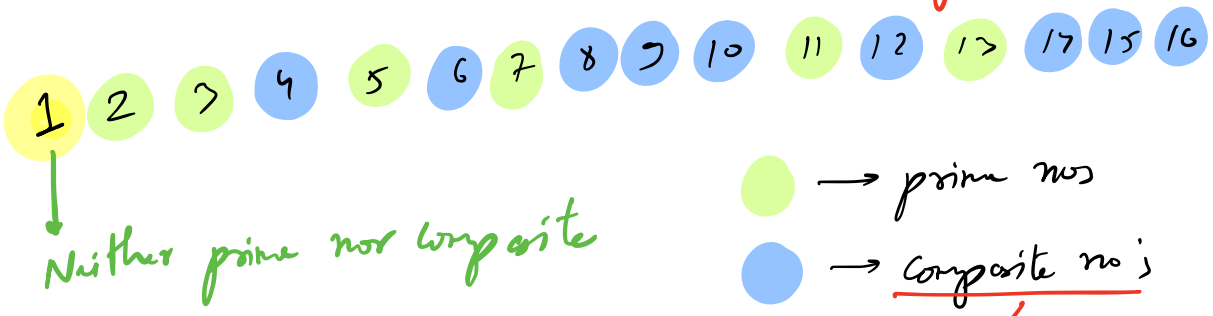
10^9 it $\longrightarrow 10^9 \times 1/10^8$ sec

317 yrs vs $\longrightarrow$ 10 sec

(8) Prime Number

→ A (pri) no having exactly 2 factors !

[1 & itself]



Q Given N.
Check if N is a prime no. or not!

N > 0

```
// N
if ( cntFactors(N) == 2) {
        ret true;
}
ret false;
```

Modify the cntFactors() to stop early → cnt > 2

Break till : 1036

(8) GAUSS SUM

Find out the sum of all no's from 1 — 100!

$$S = 1 + 2 + 3 + \cdots + 100$$
$$+\ S = 100 + 99 + 98 + \cdots + 1$$

$$2S = 101 + 101 + 101 - \cdots + 101$$

$$2S = 100 \times 101$$

$$S = \frac{100 \times 101}{2} \Rightarrow 5050$$

Q → Sum [1, N]

$$S_N = 1 + 2 + \cdots + N$$
$$+\ S_N = N + N-1 + \cdots + 1$$

$$2S_N = N+1 + N+1 + \cdots + N+1$$

$$2S_N = N(N+1)$$

$$S_N = \frac{N(N+1)}{2}$$

→ Sum of 1st N Natural no's

⊙ Log basics

"log a base b"

$\log_b a = c \Rightarrow \boxed{b^c = a}$

↳ to what power should I raise b to get a?

a, b

$\log_2 64 \rightarrow 2^{\underline{6}} = 64$

$\log_2 64 = 6$

$\log_2 32 \rightarrow 2^{\underline{5}} = 32$

$\log_2 32 = 5$

$\log_{10} 1000 \rightarrow 10^{\underline{3}} \rightarrow 1000$

$\log_{10} 1000 = 3$

$\log_2 10$

$2^3 = 8$

$2^{3\ldots} = 10$

$2^4 = 16$

$\log_2 10 = 3.\_$

$\log_2 40$

$2^5 = 32$

$2^{5\ldots} \rightarrow 40$

$2^6 = 64$

$\log_2 2^{20} \rightarrow 2^{20} = 2^{20}$

$\log_2 2^{20} \rightarrow 20$

$\log_b b^k = k$

$\log_2 N = k \iff 2^k = N$

$\log_c (a \times b) = \log_c a + \log_c b$

HW

Given N (N > 0)

Find how many times you need to divide it by 2 so that you reach 1

N = 45 → /2 → 22 → /2 → 11 → /2 → 5 → /2 → 2 → /2 → 1

① Perfect square

↳ product of 2 same integers

1, 4, 9, 16, 25, 36 —— x^2 ——

② Given N ($N > 0$)

Given N is a perfect square!

Find out it's square root!

N = 100 → 10

N = 25 → 5

N = 36 → 6

N = 25

i : [1, 2, 3, 4, 5]

1×1 2×2 3×3 4×4 5×5

N = 10000

```
for( i=1; i<=N; i++) {
    if( i*i == N) {
        ret i;
    }
}
```

i : [1 — — 100]

$\sqrt{N}$

$\sqrt{N}$ times

Q Given N (N > 0)
Find its floor($\sqrt{N}$)

$\lfloor 44.5 \rfloor \rightarrow 44$

$\lfloor 44 \rfloor \rightarrow 44$

floor(n)
↳ greatest integer <= x

N = 35 → 5
N = 36 → 6
N = 37 → 6
N = 48 → 6
N = 49 → 7

N = 27

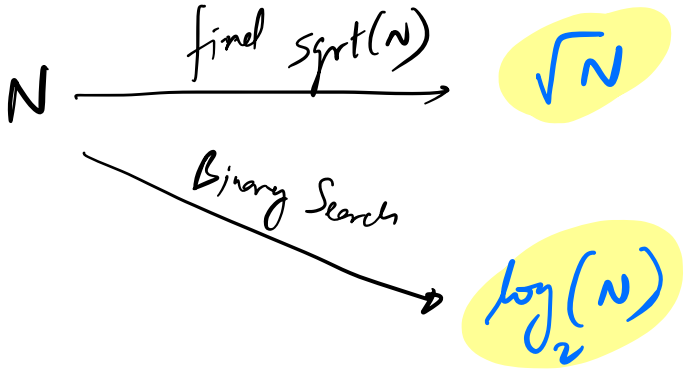
1 2 3 4 5 6 7

6 × 6 > 27

ans

```
ans = 0;
for( i=1; i<=N; i++){
    if( i*i <= N){
        ans = i;
    }else{
        break;
    }
}
return ans;
```

$\sqrt{N}$ times



$N = 10^{18}$

$\sqrt{N}$ vs $\log_2(N)$

$\sqrt{10^{18}}$ $\log_2(10^{18})$

10^9 ~ 60